THE OPTICAL REQUIREMENTS OF AIRPLANE MAPPING[1]

By I. C. Gardner

ABSTRACT

A definition of stereoscopic parallax is given and the relation between the stereoscopic parallax and the angular field of view of the airplane camera is derived. The stereoscopic parallax increases, to an important extent, as the field of view is increased. This makes the use of a large field of view, obtained by a wide angle lens or by a multiple lens camera, particularly desirable as it not only lessens the expense of flying, but increases the precision of the measurements obtained. In so far as the optical requirements are concerned, the required accuracy for the contours, together with the angular field of view, determines the necessary scale factor; and this, in turn, with the altitude of the airplane, determines the focal length of the objective which must be used in the airplane camera. The relations between the distortion and curvature of field of the camera objective and the resulting errors in the contours are considered. Measured values of the distortion of airplane photographs indicate that many of the photographic objectives in use are not sufficiently free from distortion to produce negatives satisfactory for topographic mapping. A large field of view and an increased freedom from distortion are of the greatest importance in extending the application of airplane photographs to the construction of topographic maps. If these characteristics can not be improved it may be desirable to develop a multiple lens camera having precision sufficient for topographic mapping and with a field of view, perhaps not so great as that of the present military multiple lens camera.

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I. INTRODUCTION

To increase the precision with which topographic maps[2] can be constructed from airplane photographs it is desirable that all sources of error be examined and their effect on the final map appraised. The precision of such maps may be limited by many causes, among which are defective imagery produced by the optical systems employed in exposing and measuring the negatives, inaccuracies in the measuring mechanism, film shrinkage, improper compensation of the tilt which the camera had at the time of the exposure, and the imperfect relative adjustment of the members of the successive pairs of negatives which are examined stereoscopically. In the present paper, however, all errors will be ignored except those which arise from the character of imagery registered upon the emulsion by the photographic lens used in the camera. An ideal limiting performance will be derived which can not be excelled by improvement in technique or in the mechanical

[1] An invited paper presented October 24, 1931, at the meeting of the Optical Society of America.
[2] Throughout this discussion the term "topographic map" will be understood to apply only to maps on which the elevation of all parts is indicated, either by contours or otherwise.

portions of the apparatus employed. Hope for greater precision must await further improvement of the photographic objective or of the photographic emulsion. On the other hand, the difference between present-day performance and this ideal limiting performance is a measure of the improvement which remains to be made in technique and apparatus if the full possibilities of the present photographic objectives are to be realized.

II. STEREOSCOPIC PARALLAX

When an exposure is made from an airplane, even though the camera is directed vertically downward, the resulting negative will not be a true horizontal projection or map of the region photographed if it deviates from a horizontal plane. Elevated points will be displaced outward from the center of the photograph and depressions will be drawn inward. This apparent displacement arises because the photograph is a projection on a plane from a point at a finite distance from the territory mapped. This is one of the difficulties which makes it impossible in many cases to construct a satisfactory map by the assemblage of separate photographs into a mosaic. When, however, topographic maps are to be made from airplane photographs this apparent displacement of elevated or depressed points not only ceases to be a detriment, but actually becomes the significant characteristic which makes topographic mapping possible.

The manner in which this apparent displacement of elevated or depressed points gives rise to stereoscopic parallax is illustrated by Figure 1. It will be assumed that AB represents the datum plane

