

# The Viscosity and Thermal Conductivity of Dilute Gaseous Hydrogen from 15 to 5000 K\*

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(November 26, 1969)

Measurements of the viscosity and thermal conductivity of dilute gaseous para and normal hydrogen are critically evaluated and correlated by means of dilute gas kinetic theory. Numerical results are presented from 15 to 5000 K including the dissociation region.

Key words: Dissociation; hydrogen; intermolecular potential functions; kinetic theory; quantum gas; thermal conductivity; viscosity.

We discuss the critical evaluation of the viscosity and thermal conductivity coefficients of dilute *p*-, *o*-, and *n*-hydrogen and present tables of values for para and normal hydrogen. New information—particularly with regard to experimental data from our laboratory—justifies a reevaluation of these properties of this well-known fluid. The tables given here differ significantly at the extreme temperatures from those published previously.

The coefficients are represented by standard kinetic theory expressions but the calculations are more than mere routine; to give two examples, at low temperatures, hydrogen behaves as a quantum fluid and at high temperatures it dissociates. Furthermore, to apply the theoretical expressions in practice one needs to evaluate critically experimental data in the first place, which is not a cut-and-dried problem at all.

## 1. Formal Equations

The required kinetic theory equations are given and briefly discussed in this section [1–4]<sup>1</sup>.

### 1.1. Viscosity of Molecular Hydrogen

The viscosity,  $\eta$ , is given to a second approximation by

$$\eta = \eta(1) \left\{ 1 + \frac{3}{196} \left[ 8 \frac{\Omega^{(2,3)*}}{\Omega^{(2,2)*}} - 7 \right]^2 \right\} \text{ g cm}^{-1} \text{ s}^{-1} \quad (1a)$$

\*Work carried out at the National Bureau of Standards under the sponsorship of the Office of Standard Reference Data and the National Aeronautics and Space Administration (SNPO-C).

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<sup>1</sup>Figures in brackets refer to the references given at the end of the paper.

with

$$\eta(1) = \frac{5}{16} \left( \frac{\sqrt{\pi m k T}}{\pi \sigma^2 \Omega^{(2,2)*}} \right) \text{ g cm}^{-1} \text{ s}^{-1} \quad (1b)$$

where  $T$  is the temperature (K),  $m$  the weight of a molecule,  $k$  Boltzmann's constant,  $\sigma$  a cross-section parameter, and  $\Omega^{(l,s)*}$  ( $l, s = 1, 2, 3$ ) are the collision integrals. Equation (1) is applicable to both classical and quantum fluids provided one properly modifies the collision integrals. For example, in the classical case the collision integrals involve the classical cross section  $Q^{(l)}$  where

$$Q^{(l)} \sim \int_0^\infty (1 - \cos^l \chi) b db \quad (2a)$$

$\chi$  is the angle of deflection and  $b$  is the impact parameter, and are computed using classical statistical mechanics. In the quantum case, however, the collision integrals involve the quantum mechanical cross section given by

$$Q^{(l)} \sim \int_0^\pi (1 - \cos^l \chi) I(\chi) \sin \chi d\chi \quad (2b)$$

and are computed via the quantum mechanical phase shifts.  $I(\chi)$  is the differential scattering cross section. In both cases the viscosity involves the intermolecular potential function  $\phi(r)$  through the scattering angle.

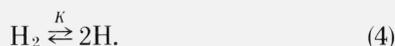
The integrals of eq (1) have been reduced by dividing by the values for a hard sphere of diameter  $\sigma$ . To agree with previous work on hydrogen [4] we define here a new parameter  $\bar{\Omega}^{(l,s)*}$  by incorporating the cross section parameter with the collision integral:

$$\bar{\Omega}^{(l,s)*} = \sigma^2 \Omega^{(l,s)*} \quad (3)$$

Strictly speaking, the viscosity equation (1) applies only to a gas whose molecules interact according to a central force law and collide in a way so that the collisions are elastic. In other words, the molecules should be spherically symmetric and not have an internal structure. This is not the case for the polyatomic hydrogen, of course, but it appears that an effective spherically symmetric form of  $\phi$  can be used for the viscosity with eq (1) and give a proper comparison with experimental data.

## 1.2. Viscosity of Atomic Hydrogen and the Dissociating Mixture

Dissociating hydrogen can be regarded as a mixture of molecular and atomic hydrogen, the proportions of each species depending on the pressure,  $p$ , and the temperature  $T$  via the dissociation constant  $K$ :



Specifically, the mole fraction of H,  $x_{\text{H}}$ , is given by [5]

$$x_{\text{H}} = 2/\{1 + [1 + (4p/K)]^{1/2}\} \quad (5)$$

and

$$x_{\text{H}_2} + x_{\text{H}} = 1.$$

The viscosity of the dissociating mixture,  $\eta_{\text{mix}}$ , can be expressed by the following kinetic theory equation [1, 4, 6]:

$$\eta_{\text{mix}} = \left[ \frac{x_1^2}{H_{11}} + \frac{x_2^2}{H_{22}} - \frac{2x_1x_2H_{12}}{H_{11}H_{22}} \right] \cdot \left[ 1 - \frac{H_{12}^2}{H_{11}H_{22}} \right]^{-1} \quad (6)$$

where

$$H_{11} = \frac{x_1^2}{\eta_1} + \left( \frac{2x_1x_2}{M_1 + M_2} \right) \frac{RT}{pD_{12}} \left[ 1 + \frac{3M_2 \langle A_{12}^* \rangle}{5M_1} \right]. \quad (7)$$

In this equation we follow the work of reference [4] and adopt the convention that the subscript 1 refers to the molecular species  $\text{H}_2$  and the subscript 2 refers to the atomic species H.

A similar expression can be written for  $H_{22}$  with the subscripts 1 and 2 interchanged. For  $H_{12}$  we have

$$H_{12} = - \left( \frac{2x_1x_2}{M_1 + M_2} \right) \frac{RT}{pD_{12}} \left[ 1 - \frac{3}{5} \langle A_{12}^* \rangle \right]. \quad (8)$$

In the above equations  $M_1$  and  $M_2$  are the molecular weights of the species and the other symbols are explained shortly. When  $x_2$  (i.e.,  $x_{\text{H}}$ ) tends to zero—negligible dissociation—eq (6) reduces to eq (1) as it should [7].

In equations (6)–(8) the entities  $\eta_1$ ,  $\eta_2$ ,  $D_{12}$  and  $\langle A_{12}^* \rangle$  appear.  $\eta_1$  is the viscosity of molecular hydrogen

and involves the  $\text{H}_2$ – $\text{H}_2$  interaction. We already have

$$\eta_1 \sim 1/\bar{\Omega}_{11}^{(2,2)*}.$$

However, the entities  $\eta_2$ ,  $D_{12}$ , and  $\langle A_{12}^* \rangle$  involve the interactions of H–H or H– $\text{H}_2$ , and the appropriate collision integrals need to be used. For example  $\eta_2$  is given by an expression equivalent to eq (1)

$$\eta_2 \sim 1/\langle \bar{\Omega}_{22}^{(2,2)*} \rangle \quad (9)$$

and  $D_{12}$ , the diffusion coefficient is given by

$$pD_{12} = \frac{3}{16} \frac{\sqrt{2\pi k^3 T^3/\mu}}{\pi \langle \bar{\Omega}_{12}^{(1,1)*} \rangle} \quad (10)$$

where  $\mu$  is the reduced mass for the H– $\text{H}_2$  mixture and the collision integral in eq (10) is appropriate for diffusion in this H– $\text{H}_2$  mixture. The term  $\langle A_{12}^* \rangle$  is simply the ratio of the collision integrals

$$\langle A_{12}^* \rangle = \langle \bar{\Omega}_{12}^{(2,2)*} \rangle / \langle \bar{\Omega}_{12}^{(1,1)*} \rangle \quad (11)$$

for the H– $\text{H}_2$  interactions. To explain the angular brackets: for the H–H or H– $\text{H}_2$  interactions the collision integrals will not be a function of a *single* potential since a single intermolecular potential energy curve will not represent a collision. Specifically one has to take into account the fact that a number of force laws will be followed as two species approach each other; each law corresponding to a different alinement of electron spins. It might be supposed that the determination of the resulting collision integrals is almost an impossible task, but the problem can be bypassed. Mason and Monchick [3], for example, show that the kinetic theory format can remain straightforward and the transport coefficient formulas can remain essentially the same, provided the appropriate collision integrals are replaced by a weighted average over the possible force laws, thus they write

$$\bar{\Omega}^{(l,s)*} \rightarrow \sum r_i \sigma_i^2 \bar{\Omega}_i^{(l,s)*} \equiv \langle \bar{\Omega}^{(l,s)*} \rangle \quad (12)$$

where  $r_i$  is the statistical weight of the  $i$ th force law. In the case of atomic hydrogen,  $r_i$  has the values 1/4 and 3/4 corresponding to the  $^1\Sigma$  state and the  $^3\Sigma$  state respectively. The same line of reasoning applies to the H– $\text{H}_2$  interaction (and by implication, to the  $\text{H}_2$ – $\text{H}_2$  interaction, although it is not necessary from a practical point of view).

## 1.3. *p*- and *n*-Hydrogen

In the above section we have assumed no difference between dissociating *p*- $\text{H}_2$  and dissociating *n*- $\text{H}_2$ ; the comments on the weighted collision integrals refer to *n*- $\text{H}_2$ . It has thus been assumed that enough of the *p*- $\text{H}_2$  has already been converted to *o*- $\text{H}_2$  to give a normal mixture at temperatures where dissociation becomes significant [8].

In principle one should also take into account dif-

ferences in the viscosity of the undissociated isotopes since the collision integrals (i.e., the intermolecular potentials) must be different. Several authors have discussed this. In particular, if the intermolecular potential is represented by a two-parameter 12-6 function with  $\epsilon$  and  $\sigma$  the energy and distance parameter respectively, Knaap et al. [9] show that one may expect a difference in these parameters given by  $\Delta\epsilon/\epsilon \sim 0.6 \times 10^{-2}$  and  $\Delta\sigma/\sigma \sim 0.03 \times 10^{-2}$ . They find for the second virial coefficient a difference  $(B_n - B_p)/B_n \sim 1.0\%$ . A similar difference in viscosities would also be expected only with  $\eta_p > \eta_n$ . In fact, Becker and Stehl [10] find a small difference. Unfortunately since the scatter in experimental viscosity coefficients is  $\sim 2.0$  percent and since a model potential function has to be used to obtain theoretical viscosity coefficients, it is difficult to judge if the difference is significant.

#### 1.4. Thermal Conductivity of Molecular Hydrogen

The simple kinetic theory formula for the thermal conductivity,  $\lambda$ , of a gas with no internal structure is

$$\lambda = \frac{5}{2} c'_v \eta \quad (13)$$

where  $c'_v$  is the specific heat per gram at constant volume for the translational degrees of freedom:  $c'_v = (3/2)k/m$ . But equation (13) cannot be used directly for a polyatomic gas because the internal structure substantially influences the thermal conductivity, specifically because inelastic collisions have to be taken into account. Thus the kinetic theory treatment needs appropriate modification. The effect of inelastic collisions has been discussed formally by several authors [11] but their resulting formal equations are too esoteric for practical calculations. However, Mason and Monchick [3] have looked at the formal theory and have simplified its application. They recognized that three mechanisms contribute to the transfer of heat: (1) the translational motion of the molecules, (2) an effective internal diffusion of the internal degrees of freedom, and (3) a coupling between the translational and internal contributions. If the total conductivity is written as the sum of contributions (1) and (2), i.e.,

$$\lambda = \lambda' + \lambda'' \quad (14)$$

where  $\lambda'$  is the translational contribution and  $\lambda''$  the internal contribution, then Mason and Monchick show that

$$\lambda' = \frac{5}{2} \eta (c'_v - \Delta) \quad (15)$$

$$\lambda'' = \rho D_{\text{int}} (c''_v + \Delta) \quad (16)$$

where the total specific heat per gram has also been written as a sum of the translational and internal parts:

$$c_v = c'_v + c''_v \quad (17)$$

In eq (16),  $D_{\text{int}}$  is the effective diffusion coefficient for the transfer of internal energy and  $\Delta$  is a term accounting for the interchange of the translational and internal energies. From eqs (15) and (16),

$$\lambda = \frac{5}{2} \eta \left( \frac{3}{2} \frac{k}{m} \right) + \rho D_{\text{int}} c''_v - \eta \left( \frac{5}{2} - \frac{\rho D_{\text{int}}}{\eta} \right) \Delta \quad (18)$$

One can further show that

$$\Delta = \frac{2}{\pi} \sum_k \frac{c''_k}{Z_k} \left( \frac{5}{2} - \frac{\rho D_{\text{int}}}{\eta} \right) + \text{higher terms} \quad (19)$$

where  $c''_k$  is the internal specific heat for the  $k$ th internal mode, and  $Z_k$  is the collision number associated with that mode. Equation (18) consequently becomes,

$$\lambda = \frac{15}{4} \frac{k}{m} \eta + \rho D_{\text{int}} c''_v - \frac{2}{\pi} \eta \left( \frac{5}{2} - \frac{\rho D_{\text{int}}}{\eta} \right)^2 \sum_k \frac{c''_k}{Z_k} \quad (20)$$

Note that if the molecule has no internal structure, eq (20) reduces to eq (13) as it should.

