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LEAKAGE OF CURRENTS FROM ELECTRIC RAILWAYS

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LEAKAGE OF CURRENTS FROM ELECTRIC RAILWAYS

By Burton McCollum and K. H. Logan

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I. INTRODUCTION

The leakage of electric current from street railway tracks used as return conductors is the principal cause of electrolytic corrosion of underground metallic structures. Occasionally electrolysis is due to other causes, such as the leakage of current from power or lighting circuits, but such corrosion is infrequent and usually confined to small areas.

This paper is intended primarily for electric railway engineers, and others familiar with electrolysis problems and a general knowledge of the phenomena and terms of electrolysis is assumed.

It is the purpose of this paper to discuss the factors which influence the escape of current from street railway tracks, indicating the importance of each factor from electrolysis standpoint, and to draw certain conclusions on the electrolysis problem.

A development of the theory of leakage currents is presented first for isolated linear lines and later for any portion of a track

network. The equations developed for the more simple case of an isolated railway line are plotted in the form of curves, which can be more readily interpreted than the equations themselves. In addition to these curves a series of conclusions have been drawn to assist the reader in interpreting the significance of the equations. In another publication [1] of the Bureau of Standards it is shown that one of the most important features of any effective plan of electrolysis mitigation is the reduction of leakage current to the lowest practicable amount. A thorough understanding of the laws governing the leakage of current from railway lines is therefore very important. It is hoped that the theoretical development presented will be of value in assisting the reader to secure a clear idea of the general principles governing leakage of stray currents into the earth.

In developing the theory of leakage currents it is assumed that any polarization effects between tracks and ground are equivalent to a resistance. This assumes that the counter electromotive force of polarization is proportional to the current flowing. In many cases this is approximately true, and since the polarization potentials are in general small compared with the resistance drop between tracks and ground the assumption will introduce no material error. It is further assumed in this theory that substantially all of the resistance between tracks and earth is encountered within a distance from the track which is quite small compared with the length of the railway line under consideration. Numerous experiments have shown that the greater part of the resistance between tracks and remote ground is found within 200 or 300 feet of the track. When we are considering sections of track several thousand feet or more in length, we can, therefore, safely make the above assumption.

II. GENERAL EQUATION FOR LEAKAGE CURRENTS

We have two cases to consider, namely, case I, in which the negative bus is substantially insulated from the earth except through its connection with the railway tracks, and case II, in which the negative bus is grounded at the power house, which in practice would correspond substantially to metallic connection

[1] E. B. Rosa and Burton McCollum, Electrolysis and Its Mitigation, Technologic Paper No. 52, Bureau of Standards.

between pipes and tracks at that point. We shall first develop the general equation which applies to both cases and then determine the constants for the two cases separately.

The following notation is used:

i_0 = originating current per unit length of line assumed uniformly distributed.

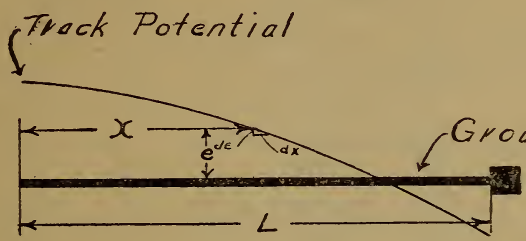

FIG. 1*a*

i = total current in rails at any point distant x from the outer end of the line.

e = potential difference between tracks and ground at any point distant x from the end of the line.

i_1 = total leakage current up to any point.

r = leakage resistance between tracks and remote earth per unit length of line.

δ = resistance of track per unit length of line.

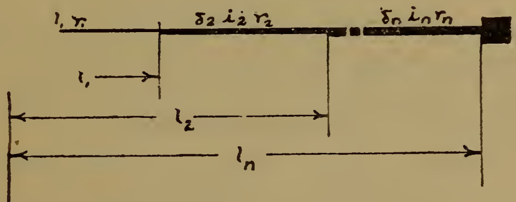

FIG. 1*b*

x = distance from outer end of line of any point under consideration.

L = total length of line.

Referring to Fig. 1*a*, the increase in potential difference de in a length dx at any point x on the line will be

$$de = -i\delta dx$$

$$\therefore \frac{de}{dx} = -i\delta \qquad (1)$$

The increase in current in the element of length dx is equal to the originating current in that length less the leakage current. The originating current is equal to $i_0 dx$, and the leakage current is equal to $\frac{e}{r}dx$. Hence we have

$$di = i_0 dx - \frac{e}{r}dx$$

$$\therefore \frac{di}{dx} = i_0 - \frac{e}{r}$$

$$\frac{d^2 i}{dx^2} = -\frac{1}{r}\frac{de}{dx}$$

$$\therefore \frac{de}{dx} = -r\frac{d^2 i}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1) and (2)

$$r\frac{d^2 i}{dx^2} - i\delta = 0$$

$$\therefore \frac{d^2 i}{dx^2} - i\frac{\delta}{r} = 0$$

This equation readily integrates into

$$i = Ae^{\sqrt{\frac{\delta}{r}}(x)} + Be^{-\sqrt{\frac{\delta}{r}}(x)} \quad \ldots\ldots\ldots\ldots (3)$$

Equation (3) is the general equation giving the current strength at any point in the tracks distant x from the end of the railway line, and applies to both case I and case II, the difference in the two cases being only in the constants of integration A and B.

Equation (3) shows that for a track of given length and originating current per unit length, the total current in the rails at any point depends upon three factors, namely, the distance of the point from the outer end of the track, the resistance of the track per unit of length, and the leakage resistance per unit length between the tracks and earth at a considerable distance from the rail.

The resistance of the rail varies with its temperature and depends also upon the size of the rail, the material from which it is rolled, and the treatment during manufacture. For rough calcu-

lations we have assumed a resistance of 0.01 ohm per 1000 feet of 100-pound rail and taken the resistance of other rails as inversely proportional to their weights. This figure represents an average of a large number of values of resistances of rails of various sizes and from several makers.

The resistance of the roadbed varies between wide limits, and experiments are now in progress for the study of roadbed characteristics. These experiments so far indicate that the roadbed resistance varies with the type of construction, the weather, and the kind of soil upon which it is laid.

The leakage resistance of single track ranges between about 0.2 and 12 ohms per 1000 feet for constructions usually employed. In special cases, where the rails are laid in moist earth of high conductivity or crushed rock in a very dry region, the resistance may not fall within these limits. For double track the resistance may theoretically vary from 50 to 100 per cent of this, according to whether most of the resistance is near the rails or remote therefrom, and in most practical cases it will vary from 60 to 80 per cent of that for single track of similar construction.

Since the leakage current up to any point is equal to i_0x-i, the effect of track resistance on leakage current is seen from equation (3) to be exactly the inverse of the leakage resistance; hence an increase in leakage resistance in any given ratio reduces the leakage currents in the same degree as increasing the conductance of the tracks in the same ratio. This emphasizes the importance of so constructing the roadbed as to give the highest practicable leakage resistance. Since in the equations which follow the factor $\sqrt{\frac{\delta}{\mathrm{r}}}$ occurs repeatedly, the equations will be simplified in form by letting $\sqrt{\frac{\delta}{\mathrm{r}}}=a$. Making this substitution in equation (3) we get the simplified form:

$$i=Ae^{\mathrm{ax}}+Be^{-\mathrm{ax}}. \qquad (3a)$$

It is apparent that the form of the curves is determined not by the numerical values of δ and r, but by the ratio of these factors. In order to facilitate the interpretation of the equations there is given in each case certain combinations of δ and r which might

be encountered in practice, and which would give the values of a used.

