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The Use of Finite Polynomial Rings in the Factorization of the General Polynomial

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The problem of decomposing the general polynomial over the integers into its prime polynomial factors is solved by the use of congruential functions.

The following principles are used:

I. If a polynomial P is reducible into rational integral factors, then P_m is similarly reducible in J_m, where P_m is the polynomial P with its coefficients reduced modulo m (m a prime).

II. The factors of P_m are congruent modulo m to the factors of P.

III. If P_m is irreducible in J_m, then P is irreducible over the rational integers. (Contrapositive of I.)

Accompanying tables are provided which give the factors of all congruential polynomials over the ring of integers modulo 2, 3, 5, up through degrees 11, 11, and 8, respectively.

Introduction

1. Foreword

The first set of tables of this kind were designed several years ago by the author at the University of Oklahoma. At their computing center they were computed through degree 6 for moduli 2, 3, 5, and 7, under the direction of R. V. Andree, and C. E. Maudlin, programmer. Based on those tables, the present tables were designed with a more concise format. The latter were computed in 1964 through the courtesy of the University of Maryland Computer Science Center, Werner C. Rheinboldt, Director, and Harry Remmers, Jr., programmer. They were programmed in Fortran IV on a No. 7090 IBM machine. The actual computing time on the machine was as follows:

Modulus 2, degrees 1–11	0 h	1.6 m
Modulus 3, degrees 1–11	2 h	7.6 m
Modulus 5, degrees 1–8	6 h	20.0 m
Total Time	8 h	29.2 m

Due to the extensive size of the tables for moduli 3 and 5, only the modulus 2 tables are included in full in this paper. However, sample extracts from the moduli 3 and 5 tables are given to show their use in conjunction with the illustrative examples given below.

These factor tables have been computed for moduli 2 and 3 through degree 11 for modulus 5 through degree 8, and modulus 7 through degree 6. These have not as yet been published but persons desiring them may contact the author in Washington, D.C.

No other factor tables for all polynomials, reducible and irreducible have heretofore been published. Previously there have been published tables of *irreducible* polynomials, notably the following:

(a) Church, Randolph. Tables of Irreducible Polynomials for the First Four Prime Moduli. Annals of Mathematics **36**, 198–209. (1935). They include:

Mod 2, Degrees 1–11	Mod 5, Degrees 1–5
Mod 3, Degrees 1–7	Mod 7, Degrees 1–4.

(b) Marsh, R. W. Table of Irreducible Polynomials Over GF(2) Through Degree 19. National Security Administration, Washington, D.C. (1957).

(c) Peterson, W. W. "Error-Correcting Codes." M. I. T. (1961). An appendix, pp. 251–270, lists the irreducible polynomials, degrees 2–16; for degrees 17–34 are listed only certain primitive polynomials belonging to various possible exponents.

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2. Description of the Factor Tables

The tables list consecutively the monic congruential polynomials with respect to moduli 2, 3, and 5. Opposite each polynomial a prime factor of least degree, and the resulting quotient, are given. For example, in modulus 3, degree 6:

$$x^6+2x^5+2x^3+x^2+x+1=(x^2+2x+2)(x^4+x^2+2).$$

The second factor may or may not be prime, and would require further investigation.

However, to produce a concise format, only the detached coefficients are printed. These are further abbreviated by a condensed system of numeration, octic for modulus 2, and a modified form for mod-

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ulus 3. For example, "273" means 1 0 1 1 1 0 1 1, mod 2. In modulus 3, "1675" means 1 2 0 2 1 1 2, wherein each two digits condense ternary-wise into one. A condensed notation is not used in modulus 5.

A further shortening of the tables is effected by omitting reciprocal polynomials. Every polynomial has a "reciprocal," formed by writing the coefficients in the reverse order. This reciprocal has factors which are the reciprocals of those of the original, and hence need not be listed.

Thus, in degree 4, modulus 3, 1 1 1 1 2=(1 2) (1 2 0 1), while its reciprocal (not listed), 1 2 2 2 2 =(1 2)(1 0 2 1). And, in modulus 5, 1 1 3 1 2 1 =(1 3 4)(1 3 0 4). While its reciprocal (not listed), 1 2 1 3 1 1=(1 2 4)(1 0 2 4). It may be observed that (1 2) is its own reciprocal in mod 3, while in modulus 5 coefficients (1, 4), (2, 3), (0 0) are "reciprocal" pairs.

The second polynomial of each of these pairs is omitted from the table, being of greater "height" of the pair. In using the tables one must thus seek the lesser of the pair, reciprocating and reversing the coefficients as necessary. As only monic polynomials are listed, it will be necessary to reduce coefficients in this process, unless the constant term is unity. The illustrative problems below clarify this procedure.

Irreducible polynomials are listed without factors.

3. Present Status of the Factoring Problem

The problem of decomposing the general rational polynomial into its prime polynomial factors has vexed mathematicians for a long time. It has never been completely solved except by tedious empirical methods. Typical of these are the methods advanced by Kronecker [8],[3] Runge [17], Mandl [11], Glenn [6], and Frumveller [5], but the computational procedures involved in these were lengthy and onerous. A critique of these methods is given in [9] and need not be repeated here.

The related problem of determining the reducibility or irreducibility of a polynomial has also attracted much attention. Success in this effort has been confined to special cases only. Some necessary conditions and some sufficient ones have been found, but no conditions that are both necessary *and* sufficient. A necessary and sufficient condition for reducibility of the quartic was found by Varnhorn [20] in 1939, and by the author [9] for the quintic in 1940. Their methods also led to the actual determination of the factors. But no absolute criterion for the reducibility of the *general* polynomial exists in the literature.

A number of writers have presented criteria for the *irreducibility* of certain classes of polynomials. Among these are Schoenemann [18], Eisenstein [4], Königsberger [7], Netto [12], Bauer [1], Perron [14], Dumas [3], Ore [13], Stäckel [19], and Polya and Szego [16]. It will not be necessary to outline their work here as such a historical survey was made by Dorwart [2] in 1935. Whereas the above schemes are helpful for certain kinds of polynomials, actually they are applicable to only a small percentage of cases.

[3] Figures in brackets indicate the literature references at the end of this paper.

From the above survey it is clear that a broader attack on the related problems of reducibility and decomposition of the general polynomial is highly desirable.

4. The Theory of the Proposed Method

We shall consider the general polynomial over J, the ring of rational integers. Further, without loss of generality, we limit our discussion to monic polynomials (leading coefficient unity), for a simple transformation will render monic any given polynomial.

We establish the correspondence $a \rightarrow a_m$, which carries each integer a into its residue class modulo m, where m is a prime. This is a homomorphism of the ring of rational integers to the finite ring of integers, J_m. We then consider the general polynomial P with rational integral coefficients. The correspondence $P \rightarrow P_m$ carries P into a finite ring of polynomials, with coefficients in J_m. We present the following three theorems with proof below (cf. reference [10]):

I. If a polynomial P is reducible into rational integral factors, then P_m is similarly reducible in J_m, where P_m is the polynomial P with its coefficients reduced modulo m, (m a prime).

II. The factors of P_m are congruent modulo m to the factors of P.

III. If P_m is irreducible in J_m, then P is irreducible over the rational integers. (Contrapositive of I.)

PROOF:

Let $$P=\sum_{i=0}^{n} C_i x^i$$

and the corresponding reduced polynomial

$$P_m=\sum_{i=0}^{n} c_i x^i$$

where each $C_i = mq_i + c_i$

$$0 \leq c_i < m.$$

Assume $P=Q \cdot R$ the product of two rational integral polynomials, where $Q=\sum_{i=0}^{k} M_i x^i$ and $R=\sum_{i=0}^{n-k} N_i x^i$ $0<k<n$.

And their corresponding reduced polynomials

$$Q_m=\sum_{i=0}^{k} m_i x^i \text{ and } R_m=\sum_{i=0}^{n-k} n_i x^i$$

when $M_i = ml_i + m_i$, $N_i = mt_i + n_i$, $0 \leq m_i$, $n_i < m$.

