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Accurate Measurements of and Corrections for Nonlinearities in Radiometers

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The methods described in the literature for accurately measuring photocell linearity are surveyed and assessed. The effect of not measuring photocell linearity under the conditions used in the final apparatus are considered. Some of the conditions necessary for accurate assessment of the nonlinearity under working conditions are specified. The use of the NRC "Photocell Linearity Tester" to measure and correct for the nonlinearity of various receivers is described.

Key words: Nonlinearity; photocell linearity; photometric accuracy; radiation addition method.

I. Introduction

A radiometer is linear if at time, t_0, the response, $N(t_0)$, indicated by the radiometer is exactly proportional to the incident radiant flux, $\phi(t_0)$, i.e., $N(t_0) = \alpha\phi(t_0)$, where α is a constant. This is the ideal and is never exactly realized.

Our problem is to measure one radiant flux, ϕ_1, relative to another, ϕ_2. Thus, we want to know ϕ_1/ϕ_2 from the measurements N_1, and N_2. In general, ϕ_1 and ϕ_2 are functions of time since we must compare the two fluxes and thus cannot irradiate the receptor indefinitely with either flux.

In general, the response, $N(t_0)$, depends on the flux which fell on the receptor from the time the receiver was made up to time, t_0. It also depends on the electronic circuit associated with the receptor and on all of the things which control its behavior, such as component temperatures, power supply voltages, charges on its capacitors, magnetic fields, etc. Again, in the ideal case the response of the electronic circuit should depend only on $\phi(t_o)$, but in practice it will depend on $\phi(t)$ where t goes from $-\infty$ to t_0.

For accurate measurements, we must apply the two fluxes to be compared in a way which will, for the receptor and electronics used, produce a ratio which is precisely reproducible. We also must devise a method to measure and correct for the nonlinearity which is present at the time of the measurement.

The purpose of this paper is to describe the principal methods of reducing and measuring nonlinearity, to indicate the accuracies attained by each, and to point out a few of the merits or drawbacks in each. Hopefully this will aid in deciding which method is appropriate for a particular task, and also in deciding how many precautions are required to attain a specified accuracy. The reference list given is not intended to be complete, but rather to indicate a sufficient variety of methods.

II. List of Methods of Measuring and Correcting for Nonlinearity

1. Superposition (Additive) method. The superposition method is a basic physical method and is useful in all radiometric applications. It may be divided into two classes: (a) Multiple sources, (b) One source providing several independently interceptible beams.
2. Bouguer's Law.
3. Beer's Law.
4. A combination of the superposition method and Bouguer's Law as described by Hawes [1].[1]
5. Inverse Square Law.
6. Standard absorbing filters of either liquid or glass.
7. Standard reflecting materials.
8. Rotating sectors. WARNING: This does not measure all types of linearity because in the extreme case the flux is either ϕ or zero and nothing can be deduced about linearity.
9. Measurement of amplitude of harmonics or beat frequencies. Increases precision of measurements and speed of measuring nonlinearities.
10. Measurement of known radiant fluxes. Erminy [2] has shown how to provide the known radiant fluxes by the superposition method using three sources.

[1] Figures in brackets indicate the literature references at the end of this paper.

11. Null method eliminates nonlinearity. Lee and others use it in optical pyrometry by providing standard sources to match the test source.

12. Use of a linear receptor to calibrate a test receptor.

III. Summary of Methods of Reducing Nonlinearities and Increasing Accuracy

1. Kunz [3] has shown that one can attenuate the flux by a known amount using a sector and thus improve the results by keeping the average anode current in a photomultiplier low enough to make negligible the drift of dark current.

2. Jung [4] has demonstrated improved linearity by attenuating the larger flux with a sector. This is effective if the chopping rate is much faster than the time constant of the nonlinearity.

3. Jung [5] has demonstrated improved linearity by chopping the flux to be measured and then adding a steady flux, P_0, to the chopped flux to keep the average flux constant. The mixture of chopped and steady flux is then passed to a phase sensitive rectifier and the output caused by the chopped radiation is shown to be very linear provided the rectifier is linear. P_0 need not be measured.

4. Special circuits in photomultipliers to keep the dynode voltage independent of anode current have been developed. The linearity is improved by using an adequate cathode to first dynode voltage and maintaining a constant last dynode to anode voltage.

5. Low or zero load resistance for selenium (barrier layer) cells and silicon diodes improve the linearity.

6. Jones and Clarke [6] used a photocell obeying Talbot's Law and a variable sector measured by time ratio photometry to reduce nonlinearities in photometric measurements.

7. Potentiometric or feedback system to keep constant the input to electrometers increases the stability and precision of measurements. Vacuum photoemission diodes with a cylindrical anode are extremely linear.

A. Detailed Descriptions of Some Methods for Measuring Nonlinearities

1. *Superposition (Additive) Method*—In the superposition method with two sources fluxes ϕ_1 and ϕ_2 produce the responses N_1 and N_2. The combined flux, $\phi_1+\phi_2$, produces the response N_{12}. If $N_1+N_2=N_{12}$ then the photometer is linear. If $N_1+N_2\neq N_{12}$ the nonlinearity may be given by the factor $N_{12}/(N_1+N_2)$, as described by Sanders [7], where this factor may be used to correct the response at the scale position $(N_1+N_2)/2$. If $N_1\neq N_2$ the measured correction is only an average correction factor, which will have an error dependent on how the nonlinearity changes with response. Zero correction is assumed necessary at N_1+N_2.

Rotter [8] preferred to find the nonlinearity as a difference correction to the response. He assumed that the correction was zero at the value $N_1=N_2$, and then defined the correction at N_{12} by

$$k_2=N_1+N_2-N_{12}.$$

With Rotter's method, it is possible to make hand computations more easily.

a. Multiple Sources—Many descriptions have been given of the use of this method. Preston and McDermott [9] in 1934 reviewed some earlier papers and then described their linearity tests using six incandescent lamps inside an integrating sphere. The lamps which were screened from the diffuse viewing window could be switched on or off independently of one another. They were each adjusted to have a similar luminous flux. Each lamp was measured independently with the test cell and then in combinations of 2, 3, 4, 5, and 6. In this experiment, each lamp had to be switched on and stabilized before being used.

In most cases the photo cells tested decreased in sensitivity with the illumination on the receiver. They used blue, green, and red filters and found that the nonlinearity was worst for blue light. In some cases, increased anode to cathode voltage improved the linearity. Nonlinearities of up to 15 percent were measured. The best cell was an Osram KMV6[2] with a nonlinearity of 0.1 percent, at a ratio of 1:6 in flux. All KMV6 cells were not as linear.

It is possible that the nonlinearity was caused by decreased cathode to anode potential at the higher flux.

This type of nonlinearity can be avoided by using a cathode which is mounted at the end of a long cylindrical anode in the manner described by Boutry and Gillod [10]. The VB59 photodiode from Rank Cintel, England, which is very linear has the cathode mounted along the axis of the cylindrical tube with a small window in the metallized tube wall. The nonlinearity can also be avoided by using an auxiliary potential to cancel out the potential developed across the anode resistor, or by using the connections to an operational amplifier as described by Witherell and Faulhaber [11]. The operational amplifier decreases the anode load resistance by the factor $1/A$ where A is the open loop gain of the amplifier.

Other users of multiple sources have mounted the lamps in individual compartments and left the lamps lit and selected the flux by shutters. Erminy [2], Kunz [3], Jung [4], and Reule [12] used two or more sources which could be isolated by baffles or rotatable mirrors. See items below for more details on these applications.

Reule [12] described some linearity measurements on a Carl Zeiss DMR21 Recording Spectrophotometer. He used a method of testing the linearity of a single beam spectrophotometer which had earlier been described by Hansen [13]. This step source or supplementary light method uses the superposition principle. A supplementary source, S_S, provides an adjustable

[2] In order to adequately describe materials and experimental procedures, it is occasionally necessary to identify commercial products by manufacturer's name or label. In no instances does such identification imply endorsement by the National Bureau of Standards, nor does it imply that the particular product or equipment is necessarily the best available for that purpose.

amount of flux which is measured as N by the spectrophotometer in the single beam mode. S_S is blocked off and the internal source, S_I, in the spectrophotometer is adjusted to provide an equal reading. Both shutters are opened and the sum is noted. S_S is again blocked off and S_I is adjusted to give a reading equal to the sum. This stepping procedure is repeated until the top of the spectrophotometric scale is reached. Reule used steps of 20 percent as a compromise between good scale coverage and drifting errors.

The analysis of the measurements is complicated if there is drift of the zero, or drift of the readings caused by lamp drift. It is not possible to read below zero or above 100 percent.

Reule also described how to use a similar method for double beam operation. The agreement between the corrections for double and single beam operation was within 0.1 percent of full scale. The corrections necessary to correct for nonlinearity depended on wavelength, but were always less than 0.1 percent when a photomultiplier was used as the detector.

Reule was able to use identical geometry for the two sources so the measurements should be applicable to the measurements made with the spectrophotometer. The two sources are independent, so no interference can be caused by partial coherence.

b. One Source Providing Several Fluxes— This is a very convenient method, since it means that only one power supply is required to provide the two or more fluxes which are required, Since the source is usually a tungsten lamp, the voltage must be controlled with four times the stability which is required in the flux. The current should be controlled with about seven times more stability than that required in the flux. Thus, the power supply will be expensive and a considerable saving will result from using only one supply.

