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A CORRELATED COLOR TEMPERATURE FOR ILLUMINANTS

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ABSTRACT

As has long been known, most of the artificial and natural illuminants do not match exactly any one of the Planckian colors. Therefore, strictly speaking, they can not be assigned a color temperature. A color of this type may, however, be correlated with a representative Planckian color.

The method of determining correlated color temperature described in this paper consists in comparing the relative luminosities of each of the three primary red, green, and blue components of the source with similar values for the Planckian series.

With such a comparison three component temperatures are obtained; that is, the red component of the source corresponds with that of the Planckian radiator at one temperature, its green component with that of the Planckian radiator at a second temperature, and its blue component with that of the Planckian radiator at a third temperature. The average of these three component temperatures is designated as the correlated color temperature of the source. The mean deviation of the component temperatures from the average temperature is used as a basis for specifying the color (chromaticity) departure of the source from that of the Planckian radiator at the correlated color temperature. The conjunctive wave length indicates the kind of color departure.

CONTENTS

I. INTRODUCTION

The method of describing the color of an illuminant in terms of the temperature at which a black body will have the same color was apparently first used by Hyde (1)[1] in 1911. Since that time, this method of assigning "color temperatures" to incandescent bodies has

[1] Numbers in parentheses, followed occasionally by page references, refer to the bibliography at the end of the paper.

come into general use. It furnishes a much simpler description of the chromaticity than a table giving spectral energy values. A definition of color temperature which is in accord with the common usage of the term is that of Priest (2) which follows. The color temperature of a light source is "the temperature at which a Planckian radiator [2] would emit radiant energy competent to evoke a color of the same quality as that evoked by the radiant energy from the source in question." This definition is seldom strictly applicable because probably none of the common illuminants evokes a color matching exactly any of those evoked by the Planckian radiator; therefore, under this definition artificial and natural illuminants can not strictly be assigned color temperatures. In such cases the specification aims to fix upon that Planckian temperature which furnishes the minimum difference in color; that is, that color temperature is specified which most nearly matches the given source. For a satisfactory specification the color difference involved must then be evaluated in readily appreciable terms.

In this paper are considered particularly illuminants which do not match exactly any of the Planckian colors, but whose color may be definitely correlated with a Planckian color. The above definition of color temperature may then be extended as follows: The "ideal correlated color temperature" of a light source is the absolute temperature at which the Planckian radiator emits radiant energy competent to evoke a color which, of all Planckian colors, most closely approximates the color evoked by the source in question. Strictly speaking, ideal correlated color temperatures can not be computed because at the present time no computational method can be successfully defended on theoretical grounds. Ideal correlated color temperatures may be approximated by direct observation under certain specific conditions; that is, the values may differ from the ideal correlated color temperature by the observational uncertainty. Accordingly, we shall call all color-temperature values representing non-Planckian colors "correlated color temperatures" regardless of the method by which they were obtained provided they are intended to approximate the ideal correlated color temperature.

The correlated color temperature derived by the method described in this paper is believed to give a Planckian temperature which is at least very close to the nearest match with the given source. The description is then completed by giving the kind and magnitude of the color difference. The correlated color temperature by this method approaches the true color temperature as a limit, as an exact color match between the Planckian radiator and the given source is approached; hence, in this respect there can be no question as to the applicability of the method. The degree of color departure [3] of the given source from that of the Planckian radiator at the correlated temperature is then expressible definitely in terms of trilinear coor-

[2] By defining color temperature in terms of a Planckian radiator, Planck's distribution formula, as given in B. S. Misc. Pub. No. 56, is used as an energy basis irrespective of the question as to whether or not it perfectly describes black body radiation. See also the footnote 3.

[3] In accord with the definition of "chromaticity" given in the Report of Committee on Colorimetry for 1920–21, ((5) p. 535) "chromaticity" departure should be used instead of color departure since brightness is not to be included in this concept. But by the same token "chromaticity temperature" should then be used instead of "color temperature." The same reservation, of course, applies to the definition of color temperature and of the ideal correlated color temperature. Throughout this paper it will be understood that when the terms color departure, color match, color difference, etc., are used, chromaticity departure, chromaticity match, chromaticity difference, etc., are meant.

dinates; or, proportionately in terms of the Planckian termperature scale in the immediate vicinity; or, by the number of sensation steps (least perceptible differences) involved. The kind of color departure of the source in question from that of the Planckian radiator at the correlated temperature is given in terms of the wave length of homogeneous light which added to or subtracted from the given source will furnish a perfect match with the Planckian radiator at the correlated temperature.

To evaluate the colors of illuminants in terms of color temperature it is necessary to adopt as a reference basis, a set of spectral energy distributions as a function of the temperature of the radiator. For this purpose it is customary to use the energy distributions calculated by Planck's well-known formula. For each of these energy distributions a single variable is usually derived which serves to represent the Planckian distribution. The corresponding quantity is then computed for the energy distribution of the illuminant. By comparing this value with the series of Planckian values the color temperature of the given distribution may be estimated. Single variables which have been used are (*a*) the slope of the relative spectral energy curve in the visual part of the spectrum, (*b*) the wave-length coordinate of the maximum of the energy curve (Wien's displacement law), and (*c*) the wave-length coordinate of the center of gravity of the luminosity curve of the source (λ_c method) (2). Forsythe (3) has used the red-blue ratio, which in effect is equivalent to slope; and the method involving Wien's law is commonly used for assigning stellar temperatures. The first two of these methods are suitable only for grading energy distributions giving smooth curves similar to those of the Planckian radiator. They are not at all suitable for grading irregular or non-Planckian energy distributions in terms of color temperatures. The λ_c method employed by Priest has been used at the National Bureau of Standards and by others for some time. The proposed method appears satisfactory from the standpoint of accuracy and recommends itself through its relative completeness.

Among the light filters designed by Davis and Gibson (4) are two which were intended to duplicate more or less accurately a series of Planckian colors by varying the color temperature of the light source from 2,000° to 3,100° K. With one of the filters the color temperature range is from 2,650° to 5,000° K. and with the other, from 4,000° to 18,600° K. The locus of colors evoked by the lamp-and-filter combination crosses the Planckian locus of colors at a small angle, the point of intersection being in one case at 3,500° K., and in the other at 6,500° K. In both of these instances the light source, is, by design, at 2,450° K. Thus, at one point in each case the filter combined with the source at 2,450° K. evokes a color which coincides with a Planckian color, and for all other color temperatures of the light source the agreement is more or less imperfect, the imperfection increasing with the departure of the light source from 2,450° K. If these two filters are to serve the purpose for which they were designed, not only the correlated color temperatures obtainable therewith at the various light-source temperatures should be specified, but also the degree of the approximation. This was the immediate spur to the present undertaking.

II. THE PROPOSED METHOD

The method of correlating a non-Planckian source with a Planckian source followed here, may be described by plotting (trilinear coordinates) the locus of Planckian colors, and inscribing on this curve the corresponding temperature scale. The color of the Planckian radiator at any given temperature is then specified equally definitely by the trilinear coordinates or by the temperature.

Colors off the Planckian locus are correlated with this temperature scale by averaging the three temperatures (primary component tem-

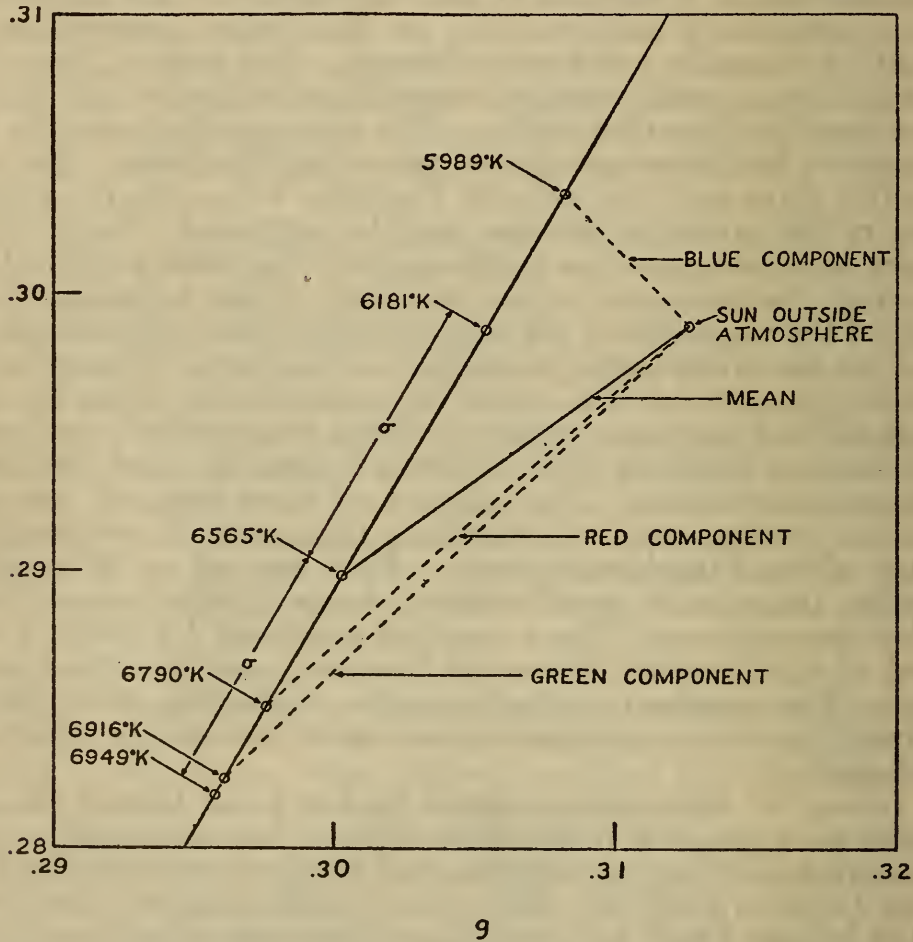

FIGURE 1.—*A section of the trilinear diagram (r-g projection) showing the colorimetric positions of the component temperatures, the mean temperature, and the mean deviation (σ) for sunlight outside the earth's atmosphere computed from Abbot's 1917 data (4)*

peratures) at which the Planckian color has, respectively, in terms of luminosity the same fractional part of primary "red," "green," and "blue" as the non-Planckian color. This average temperature (correlated color temperature) is used to associate non-Planckian colors with some definite color temperature. Points representing colors having the same correlated color temperature form lines on the trilinear diagram (isotemperature lines) intersecting the Planckian locus at the point where the temperature of the Planckian radiator and the correlated color temperature of the source are identical. The system of isotemperature lines fills the color area with nonintersecting lines which, sufficiently near the Planckian locus, may be treated as parallel straight lines. These are more widely spaced at lower temperatures (2,000° K.) crowding closely at the higher temperatures (20,000° K.). See for example Figure 3.