Then $$P=\sum_{i=0}^{n}(mq_i+c_i)x^i. \quad (1)$$

Also $$P=\sum_{i=0}^{k} M_i x^i \sum_{i=0}^{n-k} N_i x^i$$

$$=\sum_{i=0}^{k}(ml_i+m_i)x^i \sum_{i=0}^{n-k}(mt_i+n_i)x^i. \quad (2)$$

Noting the identity of (1) and (2), and by reducing coefficients, modulo m, we have the congruence:

$$\sum_{i=0}^{n} c_i x^i = \left(\sum_{i=0}^{k} m_i x^i\right) \cdot \left(\sum_{i=0}^{n-k} n_i x^i\right).$$

That is,

$$P_m \equiv \left(\sum_{i=0}^{k} m_i x^i\right) \cdot \left(\sum_{i=0}^{n-k} n_i x^i\right) = Q_m R_m$$

where Q_m and R_m are the polynomials Q, R, reduced by modulus m.

The truth of the theorems is thence immediate.

5. Illustrative Problems Applying the Tables

Principles I–III above provide a method for finding the factors of any polynomial by a study of a more limited set of congruent polynomials over finite rings, such as modulo 2, 3, or 5.

Principle III states that a polynomial is prime, if its homomorphic congruential polynomial in any finite ring (of prime modulus) is prime. It should be remembered that the converse is *not* true. To determine the reducibility of the following polynomials, we form the corresponding reduced polynomials, with respect to one or more moduli, and note from the Factor Tables whether or not the latter are prime. Thus, to determine reducibility, form $P_m \equiv P$ in the following examples:

(a) $x^3 - 11x^2 + 7x - 16 \equiv x^3 + x^2 + x \pmod 2$, which is composite, but $P_3 = x^3 + x^2 + x + 2 \pmod 3$, which is prime by the tables; hence P is prime.

(b) $x^3 - 12x^2 - 17x + 25 \equiv x^3 + x + 1 \pmod 3$, which is composite. But $P_2 = x^3 + x + 1 \pmod 2$, which is prime; hence P is prime.

(c) $x^4 - 13x^3 - 19x^2 + 21x + 45 \equiv x^4 + x^3 + x^2 + x + 1$ (mod 2) which is prime; hence P is prime. (Although P_3 and P_5 are composite).

(d) $x^4 + 7x^3 + 7x - 35 \equiv x^4 + x^3 + x + 1$ (mod 2 and 3). This is composite in A_2, A_3, and A_5, so the test is indecisive from the tables alone. It could be tested for factors, as shown below. Actually it is prime by Eisenstein's Theorem.

(e) Given $P = x^6 - 3x^5 + 5x^4 + x^3 - 4x^2 + 25$. Using the condensed numeration of the tables:

$P_2 = 1\ 1\ 1\ 1\ 0\ 0\ 1 = (1\ 7\ 1)$,

which factors into $(7)(23) = (1\ 1\ 1)(1\ 0\ 0\ 1\ 1)$

$= (x^2 + x + 1)(x^4 + x + 1)$.

Both of these factors are prime, by the tables. It is thus indicated that P, if reducible, must factor into a quadratic and a quartic, both prime.

But, $P_3 = 1\ 0\ 2\ 1\ 2\ 0\ 1 = (1\ 2\ 5\ 1)$

$= (47)(54) = (1\ 1\ 2\ 1)(1\ 2\ 1\ 1)$

$= (x^3 + x^2 + 2x + 1)(x^3 + 2x^2 + x + 1)$.

Both cubics are prime, requiring that P factor, if at all, into two prime cubics. The two factor patterns demanded by P_2 and P_3 are incompatible and hence P is irreducible.

(f) Given $P = x^7 + 13x^6 - 32x^5 + 27x^4 + 29x^3 + 23x^2 - 19x - 35$. $P_2 = 1\ 1\ 0\ 1\ 1\ 1\ 1\ 1 = (3\ 3\ 7) = (7)(51) = (1\ 1\ 1)(1\ 0\ 1\ 0\ 0\ 1)$, a quadratic and a quintic, both prime.

$P_3 = 1\ 1\ 1\ 0\ 2\ 2\ 2\ 1 = (4\ 3\ 8\ 7) = (38)\ (158)$[4]

$= (1\ 0\ 2\ 2)(1\ 1\ 2\ 2\ 2)$, a cubic and a quartic, both prime.

Since these factor patterns are incompatible, P must be irreducible. It happens here that P_5 also factors incompatibly into a linear and a sextic, both prime, as the reader may verify.[4]

Further explanations, with both theory and examples will be found in reference [10], but the following will probably prove adequate for a wide variety of factoring problems encountered.

Principles I–III above enable us to find the factors of any polynomial by a study of a more limited set of congruent factors in the finite rings, modulo 2, 3, or 5. Thus, in modulo 3, since the prime factors

$$(x^2 + x + 2)(x^3 + 2x^2 + x + 1) = x^5 + 2x^3 + 2 = P_3,$$

then any quintic polynomial P congruent to P_3, such as

$$P = x^5 - 3x^4 + 5x^3 - 6x^2 - 36x - 16,$$

if factorable at all, must have factors congruent to those of P_3. Actually, here

$$P = (x^2 - 2x - 4)(x^3 - x^2 + 7x + 4).$$

Next will be presented in detail the solution of three typical factoring problems—a quintic, an octic, and a decimic. They are found to be reducible.

EXAMPLE I. *Quintic*

Given $P = x^5 - 6x^4 + 20x^2 + 9x + 1$.

Form the congruential polynomials, moduli 2 and 3 using detached coefficients:

$$P_2 = 1\ 0\ 0\ 0\ 1\ 1.$$

Converting to condensed numeration and entering the tables

$$P_2 = (4\ 3) = (7)\ (1\ 5) = (1\ 1\ 1)\ (1\ 1\ 0\ 1)$$

We observe in the tables that both of these factors are prime. This indicates that P factors, if at all, into this pattern—a quadratic times a cubic,—both prime.

Similarly, $P_3 = 1\ 0\ 0\ 2\ 0\ 1 = (3\ 2\ 1)$

$= (1\ 5)\ (5\ 8) = (1\ 1\ 2)\ (1\ 2\ 2\ 2)$, both prime.

[4] Tables are not available in this article.

(The cubic (1 2 2 2) is found under its reciprocal (1 1 1 2)).

We note here that the factor pattern for P_3 is consistent with that for P_2. (If it had been prime, or reducible only as a linear times a quartic, this inconsistency would have necessitated the irreducibility of P.)

We now examine the factorization indicated for P:

$$P=(x^2+ax+b)(x^3+a'x^2+b'x+c'). \quad (1)$$

Multiplying out, and equating to the known coefficients of P:

$$a+a'=-6 \quad (2)$$

$$aa'+b+b'=0 \quad (3)$$

$$ab'+a'b+c'=20 \quad (4)$$

$$bb'+ac'=9 \quad (5)$$

$$bc'=1 \quad (6)$$

Since the unknown literal coefficients must agree with the known coefficients of the factors of P_2 and P_3, we form the *table of residues:*

	a	b	a'	b'	c'
P_2............	1	1	1	0	1
P_3............	1	2	2	2	2

From equation (6), and the residue table, we have $(b, c')=(-1, -1)$. Then equations (2)−(5) solved simultaneously produce the cubic $a^3+6a^2+2a-15=0$, which promptly yields $a=-5$, with the help of the residue table. Thence $a'\equiv-1$ and $b'=-4$. So

$$P=(x^2-5x-1)(x^3-x^2-4x-1)$$

in prime factors.

EXAMPLE II. *Octic*

$$P=x^8+x^7-11x^6-2x^5+36x^4-6x^3-43x^2+21x+24.$$

Construct therefrom the reduced congruential functions, for moduli 2, 3, and 5. Using detached coefficients,

$$P_2=1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0$$

$$=(1\ 0)(1\ 1\ 1\ 0\ 0\ 0\ 1\ 1)$$

we shall enter the Factor Tables to determine factors of the septic factor

$$F_2=1\ 1\ 1\ 0\ 0\ 0\ 1\ 1.$$

The reciprocal polynomial must be used here:

$F_2'=1\ 1\ 0\ 0\ 0\ 1\ 1\ 1$

$=3\ 0\ 7$, in condensed octic numeration.

$=(2\ 3)\ (1\ 5)$, prime factors

$=(1\ 0\ 0\ 1\ 1)\ (1\ 1\ 0\ 1)$

$\therefore F_2=(1\ 1\ 0\ 0\ 1)\ (1\ 0\ 1\ 1)$, quartic x cubic, both prime.

$\therefore P_2=(1\ 0)\ (1\ 1\ 0\ 0\ 1)\ (1\ 0\ 1\ 1)$ in prime factors

Next: $P_3=1\ 1\ 1\ 1\ 0\ 0\ 2\ 0\ 0$

$=(1\ 0\ 0)\ (1\ 1\ 1\ 1\ 0\ 0\ 2)$.