There is, however, a danger in deriving the two fluxes for the superposition method from a single source. The two fluxes may be coherent to some extent and this may cause errors in measuring the nonlinearity. In fact, Mallick [14] describes a method in which the departure of additivity of two intensities indicates the degree of coherence of the light vibrations at the two points. Since one cannot use the same measurement to determine both nonlinearity and coherence, it is necessary to be sure which or what combination of these phenomena one is measuring in a given experiment. The equation given by Mallick is

$$I(Q)=I_1+I_2+2(I_1I_2)^{1/2}|\gamma_{12}(\tau)|\cos[\alpha_{12}(\tau)-\delta].$$

One can see that if $\gamma_{12}=1$ and the phase is correct, we can obtain a value of $I(Q)=2(I_1+I_2)$ in the case where $I_1=I_2$. This is twice the value expected for incoherent sources. Similarly, $I(Q)$ could be zero if the phase was shifted by 180°. Admittedly these are the extreme cases. For some ideas on how to evaluate the possible errors in linearity measurements due to coherence, see Born and Wolf [15], Mielenz and Eckerle [16], and Bures and Delisle [17].

The coherence will depend on the wavelength, wavelength range, area of source, coherence in original source, angular separation of apertures, size of apertures, area of receiver, path lengths, difference in path lengths, and diffusion in the system. Thus, it would be advisable to use two different parts of the strip filament which acted as the original source. This may be accomplished in Sanders' [7] linearity tester by placing a thin prism in front of one-half of

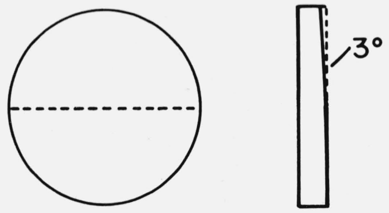

FIGURE 1. Circular glass plate with a 3 degree wedge removed from one half. This half deflects rays from 1 aperture of each pair of apertures in the linearity tester of figure 2.

the pairs of holes. As shown in figure 1, this may be easily constructed by removing a 3° wedge from one-half of a circular glass plate with the thin edge of the wedge on a diameter. The plate may be positioned in front of the lens in the linearity tester with the diameter positioned relative to the master plate, shown in figure 2, so all the rays passing the upper half of the holes are deflected downward. A strip filament lamp has replaced the lamp with diffusing bulb. A photocell placed at the image of the lamp filament, will receive images from two separate parts of the vertical strip filament lamp. There may be some slight residual coherence due to interreflections in the lamp or by diffraction or stray light at the apertures, but it will be considerably reduced. If it is desirable to measure with monochromatic light or to extremely high accuracy, it would be advisable to use a diffuser before the receiver to remove even the residual coherence.

The master plate in figure 2 has 18 apertures related in area as 1:1:2:2:4:4:8:8:16: . . . 64:128:128. This master plate is placed between a lens and the image of a lamp. Thus, each aperture as viewed from the image plane has a luminance equal to the luminance, L, of the source times, T, the transmittance of the lens. Thus, the irradiance from each aperture at the image is proportional to the area of the aperture. A rotatable disk with suitable holes can be positioned to transmit light through one hole or a pair of holes from the master plate. Thus, a range of illuminances of 1:256 are obtained at the image position. A receiver is placed at this position and the response measured for each aperture or pair of apertures, as shown in table 1, which is taken from Sanders [1]. For the whole series of measurements we must select one position which requires no correction. Sanders in 1962 used normalization at I_i, the maximum value. It is now believed that normalization at some fixed position on a scale would be more reasonable and advantageous in comparing nonlinearity measurements obtained at two different

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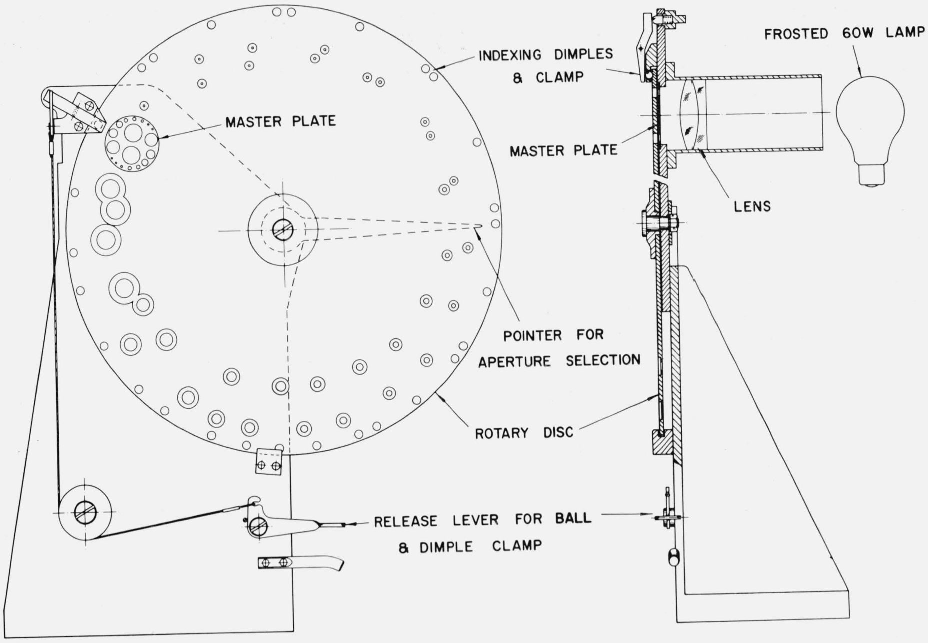

FIGURE 2. Sanders' Photocell Linearity Tester. (Courtesy, Applied Optics.)

times. With normalization at the maximum, one obtains the correction factors as shown in column 4, which are applicable to the responses shown in column 5. Using this apparatus, Sanders obtained correction factors for two detectors, the best of which was a vacuum diode photocell of Boutry and Gillod [10] which required a maximum correction factor of 0.997 over the whole range of 256:1. At the upper readings the correction required was less than 0.05 percent for a range of 30:1. The larger corrections with the smaller apertures may be caused by errors in measuring the smaller fluxes, by stray light, or by interference between partially coherent beams. The advantage of Sanders arrangement is that only one

TABLE 1. Determination of factors to correct readings to a linear scale

Net reading	Sum	Ratio	Correction factor[a]	Correction applies to reading
A			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}\cdot\frac{Gg}{G+g}\cdot\frac{Ff}{F+f}\cdot\frac{Ee}{E+e}\cdot\frac{Dd}{D+d}\cdot\frac{Cc}{C+c}\cdot\frac{Bb}{B+b}\cdot\frac{Aa}{A+a}$	$\frac{A+a}{2}$
Aa	$A+$	$\frac{Aa}{A+a}$		
a				
B			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}\cdot\frac{Gg}{G+g}\cdot\frac{Ff}{F+f}\cdot\frac{Ee}{E+e}\cdot\frac{Dd}{D+d}\cdot\frac{Cc}{C+c}\cdot\frac{Bb}{B+b}$	$\frac{B+b}{2}$
Bb	$B+b$	$\frac{Bb}{B+b}$		
b				
C			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}\cdot\frac{Gg}{G+g}\cdot\frac{Ff}{F+f}\cdot\frac{Ee}{E+e}\cdot\frac{Dd}{D+d}\cdot\frac{Cc}{C+c}$	$\frac{C+c}{2}$
Cc	$C+c$	$\frac{Cc}{C+c}$		
c				
D			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}\cdot\frac{Gg}{G+g}\cdot\frac{Ff}{F+f}\cdot\frac{Ee}{E+e}\cdot\frac{Dd}{D+d}$	$\frac{D+d}{2}$
Dd	$D+d$	$\frac{Dd}{D+d}$		
d				
E			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}\cdot\frac{Gg}{G+g}\cdot\frac{Ff}{F+f}\cdot\frac{Ee}{E+e}$	$\frac{E+e}{2}$

Ee	$E+e$	$\frac{Ee}{E+e}$		
e				
F			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}\cdot\frac{Gg}{G+g}\cdot\frac{Ff}{F+f}$	$\frac{F+f}{2}$
Ff	$F+f$	$\frac{Ff}{F+f}$		
f				
G			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}\cdot\frac{Gg}{G+g}$	$\frac{G+g}{2}$
Gg	$G+g$	$\frac{Gg}{G+g}$		
g				
H			$\frac{Ii}{I+i}\cdot\frac{Hh}{H+h}$	$\frac{H+h}{2}$
Hh	$H+h$	$\frac{Hh}{H+h}$		
h				
I			$\frac{Ii}{I+i}$	$\frac{I+i}{2}$
Ii	$I+i$	$\frac{Ii}{I+i}$	1.000	Ii
i				

[a] Assuming no correction to reading of Ii.

moving part is required.

We have now modified it by cutting gear teeth on the edge of the large disk and driving the disk with a stepping motor with 500 steps per revolution. The positioning accuracy is adequate and it is no longer necessary to use the ball in the conical indentations to locate the disk exactly. A Slo Syn [18] Tape Control System operates the linearity tester. The Slo-Syn system was available for use since it was incorporated in 1968 into the spectroradiometer described earlier by Sanders and Gaw [19]. The apertures may be selected by punched paper tape in the order described in table 1 or in any other desirable way. It takes about 5 s to rotate the disk 180° so it is faster to proceed following approximately the order of table 1. One program used is to go through the table to the bottom and then to repeat starting from the bottom up. Nine zero readings are spaced fairly uniformly through this measurement cycle. The results are recorded on punched cards by means of a digital voltmeter. The measurements are analyzed by computer and a table such as table 2 is produced for each half-cycle.