For hydrogen, both  $Z_{\text{rot}}$  and  $Z_{\text{vib}}$  are large and the last term on the right-hand side of eq (20) can be dropped. Another simplification is possible if  $D_{\text{int}}$  is approximated to the self-diffusion coefficient,  $D$ , because

$$\frac{\rho D_{\text{int}}}{\eta} \equiv \frac{\rho D_{11}}{\eta} = \frac{6}{5} A_{11}^* \quad (21)$$

Thus, for hydrogen,

$$\lambda = \frac{15}{4} \frac{R}{M} \eta + \frac{6}{5} A_{11}^* c''_v \eta \quad (22)$$

The question of the validity of the general equation (18) and of the particular equation (22) has been an object of discussion for some time. A recent review is given by Sandler [12].

#### 1.5. Thermal Conductivity of Atomic Hydrogen and the Dissociating Mixture

The thermal conductivity of a dissociating mixture is given by an expression equivalent to the formula for the viscosity, with the appropriate allowance for  $\lambda''$ . This is not all, however, for an additional feature appears: the dissociation process has an associated heat of reaction and this dissociation heat substantially contributes to the overall heat transfer [4, 6, 13]. The conductivity of the mixture,  $\lambda(\text{mix})$ , is then expressed as a sum of these contributions

$$\lambda(\text{mix}) = \lambda_f(\text{mix}) + \lambda(r) \quad (23)$$

where  $\lambda_f(\text{mix})$  is the conductivity in the absence of the chemical reaction—called the frozen conductivity—and  $\lambda(r)$  is the conductivity due to the chemical reaction. We write down the equations for these terms separately.

**Frozen Thermal Conductivity:** We have, following eq (14)

$$\lambda_f(\text{mix}) = \lambda'(\text{mix}) + \lambda''(\text{mix}) \quad (24)$$

where the prime and double primes refer to the translational and internal portions respectively. The expression for the first part is similar to the viscosity equation, viz.,

$$\lambda'(\text{mix}) = -4 \left[ \frac{x_1^2}{L_{11}} + \frac{x_2^2}{L_{22}} - \frac{2x_1x_2L_{12}}{L_{11}L_{22}} \right] \cdot \left[ 1 - \frac{L_{12}^2}{L_{11}L_{22}} \right]^{-1} \quad (25)$$

with

$$L_{11} = \left[ -\frac{4x_1^2}{\lambda'_1} - \frac{16}{25} \frac{x_1x_2}{(M_1+M_2)^2} \frac{T}{pD_{12}} \right] \left[ \frac{15}{2} M_1^2 + \frac{25}{4} M_2^2 - 3M_2^2 \langle B_{12}^* \rangle + 4M_1M_2 \langle A_{12}^* \rangle \right] \quad (26)$$

and a corresponding expression for  $L_{22}$  can be written with the subscripts 1 and 2 interchanged. The term accounting for the H-H<sub>2</sub> interactions in eq (25) is

$$L_{12} = \left[ \frac{16}{25} \frac{x_1x_2M_1M_2}{(M_1+M_2)^2} \frac{T}{pD_{12}} \right] \cdot \left[ \frac{55}{4} - 3\langle B_{12}^* \rangle - \langle A_{12}^* \rangle \right] \quad (27)$$

The symbols of eqs (25)–(27) have been defined when eqs (6)–(12) were discussed except for  $\lambda'_i$  which is given by

$$\lambda'_i = \frac{15}{4} \frac{R}{M} \eta_i \quad (28)$$

and in eqs (26) and (27) we have  $\langle B_{ij}^* \rangle$  which is given by

$$\langle B_{ij}^* \rangle = \{5\langle \bar{\Omega}_{ij}^{(1,2)*} \rangle - 4\langle \bar{\Omega}_{ij}^{(1,3)*} \rangle\} / \langle \bar{\Omega}_{ij}^{(1,1)*} \rangle. \quad (29)$$

We have again used the convention that subscripts 1 and 2 refer to the molecular and atomic species respectively.

Fortunately, the internal part  $\lambda''(\text{mix})$  is given by a fairly straight forward expression for hydrogen because Hirschfelder [14] has shown that this internal contribution can be depicted by

$$\lambda''(\text{mix}) = \frac{\lambda_1 - \lambda'_1}{1 + (D_{11}/D_{12})(x_1/x_2)} \quad (30)$$

where  $\lambda_1$  is the total conductivity for pure hydrogen molecules. From eqs (28) and (22), we get

$$\lambda''(\text{mix}) = \frac{6}{5} A_{11}^* c_v'' \eta_1 / \{1 + (D_{11}/D_{12})(x_1/x_2)\} \quad (31)$$

Thus, the total frozen conductivity of the mixture  $\lambda_f(\text{mix})$  is given by the sum of eqs (25) and (31).

**Chemical Reaction Contribution to the Thermal Conductivity:** The last term of eq (23),  $\lambda(r)$ , is given by [13].

$$\lambda(r) = \left[ \left( \frac{pD_{12}}{RT} \right) \frac{(\Delta H)^2}{RT^2} \right] \left[ \frac{x_1x_2}{(1+x_1)^2} \right] \quad (32)$$

where  $\Delta H$  is the heat of reaction for the dissociation.

Hence the total thermal conductivity of the dissociating mixture can be determined by summing the eqs (25), (31), and (32).

### 1.6. p- and n-Hydrogen

In addition to the reasons mentioned in 1.3, the thermal conductivity of the two species will differ since the internal specific heat,  $c_v''$ , appears in the formulas. Between about 40 and 300 K,  $c_v''$  for p-H<sub>2</sub> is significantly different from  $c_v''$  for n-H<sub>2</sub>[5].

## 2. Calculation of Mass Fraction, Heat of Reaction and Internal Specific Heat

We continue by discussing the calculation procedure we adopted. First let us consider the calculation of  $x$ ,  $\Delta H$  and  $c_v''$ . Actually the calculation of these properties poses no problem since the necessary variables are well known. The mass fractions at given pressures and temperatures were computed from eq (5), using values of  $K$  from reference [5]. The heat of reaction,  $\Delta H$ , was also calculated from  $K$  via the Van't Hoff isobar [4, 6].

$$\frac{\Delta H}{RT^2} = \frac{d \ln K}{dT}. \quad (33)$$

The internal specific heat at constant volume was computed from values of  $c_p$  (from reference [5]) since  $c_v''$  is given by the equation

$$c_v'' = c_p - \frac{5R}{2M}. \quad (34)$$

## 3. The Collision Integrals

Once numerical values of  $c_v''$ ,  $\Delta H$ , and  $x$  are available, the only remaining unknowns in the formulas of section 1 are the collision integrals. Unfortunately, the computation of these collision integrals is the most uncertain factor in the calculation of the transport coefficients but is at the same time the most important. Except possibly, for the H-H integrals, they have to be computed from model intermolecular potential functions. The choice of the model and its parameters has to be determined from experiment and, for hydrogen, the evaluation of experimental data is not a straightforward task.

Of the collision integrals, the most important for the wide temperature range under consideration are those for the H<sub>2</sub>-H<sub>2</sub> interaction. The others are needed only when dissociation becomes significant—and even at

a low pressure of 0.01 atm dissociation is not significant until about 2000 K. Further, dissociation between 2000 and 5000 K at pressures between 0.01 and 100 atm only changes the viscosity by about  $\pm 10$  percent from the equivalent undissociated value. For thermal conductivity the situation is different; the conductivity of the mixture is much greater than it would be for the equivalent undissociated value, but the dominant contribution to this thermal conductivity is the contribution  $\lambda(r)$  given by eq (32). This does involve the integral for the H—H<sub>2</sub> interaction yet, even so, the two terms  $(\Delta H)^2/RT^2$  and  $x_1x_2/[(1+x_1)^2]$  of eq (32) largely control the temperature dependence of  $\lambda(r)$ .

Accordingly, we did not feel it necessary to reevaluate the collision integrals for the H—H and H—H<sub>2</sub> interactions or comment on their selection. Consequently, the values used in these calculations were taken directly from table II of reference [4]. We concentrated on the selection of the  $\Omega_{11}^{(l,s)*}$  integrals necessary to compute the transport properties of the undissociated molecular hydrogen.

### 3.1. General Rules for Choosing a Model Intermolecular Potential Function

We have recently clarified the overall relation between model potential functions, theoretical expressions and experimental data [15]. We arrived at conclusions which are, in short: (1) The realistic three parameter intermolecular potential function families are equivalent with respect to the correlation of data. By equivalent we mean that one member of all the families in common use can be found that will fit a given set of data in the same way. (2) A temperature range exists over which a property is insensitive to all sensible members of all model families. For transport properties of classical fluids this range is  $2 \lesssim T_1^* \lesssim 5$  where  $T_1^*$  is the temperature reduced by the Lennard-Jones energy parameter  $(\epsilon/k)_{L-J}$ . This conclusion leads to the definition of a *high* temperature as a temperature above the insensitive range, and a *low* temperature as a temperature below the range. Because  $(\epsilon/k)_{L-J}$  for hydrogen is  $\sim 40$  K, one would expect that the range is  $80 \text{ K} \lesssim T \lesssim 200 \text{ K}$  but we are not yet clear on how quantum effects might adjust these limits [16]. (3) One member of a three parameter family is not flexible enough to simultaneously represent a property at *both* high and low temperatures. Finally, (4) a significant choice of a potential function requires that the data have a precision of about 0.5 percent at low temperatures, or about 3 percent at high temperatures. By significant choice we mean that we can distinctly select a function and attach some meaning to the selection. Of course, unless we use independent information we cannot be sure that even a distinct and proper choice does not reflect systematic error. A full discussion on these conclusions is available in reference [15].

## 4. Correlation of Hydrogen Data

In practice, application of the conclusions of the previous section is somewhat restricted. There are

two reasons. First, we really have only one model potential with which to work at low temperatures—the quantum mechanical 12–6 (or Lennard-Jones) of Munn et al. [17]. Second, the data available often do not satisfy the criterion (4) of Section 3.

### 4.1. The High Temperatures Region Without Dissociation

Conclusion (1) of section 3 states that only one model function family need be considered. Since we have to use the 12–6 at low temperatures, we studied the high temperature regions with members of the  $m$ -6 family. For this family when  $\phi(r)$  is the potential for two molecules separated by distance  $r$ , we have [15, 18],

$$\phi(r) = \epsilon \left[ \left( \frac{\sigma}{r} \right)^m - \left( \frac{\sigma}{r} \right)^6 \right] / \left[ \left( \frac{6}{m} \right)^{6/m-6} - \left( \frac{6}{m} \right)^{m/m-6} \right] \quad (35)$$

with (in our case)  $m = 9, 12, 15, 18, 21, 24$ . In eq (35),  $\epsilon$  is the depth of the energy well and  $\sigma$  is the distance separating the molecules at  $\phi(r) = 0$ .

#### a. Viscosity

We started with the viscosity equation (1b) and used it to select the potential and its parameters. The selection procedure is described in reference [19]. Basically it goes as follows:  $m$  is fixed,  $\sigma$  is fixed at a sensible value, and selected data for several temperatures are put into the left-hand side of equation (1b). We thus obtain experimental collision integrals as a function of temperature  $T$ . These experimental integrals are then compared with the theoretical integrals for the fixed  $m$  which are available as functions of the reduced temperature  $T^*$ ,  $T^* = T/(\epsilon/k)_{m-6}$ . We can thus observe how  $\epsilon/k$  varies with  $T$  for the given  $\sigma$  and  $m$ .  $\sigma$  is now varied until one finds a value for  $\sigma$  which gives the least variation of  $\epsilon/k$  with  $T$ . Finally, the family parameter  $m$  is changed and the procedure repeated. Our selection of  $m$ ,  $\sigma$ , and  $\epsilon/k$  is made by finding the  $m$  and  $\sigma$  which give the least variation of  $\epsilon/k$  with  $T$  after considering all values of  $m$ . If one member of the function family represented the data exactly,  $\epsilon/k$  would be constant. The collision integrals used were those of Klein and Smith [18].

The procedure of course relies (as does any other procedure) on the data which is why criterion (4) of section 3 needs to be stated, but while several authors have investigated the viscosity (for  $n$ -hydrogen only) in the high temperature range [20–32] the amount of *reliable* data is somewhat limited. For example, there is nothing reliable at this time between about 400 and 1100 K. Other data do exist above 400 K but we have shown [33] that they are most probably incorrect (by a large amount—almost 10 percent at 2000 K). It appears that the older measurements of viscosity by the capillary flow technique are erroneous. We have pointed out that the incorrect data form the

TABLE 1. Potential function parameters for n- and p-hydrogen selected from viscosity data

$m$	$\Lambda^*$	$\sigma(\text{\AA})$	$\epsilon/k(\text{K})$
High temperature			
9	....	3.06	30.4
Low temperature			
12	1.70	2.97	39.2

basis for most of the tabulations and correlations of transport properties at present in the literature. Hence the tables produced by our correlations will differ significantly from those available up to now. We refer to reference [33] for more details on this point.