We shall examine the sextic for factors, using the reciprocal

$F_3'=1\ 0\ 0\ 2\ 2\ 2\ 2$

$=1\ 0\ 8\ 8$

$=(5)(4\ 3\ 7)$

$=(5)(5)(1\ 6\ 2)$—all prime factors

$=(1\ 2)(1\ 2)(1\ 2\ 0\ 0\ 2)$.

Again reciprocating,

$F_3=(1\ 2)(1\ 2)(1\ 0\ 0\ 1\ 2)$

$\therefore P_3=(1\ 0\ 0)(1\ 2)(1\ 2)(1\ 0\ 0\ 1\ 2)$, in prime factors.

Next, $P_5=1\ 1\ 4\ 3\ 1\ 4\ 2\ 1\ 4$

$=(1\ 0\ 1\ 2\ 3)(1\ 1\ 3\ 0\ 3)$, both quartics prime.

Use should be made here of the modulus 5 tables, degree 8 (available as mentioned on page 1 above). Without the use of them the problem could be solved, but would involve testing out numerous alternative possibilities, uneconomical in time and effort. The factor pattern of P_5 signifies that P will factor into 2 prime quartics unless it is prime. Hence let

$$P=(x^4+ax^3+bx^2+cx+d)(x^4+a'x^3+b'x^2+c'x+d'). \quad (1)$$

Multiplying out and equating to the known coefficients of P:

$$a+a'=1 \quad (2)$$

$$aa'+b+b'=-11 \quad (3)$$

$$ab'+a'b+c+c'=-2 \quad (4)$$

$$ac'+a'c+bb'+d+d'=36 \quad (5)$$

$$ad'+a'd+bc'+b'c=-6 \quad (6)$$

$$bd'+b'd+cc'=-43 \quad (7)$$

$$cd'+c'd=21 \quad (8)$$

$$dd'=24 \quad (9)$$

Grouping the factors of P_2 and P_3 accordingly into (quartic) x (quartic):

$$P_2 = (1\ 0\ 1\ 1\ 0)(1\ 1\ 0\ 0\ 1) \quad (10)$$

$$P_{3a} = (1\ 1\ 1\ 0\ 0)(1\ 0\ 0\ 1\ 2) \quad (11)$$

$$P_{3b} = (1\ 0\ 0\ 1\ 2)(1\ 1\ 1\ 0\ 0) \quad (12)$$

$$P_{5a} = (1\ 0\ 1\ 2\ 3)(1\ 1\ 3\ 0\ 3) \quad (13)$$

$$P_{5b} = (1\ 1\ 3\ 0\ 3)(1\ 0\ 1\ 2\ 3) \quad (14)$$

It is observed that P_3 and P_5 have *two* possible ways to relate to P_2; hence (11) to (14).

The unknown coefficients in (1) must satisfy the following table of residues:

	a	b	c	d	a'	b'	c'	d'
P_2	0	1	1	0	1	0	0	1
P_{3a}	1	1	0	0	0	0	1	2
P_{3b}	0	0	1	2	1	1	0	0
P_{5a}	0	1	2	3	1	3	0	3
P_{5b}	1	3	0	3	0	1	2	3

From (9), $dd' = 24$; then referring to the table of residues, necessarily $(d, d') = (8, 3)$. Thus, P_{3a} is rejected. Thence, similarly, from (8), $(c, c') = (7, 0)$. Hence P_{5b} is rejected. $\therefore P_2$, P_{3b}, and P_{5a} will be used to determine the remaining coefficients.

From (7), $(b, b') = (-9, -2)$.

Next, from (2) and (3), $(a, a') = (0, 1)$.

The values check in (4), (5), and (6).

$\therefore P = (x^4 - 9x^2 + 7x + 8)(x^4 + x^3 - 2x^2 + 3)$, the product of two prime quartics.

EXAMPLE III. *Decimic*

$$P = x^{10} + 19x^9 + 78x^8 - 89x^7 + 15x^6 + 106x^5 - 45x^4 - 8x^3 + 23x^2 + 15x + 5.$$

First, form

$$P_2 = 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 1 = 3327$$

Using the Factor Tables, Degree 10, Mod 2: [5]

$$P_2 = (3)(1\ 1\ 1\ 5) = \text{Linear} \times \text{Nonic}$$

$$= (3)(37)(67) = \text{Linear} \times \text{Quartic} \times \text{Quintic} - \text{all prime.}$$

$$= (x+1)(x^4 + x^3 + x^2 + x + 1)(x^5 + x^4 + x^2 + x + 1).$$

[5] Table for this degree not included in this article.

Next, $P_3 = 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 2\ 0\ 2.$

In this case we must use its *reciprocal* in the table.

$$\therefore P_3' = 1\ 0\ 1\ 2\ 0\ 2\ 0\ 2\ 0\ 2\ 2 = 116668$$

$$= (5)(4\ 7\ 3\ 2\ 7) = (5)(5\ 1)(1\ 7\ 3\ 7)$$

$$= (1\ 2)(1\ 2\ 0\ 1)(1\ 2\ 1\ 1\ 0\ 2\ 1).$$

We must again reciprocate to examine the cubic and sextic of the tables.

$$\therefore (P_3')' = P_3 = (1\ 2)(1\ 0\ 2\ 1)(1\ 2\ 0\ 1\ 1\ 2\ 1)$$

$= (5)(3\ 7)(1\ 6\ 4\ 7)$. These factors are found to be prime in the tables

$$\therefore P_3 = (x+2)(x^3 + 2x + 1)(x^6 + 2x^5 + x^3 + x^2 + 2x + 1)$$

$$= \text{Linear} \times \text{Cubic} \times \text{Sextic}, \text{— all prime.}$$

Comparing P_2 and P_3 we deduce that the only possible factor pattern for P is Sextic × Quartic,—both prime. (Since, by synthetic division, P has no linear factor.)

Next form

$$P_5 = 1\ 4\ 3\ 1\ 0\ 1\ 0\ 2\ 3\ 0\ 0$$

$$= 1\ 0\ 0(1\ 4\ 3\ 1\ 0\ 1\ 0\ 2\ 3).$$

Examining the octic factor, in the tables for

$$F_5 = 1\ 4\ 3\ 1\ 0\ 1\ 0\ 2\ 3.$$

Reciprocating,

$$F_5' = 1\ 4\ 0\ 2\ 0\ 2\ 1\ 3\ 2$$

$$= (1\ 3)(1\ 1\ 2\ 1\ 2\ 1\ 3\ 4)$$

$$= (1\ 3)(1\ 4)(1\ 2\ 4\ 0\ 2\ 3\ 1)$$

$$= (1\ 3)(1\ 4)(1\ 1\ 0\ 1)(1\ 1\ 3\ 1)$$

$$\therefore F_5 = (1\ 2)(1\ 4)(1\ 0\ 1\ 1)(1\ 3\ 1\ 1), \text{ all prime}$$

$$\therefore P_5 = 1\ 0\ 0\ (1\ 2)(1\ 4)(1\ 0\ 1\ 1)(1\ 3\ 1\ 1).$$

Combining the factors of P_2, P_3, and P_5 into a sextic $\times$ quartic pattern:

$$P_2=(1\ 0\ 1\ 1\ 0\ 0\ 1)(1\ 1\ 1\ 1\ 1) \tag{1}$$

$$P_3=(1\ 2\ 0\ 1\ 1\ 2\ 1)(1\ 2\ 2\ 2\ 2) \tag{2}$$

$$P_{5a}=(1\ 0\ 2\ 3\ 2\ 0\ 0)(1\ 4\ 1\ 0\ 4) \tag{3}$$

$$P_{5b}=(1\ 2\ 3\ 0\ 4\ 0\ 0)(1\ 2\ 1\ 3\ 2) \tag{4}$$

$$P_{5c}=(1\ 4\ 2\ 1\ 4\ 3\ 0)(1\ 0\ 1\ 1\ 0) \tag{5}$$

$$P_{5d}=(1\ 2\ 1\ 3\ 2\ 0\ 0)(1\ 2\ 3\ 0\ 4) \tag{6}$$

$$P_{5e}=(1\ 4\ 1\ 0\ 4\ 0\ 0)(1\ 0\ 2\ 3\ 2) \tag{7}$$

$$P_{5f}=(1\ 1\ 4\ 2\ 4\ 3\ 0)(1\ 3\ 1\ 1\ 0) \tag{8}$$

$$P_{5g}=(1\ 3\ 2\ 0\ 4\ 2\ 1)(1\ 1\ 3\ 0\ 0). \tag{9}$$

And $P=(x^6+ax^5+bx^4+cx^3+dx^2+ex+f)$

$$\times(x^4+a'x^3+b'x^2+c'x+d') \tag{10}$$

The unknown coefficients of P must satisfy the following table of residues (using (1)–(9)):

	a	b	c	d	e	f	a'	b'	c'	d'
P_2	0	1	1	0	0	1	1	1	1	1
P_3	2	0	1	1	2	1	2	2	2	2
P_{5a}	0	2	3	2	0	0	4	1	0	4
P_{5b}	2	3	0	4	0	0	2	1	3	2
P_{5c}	4	2	1	4	3	0	0	1	1	0
P_{5d}	2	1	3	2	0	0	2	3	0	4
P_{5e}	4	1	0	4	0	0	0	2	3	2
P_{5f}	1	4	2	4	3	0	3	1	1	0
P_{5g}	3	2	0	4	2	1	1	3	0	0