The top line gives the description of the test. The next line shows the zero readings in volts which were recorded. The next line shows the eight averages of the successive pairs of adjacent zeros. The data in this table were obtained between the first and fifth zero. The table then follows the pattern set out in table 1. The data in table 2 are for a silicon diode connected to an operational amplifier to produce an effective 0.1Ω load resistance. The diode was a Pin 10 from United Detector Technology and was connected in the photovoltaic mode to an operational amplifier as described by Witherell and Faulhaber [11]. The silicon diode had a flashed opal glass in front of its V(λ) correction filter. Each measurement made after the cell had been illuminated for about 11 s. A 1 s reading was taken by the integrating DVM. The correction ratios in column 4 for a ratio of 1 : 2 in flux show a maximum deviation of 0.0026 from 1.0 at the response of 0.00567 V. This corresponds to a photocurrent of 0.00567 μA. The correction factor based on zero correction at the maximum reading is given in colmn 5. Here the correction is farthest from unity at the middle of the range, with the maximum departure of 0.0014 at 0.011 V. There is considerable variation from one measurement series to another and a number of repeat measurements were made.

Table 3 shows the correction factors for various selected voltage outputs for twelve repeat measurements on the same silicon diode. The first six lines are for a 1.5 s delay and the next six for an 11 s delay. The first line shows the response voltages at which linear

TABLE 2. Computer Output for Nonlinearity Measurements on a silicon diode

11 2102720006 TEST DIODE X1 12.45 AMPS 10 S, 1 S COUNT
0.000100 0.000100 0.000100 0.000120 0.000150 0.000120 0.000130 0.000110 0.000110
0.000100 0.000100 0.000110 0.000135 0.000135 0.000125 0.000120 0.000110

READING NAME	NET READING	SUM	CORRECTION RATIO	FACTOR	READING AT WHICH FACTOR APPLIES
A1	0.002500		1.0000	1.0013	0.002880
A12	0.005760	0.005760			
A2	0.003260				
B1	0.005060		1.0026	1.0013	0.005670
B12	0.011370	0.011340			
B2	0.006280				
C1	0.010360		0.9996	0.9986	0.011300
C12	0.022590	0.022600			
C2	0.012240				
D1	0.024660		0.9994	0.9991	0.024600
D12	0.049170	0.049200			
D2	0.024540				
E1	0.051370		0.9997	0.9997	0.047885
E12	0.095740	0.095770			
E2	0.044400				
F1	0.103610		1.0007	1.0000	0.087310
F12	0.174750	0.174620			
F2	0.071010				
G1	0.207140		0.9997	0.9992	0.179910
G12	0.359710	0.359820			
G2	0.152680				
H1	0.309115		1.0001	0.9995	0.359185
H12	0.718455	0.718370			
H2	0.409255				
I1	0.779645		0.9994	0.9994	0.786410
I12	1.571915	1.572820			
I2	0.793175				

TABLE 3. Correction factors at selected voltages for successive measurements on a silicon diode. Measurements 1–6 with a 1.5 s exposure before the reading. Measurements 7–12 with an 11 s exposure before the reading

Meas. No. Volts	0.003	0.006	0.01	0.05	0.10	0.30	0.50	0.70	1.00	1.30	1.50
1	1.0038	0.9986	1.0006	1.0005	1.0002	0.9993	0.9993	0.9994	0.9995	0.9998	0.9999
2	1.0057	1.0022	1.0008	1.0000	0.9998	0.9992	0.9993	0.9994	0.9996	0.9998	1.0000
3	0.9930	0.9982	0.9989	0.9998	0.9994	0.9990	0.9993	0.9993	0.9995	0.9998	0.9999
4	0.9979	0.9962	0.9989	0.9999	0.9997	0.9993	0.9994	0.9994	0.9996	0.9998	0.9999
5	0.9985	1.0035	0.9967	0.9999	0.9997	0.9992	0.9993	0.9994	0.9996	0.9998	1.0000
6	0.9943	0.9960	0.9974	1.0003	1.0003	0.9996	0.9995	0.9995	0.9997	0.9998	1.0000
7	0.9951	0.9968	0.9988	0.9993	0.9994	0.9992	0.9993	0.9994	0.9996	0.9998	1.0000
8	1.0004	0.9987	0.9994	1.0006	1.0010	0.9996	0.9993	0.9995	0.9997	0.9999	1.0000
9	1.0025	1.0043	1.0022	1.0013	1.0013	1.0002	0.9999	0.9998	0.9998	0.9999	1.0000
10	1.0038	1.0003	0.9990	0.9994	0.9998	0.9991	0.9994	0.9996	0.9998	0.9999	1.0000
11	1.0013	1.0013	0.9992	0.9997	0.9999	0.9994	0.9995	0.9994	0.9996	0.9998	0.9999
12	0.9977	0.9943	0.9936	1.0012	1.0012	0.9998	0.9995	0.9996	0.9998	0.9999	1.0000
Ave. 1–6	0.9989	0.9981	0.9989	1.0000	0.9999	0.9993	0.9994	0.9994	0.9996	0.9998	1.0000
Ave. 7–12	1.0013	0.9990	0.9987	1.0002	1.0004	0.9995	0.9995	0.9996	0.9997	0.9999	1.0000

interpolations were made to find the applicable correction factor. The next line shows the measurement number in column 1, followed by the correction factors for each voltage. The correction factors are quite consistent from one run to the next at voltages from 0.3 to 2.0 V. The average correction factors are shown in the next to last line of table 3. The correction factor of 0.9995 is the farthest from unity. For this set of measurements the lamp used was a vacuum strip filament lamp.

The irradiance due to one aperture must not change when another aperture is opened. Interaction between one aperture and another may result from interreflections which depend on whether another aperture is open or closed. The precautions required to reduce these effects to the minimum possible have not been taken to date in the National Research Council (NRC) linearity tester. It seemed, previously, that the instability of the source or the receiver and its attendant electronic circuits, were the dominant sources of fluctuation and systematic error. Present developments suggest that some improvements would be warranted as follows. One could put antireflection coatings on all the optical surfaces, put light traps to collect major stray fluxes, make the aperture covers into light traps to prevent back reflected light when an aperture is closed and in general tighten up the experiment to ensure that all these effects produce negligible errors.

Nonaka and Kashima [20] made a series of measurements on several RCA photomultipliers of the types 1P21, 1P22, and 1P28. They used ten equal sized apertures in parallel light between two lenses which focussed the source on the receptor. Their measurements, made with a precision of about 0.1 percent, showed that the nonlinearity depended on the color of the light; on the position of radiation on the cathode; on the voltages (a) from the cathode to the anode, (b) from cathode to first dynode, and (c) from the last dynode to the anode. Nonaka and Kashima adjusted the radiation to obtain an anode current of 0.3 μA with all ten apertures open. They did not compensate for the voltage drop in the anode resistor and, hence, some nonlinearity will be caused by anode load feedback which results in a lower voltage from the last dynode to the anode as the anode current increases. At 500 V overall, the gain decreased with anode current, but at 1000 V overall, the gain usually increased with anode current. With 1000 V overall, the collection by the anode is complete, but the higher voltage produces a higher gain which with increasing incident flux causes enough voltage redistribution in the dynode chain to cause an increase in overall gain. This agrees with the analysis of Moatti [21] and Lush [22]. The nonlinearity was greater near the edge of cathode.

These measurements by Nonaka and Kashima, show that it is essential to measure the nonlinearities of a photomultiplier under the same optical and electrical conditions which will be used in obtaining the measurements to be corrected for nonlinearity. The nonlinearities can vary in the range ±1.0 percent when these conditions are changed.

Rotter [8] analyzed the possibilities which existed for selecting and providing the irradiances to be measured in testing the nonlinearity of receptors by the superposition method. His treatment assumed that the irradiances were provided by a single stable source. A lens system with apertures graduated in size provided irradiances related in several different ways. Rotter analyzed the system with respect to the work involved and the accuracy produced.

He divided the measuring systems into two main classes:

1. Method in which more than two irradiances may be received at the same time.
   - 1.1. Method with, n, nearly equal sized openings, i.e., 1 : 1 : 1 : 1 : . . . .
   - 1.2. Method with gradation of the openings by a factor of two, i.e., 1 : 1 : 2 : 4 : 8 : 16 : 32 : . . . .
2. Method in which, at most, two irradiances are received at one time.
   - 2.1. Method with steps of pairs of equal irradiances increasing in size from one pair to the next by a factor of two, i.e., 1 : 1 : 2 : 2 : 4 : 4 : . . . .
   - 2.2. Method of increasing the size by steps of 62 percent which was first suggested and used by Rotter. These steps are arranged in size, according to the equation

$$A_i + A_{i+1} = A_{i+2}$$

except that the first two irradiances are equal in size, i.e., $A_0 = A_1 = 1$. The series then becomes 1:1:2:3:5:8:13:21:34:55:89:144.

For each method, Rotter gave the order of measurements, and the order of calculating the additive correction. As an example of this, the measurement program for Method 2.2 follows:

Reading	Irradiance	Correction
$N_{1'}$	1′	k_1
$N_{1',1}$	1′ 1	k_2
N_1	1	k_1
$N_{1,2}$	1 2	k_3
N_2	2	k_2
$N_{2,3}$	2 3	k_5

It is noted that each aperture is used in three consecutive readings, except aperture 1′, and the largest apertures are used only twice. The corrections k_i are found from

$$k_1 = 0 \qquad \text{(arbitrarily chosen)}$$

$$k_2 = N_{1'} + N_1 + 2k - N_{1',1}$$

$$k_3 = N_1 + N_2 + k_1 + k_2 - N_{1,2}$$

$$k_5 = N_2 + N_3 + k_2 + k_3 - N_{2,3}$$

$$k_8 = N_3 + N_5 + k_3 + k_5 - N_{3,5}$$

The calculations to find the corrections, k_i, are thus easily determined by additions and subtractions in this method, whereas that presented by Sanders requires extensive mulplication.