Working with selected data [28–32], eq (1), and the collision integrals of reference [18], the parameters shown in table 1 were selected. The parameters were then used to compute the viscosity and a comparison of theory and experiment is shown as a deviation curve, figure 1. The curve is reasonably satisfactory. There is a small systematic deviation at the high temperatures but to remove this deviation rather unrealistic collision integrals would have to be used. It is also possible that some dissociation is occurring at the two highest temperatures.

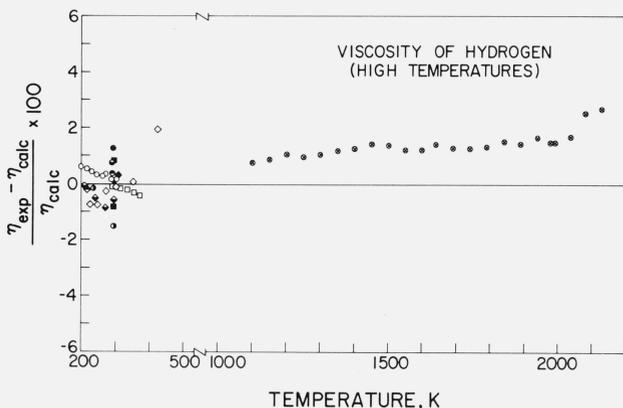


FIGURE 1. Viscosity of hydrogen at high temperatures.

Experimental data fitted to a 9-6 function with  $\sigma = 3.06 \text{ \AA}$ ,  $\epsilon/k = 30.4 \text{ K}$ .  
Key:  $\otimes$  [32],  $\diamond$  [30],  $\bullet$  [21],  $\circ$  [22],  $\square$  [24],  $\circ$  [26],  $\bullet$  [27],  $\blacklozenge$  [29],  $\blacklozenge$  [31],  $\blacksquare$  [28],  $\blacklozenge$  [20].

#### b. Thermal Conductivity

The situation with regard to the thermal conductivity measurements is not very good [34–42], accurate data are scarce and scattered. Above 400 K four authors report results and their sets of data differ systematically by more than 20 percent [35, 36, 41, 42]. Moreover, when we use the selected potential function to compute thermal conductivity via eq (22) the deviation curve produced, figure 2, is not satisfactory.

Comments are deferred until the discussion section, but we feel the conductivity data are most probably responsible for this. We cannot believe that the theoretical formulas, eqs (1) and (22), or the 9-6 function chosen to represent the viscosity are that wrong.

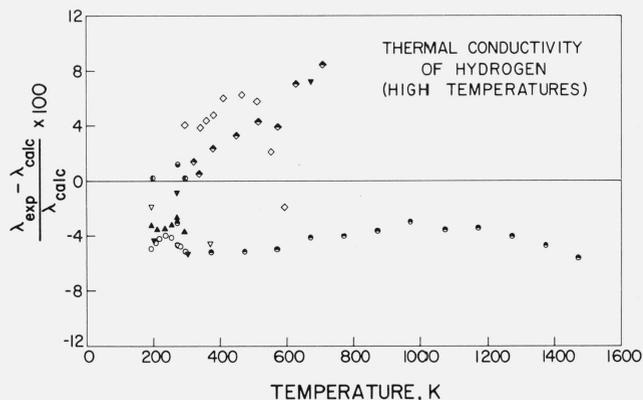


FIGURE 2. Thermal Conductivity of hydrogen at high temperatures.

Experimental data fitted on the basis of the viscosity correlation, see caption for figure 1. Omitted are data from reference [35].  
Key:  $\bullet$  [36],  $\bullet$  [38],  $\nabla$  [40],  $\circ$  [39],  $\blacktriangledown$  [37],  $\blacktriangle$  [34],  $\bullet$  [52],  $\otimes$  [60],  $\diamond$  [42],  $\blacklozenge$  [41].

#### 4.2. The Low Temperature Region

We have already mentioned that we have to work with the quantum mechanical 12-6 intermolecular potential. (Work is in progress to modify other functions for quantum effects, but it appears that results will not be available for some time.) Actually, this restriction may not matter much. Conclusion (3) of section 3.1, states that the model potential found from experiment to be suitable for the high temperature region will not be satisfactory when used in the low temperature region. Anyway, this might rule out the 9-6 for the low temperature region. Furthermore, our previous work with other gases using the  $m$ -6 family suggests that the parameter  $m$  compatible with the high temperature range is less than the corresponding  $m$  for the low temperature range. For example, for argon,  $m$  (high)  $\approx 13$ ,  $m$  (low)  $\approx 18$ . While comparison of other gases with hydrogen should be made with caution, it is quite possible that if  $m \approx 9$  for high temperature hydrogen, then  $m \approx 12$  is reasonable for low temperature hydrogen.

##### a. Viscosity

Several workers have measured hydrogen below 300 K [20–25, 27, 30, 31, 43–46] but much of the data have to be considered unreliable. However, sets of viscosity data for several gases other than hydrogen [47], and one set for hydrogen [31] have recently been published and these new results tend to be somewhat lower systematically than those reported in the past. We have studied the apparent discrepancies and show there are grounds to favor the newer data [48]. In other words, while there are little, if any, direct independent evidence that the majority of hydrogen viscosities are too high, they may well nevertheless be so. To get as much new evidence as possible on

this point we re-examined the viscosity data of Diller published in 1965 [49]. Diller did not measure the dilute gas viscosities directly but rather presented the density dependence of viscosity for the dense gas and liquid state. However, by a careful analysis of his isotherms it was possible to obtain dilute gas values with reliable error limits. The analysis procedure is discussed in reference [50] and the corresponding dilute gas viscosities are listed in table 2. We find that these data are significantly lower than data published previously. Unfortunately although this result is consistent with the point made that the latter data may be too high, it does not agree with the conclusion of Knaap et al. [9], or with the experiment of Becker and Stehl [10] which suggest that  $\eta(p\text{-H}_2) > \eta(n\text{-H}_2)$  at a given temperature; the existing  $n\text{-H}_2$  data would have to be much too high if this is to be so.

TABLE 2. Dilute gas viscosities for p-hydrogen obtained from dense gas data of Diller [49]

Temperature K	Viscosity $10^6 \text{g cm}^{-1} \text{s}^{-1}$
33.0	$16.96 \pm .35$
36.0	$16.3 \pm .2$
40.0	$20.0 \pm .5$
50.0	$23.3 \pm .5$
60.0	$28.0 \pm .3$
70.0	$31.3 \pm .1$
80.0	$34.9 \pm .2$
100.0	$40.5 \pm .5$

After considerable manipulation of the parameters of the 12-6 function and keeping the possibilities of errors in the data in mind, we eventually concluded that the parameters shown in table 1 were the best. The deviation curve, figure 3, was plotted. We confirmed that the fit for the low temperature region matched with the fit for the high temperature region.

It should be clear that the uncertainty and scatter in the data means that one has a considerable and unwelcome latitude in choosing the parameters (conclusion (4) of section 3.1) and that several combinations

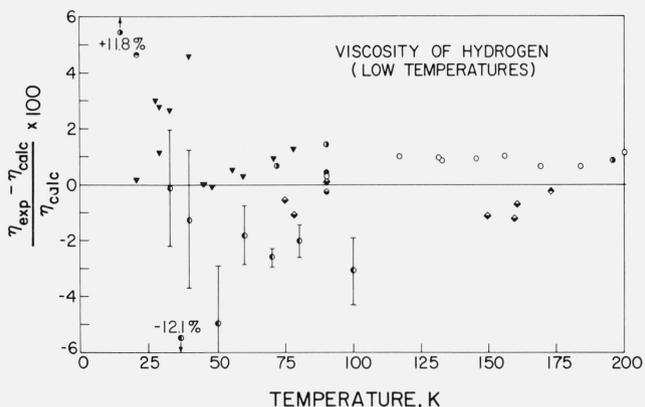


FIGURE 3. Viscosity of p- and n-hydrogen at low temperatures.

Experimental data fitted to a quantum mechanical 12-6 function with  $\Lambda^* = 1.70$ ,  $\sigma = 2.97$  Å, and  $\epsilon/k = 39.2$  K.  
Key: p-H<sub>2</sub>, ○ [49]; n-H<sub>2</sub>, ● [20], ● [21], ● [22], ● [27], ○ [24], ▼ [46], ◆ [31].

are possible, Diller and Mason [51], for example, found  $\epsilon/k = 37.2$  K,  $\sigma = 2.97$  Å, and  $\Lambda^* = 1.70$  (but by correlating the results of references 24 and 46). One should note that since  $\Lambda^* = h/(\sigma(2\mu\epsilon))^{1/2}$ , the values of  $\epsilon$  and  $\sigma$  should correspond to the value of  $\Lambda^*$  selected. This is the case for Diller and Mason, but not here.

### b. Thermal Conductivity

Up to now two main sets of data existed for the thermal conductivity of hydrogen at low temperatures, those of Golubev and Kalsina [34] and of Johnston and Grilly [39]. More recently Roder and Diller have measured both para and normal hydrogen from 17 to 200 K at pressures from about 1 to 170 atm [52]. Their dilute gas values are given in table 3.

Theoretical values of the thermal conductivity for the  $n$ - and  $p$ -modifications were calculated from eq (22) using the 12-6 function and the parameters estimated from viscosity data. Values of the internal specific heat were determined from eq (34) with data from reference [5]. Theoretical conductivities were compared with experimental at the same temperature, and a deviation curve was plotted, figure 4. The fit is low overall.

TABLE 3. Dilute gas thermal conductivity values of p- and n-hydrogen from Diller and Roder [52]

Temperature K	p-Hydrogen	n-Hydrogen
	$\text{mW cm}^{-1} \text{K}^{-1}$	
17.38	<sup>a</sup> 0.138	
20.06	.153	
22.0	.165	
25.47	.196	
29.98	.231	
40.24	.303	
59.1		$0.423 \pm 0.002$
59.2	$.440 \pm 0.001$	
79.8		$.561 \pm .001$
79.9	$.599 \pm .002$	
99.9	$.797 \pm .001$	$.687 \pm .002$
122.9	$1.033 \pm .001$	
123.0		$.851 \pm .001$
153.0		$1.052 \pm .001$
197.9		$1.331 \pm .002$

<sup>a</sup> Values without error limits were taken to be the 1 atm value. Values with error limits were obtained by analysis of conductivity as a function of density at the given temperatures. The method of reference [50] was used.

### 4.3. Thermal Conductivity and Viscosity of Dissociated Hydrogen

We compute the transport properties of the dissociated hydrogen using the formulas of section 1. The collision integrals were those listed in reference [4], except for  $\Omega^{(l,s)*}$  for the H<sub>2</sub>-H<sub>2</sub> interaction. These latter integrals are the same, of course, as those discussed in section 4.1a, i.e., the 9-6 integrals of reference [18]. The parameters are given in table 1.

There does not seem to be any viscosity data for the dissociated gas. Two sets of data exist for the

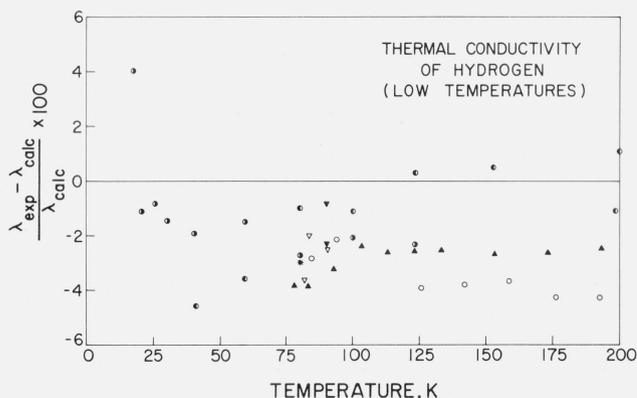


FIGURE 4. Thermal Conductivity of *p*- and *n*-hydrogen at low temperatures, experimental data fitted on the basis of the low temperature viscosity correlation, see caption to figure 3.

Key: *p*-H<sub>2</sub>, ● [52]; *n*-H<sub>2</sub>, ▽ [40], ○ [39], ▼ [37], ▲ [34], ● [52].

thermal conductivity however, those of Israel et al. [53], and those of Bethringer et al. [54]. The former set of data appears to be incorrect; dissociation seems to take place at too low a temperature. The latter data are illustrated in figure 5, together with our theoretical estimate.