The value of a will vary between wide limits. For example, if the track resistance be very low, such as when well-bonded 125-pound rails are used, the value of δ for a single track would be 0.004 ohm per 1000 feet. If, also, the leakage resistance has a high value, such as 12 ohms per 1000 feet, the value of a will be 0.018. On the other hand, if the track resistance be high, such, for example, 0.04 ohm per 1000 feet, due to very bad bonding, and if, at the same time, the leakage resistance be as low as 0.2 ohm per 1000 feet, the value of a would be 0.45. Under most practical conditions the value of a may be taken to range between 0.025 and 0.25, or in the ratio of about 10 to 1. The quantity a is called the "leakage factor" of the railway line. It will be shown later in the discussion of the theory of leakage currents that it is very important that the value of a be kept as low as possible. This can be done either by maintaining good track bonding and by the use of heavy rails, or by so constructing the roadbed that the leakage resistance will be high.

III. LEAKAGE CURRENTS FOR UNIFORM LINE

We shall now proceed to examine in detail the application of equation 3a to the two cases encountered in practice, first with the negative bus not grounded, and next with the bus grounded.

1. (CASE I) BUS NOT GROUNDED

At the outer end of the line $x=0$, and the current $i=0$. Since the bus is not grounded, all of the current which leaks off the track must return to the track. Hence, when $x=L$, the length of the line, the current in the tracks must be i_0L. Imposing these conditions on equation 3a and solving for the constants A and B we get the following:

When $x=0$ $A+B=0$, or $A=-B$.

When $x=L$, $Ae^{aL}+Be^{-aL}=i_0L$

$$i_0L=(e^{aL}-e^{-aL})A$$

$$\therefore A=\frac{i_0L}{e^{aL}-e^{-aL}}$$

From (3a) and the above relation $A = -B$ we have:

$$i = A[e^{ax} - e^{-ax}] = \frac{i_o L}{e^{aL} - e^{-aL}}[e^{ax} - e^{-ax}] \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$\therefore i = \frac{i_o L}{\sinh\ (aL)} \cdot \sinh\ (ax) \quad \ldots\ldots\ldots\ldots \quad (5)$$

The total current originating in the tracks between the end of

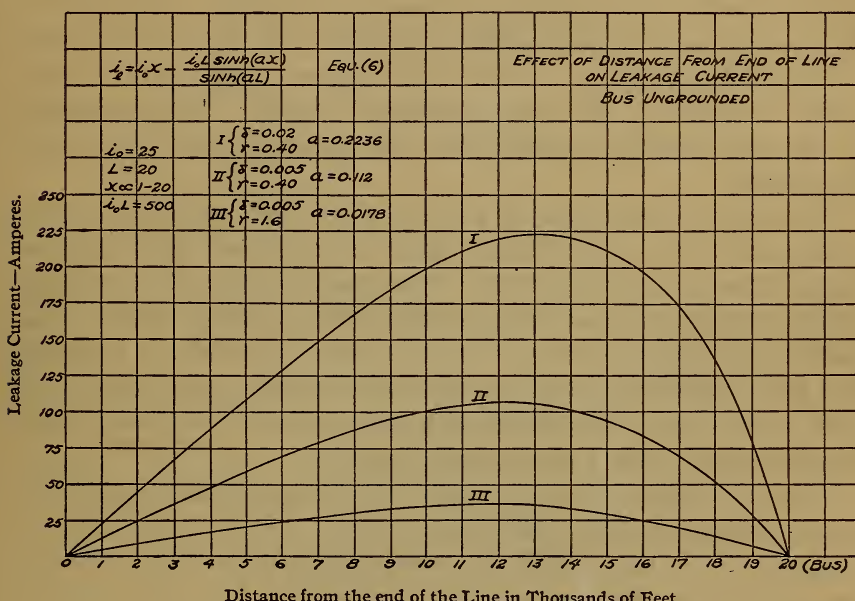

FIG. 2

the line and any point x is $i_o x$. Hence, the total leakage current up to any point x is

$$i_1 = i_o x - i$$

$$\therefore i_1 = i_o x - \frac{i_o L}{\sinh\ (aL)} \cdot \sinh\ (ax) \quad \ldots\ldots\ldots\ldots \quad (6)$$

The curves in Fig. 2 plotted from equation (6) show the magnitude of the total leakage current at each point along a 20 000-foot single track for three conditions of rail and leakage resistance, the originating current being assumed to be 25 amperes per 1000 feet of track.

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In Fig. 2 three curves are shown. Curve *I* shows the leakage current at various points on a line for which the track resistance δ is 0.02 ohm per 1000 feet due to bad bonding, and the leakage resistance r is 0.4 ohm per 1000 feet, a value not unusually low. This gives a value of a equal to 0.2236. The maximum leakage current in this case is seen to occur at about 13 000 feet from the end of the line and amounts to about 220 amperes. The current originating in the tracks being 25 amperes per 1000 feet the total originating current is $25 \times 20 = 500$ amperes. Of this, 220 amperes, or about 44 per cent, has leaked to earth. This represents a very bad leakage condition, but one not infrequently met with in practice.

In curve *II* the value of δ is 0.005 ohm per 1000 feet, which corresponds to a well-bonded track of 100-pound rails, and the same value of r as in curve *I*. This gives a value of 0.112 for a, which represents a fair average value of the leakage factor. The difference between curves *I* and *II* shows the effect of reducing the track resistance from 0.02 to 0.005 ohm per 1000 feet, and it will be seen that the maximum leakage has been reduced to less than one-half of the value shown by curve *I*.

Curve *III* represents a very good condition regarding leakage, the track resistance being fairly low and the leakage resistance 1.6 ohms per 1000 feet, which is a moderately high value, although one which may often be exceeded in practice. δ is 0.005 ohm per 1000 feet, giving a value of $a = 0.0178$. Comparison of curves *II* and *III*, which are for the same track resistance, shows the effect on the leakage current of increasing the leakage resistance.

The leakage current decreases with decreasing values of a. The point at which the leakage current is a maximum is, of course, the neutral area in the electrolysis region at which point the earth and the tracks are at the same potential. This point is seen to shift toward the negative bus as the maximum leakage increases. This change in the size of the positive and negative areas will be shown more clearly by a later curve when its significance will be discussed. For high values of the leakage factor a the leakage current will be seen to increase more rapidly than the value of a,

which emphasizes the importance of maintaining the value of a as low as practicable.

For maximum leakage

$$\frac{di_1}{dx}=0$$

From (6)

$$\frac{di_1}{dx}=i_0-\frac{i_0La}{\sinh(aL)}\cdot\cosh(ax)=0$$

$$\therefore \cosh(ax)=\frac{\sinh(aL)}{(aL)} \quad (7)$$

$$\therefore x=\frac{1}{a}\cosh^{-1}\left[\frac{\sinh(aL)}{(aL)}\right] \quad (8)$$

= value of x for neutral zone.