Figure 1 is a restricted plot which shows the correlation of the color of the sun outside the earth's atmosphere with that of the Planckian radiator at 6,565° K. As indicated, the sun's red component corresponds in luminosity to that of a Planckian radiator at 6,790°, its green component to that of a Planckian radiator at 6,916°, and its blue component to that of a Planckian radiator at 5,989° K. By the

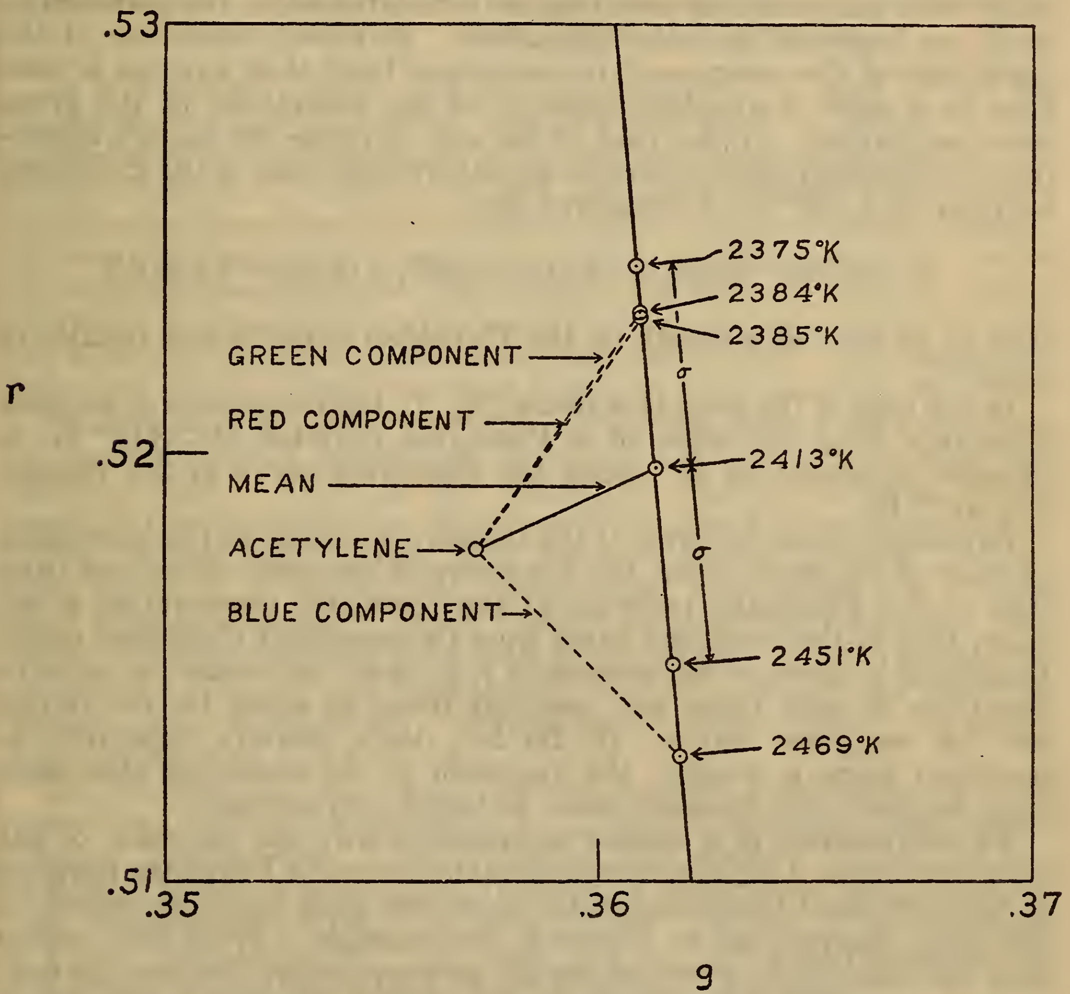

FIGURE 2.—*A section of the trilinear diagram (r-g projection) showing the colorimetric positions of the component temperatures, the mean temperature and the mean deviation (σ) for the energy distribution of the acetylene flame computed from Coblentz's data*

present method the mean of these three temperatures, 6,565° K., is the correlated color temperature of the sun. The line which joins the sun's colorimetric position with that of the Planckian radiator at 6,565° is an isotemperature line along which the color changes by magnitudes approximately proportional to distance. Likewise, Figure 2 illustrates the correlated color temperature (2,413° K.) of the acetylene flame and its color departure from the correlated Planckian color.

If these magnitudes of color departure in the specific direction and for the specific locations be evaluated once in terms of sensation units, they may be used as a basis for evaluating like directed color changes in the immediate neighborhood. There is, however, no a priori reason for applying the evaluation of sensation magnitudes in one direction to those in another direction, in fact, they are known to be different. The same reservation applies to points in different neighborhoods even though the direction of the departure be the same.

We may now consider expressing the degree of color departure of a given source from that of the Planckian radiator at the correlated temperature. The length of the line connecting the non-Planckian point with the point representing its correlated color temperature is, itself, an index of the color departure. However, the mean of the deviations of the component temperatures from their average is used here as a more convenient measure of the magnitude of the given color separation. In the case of the sun (outside the earth's atmosphere) the magnitude of its color departure from that of the Planckian radiator at 6,565° K. is measured by

$$\sigma = [(6{,}790 - 6{,}565) + (6{,}916 - 6{,}565) + (6{,}565 - 5{,}989)]/3$$

that is, by 384° as a length on the Planckian curve in the vicinity of 6,565° K.

In the case of the acetylene flame (fig. 2), the magnitude of its color departure from the color of a Planckian radiator at 2,413° K. is likewise measured by 38° along the Planckian curve in the vicinity of 2,413° K.

Expressed, then, in terms of the temperature scale in the immediate vicinity of the given color, the departure of the color of the sun from that of the Planckian radiator at the correlated temperature is ten times that of the acetylene flame from its correlated Planckian point. Expressed in terms of differences in r, g, and b primaries the relative departure is only three and one-half times as much for the sun as for the acetylene flame. If, further, their relative departure in sensation steps is sought, the variation of the sensation scale with both location and direction must be taken into account.

To characterize in a readily appreciable way the direction of the color departure of a given source from the color of a Planckian radiator at the correlated temperature, the spectrum locus also is inscribed on the color diagram, as in Figure 3, for example. From that region near the black-body curve where the isotemperature line may be considered as straight it is extended in direction to intersect the spectrum locus. This intersection gives the wave length of the homogeneous light (conjunctive wave length) which must be added to, or subtracted from, as the case may be, the given source in order to make it match exactly in chromaticity the Planckian radiator at the correlated temperature. In the case of the sun, this conjunctive wave length is about 565 mμ; for the acetylene flame it is slightly more than 580 mμ. For a color temperature of 1,800°, it is 586 mμ; and for 15,000°, it is near 560 mμ. These limits correspond to only a moderate deviation of the conjunctive wave-length lines from parallelism.

Color points beyond the region in which the isotemperature lines are practically straight are specified in exactly the same way as those falling within this region. The conjunctive wave length serves just as definitely in these cases to specify the direction of the color departure of the source in question from the color of the Planckian radiator at the correlated color temperature; but this direction is slightly different from that for nearer color points, hence could not be expected to coincide with the direction of least color difference in terms of sensation change if it does for nearer points. However, the range of colors expressible in terms of correlated color temperature should obviously be restricted to near-Planckian colors

III. PROCEDURE

1. THE PLANCKIAN RADIATOR EVALUATED IN TERMS OF RELATIVE LUMINOSITY OF THE PRIMARY COMPONENTS

As a necessary basis, trilinear coordinates, r, g, and b, for the Planckian radiator at various temperatures were computed by the method described in the report of the committee on colorimetry of the Optical Society of America (5) and elsewhere (Abney, Ives, Guild). In the paper describing Davis-Gibson filters (4) colorimetric computations on this basis, with mean noon sunlight represented at the center of the color triangle, have been carried out not only for the energy distributions obtained with the various lamp and filter combinations, but also for Planckian distributions between 1,600° and 20,000° K. Reference should be made to that paper for the basic data used here.

It is the generally accepted view that the products of the excitations by their respective luminosity coefficients should sum to yield the visibility curve. Because the luminosity coefficients given in the report of the committee on colorimetry (5) were not considered satisfactory in this respect, new ones were computed, by least squares, from the O. S. A. excitations, extrapolated (6), adjusted to "mean sun" and the standard Gibson-Tyndall recommended visibility data (7) in a manner differing from that used by Judd (8) only in that the equations contained the requirement that the sum of the three coefficients shall equal unity. The values so obtained are $L_r = 0.45014$, $L_g = 0.54417$, and $L_b = 0.00569$.