Rotter's analysis showed that the Method 1.1 using n equal sized apertures was very expensive (in the number of measurements required) compared to the other three methods, if the range was greater than 1:4

Thus, for the range 1:8, Method 1.1 required twice as many readings. For the range 1:64, it required about ten times as many. However, Method 1.1 is more likely to be used with eight sources and will cover a range of 1:8. Then the source will be raised in intensity by a factor of eight to cover the next factor of eight on the scale. With this procedure, the equal sized aperture method would only require twice as many readings as the other three.

In Rotter's analysis of the errors in the corrections relative to the errors caused by the source variations, he found that Method 1.1 produced correction values with errors between two and three times lower than those produced by the other methods. Thus, at least four repetitions of the measurements in the last three methods would be required to produce measurements with the same error as those obtained by a single set of measurements by Method 1.1.

It thus seems that the choice of the best method must be made based on other criteria, such as cost of the equipment, ease of use, or minimization of drifts.

Rotter described three physical arrangements of

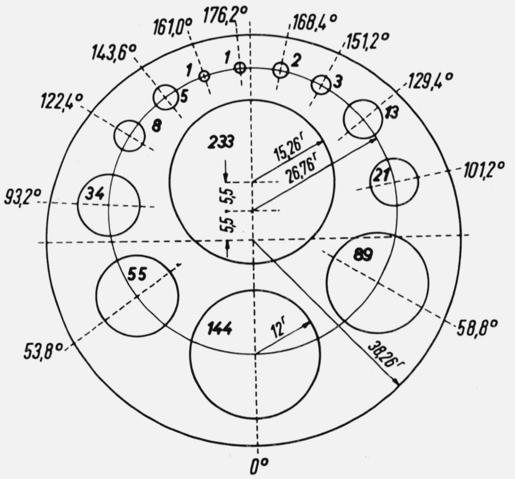

FIGURE 3. Rotter's aperture arrangement using 13 apertures with a sliding shutter on each. The angular position and relative diameter of each aperture is indicated. The outer circle indicates the outside diameter of the lens. (Courtesy, Messtechnik.)

apertures. One, reproduced in figure 3, has 13 apertures increasing in size by a factor of 62 percent, except for apertures 1 and 1′, which are equal. Each aperture is covered as required by a sliding shutter. The outer circle shows the size of the lens. This is a variation of Bischoff's [23] arrangement and makes good use of the lens area. The range covered is 1:377. The 13 sliding shutters make the method more complicated to use than that of Sanders or the one shown in figure 4 which was also given by Rotter.

This apparatus has two moving disks, B and C, with six apertures in each. The lens is outlined by the dotted circles enclosing both the smaller dotted circles D and E. These are apertures in a plate fixed in front of the lens. The numbers near the circles shown on the disks

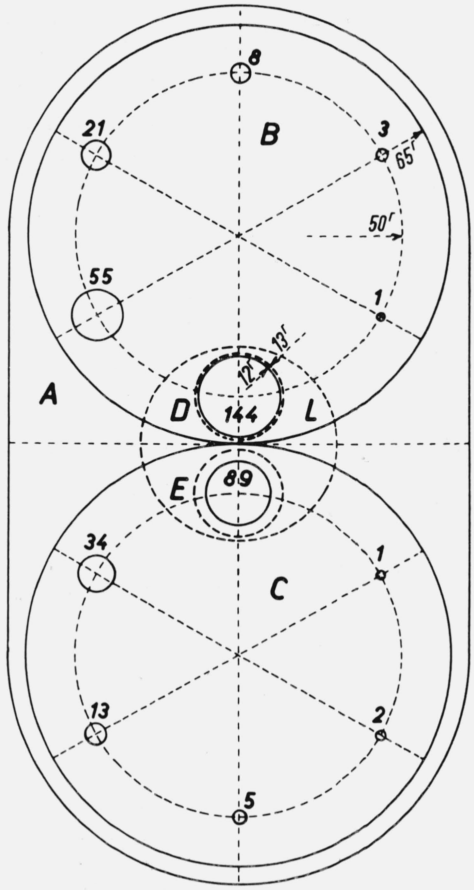

FIGURE 4. Rotter's aperture arrangement using two rotatable discs B and C with 6 apertures in each. Apertures D and E are in a disc before lens L with its outer diameter indicated by a dashed circle. (Courtesy, Messtechnik.)

indicate the relative area of these apertures. The disk B may be rotated to place any of the six apertures inside D or to cover D as required. The same applies to disk C relative to aperture E. The lens may be more effectively utilized if the holes D and E are oblong in shape and the largest holes in the disks are similarly distorted. This method of using two disks eliminates the requirement for a large number of moving parts. At the same time it requires less space and less accurate machining than Sanders linearity tester. Each disk remains stationary for three measurements as discussed above. The large disk in Sanders apparatus must be moved between the reading on A alone and the reading on A in combination with a. Thus there is a possibility of inexact reproduction of the flux through A or through a. Also, by using different disks in Rotter's apparatus, a different scaling of sizes may easily be selected. Also, disks B and C may be interchanged, which could be useful in eliminating certain errors.

2. Bouguer's Law— This law states that if filters a and b have transmittance T_a and T_b respectively, then the transmittance of the two filters placed one after the other in an optical beam, will be $T_a \times T_b$. Thus, if one knows the transmittance of each of a set of filters, one can combine them to produce various transmittances to be used to test a photometric system.

The validity of the method is not always certain. Hawes [1] noted difficulties caused by transmittances changing with change of temperature, with age, with cleaning techniques, with interreflections, with nonuniformity of the filters, and with bandwidth of the transmitted beam. He found the system useful if one did not depend on the long term stability of the filter transmittance and took the proper precautions to control temperature, interreflections, uniformity and bandwidth.

Hawes, Bischoff [23] and others have noted that Bouguer's Law does not check for nonlinearity unless the transmittance of at least one filter is known by an independent method. See below under A(4) for Hawes' method which combines superposition and Bouguer's Law. Hawes found it necessary to select filters sufficiently uniform that a 1-mm aperture moving over the filter detected no changes larger than 0.01 in absorbance. He described an apparatus to use with a Cary spectrophotometer in making these measurements.

3. Beer's Law—This law states that log $(1/T)$ is proportional to the concentration of the solute in the solution, where T is the transmittance of a constant thickness of the solution. This is only true for monochromatic light unless T is independent of wavelength over the full extent of the measured radiation. This method is used in chemical analysis where the techniques are available to make the solutions properly and where the scientists may depend on the law in their analyses and know when it will be valid. Problems have been noted with particles growing in the solutions. Interreflections may change with concentration and the law may not be quite exact in all cases.

4. Combination of the Superposition Method With Bouguer's Law to Test a Spectrophotometer—Hawes [1] gave a useful set of criteria which should be met in selecting a method for calibrating photometric linearity in spectrophotometers. Hawes' method for spectrophotometry cannot be transferred to broad band photometry because the absorbing glass filters which are available are not exactly neutral. He used three Chance ON 10 filters about 1 mm thick. Two had transmittances of about 0.49 and the third about 0.57. He made the apparatus shown in figure 5. The solid block of metal with the two large holes is screwed firmly in place in the test beam of the Cary. Any one or any combination of the three filters may be placed in slots in the metal block, B. The possible selection of holes is: both open, a open, b open, or both closed, respectively, when no slide, slide a, slide b, or slide c is in the slot. The three filters cover both beams and are oriented, by the retainers shown at R, to prevent interreflections from reentering the optical beam. The test compartment is optically remote from the cathode of the photomultiplier in the Cary, so the beams from the two apertures fall on the same area of the cathode. It was necessary to insert and remove the slides by means

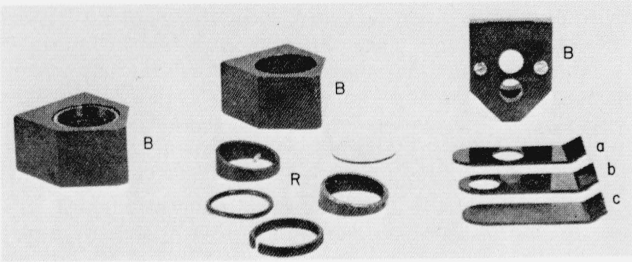

FIGURE 5. Dual aperture with filter assembly used by Hawes to determine and correct for nonlinearities. Fixed block B. Slides a, b, and c. Retaining rings R for three filters. (Courtesy, Applied Optics.)

of a flexible material in order not to disturb the position of the metal block. This procedure may also prevent temperature changes from being introduced by the fingers. It is not clear how the filters could be inserted and removed in the Bouguer Law tests without disturbing the position of the limiting aperture.

The slides produced a measurement at 100 percent on the scale and at two positions near 0.50 and at zero. Thus, using the superposition method, one could find the correction required near the middle of the scale and could use this to find corrected transmittances of filters 1 and 2. Hawes assumed that in the instrument the following relationship held over the whole scale:

$$I_{T'}/I_0 = T^{(1+e)} = T'$$

where e is the amount by which the exponent differs from unity, T is the true transmittance, $I_{T'}$ is the measured current with the filter in place, and I_0 is the current for 100 percent transmittance. Thus,

$$\log (I_{T'}/I_0) = (1+e) \log T = \log T'$$

and, from the measurements on filters 1 and 2 and those with the superposition method, e can be determined. Hawes calculated in absorbance rather than transmittance and applied a correction, eA, to each absorbance A.