Substituting from (8) and (6) we get the value of maximum leakage.

$$\text{Max } i_1=i_0\frac{1}{a}\cosh^{-1}\left[\frac{\sinh(aL)}{aL}\right]-\frac{i_0L}{\sinh(aL)}\sqrt{\frac{\sinh^2(aL)}{a^2L^2}-1}$$

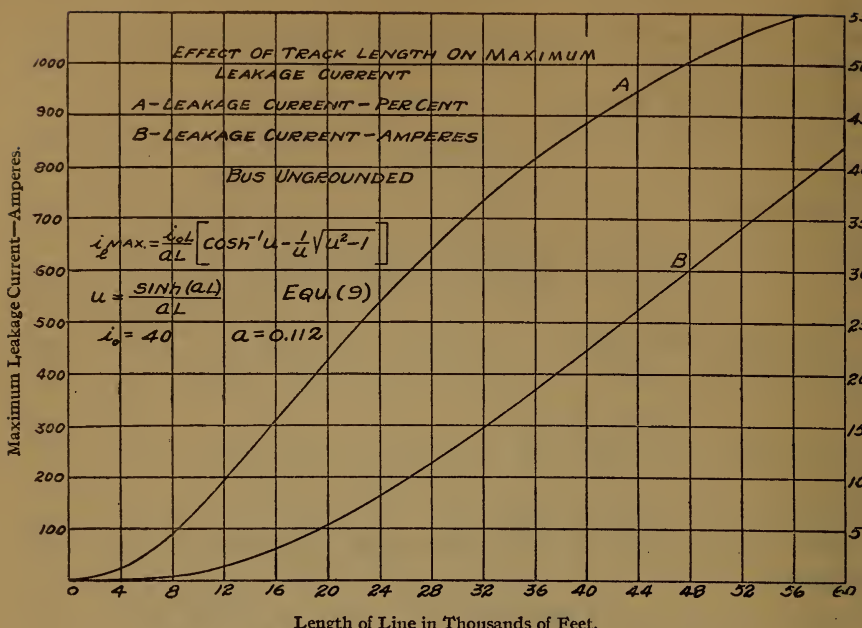
$$\frac{i_1\,max}{i_0L}=\frac{1}{aL}\left[\cosh^{-1}u-\frac{1}{u}\sqrt{u^2-1}\right] \quad (9)$$

$$\text{where } u=\frac{\sinh(aL)}{aL}$$

Equation 9 is plotted in Fig. 3 with length of line, L, as abscissae for $i_0=40$ amperes per 1000 feet, $\delta=0.005$, and $r=0.4$, which corresponds to a fair average condition.

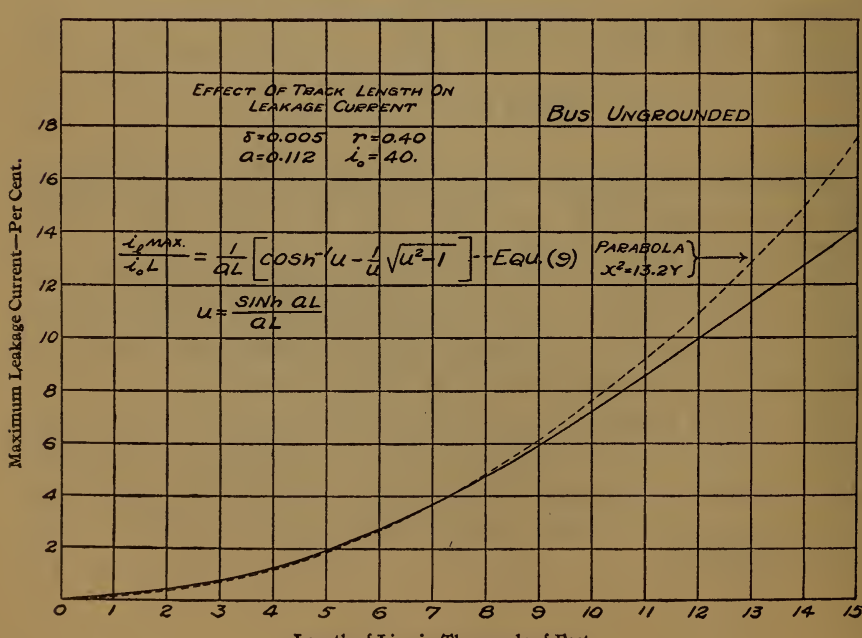
The curves indicate that the leakage current increases much faster than the length of the line, especially for lines of moderate length, and shows the importance from an electrolysis standpoint of reducing the feeding distance as much as possible.

Fig. 4 is the same as the lower part of Fig. 3 plotted on a larger scale, together with the dotted curve of a parabola. It indicates that for feeding distances up to about 10 000 feet the maximum leakage current increases approximately as the square of the feeding distance.



EFFECT OF TRACK LENGTH ON MAXIMUM LEAKAGE CURRENT
A-LEAKAGE CURRENT-PER CENT
B-LEAKAGE CURRENT-AMPERES
BUS UNGROUNDED
EQU. (9)
a=0.112
Maximum Leakage Current—Amperes.
Length of Line in Thousands of Feet.

FIG. 3



EFFECT OF TRACK LENGTH ON LEAKAGE CURRENT
BUS UNGROUNDED
EQU. (9)
PARABOLA
Maximum Leakage Current—Per Cent.
Length of Line in Thousands of Feet.

FIG. 4

For long feeding distances the rate of change of maximum leakage current is much less than for short feeding distances.

Fig. 5 represents equation 9 with the rail resistance as the independent variable. The maximum leakage current increases much less rapidly than the track resistance, except where the track resistance is very low.

In Fig. 6, equation 9, is plotted with the leakage resistance as the independent variable. If the leakage resistance is small,

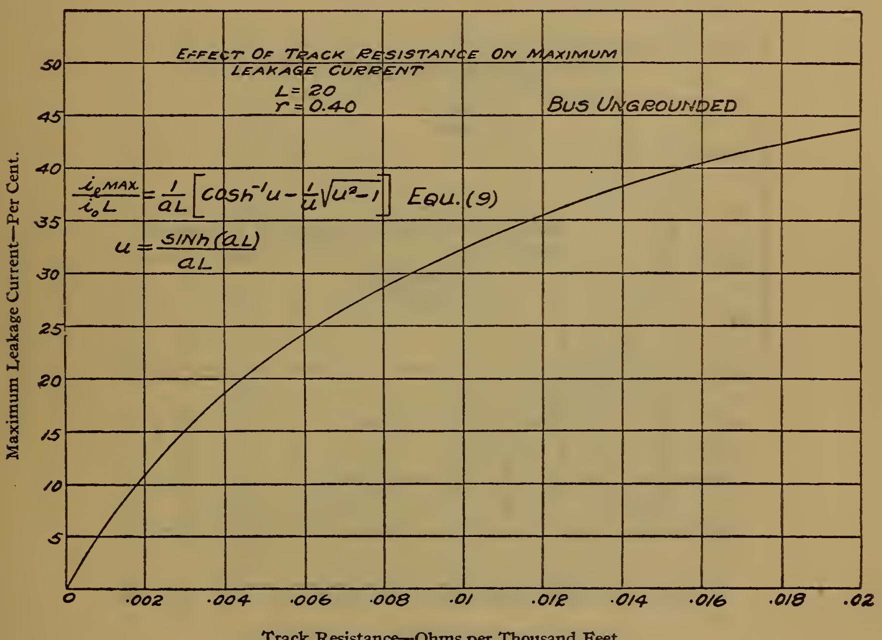


FIG. 5

such as that corresponding to an average concrete roadbed or track embedded in damp soil, the leakage current decreases very rapidly with increased leakage resistance.