The products of r, g, and b for any energy distribution by their respective luminosity coefficients L_r, L_g, and L_b give the three relative primary luminosities. Computing in this manner values for the Planckian series, it is seen that the sum of these three varies with the color temperature; which is to be expected. To give them on a basis of equal luminosities rL_r, gL_g, and bL_b are, for each temperature, multiplied by a factor ω such that the sum is one-third.

In Table 1 are given then ωrL_r, ωgL_g, and ωbL_b for Planckian energy distributions obtained by computation and by graphical interpolation from their respective r, g, and b values.

TABLE 1.—*Table to facilitate the computation of correlated color temperatures*

The *r*, *g*, and *b* values of the energy distribution (computed as illustrated in B. S. Mis. Pub. No. 114, Figure 12) are multiplied by the respective luminosity coefficients (L_r=0.45014, L_g=0.54417, L_b=0.00569). The resulting values are adjusted by a factor (ω) so that their sum is 0.33333.

Color temperature	ωrL_r	ωgL_g	ωbL_b	Color temperature	ωrL_r	ωgL_g	ωbL_b
° *K*				° *K*			
1, 600	0. 20788	0. 12530	0. 000152	2, 500	0. 17895	0. 15382	0. 000571
10	. 20741	. 12577	. 000156	20	. 17854	. 15421	. 000582
20	. 20693	. 12624	. 000159	40	. 17813	. 15461	. 000593
30	. 20646	. 12672	. 000162	60	. 17773	. 15500	. 000604
40	. 20599	. 12718	. 000166	80	. 17733	. 15539	. 000616
1, 650	. 20552	. 12765	. 000169	2, 600	. 17693	. 15577	. 000627
60	. 20505	. 12812	. 000173	20	. 17654	. 15614	. 000639
70	. 20459	. 12857	. 000176	40	. 17615	. 15652	. 000650
80	. 20413	. 12902	. 000180	60	. 17578	. 15688	. 000661
90	. 20369	. 12946	. 000183	80	. 17541	. 15725	. 000673
1, 700	. 20324	. 12990	. 000187	2, 700	. 17505	. 15760	. 000684
10	. 20281	. 13034	. 000191	20	. 17470	. 15794	. 000696
20	. 20237	. 13077	. 000194	40	. 17435	. 15828	. 000707
30	. 20194	. 13120	. 000198	60	. 17401	. 15861	. 000719
40	. 20152	. 13161	. 000202	80	. 17368	. 15893	. 000730
1, 750	. 20110	. 13203	. 000206	2, 800	. 17334	. 15925	. 000742
60	. 20069	. 13244	. 000210	20	. 17301	. 15957	. 000753
70	. 20028	. 13284	. 000214	40	. 17268	. 15989	. 000765
80	. 19988	. 13324	. 000218	60	. 17236	. 16020	. 000777
90	. 19948	. 13363	. 000222	80	. 17203	. 16051	. 000788
1, 800	. 19909	. 13402	. 000226	2, 900	. 17172	. 16081	. 000800
10	. 19870	. 13440	. 000230	20	. 17142	. 16111	. 000812
20	. 19831	. 13478	. 000234	40	. 17112	. 16140	. 000824
30	. 19793	. 13516	. 000238	60	. 17082	. 16168	. 000835
40	. 19755	. 13554	. 000243	80	. 17053	. 16196	. 000847
1, 850	. 19716	. 13591	. 000247	3, 000	. 17023	. 16225	. 000859
60	. 19679	. 13628	. 000251	20	. 16994	. 16252	. 000871
70	. 19643	. 13664	. 000255	40	. 16965	. 16280	. 000883
80	. 19606	. 13700	. 000259	60	. 16937	. 16307	. 000895
90	. 19570	. 13736	. 000264	80	. 16908	. 16334	. 000907
1, 900	. 19535	. 13772	. 000268	3, 100	. 16880	. 16361	. 000919
10	. 19499	. 13807	. 000272	20	. 16853	. 16387	. 000930
20	. 19464	. 13842	. 000277	40	. 16827	. 16413	. 000942
30	. 19429	. 13877	. 000281	60	. 16800	. 16438	. 000954
40	. 19395	. 13910	. 000286	80	. 16773	. 16464	. 000966
1, 950	. 19361	. 13944	. 000290	3, 200	. 16747	. 16488	. 000977
60	. 19327	. 13977	. 000295	20	. 16722	. 16512	. 000989
70	. 19293	. 14010	. 000299	40	. 16697	. 16537	. 001001
80	. 19260	. 14043	. 000304	60	. 16672	. 16560	. 001013
90	. 19227	. 14075	. 000308	80	. 16648	. 16583	. 001024
2, 000	. 19195	. 14107	. 000313	3, 300	. 16623	. 16607	. 001036
20	. 19129	. 14171	. 000322	20	. 16599	. 16629	. 001048
40	. 19066	. 14233	. 000332	40	. 16576	. 16652	. 001059
60	. 19004	. 14294	. 000342	60	. 16553	. 16674	. 001071
80	. 18944	. 14354	. 000351	80	. 16529	. 16696	. 001082
2, 100	. 18885	. 14412	. 000361	3, 400	. 16507	. 16718	. 001094
20	. 18827	. 14470	. 000371	20	. 16484	. 16739	. 001106
40	. 18770	. 14525	. 000381	40	. 16462	. 16759	. 001117
60	. 18714	. 14579	. 000391	60	. 16440	. 16780	. 001129
80	. 18659	. 14634	. 000401	80	. 16419	. 16800	. 001141
2, 200	. 18606	. 14686	. 000411	3, 500	. 16398	. 16820	. 001152
20	. 18553	. 14738	. 000421	20	. 16377	. 16841	. 001164
40	. 18499	. 14790	. 000432	40	. 16355	. 16860	. 001176
60	. 18447	. 14841	. 000442	60	. 16335	. 16879	. 001187
80	. 18397	. 14891	. 000452	80	. 16315	. 16898	. 001199
2, 300	. 18347	. 14939	. 000463	3, 600	. 16295	. 16917	. 001210
20	. 18298	. 14987	. 000473	20	. 16274	. 16937	. 001222
40	. 18252	. 15033	. 000484	40	. 16255	. 16955	. 001233
60	. 18204	. 15080	. 000494	60	. 16236	. 16973	. 001245
80	. 18157	. 15124	. 000505	80	. 16217	. 16991	. 001256
2, 400	. 18112	. 15169	. 000516	3, 700	. 16197	. 17009	. 001268
20	. 18068	. 15212	. 000527	20	. 16178	. 17027	. 001280
40	. 18024	. 15255	. 000538	40	. 16160	. 17044	. 001291
60	. 17980	. 15298	. 000549	60	. 16141	. 17062	. 001303
80	. 17937	. 15340	. 000560	80	. 16123	. 17079	. 001314

TABLE 1.—*Table to facilitate the computation of correlated color temperatures*—Con.

Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$	Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$
° K				° K			
3, 800	0. 16104	0. 17096	0. 001326	5, 100	0. 15232	0. 17900	0. 002007
20	. 16086	. 17113	. 001337	20	. 15222	. 17908	. 002017
40	. 16068	. 17130	. 001348	40	. 15213	. 17917	. 002027
60	. 16050	. 17147	. 001360	60	. 15203	. 17926	. 002036
80	. 16033	. 17163	. 001371	80	. 15193	. 17935	. 002046
3, 900	. 16016	. 17179	. 001382	5, 200	. 15184	. 17943	. 002055
20	. 15998	. 17196	. 001394	20	. 15174	. 17952	. 002064
40	. 15981	. 17212	. 001405	40	. 15165	. 17960	. 002073
60	. 15964	. 17227	. 001416	60	. 15156	. 17968	. 002083
80	. 15947	. 17243	. 001427	80	. 15147	. 17977	. 002093
4, 000	. 15931	. 17258	. 001438	5, 300	. 15138	. 17986	. 002102
20	. 15914	. 17274	. 001450	20	. 15128	. 17994	. 002111
40	. 15898	. 17288	. 001461	40	. 15119	. 18002	. 002120
60	. 15882	. 17304	. 001472	60	. 15111	. 18010	. 002129
80	. 15866	. 17318	. 001483	80	. 15102	. 18018	. 002138
4, 100	. 15850	. 17334	. 001494	5, 400	. 15093	. 18026	. 002148
20	. 15834	. 17349	. 001505	20	. 15084	. 18034	. 002157
40	. 15818	. 17363	. 001516	40	. 15075	. 18042	. 002166
60	. 15803	. 17377	. 001527	60	. 15066	. 18050	. 002175
80	. 15788	. 17391	. 001538	80	. 15057	. 18058	. 002184
4, 200	. 15773	. 17405	. 001549	5, 500	. 15049	. 18065	. 002193
20	. 15758	. 17419	. 001560	20	. 15041	. 18073	. 002203
40	. 15743	. 17433	. 001571	40	. 15032	. 18081	. 002212
60	. 15729	. 17446	. 001581	60	. 15023	. 18088	. 002221
80	. 15715	. 17459	. 001592	80	. 15016	. 18095	. 002229
4, 300	. 15701	. 17472	. 001603	5, 600	. 15007	. 18102	. 002237
20	. 15687	. 17485	. 001613	20	. 14999	. 18110	. 002246
40	. 15673	. 17498	. 001624	40	. 14991	. 18117	. 002255
60	. 15659	. 17511	. 001635	60	. 14983	. 18124	. 002264
80	. 15646	. 17523	. 001645	80	. 14975	. 18132	. 002273
4, 400	. 15632	. 17536	. 001656	5, 700	. 14967	. 18138	. 002282
20	. 15618	. 17548	. 001666	20	. 14959	. 18145	. 002289
40	. 15605	. 17560	. 001677	40	. 14951	. 18152	. 002297
60	. 15593	. 17572	. 001687	60	. 14943	. 18159	. 002306
80	. 15579	. 17584	. 001698	80	. 14936	. 18166	. 002315
4, 500	. 15568	. 17595	. 001708	5, 800	. 14928	. 18173	. 002324
20	. 15554	. 17607	. 001718	20	. 14920	. 18179	. 002333
40	. 15542	. 17618	. 001729	40	. 14913	. 18186	. 002341
60	. 15529	. 17630	. 001739	60	. 14906	. 18193	. 002349
80	. 15517	. 17641	. 001749	80	. 14899	. 18199	. 002357
4, 600	. 15504	. 17653	. 001759	5, 900	. 14891	. 18205	. 002366
20	. 15493	. 17663	. 001770	20	. 14884	. 18211	. 002374
40	. 15481	. 17674	. 001780	40	. 14877	. 18218	. 002382
60	. 15468	. 17686	. 001790	60	. 14869	. 18224	. 002390
80	. 15456	. 17697	. 001800	80	. 14863	. 18230	. 002398
4, 700	. 15445	. 17707	. 001810	6, 000	. 14856	. 18237	. 002407
20	. 15433	. 17718	. 001820	20	. 14848	. 18243	. 002415
40	. 15422	. 17728	. 001830	40	. 14841	. 18249	. 002423
60	. 15410	. 17739	. 001840	60	. 14834	. 18255	. 002430
80	. 15398	. 17750	. 001850	80	. 14827	. 18261	. 002438
4, 800	. 15387	. 17760	. 001860	6, 100	. 14821	. 18267	. 002446
20	. 15377	. 17770	. 001870	20	. 14814	. 18273	. 002454
40	. 15366	. 17779	. 001880	40	. 14807	. 18279	. 002462
60	. 15355	. 17789	. 001890	60	. 14801	. 18285	. 002470
80	. 15344	. 17799	. 001900	80	. 14794	. 18291	. 002478
4, 900	. 15333	. 17809	. 001910	6, 200	. 14788	. 18297	. 002486
20	. 15323	. 17818	. 001920	20	. 14782	. 18302	. 002494
40	. 15312	. 17828	. 001930	40	. 14775	. 18309	. 002502
60	. 15302	. 17837	. 001940	60	. 14768	. 18314	. 002509
80	. 15292	. 17846	. 001950	80	. 14762	. 18320	. 002517
5, 000	. 15282	. 17855	. 001960	6, 300	. 14755	. 18326	. 002525
20	. 15272	. 17864	. 001969	20	. 14749	. 18332	. 002533
40	. 15262	. 17873	. 001978	40	. 14743	. 18337	. 002541
60	. 15252	. 17882	. 001988	60	. 14737	. 18342	. 002549
80	. 15242	. 17891	. 001998	80	. 14731	. 18347	. 002556