Using two different photomultiplier tubes he found a difference of 0.07 percent in the transmittance when the two photomultipliers were used in the same instrument to measure the three component filter with a transmittance of 13.59 percent. The use of the value of e determined by the superposition method did not reduce the discrepancy. Hawes felt that the transmittance of one filter probably changed between the two sets of measurements. It thus remains to be shown whether the assumption of a constant value for e is valid and useful. If no sufficiently stable filters can be found, then it is questionable whether the corrections are necessary.

5. Inverse Square Law—the accuracies attainable with the inverse square law depend on the accuracy available in measuring: the distance from the source to the receiver, the uniformity with angle of the intensity of the source in the direction of the receiver, the size of the receiver, and the range of intensities required. It should be possible to refine the inverse square law method by measuring the angular distribution of intensity of the source over the largest solid angle which the receiver will subtend at the source. With this distribution and the cosine law to allow for the angle of incidence of the receiver surface, the flux incident on the receiver could be calculated for a number of distances. Changing the voltage on the lamp or inserting a filter would extend the range. Stray light due to reflections and due to diffraction caused by the baffles (see Blevin [24] would need to be given careful consideration.

6. *Standard Absorbing Filters*—See comments under Bouguer's Law. NBS filter sets described by Keegan, Schleter and Judd [25] are useful for detecting errors in a spectrophotometer and in keeping two or more spectrophotometers of similar geometry in agreement within a few tenths of a percent provided that the same filter set is used on both instruments.

7. *Standard Reflecting Materials*—Robertson and Wright [26] reported the results of measurements on grey ceramic tiles in a number of different laboratories. Standardized ceramic tiles [27], now available from National Physical Laboratory (NPL) or from the British Ceramic Tile Council, may be used for checking photometric linearity if the geometry of the spectrophotometer being used matches the geometry provided during the calibration.

8. *Rotating Sectors*—In some cases, sectors are used to reduce the average flux incident on a receptor or to measure the nonlinearity of a receptor. If the response produced by the intermittent flux is identical with that produced by a steady flux whose magnitude equals the mean magnitude of the intermittent flux taken over one period then the receptor is said to obey Talbot's Law. This does not necessarily mean that the receptor is linear. It just means that the nonlinearity is independent of whether the flux is chopped or steady. The faster the sector rotates the more likely it is that Talbot's Law will be valid. See parts B(1) and B(2) for applications of sectors to improve linearity of measurements. Kunz [3] describes methods used in accurate construction and calibration of sectors.

9. *Measurement of Amplitude of a Harmonic or the Beat Frequency*—Jung [4] described a very interesting, useful and rapid method of measuring the nonlinearity of receivers. In this method, two chopped beams of slightly different frequencies are combined on the receptor and the output of the receptor is examined at the difference frequency. Jung showed that the amplitude of this difference frequency is proportional to the nonlinearity if the nonlinearity is proportional to the flux. He also demonstrated that one could determine the time constant of the nonlinearity even if it was as small as a few milliseconds.

Jung assumed that

$$i = BP + CP^2 + DP^3 \tag{1}$$

with $|CP^2|$ and $|DP^3| \ll BP$ where B, C and D are constants, and i is the photocurrent and P is the incident flux. He defined NL_G as the nonlinearity measured by the superposition method at $i_1 + i_2$, where

$$NL_G\,(i_{1+2}) = 2\,\frac{i_{1+2} - (i_1 + i_2)}{i_{1+2}} \tag{2}$$

Using eq (1) to find the current, i_1 and i_2, for two fluxes $P_1 = P_2 = 0.5\ P$ and placing the currents in eq (2), we get

$$NL_G\,(i_{1+2}) \approx \frac{1}{B}\,(CP + \frac{3}{2}DP^2) \tag{3}$$

In the dynamic method, the flux, P_1, is modulated at frequency ω, and P_2 at ω_2, where $\omega_1 > \omega_2$. Jung proceeded to use a Fourier expansion to show that $A\ (\omega_1 - \omega_2)$, the amplitude of the frequency $\omega_1 - \omega_2$ is given by

$$A(\omega_1 - \omega_2) = \frac{1}{\pi^2}\,(CP^2 + \frac{3}{2}DP^3) \tag{4}$$

where the basic frequency ω_1, had the amplitude $A\ (\omega_1) = BP/\pi$. Jung then defined $NL_D(\hat{\imath})$ the nonlinearity at the maximum current, $i = i_{1+2}$, by

$$\begin{aligned} NL_D(\hat{\imath}) &= \pi A(\omega_1 - \omega_2)/A(\omega_1) \\ &= (1/B)\ (CP + (3/2)\ DP^2 \end{aligned} \tag{5}$$

under this assumption $NL_D = NL_G$ but the assumption of eq (1) is too simple because the photocurrent does not instantaneously change when P changes. Thus, Jung found it necessary to postulate

$$i = BP + CP^2\ (1 - \epsilon^{-(t/\tau)}) \tag{6}$$

where P is a function of time, τ is the time constant of the nonlinearity, and t is the time from commencement of the steady signal. By use of suitable expressions for i and P as a function of time and applying a Laplace transformation, Jung found that

$$NL_D\ (i, \omega\tau) = CP/B\ (1 + (\omega t)^2)$$

Thus by eq (3) with $D = 0$ and assuming that in the superposition method $t > \tau$, the relationship between NL_G and NL_D is

$$NL_D = NL_G/(1 + (\omega\tau)^2) \tag{7}$$

If the frequency of the chopper is 200 Hz and $\tau = 1$

ms, then $NL_D/NL_G = 0.4$. Thus one can find τ from the ratio of NL_D/NL_G.

The experimental arrangement used by Jung is illustrated in figure 6. The oscillating shutters, S_1 and S_2, operate at frequencies of 213.5 Hz and 200 Hz, respectively. The semitransparent plate, SP, combines the beams from the lamps, L_1 and L_2, so they illuminate the receiver, R, identically. An interference filter, IF, could be used to isolate a narrow band of wavelengths.

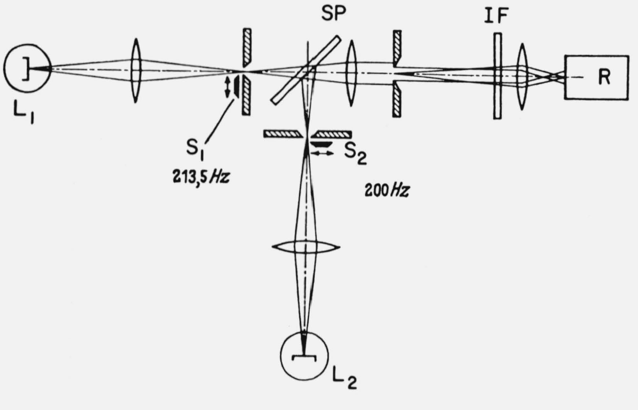

FIGURE 6. Jung's optical apparatus for dynamic measurement of nonlinearities. The semireflecting plate SP combines radiation from sources L_1 and L_2. The radiation is chopped by shutters S_1 and S_2. (Courtesy, Z. Angew. Physik.)

The measuring circuit Jung used is shown in figure 7. The oscillating shutters, S_1 and S_2, were driven by independent generators, G_1 and G_2, through power amplifiers, A_1 and A_2. The signals from G_1 and G_2 were fed to a mixer to produce the beat frequency at 13.5 Hz which was filtered by a filter, RC, to attentuate the base frequencies by 80 db. A phase shifter, PS, was required before the phase sensitive rectifier, PSR, and was adjusted to give maximum signal at the digital voltmeter, DVM. Provision was made for using either a photomultiplier, PMT, or a photodiode, Si. They could be connected at points A and B. In addition, B could be connected to either terminal 1 or 2. The first of these connections produces a voltage in the impedance transformer which keeps the voltage constant from the anode of the photomultiplier to the last dynode. The input impedance of the transformer is greater than 10^{11} Ω. This connection keeps the collection efficiency of the anode constant. The nonlinearity obtained by Jung using connection 1 for B is as shown by the triangles in figure 8. With B connected to position 2, there is a voltage drop produced in R_A by the anode current which causes the collection efficiency of the anode to decrease and causes nonlinearities, as shown by the crosses in figure 8. Jung's measurements by the dc superposition method with the point A connected to position 2 gave the results shown by circles.

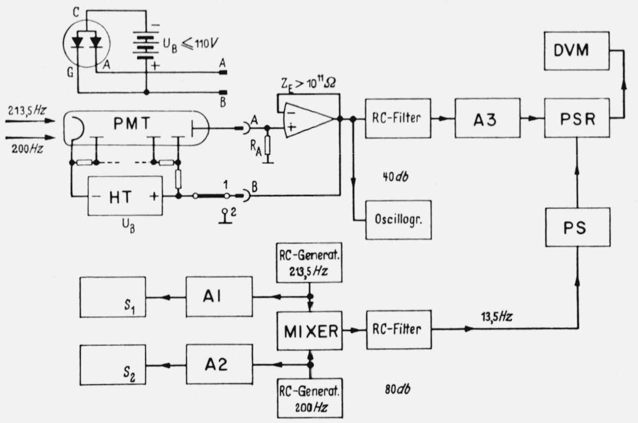

FIGURE 7. Jung's electronic apparatus for dynamic measurement of nonlinearities. A1, A2, and A3 are amplifiers. S_1 and S_2 are shutters. PS is a phase shifter and PSR a phase sensitive rectifier. (Courtesy, Z. Angew. Physik.)