## 5. Discussion

The deviation curves, figures 1 to 4, indicate that the fits are not completely satisfactory overall. Since we based the initial correlations on viscosity measure-

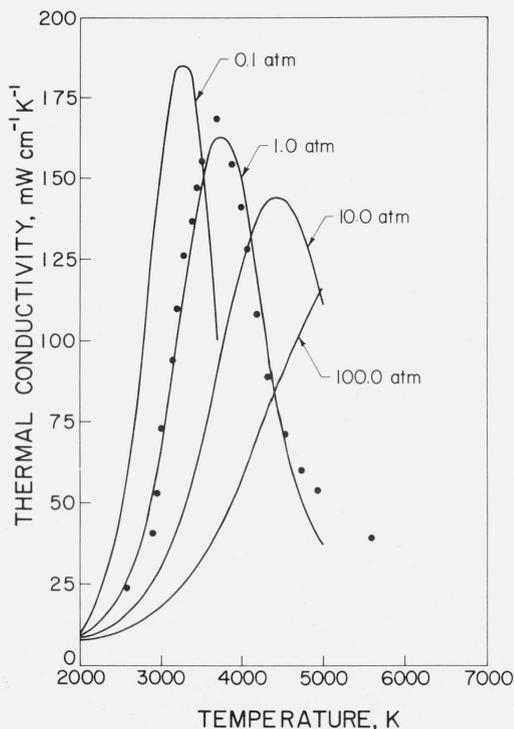


FIGURE 5. Thermal conductivity of dissociating hydrogen at various pressures.

See section 4.3. Data from reference [54] shown as points.

ments, a reasonable looking deviation curve for the viscosity will always result, the disturbing fact is that the calculated values of the thermal conductivity are somewhat too high when compared with most of the experimental data especially at low temperatures. However, we feel we may explain this relatively poor correlation of experimental thermal conductivity data by blaming the experimental data itself. Our conclusion is backed by the arguments that follow.

It is fairly obvious that the discrepancies between theoretical and experimental conductivities, and by discrepancies we mean first order systematic deviations between theory and experiment of about 1–2 percent, can be due to at least one of these reasons:

- The viscosity equation (1) is incomplete leading to an incorrect choice of a model potential function and its parameters.
- The viscosity equation is correct, but the viscosity data are inaccurate. As in (a) this will also mean that the selected potential function is wrong.
- The thermal conductivity equation (22) is incomplete.
- The equation (22) is correct but the thermal conductivity data are inaccurate.

(One could add to (a) and (b) that while *both* the viscosity equation and the viscosity data could be correct, the selected potential function may not be flexible enough to represent another property, such as the thermal conductivity. This cannot be the case here, however, since the same collision integrals are prominent for both viscosity and thermal conductivity.)

It should be stated that one could go a long way towards deciding which of the reasons (a) through (d) are the cause of the apparent errors if we had more independent (non-transport) information to check on the potential functions selected, or if we had definite evidence on systematic errors in the experimental measurements. For instance, we have for other gases been able to check the appropriate potential functions using results from molecular beam data, x-ray scattering data, and optical studies [33, 48]. We cannot do this for hydrogen because the necessary experiments have not been done. Hence, here we have to examine points (a) to (d) on the basis of the transport data alone which is obviously not a really satisfactory procedure. Nevertheless we will argue that for our purpose all reasons but (d) can probably be eliminated.

*High Temperature Region:* By and large the poor correlation of conductivities at high temperature appears to be due to random scatter in the data. Systematic deviations are not so prominent as they are at low temperatures. But it is worthwhile to go through the points (a) to (d) if only to set up our reasoning for the low temperature region. The high temperature viscosities (without dissociation) are first considered. It is assumed that the fundamental viscosity formula is valid. While such an assumption might provoke discussion, there is no evidence from any experiments for any gas that the formula is *not* correct. In fact, Klein and Hanley have demonstrated [15, 55] that from the statistical mechanical point of view the

viscosity equation is at least *consistent* with the equation for the second virial coefficient (that is, if the viscosity equation is correct, the second virial equation is also correct, and vice versa).

With regard to part (b) and the viscosity experimental data, we have confidence in at least two sets of data, those of Kestin et al. [28, 29] and of Guevara et al. [32]. The data of these authors have been found very satisfactory for other gases [33] and we see no reason why hydrogen should be an exception.

Let us now discuss point (c), i.e., the validity of the thermal conductivity equation (22)

$$\lambda = \frac{15}{4} \frac{R}{M} \eta + \frac{6}{5} A^* c_v'' \eta.$$

Note that if experimental viscosities are used, eq (22) only weakly involves the potential function through  $A^*$ . Equation (22) was obtained from the more general equations (19) and (20):

$$\lambda = \frac{15}{4} \frac{R}{M} \eta + \rho D_{\text{int}} c_v'' - \frac{2}{\pi} \eta \left( \frac{5}{2} - \frac{\rho D_{\text{int}}}{\eta} \right)^2 \sum_k \frac{c_k''}{Z_k}. \quad (36)$$

It is well known that this equation is not complete [12] because Mason and Monchick derived it from the Wang-Chang-Uhlenbeck Theory [11], and in this theory only the first approximation expression for the thermal conductivity was obtained. In other words only one Sonine polynomial was used to compute the perturbation term in the appropriate solution of the Boltzmann equation for polyatomic molecules. If we were dealing with a monatomic gas this restriction would not be necessary and the higher approximations can be determined without too much trouble. In particular, eq (36), with the second approximation, becomes

$$\lambda = \frac{15}{4} \frac{R}{M} \eta [1 + (70/8232)y] \quad (37)$$

where

$$y = [(8\Omega^{(2,3)}/\Omega^{(2,2)}) - 7]^2.$$

We write down eq (37) to show that the calculated conductivity value for a monatomic gas *increases* if eq (37) is used in preference to the first approximation eq (13). Although the effects of higher approximations on the other terms of eq (36) are not yet known, for real molecules it seems reasonable to suppose that if they were included, the calculated thermal conductivity would also increase for a polyatomic gas [56]. If this were so, the systematic deviations of figure 2 generally would be worse. Hence, neglect of higher approximations in the general equation (36) does not appear to account for the fact that the calculated thermal conductivities are generally too high when compared to most of the (presumably) more reliable data, (we exclude that data of reference [35]).

It is clear that two simplifications have been made

when eq (22) is derived from eq (36);  $D_{\text{int}}$  has been taken equal to the ordinary self-diffusion coefficient  $D$ , and terms in  $1/Z$ , and terms with higher powers of  $Z$ , have been neglected. The two approximations can be related, however. The replacement of  $D_{\text{int}}$  by  $D$  has been a subject of some discussion and it is known that it is not a proper thing to do for many gases. This has come up again recently because Sandler [12] has compared polyatomic gas theory for model molecules with other theories such as that of Mason and Monchick [3]. One result of the comparison is the substitution,

$$D_{\text{int}} = D[1 + 0.27/Z \dots] \quad (38)$$

should be made rather than merely equating  $D_{\text{int}}$  with  $D$ .

This substitution also gives improved agreement of theory with experiment although one should note that gases other than hydrogen were tested.

Accepting eq (38) as a reasonable representation of  $D_{\text{int}}$ , we see that it is only necessary to drop terms in  $1/Z$  to get eq (22) from eq (36). Such a step appears justified for hydrogen because all appropriate experimental evidence available indicates that  $Z$  for hydrogen is at least around 300 [3, 57]. To sum up, we feel eq (22) is satisfactory for hydrogen and thus rule out point (c). This leaves point (d) as the most likely cause for the systematic discrepancies at high temperatures.

*Low Temperature Region:* We are not in a particularly strong position to evaluate all the points (a)–(d) when the low temperature region is considered. Points (a) and (c) can be ruled out for the same reason as discussed above, and with regard to (c), we can further show that the question of the validity of the *particular* thermal conductivity eq (22) is not a factor of great importance when correlating data at low temperatures. Unfortunately, it is presently impossible to confidently dismiss either reason (b)—errors in viscosity or reason (d)—errors in thermal conductivity; we are not sure if any of the low temperature transport measurements are really reliable. Yet it is possible to make a sensible guess as to which of the two is more likely to be the cause of the discrepancies. Because we suspect from independent studies that the older viscosity data tend to be somewhat high [48], and because we give some weight to the apparently low results of Diller, we feel that our viscosity correlation is at least reasonable.

This all means that if a *single* factor has to be selected as a cause for the major discrepancies, we are suggesting that this factor is the experimental thermal conductivity data.

### 5.1. Theoretical Thermal Conductivities at Low Temperatures

The point was made above that the particular thermal conductivity expression was not too important at low temperatures. One sees this at very low temperatures because  $c_v'' \approx 0$  below about 40 K. Thus the terms in eq (36) involving the internal degrees of free-

dom can be dropped. The monatomic gas equation results

$$\lambda = \frac{15}{4} \frac{R}{M} \eta \quad (28)$$

[or eq (37) results if the expression is taken to the second approximation]. This type of simplification can be carried to higher temperatures if a procedure described by Harris [58] is used. It is interesting to follow this procedure up.

One can write a very general expression,

$$\lambda = \lambda' [1 + a c_v'' \delta] \quad (39)$$

where as before  $\lambda$  is the total thermal conductivity and  $\lambda'$  is the translational conductivity,  $a$  is a constant and  $\delta$  is a variable. If

$$\lambda = \lambda' [1 + 2/5 (c_v''/R) \delta] \quad (40)$$

$$\delta = \delta_H - \frac{25}{3\pi Z} \left[ 1 - \frac{3}{5} \delta_H \right]^2 \quad (41)$$

$$\delta_H = \frac{2}{3} \rho D / \eta \quad (42)$$

then eq (39) becomes the Mason, Monchick equation (36). But we do not require in the development that follows  $a$  or  $\delta$  to have these particular forms.

We will show that eq (39) for  $p$ -hydrogen can be expressed in terms of experimental quantities. Consider a mixture of  $o$ - and  $p$ -hydrogen. The thermal conductivity of the mixture of  $o$ - and  $p$ -hydrogen can be expressed by the mixture equations discussed in section 1.5. Specifically,

$$\lambda_{op} = \lambda'_{op} - (\lambda_p - \lambda'_p) / [1 + (x_o/x_p)(D_{pp}/D_{op})] \\ + (\lambda_o - \lambda'_o) / [1 + (x_p/x_o)(D_{oo}/D_{op})] \quad (43)$$

where the subscripts  $o$ , and  $p$ , or  $op$  refer to the  $o$ -,  $p$ - or  $op$ -mixture of hydrogen respectively. Assuming (and this is an assumption, see section 1.3) that

$$\lambda'_{op} \equiv \lambda'_o \equiv \lambda'_p = \lambda'; \\ D_{oo} \equiv D_{pp} \equiv D_{op} \quad (44)$$

and using  $x_o + x_p = 1$ , eq (43) then becomes

$$\lambda_{op} = \lambda' + x_o(\lambda_o - \lambda'_o) + x_p(\lambda_p - \lambda'_p). \quad (45)$$

Writing eq (40) for  $o$ - and for  $p$ -hydrogen and using eq (45), we obtain

$$\lambda_{op} = \lambda' \left[ 1 + \frac{2}{5R} c_v''(o) \delta(o) + x_p b \right] \quad (46)$$

where

$$b = \frac{2}{5R} [c_v''(p) \delta(p) + c_v''(o) \delta(o)]. \quad (47)$$

Here the  $o$  and  $p$  in parentheses signify the particular hydrogen isotope. Now for 100 K or less,  $c_v''(o) \approx 0$ , therefore

$$b \approx \frac{2}{5} \frac{c_v''(p) \delta(p)}{R} : \lambda_{op} \approx \lambda' [1 + x_p b]. \quad (48)$$

The key to the procedure is to consider two mixtures of hydrogen with different concentrations of  $p$ -H<sub>2</sub>, designated as 1 and 2 respectively. One then obtains from eq (48) that

$$\lambda_{op}(1) - \lambda_{op}(2) = \lambda' b [x_p(1) - x_p(2)]. \quad (49)$$

In the special case that mixture 1 is pure  $p$ -hydrogen, and mixture 2 is  $n$ -hydrogen (i.e.,  $x_p(2) = 0.25$ ), then

$$\lambda_p - \lambda_n = 0.75 b \lambda'. \quad (50)$$

Hence, writing eq (46) for  $p$ -hydrogen, with eq (50) we find

$$\lambda_p = \lambda' + 4/3 (\lambda_p - \lambda_n). \quad (51)$$

Further, if  $c_v''(o) \approx 0$  we obtain from eq (48) that

$$\lambda' = 1/3 (4\lambda_n - \lambda_p). \quad (52)$$

Thus the total measured thermal conductivity of  $p$ -hydrogen can be written in terms of itself and of the total measured conductivity of  $n$ -hydrogen at the same temperature:

$$\lambda_p = 1/3 [4\lambda_n - \lambda_p] + 4/3 [\lambda_p - \lambda_n] \quad (53)$$

This particular breakdown is only possible if  $c_v''(o) \approx 0$  but  $c_v''(p) \neq 0$ . Equation (53) is thereby restricted to the approximate temperature range of 40 to 100 K.

