DEPARTMENT OF COMMERCE
BUREAU OF STANDARDS
George K. Burgess, Director

TECHNOLOGIC PAPERS OF THE BUREAU OF STANDARDS, No. 333
[Part of Vol. 21]

TRANSMISSION OF SOUND THROUGH VOICE TUBES

BY

E. A. ECKHARDT, Physicist
V. L. CHRISLER, Assistant Physicist
P. P. QUAYLE, Assistant Physicist
M. J. EVANS, Junior Scientific Aid
With a note on the Absorption of Sound in Rigid Pipes
By EDGAR BUCKINGHAM, Physicist
Bureau of Standards

November 8, 1926

PRICE 15 CENTS
$1.25 PER VOLUME ON SUBSCRIPTION
Sold only by the Superintendent of Documents, Government Printing Office
Washington, D. C.

WASHINGTON
GOVERNMENT PRINTING OFFICE
1926

TRANSMISSION OF SOUND THROUGH VOICE TUBES

By E. A. Eckhardt, V. L. Chrisler, P. P. Quayle, and M. J. Evans

WITH A NOTE ON THE ABSORPTION OF SOUND IN RIGID PIPES

By Edgar Buckingham

ABSTRACT

This paper gives the results of an investigation made by the Bureau of Standards of the transmission of sound through voice tubes such as used on board ship. The work was initiated by the Bureau of Construction and Repair of the Navy Department.

A good voice tube should transmit speech with as little diminution as possible in intensity and with a minimum of distortion of articulation. Each of these qualities was separately tested, and tables and diagrams of comparative results are given.

Tests were made upon straight tubes, curved sweeps, flexible tubes, fittings, elbows, and terminal cones.

A brief discussion of sound filters is given and a note on the absorption in rigid pipes is appended.

CONTENTS

I. INTRODUCTION

The present paper describes work done at the sound laboratory of the Bureau of Standards on the transmission of sound through voice tubes for the Bureau of Construction and Repair of the Navy Department. The work was initiated by Capt. Elliott Snow, who carried on the negotiations with the Bureau of Standards which led to an extended program of experimental research.

Two factors enter into satisfactory voice-tube transmission—intensity and articulation. Both must be adequate in order to secure satisfactory communication. These factors are measured separately by radically different methods. Intensity is measured directly and articulation statistically.

The material studied was standard Navy equipment, as used in installations in ships, and consisted of straight tubes of different materials, curved sweeps, flexible tubes, fittings, and terminal cones. In addition, a brief discussion of sound filters is appended.

II. MEASUREMENT OF INTENSITY

1. MATHEMATICAL THEORY

In the following discussion we shall consider only that part of the loss of sound energy in a voice-tube installation which is due to absorption. As a working hypothesis we shall assume that the loss of intensity in a section of tube is proportional to the intensity entering. This assumption is stated mathematically by the equation

$$\frac{dI}{dx} = -\alpha I \tag{1}$$

where dI is the change in intensity as we progress a distance, dx, along the tube. The negative sign is used because there is a decrement of I for an increment in x. The quantity α may be called the absorption constant and its value is characteristic for a given tube of definite material, finish of wall, etc.

Putting (1) into the form

$$-\alpha dx = \frac{dI}{I} = d(\log_e I) \tag{2}$$

and integrating from $I = I_0$ at $x = x_0$, to $I = I$ at $x = x_0 + L$, we have

$$-\alpha L = \log_e I - \log_e I_0$$

or, changing signs and changing to the more convenient common logarithms by means of the relation $\log_e N = 2.303 \log_{10} N$,

$$\alpha L = 2.303 \ (\log_{10} I_0 - \log_{10} I) \tag{3}$$

If put into the form

$$\alpha = \frac{2.303}{L} \log_{10} \frac{I_0}{I} \tag{4}$$

this permits of computing the value of the absorption constant for a given tube of length L from a measurement of the ratio of the initial and final intensities of a sound transmitted along the tube.

If the value of α has already been determined for a particular kind of tube, and if (4) is put into the form

$$\log_{10} \frac{I_0}{I} = 0.4343\alpha L \tag{5}$$

the attenuation over a length L of the tube may be found by inserting the values of α and L and looking up in a table of logarithms the corresponding value of I_0/I, or of its reciprocal I/I_0, which is the fraction of the initial intensity remaining at the end of the length L.

The above theory provides the groundwork for measuring the quantity α for typical voice-tube material and makes it possible to compute in advance the attenuation for a given installation. Terminal effects are eliminated by the process of measurement and are, therefore, not included in any results computed by use of formula (4).

2. DESCRIPTION OF APPARATUS

The apparatus used (for straight tubes) is shown diagrammatically in Figure 1. The arrangements used for sweeps, fittings, flexible tubes, and cones differed only in the arrangement of the piping under test, and will be further described where necessary in stating the several results obtained.

Pure tones were used except as otherwise stated. The sound source, S, was a loud-speaking telephone operated by a vacuum-tube oscillator. This arrangement provides relatively convenient control of intensity, frequency, and purity of the sound output. No horn was used with the loud-speaker. The sound source was placed at the transmitting end of a long line of voice tubing and a telephone receiver, R, was placed at the receiving end and constituted the receiver of the device for measuring the transmitted sound intensity. Errors which might have arisen from changes in the relative position of the source and the mouth of the tube were made negligible by adopting a position, found by trial, in which the sensitiveness to small changes was a minimum.

The current induced by the received sound in R was amplified and rectified and measured in the way described in Bureau of Standards Scientific Paper No. 526, Transmission and Absorption of Sound of Some Building Materials, by E. A. Eckhardt and V. L.

Chrisler. A comparison of the emf's thus obtained from the receiver is known to afford a relative determination of the intensities of the received sound, if the amplitudes are not too great.

3. EXPERIMENTAL PROCEDURE

The voice tube was used in sections of equal length, l. A sound source was placed at one end of the line of length L, and the sound-receiving device at the other, as shown in Figure 1. The received intensity was measured and recorded as I_1. One section was removed from the line and the sound source moved to the new input end, while the sound receiver remained fixed at the output end. This rearrangement was so made that the input intensity remained unchanged, the realization of this condition being checked by experi-

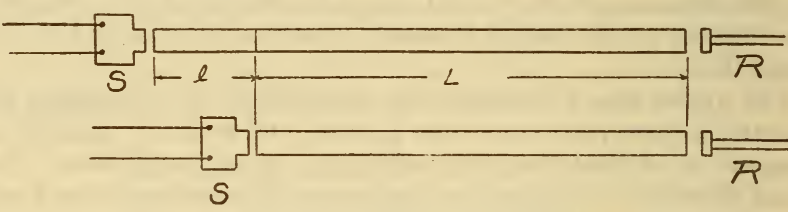

FIG. 1.—*Arrangement of voice tubes for intensity test*

ment. The transmitted intensity was now measured and recorded as I_2. Upon substituting the two results in (3) we have

$$\alpha L = 2.303\ (\log_{10} I_0 - \log_{10} I_1)$$

$$\alpha (L-l) = 2.303\ (\log_{10} I_0 - \log_{10} I_2)$$

Where upon subtracting the second from the first, dividing by l, and combining the logarithms, we have

$$\alpha = \frac{2.303}{l} \log_{10} \frac{I_2}{I_1}$$

It may be noted that $\log_{10} I_0$ has been eliminated, so that I_0 need not be measured, as long as it is known to have been constant.

By removing additional sections and proceeding in a similar manner additional values I_3, I_4, etc., were obtained. These values substituted in equation (4) lead to other values of the quantity α. The values of α thus obtained have been found to be constant within the errors of measurements for the greatest length of line which it has been practicable to test, consisting of ten 10-foot sections. The fact that α is practically constant over this range substantiates the assumptions which underlie equation (1). Progressive removal of sections was used instead of addition because the removal could be effected more readily without disturbing the rest of the line.

We may illustrate the use of these equations by a concrete case.