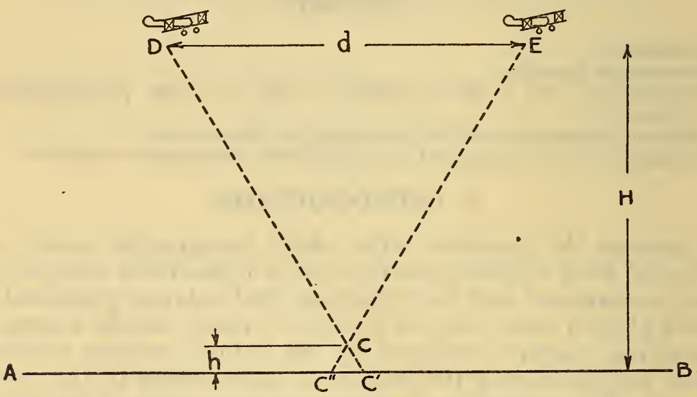

FIGURE 1.—*The derivation of stereoscopic parallax*

and that the point C lies h units above this plane. From the airplane two exposures are made, the first when the plane is at D and the second when it is at E. On the first negative the point C will be positioned as though it lay in the datum plane at C'. On the second negative it will appear to lie in the datum plane at C''. The relative displacement $C'\,C''$ will be termed the stereoscopic parallax. It is evident that the stereoscopic parallax may also be measured as an angular displacement or as a displacement on the negative, but, for the present purposes, it will be convenient to designate it as an apparent displacement measured in the datum plane. For all points common to the pair of photographs and at a height h above the datum plane it is evident that the stereoscopic parallax has the same value for it is equal to

$$\frac{d\,h}{H-h} \tag{1}$$

where d is the distance from D to E and H is the height of the airplane when the two exposures are made. In a great number of cases, to a satisfactory degree of approximation, one may write

$$\text{Stereoscopic parallax} = h\frac{d}{H} \tag{2}$$

and, for moderate elevations, to an approximation satisfactory for the present purposes, it may be assumed that the stereoscopic parallax is proportional to the elevation of the point in question.

It is evident that the stereoscopic parallax of a point can not be measured unless the point appears on two different photographs. Consequently, if an airplane flies in a straight line, making exposures at regular intervals, it is essential that successive exposures overlap 50 per cent in order that the stereoscopic parallax of any desired point can be determined. Fifty per cent is the minimum overlap. As a matter of fact, it is desirable in all processes, and necessary in some, that a fair proportion of the points appear in three successive photographs. Consequently the nominal overlap is commonly 60 per cent. With this information the equation for the stereoscopic parallax may be advantageously modified. If the half-angular field of view of the camera, measured in the plane which includes the median of the photographic plate parallel to the direction of flight is α degrees, then $2H \tan \alpha$ is the distance, measured parallel to the travel of the plane, of two extreme points included in a single photograph. Consequently, if there is to be a 60 per cent overlap, d, the distance traveled between two successive exposures will be $0.8\ H \tan \alpha$. Substituting this value of d in the approximate equation for the stereoscopic parallax, it becomes

$$\text{Stereoscopic parallax} = 0.8\ h \tan \alpha \tag{3}$$

With the definition of stereoscopic parallax which has been given, it will be noted that the stereoscopic parallax of a given point is proportional to its elevation and proportional to the angular field of view of the camera but is independent of the height of the plane. If all other conditions can be properly controlled it is evident that the precision can be improved by increasing α, the half-angular field of view of the camera. For the single-lens camera at present available the limit in this direction is soon reached at about 60° for 2α, but the multiple-lens camera is a means by which the value of α can be considerably increased. With a camera of this sort exposures are taken simultaneously from the airplane with 3, 5, or 9 lenses mounted in a fixed relation to each other. A knowledge of the angles between the axes of the different lenses enables the different photographs to be copied and rectified in such a way that they may be combined to form a single large photograph. If the rectification has been correctly done this single large photograph, except for the effects of aberrations and other secondary defects, shows the same perspective as though a single photograph had originally been taken with one lens embracing all of the fields of view of the separate lenses. The outstanding advantage of the multiple-lens camera is that it enables an extremely large area to be reproduced according to a single perspective system.

As has already been mentioned, a second advantage of the multiple-lens camera lies in the great increase of the stereoscopic parallax by reason of the increased angular field of view. It would seem that the 9-lens camera should give a field of view having its angular diameter approximately three times that of the single-lens camera. However, there are several characteristics which prevent the use of such a large field for topographic work. In the process of rectification the peripheral portions of the picture are greatly enlarged and there is a corresponding loss of detail. These portions are also likely to be adversely affected by atmospheric haze because of the long light path. Furthermore, even if the negatives were sharply defined over the extremely large field, there is serious danger that the measurement of the stereoscopic parallax will be incorrectly made because of the greatly different aspects of the object points from the two widely separated view points. Then, too, inaccuracies in the rectification and assembly of the different exposures introduce errors. These various difficulties have prevented the use of the multiple-lens camera for precise topographic surveying. However, the larger field of view has the advantage of the greater stereoscopic parallax and, if the field of view of the single-lens camera can not be greatly increased, it seems not unreasonable to believe that economic conditions, in time, will justify the construction of multiple-lens cameras sufficiently precise for topographic work.

