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STRENGTH OF STEEL TUBING UNDER COMBINED COLUMN AND TRANSVERSE LOADING, INCLUDING TESTS OF COLUMNS AND BEAMS

BY

TOM W. GREENE, Assistant Engineer Physicist
Bureau of Standards

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STRENGTH OF STEEL TUBING UNDER COMBINED COLUMN AND TRANSVERSE LOADING, INCLUDING TESTS OF COLUMNS AND BEAMS.

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ABSTRACT.

This investigation was made for the purpose of determining whether experimental data confirmed the theory of struts subjected to combined column and transverse loading. A number of tests were made on steel-tubing struts ranging from that of a column with no transverse load to that of a beam with no column load.

A study was made of the conditions contributing to the strength of a strut and a method devised for measuring eccentricity. It was found that the eccentricity due to variation in wall thickness and to deviation from straightness is an important factor and should be taken into account. The results show that the commonly used formulas, which neglect the effect of eccentricity of loading, do not represent actual strut condition and are liable to give dangerously high results.

A modified rational formula based upon consideration of the effect of eccentricity was found to fit experimental results very closely and is the preferable one for design. Failure of a strut will occur when the maximum compressive stress computed by this modified formula is approximately equal to the yield point. The modified rational formula also applies to columns as it reduces to the "secant" column formula when the transverse load is equated to zero. Failure of a column will occur when the extreme fiber stress computed by the "secant" formula is equal to the yield point of the material.

A reasonably accurate computation of the stress for a strut under transverse load can be made by summing the bending stress due to the transverse load and the column stress obtained by the "secant" formula if for the latter the effective eccentricity is taken as the sum of the original eccentricity, due to tube irregularities, and the deflection of the strut at the center resulting from the transverse load.

CONTENTS.

I. INTRODUCTION.

1. PURPOSE OF THE INVESTIGATION.

In airplane construction there are many members, such as struts, which are subjected to a lateral or transverse loading in addition to an axial or column loading. This investigation was made at the request of the Bureau of Aeronautics, Navy Department, for the purpose of determining whether experimental data confirmed the approximate theory of struts subjected to combined axial and transverse forces or whether it would be necessary to devise new formulas.

A large number of tests were made on steel-tubing struts of different lengths with various intensities of transverse loading. The tests included different ratios of direct compression to transverse loading, ranging from that of a column with no transverse load to that of a beam with no column load.

2. ACKNOWLEDGMENTS.

The funds and material for this investigation were furnished by the Bureau of Aeronautics, Navy Department. Acknowledgments are also due Lieut. C. J. McCarthy, Bureau of Aeronautics, for his assistance, cooperation, and suggestions.

II. TEST PROCEDURE.

1. PHYSICAL PROPERTIES OF THE MATERIAL.

Steel tubing made in England of 1½-inch diameter, 20 gauge, and 1½-inch diameter, 16 gauge, was used for all struts and beams in this investigation. The physical properties of the material were accurately determined by tensile and compression tests of short specimens cut from every 15-foot length or section of tubing. The physical properties of each section of tubing used are given in Table 1, and a few typical stress strain curves are shown in Figure 1.

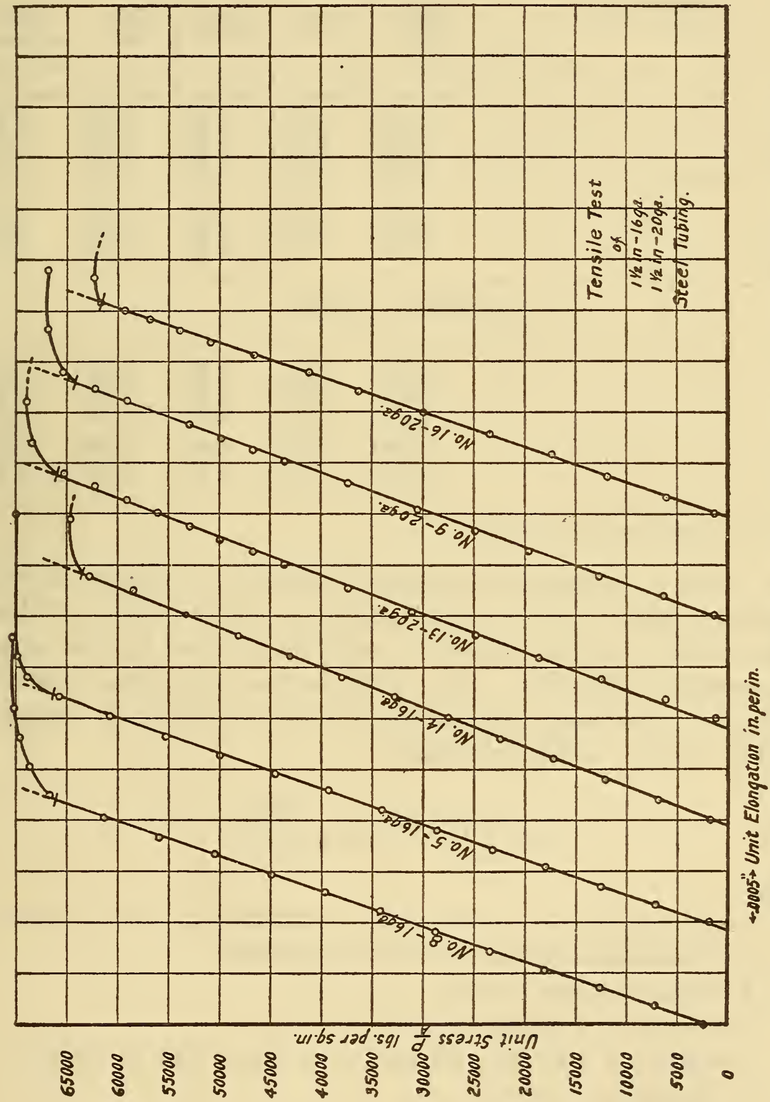

FIG. 1.—*Typical stress-strain curves (tensile test) of short specimens cut from each section of tubing.*

A comparison of the yield point values in tension with the ultimate compressive stress at crinkling shows considerable variations. It is believed that a more accurate determination of the ultimate compressive stress to cause failure for a short column is given by the "secant" column formula in which the effect of eccentricity of loading is considered.

TABLE 1.—Physical Properties of Each Section of Tubing.

1½ INCHES, 20 GAUGE.

Section number.	Tension.			Compression.[1]	
	Proportional limit.	Yield point.	Ultimate strength.	Proportional limit.	Ultimate stress (crinkling).
	Lbs./in.²	Lbs./in.²	Lbs./in.²	Lbs./in.²	Lbs./in.²
7	66,000	72,100	78,900	58,500	67,700
15	64,000	69,900	76,400	54,000	65,800
13	66,000	68,900	74,900	56,500	65,000
11	67,500	69,000	74,000	58,000	64,800
17	66,000	67,400	73,400	56,000	68,400
9	64,000	66,800	73,400	56,000	64,800
4	54,000	63,400	70,800	53,000	65,200
1	59,000	61,200	69,600	53,500	59,000
16	61,500	62,200	68,700	52,600	66,200

1½ INCHES, 16 GAUGE.

3	67,000	71,300	75,300	59,000	75,500
8	66,000	70,200	74,600	54,000	71,800
5	66,000	70,000	73,800	55,000	67,800
9	65,000	69,100	73,400	55,000	69,500
7	62,000	67,500	71,400	54,000	67,400
14	63,500	64,600	69,600	51,000	65,300
10	59,000	61,300	67,100	51,000	65,800

[1] $L/r = 10$ for short compression specimens.

In Table 2 are given the eccentricities due to variation in wall thickness, which will be explained later, and the effect of these eccentricities on the ultimate compressive stress for the short compression specimens. The values in the last column are the ultimate or maximum compressive stresses at failure S_c obtained by the "secant" column formula,

$$S_c = \frac{P}{A}\left(1 + \frac{ec}{r^2} \sec \sqrt{\frac{PL^2}{4EI}}\right)$$

where

e = eccentricity in inches due to variation in wall thickness,
P = maximum compressive load in pounds,
A = area in square inches,
r = radius of gyration in inches,
c = distance from neutral axis to extreme fiber in inches,
L = length in inches,
I = moment of inertia,
E = modulus of elasticity (29,000,000 lbs./in.²).

FIG. 2.—*Typical failures of short compression specimens cut from each section of tubing.*

TABLE 2.—Ultimate or Maximum Compressive Stresses for Short Compression Specimens.

1½ INCHES, 20 GAUGE.

Section number.	Variation in thickness of wall.	Eccentricity. e	Ultimate compressive stress at failure. S_c [1]
	Inches.	Inches.	Lbs./in.[2]
15	0.0325–0.035	0.0153	68,800
13	.033 – .037	.022	69,300
11	.0325– .035	.014	67,600
17 [2]	.035	.000	68,400
9	.0345– .0355	.0083	66,400
4 [2]	.035	.000	65,200
1	.034 – .040	.032	64,700
16 [2]	.036	.000	66,200

1½ INCHES, 16 GAUGE.

Section number.	Variation in thickness of wall.	Eccentricity. e	Ultimate compressive stress at failure. S_c
3	0.059 –0.066	0.0215	80,400
8	.0595– .064	.014	74,800
5	.059 – .066	.0215	71,600
7	.060 – .063	.0095	69,300
14	.061 – .067	.018	68,800

[1] $S_c=\frac{P}{A}\left(1+\frac{ec}{r^2}\sec\sqrt{\frac{PL^2}{4EI}}\right)$ where e is eccentricity due to variation in wall thickness and P is the maximum compressive load (crinkling).

[2] Short compressive specimens crinkled near end.

It will be seen that for tubes with eccentricity of loading the values for the ultimate stress in compression are raised and approximate very closely the yield point of the material in tension. Although the "secant" column formula is not exact above the proportional limit, it is believed that for the material used, in which the proportional limit and yield point are nearly the same, the ultimate compressive stress value S_c in the table is probably the yield point in compression and represents very closely the stress to cause failure in compression for any strut cut from these sections.

The effect of eccentricity is also indicated in the type of failure for the short columns. In specimens Nos. 17, 4, and 16, where the eccentricities were zero, the specimens failed by crinkling near the ends. In the other short columns the crinkling occurred on the thin side at the center where the maximum compressive stress occurs from bending due to the eccentricities. Figure 2 shows a few of the typical failures.

2. BEAM TESTS.

To obtain conditions where the bending stress is a maximum with no column load, transverse tests were made on three specimens of each gauge thickness. The tubing was tested as a simple

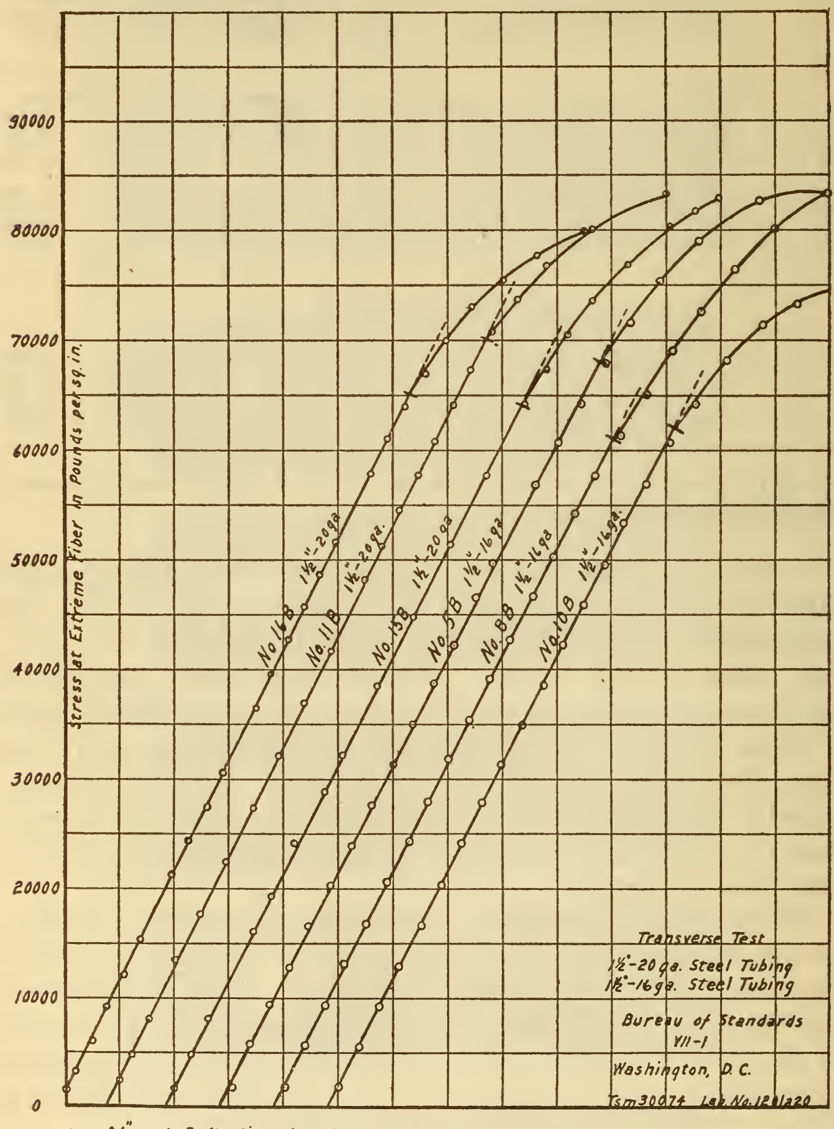

FIG. 3.—*Stress-deflection curves, transverse test.*

beam of 36-inch span. The load was applied at the center of the span by means of a wooden block one-half inch thick cut to fit the tubing. The results of the beam tests are given in Table 3, and the stress deflection curves are shown plotted in Figure 3.