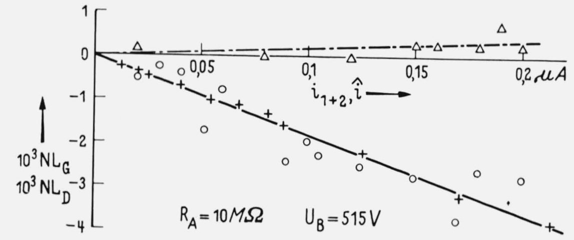

FIGURE 8. Jung's measurements on an EMI 9558 photomultiplier. Measurements without anode voltage compensation by the dc and dynamic method are shown respectively by circles and crosses. Those by the dynamic method with anode voltage compensation are shown by triangles. (Courtesy, Z. Angew. Physik.)

There is much more scatter in the dc measurements than in the dynamic measurements. This instability may be partly caused by instability in the source, but also by drifting dark current caused by the large changes in average flux incident on the photomultiplier. Kunz [3] describes this effect in detail and his findings will be discussed further in B(1). Since the two measurements made without compensation of anode drop by Jung agree within the scatter, it is not possible to estimate a value of τ from eq (7). It only suggests that $\omega\tau \leq 1$.

As shown by the triangles in figure 7, the nonlinearity is much less when the change in the voltage across the anode resistor is compensated by connecting B to position 1. The sign of the nonlinearity is reversed as one would expect from Moatti [21] (see B4 below).

Jung's measurements on a silicon photodiode with 24 V bias and connected to give compensation of the voltage drop showed good agreement between the dynamic and dc method. The nonlinearity in this case was about 0.8 percent at 1 μA photocurrent.

Bressani, Brovetto and Rucci [28] described a method of testing nonlinearity of a photomultiplier by modulating the flux on a slit in front of a photomultiplier. The method depends on the modulation being exactly a sine function. Any nonlinearities in the receiver will produce the second harmonic which is selected by a lock-in amplifier whose reference channel is driven by a frequency doubler from the oscillator, which also drives the oscillating mirror which modulates the beam. The 1 percent error in the linearity measurements was said to be due to the lock-in amplifier. This does not compare very well to the error of the phase sensitive rectifier used by Jung which Jung claimed was responsible for 0.05 percent of the error. The illumination at the slit must be uniform in Bressani's method and nonuniformity of sensitivity over the cathode must not be able to interfere, if the nonlinearity is to be measured correctly. This makes the potential of the method much lower than that of Jung's method using the beat frequency.

10. Measurement of Radiant Fluxes to Test Linearity—To the series of steps of irradiance which Rotter described, we should add that described by Erminy [2]. Erminy used three strip filament lamps,

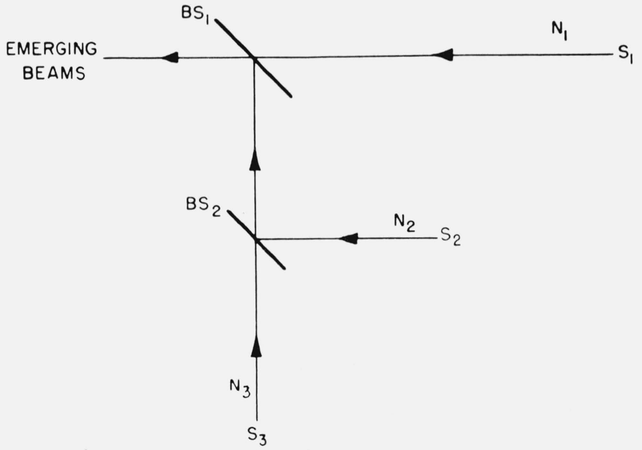

FIGURE 9. Erminy's arrangement using 3 sources for producing integral and fractional multiples of a radiance. (Courtesy, J. Opt. Soc. Amer.)

as shown in figure 9. The current on each lamp could be adjusted to provide selected levels of radiance. Each source could be blocked off with a shutter. Starting with N_0, Erminy showed how to obtain the radiances $mN_0/2^i$ where

$$m=1, 2, 3 \ldots \text{ and } i=1, 2, 3 \ldots .$$

Erminy used the device to obtain radiances which could be matched to the radiance of another lamp by a null detector. He produced in this other lamp a radiance scale obtained by additive means. The detector was not required to be linear or to have a good stability over a long period of time. Thirty seconds were required for each measurement.

In testing the nonlinearity of photocells, we are assuming their stability. The adjustment routine used by Erminy to provide a variety of radiances can also be used to provide irradiances with the same multiples, $mN_0/2^i$, and thus may be used to check the nonlinearity of photocells at a much greater variety of irradiances than can be obtained by the use of two sources. In producing an irradiance scale, one could, by appropriate optical means, adjust the irradiance from each source without changing the lamp current on the three lamps. An iris diaphragm, distance variation neutral wedge, etc. could be used with each source.

The two-step method can only produce irradiances differing by a factor of two, so one is very dependent on having a device which changes smoothly in nonlinearity as the irradiance increases. Erminy's arrangement can help to avoid such an assumption.

11. Null Method—Lee [29], in establishing the NBS pyrometric temperature scale, used Erminy's method with three strip filament lamps to calibrate a fourth lamp at 14 currents, so the radiances were in the proportion 0.007812 : 0.015625 : 0.03125 : 0.0625 : 0.125 : 0.25 : 0.5 : 0.67 : 1.0 : 1.5 : 2.0 : 3.0 : 4.0 : 6.0. Using $I=P(T_r)$, where $P(T_r)$ is a power function of T_r, he found seven coefficients to fit the 14 radiance temperatures T_r corresponding to the 14 currents. Tables were made using the equation so current could be related to temperature. A similar process could produce a (completely adjustable) radiance standard so the detector would need only serve as a null detector. It takes seconds for the lamp to stabilize at each temperature, but there may be some circumstances where this method should be used. The paper by Lee [29] describes the behaviour of strip filament lamps. Vacuum lamps are more stable than gas filled lamps, provided the temperature is kept low.

12. Use of a Linear Receptor to Calibrate a Test Receptor—Edwards and Jeffries [30] used a linear planar photodiode as a reference detector to measure the linearity of silicon diodes using a cathode ray oscilloscope as the display element. They were able to detect nonlinearities of 3 percent over a range of 10^5:1. Their applications involved measurements of pulses down to 3 ns duration. For slower tests, it should be possible to use, as a linear reference device, a vacuum photoemissive diode with an anode subtending almost all the solid angle surrounding the cathode.

B. Method of Increasing Linearity and Accuracy

1. Selection and Operation of Photomultipliers for Measurement of High Flux Ratios—Kunz [3] in a paper describing the development of a temperature scale above the gold point, listed the following 12 points to consider in the selection and use of photomultipliers:

(1) Photomultipliers with nonfocusing dynode systems are more suitable than other types.

(2) Photomultipliers with glass bases show smaller leakage currents than those with plastic bases.

(3) One should seek a photomultiplier with the smallest possible dark current leakage at the anode and dynode, e.g., 2.5×10^{-12} A at step voltages of 30 V. The insulation test should take place several hours after washing the base with very clean solvents and distilled water.

(4) Photomultipliers should be selected for the most constant possible sensitivity under changing illumina-

tion. In a check of the drift situation, particular attention should be paid to the drift following a decrease in illumination.

(5) For measuring uncertainties below 0.05 percent the EMI 9558 A photomultiplier must be operated only at photocurrents below 3×10^{-8} A.

In B2 and B3 below, Jung has shown that operation at considerably higher currents under special conditions can produce results of similar accuracy.

(6) With the EMI 9558 photomultiplier tube, operation with five (or eight) dynodes is possible, and, especially at small gains of 10 to 1,000 (or 30 to 10,000), is more advantageous if an initial drift causes interference when all stages are in use.

(7) The photomultiplier and its anode resistance must be installed in a housing with high thermal inertia or in a thermostatted housing.

(8) The anode resistance must have small voltage and small temperature coefficients.

(9) Anode feedback should be avoided by compensation or the provision of proper resistances.

(10) The capacitor in parallel with the anode resistance must have very low dielectric absorption.

(11) Interferences due to dielectric absorption in the electrometer input and its connecting cable must be avoided by suitable selecting or handling.

(12) In addition to the recommended application of rotating sectored disks for attenuation of the radiation, it may be necessary to incorporate simple supplementary filters in vibrating capacitor electrometers.

Kunz followed these rules and showed that an EMI 9558 was linear to about ± 0.005 percent at final electrode currents from 0.03 to 20 nA with an overall gain of either 30 or 300 and using the fifth dynode as the final electrode.

Kunz's paper should certainly be consulted by anyone wishing to improve their radiometric techniques.

2. Improving Linearity Using a Sector—After developing the dynamic (beat frequency) method of measuring nonlinearity and finding the relationship between NL_D and NL_G, Jung [5] proceeded to describe two methods of increasing the linearity of radiation measurements. The first method is to attenuate the strongest, P_2, of the two radiations which are being compared, with a sector with an opening which is such that the open time, D_S, relative to the period, T, is given by $D_S = P_1T/P_2$. With this arrangement, the average irradiance from P_2 which reaches the receiver is equal to $P_2D_S/T = P_1$. With the assumption of eq (6) for the type of nonlinearity and using a Fourier treatment, Jung showed that $\epsilon_D/\epsilon = 0$ for $\omega\tau \rightarrow \infty$ and $\epsilon_D/\epsilon = 1$ for $\omega\tau \rightarrow 0$. In this case, ω is 2π times the chopping frequency, and τ is the time constant of the nonlinearity. The errors, ϵ_D and ϵ, are respectively, the nonlinearity errors in the measurement with and without the attenuating sector. Thus if the nonlinearity appears much faster than the chopping period, the sector will not reduce the error at all, but if the nonlinearity appears slowly relative to the chopping period, the sector will reduce the nonlinearity. Note that the nonlinearity assumed in eq (6) is a nonlinearity proportional to the flux.