III. REQUIREMENTS FOR A GIVEN PRECISION WITH AN IDEAL PHOTOGRAPHIC OBJECTIVE

A most important portion of the equipment necessary for airplane surveying is the photographic emulsion, and some appraisal must be made of the precision with which the location of an image on a negative may be measured. The limit of resolution of a photographic film is of the order of 0.020 to 0.025 mm. In other words, a pair of lines separated this distance on a negative will be recorded as two lines just sufficiently distinct to permit a measurement to be made on each line. The probable error of a single setting on one of these lines will evidently be considerably less than the distance between the two lines. It will be assumed that the probable error of a single setting on a line is 0.004 mm which is one-fifth to one-sixth of the distance between two lines which are just resolved. This assumption is somewhat arbitrary but, if in error, it is believed that it under estimates the precision with which measurements of fairly well-defined images may be made on a photographic negative.[3] If the probable error of a single setting on a point is 0.004 mm, the probable error in the determination of a length by a single measurement at each end will be approximately 0.006 mm.

This selection of the probable error to be assumed is of fundamental importance for the entire discussion. In the laboratory, on photographs of test objects, it is not difficult to make measurements with a

[3] This uncertainty of measurement on the photographic emulsion arises from the grain of the developed image and does not include the errors introduced by film shrinkage. This latter type of error has not been included because we are here endeavoring to appraise an ideal best possible performance and film shrinkage can be eliminated by the use of glass plates.

precision considerably greater than that which has been here assumed. In airplane mapping, as practiced at present, the points on which observations are made are not always viewed against a background which provides conditions for the best definition. Furthermore, even with perfect lenses, the turbidity of the atmosphere introduces a loss of definition which tends to further increase the probable error of a setting. However, Edward H. Cahill, with Brock & Weymouth (Inc.), informs me that their operators on the stereoscopes can read the parallax of a point with a total variation of the order of 0.015 mm. As the pictures used in the stereoscope in the Brock process are magnified 2 diameters from the original negatives, this value does not differ greatly from that indicated by the assumption of a probable error of 0.006 mm on the original negative.

In order to have a concrete problem to which consideration can be directed it will be assumed that the allowable probable error in a contour is 1 foot (300 mm).[4] The stereoscopic parallax corresponding to a change of elevation of 300 mm is 240 tan α millimeters and, if this is to be the probable error of a single setting on the negative, the scale must be such that it corresponds to 0.006 mm on the plate; that is, the scale factor of the photograph must be equal to

$$\frac{0.006}{240 \tan \alpha} = \frac{1}{40{,}000 \tan \alpha} \text{ approximately} \tag{4}$$

Before evaluating this expression further it is well to note a general conclusion which is apparent from this result. The scale factor required for a given precision is not a function of the height of the plane. Consequently if α, the half-angular field of view of the camera is held constant, the precision will be unaffected by the altitude of the plane, provided that objectives of different focal lengths are selected for the different altitudes in such a manner that the scale factor is unchanged. The advantage of the greater flying height and the objective of correspondingly longer focal length lies in the fact that fewer pictures are required for a given area and consequently the labor of securing the data from the negatives is greatly reduced.

The values of the ideal scale factors corresponding to several different values of the half-angular field of view have been computed from equation (4) and are given below:

Half-angular field of view $=\alpha$	Scale factor $=\frac{1}{40{,}000 \tan \alpha}$
25°	$\frac{1}{18{,}600}$
30°	$\frac{1}{23{,}000}$
50°	$\frac{1}{48{,}000}$
60°	$\frac{1}{69{,}000}$

When using these values it must not be forgotten that they are ideal values which, in their derivation, are based upon the assumption that

[4] Commercially used specifications for airplane topographic mapping have in some cases required that 90 per cent of the contours must not be in error by more than 50 per cent of the contour interval. For a contour interval of 10 feet this corresponds to a probable error of 2 feet instead of 1 foot as has been assumed in this discussion. For a 10-foot contour interval, therefore, the probable error assumed is smaller than has been found necessary for some commercial mapping.