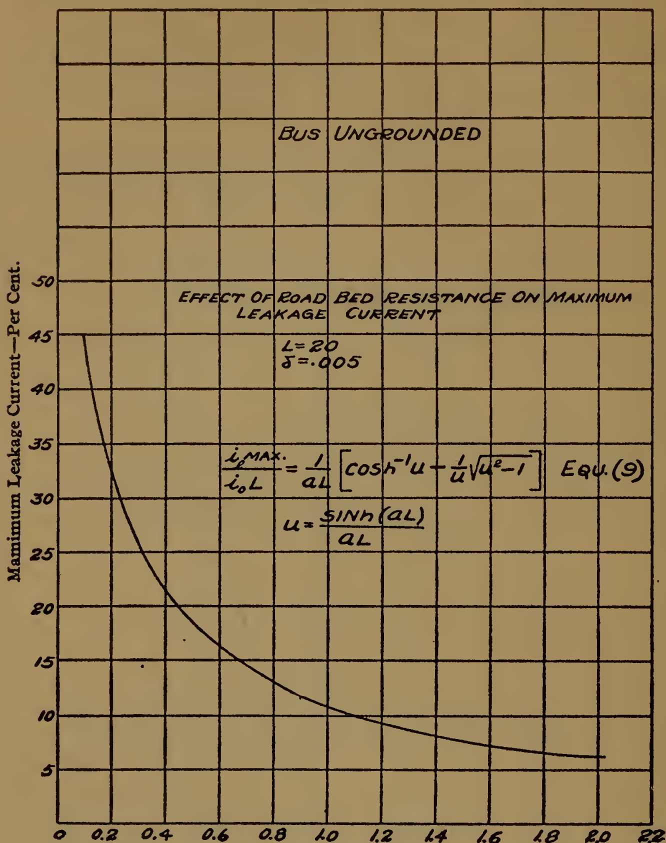
2. (CASE II) BUS GROUNDED

This differs from case I only in the terminal conditions. From case I.

$i = Ae^{ax} + Be^{-ax}$ . . . . (10)

when $x = 0$. $i = 0$. $\therefore A = -B$ . . . . (11)

Where $x = L$—that is, at the power house—we assume that the tracks are perfectly grounded so that the potential difference



Road Bed Resistance—Ohms per Thousand Feet.

FIG. 6

between tracks and earth is zero, and therefore there is no leakage at this point.

Hence, for $x = L$ we have $\frac{di}{dx} = i_o$

Differentiating 10 with respect to x

$$\frac{di}{dx} = Aa.e^{ax} - Ba.e^{-ax}$$

$\therefore$ $Aa.e^{al} - Ba.e^{-al} = i_o$ . . . . (11*a*)

From (11) and (11*a*).

$$A = \frac{i_0}{a(e^{al} + e^{-al})} = \frac{i_0}{2a \cosh (aL)}$$

$$\therefore\ i = \frac{i_0}{2a \cosh (aL)} (e^{ax} - e^{-ax}) = \frac{i_0 \sinh (ax)}{a \cosh (aL)} \quad \text{(12)}$$

$$i_1 = i_0 x - i = i_0 x - \frac{i_0 \sinh (ax)}{a \cosh (aL)} \quad \text{(13)}$$

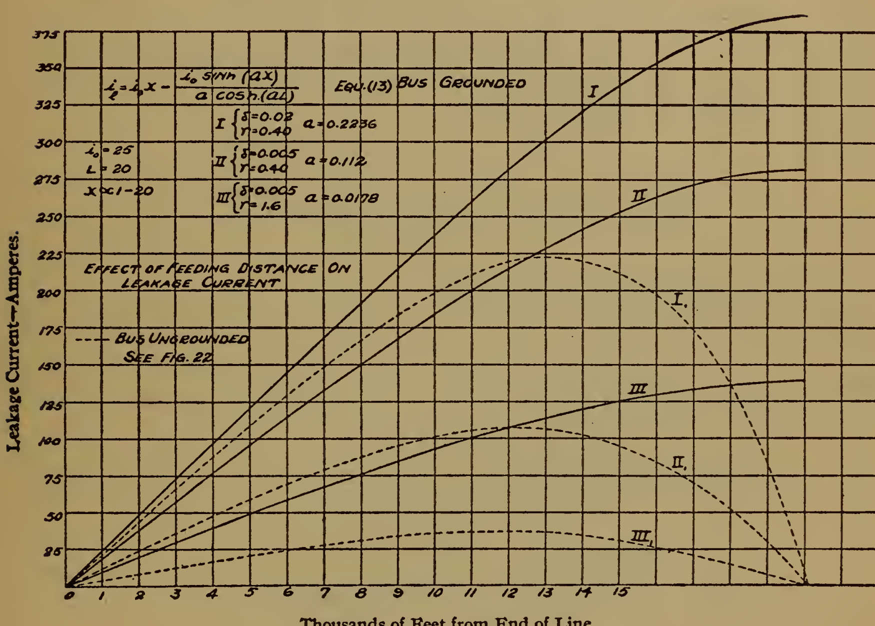


FIG. 7

Fig. 7 represents equation 13 for conditions similar to those for Fig. 2 except that the solid lines show the conditions for bus grounded. The ground connection is assumed to have negligible resistance. Fig. 7 shows that grounding the bus, as by connecting it to a buried pipe system, greatly increases the leakage current. In order to facilitate comparison the curves of Fig. 2 for the bus ungrounded are reproduced in the dotted curves of Fig. 7. The leakage current continues to increase as the power house is approached, but at a diminishing rate. It will be seen that

for moderate and good leakage conditions (curves *II* and *III*) increase in leakage resistance has a greater effect in reducing leakage currents if the bus is grounded than when it is ungrounded.

When conditions are such as to give rise to only moderate leakage currents, the maximum leakage will be more than doubled by grounding the negative bus. This is shown by curves *III* and III_1. When leakage conditions are bad the ratio of increase in leakage current due to grounding is less, but the increase is still quite marked as shown by curves *I* and I_1. Since the leakage current may not be confined to the underground structure to which the bus is connected the curves emphasize the importance of insulating the negative bus.

When

$$\frac{di_1}{dx}=0 \quad \cosh(ax)=\cosh(aL) \text{ or } x=L$$

∴ Leakage is maximum for $x=L$

. . Maximum leakage =

$$\text{Max. } i_1=i_0L-\frac{i_0 \sinh(aL)}{a\cosh(aL)} \qquad (14)$$

$$=i_0L\left(1-\frac{\tanh(aL)}{aL}\right) \qquad (15)$$

$$=i_0L\left(1-\frac{\tanh(v)}{v}\right) \qquad (16)$$

where $v=aL$.

Fig. 8 shows the relation between the length of the line and the leakage current for a rather heavily loaded line with good rails and roadbed corresponding to down-town conditions in a city of medium size. It will be seen that with the bus grounded the maximum leakage increases more rapidly than the feeding distance. This is particularly true for feeding distances up to 15 000 or 16 000 feet. It is evident from Fig. 8 that for very long feeding distances, such as are frequently encountered on interurban lines, practically all of the current may return by way of the earth.

IV. POTENTIAL GRADIENTS

From the equations for current in cases I and II we can determine the potential gradients on the tracks, the over-all potential drops and the potential difference between tracks and ground.