Multiplying out in (10) and equating to the known coefficients of P:

$$a+a'=\ 19 \tag{11}$$

$$aa'+b+b'=\ 78 \tag{12}$$

$$ab'+a'b+c+c'=-89 \tag{13}$$

$$ac'+a'c+bb'+d+d'=\ 15 \tag{14}$$

$$ad'+a'd+bc'+b'c+e=\ 106 \tag{15}$$

$$a'e+bd'+b'd+cc'+f=-45 \tag{16}$$

$$a'f+b'e+cd'+c'd=-8 \tag{17}$$

$$b'f+c'e+dd'=\ 23 \tag{18}$$

$$c'f+d'e=\ 15 \tag{19}$$

$$d'f=\ \ 5 \tag{20}$$

Using the residue table and eq (20), we have $(d', f)=(5, 1)$, or $(d', f)=(-1, -5)$,—two possibilities. The first uses P_{5g}, rejecting the other P_5's. The second uses P_{5a} or P_{5d} only. Trying the first solution, then from eq (19) we find $(c', e)=(5, 2)$. Thence (18) yields $(b', d)=(-7, 4)$. Next (17) yields $(a', c)=(11, -5)$. Finally (11) gives $a=8$, and (12) gives $b=-3$. All of these values check in the remaining equations.

$$\therefore P=(x^6+8x^5-3x^4-5x^3+4x^2+2x+1)$$
$$(x^4+11x^3-7x^2+5x+5).$$

Instead, if we had pursued the wrong possibility $(d', f)=(-1, -5)$, we would soon encounter unduly large coefficient values and ultimately reach an outright contradiction. For instance, (19) would give $(c', e)=(5, -40)$; then (18) two sets of values: $(b', d)=(-37, -38)$ based on P_{5d}, and $(b', d)=(-49, 22)$ based on P_{5a}. Then in (17) the values become outlandishly large, and ennui overcomes valor ere the final kill is attained. One should suspect such wrong choices early and thus avoid lost motion.

6. Other Long-Range Values of the Factor Tables

Further values of the factor tables for the pure mathematician might include a study of factor patterns and coefficient sequences. The density and distribution of irreducible polynomials should also be of interest. Much work has been done in this area for the rational integers, but relatively little for polynomials.

In constructing and analyzing Galois Fields, it is necessary in the decomposition of the defining polynomial to know the manner in which polynomials are reducible. The order of the elements of the field and the determination of primitive elements follows therefrom. Expressing the factors merely in terms of their degree is adequate for some purposes, but the exhibition of the actual factors throws additional light on the structure of the field. The understanding of Galois Fields is thus considerably enhanced by this approach.

A recent instance of the application of congruential polynomials is suggested in the work of W. Wesley Peterson on "Error-Correcting Codes" [15]. He deals with error-detecting and error-correcting codes which possess an algebraic structure. He uses the GF(2) irreducible polynomials in his theory and implementation. He stresses mainly binary codes, since nearly all present-day machines are binary, though there is no purely theoretical basis for this restriction. Undoubtedly the work can and will be carried still further in the future.

Polynomials, Modulo 2

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
2	5	3	3	7	225	3	163	8	463	13	51
2	7	....		7	227	7	61	8	465	7	147
3	11	3	7	7	231	3	167	8	467	3	355
3	13	....		7	233	15	37	8	471	....	
3	17	3	5	7	235	....		8	473	3	351
4	21	3	17	7	237	3	165	8	475	3	353
4	23	....		7	243	3	141	8	477	....	
4	25	7	7	7	245	3	143	8	503	3	301
4	27	3	15	7	247	....		8	505	3	303
4	33	3	11	7	253	....		8	507	13	45
4	37	....		7	255	13	23	8	513	7	1[illegible]5
5	41	3	37	7	257	3	145	8	515	....	
5	43	7	15	7	263	7	75	8	517	3	305
5	45	....		7	267	3	155	8	523	15	67
5	47	3	35	7	273	3	151	8	525	37	37
5	53	3	31	7	275	3	153	8	527	3	315
5	55	3	33	7	277	....		8	533	3	311
5	57	....		7	303	3	101	8	535	3	313
5	63	3	21	7	307	15	23	8	537	....	
5	67	....		7	313	....		8	543	....	
5	77	3	25	7	317	3	105	8	547	3	335
6	101	3	77	7	327	3	115	8	553	3	331
6	103	....		7	333	3	111	8	555	3	333
6	105	13	13	7	337	7	51	8	557	7	171
6	107	3	75	7	347	3	135	8	563	3	321
6	111	....		7	357	....		8	567	....	
6	113	3	71	7	377	3	125	8	573	....	
6	115	3	73	8	401	3	377	8	575	7	177
6	117	7	31	8	403	7	155	8	577	3	325
6	123	3	61	8	405	23	23	8	603	3	201
6	125	3	63	8	407	3	375	8	607	....	
6	127	....		8	411	13	57	8	613	....	
6	133	....		8	413	3	371	8	617	3	205
6	135	7	37	8	415	3	373	8	623	7	135
6	137	3	65	8	417	15	73	8	627	3	215
6	143	3	41	8	421	7	153	8	633	3	211
6	147	....		8	423	3	361	8	637	....	
6	153	7	25	8	425	3	363	8	647	3	235
6	157	3	45	8	427	31	37	8	653	3	231
6	167	3	55	8	431	3	367	8	657	13	75
6	177	13	15	8	433	....		8	667	7	121
7	201	3	177	8	435	....		8	673	23	31
7	203	....		8	437	3	365	8	677	3	225
7	205	7	67	8	443	3	341	8	707	3	275
7	207	3	175	8	445	3	343	8	717	....	
7	211	....		8	447	7	141	8	727	....	
7	213	3	171	8	451	3	347	8	737	3	265
7	215	3	173	8	453	....		8	757	3	245
7	217	....		8	455	....		8	777	7	111
7	223	3	161	8	457	3	345				

Polynomials, Modulo 3

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
2	11	....		5	301	4	177	5	407	....	
2	12	4	5	5	302	5	144	5	408	4	102
2	14	5	5	5	304	5	145	5	414	....	
2	15	....		5	305	4	178	5	415	5	181
2	17	4	4	5	307	....		5	417	4	104
3	31	4	17	5	308	....		5	418	....	
3	32	5	14	5	311	5	148	5	424	5	185
3	34	5	15	5	312	18	47	5	425	4	108
3	35	4	18	5	314	4	171	5	427	11	47
3	37	....		5	315	....		5	428	18	51
3	38	....		5	317	11	37	5	435	4	118
3	44	4	11	5	318	4	172	5	437	5	162
3	45	....		5	321	15	58	5	444	4	111
3	47	....		5	322	4	175	5	445	18	54
3	48	4	12	5	324	....		5	447	....	
3	55	5	11	5	325	5	141	5	448	4	112
3	57	4	14	5	327	4	174	5	454	....	
4	101	15	18	5	328	11	38	5	455	....	
4	102	4	55	5	334	....		5	457	4	114
4	104	5	45	5	335	....		5	458	5	167
4	105	....		5	337	4	184	5	465	11	45
4	107	4	54	5	338	5	157	5	467	4	124
4	108	....		5	341	4	187	5	475	4	128
4	111	4	57	5	342	....		5	477	18	58
4	112	....		5	344	18	42	5	478	5	177
4	114	....		5	345	4	188	5	484	4	121
4	115	4	58	5	347	5	152	5	485	5	171
4	117	....		5	348	....		5	487	5	172
4	118	5	47	5	351	....		5	505	5	101
4	121	11	11	5	352	5	154	5	507	4	154
4	124	4	51	5	354	4	181	5	515	4	158
4	125	5	41	5	355	15	51	5	517	15	42
4	127	5	42	5	357	....		5	525	....	
4	128	4	52	5	358	4	182	5	527	....	
4	134	4	31	5	364	4	161	5	545	....	
4	135	11	15	5	365	5	131	5	547	4	134
4	137	....		5	367	5	132	5	555	4	138
4	138	4	32	5	368	4	162	5	557	5	112
4	144	....		5	372	4	165	5	575	5	121
4	145	5	51	5	374	5	135	5	587	4	144
4	147	4	34	5	375	....		6	1001	11	121
4	148	....		5	377	4	164	6	1002	4	555
4	154	5	55	5	378	....		6	1004	5	445
4	155	4	38	5	381	4	167	6	1005	....	
4	157	18	18	5	384	....		6	1007	4	554
4	165	4	48	5	385	4	168	6	1008	....	
4	167	5	32	5	387	....		6	1011	4	557
4	175	....		5	388	5	137	6	1012	11	122
4	177	....		5	404	4	101	6	1014	....	
4	187	4	44	5	405	....		6	1015	4	558