all errors are eliminated except those introduced by the limited resolution of the photographic emulsion. The first two tabulated values may be considered as characteristic of single-lens cameras and the last two apply to multiple-lens cameras. For a probable error of 1 foot in the contours the smallest scale factor that would be used with a half-angular field of 30°, according to present commercial practice, is perhaps $\frac{1}{7{,}000}$ and probably, in most cases, a considerably larger scale factor would need to be employed to insure a probable error less than 1 foot over all parts of the map. This difference between $\frac{1}{23{,}000}$ and $\frac{1}{7{,}000}$ may be attributed, on the one hand, to the distortion and poor definition of the objective as will be shown in the following section and, on the other hand, to the several operations which intervene between the making of the exposure and the final production of the maps.

In making this comparison the discrepancy between theory and actual practice is perhaps even greater than is indicated, because the determination of the probable error of a single isolated point has been under consideration. Actually, however, measurements are taken on a very large number of points and the construction of the contours involves an averaging or smoothing process by which there is a considerable gain in precision. In deriving the scale factors tabulated above, this averaging effect has not been included and this omission tends to bring the tabulated theoretical values of the scale factor closer to the values in actual use.

IV. EFFECTS OF ABERRATIONS OF THE PHOTOGRAPHIC OBJECTIVE

In deriving the ideal scale factors which have been given as necessary for a certain precision, the consequences of the aberrations of the photographic objective have not been considered. Of these, the effects of distortion and the falling off in the quality of definition at the edge of the field are the most obvious. It is, of course, well known that the images of all points are displaced in a radial direction either toward or from the center of the field and, unfortunately, the magnitude of this displacement is often very large in comparison with the precision with which the location of a point on the negative can be determined. When making a mosaic, from photographs having an overlap of 50 or 60 per cent, this optical distortion is not particularly serious because only the central portion of each exposure need be utilized. When using the same series of negatives, however, for the construction of a topographic map, it is necessary that the marginal portions of the negatives be used and the distortion, if uncompensated, may easily be as great as the stereoscopic parallax from which the elevation of a point is to be determined. This distortion, by the lens, leads to systematic errors in the determined values of the elevation which can be compensated by the use of the proper corrections. However, the application of corrections is an added complication which introduces many difficulties and it is obviously desirable to have conditions such that the effect of the distortion may be neglected.

Some general considerations regarding the character of the distortion of a lens are now necessary. Analytically the distortion of a point may be expressed as a series of terms in α^3, α^5, etc., where α is the angular distance of the point from the center of the field; but, for points very near the center of the field, the distortion may be assumed to vary as α^3. If a lens is well corrected for distortion the higher order distortions will be balanced against the third order terms and the distortion over the central portion of the field will be negligible but for values of α greater than a given value the distortion will increase rapidly. For two lenses of the same design but differing in focal length, if the control during manufacture has been sufficiently exact, the distortions, for a given value of α, will vary directly as the two focal lengths. The Bureau of Standards has had occasion to test a large number of lenses destined for use in airplane photography by the different departments of the Government and by private firms. For the great majority of lenses tested the distortion, 25° from the axis, for a focal length of the order of 180 mm is as great as 0.1 mm. This is approximately twenty times the probable error of a single setting on a point on the negative. If exposures with such a lens are made from a height of 10,000 feet this distortion will introduce a systematic error at the edge of the field corresponding to a difference in elevation of approximately 15 feet. Most of these lenses were to be used for mapping, although probably not for topographic mapping. However, the results found from the tests of these lenses clearly indicate the need for such tests before lenses of commercial quality are used for topographic mapping.

The results of some of these tests are shown in the accompanying illustrations. Figure 2 shows the distortion found in three lenses nominally identical and of a type commonly used for airplane photography. Forty lenses of this type submitted in a single lot were tested and the curves are typical of the amount of variation which was found. Figure 3 shows the freedom from distortion of a lens selected for topographic mapping purposes. The distortion in Figure 4 is characteristic of a lens which has been specially designed for airplane mapping and which regularly shows excellent freedom from distortion. Figure 5 shows the distortion curves for a lot of 10 lenses, nominally identical. For all the lenses of this group the distortion is small and the uniformity of product is rather remarkable.