TABLE 1.—*Table to facilitate the computation of correlated color temperatures*—Con.

Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$	Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$
°K				°K			
6,400	0.14724	0.18353	0.002563	7,700	0.14398	0.18635	0.003001
20	.14718	.18358	.002570	20	.14394	.18638	.003007
40	.14712	.18363	.002577	40	.14391	.18642	.003013
60	.14706	.18369	.002586	60	.14387	.18645	.003019
80	.14701	.18374	.002593	80	.14383	.18648	.003025
6,500	.14695	.18379	.002600	7,800	.14379	.18652	.003030
20	.14689	.18384	.002607	20	.14375	.18655	.003036
40	.14683	.18390	.002615	40	.14371	.18658	.003042
60	.14677	.18395	.002623	60	.14367	.18662	.003048
80	.14671	.18400	.002630	80	.14363	.18665	.003053
6,600	.14666	.18404	.002637	7,900	.14359	.18668	.003059
20	.14659	.18410	.002644	20	.14356	.18671	.003065
40	.14654	.18415	.002652	40	.14352	.18674	.003070
60	.14648	.18420	.002659	60	.14348	.18678	.003076
80	.14643	.18424	.002666	80	.14345	.18681	.003082
6,700	.14637	.18429	.002673	8,000	.14341	.18684	.003087
20	.14632	.18434	.002680	20	.14337	.18687	.003093
40	.14626	.18439	.002687	40	.14333	.18691	.003098
60	.14621	.18444	.002694	60	.14330	.18694	.003104
80	.14616	.18448	.002701	80	.14326	.18697	.003109
6,800	.14610	.18453	.002708	8,100	.14322	.18700	.003115
20	.14604	.18458	.002715	20	.14319	.18703	.003120
40	.14599	.18462	.002722	40	.14315	.18706	.003126
60	.14594	.18467	.002729	60	.14311	.18709	.003131
80	.14588	.18471	.002736	80	.14308	.18712	.003137
6,900	.14583	.18476	.002743	8,200	.14304	.18715	.003142
20	.14578	.18481	.002750	20	.14300	.18718	.003147
40	.14573	.18485	.002756	40	.14297	.18721	.003153
60	.14568	.18490	.002763	60	.14293	.18724	.003158
80	.14563	.18494	.002770	80	.14290	.18727	.003163
7,000	.14558	.18498	.002777	8,300	.14286	.18730	.003168
20	.14553	.18502	.002784	20	.14283	.18733	.003174
40	.14548	.18506	.002790	40	.14279	.18736	.003179
60	.14543	.18510	.002797	60	.14276	.18739	.003184
80	.14538	.18515	.002804	80	.14272	.18742	.003189
7,100	.14533	.18519	.002810	8,400	.14269	.18745	.003194
20	.14528	.18523	.002817	20	.14265	.18748	.003200
40	.14523	.18527	.002824	40	.14262	.18751	.003205
60	.14518	.18531	.002830	60	.14258	.18753	.003210
80	.14513	.18536	.002837	80	.14255	.18756	.003215
7,200	.14509	.18540	.002844	8,500	.14252	.18759	.003220
20	.14504	.18544	.002850	20	.14249	.18762	.003225
40	.14499	.18548	.002857	40	.14245	.18765	.003230
60	.14494	.18552	.002863	60	.14242	.18767	.003235
80	.14490	.18556	.002870	80	.14239	.18770	.003240
7,300	.14485	.18560	.002876	8,600	.14236	.18773	.003245
20	.14480	.18564	.002883	20	.14233	.18775	.003250
40	.14476	.18568	.002889	40	.14229	.18778	.003255
60	.14471	.18572	.002896	60	.14226	.18781	.003260
80	.14467	.18576	.002902	80	.14223	.18783	.003265
7,400	.14462	.18580	.002907	8,700	.14220	.18786	.003270
20	.14458	.18584	.002915	20	.14217	.18788	.003275
40	.14453	.18588	.002921	40	.14214	.18791	.003280
60	.14449	.18591	.002928	60	.14211	.18794	.003285
80	.14444	.18595	.002934	80	.14208	.18796	.003290
7,500	.14440	.18599	.002940	8,800	.14205	.18799	003295
20	.14436	.18603	.002946	20	.14202	.18801	.003300
40	.14431	.18606	.002952	40	.14199	.18804	.003304
60	.14427	.18610	.002959	60	.14196	.18806	.003309
80	.14423	.18614	.002965	80	.14193	.18809	.003314
7,600	.14419	.18617	.002971	8,900	.14190	.18811	.003319
20	.14415	.18621	.002977	20	.14187	.18814	.003324
40	.14411	.18624	.002983	40	.14184	.18817	.003329
60	.14407	.18628	.002989	60	.14181	.18819	.003334
80	.14402	.18631	.002995	80	.14178	.18821	.003338

TABLE 1.—*Table to facilitate the computation of correlated color temperatures*—Con.

Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$	Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$
° K				° K			
9,000	0.14175	0.18824	0.003343	10,300	0.14016	0.18956	0.003616
20	.14172	.18826	.003348	20	.14013	.18958	.003620
40	.14169	.18829	.003352	40	.14011	.18959	.003624
60	.14167	.18831	.003357	60	.14009	.18961	.003627
80	.14164	.18833	.003361	80	.14007	.18963	.003631
9,100	.14161	.18835	.003364	10,400	.14005	.18965	.003634
20	.14158	.18837	.003370	20	.14003	.18966	.003638
40	.14156	.18840	.003375	40	.14001	.18968	.003641
60	.14153	.18842	.003380	60	.13999	.18970	.003645
80	.14150	.18844	.003384	80	.13997	.18971	.003649
9,200	.14148	.18846	.003389	10,500	.13995	.18973	.003652
20	.14145	.18849	.003393	20	.13993	.18975	.003656
40	.14142	.18851	.003398	40	.13991	.18976	.003659
60	.14139	.18853	.003402	60	.13989	.18978	.003663
80	.14137	.18855	.003406	80	.13987	.18980	.003666
9,300	.14134	.18857	.003411	10,600	.13985	.18981	.003670
20	.14131	.18860	.003415	20	.13983	.18983	.003673
40	.14129	.18862	.003420	40	.13981	.18985	.003677
60	.14126	.18864	.003424	60	.13979	.18986	.003680
80	.14123	.18866	.003429	80	.13977	.18988	.003684
9,400	.14121	.18868	.003433	10,700	.13975	.18990	.003688
20	.14118	.18871	.003438	20	.13973	.18991	.003691
40	.14116	.18873	.003442	40	.13971	.18993	.003695
60	.14113	.18875	.003446	60	.13969	.18994	.003698
80	.14110	.18877	.003451	80	.13967	.18996	.003702
9,500	.14108	.18879	.003455	10,800	.13965	.18998	.003705
20	.14105	.18881	.003459	20	.13963	.18999	.003709
40	.14103	.18883	.003464	40	.13961	.19001	.003712
60	.14100	.18885	.003468	60	.13959	.19003	.003716
80	.14098	.18887	.003472	80	.13957	.19004	.003719
9,600	.14095	.18889	.003477	10,900	.13955	.19006	.003723
20	.14093	.18891	.003481	20	.13953	.19008	.003726
40	.14090	.18893	.003486	40	.13951	.19009	.003730
60	.14088	.18896	.003490	60	.13949	.19011	.003733
80	.14085	.18898	.003494	80	.13947	.19012	.003737
9,700	.14083	.18900	.003498	11,000	.13945	.19014	.003740
20	.14081	.18902	.003503	20	.13943	.19015	.003744
40	.14078	.18904	.003507	40	.13941	.19017	.003747
60	.14076	.18906	.003511	60	.13939	.19019	.003750
80	.14073	.18908	.003515	80	.13938	.19020	.003754
9,800	.14071	.18910	.003519	11,100	.13936	.19022	.003757
20	.14069	.18912	.003524	20	.13934	.19023	.003761
40	.14066	.18914	.003528	40	.13932	.19025	.003764
60	.14064	.18916	.003532	60	.13930	.19026	.003768
80	.14062	.18917	.003536	80	.13928	.19028	.003771
9,900	.14059	.18919	.003540	11,200	.13927	.19029	.003774
20	.14057	.18921	.003544	20	.13925	.19030	.003778
40	.14055	.18923	.003548	40	.13923	.19032	.003781
60	.14052	.18925	.003552	60	.13921	.19033	.003784
80	.14050	.18927	.003556	80	.13920	.19035	.003788
10,000	.14048	.18929	.003560	11,300	.13918	.19036	.003791
20	.14046	.18931	.003564	20	.13916	.19038	.003794
40	.14043	.18933	.003568	40	.13914	.19039	.003798
60	.14041	.18935	.003572	60	.13913	.19040	.003801
80	.14039	.18936	.003576	80	.13911	.19042	.003804
10,100	.14037	.18938	.003579	11,400	.13909	.19043	.003807
20	.14035	.18940	.003583	20	.13908	.19045	.003810
40	.14033	.18942	.003587	40	.13906	.19046	.003814
60	.14030	.18944	.003591	60	.13904	.19047	.003817
80	.14028	.18945	.003595	80	.13903	.19049	.003820
10,200	.14026	.18947	.003598	11,500	.13901	.19050	.003823
20	.14024	.18949	.003602	20	.13899	.19051	.003826
40	.14022	.18951	.003606	40	.13898	.19053	.003829
60	.14020	.18952	.003609	60	.13896	.19054	.003832
80	.14018	.18954	.003613	80	.13894	.19055	.003835