Polynomials, Modulo 3—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
6	1017	....		6	1115	5	481	6	1224	15	175
6	1018	5	447	6	1117	5	482	6	1225	4	508
6	1021	....		6	1118	4	572	6	1227	18	148
6	1022	47	58	6	1121	....		6	1228	5	407
6	1024	4	551	6	1122	4	575	6	1234	5	415
6	1025	5	441	6	1124	5	485	6	1235	4	518
6	1027	5	442	6	1125	....		6	1237	....	
6	1028	4	552	6	1127	4	574	6	1238	18	141
6	1031	5	458	6	1128	....		6	1242	....	
6	1032	15	184	6	1134	....		6	1244	4	511
6	1034	4	531	6	1135	5	461	6	1245	....	
6	1035	....		6	1137	4	584	6	1247	....	
6	1037	....		6	1138	....		6	1248	4	512
6	1038	4	532	6	1141	4	587	6	1251	47	54
6	1041	....		6	1142	5	464	6	1254	11	114
6	1042	4	535	6	1144	5	465	6	1255	5	411
6	1044	15	185	6	1145	4	588	6	1257	4	514
6	1045	5	451	6	1147	38	38	6	1258	15	177
6	1047	4	534	6	1148	....		6	1264	....	
6	1048	....		6	1151	5	468	6	1265	15	171
6	1051	4	537	6	1152	18	137	6	1267	4	524
6	1052	5	454	6	1154	4	581	6	1268	5	427
6	1054	5	455	6	1155	11	105	6	1274	....	
6	1055	4	538	6	1157	15	162	6	1275	4	528
6	1057	....		6	1158	4	582	6	1277	5	422
6	1058	42	54	6	1164	4	561	6	1278	....	
6	1061	4	547	6	1165	....		6	1281	45	58
6	1064	....		6	1167	....		6	1284	4	521
6	1065	4	548	6	1168	4	562	6	1285	18	144
6	1067	5	432	6	1171	5	478	6	1287	11	117
6	1068	....		6	1172	4	565	6	1288	4	522
6	1071	....		6	1174	37	37	6	1304	4	301
6	1072	5	434	6	1175	....		6	1305	....	
6	1074	4	541	6	1177	4	564	6	1307	11	157
6	1075	....		6	1178	5	477	6	1308	4	302
6	1077	18	158	6	1181	4	567	6	1314	....	
6	1078	4	542	6	1184	18	132	6	1315	5	581
6	1081	5	438	6	1185	4	568	6	1317	4	304
6	1082	4	545	6	1187	5	472	6	1318	11	158
6	1084	....		6	1188	11	108	6	1324	5	585
6	1085	45	51	6	1204	4	501	6	1325	4	308
6	1087	4	544	6	1205	5	401	6	1327	54	54
6	1088	5	437	6	1207	5	402	6	1328	15	117
6	1104	42	58	6	1208	4	502	6	1334	....	
6	1105	4	578	6	1212	4	505	6	1335	4	318
6	1107	47	51	6	1214	5	405	6	1337	5	562
6	1108	5	487	6	1215	....		6	1338	51	58
6	1111	11	101	6	1217	4	504	6	1344	4	311
6	1112	37	38	6	1218	....		6	1345	....	
6	1114	4	571	6	1221	4	507	6	1347	15	112

Polynomials, Modulo 3—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
6	1348	4	312	6	1524	38	42	6	1887	4	444
6	1354	18	162	6	1525	5	541	9	47015	5	17881
6	1355	....		6	1527	4	374	9	47017	5	17882
6	1357	4	314	6	1535	....		9	47025	11	4365
6	1358	5	567	6	1537	4	384	9	47027	4	12774
6	1364	5	575	6	1545	4	388	9	47035	5	17861
6	1365	18	164	6	1547	5	552	9	47037	4	12784
6	1367	4	324	6	1548	....		9	47045	4	12788
6	1374	....		6	1554	4	381	9	47047	11	4367
6	1375	4	328	6	1555	....		9	47048	37	1355
6	1377	....		6	1557	....		9	47054	4	12781
6	1378	5	577	6	1565	5	531	9	47055	141	335
6	1384	4	321	6	1567	5	532	9	47057	164	544
6	1385	5	571	6	1575	37	42	9	47065	51	1755
6	1387	5	572	6	1577	4	364	9	47067	....	
6	1388	4	322	6	1584	11	144	9	47075	132	387
6	1405	5	501	6	1585	4	368	9	47077	4	12764
6	1407	4	354	6	1587	....		9	47078	5	17877
6	1414	5	505	6	1605	4	478	9	47084	18	5832
6	1415	4	358	6	1607	5	302	9	47085	4	12768
6	1417	11	137	6	1615	11	185	9	47087	5	17872
6	1418	....		6	1617	....		9	47105	5	17801
6	1424	4	351	6	1625	18	114	9	47107	5	17802
6	1425	54	58	6	1627	4	474	9	47115	38	1314
6	1427	....		6	1635	38	54	9	47117	4	12704
6	1428	4	352	6	1637	4	484	9	47125	4	12708
6	1435	....		6	1645	4	488	9	47127	18	5848
6	1437	....		6	1647	....		9	47135	4	12718
6	1444	....		6	1655	5	311	9	47137	....	
6	1445	....		6	1657	5	312	9	47145	15	3161
6	1447	4	334	6	1667	....		9	47147	....	
6	1448	5	517	6	1675	5	321	9	47148	4	12712
6	1454	37	47	6	1677	4	464	9	47154	....	
6	1455	4	338	6	1685	4	468	9	47155	5	17811
6	1457	5	512	6	1687	15	132	9	47157	4	12714
6	1458	....		6	1715	....		9	47165	58	1814
6	1465	4	348	6	1717	4	404	9	47167	4	12724
6	1467	15	122	6	1725	4	408	9	47175	4	12728
6	1474	4	341	6	1727	5	342	9	47177	5	17822
6	1475	5	521	6	1745	5	351	9	47178	....	
6	1477	5	522	6	1747	5	352	9	47184	4	12721
6	1478	4	342	6	1755	....		9	47185	18	5844
6	1484	5	525	6	1757	4	414	9	47187	148	348
6	1485	11	135	6	1775	4	428	9	47205	....	
6	1487	4	344	6	1777	....		9	47207	4	12754
6	1505	4	378	6	1787	38	58	9	47215	4	12758
6	1507	58	58	6	1815	4	458	9	47217	....	
6	1515	15	101	6	1827	37	51	9	47225	5	17841
6	1517	....		6	1845	....		9	47227	5	17842
6	1518	4	372	6	1857	11	177	9	47235	105	461