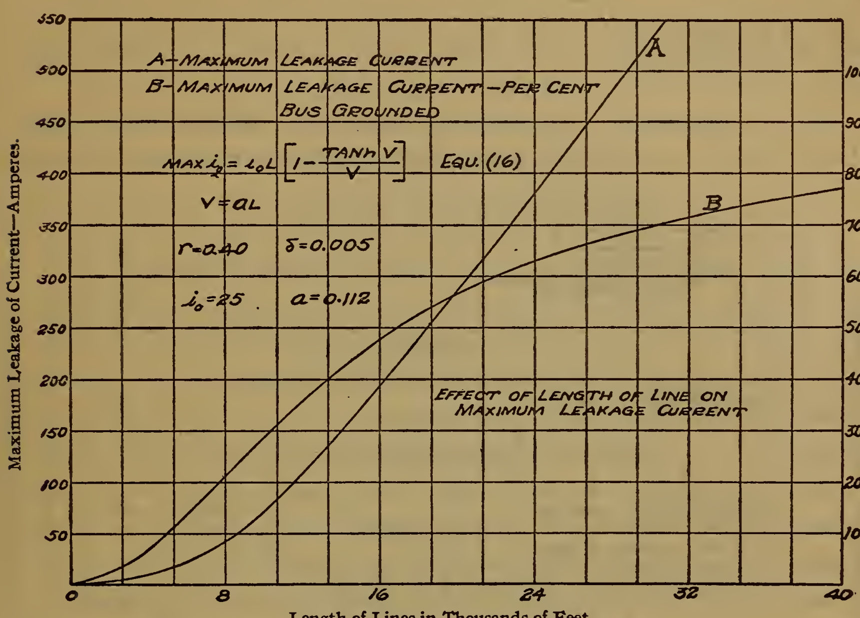


FIG. 8.

1. (CASE I) BUS NOT GROUNDED

From equation 5 we have

$$i=\frac{i_0 L \sinh (ax)}{\sinh (aL)}$$

The potential gradient in the track is $\frac{de}{dx}=i\delta=E_1$

$$\therefore E_1=\frac{i_0\delta \; L \sinh (ax)}{\sinh (aL)} \qquad (17)$$

If there were no leakage the gradient would obviously be

$$E_1{}^1 = i_0 \delta x \qquad (18)$$

2. (CASE II) BUS GROUNDED

From equation 12 after multiplying by δ we have

$$E_2 = \frac{i_0 \delta \ \sinh\ (ax)}{a\ \cosh\ (aL)} \qquad (19)$$

Fig. 9 illustrates the potential gradients indicated by equations 17 and 19 under the same track and load conditions with the

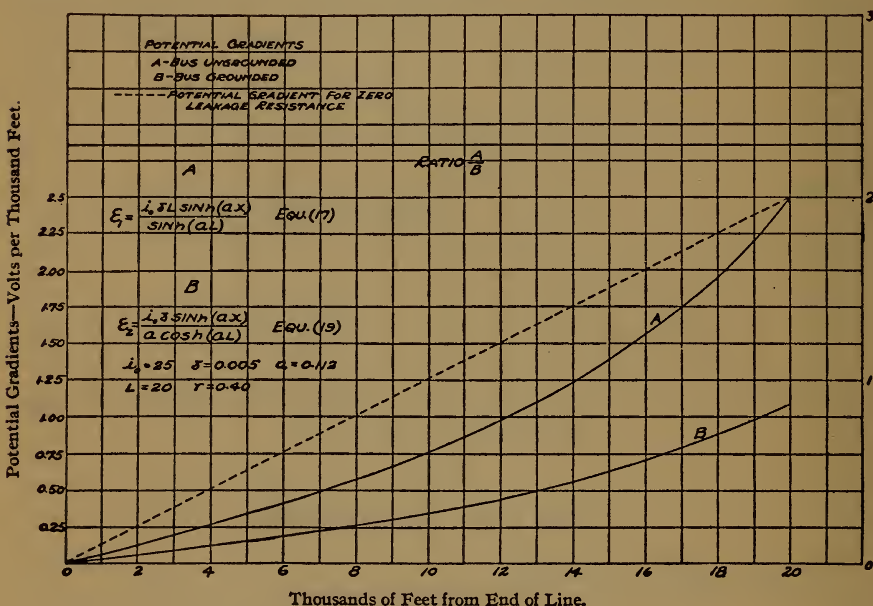


FIG. 9

bus ungrounded and grounded. The broken line indicates the gradient given by equation 18 when the entire current is confined to the rails.

It will be seen that potential gradients in the tracks may be materially reduced due to leakage currents, and this reduction is more marked if the bus is grounded.

Dividing equation 19 by 17 we get as the ratio of the gradients at any point under the two cases.

$$\frac{E_2}{E_1}=\frac{\tanh\ (aL)}{aL} \qquad (20)$$

The ratio of the gradients in the two cases is thus independent of x, and for large values of L the ratio becomes practically inversely as L. Thus, when the negative bus is ungrounded the potential gradients are greater than if the bus is grounded and the difference is very marked where the feeding distances are long.

For very long lines the ratio varies practically inversely as the length of the line.

V. OVER-ALL POTENTIALS.

1. (CASE I) BUS NOT GROUNDED

The over-all potential drop is $E=\int_0^L edx$. Hence, from (17) we have:

$$E_1=\frac{i_0\delta L}{\sinh\ (aL)}\int_0^L \sinh\ axdx$$

$$=\frac{i_0\delta L}{a\ \sinh\ (aL)}\Big[\cosh\ (ax)\Big]_0^L$$

$$\therefore E_1=\frac{i_0\delta L}{a\ \sinh\ (aL)}\Big[\cosh\ (aL)-1\Big] \qquad (21)$$

If there were no leakage, the over-all potential drop would be:

$$\therefore E_1{}^1=\int_0^L i\delta dx=i_0\delta\int_0^L xdx=\frac{i_0\delta L^2}{2} \qquad (22)$$

This value is also derivable directly from equation 21 by making $r=\infty$ whence $a=0$.

2. (CASE II) BUS GROUNDED

From equation 19 we have:

$$E_2=\int_0^L E_2dx=\frac{i_0\delta}{a\ \cosh\ (aL)}\int_0^L \sinh\ a\ xdx$$

$$\therefore E_2=\frac{i_0\delta}{a^2\ \cosh(aL)}\Big[\cosh\ (ax)\Big]_{x=0}^{x=L}$$

$$\therefore E_2=\frac{i_0\delta}{\cosh\ (aL)}\Big[\cosh\ (aL)-1\Big] \qquad (23)$$

Dividing equation 23 by 22 we get as the ratio of the over-all potentials in the two cases:

$$\frac{E_2}{E_1}=\frac{\tanh (aL)}{aL} \qquad (24)$$

This last result is, of course, deducible directly from equation 20, since if the ratio of the gradients at any point is constant through-

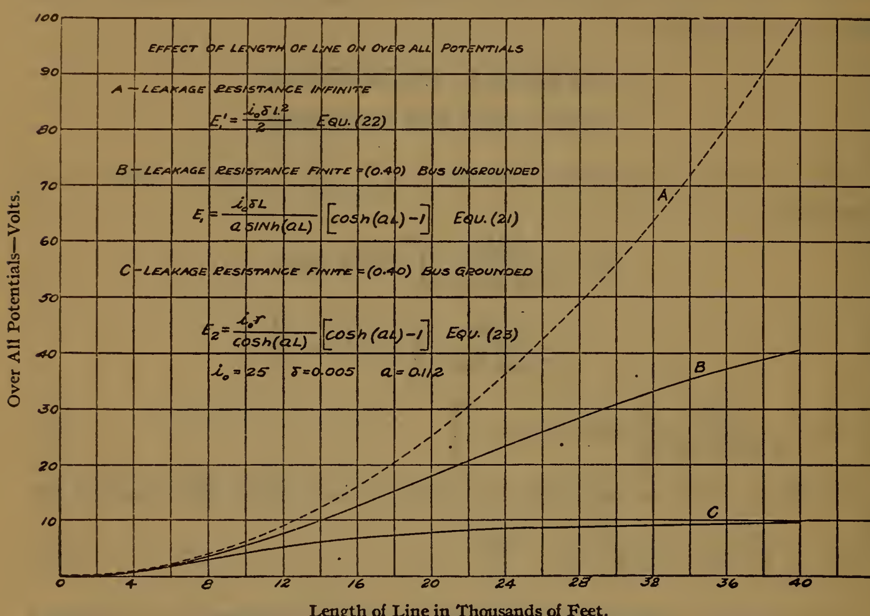


FIG. 10

out the entire line the ratio of the over-all potentials must be equal to this constant ratio.