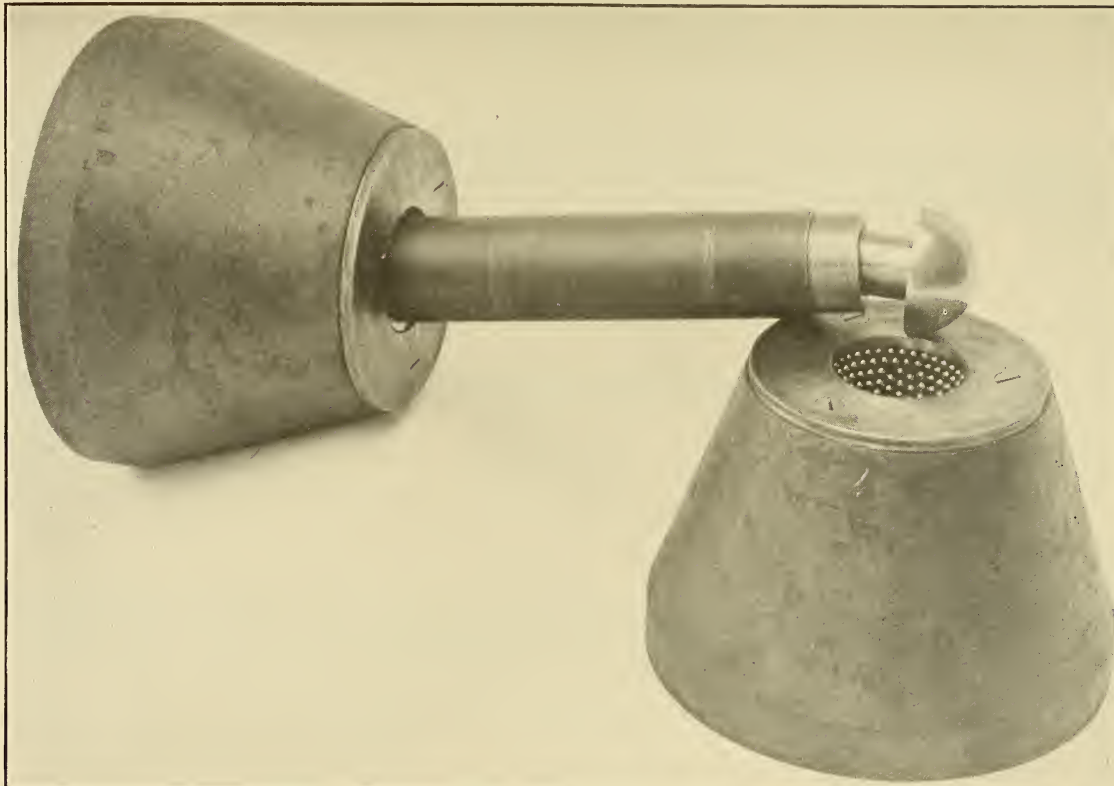

FIG. 4.—*Ball-bearing end supports used for the strut tests.*

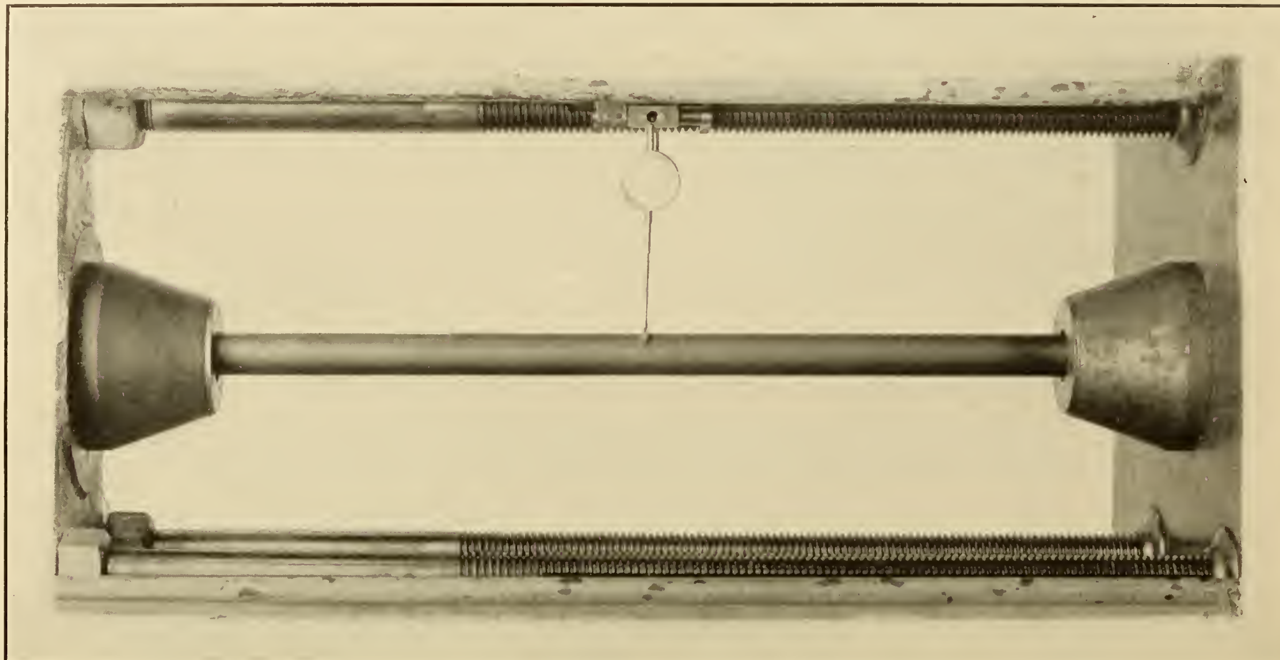

FIG. 5.—*Test of one of the struts, showing method of measuring deflection.*

TABLE 3.—Beam Test (Span 36 Inches).

1½ INCHES, 20 GAUGE.

Specimen number.	Variation in thickness of wall.	Area.	Moment of inertia. I	Sectional modulus. $\frac{I}{c}$	Proportional limit.	Modulus of rupture.	Modulus of elasticity.
	Inches.	Sq. inch.	In.[4]	In.[3]	Lbs./in.[2]	Lbs./in.[2]	Lbs./in.[2]
15-B	0.032-0.035	0.1566	0.0421	0.0562	64,000	88,080	28,400,000
11-B	.033- .035	.1566	.0421	.0562	70,000	85,670	29,020,000
16-B	.035- .037	.1656	.0444	.0592	65,000	82,080	29,140,000
1½ INCHES, 16 GAUGE.							
8-B	0.061-0.063	0.2803	0.0725	0.0966	61,000	92,600	28,080,000
5-B	.061- .065	.2846	.0735	.0980	68,000	91,800	27,500,000
10-B	.062- .064	.2846	.0735	.0980	62,000	83,560	28,080,000

3. COLUMN TESTS.

Some of the tubes were tested as round end columns without transverse loading. Free end conditions were insured by special ball-bearing end supports. Figure 4 shows the ball-bearing end supports used for all the compression tests. The small balls minimized the frictional resistance developed during the loading of the struts and permitted the struts to deflect freely in any direction, thus approaching ideal "free end" conditions.

4. COMBINED COLUMN AND TRANSVERSE TESTS.

The tube was mounted as in the column test for the combined beam and column tests. These tests were made in a horizontal position in an Emery hydraulic testing machine. Extreme care and precaution was exercised in the application of the side load. A small initial end load was first applied to the strut. The transverse load was then applied. For uniform transverse loads of 1, 5, 10, and 20 pounds per linear inch weights of 1, 5, 10, and 20 pounds, respectively, were suspended 1 inch apart throughout the length of the struts. For loads of 1.25, 3, and 6.05 pounds per linear inch weights were suspended in bags attached to the tubing. Thus, a very uniform distribution of lateral loading was obtained in all the tests.

Measurements of the original straightness of the tubes and of the vertical deflection under load at the mid length of the test piece were taken for all the tests by means of a micrometer dial reading directly to 0.001 inch. Figure 5 shows the test of one of the struts and the method of measurements.

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III. RESULTS OF STRUT TESTS.

The number assigned to the struts tested in this investigation, together with the length, L/r ratio, kind of test, the transverse load, and properties, are given in Table 4. The dimensions of each strut tested were determined from a large number of micrometer measurements of wall thickness made at each end. Measurements of the end sections showed that while the wall thickness was quite variable (see Table 4) it was practically the same for corresponding points at the two ends of a tube, so that the end measurements fairly represent the wall thickness throughout the length of a strut.

The ultimate loads of the columns and of the struts with transverse loading are given in Table 5. The table also includes the effect of the transverse loading; that is, the bending stress, the computed and measured deflection at the center of the strut produced by the transverse load alone.

The stress-deflection curves for the struts are shown in Figures 6 to 9, inclusive. The effect of increasing the transverse loading in decreasing the column strength and increasing the rate of deflection are shown in these curves.

TABLE 4.—Outline of Tests and Properties of Struts.

1½ INCHES, 20 GAUGE.

Strut number.[1]	Length.	$\frac{L}{r}$ ratio.	Kind of test.	Transverse load. w	Maximum and minimum wall thickness (average of both ends).	Area. A	Moment of inertia. I	Distance to compressive extreme fiber. c
	Inches.			Lbs. per linear inch.	Inch.	Square inch.	$In.^4$	Inch.
11-70-C	36.3	70	Column		0.035 -0.0335	0.1577	0.0424	0.743
17-70-C	36.3	70	do		.0365- .0355	.1656	.0444	.745
13-70-1	36.2	70	Combine	1.25	.034 - .036	.1588	.0427	.75
9-70-1	36.2	70	do	1.25	.034 - .035	.1597	.0429	.755
13-70-5	36.3	70	do	5.0	.033 - .037	.1611	.0432	.770
15-70-5	36.3	70	do	5.0	.0325- .035	.1552	.0418	.764
13-70-10	36.2	70	do	10.0	.033 - .036	.1579	.0424	.763
15-70-10	36.2	70	do	10.0	.034 - .0337	.1556	.0419	.749
1-110-C	57	110	Column		.034 - .038	.1656	.0444	.75
7-110-C	57	110	do		.032 - .036	.1566	.0421	.75
4-110-1	57	110	Combine	1.0	.035 - .036	.1633	.0438	.755
17-110-1	57	110	do	1.0	.036 - .035	.1633	.0438	.745
9-110-3	57	110	do	3.0	.0355- .036	.1647	.0442	.752
16-110-3	57	110	do	3.0	.035 - .0375	.1667	.0447	.763
1-110-6	57	110	do	6.05	.0393- .0337	.1678	.0450	.723
11-104-6	53.8	104	do	6.05	.0345	.1588	.0427	.75
15-B	36		Bend					
11-B	36		do					
16-B	36		do					

[1] First number denotes the section of tubing from which the strut was cut, second number denotes the L/r ratio, and the third number the transverse load.