Polynomials, Modulo 3—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
9	47237	11	4387	10	116567	5	47532	10	116647	4	56234
9	47245	5	17851	10	116568	371	378	10	116648	5	47317
9	47247	4	12734	10	116571	....		10	116651	4	56237
9	47248	11	4388	10	116572	5	47534	10	116654	15	16545
9	47254	5	17855	10	116574	4	56141	10	116655	4	56238
9	47255	4	12738	10	116575	177	1422	10	116657	5	47312
9	47257	....		10	116577	45	5342	10	116658	....	
9	47265	4	12748	10	116578	4	56142	10	116664	18	13082
9	47267	5	17832	10	116581	5	47538	10	116665	4	56248
9	47275	171	525	10	116584	15	16515	10	116667	38	3678
9	47277	15	3172	10	116585	38	3664	10	116668	5	47327
9	47278	4	12742	10	116587	4	56144	10	116671	....	
9	47284	....		10	116588	5	47537	10	116672	....	
9	47285	....		10	116604	148	1722	10	116674	4	56241
9	47287	4	12744	10	116605	5	47301	10	116675	5	47321
9	47305	5	17601	10	116607	4	56254	10	116677	5	47322
9	47307	4	12854	10	116608	37	3775	10	116678	4	56242
9	47315	4	12858	10	116612	5	47304	10	116681	132	1812
9	47317	42	1215	10	116614	5	47305	10	116684	5	47325
9	47325	....		10	116615	4	56258	10	116685	45	5351
9	47327	51	1737	10	116617	15	16542	10	116687	4	56244
9	47335	15	3111	10	116618	....		10	116688	11	10608
9	47337	171	527	10	116621	5	47308	10	116704	5	47345
9	47345	185	571	10	116624	4	56251	10	116705	4	56278
9	47347	4	12834	10	116625	....		10	116707	18	13068
9	47348	5	17617	10	116627	....		10	116708	54	4482
10	116551	4	56137	10	116628	4	56252	10	116712	....	
10	116552	5	47554	10	116634	4	56231	10	116714	4	56271
10	116554	5	47555	10	116635	....		10	116715	....	
10	116555	4	56138	10	116637	....		10	116717	158	1678
10	116557	185	1382	10	116638	4	56232	10	116718	4	56272
10	116558	42	5754	10	116642	4	56235	10	116721	11	10611
10	116564	58	4172	10	116644	11	10604	10	116724	42	5738
10	116565	4	56148	10	116645	162	1557				

Polynomials, Modulo 5

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
2	101	12	13	3	1213	....		4	10123	....	
2	102	....		3	1214	....		4	10124	12	1302
2	103	....		3	1221	11	111	4	10131	11	1421
2	104	11	14	3	1223	....		4	10132	....	
2	111	....		3	1224	12	102	4	10133	....	
2	112	....		3	1231	12	103	7	10134	13	1203
2	113	12	14	3	1232	11	112	4	10141	....	
2	114	13	13	3	1234	14	131	4	10142	11	1422
2	121	11	11	3	1241	13	142	4	10143	12	1304
2	123	....		3	1243	11	113	4	10144	14	1121
2	124	....		3	1312	....		4	10201	12	1313
2	131	14	14	3	1314	11	124	4	10203	....	
2	132	11	12	3	1323	....		4	10204	123	133
2	141	....		3	1324	14	141	4	10211	14	1134
3	1001	11	141	3	1331	11	121	4	10212	111	142
3	1002	13	124	3	1332	12	111	4	10213	11	1433
3	1003	12	134	3	1341	....		4	10214	13	1213
3	1004	14	111	3	1414	12	122	4	10221	....	
3	1011	....		3	1423	14	102	4	10222	13	1214
3	1012	11	142	3	1441	11	131	4	10223	....	
3	1013	14	112	4	10001	102	103	4	10224	11	1434
3	1014	....		4	10002	....		4	10231	....	
3	1021	....		4	10003	....		4	10232	12	1311
3	1022	12	131	4	10004	11	1414	4	10233	....	
3	1023	11	143	4	10011	12	1343	4	10234	14	1131
3	1024	....		4	10012	13	1244	4	10241	11	1431
3	1031	13	122	4	10013	14	1112	4	10242	112	141
3	1032	....		4	10014	....		4	10243	13	1211
3	1033	....		4	10021	11	1411	4	10244	12	1312
3	1034	11	144	4	10022	14	1113	4	10301	11	1441
3	1041	12	133	4	10023	12	1344	4	10302	12	1321
3	1042	....		4	10024	....		4	10311	....	
3	1043	....		4	10031	14	1114	4	10312	11	1442
3	1044	13	123	4	10032	11	1412	4	10313	....	
3	1111	11	101	4	10033	13	1241	4	10314	12	1322
3	1112	14	123	4	10034	....		4	10321	12	1323
3	1113	....		4	10041	13	1242	4	10322	123	134
3	1114	....		4	10042	12	1341	4	10323	11	1443
3	1121	14	124	4	10043	11	1413	4	10324	14	1141
3	1122	11	102	4	10044	....		4	10331	13	1222
3	1123	12	144	4	10101	111	141	4	10332	124	133
3	1124	13	133	4	10102	....		4	10333	12	1324
3	1131	....		4	10103	11	1423	4	10334	11	1444
3	1132	13	134	4	10104	103	103	4	10341	....	
3	1134	....		4	10111	....		4	10342	14	1143
3	1141	....		4	10112	12	1301	4	10343	....	
3	1142	12	141	4	10113	13	1201	4	10344	13	1223
3	1143	....		4	10114	11	1424	4	10401	124	134
3	1144	11	104	4	10121	13	1202	4	10411	11	1401
3	1212	12	101	4	10122	....		4	10412	....	

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
4	10413	....		4	11223	12	1444	4	12114	103	123
4	10414	14	1101	4	11224	13	1333	4	12121	....	
4	10421	....		4	11231	11	1021	4	12123	....	
4	10422	11	1402	4	11232	13	1334	4	12124	14	1341
4	10423	14	1102	4	11233	14	1242	4	12131	....	
4	10424	13	1233	4	11234	....		4	12132	12	1011
4	10431	....		4	11241	134	134	4	12134	....	
4	10432	13	1234	4	11242	11	1022	4	12141	13	1442
4	10433	11	1403	4	11243	123	141	4	12142	14	1343
4	10434	12	1332	4	11244	....		4	12143	102	124
4	10441	12	1333	4	11311	123	142	4	12212	103	124
4	10442	....		4	11312	12	1401	4	12213	13	1401
4	10443	....		4	11313	11	1033	4	12214	12	1022
4	10444	11	1404	4	11314	14	1201	4	12221	11	1111
4	11011	11	1001	4	11321	....		4	12223	14	1302
4	11012	14	1223	4	11322	102	111	4	12224	....	
4	11013	....		4	11323	14	1202	4	12231	133	142
4	11014	12	1422	4	11324	11	1034	4	12232	11	1112
4	11021	12	1423	4	11331	12	1403	4	12234	13	1403
4	11022	11	1002	4	11332	14	1203	4	12241	14	1304
4	11023	....		4	11334	124	141	4	12243	11	1113
4	11024	....		4	11341	11	1031	4	12312	....	
4	11031	103	112	4	11342	....		4	12314	11	1124
4	11032	....		4	11343	12	1404	4	12321	111	111
4	11034	133	133	4	11411	....		4	12323	134	142
4	11041	....		4	11412	11	1042	4	12324	....	
4	11042	....		4	11413	12	1414	4	12331	11	1121
4	11043	13	1311	4	11414	....		4	12332	....	
4	11044	11	1004	4	11421	13	1302	4	12341	12	1033
4	11111	14	1234	4	11423	11	1043	4	12342	11	1122
4	11112	133	134	4	11424	102	112	4	12343	13	1411
4	11113	....		4	11431	14	1214	4	12412	14	1323
4	11114	....		4	11432	12	1411	4	12414	....	
4	11121	11	1011	4	11434	11	1044	4	12421	14	1324
4	11122	12	1431	4	11441	....		4	12423	12	1044
4	11123	13	1321	4	11442	13	1304	4	12424	11	1134
4	11124	....		4	11443	....		4	12431	13	1422
4	11131	13	1322	4	12012	11	1142	4	12432	111	112
4	11132	11	1012	4	12013	....		4	12441	11	1131
4	11133	....		4	12014	....		4	12443	....	
4	11134	12	1432	4	12021	....		4	13012	....	
4	11141	12	1433	4	12023	11	1143	4	13014	142	142
4	11142	....		4	12024	12	1002	4	13023	....	
4	11143	11	1013	4	12031	12	1003	4	13024	11	1234
4	11144	13	1323	4	12032	13	1434	4	13031	....	
4	11211	12	1443	4	12034	11	1144	4	13032	....	
4	11212	....		4	12041	102	123	4	13041	11	1231
4	11213	....		4	12043	12	1004	4	13112	11	1242
4	11214	11	1024	4	12112	13	1444	4	13114	13	1013
4	11221	....		4	12113	12	1014	4	13123	11	1243