The systematic errors introduced by the distortion shown for some of these lenses would largely destroy the usefulness of the resulting negative for topographic purposes. There are methods by which the detrimental effects of the distortion may be reduced. If pictures to the same scale are taken from a lesser altitude, as has been shown, the stereoscopic parallax will remain the same. However, if the scale is held constant, a lens of shorter focal length will have been used and the distortion will have been decreased in the same ratio as the altitude of the plane. Consequently the net result is a lessening of the effect of the distortion. On the other hand, if the original focal length and altitude are retained the overlap may be increased. With the increased overlap the value of the stereoscopic parallax decreases, but the corresponding decrease in the distortion is greater proportionately so that there is a lessening of the errors which it introduces. Each of these methods for improving conditions carries the disadvantage that an increased number of exposures are necessary with a greatly increased cost in making the measurements on the plates. It should, perhaps, be mentioned that increasing the focal length of the objective in order to increase the scale of the picture does not, in itself, lessen the error introduced by distortion. If, for example, the focal length of the objective is doubled, the distance on the plate corresponding to the stereoscopic parallax for 1 foot is doubled. But if the type and quality of the lens remain the same the distortion is also doubled so that there is no net gain.

Aside from the systematic errors introduced by distortion there are accidental errors which arise from the failure to secure critical definition in the marginal portions of the plate because of the presence of curvature of field, astigmatism, or coma. For pictorial purposes the definition yielded by a modern lens appears to be satisfactory

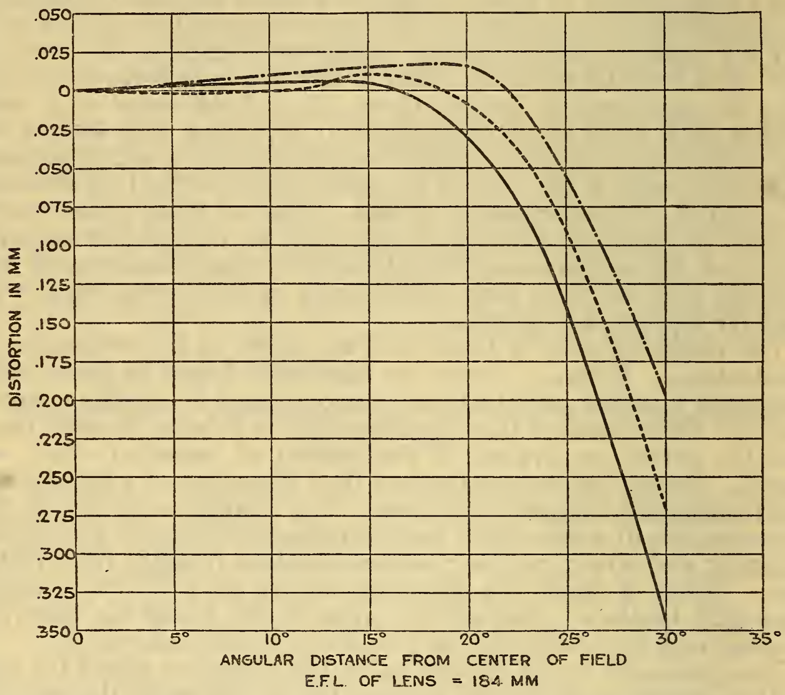

FIGURE 2.—*Distortion curves of three lenses which were nominally identical*

over all parts of the field. When, however, measurements of the resolving power are made for different portions of the field it is found that the boundary portions of the plate are not nearly so well defined as the central portions. Such a defect does not introduce systematic errors, but does greatly increase the accidental errors of observation. The difficulties of this nature are less important than those introduced by distortion because of this accidental character. Furthermore when contours are plotted the smoothing process, to which reference has already been made, tends to eliminate a portion of the errors of an accidental nature.