It is interesting to check eq (53) with eq (22). Writing eq (53) as

$$\lambda(\text{exp}) = \lambda'(\text{exp}) + \lambda''(\text{exp}) \quad (54)$$

one obtains

$$\lambda'(\text{exp}) = 1/3 [4\lambda_n(\text{exp}) - \lambda_p(\text{exp})] \equiv 15/4 \frac{R}{M} \eta \\ \lambda''(\text{exp}) = 4/3 [\lambda_p(\text{exp}) - \lambda_n(\text{exp})] \equiv 6/5 A^* c_v''(p) \eta. \quad (55)$$

Values for both  $n$ - and  $p$ -hydrogen are available for three temperatures between 40 and 100 K: 59.1 K, 79.1 K, and 99.8 K, see table 3. Table 4 was constructed in which we have designated (15/4)  $(R/M) \eta$  as  $\lambda'$  (calc) and (6/5)  $A^* c_v''(p) \eta$  as  $\lambda''$  (calc).

It is seen from table 4 that the agreement between the experimental and calculated translational and internal contributions is quite good for this limited data. It should be stressed, however, that the table does not give us definite information on the apparent discrepancy between the viscosities and thermal conductivities. Nevertheless, the rough agreement between the experimental and calculated contribu-

TABLE 4. Comparison of thermal conductivity eqs (22) and (53)

Temp K	$\lambda_n(\text{exp})$	$\lambda_p(\text{exp})$	$\lambda_p(\text{calc})$	$\lambda'_p(\text{calc})$	$\lambda'_p(\text{exp})$	$\lambda''_p(\text{calc})$	$\lambda''_p(\text{exp})$
	$mWcm^{-1}K^{-1}$						
99.9	0.687	0.797	0.814	0.646	0.649	0.168	0.147
79.9	.561	.599	.616	.551	.549	.065	.051
59.2	.423	.440	.446	.436	.416	.010	.023

tions is encouraging, especially as eq (53) is not required to have the particular form of eq (22).

### 5.2. Correlation of Diffusion Data

In principle, one can check the potential and the parameters chosen by comparing calculated and experimental diffusion and thermal diffusion coefficients. But because of the lack of experimental data, such a comparison is not very significant. For the record we correlated the self-diffusion coefficients at 1 atm pressure measured by Lipsicas [59]. The appropriate deviation curve is given as figure 6. The result is satisfactory, but does not add to, or alter our previous conclusions.

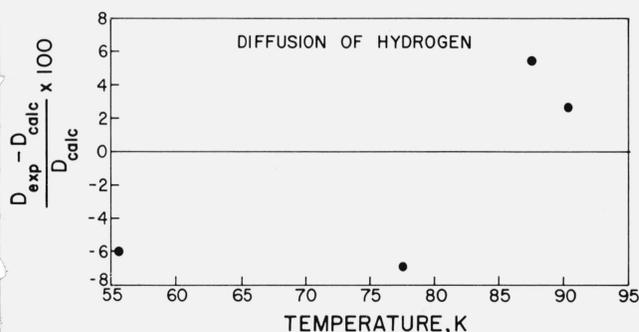


FIGURE 6. Correlation of the self-diffusion results of Lipsicas [59] based on the viscosity correlation.

See caption to figure 3.

## 6. Presentation of Results

Tables of the viscosity and thermal conductivity coefficients were constructed using the formulas and potential functions described in the previous sections. We present the results in tables 5-8. Table 5 gives the coefficients as a function of temperature in the "low temperature" region, table 6 gives the coefficients in the "high temperature" region. As previously explained, we are required at this time to use two separate functions to describe these regions - the 12-6 and 9-6 respectively. Although the regions overlap, there is a slight discontinuity at 160 K. However, we did not feel that our selection of the functions and parameters was definitive enough to justify further manipulation. Tables 7 and 8 list the coefficients for the dissociating mixture; here they are given as a function of temperature and mole fraction of the atomic species H. Such tables have the distinct advantage of compactness but, of course, mole fraction is not necessarily the most

practical variable, one will often prefer pressure. Consequently we also show in table 9, the variation of mole fraction of H as a function of pressure and temperature. In addition, we have plotted the conductivity for several pressures in figure 5 and also include a plot of the viscosity in figure 7.

The lower limit for the pressure was chosen at 0.001 atm. For pressures below this limit, it is very likely that the dimensions of the experimental apparatus would be needed to calculate the coefficients. For pressures above the upper limit, it is unlikely that the kinetic theory expressions are valid. If an extension of this range is desired, although such an extension should be made with caution, one may compute  $x_H$  from eq (5) at the given pressure, using tabulated values of  $K$ , (see, for example, reference [5]).

The upper temperature was chosen to be 5000 K. This already represents a considerable extrapolation of the data and we did not feel that our 9-6 function could be extrapolated much further. Moreover, by not going above 5000 K it is not necessary to consider ionization [6]: ionization is not taken into account in the high temperature collision integrals.

TABLE 5. Viscosity and thermal conductivities of hydrogen at low temperatures

Temperature	Viscosity	Thermal conductivity (normal)	Thermal conductivity (para)
T, K	$10^3 \eta \text{ g cm}^{-1} \text{ s}^{-1}$	$\lambda, \text{ mWcm}^{-1} \text{ K}^{-1}$	
15.0	0.00740	0.114	0.114
20.0	.01023	.158	.158
25.0	.01292	.200	.200
30.0	.01549	.240	.240
35.0	.01795	.278	.278
40.0	.02025	.313	.313
45.0	.02241	.347	.347
50.0	.02451	.380	.382
55.0	.02657	.412	.417
60.0	.02852	.444	.452
65.0	.03032	.474	.489
70.0	.03215	.505	.529
75.0	.03388	.536	.571
80.0	.03562	.568	.617
85.0	.03727	.600	.664
90.0	.03889	.632	.714
95.0	.04028	.662	.763
100.0	.04179	.695	.816
105.0	.04327	.728	.869
110.0	.04487	.763	.924
115.0	.04624	.796	.977
120.0	.04765	.829	1.028
125.0	.04900	.863	1.077
130.0	.05036	.896	1.125
135.0	.05169	.929	1.171
140.0	.05298	.962	1.213
145.0	.05426	.994	1.254
150.0	.05550	1.026	1.291
155.0	.05676	1.058	1.326
160.0	.05793	1.088	1.358
165.0	.05920	1.120	1.389
170.0	.06040	1.151	1.418

TABLE 6. *Viscosity and thermal conductivities of hydrogen at high temperatures without dissociation*

Temperature	Viscosity	Thermal conductivity (normal)	Thermal conductivity (para)	Temperature	Viscosity	Thermal conductivity (normal)	Thermal conductivity (para)
T, K	$10^3 \eta$ $\text{g cm}^{-1} \text{s}^{-1}$	$\lambda, \text{mW cm}^{-1} \text{K}^{-1}$		T, K	$10^3 \eta$ $\text{g cm}^{-1} \text{s}^{-1}$	$\lambda, \text{mW cm}^{-1} \text{K}^{-1}$	
140.0	0.05275	0.959	1.213	600.0	0.14338	3.050	3.050
150.0	.05540	1.026	1.294	610.0	.14499	3.085	3.086
160.0	.05799	1.092	1.365	620.0	.14660	3.121	3.121
170.0	.06052	1.157	1.427	630.0	.14820	3.156	3.156
180.0	.06299	1.220	1.482				
				640.0	.14980	3.191	3.191
190.0	.06541	1.282	1.530	650.0	.15138	3.226	3.226
200.0	.06777	1.342	1.574	660.0	.15296	3.261	3.261
210.0	.07010	1.401	1.614	670.0	.15453	3.296	3.296
220.0	.07239	1.458	1.651	680.0	.15608	3.331	3.331
230.0	.07464	1.514	1.687				
				690.0	.15765	3.366	3.366
240.0	.07686	1.569	1.723	700.0	.15920	3.401	3.401
250.0	.07904	1.622	1.758	720.0	.16228	3.470	3.471
260.0	.08119	1.674	1.793	740.0	.16532	3.540	3.540
270.0	.08332	1.725	1.828	760.0	.16836	3.609	3.610
280.0	.08541	1.774	1.863				
				780.0	.17137	3.678	3.679
290.0	.08748	1.823	1.899	800.0	.17433	3.747	3.748
300.0	.08953	1.870	1.935	820.0	.17730	3.817	3.818
310.0	.09156	1.917	1.972	840.0	.18023	3.886	3.887
320.0	.09356	1.962	2.010	860.0	.18314	3.955	3.956
330.0	.09554	2.007	2.047				
				880.0	.18604	4.025	4.026
340.0	.09749	2.051	2.085	900.0	.18891	4.095	4.096
350.0	.09944	2.095	2.123	920.0	.19175	4.165	4.165
360.0	.10137	2.137	2.161	940.0	.19459	4.235	4.236
370.0	.10326	2.179	2.199	960.0	.19740	4.305	4.306
380.0	.10516	2.220	2.237				
				980.0	.20019	4.376	4.376
390.0	.10703	2.261	2.275	1000.0	.20298	4.447	4.447
400.0	.10888	2.302	2.313	1050.0	.20984	4.623	4.623
410.0	.11073	2.342	2.352	1100.0	.21662	4.801	4.801
420.0	.11255	2.381	2.390	1150.0	.22331	4.980	4.981
430.0	.11436	2.420	2.428				
				1200.0	.22990	5.160	5.161
440.0	.11616	2.459	2.466	1250.0	.23641	5.341	5.343
450.0	.11795	2.498	2.504	1300.0	.24284	5.523	5.525
460.0	.11971	2.536	2.541	1350.0	.24921	5.707	5.708
470.0	.12148	2.575	2.578	1400.0	.25550	5.891	5.892
480.0	.12323	2.613	2.616				
				1450.0	.26172	6.075	6.076
490.0	.12497	2.650	2.653	1500.0	.26789	6.261	6.261
500.0	.12670	2.688	2.690	1550.0	.27398	6.445	6.441
510.0	.12841	2.725	2.726	1600.0	.28002	6.628	6.620
520.0	.13011	2.761	2.763	1650.0	.28601	6.812	6.800
530.0	.13181	2.798	2.799				
				1700.0	.29191	6.995	6.978
540.0	.13349	2.835	2.836	1750.0	.29781	7.178	7.158
550.0	.13516	2.871	2.872	1800.0	.30364	7.360	7.336
560.0	.13682	2.907	2.908	1850.0	.30939	7.541	7.514
570.0	.13848	2.943	2.944	1900.0	.31515	7.723	7.693
580.0	.14012	2.979	2.979				
				1950.0	.32085	7.903	7.871
590.0	.14175	3.014	3.015	2000.0	.32647	8.082	8.048

TABLE 7. Viscosity of dissociating hydrogen, units  $10^3 \text{ g cm}^{-1} \text{ s}^{-1}$  presented as a function of the mole fraction of atomic hydrogen

The asterisks at low temperatures indicate that the dissociation would occur at pressure less than 0.001 atm. The asterisks at high temperatures indicate that the atomic hydrogen could only be present at the particular mole fraction if the pressure were greater than 290 atm.