With a certain type of tube, for each 10-foot section the output intensity was, roughly, 85 per cent of the input intensity. Substitution in equation (4), therefore, gives

$$\alpha = \frac{2.303}{10} \log_{10}\left(\frac{100}{85}\right) \text{ per foot}$$

$$= 0.016 \text{ per foot}$$

The transmission (remaining intensity) of a 200-foot line of this pipe may be computed from equation (5) which gives us

$$\log_{10}\frac{I_0}{I} = 0.434 \times 0.016 \times 200 = 1.389$$

Upon consulting a table we find that $I_0/I = 24.5$ or $I/I_0 = 0.041$, so that the intensity at the end of 200 feet is about 4 per cent of the

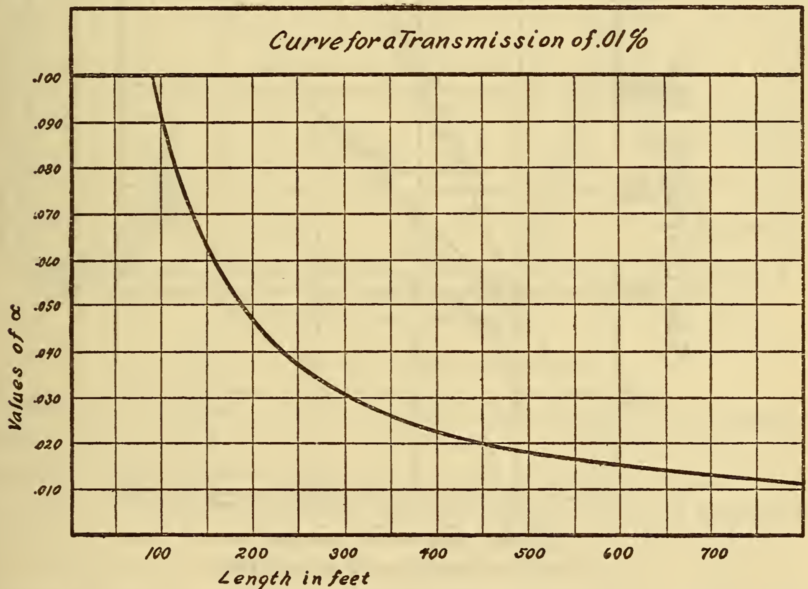

FIG. 2.—*Attenuation-length curve*

intensity at the entrance end. In the same way the transmission of a 400-foot length of the same pipe may be computed to be approximately one-sixth of 1 per cent.

From equation (4) it follows that for a given attenuation; that is, for a given ratio, I_0/I, the product, αL, is constant for all kinds of tubing. If a standard of attenuation has been decided upon α may be plotted against length and the curve may be used to determine the length of any kind of tubing to give the standard attenuation. Such a curve has been plotted in Figure 2 for a value of I/I_0 of 0.0001, which is equivalent to the attenuation of 307 feet of 2-inch brass tubing. To obtain the length of 1-inch fiber tubing of the same percentage transmission the length corresponding to the value of α for 1-inch fiber tubing (Table 1) is read from the curve. The proper value of α is 0.065, and the graph shows the corresponding length to be 142 feet.

In Figure 3 the results obtained for straight tubes are presented in a form which facilitates comparison with the graph on page 2912 of Bulletin 111.[1] In this figure are shown the computed lengths of

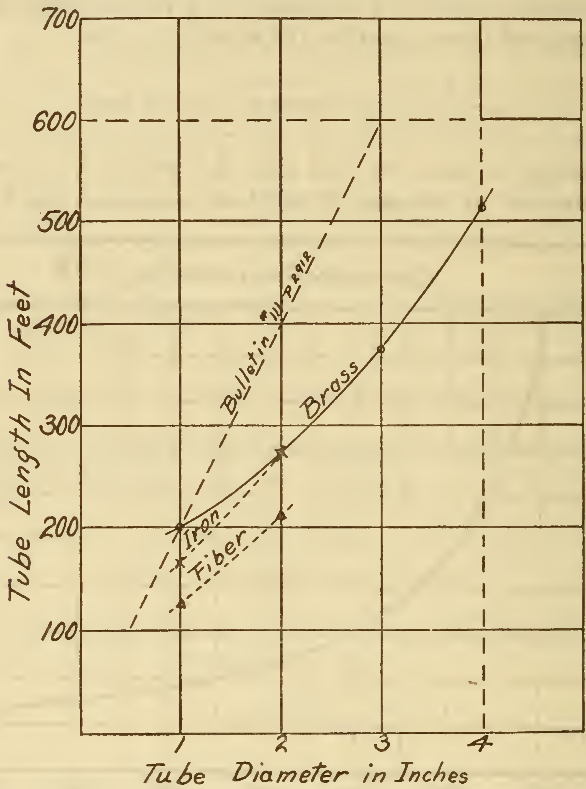

Fig. 3.—*Comparison of straight tubes*

various types of straight tubing which, on the basis of our experiments, have the same intensity attenuation as a 200-foot line of 1-inch brass tubing. This latter standard was chosen because it provides a common point for the new data and the graph from Bulletin No. 111 referred to above. The caption of this graph is "Diagram showing relation between diameters and lengths of pipe. Smooth brass standard Navy voice tubing."

Figure 3 indicates that the diagram of Bulletin No. 111, page 2912, as judged on the basis of our experiments, overestimates the

[1] Bulletin No. 111: Voice Tubes, Appendix No. 3, by Capt. Elliott Snow. Navy Department, Bureau of Construction and Repair; 1923.

gain (measured in terms of additional feet of line made available for satisfactory communication) which results from the use of larger tubing. It must be remembered, however, that the data hitherto presented in the present paper relate to intensity alone and give no information as to articulation except in so far as articulation is a matter of frequency. It is known that at the higher frequencies which are important for articulation the attenuation of intensity is greater than for the lower frequencies which are important for intensity.

In the first experiments pure sounds of fixed frequency were used and a very large number of measurements were made over the voice range of frequencies.

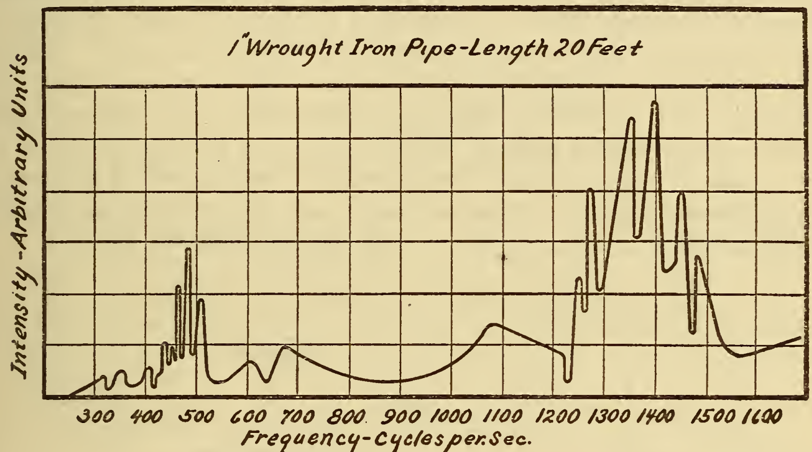

FIG. 4.—*Intensity at different frequencies*

Figure 4 is a typical curve obtained by this procedure of measurement. The general elevation around 400 to 500 cycles is due to resonance of the sound source, that at 1,200 to 1,500 cycles is due to resonance of the sound-receiving instrument, and the small pinnacles of the curve are due to resonance of the tubing under investigation. The obtaining of a single curve of this kind involved a very large amount of work. A procedure was finally adopted in which the resonance regions of sound source and receiver were avoided and the sound source, instead of emitting a single continuous pure tone, was caused to emit a sound of rising and falling pitch within certain definite frequency limits. The sound source was a loud-speaker fed by alternating current from a vacuum-tube oscillator. An air condenser formed part of the tuning system of the oscillator circuit. By driving the rotor of this air condenser by means of a motor the frequency could be varied cyclically between definite limits determined by the maximum and minimum capacities of the air condenser and the fixed capacities and inductances of the oscillating circuit.

4674°—26——2

Measurements were made during the fall and winter and a record was kept of the prevailing temperature. The results obtained were in agreement within the errors of measurement over a wide range of temperature. Humidity seems similarly to have no appreciable effect on the quantities measured.

The manner of making the joints in a line has an appreciable effect on the transmission of a voice-tube line. This was particularly noticeable in the case of the fiber tubing which, in general, is neither straight nor round.

4. RESULTS

(*a*) Straight Tubing.—The results of our completed measurements are presented for straight tubes in Table 1, which gives the values of α for each type of tubing over a frequency band, the maximum, minimum, and average frequencies of which are given. Each value of α is the mean of at least five values. In a few cases where the dispersion of the observed values was large more observations were made. The probable error of all the values of α is of the order of 2 per cent and the maximum error probably never exceeds 5 per cent.

The mean values of α given in Table 1 are the arithmetical means of the values obtained over the various frequency bands used.