The curves of Fig. 10 are plotted from equations 21, 22, and 23 and indicate the effect of the feeding distances on over-all potentials. The reduction of over-all potentials due to leakage is relatively much greater in long lines than in short lines and greater with grounded than with ungrounded bus. For very long lines and the moderate leakage and track resistance assumed in plotting these curves the over-all potentials are reduced in case of the ungrounded bus to about 40 per cent and in the case of the

grounded bus to about 10 per cent of the values they would have if there were no leakage.

If there be no leakage, the over-all potential drop is proportional to the first power of the track resistance and to the square of the feeding distance.

If there is leakage and the bus is ungrounded, then, as either the track resistance or the feeding distance increases indefinitely, the over-all potential tends to become proportional to the square root of the track resistance and to the first power of the feeding distance.

As the track resistance or the feeding distance increases indefinitely the over-all potential in the case of the grounded bus tends to become independent of both the track resistance and the feeding distance.

VI. POTENTIAL DIFFERENCE BETWEEN TRACKS AND GROUND

1. (CASE I) BUS NOT GROUNDED

The intensity of current leakage at any point is equal to $\frac{di_1}{dx}$ and the potential difference between tracks and ground is $r\frac{di_1}{dx}$, r being as before the leakage resistance per unit length.

From equation 6 we have

$$\frac{di_1}{dx} = i_o - \frac{i_0 al \cosh (ax)}{\sinh (al)}$$

$$\frac{di_1}{dx} = i_0\left[1 - \frac{al \cosh (ax)}{\sinh (al)}\right] \qquad (25)$$

The potential difference between tracks and ground is therefore

$$\Pi_1 = r\frac{di_1}{dx} = i_0 r\left[1 - \frac{al \cosh (ax)}{\sinh (al)}\right] \qquad (26)$$

The effects of leakage resistance and rail resistance on potential differences between tracks and the earth are shown by Figs. 11 and 12. If the bus is ungrounded, the intensity of the leakage current at the outer end of the line is less than at the power house

end (current returning to the tracks may be regarded as negative leakage current), and the difference is greater the lower the leakage resistance and the greater the rail resistance. It can be shown that the difference also increases with the length of track. Hence operating with the trolley negative would, in general, tend to reduce the rapidity with which trouble would become acute. The corrosion would, however, be distributed over a larger territory and its total amount would be substantially unchanged.

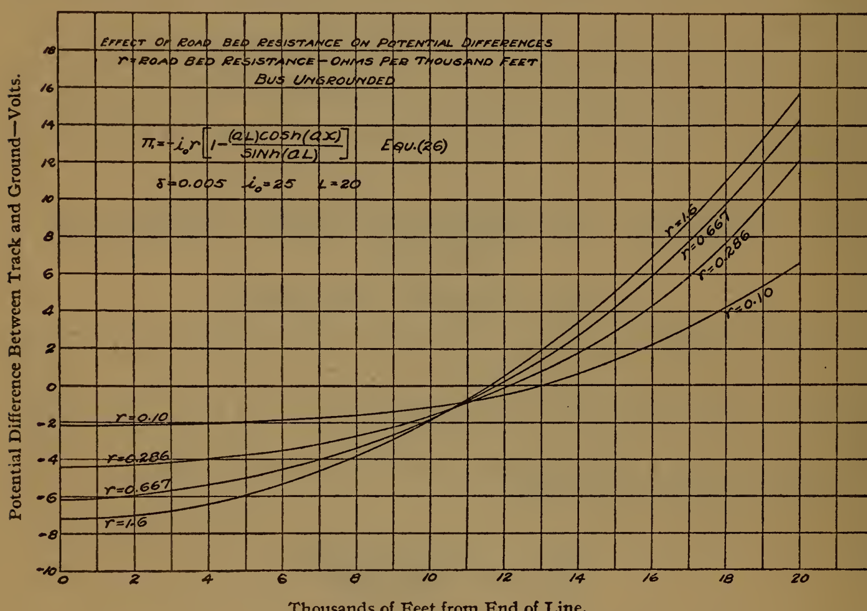


FIG. 11

Although high leakage resistance lowers the leakage current and hence the danger from electrolysis, as is shown by Fig. 6, it also increases the potential difference between the tracks and the earth, and increases the size of the positive area as is shown by Fig. 11. High potential differences are not in themselves, therefore, a definite indication of leakage current, and they may even indicate good, rather than bad, electrolysis conditions, depending upon whether the high voltage is due to high roadbed resistance, high track resistance, or overloading of rails. This is shown by Fig. 12.

As the track resistance or the feeding distance increases indefinitely, the potential difference between the tracks and the ground at the power house becomes indefinitely large and the area of the positive zone becomes indefinitely small. This is shown in Fig. 12 and by equation 26 and less clearly in Fig. 2 where the point of maximum leakage—that is, the neutral zone—was seen to shift toward the power house with increasing values of a. Thus, with

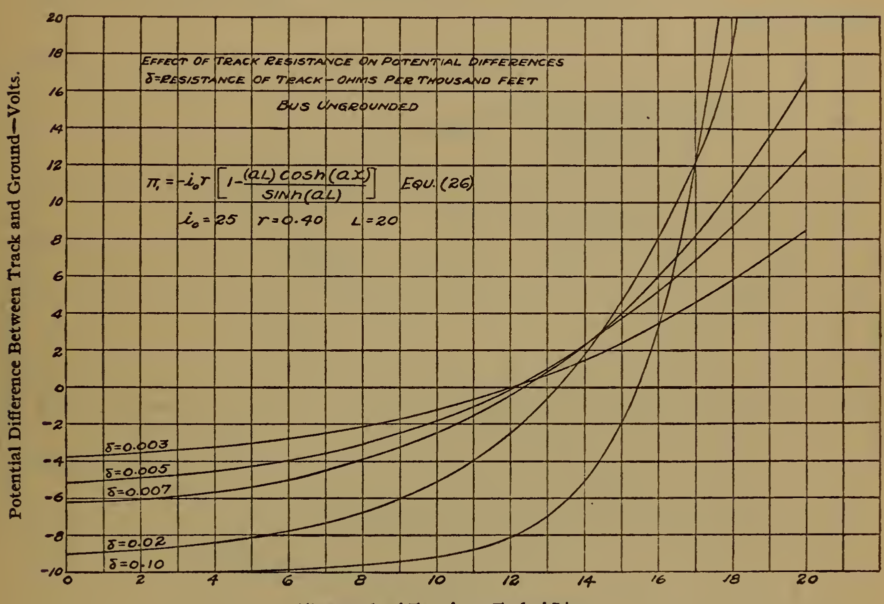


FIG. 12

high track resistance or long feeding distances there will be very severe trouble in a relatively small area.

Long feeding distances, high track resistance, and low leakage resistance all tend to reduce the size of the positive area, although tending to increase the total amount of leakage current, and hence they greatly increase the severity of the electrolysis trouble near the power house. A relatively small positive area, therefore, is an indication of bad electrolysis conditions generally. The length of the positive zone varies from a maximum of 42 per cent of the feeding distance under ideal electrolysis conditions (zero

leakage) to an indefinitely small value where electrolysis conditions are particularly bad. This is indicated by Fig. 12. At the end of the line, where $x=0$, we get the maximum value of potential difference which exists beyond the neutral point. Its value is

$$\Pi_1' = i_0 r\left[1 - \frac{aL}{\sinh(aL)}\right] \qquad (27)$$

If either δ or L is indefinitely increased, Π_1' approaches as a limit the value $i_0 r$.