3. Improving Linearity by Adding a Steady Flux to the Weakest Flux of Two Chopped Radiations—Since some receivers, such as the silicon diode measured in figure 14, have a nonlinearity which is not proportional to the irradiance, Jung developed another method of increasing linearity in a receiver which has a nonlinearity that appears very quickly. The apparatus for comparing the radiant flux from P_1 and

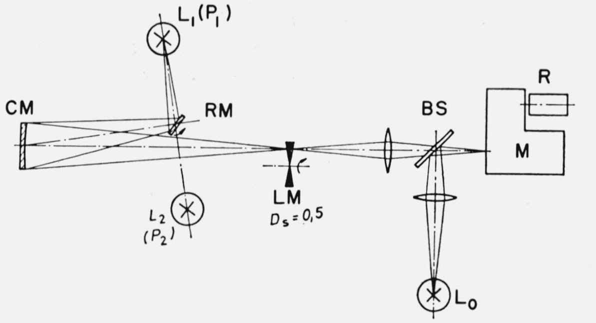

FIGURE 10. Optical arrangement for photometry of two radiances P_1 and P_2 selected by mirror RM. P_1 and P_2 are chopped. An adjustable radiation P_0 is added to the weaker radiation. (Courtesy, Z. Angew. Physik.)

P_2 by this method is illustrated in figure 10, taken from Jung. The rotatable mirror, RM, directs the radiation from either L_1 to L_2 to the concave mirror CM. The radiation is then chopped by LM, a 50 percent sector. During the time when radiation, P_1, from source, L_1, is falling on the receiver, R, radiation P_0 from L_0 is also incident on the receiver. P_0 is adjusted so the average chopped flux on the receiver from P_1 plus P_0 is equal to the average flux from P_2. The variation with

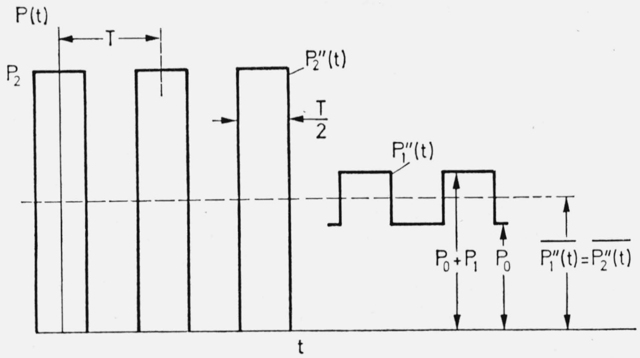

FIGURE 11. Time dependency of radiation on receptor. Radiations P_0, P_1 and P_2 are indicated. After chopping and recombination either $P_1''(t)$ or $P_2''(t)$ is incident on receptor. (Courtesy, Z. Angew. Physik.)

time of the incident flux on R is shown in figure 11. P_0, P_1, and P_2 indicate the steady fluxes. $P_1''(t)$ and $P_2''(t)$ indicate the respective incident flux depending on whether the mirror selects P_1 or P_2. The shutter on P_0 is also opened in the first case. A phase sensitive rectifier permits the chopped signal $P_1''(t)$ to be separated from the steady signal resulting from P_0. Again

Jung provided a theoretical treatment and showed that the nonlinearity should be eliminated completely if the nonlinearity is proportional to the flux.

If the photocurrent obeys eq (1), Jung showed that the quadratic part of the nonlinearity should be reduced by a factor of four.

Jung tested an EMI 9558 QB photomultiplier with the voltage divider network shown at the top of figure 12. This circuit provides a positive anode current-proportional nonlinearity of about $2\times10^{-3}\cdot\mu A^{-1}$ with a rise time of a little more than 1.3 ms. The rise time is caused by the capacitors across the last three stages. He used three methods of measurement:

(1) the usual dc superposition method,
(2) the superposition method where the combined beam was attenuated with a 50 percent sector,
(3) the dynamic method with the average frequency 206.5 Hz of the two signals.

The nonlinearity was measured over a wide range of anode currents. The respective results for the three

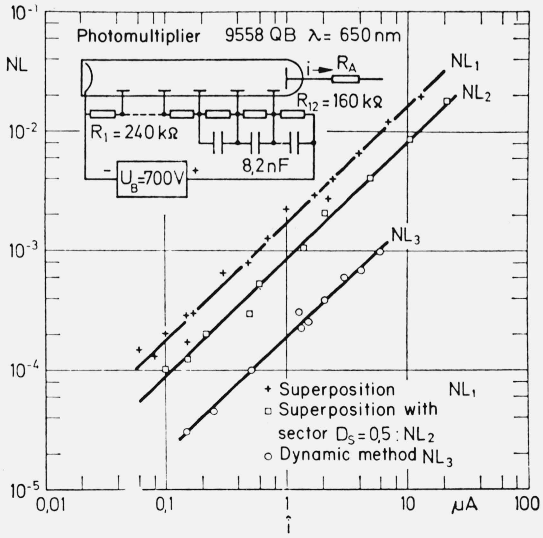

FIGURE 12. Jung's measurements of the nonlinearity of an EMI 9558 photomultiplier using 3 methods. (Courtesy, Z. Angew. Physik.)

methods are shown by NL_1, NL_2, and NL_3 in figure 12. The largest nonlinearity was measured by the first method. The sector attentuation of method 2 reduces the nonlinearity by a factor of 2. The dynamic method measures a nonlinearity less by a further factor of nearly 5. Jung showed that these results were consistent with the theory and indicated a rise time for the nonlinearity of 2.1 ms.

It remained for him to demonstrate experimentally that the method of providing equal average photocurrent by adding a steady flux to the weakest signal would produce improved linearity. For this purpose, he built the apparatus shown in figure 13. The equalization of average signal is accomplished by a voltmeter measuring the average voltage after IC, the impedance con-

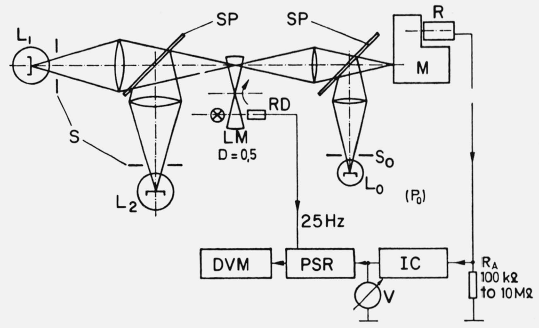

FIGURE 13. Apparatus for confirming the method of figure 10 by superposition. Radiation from sources L_1 and L_2 may be recombined or measured separately. Radiation from adjustable source L_0 is added until the voltmeter at the output of impedance converter IC reads a constant value. Phase sensitive rectifier PSR provides a signal to digital voltmeter DVM. Detector RD provides synchronization from chopper LM. (Courtesy, Z. Angew. Physik.)

verter. This is followed by PSR, the phase sensitive rectifier (controlled by a signal picked off RD, a detector, which is actuated by LM, the modulator) and a digital voltmeter. The results obtained by Jung are given

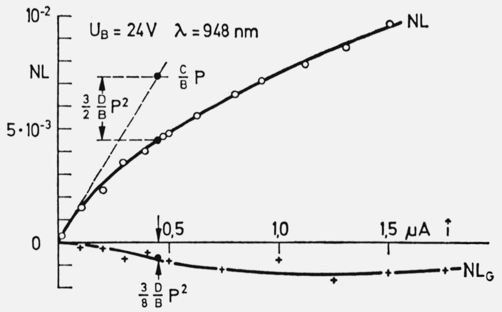

FIGURE 14. Jung's measurements of the nonlinearity of a silicon diode. NL for measurements by the dynamic method. NL_G for measurements with equalization of average current on the receptor. (Courtesy, Z. Angew. Physik.)

in table 4; column 1 shows the maximum anode current for each ratio of 2:1. Column 2 shows the nonlinearity for the method of figure 6, and column 3 the nonlinearity for the usual dc superposition method. The values in column 2 fluctuate in the range $\pm0.2\times10^{-3}$, while the values in column 3 increase with anode current to 30×10^{-3} at 19 μA. We can see that Jung achieved a really vast improvement in linearity by maintaining the average light constant by adding a steady light during the presentation of the weaker flux.

After these measurements for a ratio of 1 : 2, Jung made measurements of P_1 relative to P_2 for ratios of 1 : 30 and 1 : 100. To obtain the true ratios, he measured i for each source without a modulator and corrected the i values using the values obtained from curve NL_1 of figure 12. The corrected ratio is given by

$$\frac{P_1}{P_2}=Q_{=}=\frac{i_1(1-NL_1(i_1))}{i_2(1-NL_1(i_2))}$$

The results for $Q_=$ are shown in column 4 of table 5. The ratio $Q_\sim$ from the use of P_0 to create constant average flux is formed without a correction of any kind. The values are given in column 6 of table 5. Column 7 shows the fractional difference between the two methods for the two flux ratios. These are respectively equivalent to agreeing in the measurement of transmittance of 0.03 to within 0.04 percent of the transmittance and agreeing in measuring a transmittance of 0.01 to within 0.06 percent of the transmittance.