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
4	13124	....		5	100033	12	13424	5	100311	102	1033
4	13131	....		5	100034	11	14144	5	100312	12	13401
4	13132	14	1403	5	100041	....		5	100313	....	
4	13141	14	1404	5	100042	....		5	100314	11	14124
4	13143	13	1011	5	100043	....		5	100321	123	1312
4	13214	111	124	5	100044	....		5	100322	13	12414
4	13223	13	1021	5	100101	142	1143	5	100323	....	
4	13224	12	1102	5	100102	....		5	100324	12	13402
4	13231	12	1103	5	100103	12	13434	5	100331	11	14121
4	13232	....		5	100104	13	12443	5	100332	141	1102
4	13241	....		5	100111	11	14101	5	100333	14	11142
4	13243	12	1104	5	100112	13	12444	5	100334	....	
4	13312	14	1423	5	100113	103	1021	5	100341	....	
4	13314	....		5	100114	....		5	100342	11	14122
4	13323	....		5	100121	14	11124	5	100343	12	13404
4	13331	141	141	5	100122	11	14102	5	100344	111	1404
4	13332	11	1212	5	100123	112	1444	5	100401	12	13413
4	13341	....		5	100124	....		5	100402	11	14132
4	13414	11	1224	5	100131	124	1304	5	100403	....	
4	13423	....		5	100132	....		5	100404	112	1442
4	13431	11	1221	5	100133	11	14103	5	100411	....	
4	13432	....		5	100134	12	13432	5	100412	103	1024
4	13441	13	1042	5	100141	12	13433	5	100413	11	14133
4	14014	11	1324	5	100142	134	1203	5	100414	14	11101
4	14023	13	1121	5	100143	....		5	100421	....	
4	14041	14	1004	5	100144	11	14104	5	100422	142	1141
4	14114	12	1222	5	100201	....		5	100423	13	12421
4	14123	....		5	100202	14	11133	5	100424	11	14134
4	14132	13	1134	5	100203	123	1311	5	100431	13	12422
4	14141	11	1331	5	100204	11	14114	5	100432	12	13411
4	14214	....		5	100211	12	13443	5	100433	....	
4	14223	11	1343	5	100212	....		5	100434	134	1201
4	14241	12	1233	5	100213	13	12401	5	100441	11	14131
4	14332	102	141	5	100214	102	1032	5	100442	....	
4	14341	112	133	5	100221	11	14111	5	100443	124	1302
4	14441	....		5	100222	....		5	100444	12	13412
5	100001	11	14141	5	100223	12	13444	5	101011	134	1214
5	100002	12	13421	5	100224	133	1213	5	101012	12	13001
5	100003	13	12431	5	100231	....		5	101013	11	14233
5	100004	14	11111	5	100232	11	14112	5	101014	124	1311
5	100011	13	12432	5	100233	111	1403	5	101021	13	12002
5	100012	11	14142	5	100234	13	12403	5	101022	....	
5	100013	14	11112	5	100241	141	1101	5	101023	....	
5	100014	12	13422	5	100242	12	13441	5	101024	11	14234
5	100021	12	13423	5	100243	11	14113	5	101031	12	13003
5	100022	14	11113	5	100244	....		5	101032	....	
5	100023	11	14143	5	100301	13	12412	5	101033	....	
5	100024	13	12433	5	100302	133	1214	5	101034	13	12003
5	100031	14	11114	5	100303	11	14123	5	101041	11	14231
5	100032	13	12434	5	100304	....		5	101042	13	12004

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	101043	12	13004	5	101322	12	13031	5	102111	12	13143
5	101044	14	11221	5	101323	14	11202	5	102112	....	
5	101101	11	14241	5	101324	13	12033	5	102113	11	14333
5	101102	14	11233	5	101331	102	1043	5	102114	....	
5	101103	....		5	101332	11	14212	5	102121	....	
5	101104	....		5	101333	141	1113	5	102122	....	
5	101111	14	11234	5	101334	12	13032	5	102123	12	13144
5	101112	11	14242	5	101341	12	13033	5	102124	11	14334
5	101113	12	13014	5	101342	142	1101	5	102131	....	
5	101114	13	12013	5	101343	11	14213	5	102132	13	12134
5	101121	111	1411	5	101344	103	1033	5	102133	14	11342
5	101122	13	12014	5	101401	....		5	102134	....	
5	101123	11	14243	5	101402	....		5	102141	11	14331
5	101124	123	1323	5	101403	11	14223	5	102142	12	13141
5	101131	112	1403	5	101404	12	13042	5	102143	134	1222
5	101132	12	13011	5	101411	12	13043	5	102144	112	1412
5	101133	124	1312	5	101412	13	12044	5	102201	11	14341
5	101134	11	14244	5	101413	14	11212	5	102203	....	
5	101141	....		5	101414	11	14224	5	102204	13	12143
5	101142	....		5	101421	133	1222	5	102211	....	
5	101143	13	12011	5	101422	14	11213	5	102212	11	14342
5	101144	12	13012	5	101423	12	13044	5	102213	....	
5	101201	14	11244	5	101424	141	1114	5	102214	14	11301
5	101202	12	13021	5	101431	11	14221	5	102221	103	1042
5	101203	....		5	101432	134	1213	5	102222	134	1223
5	101204	....		5	101433	13	12041	5	102223	11	14343
5	101211	11	14201	5	101434	142	1102	5	102224	12	13102
5	101212	....		5	101441	13	12042	5	102231	12	13103
5	101213	....		5	101442	11	14222	5	102232	14	11303
5	101214	12	13022	5	101443	....		5	102233	13	12141
5	101221	12	13023	5	101444	....		5	102234	11	14344
5	101222	11	14202	5	102011	14	11334	5	102241	13	12142
5	101223	13	12021	5	102012	....		5	102242	....	
5	101224	14	11241	5	102013	....		5	102243	12	13104
5	101231	13	12022	5	102014	11	14324	5	102244	....	
5	101232	111	1412	5	102021	....		5	102301	12	13113
5	101233	11	14203	5	102022	12	13131	5	102302	....	
5	101234	102	1042	5	102023	13	12121	5	102304	14	11311
5	101241	103	1032	5	102024	....		5	102311	11	14301
5	101242	14	11243	5	102031	11	14321	5	102312	....	
5	101243	112	1404	5	102032	112	1411	5	102313	12	13114
5	101244	11	14204	5	102033	142	1114	5	102314	....	
5	101301	....		5	102034	12	13132	5	102321	13	12102
5	101302	....		5	102041	12	13133	5	102322	11	14302
5	101304	11	14214	5	102042	11	14322	5	102323	124	1322
5	101311	13	12032	5	102043	14	11332	5	102324	103	1043
5	101312	....		5	102044	13	12123	5	102331	14	11314
5	101313	....		5	102102	11	14332	5	102332	12	13111
5	101314	14	11201	5	102103	13	12131	5	102333	11	14303
5	101321	11	14211	5	102104	12	13142	5	102334	13	12103