The extent to which the systematic errors arising from distortion and the accidental errors arising from impairment of the definition for the marginal portions of the field are to be tolerated is, to a large extent, determined by the magnitude of the errors which are incident to the mechanism and methods by which the data are obtained from the negative. It appears that the errors of this sort which are made in measuring the negative are of the order of 0.015 to 0.030 mm. Such errors clearly demand that the distortion of the lens, for the portion of the field that is utilized, should be very much less than the 0.1 mm which has been previously discussed in connection with the consideration of distortion. Furthermore, if the errors of 0.015 to 0.030 mm are assumed to represent the errors which arise from defects in the measuring mechanism, the effects of these errors can be still further reduced if the measurements are made, not on the original negative, but on an enlarged copy. Of course it is necessary

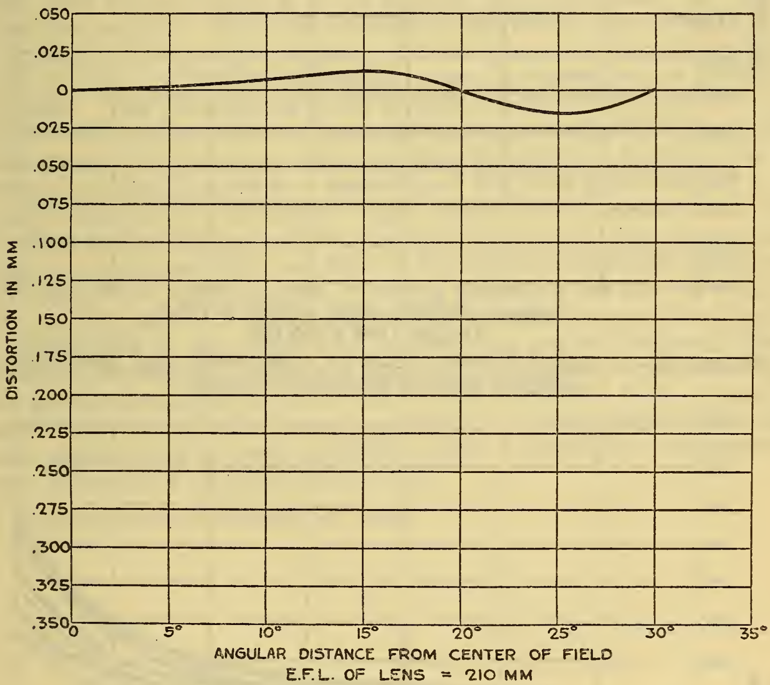

FIGURE 3.—*Distortion curve of a lens relatively free from distortion*

This particular lens was obtained by selection.

that the distortion introduced by the optical system of the enlarger be nil or, better yet, that it be so determined as to compensate the distortion of the original negative. The making of an enlarged copy on which the measurements are made is a practicable expedient which occupies a fundamental position in one commercial mapping process.

V. PROBABLE FUTURE DEVELOPMENT OF THE AIRPLANE PHOTOGRAPHIC OBJECTIVE

The scale factor found necessary in practice has been found in Section III to be approximately 3½ times that indicated by theory. When it is considered that this ratio is partially accounted for by the distortion and lack of perfect definition of the objective, this indicates that the mechanical equipment has been brought to such perfection