Temp (K)	Mole fraction										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
2000.0	0.32647	*****	*****	*****	*****	*****	*****	*****	*****	*****	0.2528
2050.0	.33210	*****	*****	*****	*****	*****	*****	*****	*****	*****	.2581
2100.0	.33766	*****	*****	*****	*****	*****	*****	*****	*****	*****	.2633
2150.0	.34317	0.3447	*****	*****	*****	*****	*****	*****	*****	*****	.2687
2200.0	.34867	.3502	*****	*****	*****	*****	*****	*****	*****	*****	.2739
2250.0	.35412	.3559	0.3596	*****	*****	*****	*****	*****	*****	*****	.2791
2300.0	.35952	.3615	.3647	*****	*****	*****	*****	*****	*****	*****	.2844
2350.0	.36492	.3678	.3704	0.3713	*****	*****	*****	*****	*****	*****	.2896
2400.0	.37027	.3727	.3762	.3775	*****	*****	*****	*****	*****	*****	.2948
2450.0	.37557	.3781	.3819	.3834	0.3821	*****	*****	*****	*****	*****	.2999
2500.0	.38086	.3837	.3887	.3893	.3883	0.3838	*****	*****	*****	*****	.3051
2550.0	.38611	.3891	.3933	.3952	.3943	.3903	0.3825	*****	*****	*****	.3101
2600.0	.39132	.3961	.3990	.4010	.4003	.3964	.3889	*****	*****	*****	.3151
2650.0	.39652	.3999	.4047	.4069	.4062	.4024	.3950	0.3836	*****	*****	.3202
2700.0	.40169	.4053	.4103	.4127	.4116	.4084	.4011	.3896	0.3735	*****	.3253
2750.0	.40681	.4107	.4159	.4185	.4182	.4145	.4072	.3956	.3794	*****	.3305
2800.0	.41193	.4160	.4225	.4243	.4241	.4203	.4132	.4016	.3853	*****	.3357
2850.0	.41701	.4213	.4270	.4300	.4300	.4266	.4189	.4076	.3911	0.3691	.3409
2900.0	.42205	.4266	.4326	.4361	.4359	.4326	.4252	.4136	.3970	.3747	.3461
2950.0	.42709	.4318	.4380	.4417	.4418	.4386	.4314	.4195	.4029	.3803	.3513
3000.0	.43210	.4370	.4435	.4472	.4476	.4445	.4373	.4256	.4086	.3858	.3564
3050.0	.43704	.4423	.4490	.4529	.4533	.4506	.4435	.4317	.4147	.3916	.3619
3100.0	.44204	.4475	.4545	.4586	.4594	.4566	.4495	.4378	.4206	.3973	.3672
3150.0	.44698	.4526	.4599	.4642	.4653	.4620	.4555	.4437	.4264	.4029	.3723
3200.0	.45190	.4578	.4653	.4698	.4710	.4684	.4615	.4496	.4322	.4084	.3775
3250.0	.45680	.4629	.4707	.4755	.4769	.4744	.4671	.4556	.4380	.4140	.3826
3300.0	.46168	.4680	.4761	.4811	.4826	.4802	.4733	.4614	.4437	.4194	.3876
3350.0	.46653	.4731	.4814	.4866	.4883	.4861	.4792	.4671	.4494	.4248	.3925
3400.0	.47136	.4782	.4868	.4922	.4941	.4919	.4851	.4729	.4550	.4301	.3975
3450.0	.47617	.4832	.4920	.4977	.4997	.4976	.4908	.4787	.4605	.4353	.4022
3500.0	.48096	.4883	.4974	.5033	.5055	.5035	.4968	.4846	.4662	.4407	.4071
3550.0	.48572	.4933	.5026	.5087	.5111	.5092	.5025	.4902	.4716	.4458	.4118
3600.0	.49046	.4983	.5079	.5142	.5167	.5149	.5082	.4958	.4771	.4509	.4164
3650.0	.49517	.5032	.5131	.5196	.5223	.5206	.5139	.5014	.4825	.4560	.4210
3700.0	.49985	.5082	.5183	.5251	.5279	.5263	.5196	.5071	.4879	.4612	.4257
3750.0	.50451	.5131	.5235	.5305	.5335	.5320	.5253	.5127	.4933	.4662	.4303
3800.0	.50914	.5180	.5287	.5359	.5390	.5376	.5309	.5182	.4987	.4713	.4348
3850.0	.51367	.5229	.5339	.5413	.5446	.5433	.5366	.5238	.5041	.4764	.4394
3900.0	.51847	.5278	.5390	.5466	.5501	.5489	.5422	.5294	.5095	.4814	.4439
3950.0	.52310	.5327	.5442	.5520	.5557	.5545	.5479	.5350	.5148	.4864	.4485
4000.0	.52772	.5375	.5493	.5573	.5611	.5601	.5535	.5405	.5201	.4914	.4529
4050.0	.53231	.5424	.5545	.5627	.5667	.5658	.5591	.5461	.5255	.4965	.4575
4100.0	.53689	*****	.5595	.5680	.5721	.5713	.5647	.5515	.5308	.5014	.4619
4150.0	.54145	*****	.5646	.5733	.5776	.5769	.5703	.5570	.5361	.5064	.4664
4200.0	.54598	*****	.5697	.5786	.5831	.5824	.5759	.5625	.5414	.5113	.4708
4250.0	.55050	*****	.5748	.5839	.5886	.5880	.5815	.5681	.5467	.5163	.4753
4300.0	.55500	*****	.5799	.5893	.5941	.5936	.5871	.5736	.5521	.5213	.4798
4350.0	.55947	*****	.5848	.5945	.5994	.5991	.5926	.5790	.5573	.5262	.4842
4400.0	.56393	*****	.5899	.5998	.6049	.6047	.5982	.5845	.5626	.5312	.4887
4450.0	.56836	*****	.5948	.6049	.6103	.6101	.6036	.5899	.5678	.5360	.4930
4500.0	.57277	*****	.5998	.6102	.6157	.6156	.6091	.5953	.5731	.5409	.4974
4550.0	.57715	*****	.6049	.6155	.6211	.6212	.6147	.6009	.5784	.5459	.5019
4600.0	.58148	*****	*****	.6206	.6265	.6266	.6202	.6063	.5836	.5508	.5063
4650.0	.58598	*****	*****	.6258	.6318	.6321	.6257	.6117	.5888	.5557	.5107
4700.0	.59038	*****	*****	.6310	.6372	.6375	.6312	.6171	.5941	.5606	.5151
4750.0	.59476	*****	*****	.6362	.6426	.6431	.6367	.6226	.5994	.5656	.5195
4800.0	.59913	*****	*****	.6414	.6479	.6485	.6423	.6280	.6046	.5705	.5239
4850.0	.60348	*****	*****	.6465	.6533	.6540	.6477	.6334	.6098	.5754	.5283
4900.0	.60782	*****	*****	.6517	.6586	.6594	.6532	.6388	.6150	.5802	.5327
4950.0	.61215	*****	*****	*****	.6640	.6649	.6587	.6443	.6203	.5852	.5371
5000.0	.61645	*****	*****	*****	.6691	.6702	.6640	.6495	.6254	.5900	.5414

TABLE 8. *Thermal conductivity of dissociating hydrogen, units  $mWcm^{-1}s^{-1}$ , presented as a function of the mole fraction of atomic hydrogen*  
 The asterisks indicate that we would have to go beyond our chosen pressure range to compute the conductivity at the particular mole fractions. See the caption to table 7

Temp (K)	Mole fraction											
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
2000.0	8.082	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****	8.702
2050.0	8.263	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****	8.424
2100.0	8.442	*****	*****	*****	*****	*****	*****	*****	*****	*****	*****	8.375
2150.0	8.620	64.866	*****	*****	*****	*****	*****	*****	*****	*****	*****	8.430
2200.0	8.798	61.665	*****	*****	*****	*****	*****	*****	*****	*****	*****	8.538
2250.0	8.975	60.599	111.300	*****	*****	*****	*****	*****	*****	*****	*****	8.671
2300.0	9.151	59.595	109.340	*****	*****	*****	*****	*****	*****	*****	*****	8.817
2350.0	9.327	58.630	107.088	153.200	*****	*****	*****	*****	*****	*****	*****	8.969
2400.0	9.502	57.906	105.150	150.211	*****	*****	*****	*****	*****	*****	*****	9.124
2450.0	9.675	56.895	103.297	147.399	188.766	*****	*****	*****	*****	*****	*****	9.281
2500.0	9.848	56.088	101.50	144.718	183.845	218.087	*****	*****	*****	*****	*****	9.440
2550.0	10.021	55.319	99.874	142.139	180.425	211.857	232.876	*****	*****	*****	*****	9.594
2600.0	10.192	54.57	98.208	139.78	177.171	207.991	228.088	*****	*****	*****	*****	9.749
2650.0	10.362	53.910	96.662	137.907	174.075	204.296	223.953	226.918	*****	*****	*****	9.906
2700.0	10.532	53.226	95.169	135.098	171.100	200.736	220.012	222.949	200.268	*****	*****	10.064
2750.0	10.701	52.607	94.239	132.879	168.471	197.402	216.232	219.111	196.847	*****	*****	10.223
2800.0	10.869	52.002	92.380	130.799	165.556	194.000	212.605	215.421	193.573	*****	*****	10.383
2850.0	11.037	51.429	91.216	128.812	162.902	190.897	209.513	211.878	190.425	*****	*****	10.543
2900.0	11.204	50.886	89.884	126.900	160.385	187.878	205.868	208.469	187.392	*****	*****	10.704
2950.0	11.370	50.367	88.653	125.000	157.959	184.987	202.547	205.118	184.473	*****	*****	10.866
3000.0	11.536	49.872	87.516	123.328	155.804	182.186	199.443	202.026	181.663	125.158	*****	11.026
3050.0	11.703	49.398	86.409	121.610	153.400	179.485	196.447	198.989	178.933	123.400	*****	11.192
3100.0	11.867	48.949	85.356	119.944	151.516	176.898	193.570	196.065	176.369	121.710	*****	11.355
3150.0	12.032	48.518	84.337	118.378	149.189	174.400	190.786	193.239	173.838	120.077	*****	11.517
3200.0	12.196	48.106	83.358	116.851	147.115	172.218	188.100	190.506	171.409	118.500	*****	11.676
3250.0	12.361	47.715	82.418	115.392	145.177	169.623	185.935	187.874	169.079	116.991	*****	11.833
3300.0	12.525	47.340	81.513	113.979	143.315	167.377	183.089	185.329	166.813	115.518	*****	11.988
3350.0	12.689	46.983	80.641	112.619	141.502	165.219	180.589	182.818	164.625	114.109	*****	12.142
3400.0	12.854	46.616	79.754	111.235	139.670	163.004	178.132	180.309	162.402	112.671	*****	12.293
3450.0	13.018	46.312	78.989	110.029	138.064	161.064	175.987	178.182	160.454	111.411	*****	12.443
3500.0	13.182	46.000	78.209	108.803	136.434	159.101	173.796	175.954	158.449	110.131	*****	12.591
3550.0	13.346	45.701	77.457	107.817	134.854	157.193	171.675	173.790	156.568	108.891	*****	12.735
3600.0	13.510	45.416	76.734	106.472	133.328	155.353	169.626	171.702	154.705	107.692	*****	12.879
3650.0	13.674	45.143	76.033	105.363	131.846	153.564	167.632	169.674	152.891	106.530	*****	13.023
3700.0	13.839	44.882	75.357	104.291	130.414	151.833	165.706	167.709	151.147	105.417	*****	13.165
3750.0	14.003	44.634	74.706	103.256	129.029	150.158	163.839	165.809	149.464	104.325	*****	13.307
3800.0	14.168	44.397	74.079	102.256	127.689	148.536	162.032	163.968	147.826	103.264	*****	13.449
3850.0	14.330	44.171	73.471	101.284	126.386	146.959	160.273	162.175	146.234	102.252	*****	13.590
3900.0	14.501	43.954	72.885	100.345	125.124	145.430	158.568	160.437	144.690	101.263	*****	13.731
3950.0	16.672	43.747	72.318	99.435	123.901	143.948	156.914	158.752	143.193	100.303	*****	13.871
4000.0	14.836	43.551	71.773	98.555	122.718	142.512	155.310	157.117	141.742	99.373	*****	14.010
4050.0	14.998	43.364	71.244	97.701	121.567	141.115	153.751	155.527	140.329	98.471	*****	14.149
4100.0	15.172	*****	70.733	96.872	120.451	139.758	152.235	153.981	138.958	97.596	*****	14.288
4150.0	15.340	*****	70.242	96.072	119.370	138.445	150.767	152.484	137.629	96.749	*****	14.426
4200.0	15.508	*****	69.765	95.294	118.317	137.165	149.336	151.024	136.334	95.924	*****	14.564
4250.0	15.676	*****	69.301	94.536	117.290	135.915	147.938	149.598	135.069	95.120	*****	14.702
4300.0	15.843	*****	68.854	93.800	116.293	134.701	146.578	148.210	133.839	94.340	*****	14.839
4350.0	16.011	*****	68.420	93.085	115.321	133.516	145.253	146.858	132.639	93.580	*****	14.976
4400.0	16.177	*****	68.002	92.394	114.380	132.368	143.967	145.545	131.476	92.843	*****	15.113
4450.0	16.343	*****	67.595	91.720	113.462	131.246	142.711	144.262	130.339	92.125	*****	15.250
4500.0	16.509	*****	67.203	91.065	112.569	130.155	141.488	143.014	129.232	91.426	*****	15.387
4550.0	16.672	*****	66.823	90.429	111.699	129.091	140.294	141.794	128.152	90.745	*****	15.522
4600.0	16.834	*****	*****	89.815	110.855	128.058	139.135	140.610	127.103	90.086	*****	15.661
4650.0	16.999	*****	*****	89.212	110.031	127.048	138.001	139.452	126.077	89.442	*****	15.797
4700.0	17.160	*****	*****	88.630	109.231	126.066	136.898	138.324	125.079	88.816	*****	15.933
4750.0	17.318	*****	*****	88.065	108.453	125.111	135.824	137.227	124.107	88.207	*****	16.069
4800.0	17.475	*****	*****	87.519	107.698	124.182	134.780	136.159	123.161	87.616	*****	16.205
4850.0	17.628	*****	*****	86.987	106.962	123.275	133.758	135.114	122.236	87.039	*****	16.340
4900.0	17.777	*****	*****	86.471	106.245	122.392	132.764	134.096	121.335	86.477	*****	16.475
4950.0	17.923	*****	*****	*****	105.551	121.534	131.797	133.106	120.459	85.932	*****	16.610
5000.0	18.064	*****	*****	*****	104.877	120.700	130.856	132.142	119.606	85.404	*****	16.747

TABLE 9. *The mole fraction of atomic hydrogen presented as a function of temperature and pressure*