Table 1.—*Mean values of α for various frequency bands*

Frequency (cycles/sec.)	Mean frequency	Material							
		Brass				Iron		Fiber	
		Nominal size (inches)							
		1	2	3	4	1	2	1	2
		Inside diameter (inches)							
		$1\frac{1}{16}$	$1\frac{15}{16}$	$2\frac{7}{8}$	$3\frac{7}{8}$	$1\frac{1}{16}$	2	1	$1\frac{15}{16}$
		Wall thickness (inches)							
		0.135	0.054	0.051	0.058	0.132	0.163	0.167	0.218
250 to 257	254	0.030	0.021	0.017		0.037	0.021	0.045	0.027
350 to 364	357				0.010	.041			
432 to 450	441	.033				.048			
700 to 791	764	.033				.036			
1,000 to 1,080	1,040		.025	.019	.015	.041	.030	.046	.041
1,350 to 1,492	1,421					.048			
1,500 to 1,890	1,695	.041				.059			
1,750 to 2,010	1,880					.050			
2,000 to 2,570	2,285	.051	.037	.027	.020	.057	.036	.077	.043
2,500 to 2,950	2,725	.051				.066			
2,500 to 3,200	2,850						.034		
3,000 to 3,500	3,250					.061			
3,000 to 3,560	3,280	.048	.036	.025	.017			.092	.046
Mean value of α		.041	.030	.022	.015	.049	.030	.065	.039

TABLE 2.—*Values of α (attenuation per foot) in long sweeps*

Frequency band	250–257	1,000–1,080	2,000–2,570	3,000–3,500	Mean value	Internal diameter	Wall thickness
	Mean frequency						
	254	1,040	2,285	3,250			
						Inches	*Inch*
2-inch brass	0.021	0.027	0.027	0.044	0.030	$1\frac{15}{16}$	0.054
3-inch brass	.019	.025	.030	.031	.026	$2\frac{7}{8}$	.051
4-inch brass	.012	.008	.030	.022	.018	$3\frac{7}{8}$	.058
Do	.013	.011	.033				

In Figure 5 the percentage transmission of 10-foot sections of the various types of straight tubing tested has been plotted against frequency.

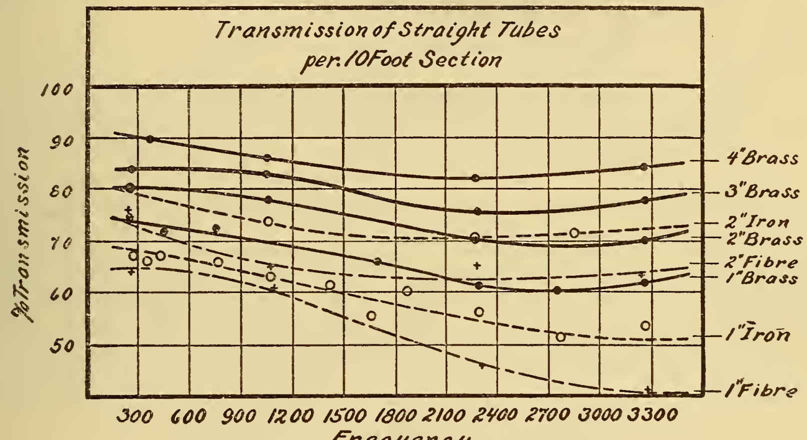

FIG. 5.—*Percentage transmission of straight tubing*

(*b*) LONG SWEEPS (90° BENDS OF 10-FOOT LENGTH).—Measurements on these bends were made in the same manner as for straight sections. Bends were removed from straight lines and the ratio of intensities determined. The values of absorption (or attenuation) per foot are given in Table 2 and are used in the same manner as the data for straight tubes.

The transmission curves giving the per cent reduction in intensity per bend at various frequencies are shown in Figure 6. The values for the 4-inch sweeps seemed peculiar and were remeasured several times. The measurements were in substantial agreement, and consequently the bowed nature of the curve is not due to errors of measurement. No explanation has so far been discovered for this peculiarity.

(*c*) Flexible Tubes.—Intensity tests were made on 40-foot sections of 2 and 3 inch standard tubing submitted by the Navy Department. The tubes were laid straight and the mean values as given in Table 3 were obtained.

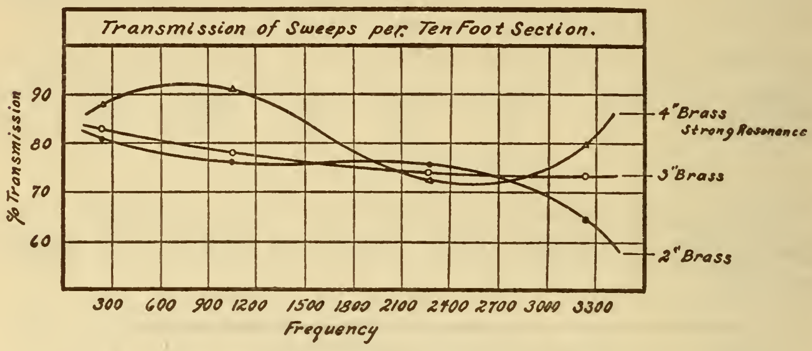

Fig. 6.—*Percentage transmission of sweeps*

Table 3.—*Attenuation in flexible tubes*

Frequency	2-inch tube			3-inch tube		
	$\frac{I}{I_0}$	$\frac{I_0}{I}$	$\log_{10}\frac{I_0}{I}$	$\frac{I}{I_0}$	$\frac{I_0}{I}$	$\log_{10}\frac{I_0}{I}$
250 to 257	0.185	5.40	0.732	0.315	3.17	0.501
1,000 to 1,087	.178	5.62	.750	.385	2.60	.415
2,000 to 2,470	.144	6.94	.841	.282	3.55	.550
3,000 to 3,470	.096	10.4	1.017	.226	4.42	.645

Now since

$$\alpha=\frac{1}{0.434\,l}\log_{10}\left(\frac{I_0}{I}\right) \text{ and } l=40 \text{ feet}$$

we have

$$\alpha=0.0576\log_{10}\left(\frac{I_0}{I}\right)$$

Table 4.—*Values of α for flexible brass tubes*

Frequency	2-inch pipe	3-inch pipe
250 to 257	0.0423	0.0287
1,000 to 1,087	.0432	[1] .0240
2,000 to 2,470	.0485	.0317
3,000 to 3,470	.0587	.0372

[1] This apparent inconsistency was not an error, but was substantiated by repeated measurements.

It will be noted that the attenuation of intensity per foot of the flexible brass tube as given in Table 4 is some 50 per cent greater than for smooth brass tubes of the same diameter (see Table 1).

The flexible tubing was also tested when laid in the form of a circle of 10-foot radius, but no change in the value of α was found.

(*d*) Y, L, AND T FITTINGS AND ELBOWS.—In making the measurements on the various fittings (both as regards intensity and articulation) four fittings were placed in the line being measured and then removed one section at a time as the measurements were made.

A "section" consisted of the fitting and a 10-foot section of the line, and in case of Y and T fittings a 10-foot section in the branch was included. It was found that if the branch of the fitting was simply left open the results were considerably different from those obtained when a line of tubing was added to the branch, but that after the first 10 feet were added, lengthening the branch did not affect the transmission of the main line appreciably.

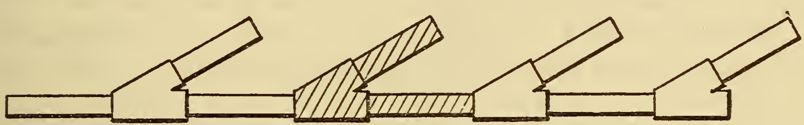

FIG. 7.—*Arrangement of test line for fittings*

A typical test line arranged for the investigation of four fittings is shown in Figure 7. The shaded unit indicates a typical unit section used in these tests. In the figure the diameter of the tubing has been exaggerated.

In a line of this character we have the usual attenuation constant and for the straight portion (of length x) and an additional constant β for each of n fittings, giving an equation of this type.

$$\frac{I}{I_0} = \mathrm{E}^{-\alpha x - n\beta} \tag{6}$$

In Tables 5 and 6 are given the results of intensity attenuation measurements on fittings and diaphragms, exclusive of the various cones and horns which are considered in a subsequent section of this paper.

TABLE 5.—*Values of β (attenuation of intensity per fitting) for brass fittings at various frequencies*

Frequency		250–257	1,000–1,087	2,000–2,570	3,000–3,470
T	2-inch	0.45	0.53	0.60	0.51
	3-inch	.63	.59	.24	.32
	4-inch	.48	.46	.30	.31
Y	2-inch	.49	.48	.65	.84
	3-inch	.64	.46	.65	.42
	4-inch	.85	.67	.33	.39
L	2-inch	.03	.03	.11	.19
	3-inch	.03	.00	.00	.23
	4-inch	.14	.13	.40	.36

TABLE 6.—*Values of β for various gas-proof diaphragms*

Frequency	3-inch tracing-cloth diaphragm	2-inch aluminum diaphragm [1]	
		Without tension	With tension
250–257	2.22	0.43	1.16
1,000–1,080	1.69	.74	1.06
2,000–2,570	2.22	.88	.76
3,000–3,470	3.42	.69	.41

[1] Two thicknesses of aluminum diaphragm were tested 0.002 and 0.0015 inch, respectively. Within the limits of experimental error no difference in attenuation of the two was observed.