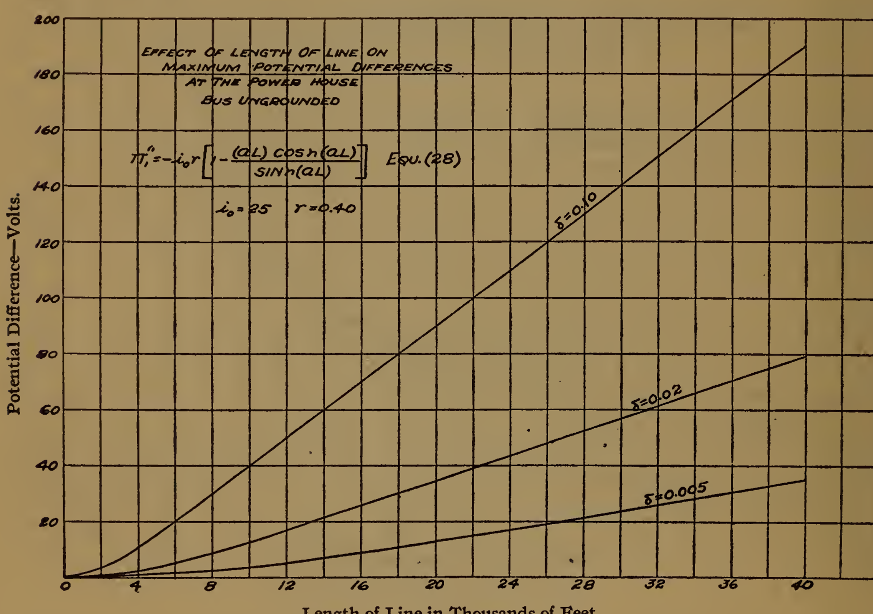


FIG. 13

At the power house end of the line, where $x=L$, the value of the potential difference is

$$\Pi_1'' = i_0 r\left[1 - \frac{aL \cosh(aL)}{\sinh(aL)}\right] \qquad (28)$$

Fig. 13 shows the variation in the maximum potential difference between the track and the ground for various feeding distances when the bus is not grounded. Three curves are given showing the effect of varying the track resistance.

2. (CASE II) BUS GROUNDED

From equation (13) we have

$$\frac{di_1}{dx} = i_0\left[1 - \frac{\cosh\ (ax)}{\cosh\ (aL)}\right]$$

$$\therefore \Pi_2 = i_0 r\left[1 - \frac{\cosh\ (ax)}{\cosh\ (aL)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

With the bus grounded the potential difference at the power house is of course zero, and the increase in leakage current shown

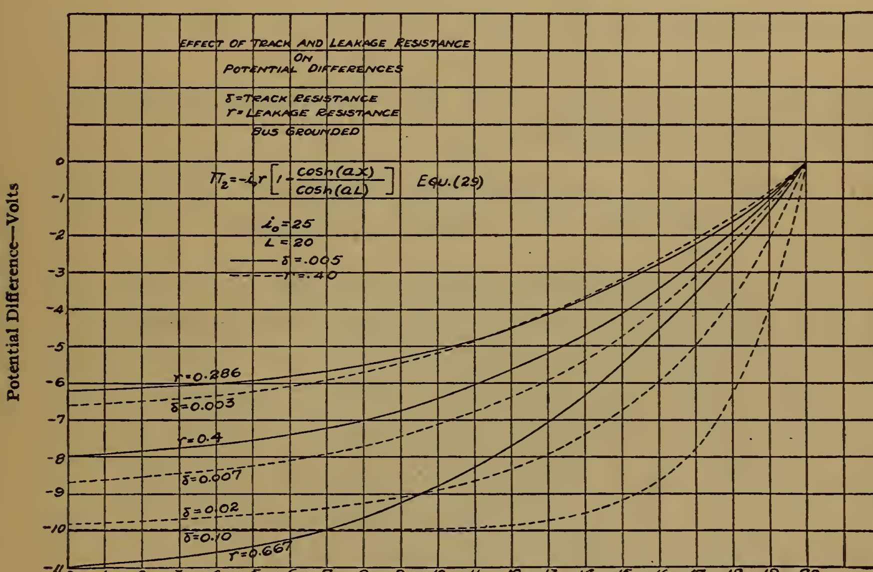


FIG. 14

in Fig. 7 indicates that the change in potential difference will also be most rapid in the region of the power house.

Increasing either the roadbed or the track resistance increases the potential difference at the end of the line, as is indicated by Fig. 14. In this figure there are shown two sets of curves, one set in solid lines plotted for a constant value of track resistance of 0.005 ohm per 1000 feet and varying values of leakage resist-

ance ranging from 0.286 to 0.667 ohm. The dotted curves show the effect of varying the track resistance from 0.003 to 0.10 ohm per 1000 feet with a constant leakage resistance of 0.4 ohm per 1000 feet. It will be observed that in this case leakage resistance is much more influential with respect to potential difference on the outer portions of the line than track resistance, a change in the leakage resistance in the ratio of 2.3 to 1 giving results of approximately the same order of magnitude as a change of 30 to 1 in the track resistance.

At end of line where $x=0$, we have:

$$\Pi_2{}^1 = i_0 r\left[1 - \frac{1}{\cosh\ (aL)}\right] \qquad (30)$$

Here also for large values of δ and L the potential difference approaches the limiting value of $i_0 r$.

VII. GENERAL EQUATIONS FOR LEAKAGE FROM ANY SECTION OF A TRACK NETWORK

In this case both leakage resistance and track resistance may be discontinuous functions of the distance along tracks, and in general the bus will not be grounded (Fig. 15). The differential equation for this case is derived in the same way as in case I and has the same form, the only difference in the solution being that in the present case the limits of the integration, instead of being throughout the line, are between the ends of the sections, the constants of the equation changing at each transition point. The equation for i, the current at any point in the n^{th} section from the outer end of line is

$$i = Ae^{a_n x} + Be^{-a_n x} \qquad (31)$$

where x is the distance from the end of the line to any point in the n^{th} section under consideration. Applying this equation to the n^{th} section where the limits of integration are L_{n-1} and L_n we have as the limiting conditions

$$x = l_{n-1} \qquad i = I_{n-1}$$
$$x = l_n \qquad i = I_n$$

Substituting these limits in equation 31 and solving for A and B, we get

$$A = \frac{I_{n-1}\, e^{a_n}\, (l_{n-1}-l_n) - I_n}{e^{a_n}\, ({}^2\, l_{n-1}-l_n) - e^{(a_n)}\, (l_n)}$$

$$B = \frac{I_n\, e^{a} l_n{}^{(n-1)} - I_{n-1}\, e^{(a_n)}\, (l_n)}{e^{\,a_n}\, (l_{n-1}-l_n) - e^{-a_n}\, (l_{n-1}-l_n)}$$

Substituting these values of A and B in equation 31, we get as the equation for the current in the tracks at any point within the n^{th} section.

$$i = \frac{(I_n\, e^{-a_n}(l_{n-1}) - I_{n-1}\, e^{-a_n}\, l_n)\, e^{a_n x} - (I_n\, e^{a_n}\, (l_{n-1}) - I_{n-1}\, e^{a_n}\, l_n)\, e^{-a_n x}}{2 \sinh\, (a_n)\, (l_n - l_{n-1})} \qquad (32)$$

If there were no leakage within the n^{th} section, the current at any point x would be $I_{n-1} + i_n\, (x - l_{n-1})$. Hence the leakage current within the section up to any point x is

$$i_1 = I_{n-1} + i_n\, (x - l_{n-1}) - i \ldots\ldots\ldots\ldots\ldots\ldots \qquad (33)$$

The value of i in this equation is given by equation 32.