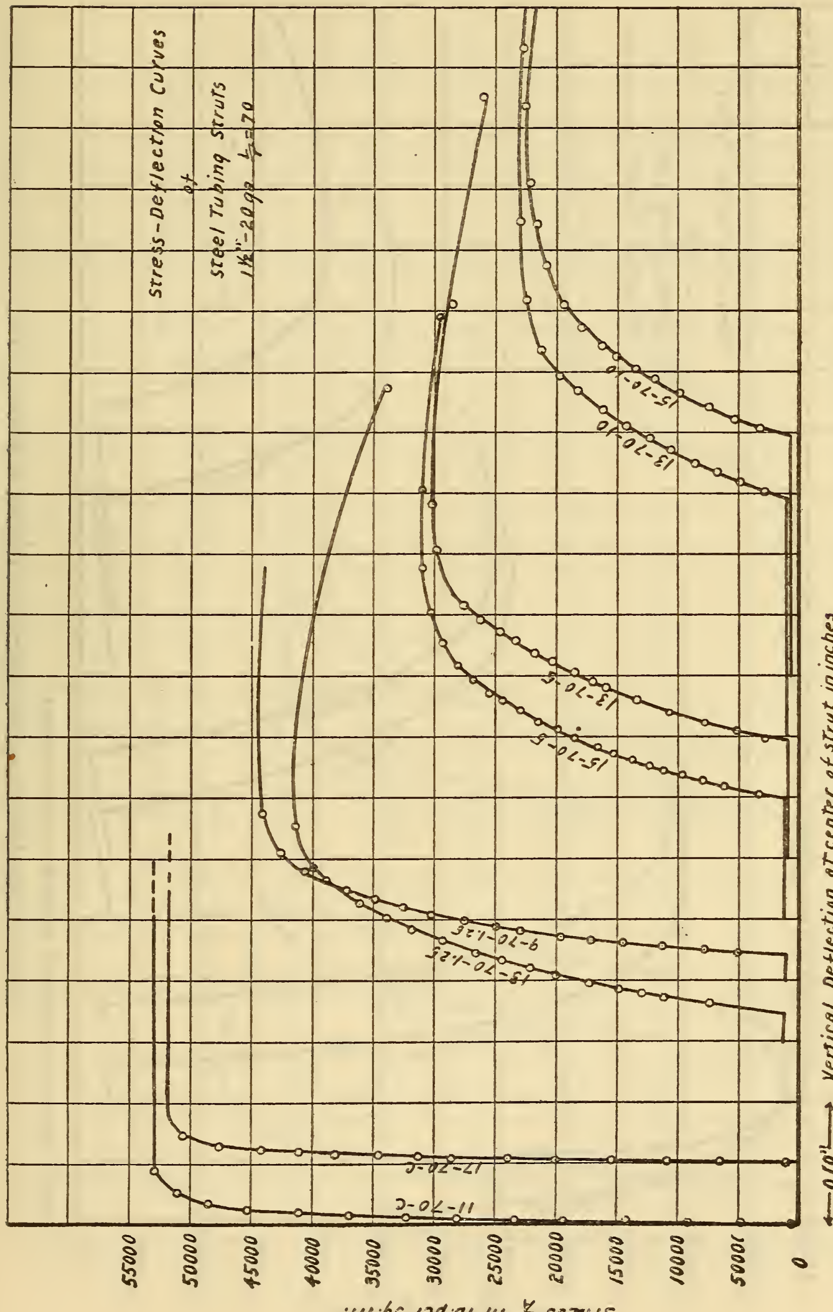

FIG. 6.—*Stress-deflection curves for struts.* 1½ *inches, 20 gauge L/r 70.*

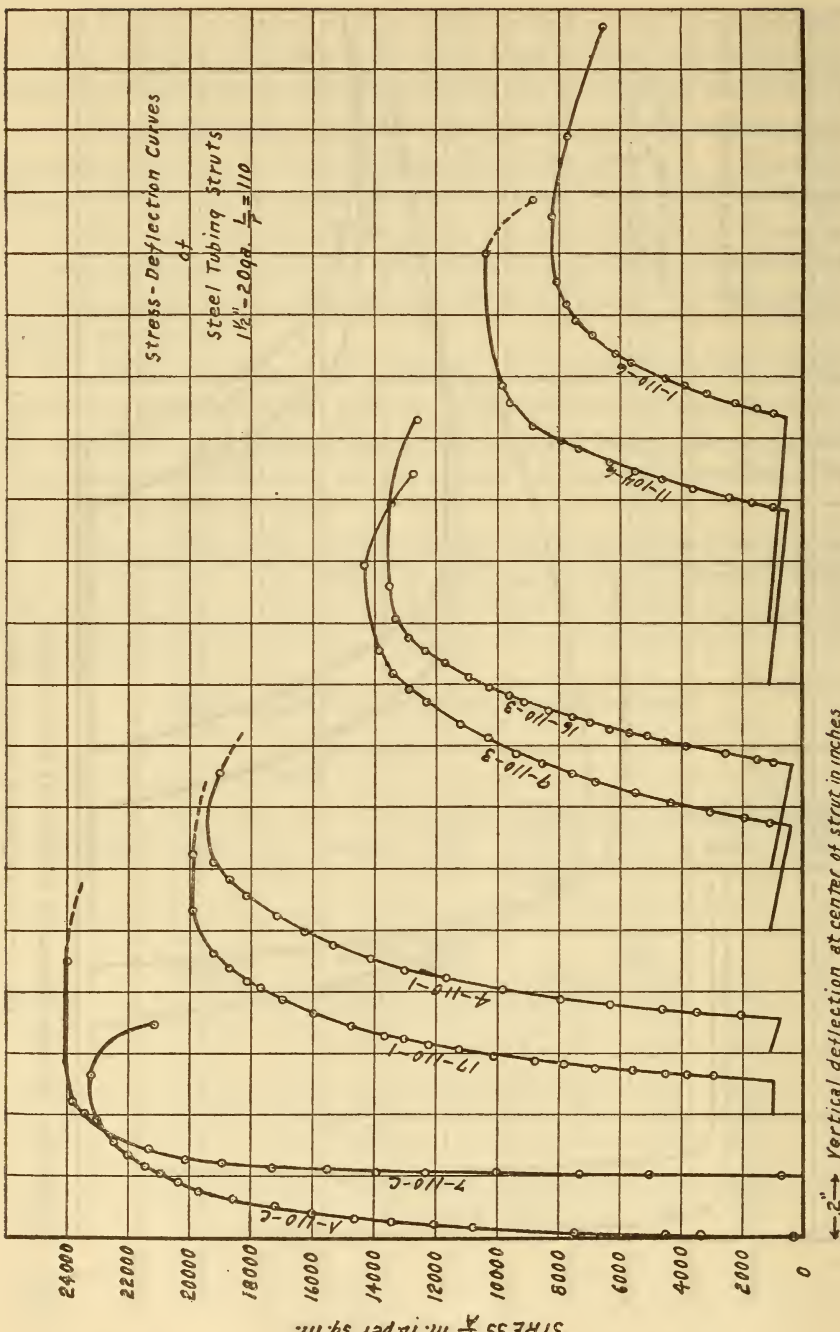

FIG. 7.—*Stress-deflection curves for struts.* 1½ *inches, 20 gauge L/r* 110.