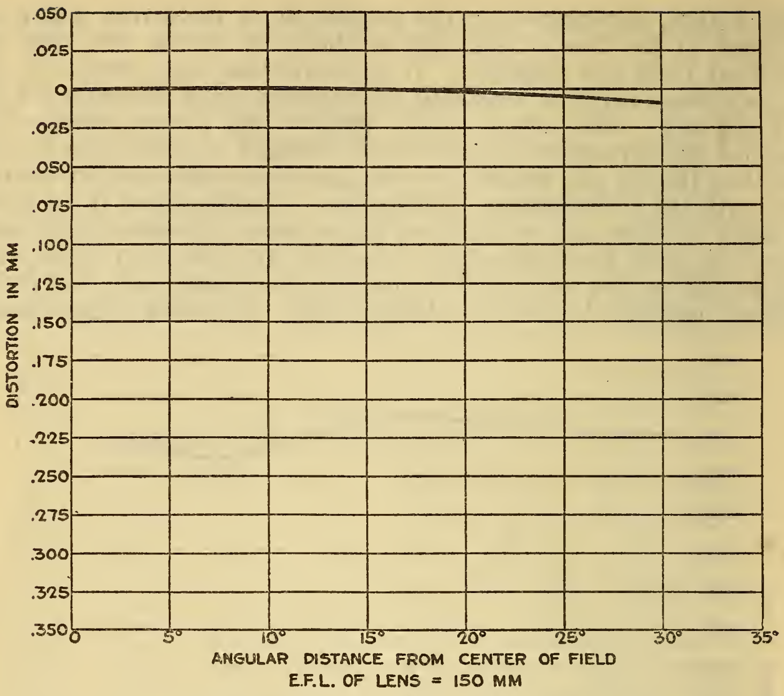

FIGURE 4.—*A typical distortion curve of a lens which was particularly designed to give a distortion-free image*

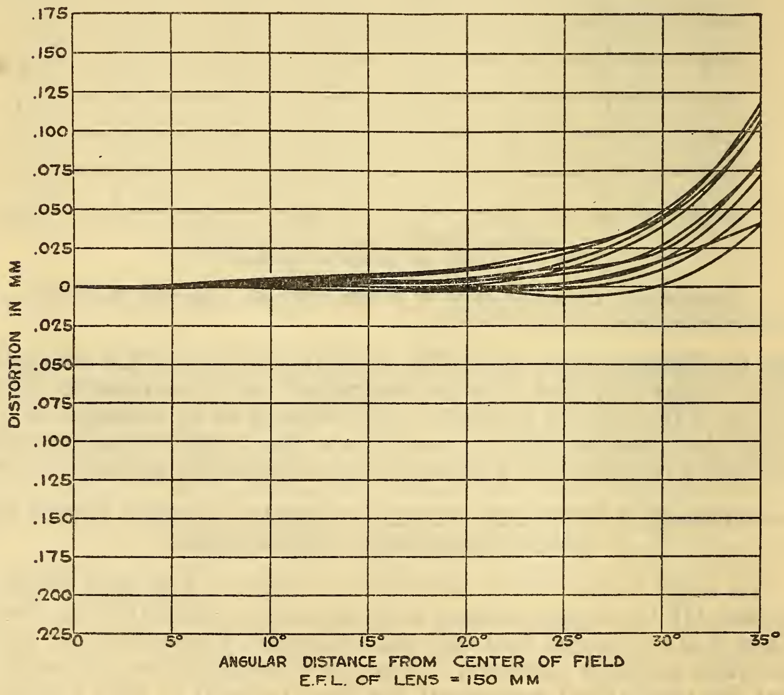

FIGURE 5.—*Distortion curves of 10 lenses which were nominally identical*

that it is fairly commensurate with the possibilities of the negatives taken with the photographic objectives and cameras which are at present available. It is obvious, upon a superficial examination, that one desires the largest possible field of view and the maximum freedom from distortion. However, by this consideration of errors, in which those arising from mechanical sources are purposely treated as absent, the requirements of the photographic objective are brought into strong relief. An increase in the useful field of view of the lens can only be brought about by an improvement in the correction for distortion because an enlarged field, over which definition may be entirely satisfactory for pictorial purposes, is quite useless for mapping unless the increased field is obtained without an increase in the linear value of the distortion on the negative. Furthermore it is important to note that the advantages to be derived from increasing the field of view are twofold. Not only is there an increase in the extent of territory included on a single negative, but, at the same time, the precision with which the vertical dimension may be measured is greatly increased. The advantages of the increased field of view, provided that it be free from distortion, are so great that is seems justifiable, certainly with lenses to be used on civil mapping projects, to increase the field even if it should result in a lessening of the true speed, either by the use of additional components or by a reduction in the aperture. When the high cost of flying and the relatively low cost of the photographic lens, as compared with the rest of the photogrammetric equipment, are taken into consideration, it seems also that the increased field of view is demanded, even at considerable increase in the cost of the lens.

If the field of the single lens camera can not be increased then it would be desirable to develop a precision multiple lens camera in which the primary purpose is, not to secure the maximum field of view, but rather to secure a field of view considerably exceeding that of the single lens camera and of such perfection that topographic measurements may be made in all parts of the field.

WASHINGTON, February 13, 1932.