Equation 33 corresponds to equation 6 and furnishes a similar basis for the development of more general equations for leakage currents, gradients, potential differences, and over-all potentials when the track under consideration is made up of sections differing in weight of rail or in the construction of the roadbed. The equations already developed will serve, however, to show the general effect of variation in rail and leakage resistance.

VIII. INTERPRETATION OF EQUATIONS

An examination of the foregoing equations permits the following deductions in regard to the effect of track resistance, leakage resistance, and feeding distances, or on the current and voltage conditions in a uniformly loaded railway line. Most of these deductions have previously been set forth more in detail in connection with the discussion of the equations but are grouped and restated here for convenience.

1. The voltage and current conditions in the return circuit are characterized by three constants, namely, the resistance of the

track per unit length, the leakage resistance between track and ground per unit length, and the feeding distance (equation 4 and following).

2. The effect of track resistance on leakage currents is exactly the inverse of leakage resistance; hence an increase in leakage resistance in any given ratio reduces leakage currents in the same degree as increasing the conductance of the tracks in the same ratio. This emphasizes the importance from an electrolysis standpoint of so constructing the roadbed as to give the highest practicable leakage resistance (equation 6 and Fig. 2).

3. The leakage current from any given line increases much faster than the length of the line (equation 6 and Fig. 3). This shows the importance of reducing feeding distances as much as practicable.

4. Where the bus is not grounded there will be distinct positive and negative areas and the relative extent of the positive and negative areas is not a constant but varies with the length of the line, the track resistance, and the leakage resistance.

5. For short track lengths the percentage of the total current which leaks from the tracks increases practically as the square of the feeding distance (equation 9 and Fig. 4).

6. For long feeding distances the rate of change of leakage current with distance is much less than for short feeding distances (equation 9 and Figs. 3 and 4).

7. The maximum leakage current increases less rapidly than the track resistance, except where the track resistance is very low (equation 9 and Fig. 5).

8. If the leakage resistance is small, such as that corresponding to an average concrete roadbed or track embedded in damp soil, the leakage current decreases very rapidly with increase in leakage resistance. For high values of leakage resistance, however, the effect of increasing the leakage resistance on the total leakage current is much less (equation 9 and Fig. 6).

9. If the bus be grounded, as by connecting it to the buried pipe systems the total leakage current is greatly increased (equations 9 and 13 and Fig. 7).

10. With grounded bus the rate of increase in leakage current becomes relatively small as the power house is approached and becomes zero at the negative bus (equation 13 Fig. 7).

11. Where conditions are relatively good increase in leakage resistance has a greater effect in reducing leakage currents if the bus is grounded than when it is ungrounded (Fig. 7).

12. When the conditions are such as to give rise to only moderate leakage currents, the maximum leakage may be more than doubled by grounding the negative bus (equation 13 and Fig. 7, curves III and III_1). Where leakage conditions are bad the ratio of increase in leakage current due to grounding is less but the increase is still quite marked (Fig. 7, curves I and I_1). These curves emphasize the importance of insulating the negative bus.

13. If the bus be grounded, the maximum leakage increases more rapidly than the feeding distance. For ordinary values of track resistance and leakage resistance this is particularly true for feeding distances up to about 15 000 or 16 000 feet (equation 16 and Fig. 8).

14. For very long feeding distances, such as are frequently encountered on interurban lines, practically all of the current may return by way of the earth (Fig. 8).

15. Potential gradients in the tracks may be materially reduced due to leakage currents, and this reduction is more marked if the bus is grounded (equations 17, 18, and 19, and Fig. 9). Low potential gradients are not in themselves, therefore, a definite indication of good electrolysis conditions, but on the contrary may be due to excessive leakage of current from the tracks. Other factors must be considered, therefore, in interpreting gradient measurements.

16. For any given line the relative value of the gradients for grounded and ungrounded bus is the same for all points on the line. For very long lines the ratio varies practically inversely as the length of the line (equation 20).

17. The reduction of over-all potentials due to leakage currents is relatively much greater in long lines than in short lines and greater with grounded bus than with ungrounded bus. For very long lines and the moderate leakage and track resistance assumed

in plotting the curves of Fig. 10 the over-all potentials are reduced in case of the ungrounded bus to about 20 per cent, and in case of the grounded bus to about 5 per cent of the values they would have if there were no leakage (equations 21, 22, and 23, and Fig. 10). It is evident, therefore, that low over-all potentials, like low potential gradients, are not a positive indication of good electrolysis conditions. It is necessary to know the cause of the low values before their significance can be determined. Certain measures, such as insulating tracks, that can be taken to reduce leakage currents may greatly increase both gradients and over-all potentials, although they would greatly improve electrolysis conditions.

18. If there be no leakage the over-all potential drop is proportional to the first power of the track resistance and to the square of the feeding distance (equation 22 and Fig. 10).

19. If there is leakage and the bus is ungrounded, then as either the track resistance or feeding distance increases indefinitely the over-all potential tends to become proportional to square root of the track resistance and the first power of the feeding distance (equation 21).

20. As the track resistance or feeding distance increases indefinitely the over-all potential, in the case of the grounded bus, tends to become independent of both the track resistance and feeding distance (equation 23 and Fig. 10).

21. If the bus is ungrounded, the intensity of the leakage current at the outer end of the line is less than at the power house end (current returning to the tracks may be regarded as negative leakage current); and the difference is greater the lower the leakage resistance and the greater the length. Hence, operating with trolley negative would, in general, tend to reduce the rapidity with which trouble would become acute. The corrosion would, however, be distributed over a larger territory and its total amount would be substantially unchanged (equation 26 and Fig. 11).

22. As the track resistance is increased indefinitely the potential difference at the outer end of the line approaches a finite maximum value which is $i_0 \delta$. A similar result follows from an indefinite increase in the feeding distance (equation 26).

23. Although high leakage resistance lowers the leakage current, as shown by Fig. 16, it also increases the potential difference between tracks and earth. High potential differences are not in themselves, therefore, a definite indication of leakage current (equation 26 and Fig. 11).

24. As the track resistance or feeding distance increases indefinitely, the potential difference between tracks and ground at power house becomes indefinitely large, and the area of the positive zone becomes indefinitely small. Thus, with high track resistance or long feeding distances there will be very severe trouble in a relatively small area (equation 26 and Fig. 12).

25. High track resistances and low leakage resistance both tend to reduce the size of the positive area, although tending to increase the total amount of leakage current, and hence they greatly increase the severity of the electrolysis trouble near the power house. A relatively small positive area, therefore, is an indication of bad electrolysis conditions generally. The length of the positive zone varies from a maximum of 42 per cent of the feeding distance under ideal electrolysis conditions to an indefinitely small value where electrolysis conditions are particularly bad (equations 8 and 26 and Fig. 12).

26. With the bus ungrounded the potential difference at the power house is nearly proportional to the length of the line except for short lines (equation 28 and Fig. 13).

27. With the bus grounded the potential difference at the power house is zero and the change in potential difference is most rapid in the region of the power house.

28. Increase in either the roadbed or the track resistance increases the potential difference at the end of the line (equation 29 and Fig. 14).

29. Leakage resistance is much more influential with respect to potential differences than track resistance (equation 29 and Fig. 14).

WASHINGTON, September 8, 1915.