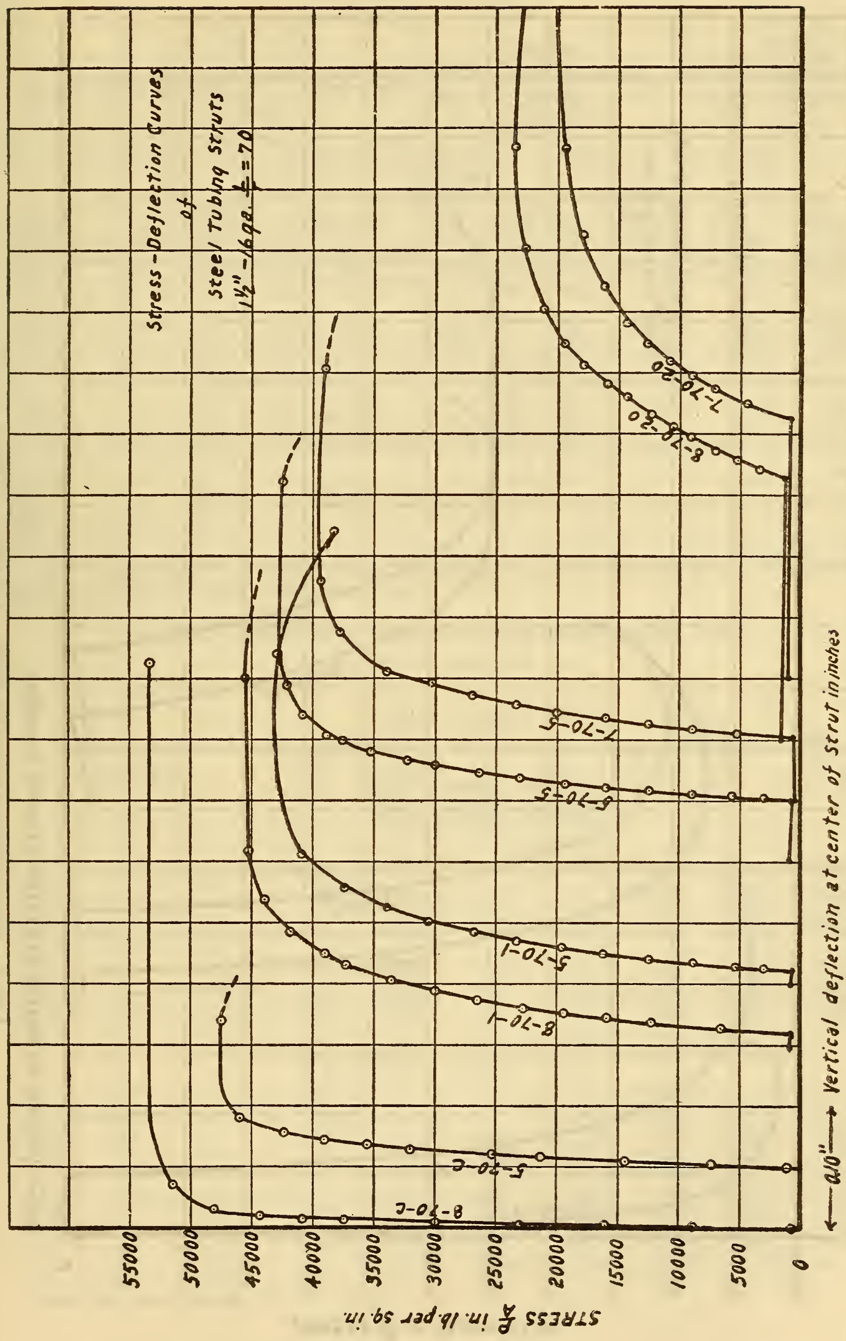

FIG. 8.—*Stress-deflection curves for struts. 1½ inches, 16 gauge L/r 70.*

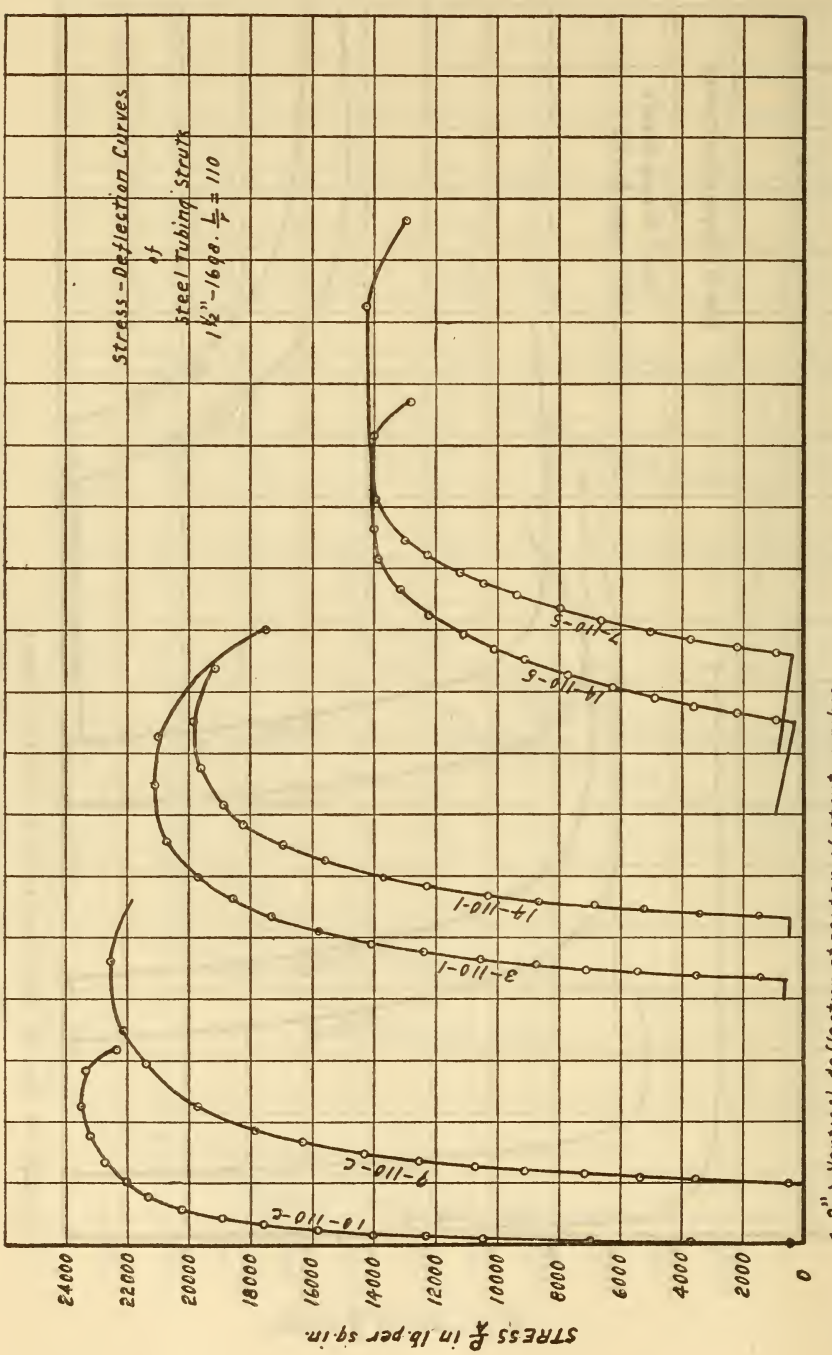

FIG. 9.—Stress-deflection curves for struts. 1½ inches, 16 gauge L/r 110.

TABLE 4.—Outline of Tests and Properties of Struts—Continued.

1½ INCHES, 16 GAUGE.

Strut number.[1]	Length.	$\frac{L}{r}$ ratio.	Kind of test.	Transverse load. w	Maximum and minimum wall thickness (average of both ends).	Area. A	Moment of inertia. I	Distance to compressive extreme fiber. c
	Inches.			Lbs. per linear inch.	Inch.	Square inch.	In.[4]	Inch.
8-70-C	35.7	70	Column		0.0635-0.0615	0.2822	0.073	0.744
5-70-C	35.7	70	do		.0655- .0605	.2832	.0735	.739
5-70-1	35.7	70	Combine	1.0	.060 - .065	.2822	.0730	.763
8-70-1	35.7	70	do	1.0	.0623- .063	.2844	.0735	.755
5-70-5	35.7	70	do	5.0	.066 - .060	.2844	.0735	.734
7-70-5	35.7	70	do	5.0	.064 - .060	.2801	.0725	.740
8-70-20	35.7	70	do	20.0	.063	.2846	.0735	.74
7-70-20	35.7	70	do	20.0	.059 - .065	.2803	.0725	.766
10-110-C	56.2	110	Column		.062 - .065	.287	.074	.75
9-110-C	56.1	110	do		.058 - .064	.2803	.0725	.75
3-110-1	56.1	110	Combine	1.0	.0655- .061	.2855	.0738	.738
14-110-1	56.1	110	do	1.0	.0615- .0685	.2920	.0754	.767
14-110-5	56.1	110	do	5.0	.062 - .056	.2887	.0745	.762
7-110-5	56.1	110	do	5.0	.063 - .060	.2780	.0720	.742
5-B	36		Bend					
8-B	36		do					
10-B	36		do					

[1] First number denotes the section of tubing from which the strut was cut, second number denotes the L/r ratio, and the third number the transverse load.

TABLE 5.—Results of Column and Combined Loading Tests.

1½ INCHES, 20 GAUGE $\frac{L}{r}=70$.

Strut number.	Transverse load.	Effect of transverse load. Bending, stress at extreme fiber.[1]	Computed deflection at center.[2]	Measured deflection at center.	Ultimate end load. P	Compressive stress. $\frac{P}{A}$	Euler load. $P_E=\frac{\pi^2 EI}{L^2}$
	Pounds.	Lbs./in.[2]	Inch.	Inch.	Pounds.	Lbs./in.[2]	Pounds.
11-70-C	Col.				8,340	52,900	9,230
17-70-C	Col.				8,600	51,900	9,665
13-70-1	1.25	3,600	0.023	0.023	6,540	41,200	9,300
9-70-1	1.25	3,600	.0215	.020	7,100	44,500	9,370
13-70-5	5.0	14,600	.090	.097	4,850	30,100	9,435
15-70-5	5.0	14,950	.093	.099	4,800	30,900	9,130
13-70-10	10.0	29,400	.183	.192	3,620	22,900	9,261
15-70-10	10.0	29,200	.196	.198	3,500	22,500	9,151
1½ INCHES, 20 GAUGE $\frac{L}{r}=110$.							
1-110-C	Col.				3,840	23,200	3,910
7-110-C	Col.				3,770	24,000	3,710
4-110-1	1.0	7,000	0.1085	0.113	3,170	19,400	3,858
17-110-1	1.0	6,900	.108	.111	3,250	19,900	3,858
9-110-3	3.0	20,700	.322	.340	2,360	14,300	3,894
16-110-3	3.0	20,800	.318	.340	2,260	13,550	3,938
1-110-6	6.05	39,500	.638	.666	1,380	8,200	3,964
11-104-6	6.05	38,100	.534	.566	1,640	10,300	4,222

[1] Bending stress at extreme fiber $S_B=\frac{1}{8}\frac{wL^2}{\frac{I}{c}}$.

[2] Deflection at center of strut $l_B=\frac{5}{384}\frac{wL^4}{EI}$.

TABLE 5.—Results of Column and Combined Loading Tests—Continued.

1½ INCHES, 16 GAUGE $\frac{L}{r}=70$.

Strut number.	Transverse load.	Effect of transverse load.			Ultimate end load. P	Compressive stress. $\frac{P}{A}$	Euler load. $P_E=\frac{\pi^2 EI}{L^2}$
		Bending, stress at extreme fiber.[1]	Computed deflection at center.[2]	Measured deflection at center.			
	Pounds.	Lbs./in.	Inch.	Inch.	Pounds.	Lbs./in.	Pounds.
8-70-C	Col.				14,900	52,800	16,390
5-70-C	Col.				13,500	47,650	16,510
5-70-1	1.0	1,660	0.010	0.010	12,100	42,900	16,390
8-70-1	1.0	1,630	.010	.010	12,950	45,500	16,510
5-70-5	5.0	7,900	.049	.051	12,100	42,500	16,510
7-70-5	5.0	8,100	.050	.053	11,000	39,300	16,280
8-70-20	20.0	32,500	.199	.214	6,620	23,250	16,510
7-70-20	20.0	33,700	.201	.213	5,540	19,780	16,280

1½ INCHES, 16 GAUGE $\frac{L}{r}=110$.

Strut number.	Transverse load.	Bending stress	Computed deflection	Measured deflection	Ultimate end load.	Compressive stress.	Euler load.
10-110-C	Col.				6,740	23,500	6,730
9-110-C	Col.				6,300	22,500	6,593
3-110-1	1.0	3,930	0.059	0.062	6,020	21,100	6,715
14-110-1	1.0	4,000	.059	.062	5,800	19,870	6,855
14-110-5	5.0	10,100	.299	.302	4,050	14,050	6,775
7-110-5	5.0	20,300	.309	.318	3,960	14,250	6,548

[1] Bending stress at extreme fiber $S_B=\frac{1}{8}\frac{wL^2}{\frac{I}{c}}$.

[2] Deflection at center of strut $l_B=\frac{5}{384}\frac{wl^4}{EI}$.

IV. THEORY AND APPLICATION OF FORMULAS.

1. APPLICATION OF FORMULAS FOR COMBINED LOADING.

The theory and generally applied formulas for determining the strength of struts subjected to combined axial and transverse loading are given by the following equations. The ultimate strength of the strut is reached when the maximum compressive stress at the extreme fiber

$$f_c=\frac{M_o}{\frac{I}{c}}+\frac{P}{A} \text{ approximates the yield point of the material} \quad (1)$$

where

f_c = maximum compressive stress at the extreme fiber,
M_o = maximum bending moment at the center of the strut,
I = moment of inertia of the strut cross section,
c = distance of extreme fiber in compression from the neutral axis,
P = ultimate end load,
A = area of cross section of the strut.

In the above equation $\frac{M_o}{\frac{I}{c}}$ is the maximum bending stress in compression S_B, and $\frac{P}{A}$ is the mean compressive stress on the section. The sum of these stresses is the maximum compressive stress acting on the strut, provided the stress remains proportional to the strain at these high stresses. It is to be expected that when this maximum intensity of compressive stress is about equal to the yield point of the material the strut will fail.

The value of M_o has been commonly computed by the rational formula [1]

$$-M_o = \frac{wEI}{P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}} - 1\right) \qquad (2)$$

or by Perry's approximate formula [1]

$$-M_o = \frac{1wL^2}{8}\left(\frac{P_E}{P_E - P}\right) \qquad (3)$$

where

w = uniform transverse load in pounds per linear inch,
L = length of strut,
E = modulus of elasticity,
P = ultimate load,
$P_E = \frac{\pi^2 EI}{L^2}$, Euler's limiting value for ideal column.

If different values of ratios P/P_E be substituted in the formulas (2) and (3) above, the results show that Perry's approximate formula agrees very closely with the exact formula for ratios of P/P_E up to 0.75.

The results obtained by application of the experimental data to the above formulas are given in Table 6. The table shows that the exact formula for computing the bending moment and Perry's approximate formula give practically identical results.

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[1] "Morley's Strength of Materials, 1916," p. 282. In this edition Perry's formula is incorrectly written. It should read, $-M_o = 1/8wL^2\left(\frac{P_E}{P_E - P}\right)$

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TABLE 6.—Results by Commonly Used Formulas for Combined Loading.

Strut number.	Transverse load.	Exact formula. Maximum— Bending stress.[1] S_B	Exact formula. Maximum— Compressive stress.[2] f_c	Perry's formula. Maximum— Bending stress.[3] S_B	Perry's formula. Maximum— Compressive stress.[2] f_c	Ratio. $\frac{S_B}{f_c}$	Yield point of material (tension).
1½ INCHES, 20 GAUGE $\frac{L}{r}$=70.							
	Pounds.	Lbs./in.²	Lbs./in.²	Lbs./in.²	Lbs./in.²		Lbs./in.²
13-70-1	1.25	12,350	53,550	12,100	53,300	0.227	68,900
9-70-1	1.25	15,100	59,600	14,900	59,400	.25	66,800
13-70-5	5.0	30,450	60,550	30,050	60,150	.50	68,900
15-70-5	5.0	32,000	62,900	31,500	62,400	.505	69,900
13-70-10	10.0	48,700	71,600	48,300	71,200	.68	68,900
15-70-10	10.0	48,300	70,800	47,400	69,900	.68	69,900
1½ INCHES, 20 GAUGE $\frac{L}{r}$=110.							
4-110-1	1.0	39,600	59,100	39,300	58,700	0.67	63,400
17-110-1	1.0	44,870	64,770	43,800	63,700	.68	67,400
9-110-3	3.0	53,300	67,600	52,700	67,000	.78	66,800
16-110-3	3.0	49,570	63,000	48,800	62,350	.78	62,200
1-110-6	6.05	61,160	69,360	60,600	68,800	.88	61,200
11-104-6	6.05	63,200	73,500	62,900	73,200	.86	69,000
1½ INCHES, 16 GAUGE $\frac{L}{r}$=70.							
5-70-1	1.0	6,500	49,400	6,400	49,300	0.13	70,000
8-70-1	1.0	7,740	53,200	7,600	53,100	.14	70,200
5-70-5	5.0	30,300	72,800	29,500	72,000	.41	70,000
7-70-5	5.0	25,600	64,900	25,030	64,350	.39	67,500
8-70-20	20.0	54,400	77 650	54,200	77,450	.70	70,200
7-70-20	20.0	51,900	71,670	51,000	70,800	.72	67,500
1½ INCHES, 16 GAUGE $\frac{L}{r}$=110.							
3-110-1	1.0	38,300	59,400	38,000	59,100	0.64	71,300
14-110-1	1.0	26,550	46,420	25,950	45,800	.57	64,600
14-110-5	5.0	50,940	65,000	50,100	64,150	.78	64,600
7-110-5	5.0	52,290	66,540	51,300	65,600	.78	67,500

[1] $S_B=\frac{M_o c}{I}=\frac{wEc}{P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}-1\right)$.