The fact that the values of β for the tracing-cloth diaphragm are greater than unity while those for other materials are less than unity does not mean that the tracing cloth acts as an amplifier. The explanation is found in the form of equation (6). For example, with a 2-inch brass tube at 1,000 cycles the value of α is 0.025. Let the length of the straight parts of the tube total 100 feet, and let there be two tracing-cloth diaphragms for each of which $\beta = 2.22$. Then by equation (6)

$$\frac{I}{I_0} = E^{-2.5-4.44} \quad = E^{-6.94} \quad = 0.0009$$

approximately one-tenth of 1 per cent.

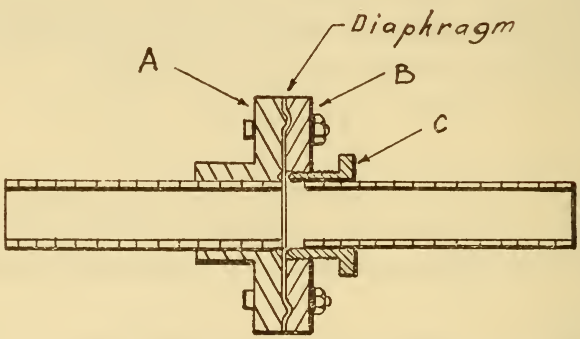

FIG. 8.—*Arrangement for mounting diaphragms*

By the heading "without tension" in Table 6 is meant that the diaphragm was clamped at the edges by the rings A and B (see fig. 8) but was not drawn sufficiently tight to remove the wrinkles. The tension sleeve C had a pointer and graduated scale by means of which the tension setting could be duplicated. When tension was applied the tension sleeve was screwed in against the diaphragm until the wrinkles were removed and the index setting recorded. In this way conditions could be easily repeated. The diaphragm thus mounted was placed near the middle of a line several hundred feet in length, and from data thus obtained, together with attenuation data of straight tubes, the attenuation of the fitting may be computed.

At frequencies above 1,000 cycles the "flat" elbows (see fig. 9) are apparently equal to the regular right-angled elbow, but at frequencies of the order of 250 they transmit only 50 per cent of the sound energy normally transmitted by the regular right-angled elbow.

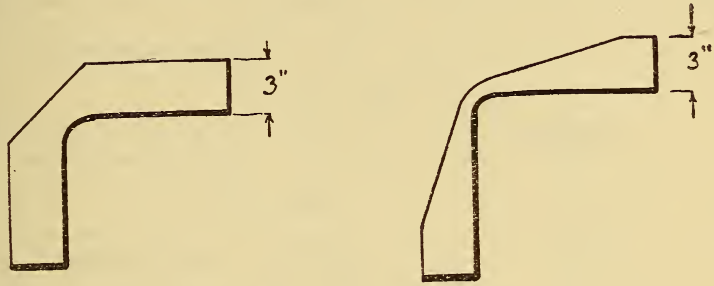

FIG. 9.—*Flat elbows*

(*e*) TERMINAL CONES.—The method of testing the cones was similar to that for testing straight pipes above described. A 40-foot line of pipe was set up, of diameter equal to that of the small end of the cone to be tested. A loud-speaker was used as a source of sound, placed in front of the cone, and a telephone receiver was placed at the other end. From the electromotive force produced in the telephone receiver a measure of the sound energy impinging upon the diaphragm could be obtained.

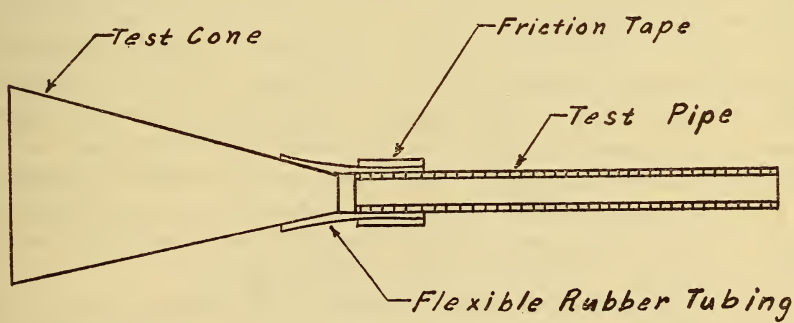

FIG. 10.—*Method of attaching cones*

Each cone in turn was placed in position at the transmitting end of the pipe line, and readings of the electromotive force in the telephone were taken, which, after proper calculation, gave numbers expressing the relative intensities of the received sound.

The method of attaching the cones to the test pipe is shown in Figure 10.

Table 7 gives the results of relative intensity tests made on five cones of enameled sheet steel, differing principally in their angular opening.

TABLE 7.—*Relative intensities*

Frequency	Enameled iron cones				
	Cone 1	Cone 2	Cone 3	Cone 4	Cone 5
	Diameter, small end (inch)				
	1	1	1	1	1
	Diameter, large end (inches)				
	14¼	13	11	9½	8
	Slant height (inches)				
	29	28½	27½	27	26
250–257	17.1	18.7	24.3	30.0	43.8
1,000–1,087	15.2	16.4	20.4	22.4	31.5
2,000–2,570	1.87	2.31	3.84	7.78	24.6
3,000–3,470	1.10	1.76	4.75	1.58	1.82

It will be noticed that at the lowest pitch the most efficient cone is that which has the smallest angular opening. Cones of large angular opening seem to reflect more sound backward toward the source as their shape approaches that of a flat wall with a hole in it. This effect persists for a time as the frequency rises, becoming quite pronounced from 2,000 to 2,750 cycles per second. For frequencies from 3,000 to 3,470 it seems that this effect suddenly disappears, leaving, however, an anomalous value for cone 3, probably a resonance effect. Frequencies of 2,000 cycles and higher are of practical importance in the case of whistles and in those higher harmonics of the voice which determine quality and enable us to recognize the speaker.

In this table no comparison is to be drawn between numbers in the same vertical column, as a change of pitch of the sound source carries with it a change of intensity of unknown magnitude.

Table 8 gives similar results obtained with several sizes of brass cones. These cones differed, first, in the diameter at the small end, and, second, in length. The angular opening of any two cones of the same diameter at the small end was substantially the same. The cones are designated in the table as 2, 3, or 4 inch cones, according to the diameter at the small end, corresponding to the size of pipe to which they were affixed.

In addition to these cones with straight sides, tests were made on one brass cone of flaring or "exponential" form, of small diameter, 2 inches; large diameter, 15 inches; and altitude, 12 inches. In Table 8 the different cones are designated as "short" or "long" (according to slant height), and "exponential."

TABLE 8.—*Dimensions of the various types of cones*

Diameter		Slant height
Small end	Large end	
Inches	*Inches*	*Inches*
2	8½	18
2	6	12
3	9	18
3	7	12
4	10	18
4	8	12

Relative intensities

2-INCH BRASS CONES

Frequency	Short cone	Long cone	Exponential cone
250–257	9. 2	7. 4	2. 1
1,000–1,087	36. 2	24. 8	14. 4
2,000–2,570	1. 90	1. 69	0. 67
3,000–3,470	5. 80	2. 40	1. 27

3-INCH BRASS CONES

Frequency	Short cone	Long cone
250–257	66. 0	47. 0
1,000–1,087	25. 3	12. 4
2,000–2,570	26. 0	5. 6
3,000–3,470	34. 0	11. 8

4-INCH BRASS CONES

Frequency	Short cone	Long cone
250–257	19. 0	10. 1
1,000–1,087	32. 4	16. 8
2,000–2,570	84. 8	48. 8
3,000–3,470	16. 4	12. 2

It will be seen that in every case the short horn is the most efficient. The exponential horn, in spite of the theoretical claims made for it, gave very poor results. As in Table 8 no comparison is to be made between numbers in the same vertical column.

4674°—26——3

III. MEASUREMENT OF ARTICULATION

1. GENERAL CONSIDERATIONS

The transmission of articulation has been studied extensively in connection with telephone practice and much of the information thus obtained is applicable to the similar voice-tube problem. This is especially true of the studies that have been made of the relative importance of different frequencies with respect to the intensity and

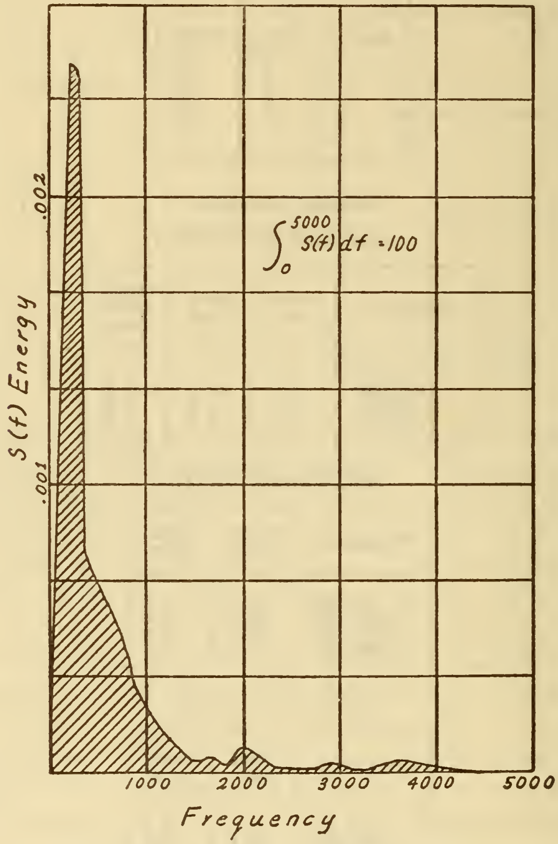

Fig. 11.—*Distribution of energy in speech*

articulation of speech. This is illustrated by the curve in Figure 11,[2] which gives the distribution of energy in speech, and shows clearly the concentration of energy in the lower frequencies, over 80 per cent of the area of this curve lying below the frequency of 1,000 cycles.