TABLE 1.—*Table to facilitate the computation of correlated color temperatures*—Con.

Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$	Color temperature	$\omega r L_r$	$\omega g L_g$	$\omega b L_b$
° K				° K			
11,600	0.13893	0.19057	0.003838	15,000	0.13684	0.19223	0.004248
20	.13891	.19058	.003841	100	.13680	.19227	.004257
40	.13889	.19059	.003844	200	.13675	.19231	.004267
60	.13888	.19060	.003847	300	.13671	.19234	.004276
80	.13886	.19062	.003850	400	.13666	.19238	.004284
11,700	.13884	.19063	.003853	15,500	.13662	.19241	.004293
20	.13883	.19064	.003856	600	.13658	.19245	.004302
40	.13881	.19066	.003860	700	.13653	.19248	.004311
60	.13880	.19067	.003863	800	.13649	.19251	.004318
80	.13878	.19068	.003866	900	.13645	.19255	.004327
11,800	.13877	.19069	.003869	16,000	.13641	.19258	.004337
20	.13875	.19071	.003872	100	.13637	.19261	.004343
40	.13873	.19072	.003874	200	.13633	.19264	.004352
60	.13872	.19073	.003877	300	.13630	.19267	.004360
80	.13870	.19074	.003880	400	.13626	.19270	.004368
11,900	.13869	.19076	.003883	16,500	.13622	.19273	.004377
20	.13867	.19077	.003886	600	.13618	.19276	.004384
40	.13866	.19078	.003889	700	.13615	.19279	.004392
60	.13864	.19079	.003892	800	.13611	.19282	.004400
80	.13862	.19081	.003895	900	.13608	.19285	.004407
12,000	.13861	.19082	.003898	17,000	.13604	.19288	.004414
100	.13854	.19088	.003911	100	.13601	.19290	.004422
200	.13847	.19093	.003925	200	.13597	.19293	.004429
300	.13839	.19099	.003938	300	.13594	.19296	.004437
400	.13832	.19105	.003952	400	.13591	.19298	.004444
12,500	.13825	.19111	.003966	17,500	.13587	.19301	.004451
600	.13819	.19116	.003978	600	.13584	.19303	.004458
700	.13812	.19122	.003992	700	.13581	.19306	.004464
800	.13805	.19127	.004004	800	.13578	.19308	.004471
900	.13799	.19132	.004017	900	.13575	.19311	.004478
13,000	.13792	.19137	.004029	18,000	.13572	.19313	.004484
100	.13786	.19142	.004042	100	.13569	.19315	.004491
200	.13780	.19147	.004054	200	.13566	.19318	.004497
300	.13774	.19152	.004067	300	.13563	.19320	.004503
400	.13768	.19157	.004078	400	.13561	.19322	.004510
13,500	.13762	.19162	.004090	18,500	.13558	.19324	.004516
600	.13756	.19166	.004102	600	.13555	.19326	.004522
700	.13751	.19171	.004113	700	.13553	.19328	.004528
800	.13745	.19175	.004124	800	.13550	.19330	.004533
900	.13740	.19180	.004135	900	.13547	.19332	.004540
14,000	.13734	.19184	.004146	19,000	.13545	.19334	.004546
100	.13729	.19188	.004157	100	.13542	.19336	.004552
200	.13724	.19192	.004167	200	.13540	.19338	.004556
300	.13718	.19196	.004177	300	.13537	.19340	.004563
400	.13713	.19200	.004188	400	.13535	.19342	.004568
14,500	.13708	.19204	.004198	19,500	.13532	.19344	.004573
600	.13703	.19208	.004208	600	.13530	.19346	.004579
700	.13699	.19212	.004218	700	.13527	.19348	.004584
800	.13694	.19216	.004228	800	.13525	.19349	.004590
900	.13689	.19220	.004238	900	.13522	.19351	.004595
				20,000	.13520	.19353	.004600

2. COMPUTATION OF THE CORRELATED COLOR TEMPERATURE

To make the directions more specific the correlated color temperature of sunlight outside the earth's atmosphere will be computed. Using Abbot's 1917 data (4, Table 1) and following the form given in (4, fig. 12) the values of *r*, *g*, and *b* were calculated.

Thus

$$r = 0.29883;\ L_r = 0.45014;\ rL_r = 0.134515$$
$$g = 0.31261;\ L_g = 0.54417;\ gL_g = .170113$$
$$b = 0.38856;\ L_b = 0.00569;\ bL_b = .002211$$

$$rL_r + gL_g + bL_b = .306839$$

$$\omega = \frac{0.333333}{0.306839} = 1.08635$$

$$\omega rL_r = .14613$$
$$\omega gL_g = .18480$$
$$\omega bL_b = .002402$$

$$(\text{Check:} - \ \omega rL_r + \omega gL_g + \omega bL_b = .33333)$$

Seeking these values of ωrL_r, ωgL_g, and ωbL_b in the respective columns of Table 1 we obtain the following component temperatures:

For

	°*K*
ωrL_r,	6,790
ωgL_g,	6,916
ωbL_b,	5,989
Correlated color temperature (mean)	6,565

Table 2 gives for a number of sources a comparison of the values calculated by the λ_c method, the present method and also what may be designated as observed magnitudes.

TABLE 2.—*Comparison of correlated color temperatures obtained by computation and observation*

Energy distribution used in computation	Correlated color temperatures					
	Computed values			Observed values		
	Reference	λ_c method	Present method	Reference	Time observed	Value
		°*K.*	°*K.*			°*K.*
Mean noon sun	(4)	5,578	5,394	(12)	2/20/20 noon	[1] 5,200
Mean noon sun summer solistice	(4)	5,918	5,740	(12)	6/26/22 12:30 p. m.	[1] 5,300
Mean noon sun winter solstice	(4)	5,275	5,077	(12)	1/24/25 1:30 p. m.	[1] 5,212
Sun outside earth's atmosphere		6,724	6,565			
Abbot-Priest sun	(12)	5,323	5,229	(12)		5,323 5,216
Equal energy spectrum		5,212	5,357			
Acetylene flame	(4)	2,388	2,413	(4)		[2] 2,420
B. S. lamp No. 1717	(11)	2,848	2,860	(11)		2,848
Blue glass R4-28 (source at 2,077°K.)		2,358	2,386			
Blue glass R4-28 (source at 2,080°K.)			2,389			[3] 2,390
Blue glass R4-28 (source at 2,360°K.) [4]			2,779			

[1] These values are isolated measurements, and are not necessarily representative of "mean" atmospheric conditions. Furthermore, the days on which these measurements were made are not wholly consistent with the dates assumed for the computation.

[2] For a discussion of the color of the acetylene flame, see B. S. Misc. Pub. No. 114, Secs. III, 4; V, 6, (*a*); VIII, 6.

[3] Quoting from National Physical Laboratory's Report dated January, 1930 (International Comparison of the Transmissions of Four Blue Glasses): "It was found by color matching experiments at the National Physical Laboratory that the glasses would change the light from a tungsten lamp operating at a color temperature of 2,080°K. to light having a color temperature of 2,390°K."

[4] Quoting again the National Physical Laboratory Report: "By calculation it was then found that the same glasses would change the light from a tungsten lamp operated at a color temperature of 2,360° K. to light having a color temperature of 2,770°K. which corresponds to about 13 lumens per watt for a gas filled lamp."