[2] $f_c=\frac{M_o}{\frac{I}{c}}+\frac{P}{A}$.

[3] $S_B=\frac{M_o c}{I}=\frac{c}{I}\frac{wL^2}{8}\left(\frac{P_E}{P_E-P}\right)$.

A comparison of the maximum compressive stress values at failure, obtained by the commonly used formulas for struts with transverse loading with the yield point of the material, shows extreme inconsistency and wide variation. This variation is shown graphically in the left half of Figure 10. In this figure the maximum compressive stresses at failure computed by either of the commonly used formulas are plotted as ordinates. The plotted values represent either of the formulas shown on the figure, the difference between results obtained with each being too

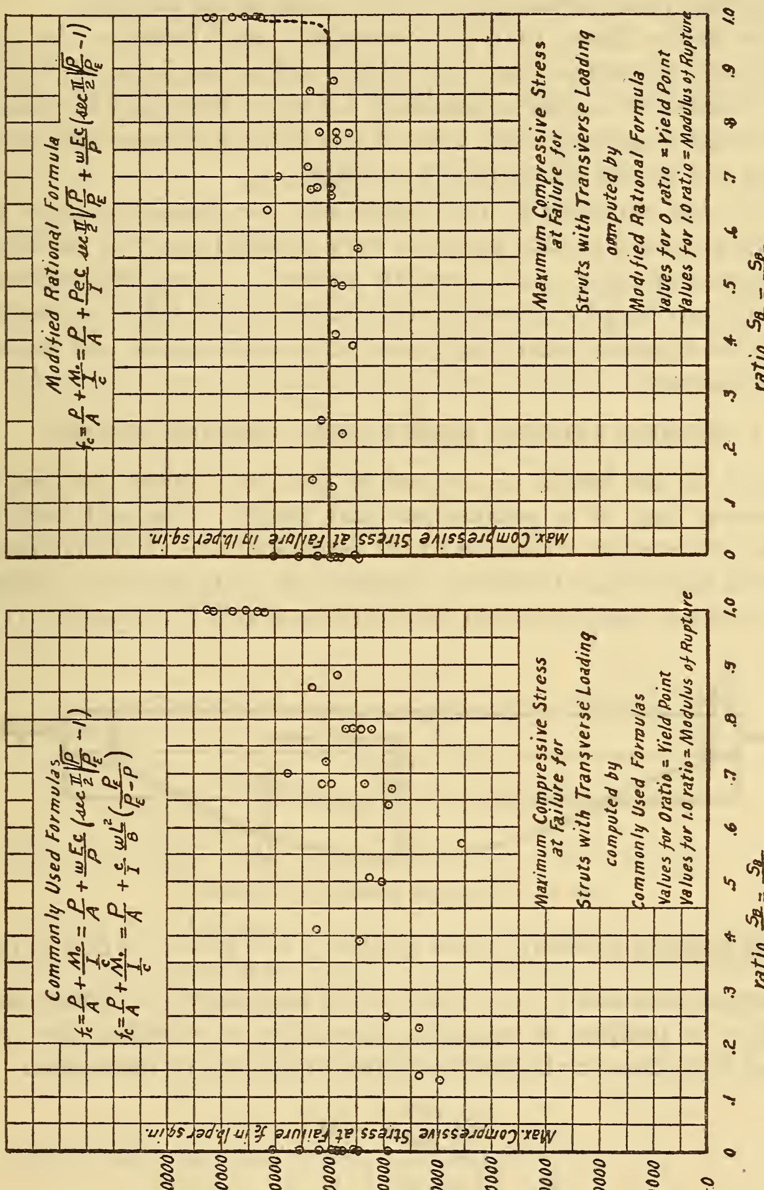

FIG. 10.—*Results obtained by the application of the formulas for combined column and transverse loading.*

small to need separate designation. The ratios $\frac{S_b}{f_c}$ of bending stress to maximum compressive stress are plotted as abscissas, the ratio unity being the condition of pure transverse loading (modulus of rupture) and the axis of coordinate values 0.0 representing pure compression on short lengths. The maximum compressive stress (ordinate values) for the latter are practically the yield-point values obtained by tensile tests of the material.

This figure shows that the commonly used formulas do not accurately determine the load which will cause failure. For a large number of the tests, especially for short struts and for struts with small transverse load, these formulas give dangerously high values and should not be used for design.

The wide variation of the results and the possible danger in applying these formulas lead to a very detailed study of the conditions contributing to the strength of struts. A modified rational formula was found that will more accurately and safely represent stress conditions of struts subjected to combined axial and transverse loading.

2. MODIFIED RATIONAL FORMULA FOR COMBINED LOADING.

Let L be the length of a round or free end column carrying a transverse load of w pounds per unit length. The end load P has an eccentricity e relative to the centroidal axis of the column. Assume the origin O midway between the ends, the line joining the centroids being taken as the coordinate axis X. (See fig. 11.)

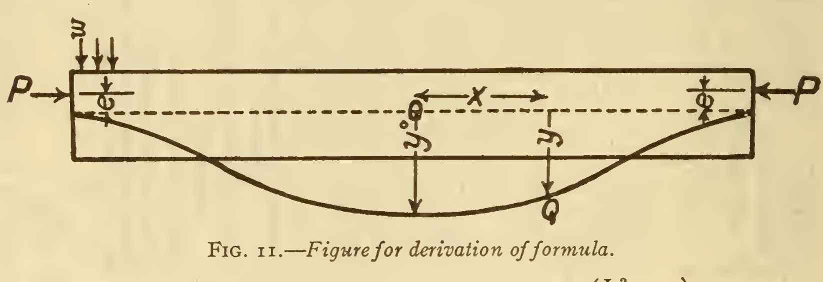

FIG. 11.—*Figure for derivation of formula.*

The bending moment at any section Q is $\frac{w}{2}\left(\frac{L^2}{4}-x^2\right)$ due to the transverse load and $P\,(y+e)$ due to the end thrust P. The eccentricity e is positive or negative, depending on whether the end thrust P is above or below the center of gravity of the section.

$$EI\,\frac{d^2y}{dx^2}=-M$$

$$EI\,\frac{d^2y}{dx^2}=-\frac{w}{2}\left(\frac{L^2}{4}-x^2\right)-P\,(y+e) \qquad (1)$$

$$\frac{d^2y}{dx^2}+\frac{Py}{EI}=+\frac{wx^2}{2\,EI}-\left(\frac{wL^2}{8EI}+\frac{Pe}{EI}\right)$$

The solution of this differential equation is

$$y=\frac{wx^2}{2P}-\frac{wL^2}{8P}-\frac{wEI}{P^2}-e+A\cos\sqrt{\frac{P}{EI}}\,x+B\sin\sqrt{\frac{P}{EI}}\,x.$$

The condition $\frac{dy}{dx}=O$ for $x=O$ and $y=O$ for $x=\frac{L}{2}$ gives $B=O$

$$A=\left(\frac{wEI}{P^2}+e\right)\sec\frac{L}{2}\sqrt{\frac{P}{EI}}$$

hence,

$$y=\frac{wx^2}{2P}-\frac{wL^2}{8P}-\frac{wEI}{P^2}-e+\left[\left(\frac{wEI}{P^2}+e\right)\sec\frac{L}{2}\sqrt{\frac{P}{EI}}\cos\sqrt{\frac{P}{EI}}\,x\right]$$

and at the origin or point of maximum moment where $x=O$

$$y_0=-\frac{wL^2}{8P}-\left(\frac{WEI}{P^2}+e\right)\left(1-\sec\frac{L}{2}\sqrt{\frac{P}{EI}}\right). \qquad (2)$$

The maximum moment at O is

$$-M_0=P\,(y_0+e)+\frac{1}{8}\,wL^2 \qquad (3)$$

$$-M_0=Pe-\left[\frac{wEI}{P}+Pe\right]\left[1-\sec\frac{L}{2}\sqrt{\frac{P}{EI}}\right]$$

$$-M_0=Pe\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}+\frac{wEI}{P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}-1\right) \qquad (4)^{2}$$

where $P_E=\frac{\pi^2EI}{L^2}$, Euler's limiting value for ideal column.

Therefore, to cause failure of a strut subjected to combined column and transverse loading, the maximum compressive stress at the extreme fiber is

$$f_c=\frac{M_0}{\frac{I}{c}}+\frac{P}{A}$$

approximates the yield point of the material, or

$$f_c=\frac{P}{A}+\frac{Pec}{I}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}+\frac{wEc}{P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}-1\right) \qquad (5)$$

[2] See Church's Mechanics of Engineering, p. 382; 1908.

An examination of formula (4) for the bending moment shows that the first term takes account of the bending moment due to the eccentricity of the end load, and that the second term is the expression for a column with a transverse load combined with an axial end load. The formula also explains why the two commonly used formulas are undesirable for short struts and struts with small side loads. For short struts the end load P is very large, so that for a given eccentricity the factor $Pe \sec \frac{\pi}{2}\sqrt{\frac{P}{P_E}}$ becomes a very large and important factor in computing the bending moment and consequently the bending stress. Also, when w is small this factor is relatively large and important. On the other hand, when the struts are long P is very small, making the factor Pe relatively less important. Likewise, when the transverse load w is large, the second factor of the formula is large, and the eccentric factor has a relatively less influence on the strength of a strut.

3. ECCENTRICITIES.

(a) CAUSES OF ECCENTRICITY.

The modified rational formula indicates the importance of eccentricities. Such eccentricities are chiefly caused by the following conditions found in commercial tubing: (1) Deviation of the shape of the tubing from a circular section, (2) variation in wall thickness, and (3) deviation from straightness.

Measurements made of the external diameters of tubing used in the investigation indicated that the first condition—deviation from circular shape—is comparatively small, and this cause, consequently, was not considered. The other two conditions—variation in wall thickness and deviation from straightness—are perceptible to the eye, and the eccentricities resulting from these conditions proved to be important factors in determining the strength of struts. Figure 12 shows the variation in wall thickness of three of the struts tested.

(b) DETERMINATION OF ECCENTRICITY.

1. Eccentricity Due to Variation in Wall Thickness.—Consider a cross section of the tube in which the wall thickness varies from an average minimum thickness of t_{min} to an average maximum thickness of t_{max}. (See fig. 13.)