The same information is presented in somewhat different form in the lower graph of Figure 12.[3] Any ordinate of the L (low) curve

[2] Crandall and Mackenzie, "Analysis of the Energy Distribution in Speech," Phys. Rev.; March, 1922.
[3] Fletcher, "The Nature of Speech and its Interpretation," J. Frank. Inst.; June, 1922.

gives the per cent of the total energy of speech accounted for by frequencies below that to which the ordinate corresponds; and any ordinate of the *H* (high) curve gives the per cent of energy of speech of frequencies higher than that of the ordinate in question. This graph is derived from Figure 11, and is somewhat more convenient for practical use.

The upper graph of Figure 12 conveys analogous information with respect to distinctness of articulation. For example, from the *L*

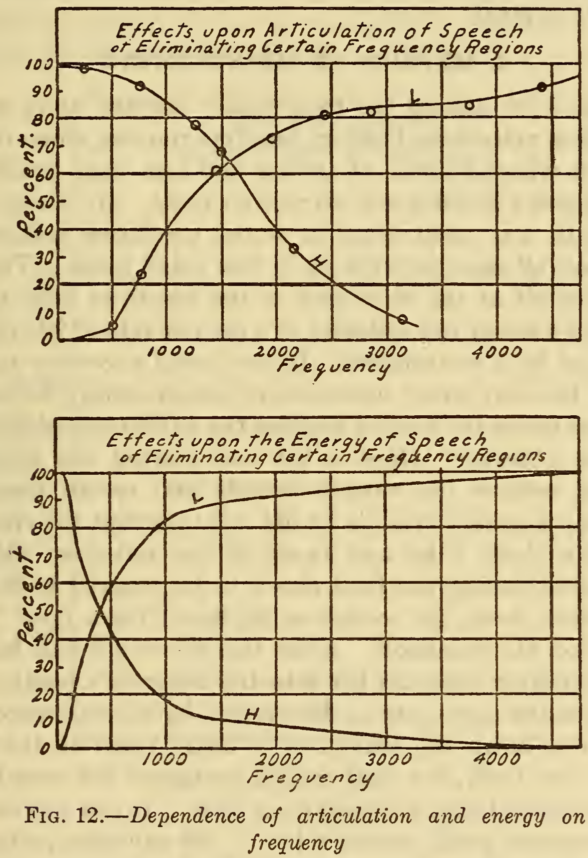

Fig. 12.—*Dependence of articulation and energy on frequency*

curve it follows that frequencies below 1,000 cycles give an articulation of only 40 per cent. In other words, if speech were transmitted through a sound filter cutting off all frequencies above 1,000 cycles and transmitting those below (a low-pass filter) 40 per cent of the syllables would be intelligible. For connected discourse, the apparent percentage would be higher, since some syllables imperfectly heard are understood by their relation to the context. The *H* curve, on the other hand, indicates that if all frequencies below 1,000 cycles are cut off the articulation is still 85 per cent. This leads to the conclusion that the frequencies below 1,000 cycles contribute most of the intensity and those above most of the intelligbility. Experimental verification of this is possible in a telephone circuit, for by the use of electric filters any desired band of frequencies may be suppressed. Speech from which all frequencies above 1,000 cycles have been eliminated is sonorous but unintelligible, while speech that lacks the lower frequencies sounds thin and without body, but its intelligibility is good.

2. METHOD OF MEASUREMENT

The method of making the tests finally decided upon was to set up the various voice-tube lines on brackets running along one side of a long room where lengths of tubing 400 feet long could be used and where there was not much extraneous noise. At one end of these tubes a booth was constructed of sound insulating material. The observer took all his observations in this small room. The speaker stationed himself at the other end of the length of tube under test and repeated a list of 100 syllables at a regular rate of 20 syllables per minute timed by a metronome. It was found necessary to time the speaker in this way, since otherwise he unconsciously either speeded up or slowed down the rate of sending the various syllables and thus affected the observer. Once a list was started the observer did nothing but receive the various sounds and record them. If he failed to hear a certain syllable he did not interrupt the speaker, but merely left a blank if he was aware of the omission; otherwise it was discovered during the final check to be referred to later. The observer wrote down the sounds as he heard them upon the right-hand page of his notebook. After the whole list had been transmitted the speaker took his list into the observer's booth, where he repeated it at the same rate as determined by the metronome. The observer wrote down the sounds as he heard them on the left-hand page of his note book, the right (which contained the record obtained through the tube) being screened from view. It can not be assumed that the observer would record a list of 100 syllables perfectly even when the transmission was perfect. For this reason the speaker read the list to the observer in the same room under ideal transmission conditions and the percentage recorded correctly was taken as a measure of what should be regarded as a perfect result with perfect transmission. The lists of syllables were of such a nature that memory effects were almost entirely eliminated. When the two series of records above referred to were completed they were both checked against the original and the result was stated in the per cent of syllables recorded correctly. The percentage of the record obtained by transmission through the tube was then divided by the percentage of the record called off in the booth directly, the quotient being taken as a measure of the articulation.

Each list of syllables was transmitted twice with the speaker and observer changing places for the second transmission.

About 8,000 syllables were constructed, involving the following combinations: (*a*) Consonant followed by vowel, (*b*) vowel followed by consonant, and (*c*) consonant-vowel-consonant. These syllables were then carefully checked over and those likely to give trouble due to memory effects were, as far as possible, eliminated. Each of the syllables finally chosen was then written on an index card so

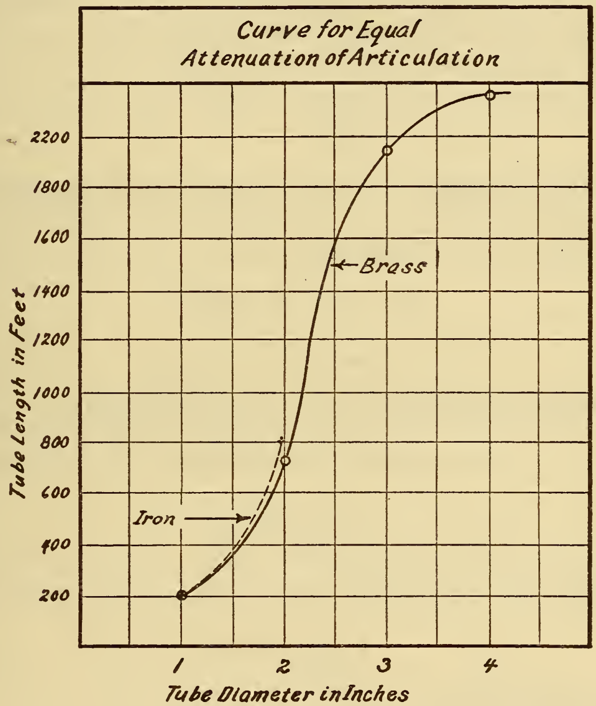

FIG. 13.—*Curve for equal attenuation of articulation*

that the lists could be shuffled conveniently and handled by the speaker. Each list of 100 syllables to be used for a transmission test was obtained by drawing 100 cards at random from the complete list. Typical lists of syllables actually used are found in the Appendix.

In order to make a comparison of the relative articulation of the various tubes under test, computations have been made in which all tubes are referred to a 1-inch brass tube 200 feet long as the standard of comparison; that is to say, the data are so presented that by consulting the curve (fig. 13) we may see at a glance just how long a tube of given diameter must be in order to have the same attenuation of intelligibility as a brass tube 1-inch in diameter and 200 feet long.

The reduction of intelligibility of speech transmitted through a communication line is what we call the articulation. It is due in part to distortion and in part to reduction in intensity. The articulation of such a line is 100 per cent if the intelligibility is perfect. We have assumed that the reduction of intelligibility follows an absorption law and the results obtained indicate that within the errors of measurement the assumption is justified. We will pick out a section of small length, dx, taken anywhere along a transmission line. Let A = the articulation remaining at the input end of the small section, dA = the change in articulation due to the small section, then the absorption law is expressed by

$$\frac{dA}{dx} = -\gamma A \tag{7}$$

where γ is the attentuation constant of articulation per unit length of tubing.