TABLE 3.—*Correlated color temperature data for well-known energy distributions* [1]

Source of energy	Correlated color temperature (θ_i)	Mean deviation (σ)	Sensation steps $\frac{\sigma}{\delta}$	Conj. wave length (λ_j)
	°K.	°K.		μ
Mean noon sun	5,394	347	11.9	566.8
Mean noon sun, summer solstice	5,740	365	11.1	566.3
Mean noon sun, winter solstice	5,077	345	13.4	567.6
Abbot-Priest sun	5,229	219	8.0	567.0
Acetylene, from Coblentz' data	2,413	38	6.6	[2] −580.5
Lamp 1717 (bib. ref. 11), from Coblentz data	2,860	18	2.2	576.7
Equal energy spectrum	5,357	291	10.1	[2] −565.7

[1] The specific energy distributions for which these values apply may be found in B. S. Misc. Pub. No. 114 (4)—the noon sun data in Table 1, the acetylene data in Table 3, Abbot-Priest sun in fig. 3, and Lamp 1717 in fig. 7.
[2] The negative sign indicates that light of the conjunctive wave length must be added to the stimulus in question in order to match the Planckian color.

3. CALCULATION OF COLOR DEPARTURE IN TERMS OF SENSATION STEPS

Within any small region near the Planckian locus the ratio of the color departure to the mean deviation should be constant. Since, as is well known, the perceptibility of a unit increment in color temperature decreases rapidly with increase in temperature, it is desirable to express mean deviation in "sensation" steps rather than in degrees of temperature.

Priest (2) has shown that between 3,000° and 20,000° K. the probable error of a single observation in color matching is between 0.1 and 0.2 mμ as expressed on the λ_c scale, without showing any systematic variation with color temperature. The reasonable assumption is made that the number of degrees just perceptibly different in color (a sensation step) at any color temperature, corresponds to a constant difference in λ_c values.

An attempt made to obtain a more convenient method resulted in the discovery that the number of degrees represented by a constant difference in λ_c over the range 2,000° to 24,000° K. varies quite closely as the square of the color temperature.[4]

Hence, a "sensation step" in degrees along the Planckian locus for any color temperature, may be calculated approximately from the relation

$$\delta = K\Theta^2$$

where:

θ = a sensation step measured in degrees, along the Planckian locus.

K = a constant evaluated by experiment.

Θ = the color temperature in degrees Kelvin.

At 2,360° K., a point much used in color matching of incandescent electric lamps, a difference of 5° or 6° in color temperature of the two fields of the photometer is roughly the magnitude of the "least perceptible difference" in color under the best observing conditions (9). Substituting 5.6° for δ and 2,360° for Θ gives $K = 1 \times 10^{-6}$.[5]

[4] Verification of this statement may be obtained by utilizing the λ_c data in Tables 1, 2, 3, and 4 of the Appendix to Priest's paper (2).
[5] A value of $K = 1 \times 10^{-6}$ corresponds to $\delta = 0.07$ mμ on the λ_c scale. Footnote 15 of Priest's paper (2), gives the values 0.1 and 0.2 mμ as representing on the λ_c scale the probable error of a single observation, stating however: "These values apply only to certain experimental conditions. Absolutely smaller probable errors can probably be obtained under improved conditions of observation."

The above relation permits the calculation of δ (the magnitude of one "sensation step") in degrees along the Planckian locus at any temperature. The ratio $\frac{\sigma}{\delta}$, then, expresses mean deviation in "sensation steps" whose size must necessarily be constant for sufficiently restricted regions near the Planckian locus. Whether the size of these "sensation steps" varies appreciably with temperature is not known; however, we assume in what follows that their size may be considered constant within the limits 1,600° to 20,000° K. Whether this assumption proves to be wholly justified or not is of little practical importance because accurate comparisons in terms of "sensation steps" are of interest chiefly within restricted ranges of correlated color temperature.

These steps are considerably smaller than the satron as defined by Priest (4; p. 121). For example, it is estimated that Abbot-Priest sunlight is about 1.6 satrons from 5,229° K., the correlated color temperature of Abbot-Priest sunlight. This compared with 8 steps $\left(\frac{\sigma}{\delta}\right)$ as given in Table 3 shows that a satron at this point is approximately equivalent to five of the units used in this paper for describing the magnitude of the color difference. While these units may seem quite small compared with the satron it should be pointed out that an absolutely smaller satron locus could be obtained under improved observing conditions. For the purpose used in this paper the size of the unit is of little consequence.

In the example of the correlated color temperature of sunlight outside the earth's atmosphere, given at the end of the previous section, we had obtained component temperatures of 6,790°, 6,916°, and 5,989° K., respectively. The average of these is 6,565° K. Taking the mean of the deviations of the component temperatures from the average we have

$$\sigma=\frac{(6{,}790^\circ-6{,}565^\circ)+(6{,}916^\circ-6{,}565^\circ)+(6{,}565^\circ-5{,}989^\circ)}{3}=384^\circ$$

To determine the divergence of the color match in least perceptible differences in color, we find from the relation

$$\delta=K\theta^2$$

That

$$\delta=\frac{6{,}565^2}{1{,}000{,}000}=43.1^\circ \text{ (a sensation step at 6,565° K).}$$

$$\frac{\sigma}{\delta}=\frac{384}{43.1}=8.9$$ sensation steps (a measure of the color departure of sunlight outside the earth's atmosphere from the Planckian radiator at 6,565° K).

Table 3 gives for a number of sources the correlated color temperatures, the values of mean deviation σ, their color departure $\frac{\sigma}{\delta}$ in sensation steps, together with the conjunctive wave length λ_j.

4. DETERMINATION OF THE CONJUNCTIVE WAVE LENGTH

In homo-hetero-analysis, a color stimulus is specified by giving its dominant wave length and its purity. The dominant wave length may be computed or it may be found graphically from the trilinear diagram by drawing a straight line through the center of the color triangle and that point representing the color of the stimulus to be specified and extending it until it cuts the spectrum locus.

In the present case, it is desired to specify a non-Planckian stimulus, not with reference to a fixed "neutral" standard, but rather with reference to a particular Planckian distribution as standard; that is, we wish to find the homogeneous stimulus which, when added to or subtracted from (as the case may be) the non-Planckian stimulus will produce a perfect color match. This is found graphically from the familiar trilinear diagram as the intersection of the spectrum locus and a straight line passing through the points representing the two energy distributions. It would appear that this color difference between two light qualities, neither of which is the "neutral" standard, has been so seldom used that it has not been given a name to distinguish it from dominant wave length. The term conjunctive wave length (λ_j) is used in the present discussion.[6] Conjunctive wave lengths for several non-Planckian colors are given in Table 3 and are considered in more detail below.

IV. CHARACTERISTICS OF CORRELATED COLOR TEMPERATURES

1. COLOR DEPARTURE AND THE ISOTEMPERATURE LINE

The work involved in carrying out the computations necessary to make a large-scale graph showing the system of isotemperature lines on the trilinear diagram is so great that a less laborious procedure illustrating the characteristics of the method was adopted. These data are given in Tables 4 and 5. A group of colors representing hypothetical energy distributions were selected as follows: The r, g, and b coordinates for 2,000°, 3,000°, and 10,000° K. were changed to represent non-Planckian colors by adding an increment, 0.0070, to the g coordinate and subtracting a similar value from the b coordinate in each case. The r coordinate was, of course, unchanged since r, g, and b sum to unity by definition. In this way colors were selected which fall on a curve approximately parallel to the Planckian locus on the spectrum-locus side. Correlated color temperatures were then computed for the colors represented by these new r, g, and b coordinates. The values were 2,011°, 3,053°, 6,359°, and 11,587° K. The r, g, and b coordinates were then obtained by interpolation from a previously published table (4, Table 11) for each of these color temperatures. The differences were obtained between the

[6] The term conjunctive wave length need not be restricted to the particular use made of it in this paper, but can be employed in all cases where an expression of the chromaticity difference between any two stimuli (neither of which is the "neutral" standard) is desired in terms of the addition (or subtraction) of a certain amount of homogenous light to (or from) one of the stimuli so that it will match the other. It may frequently happen that a line passing through the 2 points (r_1, g_1, b_1; r_2, g_2 and b_2) representing the given stimuli will intersect the spectrum locus at 2 points, then each stimulus may be said to have 2 conjunctive wave lengths with respect to the other. However, this condition will not be found in any case where a non-Planckian stimulus is correlated by the present method with a Planckian stimulus. Conjunctive wave length approaches dominant wave length as a limit as the trilinear coordinates (r_1, g_1, b_1, or r_2, g_2, b_2) of one of the colors approach those corresponding to the "neutral" stimulus for which $r=g=b=⅓$.

respective *r* coordinates, *g* coordinates, and *b* coordinates of each of the non-Planckian colors and those of the corresponding correlated color temperatures. By adding 0. 5 1, 2, 4, and in two instances, 8 times these differences, according to sign, to the Planckian coordinates new *r*, *g*, and *b* coordinates were obtained for a series of colors which plot on a straight line; that is, the original conjunctive wave-length line. Those colors at the same number of increments away from their respective Planckian color plot on a locus approximately paralleling the Planckian locus. With these new coordinates the data given in Tables 4 and 5 for both sides of the Planckian locus were obtained.