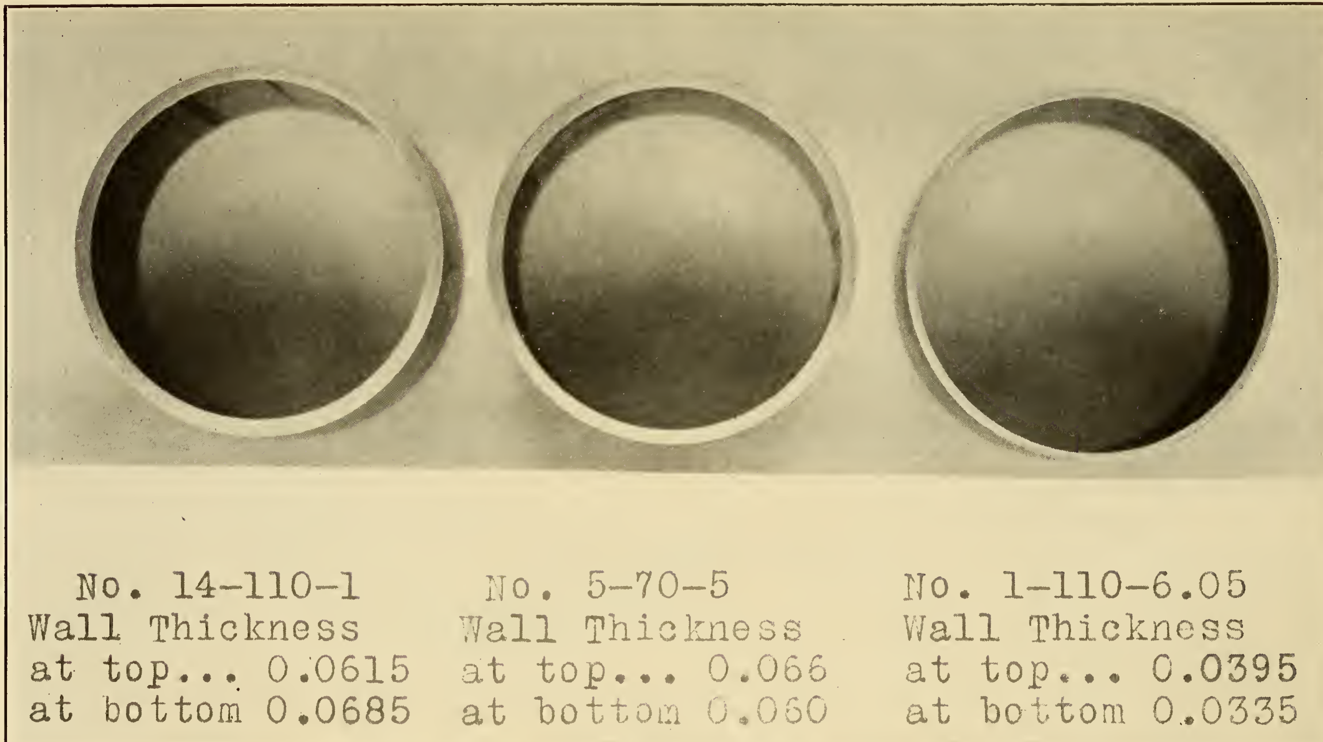

FIG. 12.—*End view of three of the struts, showing variation in wall thickness.*

Let

D = outside diameter of tube,

k = distance from center of outer circle to center of inner circle,

y = distance from center of outer circle to center of gravity of the section,

$$k = \frac{t_{max} - t_{min}}{2}$$

$$y = \frac{A_1}{A_0 - A_1} k$$

where

$$A_0 = \frac{\pi D^2}{4} \text{ and } A_1 = \frac{\pi}{4} [D - (t_{max} + t_{min})]^2$$

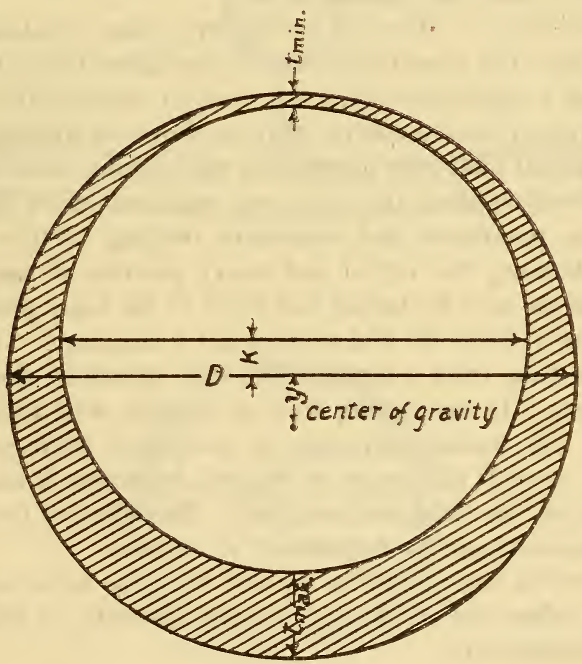

FIG. 13.—*Method of determining eccentricity.*

Section of tubing showing eccentricity due to variation in wall thickness.

In these tests the center of thrust was at the center of the inner circle, due to the fact that the lug on the hemispherical ball placed on the end of the strut filled the inside of the tubing.

Therefore the eccentricity e_w due to variation of wall thickness is

$$e_w = k + y = k\left(1 + \frac{A_1}{A_0 - A_1}\right) = \frac{kA_0}{A_0 - A_1} = \frac{kD^2}{2D(t_{max} + t_{min}) - (t_{max} + t_{min})^2}$$

Should the minimum thickness be at a point other than at the top or bottom of the tube, the eccentricity about the horizontal axis is

$$e_w = (k+y) \sin \alpha = \frac{kA_0}{A_0 - A_1} \sin \alpha$$

where α is the angle that the diameter connecting the points of minimum and maximum wall thickness makes with the horizontal axis. In this investigation practically all of the specimens had the minimum thickness either at the top or bottom when tested. In the few cases where this condition did not exist the value of $\sin \alpha$ was considered as unity and the average thickness at the top and bottom used for t_{min} and t_{max} in the formula to determine the eccentricity about the horizontal axis.

2. Eccentricity Due to Deviation from Straightness.—To determine the eccentricity due to deviation from straightness of the tube a micrometer dial was used to measure the deflection at the center of the column or strut, as shown in Figures 5 and 14. A small initial load was applied to the column, and before any side load was applied the tube was revolved in its ball-bearing ends. The maximum and minimum readings of the dial were noted, indicating the dotted and heavy position shown in Figure 14. *In all the tests the tubing was tested in the lower position illustrated—that is, where the dial reading was a minimum.* The deflection of the tube from a straight line was, therefore, downward in all the tests. If the tubing was of uniform wall thickness and diameter, the eccentricity due to deviation from straightness would be one-half the range of original deflection R indicated by the dial when the strut was revolved. The effect of the variation of wall thickness can be determined as follows:

Consider the cross-sectional area of the tube to be as shown in Figure 14 when the strut was concave upwards, as indicated by the micrometer dial.

Let

R = difference in maximum and minimum deflection, as indicated by micrometer dial,

e_x = distance from center of thrust to center of inner circle.

$$R = \frac{D}{2} + e_x + k - \left[\frac{D}{2} - k - e_x\right] = 2e_x + 2k$$

$\therefore e_x = \dfrac{R-2k}{2}$ the eccentricity due to deviation from straightness of the tube.

The total eccentricity e—that is, the distance from the center of thrust to center of gravity of the section—is the sum of the eccentricities due to variation in wall thickness and deviation from straightness, or

$$e = e_w + e_x = (k + y) + e_x = k\left(1 + \frac{A_1}{A_0 - A_1}\right) + \frac{R - 2k}{2}$$

Should the maximum thickness of the wall t_{max} be up when the tube was tested, k is negative and the eccentricity is

$$e = -(k + y) + e_x = -k\left(1 + \frac{A_1}{A_0 - A_1}\right) + \frac{R + 2k}{2}$$

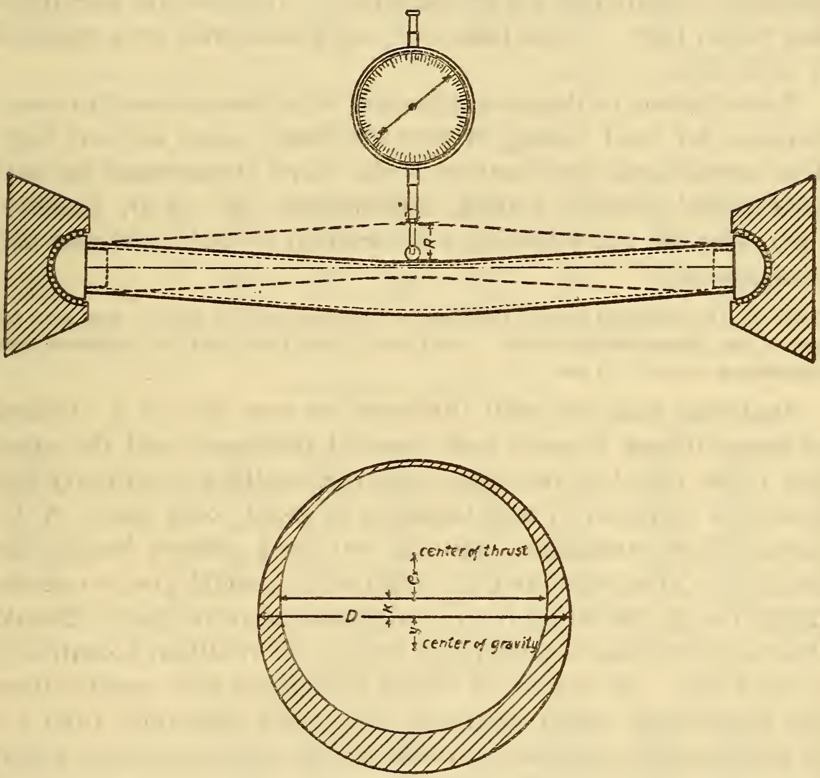

FIG. 14.—*Method of determining eccentricity due to deviation from straightness.*

(*c*) DISCUSSION.

Measurements of the eccentricities in the tubing showed that practically in every case the diameter connecting the minimum and maximum wall thickness was in the plane of deflection or warping of the tubing. Thus, the total effective eccentricity is either the sum or difference of the eccentricity due to variation in wall thickness and eccentricity due to variation in straightness. This is to be expected, because any condition during the drawing of the tube that would cause the wall to be thin on one side and thick on the other would tend to warp the tube in the plane of symmetry; that is, the plane of minimum and maximum wall thickness. Moreover, any heat treatment that would relieve the unequal stresses in a tube of varying wall thickness would tend to produce warping in the plane of symmetry.

In Tables 7 and 8 are given the eccentricities resulting from wall variation and deviation from straightness, together with the total effective eccentricities for all the struts. The effective eccentricities varied from −0.008 inch to +0.039 inch, with an average of +0.021 inch.

A comparison of these eccentricities with those allowed in specifications for steel tubing shows that these values are not high. The aeronautical specifications of the Navy Department for mild carbon-steel seamless tubing, Specification No. 58–B, January, 1920, specifies the following for variation in wall thickness and straightness:

Par. 8. The variation in wall thickness of the tubes may be plus or minus 10 per cent of the dimensions specified. In no part of any tube shall the departure from straightness exceed 1 in 600.

Assuming that the wall thickness on one side of a 1½-inch 16-gauge tubing is 0.065 inch (normal thickness) and the other side 10 per cent less, or 0.0585 inch, the resulting eccentricity due from this variation in wall thickness is about 0.018 inch. A departure from straightness of 1 in 600 for a 36-inch length, the length of a strut with and L/r ratio of 70, would give an eccentricity due to deviation from straightness of 0.060 inch. Should these eccentricities be additive in a strut, the resultant eccentricity is 0.078 inch. In any lot of tubing complying with specifications the eccentricity would probably vary fairly uniformly from 0.0 to the extreme case above, so that the average eccentricity would be approximately 0.039 inch. This value is about twice the general average of 0.021 inch for the struts tested in this investigation.

TABLE 7.—Eccentricities.

1½ INCHES, 20 GAUGE $\frac{L}{r}=70$.

Strut number.	Average wall thickness.		k[1]	y[2]	Eccentricity from wall variation. $e_w=k+y$	Dial deflection. R	Eccentricity due to crookedness. e_x[3]	Total eccentricity. $e=e_w+e_x$
	Top side.	Bottom side.						
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
11-70-C	0.035	0.0335	−0.0007	−0.007	−0.0077	0.025	0.0132	0.0055
17-70-C	.0365	.0355	−.0006	−.006	−.0066	.029	.015	.008
13-70-1	.0347	.0347	.000	.000	.000	.070	.035	.035
9-70-1	.034	.0347	+.0004	+.0041	+.0045	.031	.015	.019
13-70-5	.033	.037	+.0020	+.020	+.022	.033	.0145	.036
15-70-5	.0325	.035	+.0013	+.0135	+.0148	.030	.0137	.0285
13-70-10	.033	.0355	+.0012	+.012	+.0132	.010	.0035	.016
15-70-10	.0339	.0337	−.0001	−.001	−.0011	.026	.013	.012
Average					+.0050		.0154	.0200

1½ INCHES, 20 GAUGE $\frac{L}{r}=110$.