TABLE 4

Max. Anode Current	Nonlinearity	
i μA	NL_0 (i = const)	NL_1 ($i \neq$ const)
0.20	$-0.18 \cdot 10^{-3}$	$0.35 \cdot 10^{-3}$
.40	.00	.70
.80	−.02	1.35
1.00	−.48	1.70
1.46	−.20	2.50
1.72	−.09	2.90
1.98	.21	3.30
2.78	.07	4.6
4.96	−.09	8.2
19.05	.10	30.0

pliers in photometry.

Finally, Jung applied the constant average light method to a silicon diode with a bias voltage of 24 V and obtained the points marked by crosses and the curve labelled NL_G in figure 13. The measurements for a ratio of 1:2 and using the usual dc superposition method are shown in figure 14 by circles and a curve marked NL. The values, NL_G, with a maximum of about 1.0×10^{-3} at 1 μA are much less in absolute value than the values NL. This shows that the equalization method also increases the linearity of receivers with curved nonlinearity characteristics. The nonlinearity may be high for a diode with a bias voltage applied. The bias makes the response fast and Jung may have used it biased for that reason.

4. Moatti [21] showed that in using a voltage divider network of equal resistors, to provide the voltages to the dynodes, one should expect a decrease of gain with anode current. Moatti suggested fixing the voltage from last dynode to anode. Lush [22] showed that this would produce an increase of gain with anode current and suggested introducing some resistance, in series with this fixed voltage, in the circuit from last dynode to anode. He showed that the optimum value of this resistance depended on the maximum anode current which would be drawn relative to the current in the voltage divider network. Land [31] experimented with the straight resistance network

TABLE 5

	D.C. method ($i \neq$ const)			Chopping method (i = const)		
	i μA	NL(i)	$Q_=$	i' μA	$Q_\sim = \frac{i_1}{i_2}$	$\frac{Q_\sim}{Q_-} - 1$
1	10.0909	$16.2 \cdot 10^{-3}$	32.295	10.0710	32.306	$0.34 \cdot 10^{-3}$
2	0.30756	$0.53 \cdot 10^{-3}$		0.31174		
1	12.2923	$20.0 \cdot 10^{-3}$	96.524	12.2048	96.580	$0.58 \cdot 10^{-3}$
2	0.12483	$0.22 \cdot 10^{-3}$		0.12637		

This is indeed a remarkable achievement, especially since one method requires no corrections for nonlinearity and the other requires a series of measurements of nonlinearity which must be combined to produce the final correction. It should be noted that Jung used vacuum tungsten strip lamps as sources with currents stabilized so their irradiance have a stability with time of 2×10^{-5}. The measurements were made with a photomultiplier at maximum currents up to 12 μA. Jung felt that the maximum measurement uncertainty was $\pm 0.7 \times 10^{-3}$ and resulted from noise and the nonlinearity of the phase-sensitive rectifiers.

If the procedure had been to add steady light to the weakest of two unchopped signals, in order to make a null reading, the added flux would need to be known to 0.0006 percent to achieve the same accuracy, but, in this constant average light method, P_0 need not be known. With the useful detail given by Jung we should all be able to make better use of photomulti-

and found that the changes in gain were about 1 percent if the ratio of the anode current to the current in the resistance network was less than 0.1. He suggested that this was good enough since instabilities would be this large.

W. Van de Stadt [32] in a short letter noted that "after exposure to a relatively high light flux (say, upwards of 10^3 times the NEP) the tubes exhibit 'memory', i.e. the anode current does not decrease to its previous dark current level after removal of the incident light flux. This memory effect may take several seconds to decay and is dependent in magnitude and duration on the light flux that generated it. The tube does recover to its original NEP given enough time."

Kunz [3] gave a detailed report on his experiences with dark current. His measurements on EMI 9558 photomultipliers gave results which indicate a time constant, of change in sensitivity after changing the incident flux, of between 8 and 16 min depending to some extent on the anode current. The change of sensitivity was − 0.1 percent at 80 nA, − 0.4 percent at 330 nA and − 0.7 percent at 860 nA.

Kunz measured the dark current at the anode of an EMI 9558 photomultiplier tube with a gain of 300 and found that the dark current increased rapidly as the anode to last dynode voltage was increased. The anode current at 30 V was 1×10^{-12} A increasing to 5×10^{-12} A at 50 V. He found hysteresis with the current remaining higher, as the voltage was reduced, than it had been at the same voltage as the voltage was increased. When he measured the dark current at any of the dynodes from 5 to 10 he found that the hysteresis was reversed. The sign may be reversed because the dynodes emit photoelectrons, but the anode does not. However, on the dynodes the increase of dark current was fairly linear with last stage voltage. Because of the nonlinearity of dark current on the anode and because he found abrupt changes of anode dark current, Kunz preferred to use the measurements of current at one of the dynodes, rather than at the anode. He also wanted to use a low gain in most cases because he found it desirable to keep the current at the final stage below 30 nA.

5. Witherell and Faulhaber [11] discussed the operation of silicon diodes for photometric applications and gave data for silicon diode nonlinearity as a function of load resistance and cell current. They use the cell in the photovoltaic mode, i.e. without the bias used by Jung. They showed that the linearity improved by a factor of almost 15 for every decade decrease in load resistance. The nonlinearity also increased proportionally with cell current. With a load resistance of 1 Ω they found that the maximum nonlinearity was 0.1 percent for cell currents ranging from 50,000 nA to 0.5 nA.

For the short circuit condition the device is insensitive to temperature, about −0.1 percent/°C at 550 nm. They give the connections of the operational amplifier and also the properties of the amplifier and the cell which must be considered in using the cell in photometric applications. The bias voltage speeds up the response time so this will make Jung's measurements on the response time of the biased silicon diode inapplicable to the circuit used by Witherell and Faulhaber. Jung [4] measured at several wavelengths and found the nonlinearity of silicon diodes much more severe at long wavelengths. It would be interesting to know whether the same applies in the photovoltaic mode of operation.

6. Jones and Clarke [6] showed that it was possible to use a photomultiplier as a null device and thus to compare a larger flux, after transmission by a sector, with a weaker light which was unattenuated. They measured the opening in a adjustable sector by a photoelectric timing device. This method takes advantage of the fact that some receivers obey Talbot's Law, although they may be nonlinear when used to compare two steady irradiances of different size. The Jones and Clarke method can produce digital data directly from the timing procedure.

7. Jones and Sanders [33] showed that keeping the input voltage to an electrometer constant would improve the precision of measurements. This potentiometric method used an electrometer amplifier as a null device. It served at the same time to keep the voltage constant from cathode to anode of the vacuum photocell. This increased its linearity. The vacuum cell used had a cylindrical anode with the cathode at one end of the cylinder. The resultant device was precise and linear, but inconvenient to use since some knowledge of the signal was required in advance. Jones [34] later described a circuit which avoided this requirement and made it possible to measure accurately using a digital voltmeter. Similar techniques now use operational amplifiers.

8. Sauerbrey [35], in a private communication of his latest extension of Jung's method of constant average anode current, was able to show that with an average anode current of 10 μA on an EMI 9558 photomultiplier tube, it can be used with a nonlinearity of less than 0.1 percent. He investigated the effect of cathode-first dynode voltage, last dynode-anode voltage and second last dynode-last dynode voltage.

9. Schanda and Szigeti [36] showed that 50-W tungsten-halogen lamps with a single coil made by Tungsram were more stable in short term use than a 200-W lamp with a coiled-coil filament. The use of the former type could, therefore, improve results of spectrophotometry or spectroradiometry.

10. Davies [37] has recently shown that photomultipliers cleaned thoroughly, painted with silver paint which is connected to the cathode by a 10 MΩ resistor, and the use of the tube with no socket will reduce the dark current by as much as 1000 times. The dark current is more stable and is not increased after operation with anode currents up to 10 μA.

IV. Conclusions

The fundamental method of testing linearity is the superposition method. Several methods have been described and used to produce accuracies of 0.05 percent or better. Jung [4] has shown that nonlinearities can be measured quickly and precisely by a beat frequency method. His measurements do not always apply to the receptor as actually used, but his measurements suggested a method [5] of using receptors so they behave linearly or very nearly so. This method involves keeping the average anode current constant. The flux to be measured is chopped, a steady flux is added to the weaker and the alternating response is separated by a phase sensitive rectifier. The results are linear to 0.05 percent without correction for nonlinearity.

The accuracy of the phase sensitive rectifier may limit the accuracy of Jung's method. The accuracy and nonlinearities of the rectifier system would need to be less than 0.01 percent if the error from this source is to be negligible.

Jung's method can only be used to measure steady radiation. For discharge lamps operated by ac, one might make a null comparison between a steady source and the periodic test source using one photometer. The steady source could then be measured using Jung's system. These measurements would need to be corrected for the residual nonlinearity as measured by the superposition method. This residual nonlinearity would be caused mainly by the increased space charges at higher photocurrents.

Sauerbrey has recently shown that with appropriate methods, a photomultiplier can be made to behave linearly, within 0.1 percent, at anode currents up to 10μA. This is in contrast to Kunz who used dc measurements and found that he needed to limit the maximum current to 0.03 μA on the last electrode to obtain results accurate to ± 0.005 percent.

Sauerbrey indicated that a photomultiplier tube which he tested did not obey the $1+e$ power function assumed by Hawes. It would, therefore, be advisable to check Hawes' assumption by the superposition method at several levels before using it in the Hawes' combination method.

For accurate measurements, tungsten strip filament lamps operating in a vacuum have been found necessary because of their better stability compared to gas filled lamps.

The receptor and its associated electronic circuit should be tested and used under the same conditions, if the measured nonlinearities are to be applicable. The nonlinearities are considerably altered by changing the resistance circuit in the dynode chain of the photomultiplier. Different resistance ratios are optimum for different types of tubes.

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