TABLE 4.—*Illustrating the characteristics of the method for computing correlated color temperature*

The colors chosen for these examples are on the spectrum side of the Planckian locus

	Trilinear coordinates			Component temperatures			Component temperature differences	Correlated color temperature	Conjunctive wave length	Mean deviation	Sensation steps in °K along Planckian locus at θ_i	Sensation steps in mean deviation (a measure of color departure)
	r	g	b	Θ_r	Θ_g	Θ_b	$\Theta_r-\Theta_b$	Θ_i	λ_i	σ	δ	$\frac{\sigma}{\delta}$
				°*K.*	°*K.*	°*K.*	°*K.*	°*K.*	$m\mu$	°*K.*		
1/2 increment	0.5749	0.3537_5	0.0713_5	2,030	2,031	1,972	58	2,011	584. 2	26. 0	4. 0	6. 5
1 increment	.5757	.3570	.0673	2,050	2,052	1,930	120	2,011	584. 2	53. 7	4. 0	13. 4
2 increments	.5773	.3635	.0592	2,090	2,094	1,845	245	2,010	584. 1	109. 7	4. 0	27. 4
4 increments	.5805	.3765	.0430	2,175	2,183	1,667	508	2,008	584. 1	227. 7	4. 0	56. 9
1/2 increment	.4547	.3636	.1817	3,081	3,086	2,992	89	3,053	575. 5	40. 7	9. 3	4. 4
1 increment	.4572	.3674	.1754	3,110	3,118	2,930	180	3,053	575. 5	81. 7	9. 3	8. 8
2 increments	.4622	.3750	.1628	3,167	3,183	2,809	358	3,053	575. 5	162. 7	9. 3	17. 5
4 increments	.4722	.3902	.1376	3,282	3,318	2,575	707	3,058	575. 7	322. 3	9. 4	34. 3
8 increments	.4922	.4206	.0872	3,519	3,602	2,112	1,407	3,078	575. 7	643. 7	9. 5	67. 8
1/2 increment	$.2988_5$	.3090	$.3921_5$	6,457	6,516	6,082	375	6,352	564. 0	179. 7	40. 3	4. 5
1 increment	.3033	.3150	.3817	6,567	6,684	5,825	742	9,359	564. 8	355. 7	40. 4	8. 8
2 increments	.3122	.3270	.3608	6,783	7,035	5,373	1,410	6,397	565. 1	682. 7	40. 9	16. 7
4 increments	.3300	.3510	.3190	7,216	7,795	4,624	2,592	6,545	565. 7	1,281. 0	42. 8	29. 9
8 increments	.3656	.3990	.2354	8,100	9,653	3,522	4,578	7,092	565. 9	2,380. 0	50. 3	47. 3
1/2 increment	$.2371_5$	.2701	$.4927_5$	11,870	12,283	10,450	1,420	11,534	560. 0	723. 0	133. 0	5. 4
1 increment	.2428	.2776	.4796	12,186	13,060	9,515	2,671	11,587	561. 0	1,381. 0	134. 0	10. 3
2 increments	.2541	.2926	.4533	12,800	14,825	8,093	4,707	11,906	561. 5	2. 542. 0	142. 0	17. 9
4 increments	.2767	.3226	.4007	14,080	19,950	6,270	7,810	13,433	562. 5	4,776. 0	180. 0	26. 5

TABLE 5.—*Illustrating the characteristics of the method for computing correlated color temperature*

The colors chosen for these examples are on the purple side of the Planckian locus

	Trilinear coordinates			Component temperatures			Component temperature differences	Correlated color temperature	Conjunctive wave length	Mean deviation	Sensation steps in °K along Planckian locus at θ_j	Sensation steps in mean deviation (a measure of color departure)
	r	g	b	Θ_r	Θ_g	Θ_b	$\Theta_r-\Theta_b$	Θ_j	λ_j	σ	δ	$\frac{\sigma}{\delta}$
				°K.	°K.	°K.	°K.	°K.	mμ	°K.		
½ increment	0.5732$_5$	0.3472$_5$	0.0795	1,991	1,991	2,050	59	2,011	584.1	26.3	4.0	6.6
1 increment	.5725	.3440	.0835	1,972	1,970	2,090	118	2,011	584.1	53.0	4.0	13.2
2 increment	.5710	.3375	.0915	1,933	1,929	2,166	233	2,009	584.3	104.3	4.0	26.1
4 increment	.5680	.3245	.1075	1,858	1,852	2,316	458	2,009	584.2	205.0	4.0	51.2
8 increment	.5620	.2985	.1395	1,715	1,704	2,615	900	2,011	584.1	402.3	4.0	100.6
½ increment	.4496	.3560	.1944	3,027	3,024	3,116	89	3,056	575.5	40.3	9.3	4.3
1 increment	.4472	.3522	.2006	2,998	2,990	3,178	180	3,055	575.4	81.7	9.3	8.8
2 increment	.4424	.3446	.2130	2,939	2,923	3,310	371	3,057	575.3	168.3	9.3	18.1
4 increment	.4328	.3294	.2378	2,822	2,794	3,584	762	3,067	574.9	345.0	9.4	36.7
8 increment	.4136	.2990	.2874	2,594	2,546	4,211	1,617	3,117	573.8	729.3	9.7	75.2
½ increment	.2896$_5$	.2968	.4135$_5$	6,254	6,197	6,680	426	6,377	564.1	202.0	40.7	5.0
1 increment	.2855	.2910	.4235	6,134	6,033	7,000	866	6,389	563.7	407.3	40.8	10.0
2 increment	.2772	.2794	.4434	5,903	5,714	7,733	1,830	6,450	562.5	855.3	41.6	20.6
4 increment	.2606	.2562	.4832	5,451	5,124	9,860	4,409	6,812	558.4	2,032.0	46.4	43.8
½ increment	.2241$_5$	.2540$_5$	.5218	11,620	11,260	13,483	1,863	12,121	557.0	908.0	147.0	6.2
1 increment	.2202	.2479	.5319	11,080	10,450	15,100	4,020	12,210	555.3	1,927.0	149.0	12.9
2 increment	.2123	.2356	.5521	10,090	9,080	20,260	10,170	13,143	548.2	4,744.0	173.0	27.4

The first column of these tables serves to identify the points chosen by the increments as described; the next three columns contain their trilinear coordinates; columns 5, 6, and 7 headed θ_r, θ_g, θ_b, respectively, are the component temperatures; column 8, $\theta_r-\theta_b$, shows that the separation of the component temperatures along the black body locus is roughly proportional to the increments given in column 1; column 9 gives the correlated color temperatures for colors having the coordinates given in columns 2, 3, and 4; column 10 gives the conjunctive wave length, column 11, the mean deviations of the component temperatures from the correlated color temperature (θ_j); column 12, the number of degrees the Planckian radiator would have to be increased or decreased in temperature to give a difference in color which would be just perceptible, as given by the expression $\delta=K\theta^2$; and column 13, the mean deviation divided by δ. Note that these last values also increase roughly by powers of 2.

These tables show that the spread of the component temperatures along the Planckian locus, the mean deviation, and $\frac{\sigma}{\delta}$ are approximately proportional to the difference between the trilinear coordinates of the Planckian and the non-Planckian colors. In Table 4 the λ_j values increase slightly with the number of "increments" indicating that the isotemperature line is slightly curved. Furthermore, the curvature becomes more pronounced as the temperature increases, and λ_j changes with θ_j. In Table 5, referring to colors on the purple side of the Planckian locus, the conjunctive wave length decreases as the distance from the Planckian locus increases; indicating that the isotemperature line continues to curve in the same direction as on the spectrum side of the Planckian locus.

2. CONJUNCTIVE WAVE LENGTH AS A FUNCTION OF CORRELATED COLOR TEMPERATURE

If the conjunctive wave lengths corresponding to a series of given Planckian colors be marked on the spectrum locus, (on a r–b diagram) lines joining these points with the corresponding Planckian points

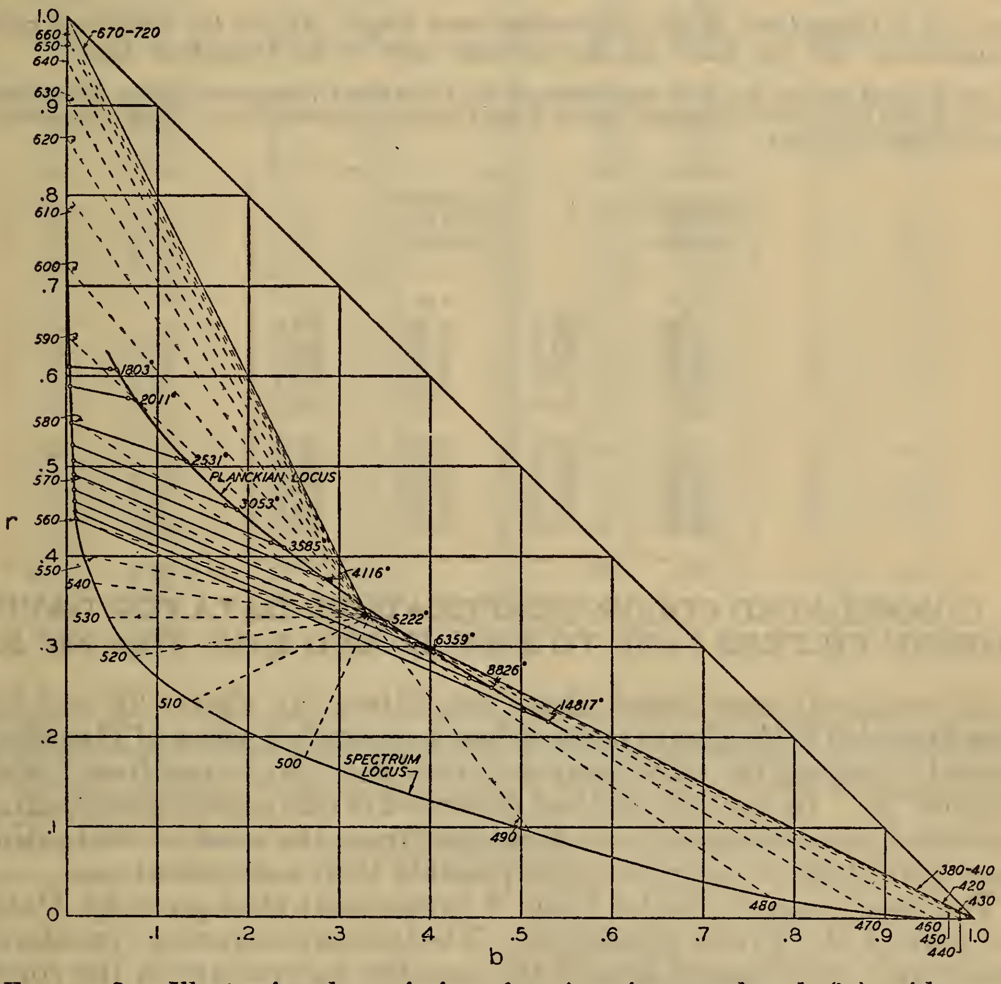

FIGURE 3.—*Illustrating the variation of conjunctive wave length* (λ_i) *with correlated color temperature using the data in Table 6 and plotted on a r-b diagram with mean sun as the "neutral" stimulus*

will indicate roughly the direction of color departure. A diagram of this type (shown in fig. 3) is of practical utility because, since these lines on the plot are nearly parallel, one may obtain, graphically, an approximate value of the correlated color temperature of any color lying in the neighborhood of one of these lines and near the Planckian locus.