Strut number.	Top side.	Bottom side.	k	y	e_w	R	e_x	e
4-110-1	0.035	0.036	+0.0005	+0.005	+0.0055	0.044	0.0215	0.027
17-110-1	.036	.035	−.0005	−.005	−.0055	.031	.016	.011
9-110-3	.0355	.036	+.0002	+.002	+.0022	.068	.034	.036
16-110-3	.035	.0375	+.0013	+.0125	+.0138	.034	.0157	.0295
1-110-6	.0393	.0337	−.0028	−.0267	−.0295	.050	.0275	−.002
11-104-6	.0345	.0345	.000	.000	.000	.017	.008	.008
Average					−.0022		.0205	.0182

1½ INCHES, 16 GAUGE $\frac{L}{r}=70$.

Strut number.	Top side.	Bottom side.	k	y	e_w	R	e_x	e
8-70-C	0.0635	0.0615	−0.001	−0.0052	−0.0062	0.028	0.015	0.009
5-70-C	.065	.0605	−.0022	−.011	−.013	.054	.0295	.017
5-70-1	.060	.065	+.0025	+.0131	+.0156	.045	.020	.0356
8-70-1	.0623	.0630	+.0003	+.0015	+.0018	.052	.0255	.027
5-70-5	.066	.060	−.003	−.0156	−.0186	.015	.0105	−.008
5-70-5	.064	.060	−.002	−.0106	−.0126	.022	.013	.0004
8-70-20	.063	.063	.000	.000	.000	.025	.0125	.0125
7-70-20	.059	.065	+.003	+.0150	+.0180	.023	.0085	.026
Average					−.0019		.0168	.0149

1½ INCHES, 16 GAUGE $\frac{L}{r}=110$.

Strut number.	Top side.	Bottom side.	k	y	e_w	R	e_x	e
3-110-1	0.0655	0.0610	−0.0023	−0.0121	−0.0145	0.088	0.046	0.031
14-110-1	.0615	.0685	+.0036	+.0182	+.022	.040	.0166	.039
14-110-5	.062	.066	+.002	+.0105	+.0125	.043	.0195	.036
7-110-5	.063	.060	−.0015	−.008	−.0095	.048	.0255	.016
Average					+.0026		.0269	.0305

[1] $k=\frac{t_{max}-t_{min}}{2}$.

[2] $y=\frac{A_1}{A_0-A_1}k$.

[3] $e_x=\frac{R-2k}{2}$.

(d) ACCURACY OF METHOD OF DETERMINING ECCENTRICITY.

From the elastic theory one may express the relation between the lateral deflection at the middle of a column y_m and the initial eccentricity e for round end columns, as follows:

$$y_m = e\left(\sec\sqrt{\frac{P}{EI}}\frac{L}{2} - 1\right)$$

It can be shown that when the load P is 4/9 P_E—that is, 4/9 of Euler's maximum load—the deflection at the center of the column is equal to the initial eccentricity, provided P does not stress the material beyond the proportional limit. In Table 8 are given the eccentricities and the deflection at a load of 4/9 Euler's load for the eight columns tested. There is practically exact agreement in all except column 9–110–C, indicating that the method of determining the eccentricities was very accurate.

TABLE 8.—Eccentricities of Columns.

Strut number.	Average wall thickness. Top side.	Average wall thickness. Bottom side.	k	y	Eccentricity from wall variation. $e_w=k+y$	Dial deflection. R	Eccentricity due to crookedness. e_x	Total eccentricity. $e=e_w+e_x$	Deflection of column at load. $P=\frac{4}{9}P_E$
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1½ INCHES, 20 GAUGE $\frac{L}{r}=110$.									
1–110–C	0.039	0.036	−0.0015	−0.0144	−0.0159	0.104	0.0535	0.038	0.040
7–110–C	.035	.035	.00	.00	.00	.016	.008	.008	.009
1½ INCHES, 16 GAUGE $\frac{L}{r}=110$.									
10–110–C	0.0637	0.064	+0.0004	+0.002	+0.0024	0.037	0.018	0.020	0.019
9–110–C	.060	.064	+.0020	+.010	+.012	.100	.048	.060	.051
1½ INCHES, 20 GAUGE $\frac{L}{r}=70$.									
11–70–C	0.035	0.0335	−0.0007	−0.007	−0.0077	0.025	0.0132	0.0055	0.004
17–70–C	.0365	.0355	−.0006	−.006	−.0066	.029	.015	.008	.006
1½ INCHES, 16 GAUGE $\frac{L}{r}=70$.									
8–70–C	0.0635	0.0615	−0.001	−0.0052	−0.0062	0.028	0.015	0.009	0.003
5–70–C	.065	.0605	−.0022	−.011	−.013	.054	.0295	.017	.017

4. APPLICATION OF MODIFIED RATIONAL FORMULA.

The results obtained by applying the eccentricities caused by tube irregularities to the modified rational formula

$$f_c = \frac{P}{A} + \frac{Pec}{I} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_E}} + \frac{wEc}{P} \left(\sec \frac{\pi}{2} \sqrt{\frac{P}{P_E}} - 1 \right)$$

are given in Table 9. The maximum compressive stress f_c to cause failure of the struts subjected to combined column and transverse loading are in very close agreement with the yield point of the material. The small discrepancies that exist are on the side of safety. In the next to last column in the table are given the ultimate compressive stress S_c, approximately the yield point in compression for the material, determined from the compression test of short specimen (see Table 2). The maximum compressive stress at failure f_c for the struts are in extremely close agreement with these values. The general average of the maximum compressive stress at failure for the struts is 70,600 lbs./in.[2] and the average ultimate compressive stress for short column, or approximate yield point in compression for the material, is 69,700, an error of about 1 per cent, which is remarkably close for experimental data involving so many variables.

TABLE 9.—Results Obtained by Use of Modified Rational Formula for Combined Loading.

1½ INCHES, 20 GAUGE $\frac{L}{r}=70$.

Strut number.	Transverse load.	Maximum bending stress.[1] S_B	Compressive stress $\frac{P}{A}$	Maximum compressive stress at failure.[2] f_c	Yield point of material (tension).	Ratio. $\frac{f_c}{YP}$	Ultimate compressive stress (short column). S_c	Ratio. $\frac{f_c}{S_c}$
	Pounds.	Lbs./in.[2]	Lbs./in.[2]	Lbs./in.[2]	Lbs./in.[2]	Per cent.	Lbs./in.[2]	Per cent.
13-70-1	1.25	26,550	41,200	67,750	68,900	98	69,300	98
9-70-1	1.25	26,800	44,400	71,200	66,800	106	66,400	107
13-70-5	5.0	37,700	30,100	67,800	68,900	98	69,300	98
15-70-5	5.0	38,000	30,900	68,900	69,900	99	68,800	100
13-70-10	10.0	50,700	22,900	73,600	68,900	107	69,300	106
15-70-10	10.0	49,700	22,500	72,200	69,900	103	68,800	105
Average				70,200	69,000	102	68,600	102

1½ INCHES, 20 GAUGE $\frac{L}{r}=110$.

Strut number.	Transverse load.	Maximum bending stress.[1] S_B	Compressive stress $\frac{P}{A}$	Maximum compressive stress at failure.[2] f_c	Yield point of material (tension).	Ratio. $\frac{f_c}{YP}$	Ultimate compressive stress (short column). S_c	Ratio. $\frac{f_c}{S_c}$
4-110-1	1.0	50,100	19,400	69,500	63,400	109	65,200	106
17-110-1	1.0	49,600	19,900	69,500	67,400	103	68,400	101
9-110-3	3.0	57,500	14,300	71,800	66,800	107	66,400	108
16-110-3	3.0	52,560	13,550	66,110	62,200	106	66,200	100
1-110-6	6.05	61,090	8,200	69,300	61,200	113	64,700	107
11-110-6	6.05	63,600	10,300	73,900	69,000	107	67,500	109
Average				70,000	65,000	107	66,400	105

See footnotes on p. 270.

TABLE 9.—Results Obtained by Use of Modified Rational Formula for Combined Loading—Continued.

1½ INCHES, 16 GAUGE $\frac{L}{r}=70$.

Strut number.	Transverse load.	Maximum bending stress.[1] S_B	Compressive stress. $\frac{P}{A}$	Maximum compressive stress at failure.[2] f_c	Yield point of material (tension).	Ratio. $\frac{f_c}{YP}$	Ultimate compressive stress (short column). S_c	Ratio. $\frac{f_c}{S_c}$
	Pounds.	Lbs./in.²	Lbs./in.²	Lbs./in.²	Lbs./in.²	Per cent.	Lbs./in.²	Per cent.
5-70-1	1.0	27,040	42,900	69,940	70,000	100	71,600	98
8-70-1	1.0	27,700	45,500	73,200	70,200	104	74,800	98
5-70-5	5.0	25,960	42,500	68,460	70,000	98	71,600	96
7-70-5	5.0	25,780	39,300	65,080	67,500	96	69,300	94
8-70-20	20.0	56,000	23,250	79,250	70,200	113	74,800	106
7-70-20	20.0	54,240	19,780	74,000	67,500	109	69,300	107
Average				71,650	69,200	103	71,900	100

1½ INCHES, 16 GAUGE $\frac{L}{r}=110$.

Strut number.	Transverse load.	Maximum bending stress.	Compressive stress.	Maximum compressive stress at failure.	Yield point of material (tension).	Ratio.	Ultimate compressive stress (short column).	Ratio.
3-110-1	1.0	60,200	21,100	81,300	71,300	114	80,400	101
14-110-1	1.0	44,770	19,870	64,700	64,600	100	68,800	94
14-110-5	5.0	54,740	14,050	68,800	64,600	106	68,800	100
7-110-5	5.0	53,810	14,250	68,060	67,500	100	69,300	98
Average				70,700	67,000	105	71,800	98

[1] $S_B=\frac{M_o}{\frac{I}{c}}=\frac{Pec}{I}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}+\frac{wEc}{P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}-1\right)$.

[2] $f_c=\frac{M_o}{\frac{I}{c}}+\frac{P}{A}$.

The accuracy and safety with which this modified rational formula determines the stress and strength of struts subjected to transverse loading is graphically shown in the right half of Figure 10. In this figure the same values for abscissas are used as in the left half of the figure. The ordinate values f_c, the maximum compressive stresses at failure, were computed by the modified formula. It will be seen that the small variations which exist are less than the variations in the short compression tests. The two high values are for tubing Nos. 3 and 8, which had a high yield point in compression.

The relation between the maximum compressive stress at failure determined by the formula and the yield point of the material is shown in Figure 15. The numbers assigned to the different sections of tubing used in these tests are plotted as abscissas and the maximum compressive stress at failure for the struts cut from these sections as ordinates. The heavy line connects the value of ultimate or maximum compressive stress, approximate yield point in compression obtained from short compression tests of each section of tubing. This figure also shows the variation in per cent of maximum compressive stress from the yield point of the material. It will be noted that the failing stress of struts computed by the formula agrees very closely with the failing stress of the short compression pieces.

Theoretically, the modified formula is not exact above the proportional limit, as the formula involves the modulus of elasticity E. In this investigation the material used had very nearly the same proportional limit and yield point values. The assumption of strict proportionality between stress and strain up to failure, although not theoretically exact, gives results that agree within a very small percentage error with actual conditions, and for all practical purposes can be used in design.

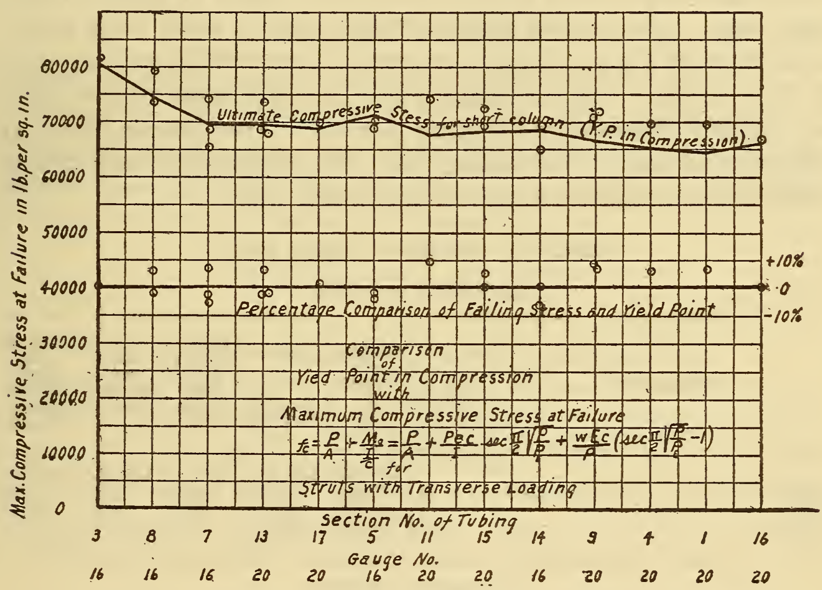

FIG. 15.—*Comparison of the maximum compressive stress at failure by modified rational formula with the yield point of the material.*

5. APPLICATION OF "SECANT" COLUMN FORMULA.

(a) TO COLUMNS.