Integrating equation (5) gives

$$\log_\epsilon A = -\gamma x + \text{const.}$$

when

$$x = O \text{ then const.} = \log_\epsilon A_0$$

where A_0 is the articulation at the transmitting end,

$$\log_\epsilon A = -\gamma x + \log_\epsilon A_0$$

or

$$\log_\epsilon \frac{A}{A_0} = -\gamma x \tag{7a}$$

and passing to the exponential form

$$\frac{A}{A_0} = \epsilon^{-\gamma x} \tag{8}$$

A useful form of (7a) is

$$\gamma = \frac{1}{x} \log \frac{A_0}{A} \tag{9}$$

The base 10 is preferable for practical computations and by using the relation

$$\log_\epsilon N = \frac{\log_{10} N}{\log_{10} \epsilon}$$

we may convert (9) to

$$\gamma = -\frac{1}{(0.434)X} \log_{10}\left(\frac{A_0}{A}\right) \tag{9a}$$

This is the form used for computing γ for a length of tube X where $\frac{A}{A_0}$ has been measured.

Another very useful form is obtained by operating on (8). Taking logs to the base 10 of both sides of the equation and removing the negative sign by reversing the ratio $\frac{A_0}{A}$:

$$\log_{10}\left(\frac{A_0}{A}\right)=\gamma x \log_{10}=0.434\gamma x \tag{10}$$

from which, given γ and x, the value of $\frac{A_0}{A}$ can be found by consulting a table of logarithms.

Since some of the tubes furnished for test required measurements to be made over comparatively short lengths it was found advisable to add such tubes to the end of a longer length of the same diameter. This was necessary in order that there might be no possibility of the observer hearing any sound directly through the air and the walls of his booth.

In this case, if A_0 is the articulation at the beginning of a tube of length x_1, A the final articulation, and γ_1 the attenuation constant for that particular tube, we have by formula (5b)

$$\frac{A}{A_0}=E^{-\gamma_1 x_1} \tag{5c}$$

or $A=A_0E^{-\gamma_1 x_1}$ as the articulation at the end of the first tube of length x_1 and articulation constant γ_1; and for tube number two of length x_2 and articulation constant γ_2 we have as above

$$A=A_0E^{-\gamma_2 x_2}$$

In this case the articulation entering the second tube is obtained from (5c). Substituting this we obtain

$$A=A_0E^{-\gamma_1 x_1}E^{-\gamma_2 x_2}$$

or

$$\frac{A}{A_0}=E^{-\gamma_1 x_1-\gamma_2 x_2} \tag{5d}$$

the form used for the computation referred to.

Although conditions in the room where the experiments were carried out were rather above the average as regards freedom from extraneous sounds, nevertheless it was found early in the work that some form of head set would have to be used by the observer to get reliable results. After experimenting with various kinds of terminal apparatus the head set furnished by the Bureau of Construction and Repair was adopted for this work. This consisted of a canvas head covering containing ear pieces which formed the terminals of the tubes. The canvas covering was removed and earpieces fastened to the head-band of an ordinary telephone receiving set. This arrangement has been found quite satisfactory and has been used throughout these tests.

3. RESULTS

(*a*) STRAIGHT TUBES AND SWEEPS.—

Values of γ (attenuation of articulation per foot)

Straight tube		γ per foot
Material	Size	
	Inches	$x\ 10^{-4}$
Brass	1	18.5
Do	2	5.13
Do	3	1.92
Do	4	1.73
Iron	1	19.0
Do	2	4.69

Sweeps and straight pipes have same coefficients within the limits of these measurements.

The differences between the values of γ for brass and iron tubes of the same diameter are practically insignificant.

Fiber tubes were not measured because a length of tubing sufficient for making the measurements was not available.

(*b*) DIAPHRAGMS AND BRASS FITTINGS.—

Values of δ (attenuation of articulation per fitting)

Fitting		δ
4-inch	T	0.007
	Y	.007
	L	.000
3-inch	T	.031
	Y	.049
	L	.000
2-inch	T	.041
	Y	.086
	L	.000

Diaphragm	δ for diaphragms
3-inch tracing cloth	0.057
2-inch aluminum	.177

(*c*) FLEXIBLE TUBES.—Forty-foot lengths of 2 and 3 inch flexible tubes were tested in a straight position. The percentage results were as follows:

	Per cent
2-inch tube	89
3-inch tube	97

As with the sweeps compared with straight tubing, no change in results was observable when the flexible tube was coiled with a radius of 10 feet.

(*d*) Terminal Cones.—These tests were carried out by the use of a set of selected test syllables spoken through the pipe line with different cones attached. The percentage of syllables correctly heard by the person at the receiving end is tabulated, and a comparison of these numbers as given in Table 9 will give the relative effect of different cones upon articulation.

Table 9.—*Relative effect of cones on articulation*

[Perfect articulation=100]

370 FEET 2-INCH BRASS PIPE	
No cone	0.75
Short cone	.81
Long cone	.84
390 FEET 3-INCH BRASS PIPE	
No cone	0.91
Short cone	.92
Long cone	.90
400 FEET 4-INCH BRASS PIPE	
No cone	0.93
Short cone	.93
Long cone	.93

It will be seen that with the larger size of pipes the presence of the cone on the pipe has no appreciable effect on the articulation. Only in the case of the smallest pipe (2-inch diameter) does the presence of the cone produce any improvement; and here there is no appreciable difference, within the limits of experimental error, between the effects of the long and the short cones.

(*e*) Accuracy of Articulation Experiments.—Although the measurement of articulation by the use of syllables as described above is probably the best method for the purpose which has so far been devised, it is subject to considerable uncertainty. Work of this kind can not be carried out with the precision or the reproducibility of physical measurements. Average deviations from the mean values given in the articulation experiments seldom exceed 5 per cent, and on the average was about 3 per cent.

IV. SOUND FILTERS

Two forms of sound filters have been tested at the Bureau of Standards—those devised by G. W. Stewart (Phys. Rev., Dec., 1922, and Nov., 1923) and by Quincke (Rayleigh: Theory of Sound, 2 par. 318, p. 210).

The purpose of such filters is to remove certain wave bands or frequencies from a complex tone. It is a desirable object on board ship to remove noise and rattle that may find entrance to the voice tubes and leave the articulate tones of the voice. Unfortunately, it is probable that in most cases the voice tones are of the same frequency as the noise, so that it is hopeless to attempt to remove one and leave the other. The best treatment for noise and rattle is prevention rather than cure.

It may, however, be of interest to describe briefly the filters tested. Photographs of these are shown in Figure 14. Numbers 1, 2, and 3 are of the Stewart type, and No. 4 is a Quincke filter.

The Stewart filter was designed on the basis of the analogy of transmission of sound along a tube and an electric current along a wire; but the resemblance of the two phenomena seems rather imperfect, and the filters were not found to be very efficient.

An inner tube carries the sound and openings at intervals allow the sound to pass into an outer tube, divided into compartments and sometimes provided with exit tubes, to the outer air.

The best results obtained at the Bureau of Standards were furnished by the Quincke filter. Side tubes of different length provided with movable pistons for tuning open out from the main sound tube. By proper tuning a component of any particular frequency may be at least partially destroyed by interference. For complete removal a second tube tuned to the same frequency is sometimes necessary. This form of filter is reasonably efficient for the purpose for which it is intended.

TABLE 10

[88 per cent correct]

1	pah	26	guh	51	id*	76	ef
2	vaw	27	vid	52	can	77	ime
3	fud	28	chuh	53	jit	78	eep
4	ite	29	coe	54	ve	79	nuh
5	loo	30	guh	55	oze	80	toll*
6	ow	31	pie	56	rye	81	seh
7	fah	32	vah	57	chew	82	aw
8	zen	33	fay*	58	co	83	ube
9	ip	34	taw*	59	gew	84	eek
10	ive	35	nigh	60	ick	85	ick*
11	oot	36	fane	61	faw	86	sha
12	voe	37	ane	62	nih	87	ow
13	mud*	38	buh	63	et	88	zeer*
14	mud	39	ack	64	duh	89	vah
15	oo	40	chi	65	ane	90	ick
16	hoff*	41	zie	66	gew	91	rosh
17	ung	42	zah	67	oom	92	zah
18	chuh	43	ikes	68	huh	93	code*
19	char	44	ood	69	dut	94	neh
20	fak	45	shif	70	shon	95	shir
21	naw	46	seeth*	71	fud	96	tuh
22	zih	47	uck*	72	eef	97	bach
23	acq	48	ject	73	pah	98	shuh
24	zith	49	idge	74	ouse*	99	hons
25	abe	50	hob	75	ene	100	rih