Note: Since the reference dissociation constant is given with pressure units of atmospheres we use atmospheres here. To convert pressure to the S. I. system the following conversion is required:

$$1 \text{ atm.} = 0.101325 \text{ MN/m}^2.$$

Temp (K)	Pressure (atm)									
	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.010
2000.0	0.050	0.036	0.029	0.025	0.023	0.021	0.019	0.018	0.017	0.016
2050.0	.069	.049	.041	.035	.032	.029	.027	.025	.024	.022
2100.0	.094	.067	.055	.048	.043	.039	.037	.034	.032	.031
2150.0	.125	.090	.074	.064	.058	.053	.049	.046	.043	.041
2200.0	.163	.118	.098	.085	.076	.070	.065	.061	.058	.055
2250.0	.209	.153	.127	.111	.100	.091	.085	.080	.075	.072
2300.0	.263	.194	.162	.142	.128	.117	.109	.102	.097	.092
2350.0	.324	.243	.203	.179	.161	.148	.138	.130	.123	.117
2400.0	.392	.298	.251	.222	.201	.185	.173	.163	.154	.147
2450.0	.465	.359	.305	.271	.246	.228	.213	.201	.190	.182
2500.0	.539	.426	.365	.326	.298	.276	.258	.244	.232	.222
2550.0	.613	.495	.430	.386	.354	.329	.310	.293	.279	.267
2600.0	.683	.565	.497	.450	.415	.387	.365	.347	.331	.317
2650.0	.746	.634	.564	.515	.478	.449	.425	.404	.387	.371
2700.0	.801	.698	.630	.581	.543	.512	.486	.464	.446	.429
2750.0	.847	.756	.692	.644	.606	.575	.549	.526	.506	.489
2800.0	.883	.806	.748	.704	.667	.636	.610	.587	.567	.549
2850.0	.912	.848	.798	.757	.723	.694	.669	.646	.626	.608
2900.0	.934	.882	.840	.804	.773	.747	.723	.702	.682	.665
2950.0	.950	.909	.874	.844	.817	.793	.771	.752	.734	.718
3000.0	.963	.930	.902	.877	.854	.833	.814	.796	.780	.765
3050.0	.972	.947	.924	.903	.884	.866	.850	.835	.820	.807
3100.0	.979	.959	.941	.924	.909	.894	.880	.867	.855	.843
3150.0	.984	.969	.954	.941	.928	.916	.905	.894	.883	.873
3200.0	.988	.976	.965	.954	.944	.934	.924	.915	.906	.898
3250.0	.990	.981	.972	.964	.956	.948	.940	.933	.925	.918
3300.0	.993	.985	.978	.972	.965	.959	.952	.946	.940	.935
3350.0	.994	.989	.983	.978	.972	.967	.962	.957	.952	.948
3400.0	.995	.989	.984	.979	.974	.970	.965	.960	.956	.951
3450.0	.996	.993	.989	.986	.983	.979	.976	.973	.969	.966

TABLE 9. *The mole fraction of atomic hydrogen presented as a function of temperature and pressure—Continued*

Note: Since the reference dissociation constant is given with pressure units of atmospheres we use atmospheres here. To convert pressure to the S. I. system the following conversion is required:

$$1 \text{ atm.} = 0.101325 \text{ MN/m}^2.$$

Temp (K)	Pressure (atm)									
	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100
2250.0	0.072	0.051	0.042	0.036	0.033	0.030	0.028	0.026	0.024	0.023
2300.0	.092	.066	.054	.047	.042	.039	.036	.034	.032	.030
2350.0	.117	.084	.069	.060	.054	.050	.046	.043	.041	.039
2400.0	.147	.106	.088	.076	.069	.063	.058	.055	.052	.049
2450.0	.182	.132	.109	.095	.086	.079	.073	.069	.065	.062
2500.0	.222	.163	.135	.118	.106	.097	.091	.085	.080	.076
2550.0	.267	.197	.164	.144	.130	.119	.111	.104	.099	.094
2600.0	.317	.237	.198	.174	.157	.145	.135	.127	.120	.114
2650.0	.371	.281	.236	.208	.189	.174	.162	.152	.144	.138
2700.0	.429	.329	.278	.246	.224	.206	.193	.182	.172	.164
2750.0	.489	.380	.324	.288	.263	.243	.227	.214	.203	.194
2800.0	.549	.435	.374	.334	.305	.283	.265	.250	.238	.227
2850.0	.608	.481	.425	.382	.350	.326	.306	.290	.276	.264
2900.0	.665	.547	.479	.433	.398	.372	.350	.332	.316	.303
2950.0	.718	.602	.533	.485	.448	.420	.396	.377	.360	.345
3000.0	.765	.655	.586	.537	.499	.469	.444	.424	.406	.390
3050.0	.807	.705	.638	.589	.551	.520	.494	.472	.453	.436
3100.0	.843	.751	.687	.639	.601	.570	.543	.521	.501	.483
3150.0	.873	.791	.732	.686	.649	.618	.592	.569	.549	.531
3200.0	.898	.827	.773	.730	.694	.664	.638	.616	.595	.577
3250.0	.918	.857	.809	.770	.737	.708	.683	.661	.641	.623
3300.0	.935	.883	.841	.806	.775	.748	.725	.704	.684	.667
3350.0	.948	.905	.868	.837	.809	.785	.763	.743	.725	.708
3400.0	.951	.911	.876	.846	.819	.796	.774	.755	.737	.721
3450.0	.966	.937	.910	.887	.865	.845	.827	.810	.795	.780
3500.0	.973	.948	.926	.906	.887	.870	.854	.839	.825	.811
3550.0	.978	.958	.939	.922	.906	.891	.877	.863	.851	.839
3600.0	.982	.966	.950	.935	.922	.909	.896	.885	.873	.863
3650.0	.986	.972	.959	.947	.935	.924	.913	.903	.893	.883
3700.0	.988	.977	.966	.956	.946	.936	.927	.918	.910	.901
3750.0	.990	.981	.972	.963	.955	.947	.939	.931	.924	.917
3800.0	.992	.984	.977	.969	.962	.955	.949	.942	.936	.929
3850.0	.993	.987	.981	.974	.968	.962	.957	.951	.946	.940
3900.0	.994	.989	.984	.979	.973	.968	.964	.959	.954	.949
3950.0	.995	.991	.986	.982	.978	.973	.969	.965	.961	.957
4000.0	.996	.992	.988	.985	.981	.977	.974	.970	.967	.964

TABLE 9. *The mole fraction of atomic hydrogen presented as a function of temperature and pressure—Continued*

Note: Since the reference dissociation constant is given with pressure units of atmospheres we use atmospheres here. To convert pressure to the S. I. system the following conversion is required:

$$1 \text{ atm.} = 0.101325 \text{ MN/m}^2.$$

Temp (K)	Pressure (atm)									
	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
2500.0	0.076	0.055	0.045	0.039	0.035	0.032	0.030	0.028	0.026	0.025
2550.0	.094	.067	.055	.048	.043	.039	.037	.034	.032	.031
2600.0	.114	.082	.068	.059	.053	.048	.045	.042	.040	.038
2650.0	.138	.099	.082	.071	.064	.059	.054	.051	.048	.046
2700.0	.164	.119	.098	.086	.077	.071	.066	.061	.058	.055
2750.0	.194	.142	.117	.102	.092	.084	.078	.074	.070	.066
2800.0	.227	.167	.139	.121	.109	.100	.093	.087	.083	.078
2850.0	.264	.195	.162	.142	.128	.118	.110	.103	.097	.093
2900.0	.303	.226	.189	.166	.150	.138	.128	.120	.114	.108
2950.0	.345	.260	.218	.192	.174	.160	.149	.140	.133	.126
3000.0	.390	.296	.250	.220	.200	.184	.172	.162	.153	.146
3050.0	.436	.335	.284	.251	.228	.211	.197	.185	.176	.168
3100.0	.483	.376	.320	.284	.259	.239	.224	.211	.200	.191
3150.0	.531	.418	.358	.320	.292	.270	.253	.239	.227	.217
3200.0	.577	.461	.398	.356	.326	.303	.284	.269	.255	.244
3250.0	.623	.505	.439	.395	.363	.337	.317	.300	.286	.274
3300.0	.667	.549	.481	.435	.400	.373	.352	.334	.318	.305
3350.0	.708	.592	.523	.475	.439	.411	.387	.368	.352	.337
3400.0	.721	.606	.536	.488	.452	.423	.399	.380	.363	.348
3450.0	.780	.673	.604	.555	.517	.487	.462	.440	.422	.406
3500.0	.811	.711	.644	.595	.556	.525	.499	.477	.458	.441
3550.0	.839	.745	.681	.633	.595	.563	.537	.514	.495	.477
3600.0	.863	.777	.716	.669	.632	.601	.574	.551	.531	.514
3650.0	.883	.806	.749	.704	.667	.637	.611	.588	.567	.549
3700.0	.901	.832	.779	.736	.701	.671	.646	.623	.603	.585
3750.0	.917	.855	.806	.766	.733	.704	.679	.657	.637	.619
3800.0	.929	.875	.831	.794	.762	.735	.711	.689	.670	.652
3850.0	.940	.892	.853	.819	.789	.764	.741	.720	.701	.684
3900.0	.949	.908	.872	.842	.814	.790	.768	.748	.730	.714
3950.0	.957	.921	.889	.861	.836	.814	.794	.775	.758	.742
4000.0	.964	.932	.904	.879	.856	.836	.817	.799	.783	.768
4050.0	.969	.941	.917	.894	.874	.855	.838	.822	.807	.792
4100.0	.974	.950	.928	.908	.890	.872	.857	.842	.828	.815
4150.0	.977	.957	.937	.920	.903	.888	.873	.860	.847	.835
4200.0	.981	.963	.946	.930	.915	.902	.888	.876	.864	.853
4250.0	.983	.968	.953	.939	.926	.914	.902	.890	.880	.869
4300.0	.986	.972	.959	.947	.935	.924	.913	.903	.894	.884
4350.0	.988	.976	.964	.954	.943	.933	.924	.915	.906	.897
4400.0	.989	.979	.969	.959	.950	.941	.933	.925	.917	.909
4450.0	.991	.982	.973	.964	.956	.948	.941	.933	.926	.919
4500.0	.992	.984	.976	.969	.962	.955	.948	.941	.935	.928
4550.0	.993	.986	.979	.973	.966	.960	.954	.948	.942	.936
4600.0	.994	.988	.982	.976	.970	.965	.959	.954	.949	.944
4650.0	.994	.989	.984	.979	.974	.969	.964	.959	.954	.950
4700.0	.995	.990	.986	.981	.977	.972	.968	.964	.959	.955
4750.0	.996	.992	.987	.983	.979	.975	.971	.968	.964	.960
4800.0	.996	.992	.989	.985	.982	.978	.975	.971	.968	.964

TABLE 9. *The mole fraction of atomic hydrogen presented as a function of temperature and pressure—Continued*

Note: Since the reference dissociation constant is given with pressure units of atmospheres we use atmospheres here. To convert pressure to the S. I. system the following conversion is required:

$$1 \text{ atm.} = 0.101325 \text{ MN/m}^2.$$

Temp (K)	Pressure (atm)									
	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000
2800.0	0.078	0.056	0.046	0.040	0.036	0.033	0.030	0.028	0.027	0.026
2850.0	.093	.066	.055	.047	.043	.039	.036	.034	.032	.030
2900.0	.108	.078	.064	.056	.050	.046	.042	.040	.038	.036
2950.0	.126	.091	.075	.065	.059	.054	.050	.047	.044	.042
3000.0	.146	.106	.087	.076	.068	.062	.058	.054	.051	.049
3050.0	.168	.122	.101	.088	.079	.072	.067	.063	.059	.056
3100.0	.191	.139	.115	.101	.091	.083	.077	.072	.068	.065
3150.0	.217	.159	.132	.115	.104	.095	.088	.083	.078	.075
3200.0	.244	.180	.150	.131	.118	.108	.101	.094	.089	.085
3250.0	.274	.203	.169	.148	.134	.123	.114	.107	.101	.097
3300.0	.305	.227	.190	.167	.151	.138	.129	.121	.115	.109
3350.0	.337	.253	.212	.187	.169	.155	.145	.136	.129	.123
3400.0	.348	.262	.220	.194	.175	.161	.150	.141	.134	.127
3450.0	.406	.309	.261	.231	.209	.193	.180	.170	.161	.153
3500.0	.441	.339	.288	.255	.232	.214	.200	.188	.178	.170
3550.0	.477	.371	.315	.280	.255	.236	.220	.208	.197	.188
3600.0	.514	.402	.344	.307	.279	.259	.242	.229	.217	.207
3650.0	.549	.435	.374	.334	.305	.283	.265	.251	.238	.227
3700.0	.585	.468	.404	.362	.332	.308	.289	.273	.260	.249
3750.0	.619	.501	.435	.391	.359	.334	.314	.297	.283	.271
3800.0	.652	.534	.466	.421	.387	.361	.340	.322	.307	.294
3850.0	.684	.566	.498	.451	.416	.388	.366	.347	.332	.318
3900.0	.714	.598	.529	.481	.445	.416	.393	.374	.357	.342
3950.0	.742	.629	.560	.511	.474	.445	.420	.400	.383	.367
4000.0	.768	.659	.590	.541	.503	.473	.448	.427	.409	.393
4050.0	.792	.686	.620	.570	.532	.501	.476	.454	.436	.419
4100.0	.815	.715	.648	.599	.561	.530	.504	.482	.462	.446
4150.0	.835	.740	.676	.627	.589	.558	.531	.509	.489	.472
4200.0	.853	.764	.702	.654	.616	.585	.559	.536	.516	.498
4250.0	.869	.786	.727	.680	.643	.612	.586	.563	.543	.525
4300.0	.884	.807	.750	.705	.669	.638	.612	.589	.569	.551
4350.0	.897	.826	.772	.729	.693	.663	.637	.614	.594	.576
4400.0	.909	.843	.792	.751	.717	.687	.662	.639	.619	.601
4450.0	.919	.859	.811	.772	.739	.710	.686	.663	.644	.626
4500.0	.928	.873	.829	.792	.760	.732	.708	.687	.667	.649
4550.0	.936	.886	.845	.810	.780	.753	.730	.709	.690	.672
4600.0	.944	.898	.859	.827	.798	.773	.750	.730	.711	.694
4650.0	.950	.908	.873	.842	.815	.791	.769	.750	.732	.715
4700.0	.955	.918	.885	.856	.831	.808	.788	.769	.751	.735
4750.0	.960	.926	.896	.869	.846	.824	.804	.786	.770	.754
4800.0	.964	.933	.906	.881	.859	.839	.820	.803	.787	.772
4850.0	.968	.940	.915	.892	.871	.852	.835	.819	.803	.789
4900.0	.972	.946	.923	.902	.883	.865	.848	.833	.819	.805
4950.0	.975	.951	.930	.911	.893	.877	.861	.846	.833	.820
5000.0	.977	.956	.937	.919	.903	.887	.873	.859	.846	.834

TABLE 9. The mole fraction of atomic hydrogen presented as a function of temperature and pressure—Continued

Note: Since the reference dissociation constant is given with pressure units of atmospheres we use atmospheres here. To convert pressure to the S. I. system the following conversion is required:

$$1 \text{ atm.} = 0.101325 \text{ MN/m}^2.$$

Temp (K)	Pressure (atm)									
	10.000	20.000	30.000	40.000	50.000	60.000	70.000	80.000	90.000	100.000
3200.0	0.085	0.061	0.050	0.043	0.039	0.036	0.033	0.031	0.029	0.028
3250.0	.097	.069	.057	.049	.044	.041	.038	.035	.033	.032
3300.0	.109	.078	.065	.056	.050	.046	.043	.040	.038	.036
3350.0	.123	.088	.073	.063	.057	.052	.048	.045	.043	.041
3400.0	.127	.092	.076	.066	.059	.054	.050	.047	.044	.042
3450.0	.153	.111	.092	.080	.072	.066	.061	.057	.054	.051
3500.0	.170	.124	.102	.089	.080	.073	.068	.064	.060	.057
3550.0	.188	.137	.114	.099	.089	.082	.076	.071	.067	.064
3600.0	.207	.152	.126	.110	.099	.091	.084	.079	.075	.071
3650.0	.227	.167	.139	.121	.109	.100	.093	.087	.083	.079
3700.0	.249	.201	.153	.134	.120	.110	.103	.096	.091	.087
3750.0	.271	.261	.167	.147	.132	.121	.113	.106	.100	.095
3800.0	.294	.219	.183	.160	.145	.133	.124	.116	.110	.105
3850.0	.318	.238	.199	.175	.158	.145	.135	.127	.120	.114
3900.0	.342	.257	.216	.190	.172	.158	.147	.139	.131	.125
3950.0	.367	.278	.234	.206	.186	.172	.160	.151	.143	.136
4000.0	.393	.299	.252	.223	.202	.186	.173	.163	.155	.147
4050.0	.419	.321	.271	.240	.218	.201	.187	.176	.167	.159
4100.0	.446	.343	.291	.258	.234	.216	.202	.190	.181	.172
4150.0	.472	.366	.311	.276	.251	.232	.217	.205	.194	.185
4200.0	.498	.389	.332	.295	.269	.249	.233	.220	.209	.199
4250.0	.525	.412	.353	.315	.287	.266	.249	.235	.224	.213
4300.0	.551	.436	.375	.335	.306	.284	.266	.251	.239	.228
4350.0	.576	.460	.397	.355	.325	.302	.283	.268	.255	.243
4400.0	.601	.484	.419	.376	.345	.320	.301	.285	.271	.259
4450.0	.626	.508	.441	.397	.365	.339	.319	.302	.288	.275
4500.0	.649	.531	.464	.418	.385	.359	.337	.320	.305	.292
4550.0	.672	.554	.486	.440	.405	.378	.356	.338	.322	.309
4600.0	.694	.577	.508	.461	.426	.398	.375	.356	.340	.326
4650.0	.715	.600	.530	.482	.446	.418	.394	.375	.358	.344
4700.0	.735	.622	.552	.504	.467	.438	.414	.394	.376	.361
4750.0	.754	.643	.574	.525	.487	.458	.433	.412	.395	.379
4800.0	.772	.664	.595	.546	.508	.478	.453	.431	.413	.397
4850.0	.789	.684	.615	.566	.528	.497	.472	.451	.432	.416
4900.0	.805	.703	.636	.586	.548	.517	.491	.470	.451	.434
4950.0	.820	.721	.655	.606	.568	.537	.511	.489	.469	.452
5000.0	.834	.739	.674	.626	.587	.556	.530	.507	.488	.471

TABLE 9. *The mole fraction of atomic hydrogen presented as a function of temperature and pressure—Continued*

Note: Since the reference dissociation constant is given with pressure units of atmospheres we use atmospheres here. To convert pressure to the S. I. system the following conversion is required:

$$1 \text{ atm.} = 0.101325 \text{ MN/m}^2.$$

Temp (K)	Pressure (atm)									
	100.000	110.000	120.000	130.000	140.000	150.000	160.000	170.000	180.000	190.000
3750.0	0.095	0.091	0.087	0.084	0.081	0.079	0.076	0.074	0.072	0.070
3800.0	.105	.100	.096	.092	.089	.086	.084	.081	.079	.077
3850.0	.114	.109	.105	.101	.098	.095	.092	.089	.087	.084
3900.0	.125	.119	.115	.110	.107	.103	.100	.097	.095	.092
3950.0	.136	.130	.125	.120	.116	.112	.109	.106	.103	.101
4000.0	.147	.141	.135	.131	.126	.122	.118	.115	.112	.109
4050.0	.159	.153	.147	.141	.137	.132	.128	.125	.122	.118
4100.0	.172	.165	.158	.153	.148	.143	.139	.135	.131	.128
4150.0	.185	.178	.171	.165	.159	.154	.150	.146	.142	.138
4200.0	.199	.191	.184	.177	.171	.166	.161	.157	.153	.149
4250.0	.213	.205	.197	.190	.184	.178	.173	.168	.164	.160
4300.0	.228	.219	.211	.203	.197	.191	.185	.180	.176	.172
4350.0	.243	.234	.225	.217	.210	.204	.198	.193	.188	.184
4400.0	.259	.249	.240	.232	.224	.217	.211	.206	.201	.196
4450.0	.275	.264	.255	.246	.239	.232	.225	.219	.214	.209
4500.0	.292	.281	.270	.261	.253	.246	.239	.233	.227	.222
4550.0	.309	.297	.286	.277	.268	.261	.254	.247	.241	.236
4600.0	.326	.314	.303	.293	.284	.276	.268	.262	.255	.250
4650.0	.344	.331	.319	.309	.300	.291	.284	.277	.270	.264
4700.0	.361	.348	.336	.326	.316	.307	.299	.292	.285	.279
4750.0	.379	.366	.353	.342	.332	.323	.315	.307	.300	.293
4800.0	.397	.383	.371	.359	.349	.340	.331	.323	.316	.309
4850.0	.416	.401	.388	.376	.366	.356	.347	.339	.331	.324
4900.0	.434	.419	.406	.394	.383	.373	.364	.355	.347	.340
4950.0	.452	.437	.423	.411	.400	.390	.380	.372	.363	.356
5000.0	.471	.455	.441	.429	.417	.407	.397	.388	.380	.372

TABLE 9. The mole fraction of atomic hydrogen presented as a function of temperature and pressure—Continued

Note: Since the reference dissociation constant is given with pressure units of atmospheres we use atmospheres here. To convert pressure to the S. I. system the following conversion is required:

$$1 \text{ atm.} = 0.101325 \text{ MN/m}^2.$$

Temp (K)	Pressure (atm)									
	200.000	210.000	220.000	230.000	240.000	250.000	260.000	270.000	280.000	290.000
3900.0	0.090	0.088	0.086	0.084	0.083	0.081	0.079	0.078	0.077	0.075
3950.0	.098	.096	.094	.092	.090	.088	.087	.085	.084	.082
4000.0	.107	.104	.102	.100	.098	.096	.094	.093	.091	.089
4050.0	.116	.113	.111	.108	.106	.104	.102	.100	.099	.097
4100.0	.125	.122	.120	.117	.115	.113	.111	.109	.107	.105
4150.0	.135	.132	.129	.127	.124	.122	.120	.117	.115	.114
4200.0	.145	.142	.139	.136	.134	.131	.129	.127	.124	.122
4250.0	.156	.153	.150	.147	.144	.141	.139	.136	.134	.132
4300.0	.168	.164	.160	.157	.154	.151	.149	.146	.144	.141
4350.0	.179	.175	.172	.168	.165	.162	.159	.156	.154	.151
4400.0	.191	.187	.183	.180	.176	.173	.170	.167	.164	.162
4450.0	.204	.200	.196	.192	.188	.185	.181	.178	.176	.173
4500.0	.217	.212	.208	.204	.200	.197	.193	.190	.187	.184
4550.0	.230	.226	.221	.217	.213	.209	.205	.202	.199	.196
4600.0	.244	.239	.234	.230	.226	.222	.218	.214	.211	.208
4650.0	.258	.253	.248	.243	.239	.235	.231	.227	.223	.220
4700.0	.273	.267	.262	.257	.252	.248	.244	.240	.236	.233
4750.0	.287	.282	.276	.271	.266	.262	.257	.253	.249	.245
4800.0	.302	.296	.291	.285	.280	.276	.271	.267	.263	.259
4850.0	.318	.311	.305	.300	.295	.290	.285	.281	.276	.272
4900.0	.333	.327	.320	.315	.309	.304	.299	.295	.290	.286
4950.0	.349	.342	.336	.330	.324	.319	.314	.309	.305	.300
5000.0	.364	.358	.351	.345	.339	.334	.329	.324	.319	.314

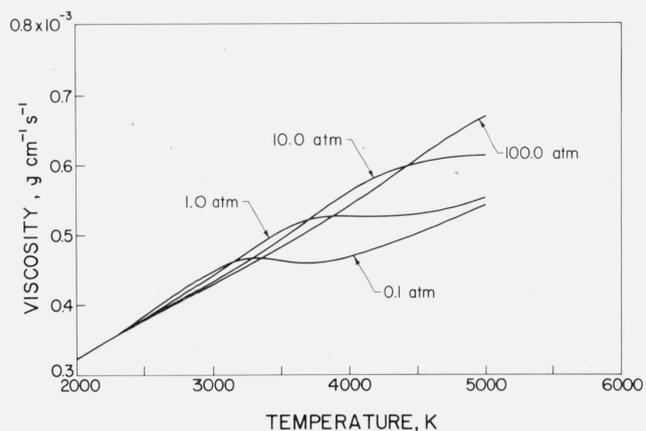


FIGURE 7. The viscosity of dissociating hydrogen at various pressures.

We place an error bar on the values tabulated of  $\pm 3$  percent up to 2000 K for viscosity and 4 percent for thermal conductivity, and  $\pm 10$  percent on both coefficients for the dissociating mixture.

### 7. Conclusion

We have correlated the viscosity and thermal conductivity coefficients of normal and para hydrogen. We hope we have reduced the considerable uncertainties and variations in data which existed in the literature until recently but clearly the situation is not very satisfactory. However, the state of the art is such

that experimental measurements could be produced which would be superior to those available, both in extended temperature range and in accuracy.

This work was supported by the Office of Standard Reference Data. In addition, R. D. McCarty was partially supported by the National Aeronautics and Space Administration (SNPO-C), Contract No. R-45.

We are very grateful to D. E. Diller and H. M. Roder, NBS-Cryogenics Division, for valuable discussions and for allowing us to use their data prior to publication. We also acknowledge the help given us by reading the relevant publications of E. A. Mason of Brown University and of R. Brokaw at NASA Lewis Research Center.

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(Paper 74A3-605)