The modified rational formula may be readily applied to columns to determine the maximum compressive stress to cause failure. By this formula the maximum intensity of compressive stress is

$$f_c = \frac{P}{A} + \frac{Pec}{I} \sec \frac{\pi}{2}\sqrt{\frac{P}{P_E}} + \frac{wEc}{P}\left(\sec \frac{\pi}{2}\sqrt{\frac{P}{P_E}} - 1\right)$$

For columns the transverse load w is zero, so that the above formula becomes

$$f_c = \frac{P}{A} + \frac{Pec}{I} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_E}}$$

or

$$f_c = \frac{P}{A} \left(1 + \frac{ec}{r^2} \sec \frac{L}{2} \sqrt{\frac{P}{EI}} \right)$$

which is the well-known "secant" formula for columns with eccentric loading.

The results of the column tests are given in Table 10. The eccentricities about the horizontal axis have been used to determine the maximum compressive stress for the shorter columns. The long columns with an L/r ratio of 110 failed at Euler's maximum load. The results indicate that there is fairly close agreement between the maximum compressive stress at failure computed by the "secant" column formula and the yield point of the material with the exception of column 5–70–C. This column failed by deflecting sideways, indicating that the eccentricity was greater than the value taken about the horizontal axis.

TABLE 10.—Results of Column Tests.

1½ INCHES, 20 GAUGE.

Strut number.	$\frac{L}{r}$ ratio.	Column unit stress. $\frac{P}{A}$	Euler $\frac{P}{A} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$.	Eccentricity e about horizontal axis.	Maximum compressive stress.[1] f_c	Ultimate compressive stress S_c (short column).
		Lbs./in.²	Lbs./in.²	Inch.	Lbs./in.²	Lbs./in.²
11-70-C	70	52,900	58,500	0.0055	71,400	67,600
17-70-C	70	51,930	58,400	.008	72,400	68,400
1-110-C	110	23,200	23,600			
7-110-C	110	24,000	23,700			
1½ INCHES, 16 GAUGE.						
8-70-C	70	52,800	58,100	0.009	72,300	74,800
5-70-C	70	47,650	58,300	.0017	63,600	71,600
10-110-C	110	23,500	23,500			
9-110-C	110	22,500	23,500			

[1] $f_c = \frac{P}{A}\left(1 + \frac{ec}{r^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_E}}\right)$.

(b) TO STRUTS WITH TRANSVERSE LOADING.

The "secant" column formula can be applied to struts subjected to transverse loading with safety and a reasonable degree of accuracy if certain modifications in the determination of the effective eccentricity are made. Consider, first, a strut to be under a very small end load and subjected to a uniform transverse load of w pounds per linear inch. The strut, by the deflection formula for a uniform transverse load, will be deflected at the center a distance

$$e_B = \frac{5}{384} \frac{wL^4}{EI} \tag{1}$$

and the bending (flexual) stress S_B in the extreme fiber at the mid-length section resulting from the transverse loading will be

$$S_B = \frac{1}{8} \frac{wL^2c}{I} \tag{2}$$

On application of the end load the section at the mid length of the strut will have an effective eccentricity e with reference to the line of load of

$$e = e_0 + e_B = e_0 + \frac{5}{384} \frac{wL^4}{EI} \tag{3}$$

where e_0 is the original eccentricity of the strut in the plane of deflection due to irregularities in the tube, and e_B is the deflection of the tube produced by the transverse loading. The maximum column stress S_u in the extreme fiber of the section at the middle of the strut is by the "secant" column formula

$$S_u = \frac{P}{A}\left(1 + \frac{ec}{r^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_E}}\right) \tag{4}$$

where e is the effective eccentricity given in equation (3).

The maximum compressive stress at the extreme fiber of the mid section is the sum of the maximum column stress S_u and the bending (flexual) stress S_B. It is to be expected that the strut will fail when the sum of these stresses is approximately equal to the yield point of the material.

The maximum compressive stress at failure is therefore

$$f_c = S_u + S_B = \text{yield point (approximately)}$$

or

$$f_c = \frac{P}{A}\left(1 + \frac{ec}{r^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_E}}\right) + \frac{1}{8} \frac{wL^2c}{I} \tag{5}$$

the effective eccentricity e to be taken as the sum of the original eccentricity and the deflection of the strut resulting from the transverse load.

The results obtained by applying the above formula to the combined tests are given in Table 11. The table shows that this method of computation gives values that are, on the average, about 6 per cent higher than those determined by the more exact formula. The error, however, is on the side of safety. Where extreme accuracy is not required, this formula can be safely used and is more reliable for designing than Perry's formula, which neglects the effect of eccentricities.

TABLE 11.—Results Obtained by Applying "Secant" Column Formula for Combined Loading.

1½ INCHES, 20 GAUGE $\frac{L}{r}=70$.

Strut number.	Transverse load.	Effective eccentricity. e_0+e_B[1]	Maximum column stress. S_u[2]	Bending stress due to transverse load. S_B[3]	Maximum compressive stress at failure. f_c[4]	Ultimate compressive stress (short column). S_c	Ratio. $\frac{f_c}{S_c}$
	Pounds.	Inch.	Lbs./in.²	Lbs./in.²	Lbs./in.²	Lbs./in.²	Per cent.
13-70-1	1.25	0.058	67,700	3,600	71,300	69,300	103
9-70-1	1.25	.0405	69,300	3,600	72,900	66,400	110
13-70-5	5.0	.126	55,400	14,600	70,000	69,300	101
15-70-5	5.0	.121	56,200	14,900	71,100	68,800	103
13-70-10	10.0	.198	46,100	29,400	75,500	69,300	109
15-70-10	10.0	.199	44,700	29,200	73,900	68,800	107
Average					72,400	68,600	105

1½ INCHES, 20 GAUGE $\frac{L}{r}=110$.

Strut number.	Transverse load.	Effective eccentricity.	Maximum column stress.	Bending stress due to transverse load.	Maximum compressive stress at failure.	Ultimate compressive stress (short column).	Ratio.
4-110-1	1.0	0.135	69,400	7,000	76,400	65,200	117
17-110-1	1.0	.119	70,800	6,900	77,700	68,400	113
9-110-3	3.0	.358	56,200	20,700	76,900	66,400	116
16-110-3	3.0	.347	49,500	20,800	70,300	66,200	106
1-110-6	6.05	.644	31,900	39,500	71,400	64,700	110
11-104-6	6.05	.542	38,200	38,000	76,200	67,600	113
Average					74,800	66,400	112

1½ INCHES, 16 GAUGE $\frac{L}{r}=70$.

Strut number.	Transverse load.	Effective eccentricity.	Maximum column stress.	Bending stress due to transverse load.	Maximum compressive stress at failure.	Ultimate compressive stress (short column).	Ratio.
5-70-1	1.0	0.045	68,900	1,660	70,560	71,600	98
8-70-1	1.0	.037	73,800	1,630	75,400	74,800	101
5-70-5	5.0	.042	65,000	7,900	72,900	71,600	102
7-70-5	5.0	.050	59,650	8,130	67,800	69,300	98
8-70-20	20.0	.212	49,400	32,500	81,900	74,800	109
7-70-20	20.0	.225	41,400	33,700	75,100	69,300	108
Average					72,300	71,900	102

1½ INCHES, 16 GAUGE $\frac{L}{r}=110$.

Strut number.	Transverse load.	Effective eccentricity.	Maximum column stress.	Bending stress due to transverse load.	Maximum compressive stress at failure.	Ultimate compressive stress (short column).	Ratio.
3-110-1	1.0	0.091	85,700	3,900	89,600	80,400	111
14-110-1	1.0	.099	66,200	4,000	70,200	68,800	102
14-110-5	5.0	.337	54,100	20,100	74,100	68,800	107
7-110-5	5.0	.327	53,300	20,300	73,600	69,300	106
Average					76,800	71,800	106

[1] e_0=original eccentricity due to tube irregularities.

$$e_B=\frac{5}{384}\,\frac{WL^4}{E\,I}.$$

[2] $S_u=\frac{P}{A}\left(1+\frac{ec}{r^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right).$

[3] $S_B=\frac{1}{8}\,\frac{wL^2c}{I}.$

[4] $f_c=S_u+S_B.$

V. CONCLUSIONS.

The results of this investigation warrant the following conclusions:

1. For determining the strength of a strut account must be taken of the effect of eccentricity. For steel tubing struts the eccentricity resulting from (*a*) variation in wall thickness and (*b*) deviation from straightness are very important factors in determining the strength.

2. For struts subjected to combined column and transverse loading it is assumed that failure occurs when the maximum compressive fiber stress approximates the yield point of the material. The commonly used formulas which neglect the effect of eccentricity are

$$f_c = \frac{P}{A} + \frac{M_0}{\frac{I}{c}} = \frac{P}{A} + \frac{wEc}{P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}} - 1\right)$$

$$f_c = \frac{P}{A} + \frac{M_0}{\frac{I}{c}} = \frac{P}{A} + \frac{c}{I}\,\frac{wL^2}{8}\left(\frac{P_E}{P_E - P}\right).$$

These do not represent actual strut condition and are not confirmed by experimental data. The use of these formulas for design purposes is shown by the data of this investigation to be inadvisable and possibly dangerous, especially for short struts or struts with small transverse loads.

3. A modified rational formula based upon consideration of the effect of eccentricity of loading,

$$f_c = \frac{P}{A} + \frac{M_0}{\frac{I}{c}} = \frac{P}{A} + \frac{Pec}{I}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}} + \frac{wEc}{P}\left(\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}} - 1\right)$$

was found to fit the experimental results very closely, the agreement being such as to indicate that it is the preferable formula for design where accuracy and safety are essential.

4. Failure of a strut subjected to combined column and transverse loading will occur when the maximum compressive stress f_c computed by this formula is approximately equal to the yield point of the material.

5. The results of the few tests made on tubes as columns indicate that failure of a column will occur when the extreme fiber stress is equal to the yield point of the material. For column loading the modified rational formula also applies, as it reduces to the "secant" column formula

$$f_c = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)$$

for such conditions of loading, since the transverse load w is zero.

6. A safe and reasonably accurate computation of stress for strut under transverse loading can be obtained by summing the bending stress S_B due to the transverse load as computed by the ordinary formula $\left(S_B = \frac{1}{8}\frac{wL^2c}{I}\right)$ and the column stress S_u obtained by the "secant" column formula. For failure

$$f_c = S_B + S_u = \frac{1}{8}\frac{wL^2c}{I} + \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)$$

approximates the yield point of the material, where the effective eccentricity e is the sum of the original eccentricity e_c due to tube irregularities, and the deflection at the middle of the strut e_B due to the applied transverse load, the latter for a uniformly distributed load being $e_B = \frac{5}{384}\frac{wL^4}{EI}$. The results obtained by this formula are shown by the data to be on the average about 6 per cent too high; the error, however, is on the side of safety.

VI. RECOMMENDATIONS.

1. The results show that there exists quite a wide deviation from straightness and wide variation in wall thickness in commercial tubing. Differences in wall thickness may cause variation in the area of two different pieces of tubing of the same gauge and diameter of 8 per cent, with corresponding variation in other properties. The data also show that stresses produced by eccentricities resulting from these variations are in some cases very high and unless known and considered are liable to be dangerous. These variables should, therefore, preferably be limited to as narrow a range as possible in the specifications for commercial tubing and enforced by careful and rigid inspection.

2. A new empirical formula for steel-tubing struts under transverse loading may be obtained by assigning a numerical value to the effective eccentricity in the modified rational formula. This value may be determined either by average measurements of commercial tubing or from the limits stated in the specification. With this procedure, the formula would probably be of assistance for design purposes when the actual eccentricity can not be determined.

3. The same numerical value for eccentricity, determined and corroborated by further experiments on columns, when applied to the "secant" formula, would probably give a satisfactory and accurate column formula for design purposes.

WASHINGTON, February 10, 1924.