V. LIST OF TEST SYLLABLES

Tables 10 and 11 contain a test list of syllables typical of those employed in articulation measurements. Table 10 is the record as

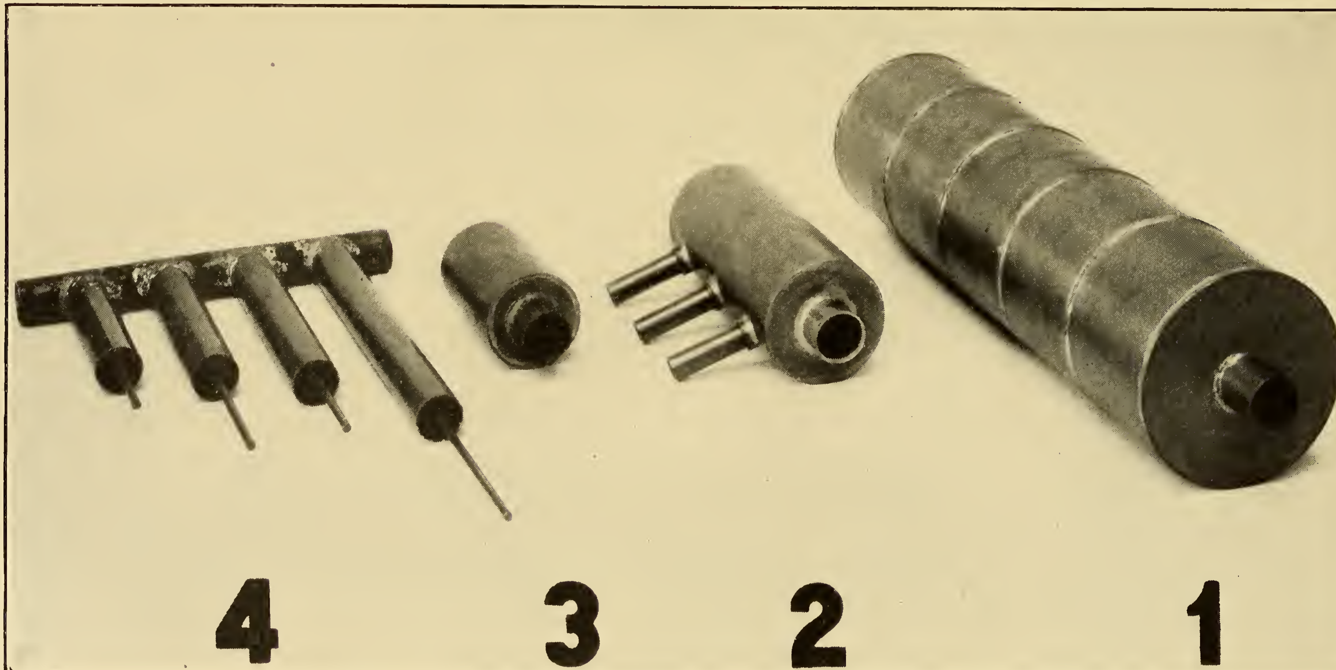

FIG. 14.—*Sound filters*

1, 2, 3, Stewart sound filters. 4, Quincke sound filter

heard by the observer through 400 feet of 4-inch brass tube. The syllables incorrectly heard are marked by an asterisk. Table 11 is the same list of syllables as heard by the same observer when read to him in the immediate presence of the transmitter. It may be worth while to add that where the syllable recorded in the test through the tube would evidently be pronounced the same as the original, although spelled slightly differently, it has been considered correct.

In comparing Table 10 with the original it is found to be 88 per cent correct, while Table 11 is 96 per cent correct. Dividing the former figure by the latter gives a quotient of 92 per cent, which is taken as the measure of the attenuation of articulation expressed in the form $\frac{A}{A_0}$ of formula (5c).

TABLE 11

[96 per cent correct]

1	pah	26	guh	51	idge	76	f
2	vaw	27	vid	52	caw	77	ime
3	fud	28	buh*	53	jit	78	eeh
4	ite	29	coe	54	be*	79	nuh
5	loo	30	guh	55	oze	80	cul
6	ow	31	pie	56	ri	81	seh
7	pah*	32	vah	57	chew	82	aw
8	zen	33	say	58	co	83	ube
9	ip	34	chah	59	gew	84	eek
10	iv	35	nigh	60	ick	85	icks
11	oot	36	fane	61	faw	86	sha
12	voe	37	ave	62	nih	87	ow
13	mudge	38	buh	63	et	88	zir
14	mud	39	uck	64	duh	89	vah
15	oo	40	chi	65	ave	90	ick
16	bah	41	zie	66	gew	91	rosh
17	ung	42	zah	67	oom	92	zah
18	chuh	43	ikes	68	huh	93	coge
19	chah	44	ood	69	dut	94	neh
20	fack	45	shif	70	shon	95	shir
21	naw	46	feas*	71	fud	96	tuh
22	zih	47	ucks	72	eef	97	bash
23	acq	48	ject	73	pah	98	shuh
24	zith	49	idge	74	outh	99	hons
25	abe	50	hob	75	een	100	rih

VI. APPENDIX.—NOTE ON THE ABSORPTION OF SOUND IN RIGID PIPES

1. GENERAL REMARKS

The results of the measurements of the decrease of intensity of a sound during transmission along straight pipes are summarized in

Table 1 of the foregoing paper, which gives the values of the absorption constant or absorptivity α occurring in the equation

$$\frac{dI}{dx} = -\alpha I$$

The value of α varies with the mean frequency of the sound and with the diameter and material of the pipe; but the physical factors that affect the absorption are not immediately evident from the table, and it seems desirable to study the figures in the hope of getting some light on this question. The phenomena are presumably simpler in the brass and iron than in the weaker and more irregular fiber pipes, and we shall confine our attention to the former.

There are three obvious ways in which the energy of a sound wave may be dissipated during transmission along a pipe: (*a*) Inelastic yielding of the pipe to the variations of pressure; (*b*) heat transmission between the air and the pipe, as the air is heated and cooled by compression and rarefaction; and (*c*) skin friction against the wall, retarding the to and fro motion of the air. We may consider these successively.

(*a*) Perfectly elastic yielding of the pipe wall might change the speed of propagation, but would not dissipate energy. Imperfectly elastic yielding would dissipate energy taken from the sound wave and would increase the absorption per unit length. If this effect is appreciable it will evidently be greater with thin than with thick pipes; but if we plot $\alpha = f(D)$, for any given frequency n, we find that the points for the four brass pipes lie along a smooth curve, although the 1-inch pipe was about two and one-half times as thick as the other three and must have yielded very much less. From this we conclude that in these pipes, and still more in the stronger iron pipes, inelastic yielding of the walls did not have any appreciable effect on the transmission of sounds of such intensities as were employed.

(*b*) In view of the low thermal conductivity of air and the very small temperature differences set up in sound waves of ordinary intensity, it seems improbable that heat transmission can have any sensible effect on the dissipation of sound energy, unless the pipe is very small. The effect, if present, must increase with the thermometric conductivity of the metal, and since this is about three times as great for brass as for iron or steel, the dissipation due to heat transmission must be considerably greater in brass than in iron. In reality, the observed values of α, at a given frequency and diameter, were smaller for the brass pipes, and we conclude that if heat transmission played any appreciable part in the absorption, its effects were overpowered and masked by something more important.

(c) This more important source of dissipation seems to be skin friction. Skin friction increases with roughness of wall, and since the iron pipes were considerably rougher than the brass, we might expect them to offer more resistance to the vibrations of the air and show higher values of the absorptivity α; and this is confirmed by the values in the table.

From the foregoing discussion it seems safe to conclude that in metallic pipes, such as were used in these experiments, the main reason for the attenuation of the intensity of a sound as it travels along the pipe is simply skin friction, and that the difference of material is of importance only because it influences the smoothness of the inside of the pipe.

2. RELATION BETWEEN ABSORPTIVITY AND MEAN FREQUENCY

Using the data in Table 1, we may plot the values of α for each pipe against the values of the mean frequency n, and the result is a set of curves like those in Figure 5 of the foregoing paper, but turned upside down (fig. 15). All the curves have maxima at about the same frequency of 2,600 per second, and from this it is clear that somewhere in the apparatus there was the possibility of a free vibration with a natural frequency in the vicinity of 2,600, so that as the mean frequency approached this value, there was resonance and an increased dissipation of energy, which in some way increased the apparent absorptivity of the pipe. The question is next, what determined this resonance frequency?

Since it occurred always at about the same point, while the pipe diameters varied from 1 to 4 inches, the resonance frequency was evidently not determined by the diameter. It was also not fixed by the length of the pipe. For at 20° C. the wave length in air at the frequency of 2,600 per second is about 5.2 inches; and the lengths of the pipes were always such large multiples of this that they could not have been responsible for any pronounced resonance at this frequency alone. Neither does it appear that the 10-foot length of the removable sections can have played any part; for at $n=2{,}600$ the wave lengths of longitudinal vibrations in brass and iron are about 4.4 and 6.3 feet, respectively, and neither of these is simply related to 10 feet.