To obtain this relation a series of non-Planckian colors was selected (by the method previously described for Table 5) the locus of which parallels the Planckian locus at a distance of "one increment."

From the equation of the line joining the points, representing respectively the Planckian and the non-Planckian colors, the intercept on the r axis is found. Drawing a line from this point through the two given points, its intersection with the spectrum locus is found graphically. From the coordinates of this intersection the corresponding spectrum wave length (conjunctive wave length) is found by way of a table (not published) containing values of $\frac{g-b}{r-b}$, $\frac{g-r}{b-r}$ or $\frac{r-g}{b-g}$ for each millimicron of wave length. In this way the values given in Table 6 were found. The material of Table 6 is applied in figure 3, an $r-b$ projection of the trilinear diagram.

TABLE 6.—*Dependence of the conjunctive wave length* (λ_i) *on the correlated color temperature* (θ_i) *for colors on the spectrum side of the Planckian locus*

It will be noted that the hue of the color corresponding to the added homogeneous radiant energy does not vary greatly (green yellow to orange); that is, λ_i has a range of only about 30mμ. These values are further illustrated in Figure 3.

θ correlated color temperature	λ_i	θ correlated color temperature	λ_i
°*K*	*mμ*	°*K*	*mμ*
1,803	586.5	6,945	564.0
2,011	584.2	7,549	563.2
2,532	579.5	8,181	562.6
3,053	575.5	8,826	562.1
3,585	572.9	9,483	561.4
4,116	570.2	10,187	561.0
4,663	568.8	10,876	561.2
5,222	567.0	11,587	561.0
5,781	565.8	13,161	560.3
6,359	564.8	14,817	560.3

V. CORRELATED COLOR TEMPERATURE DATA FOR DAVIS-GIBSON FILTERS 2,450° TO 3,500° K. AND 2,450° TO 6,500° K.

As previously mentioned, these two filters (4; charts 37 and 38) were intended to duplicate more or less accurately a series of Planckian colors by varying the color temperature of the light source from 2,000° to 3,100° K. In fact, the method described in this paper for computing correlated color temperatures developed from the need of calibrating these two filters in order to make possible their convenient use.

The data given in Tables 7 and 8 supplement that given by Davis and Gibson (4; Tables 15 and 16). The last three columns, considered collectively, give a good idea of the possible uncertainty in the correlated color temperatures given in the second column.

TABLE 7.—*Correlated color temperature data as functions of the light-source temperature of the Davis-Gibson filter (chart No. 37 and Table 14 of B. S. Misc. Pub. No. 114) designed to convert 2,450° to 3,500° K*

Color temperature of light source	Correlated[1] color temperature of source and filter combination	σ (mean[2] deviation)	δ ($K\theta^2$)	$\frac{\sigma}{\delta}$ sensation steps[3]	Color temperature of light source	Correlated[1] color temperature of source and filter combination	σ (mean[2] deviation)	δ ($K\theta^2$)	$\frac{\sigma}{\delta}$ sensation steps[3]
°K	°K	°K			°K	°K	°K		
2, 000	2, 649	8	7. 0	1. 1	2, 600	3, 817	7	14. 6	. 5
20	2, 685	7	7. 2	1. 0	20	3, 859	8	14. 9	. 5
40	2, 721	7	7. 4	. 9	40	3, 903	9	15. 2	. 6
60	2, 757	6	7. 6	. 8	60	3, 947	9	15. 6	. 6
80	2, 793	5	7. 8	. 7	80	3, 992	10	15. 9	. 6
2, 100	2, 829	5	8. 0	. 7	2, 700	4, 038	10	16. 3	. 6
20	2, 865	4	8. 2	. 6	20	4, 082	11	16. 7	. 6
40	2, 901	4	8. 4	. 5	40	4, 126	11	17. 0	. 7
60	2, 939	3	8. 6	. 5	60	4, 171	12	17. 4	. 7
80	2, 976	3	8. 9	. 4	80	4, 216	13	17. 8	. 7
2, 200	3, 012	3	9. 1	. 4	2, 800	4, 262	14	18. 2	. 7
20	3, 051	3	9. 3	. 4	20	4, 309	14	18. 6	. 7
40	3, 088	3	9. 5	. 3	40	4, 356	15	19. 0	. 8
60	3, 127	2	9. 8	. 3	60	4, 404	16	19. 4	. 8
80	3, 165	2	10. 0	. 3	80	4, 453	17	19. 8	. 9
2, 300	3, 202	2	10. 3	. 2	2, 900	4, 502	18	20. 3	. 9
20	3, 242	2	10. 5	. 2	20	4, 551	18	20. 7	. 9
40	3, 281	1	10. 8	. 2	40	4, 600	19	21. 2	. 9
60	3, 320	1	11. 0	. 1	60	4, 650	20	21. 6	1. 0
80	3, 359	1	11. 3	. 1	80	4, 700	21	22. 1	1. 0
2, 400	3, 398	1	11. 5	. 1	3, 000	4, 751	22	22. 6	1. 0
20	3, 439	0	11. 8	. 1	20	4, 803	23	23. 1	1. 0
40	3, 480	0	12. 1	. 0	40	4, 855	25	23. 6	1. 0
60	3, 521	0	12. 4	. 0	60	4, 908	28	24. 1	1. 1
80	3, 563	1	12. 7	. 1	80	4, 961	31	24. 6	1. 2
2, 500	3, 605	2	13. 0	. 1	3, 100	5, 015	34	25. 2	1. 3
20	3, 647	3	13. 3	. 2					
40	3. 688	4	13. 6	. 3					
60	3, 730	5	13. 9	. 4					
80	3, 773	6	14. 2	. 5					

[1] The values in the second and third columns were obtained from large-scale graphs plotted from data computed for every 100° color temperature of the light source. It is impossible at this time to state the accuracy of the color temperatures given in the second column. They are given to the nearest degree to facilitate interpolation.

[2] The mean deviations (σ) plot somewhat irregularly and their accuracy is not, in general, considered to be better than ±1°.

[3] To preserve continuity of the values $\frac{\sigma}{\delta}$, the mean deviations (σ) used were smoothed to an extent beyond those given in the third column.

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TABLE 8.—*Correlated color temperature data as functions of the light source temperature of the Davis-Gibson filter (chart No. 38 and Table 15 of B. S. Misc. Pub. No. 114) designed to convert 2,450° to 6,500° K.*

Color temperature of light source	Correlated [1] color temperature of source and filter combination	σ (mean [2] deviation)	δ ($K\theta^2$)	$\frac{\sigma}{\delta}$ sensation step	Color temperature of light source	Correlated [1] color temperature of source and filter combination	σ (mean [2] deviation)	δ ($K\theta^2$)	$\frac{\sigma}{\delta}$ sensation step
° K.	° K.	° K.			° K.	° K.	° K.		
2,000	4,031	47	16	2.9	2,600	7,773	66	60	1.1
20	4,116	47	17	2.8	20	7,976	78	64	1.2
40	4,201	47	18	2.6	40	8,185	91	67	1.4
60	4,289	47	18	2.6	60	8,404	105	71	1.5
80	4,378	46	19	2.4	80	8,632	121	75	1.6
2,100	4,468	45	20	2.2	2,700	8,869	139	79	1.8
20	4,560	45	21	2.1	20	9,123	160	83	1.9
40	4,656	44	22	2.0	40	9,387	183	88	2.1
60	4,752	42	23	1.8	60	9,665	211	93	2.3
80	4,853	41	24	1.7	80	9,957	244	99	2.5
2,200	4,956	39	25	1.6	2,800	10,252	278	105	2.6
20	5,060	37	26	1.4	20	10,600	318	112	2.8
40	5,168	35	27	1.3	40	10,956	360	120	3.0
60	5,279	32	28	1.1	60	11,320	404	128	3.2
80	5,393	28	29	1.0	80	11,696	450	137	3.3
2,300	5,511	25	30	.8	2,900	12,090	500	146	3.4
20	5,632	23	32	.7	20	12,517	559	157	3.6
40	5,757	21	33	.6	40	12,979	624	168	3.7
60	5,884	18	35	.5	60	13,482	700	182	3.8
80	6,014	16	36	.4	80	14,030	789	197	4.0
2,400	6,143	13	38	.3	3,000	14,631	892	214	4.2
20	6,286	8	40	.2	20	15,299	1,020	234	4.4
40	6,430	2	41	.0	40	16,032	1,169	257	4.5
60	6,578	4	43	.1	60	16,844	1,346	284	4.7
80	6,728	11	45	.2	80	17,737	1,557	315	4.9
2,500	6,884	16	47	.3	3,100	18,667	1,778	348	5.1
20	7,047	23	50	.5					
40	7,217	32	52	.6					
60	7,394	43	55	.8					
80	7,580	54	57	.9					

[1] The values in the second and third columns were obtained from large-scale graphs plotted from data computed for every 100° color temperature of the light source. It is impossible at this time to state the accuracy of the color temperatures given in the second column. They are given to the nearest degree to facilitate interpolation.

[2] The mean deviations (σ) plot somewhat irregularly and their accuracy is not, in general, considered to be better than ±1°.

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