The most obvious supposition is that the receiving apparatus had a natural frequency of 2,600, and at all events it seems fairly certain that this frequency was not characteristic of the pipes by themselves but was determined by something in the apparatus or the method of experiment. If this is so, a part of the observed absorption was due to something else than the pipes alone, and if there had been no resonance, the values obtained for the absorptivity would have been lower. Thus the resulting tabulated values of α are too high by an amount which increases as n approaches 2,600, and the true absorptivities are best represented by the results obtained at frequencies far from 2,600, where the disturbing effect of the resonance was least.

What we should like to do is to disentangle the mixed effects of the resonance and the true absorptivity of the pipe, but since we are completely in the dark as to the mechanism of the resonance and the manner in which it influences the apparent absorption of the pipe, all we can do is to proceed empirically from very simple assumptions.

3. QUANTITATIVE REPRESENTATION OF THE RELATION $\alpha=f(n)$

Since there is no obvious reason why the absorptivity of a rigid pipe should depend on the mean frequency, when a wide band of frequencies is used, we start by assuming that the observed variable value of α for a given pipe is the sum of a constant part β, the true absorptivity, and a variable part $(\alpha-\beta)$ due to the resonance, which has a maximum value $(\alpha_0-\beta)$ when $n=n_0=2{,}600$. And we then attempt to represent the variation with frequency by the empirical equation

$$\frac{\alpha-\beta}{\alpha_0-\beta}=e^{-C(n_0-n)^2} \tag{1}$$

in which the maximum absorptivity α_0 is to be found from a smooth curve drawn to represent the observed points, and the constants β and C are to be determined by trial so as to make the equation fit the observations as well as may be.

The constants actually found by trial are shown in the accompanying table:

	Brass				Iron	
	Nominal diameter					
	1 inch	2 inches	3 inches	4 inches	1 inch	2 inches
α_0	0.051	0.038	0.028	0.020	0.066	0.035
β	.0318	.0202	.0149	.0119	.040	.0248
$10^7\ C$	7.10	5.35	4.44	3.86	7.10	5.27

Curves plotted from equation (1) by using these constants are shown in the figure. The observed values are put in as separate points, and it appears that on the whole the curves do represent the observations fairly well—probably within the errors of the determinations.

In view of the obviously rather large uncertainties of the determination of α, the constants might be varied slightly without much effect on the agreement of the computed curves with the observed points. Preliminary trials showed that the best values of β and C satisfied, approximately, the following equations,

$$\beta = \frac{0.03335}{D^{0.76}} \text{ for the brass pipes} \tag{2}$$

$$\beta = \frac{0.0419}{D^{0.76}} \text{ for the iron pipes} \tag{3}$$

$$C = \frac{7.3 \times 10^{-7}}{D^{0.47}} \text{ for all six pipes} \tag{4}$$

and the values given in the table were found from these equations.

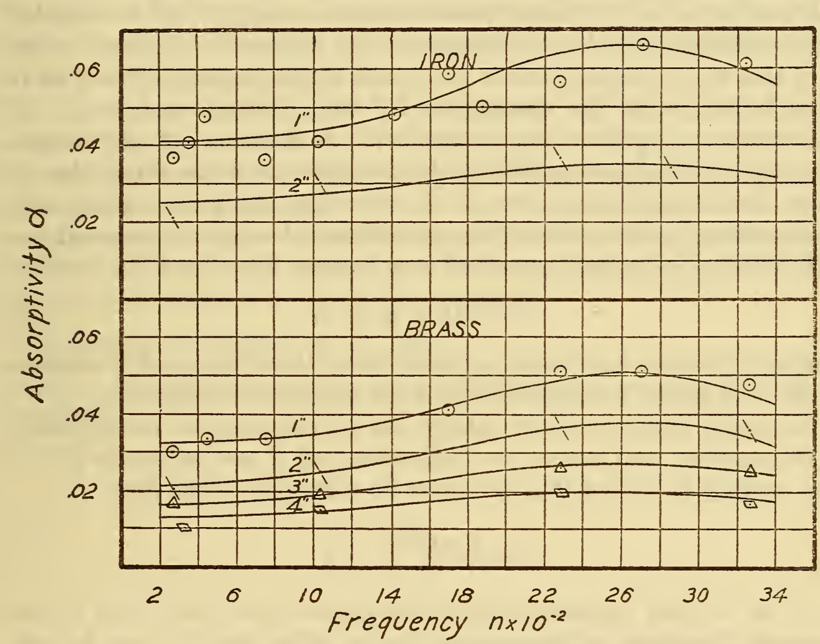

FIG. 15.—*Absorptivity at different frequencies*

Accepting this representation of the whole body of results as substantially correct though purely empirical, we may go on to inquire whether any further information is obtainable from dimensional reasoning.

4. THE TRUE ABSORPTIVITY β

On the view adopted here, the true absorptivity of these rigid pipes is independent of the frequency and is represented by β, while the additional absorption observed is due to some peculiarity of the apparatus or method and would be absent if the extraneous resonance could be eliminated.

The maximum effect of resonance is measured by $(\alpha_0 - \beta)$ and the rate at which the effect falls off as the frequency n departs from the resonance frequency n_0 is measured by the value of C. If we knew more about the details of what takes place, we might be able to give some physical interpretation of these quantities, but without such knowledge the attempt seems useless. The values of β, however, are more interesting and important and are worth further study.

The value of β depends on the diameter of the pipe in the way shown by equations (2) and (3), which also show the difference between brass and iron. Since the dissipation of the energy of the sound is due, partly at all events, to skin friction, β must be supposed to depend on the viscosity μ and density ρ of the air, and on the roughness of the pipe wall. The latter may be specified by the values of a number of dimensionless length ratios; it will be denoted by the single symbol r, standing for the aggregate of these ratios. It is also to be presumed that the speed of propagation S may have some influence on the dissipation by skin friction, and on β. If heat transmission is an important factor in determining the dissipation, the thermal properties of the air and the pipes must also be taken into consideration; but in so far as the dissipation arises only from friction at the walls, the quantities already enumerated are sufficient to determine it, and we may assume that there is a relation

$$\beta = f\,(D, \rho, \mu, S, r) \tag{5}$$

which, if its form were known, would show the effect on β of changes in D, S, ρ, and μ, for pipes of any fixed degree of roughness.

Any such equation must satisfy the requirement of homogeneity in dimensions, and since the dimensions of β are evidently $[l^{-1}]$ it may readily be shown that (5) must be reducible to the form

$$\beta = \frac{1}{D} F\left(\frac{DS\rho}{\mu}, r\right) \tag{6}$$

So far as the argument from dimensions goes, the form of the operator F remains indeterminate; but in order that (6) may be consistent with the experimental results expressed by (2) and (3), F must obviously contain the factor $D^{0.24}$. And since D appears in F only in the combination $DS\rho/\mu$, the form of F must be

$$F\left(\frac{DS\rho}{\mu}, r\right) = \left(\frac{DS\rho}{\mu}\right)^{0.24} \phi\,(r) \tag{7}$$

in which only $\phi(r)$, which expresses the effect of changes in roughness, remains indeterminate.

Substituting from (7) into (6) we now have, as the general equation of which (2) and (3) are particular forms,

$$\beta = \frac{\left(\frac{S\rho}{\mu}\right)^{0.24} \phi\,(r)}{D^{0.76}} \tag{8}$$

The factor $(S\rho/\mu)^{0.24}$ expresses the effect of changes in atmospheric conditions, and upon inserting known values it will be found that any ordinary variations of pressure and temperature can not affect the value of this factor by more than a very few per cent, so that in work of this degree of accuracy $(S\rho/\mu)^{0.24}$ may be regarded as a constant.

As regards roughness, all we can say is that

$$\frac{\phi(r, \text{iron})}{\phi(r, \text{brass})}=\frac{4{,}190}{3{,}335}=1.26 \tag{9}$$

and until we have some way of defining "roughness" so that we can specify it quantitatively, we can go no further in investigating the form of ϕ, even if we have experimental data on many more pipes of different degrees of roughness.

It is interesting to note, from equation (8), that increasing μ *decreases* β; that is, that if the dissipation of energy represented by β is really due to skin friction, increasing the viscosity decreases the skin friction. At first sight this looks paradoxical, and if no other such case were known, it would seem that the reasoning which led to this result must be erroneous. But in reality, though such cases are unusual they are by no means unknown.[4]

WASHINGTON, February 11, 1926.

[4] See, for example, A. H. Gibson, Phil. Mag. (6), 50, p. 199, July, 1925, on "The flow of water in a corrugated pipe."