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Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	102341	....		5	103133	12	13224	5	103424	12	13202
5	102342	13	12104	5	103134	13	12203	5	103431	12	13203
5	102343	....		5	103141	14	11404	5	103432	13	12234
5	102344	11	14304	5	103142	11	14422	5	103433	11	14403
5	102401	13	12112	5	103143	....		5	103434	14	11431
5	102403	14	11322	5	103144	....		5	103441	....	
5	102411	....		5	103203	12	13234	5	103442	....	
5	102412	14	11323	5	103204	14	11411	5	103443	12	13204
5	102413	....		5	103211	....		5	103444	11	14404
5	102414	12	13122	5	103212	....		5	104011	11	14001
5	102421	11	14311	5	103213	11	14433	5	104012	111	1442
5	102422	13	12114	5	103214	13	12213	5	104013	141	1143
5	102423	....		5	103221	....		5	104014	13	12313
5	102424	....		5	103222	12	13231	5	104021	....	
5	102431	....		5	103223	....		5	104022	11	14002
5	102432	11	14312	5	103224	11	14434	5	104023	12	13344
5	102433	12	13124	5	103231	14	11414	5	104024	....	
5	102434	....		5	103232	....		5	104031	....	
5	102441	142	1113	5	103233	....		5	104032	14	11003
5	102442	124	1323	5	103234	12	13232	5	104033	11	14003
5	102443	11	14313	5	103241	11	14431	5	104034	....	
5	102444	14	11321	5	103242	133	1244	5	104041	14	11004
5	103011	....		5	103243	13	12211	5	104042	12	13341
5	103012	13	12244	5	103244	141	1134	5	104043	13	12311
5	103013	12	13214	5	103301	11	14441	5	104044	11	14004
5	103014	....		5	103302	13	12224	5	104104	11	14014
5	103021	11	14411	5	103311	12	13243	5	104111	....	
5	103022	....		5	103312	11	14442	5	104112	12	13301
5	103023	....		5	103313	....		5	104113	14	11012
5	103024	14	11441	5	103314	....		5	104114	....	
5	103031	133	1242	5	103321	14	11424	5	104121	11	14011
5	103032	11	14412	5	103322	....		5	104122	14	11013
5	103033	13	12241	5	103323	11	14443	5	104123	13	12321
5	103034	123	1343	5	103324	....		5	104124	12	13302
5	103041	13	12242	5	103331	13	12222	5	104131	12	13303
5	103042	14	11443	5	103332	....		5	104132	11	14012
5	103043	11	14413	5	103333	....		5	104133	103	1011
5	103044	12	13212	5	103334	11	14444	5	104134	124	1341
5	103102	12	13221	5	103341	111	1431	5	104141	123	1302
5	103104	....		5	103342	12	13241	5	104142	102	1021
5	103111	....		5	103343	123	1341	5	104143	11	14013
5	103112	....		5	103344	13	12223	5	104144	13	12323
5	103113	13	12201	5	103401	....		5	104203	11	14023
5	103114	11	14424	5	103411	11	14401	5	104211	13	12332
5	103121	12	13223	5	103412	12	13201	5	104212	14	11023
5	103122	102	1011	5	103413	....		5	104213	12	13314
5	103123	14	11402	5	103414	....		5	104214	11	14024
5	103124	111	1434	5	103421	141	1131	5	104221	14	11024
5	103131	11	14421	5	103422	11	14402	5	104222	112	1431
5	103132	14	11403	5	103423	102	1014	5	104223	134	1242

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	104224	13	12333	5	110031	13	13122	5	110334	13	13103
5	104231	11	14021	5	110032	12	14211	5	110341	11	10031
5	104232	12	13311	5	110034	112	1032	5	110342	12	14241
5	104233	133	1201	5	110041	....		5	110343	....	
5	104234	111	1444	5	110042	142	1201	5	110344	124	1431
5	104241	....		5	110043	134	1322	5	110411	....	
5	104242	11	14022	5	110044	11	10004	5	110412	11	10042
5	104243	....		5	110111	13	13132	5	110413	14	12212
5	104244	12	13312	5	110112	111	1042	5	110414	13	13113
5	104311	14	11034	5	110113	102	1134	5	110421	....	
5	104312	13	12344	5	110114	12	14222	5	110422	13	13114
5	104313	11	14033	5	110121	11	10011	5	110423	11	10043
5	104314	12	13322	5	110122	134	1323	5	110424	12	14202
5	104321	12	13323	5	110123	....		5	110431	12	14203
5	104322	124	1343	5	110124	13	13133	5	110432	....	
5	104323	142	1134	5	110131	....		5	110433	141	1213
5	104324	11	14034	5	110132	11	10012	5	110434	11	10044
5	104331	141	1141	5	110133	12	14224	5	110441	....	
5	104332	123	1304	5	110134	14	12231	5	110442	....	
5	104333	12	13324	5	110141	112	1033	5	110443	12	14204
5	104334	14	11031	5	110142	....		5	110444	....	
5	104341	11	14031	5	110143	11	10013	5	111012	11	10142
5	104342	....		5	110144	....		5	111013	....	
5	104343	14	11032	5	110211	123	1442	5	111014	123	1403
5	104344	....		5	110212	13	13144	5	111021	....	
5	104401	11	14041	5	110213	....		5	111023	11	10143
5	104411	....		5	110214	11	10024	5	111024	....	
5	104412	11	14042	5	110221	142	1203	5	111031	102	1143
5	104413	13	12301	5	110222	12	14231	5	111032	....	
5	104414	....		5	110223	111	1043	5	111034	11	10144
5	104421	13	12302	5	110224	14	12241	5	111041	13	13242
5	104422	12	13331	5	110231	11	10021	5	111042	12	14341
5	104423	11	14043	5	110232	....		5	111043	14	12332
5	104424	14	11041	5	110233	13	13141	5	111111	11	10101
5	104431	134	1244	5	110234	12	14232	5	111112	12	14301
5	104432	103	1014	5	110241	12	14233	5	111113	13	13201
5	104433	14	11042	5	110242	11	10022	5	111114	....	
5	104434	11	14044	5	110243	....		5	111121	13	13202
5	104441	12	13333	5	110244	....		5	111122	11	10102
5	104442	13	12304	5	110311	12	14243	5	111123	....	
5	104443	102	1024	5	110312	102	1131	5	111124	12	14302
5	104444	133	1203	5	110313	11	10033	5	111131	12	14303
5	110011	11	10001	5	110314	14	12201	5	111132	123	1404
5	110012	14	12223	5	110321	13	13102	5	111133	11	10103
5	110013	12	14214	5	110322	....		5	111134	13	13203
5	110014	....		5	110323	12	14244	5	111141	142	1213
5	110021	14	12224	5	110324	11	10034	5	111142	13	13204
5	110022	11	10002	5	110331	....		5	111143	12	14304
5	110023	13	13121	5	110332	14	12203	5	111144	11	10104
5	110024	141	1214	5	110333	....		5	111211	112	1043

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Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	111212	....		5	112023	....		5	112342	....	
5	111213	12	14314	5	112024	11	10234	5	112343	11	10213
5	111214	13	13213	5	112031	124	1404	5	112344	14	12421
5	111221	11	10111	5	112032	....		5	112411	14	12434
5	111222	13	13214	5	112034	....		5	112412	124	1403
5	111223	14	12302	5	112041	11	10231	5	112413	12	14414
5	111224	....		5	112042	14	12443	5	112414	11	10224
5	111231	....		5	112043	13	13311	5	112421	13	13302
5	111232	11	10112	5	112112	11	10242	5	112423	103	1141
5	111233	142	1214	5	112113	....		5	112424	142	1222
5	111234	....		5	112114	14	12401	5	112431	11	10221
5	111241	14	12304	5	112121	123	1412	5	112432	12	14411
5	111242	103	1134	5	112122	12	14431	5	112433	....	
5	111243	11	10113	5	112123	11	10243	5	112434	13	13303
5	111244	12	14312	5	112124	103	1143	5	112441	....	
5	111311	....		5	112131	13	13322	5	112442	11	10222
5	111312	....		5	112132	14	12403	5	112443	14	12432
5	111313	14	12312	5	112133	....		5	113012	12	14001
5	111314	11	10124	5	112134	11	10244	5	113013	112	1014
5	111321	12	14323	5	112141	12	14433	5	113014	11	10324
5	111322	14	12313	5	112142	....		5	113021	102	1113
5	111323	13	13221	5	112143	....		5	113023	14	12002
5	111324	....		5	112144	13	13323	5	113024	12	14002
5	111331	11	10121	5	112211	11	10201	5	113031	11	10321
5	111332	102	1141	5	112212	....		5	113032	13	13434
5	111334	....		5	112213	14	12412	5	113034	....	
5	111341	124	1444	5	112214	....		5	113041	14	12004
5	111342	11	10122	5	112221	111	1011	5	113042	11	10322
5	111343	141	1223	5	112223	12	14444	5	113043	12	14004
5	111344	13	13223	5	112224	13	13333	5	113112	13	13444
5	111411	13	13232	5	112231	14	12414	5	113113	11	10333
5	111412	14	12323	5	112232	13	13334	5	113114	111	1024
5	111413	11	10133	5	112233	11	10203	5	113121	134	1304
5	111414	133	1343	5	112234	....		5	113122	14	12013
5	111421	14	12324	5	112241	....		5	113123	102	1114
5	111422	12	14331	5	112242	12	14441	5	113124	11	10334
5	111423	....		5	112243	....		5	113131	14	12014
5	111424	11	10134	5	112244	11	10204	5	113132	12	14011
5	111431	....		5	112311	....		5	113134	....	
5	111432	13	13234	5	112312	12	14401	5	113141	11	10331
5	111433	....		5	112313	....		5	113142	....	
5	111434	12	14332	5	112314	....		5	113143	....	
5	111441	11	10131	5	112321	11	10211	5	113144	12	14012
5	111442	....		5	112322	133	1304	5	113212	11	10342
5	111443	103	1131	5	112323	....		5	113213	13	13401
5	111444	14	12321	5	112324	12	14402	5	113214	12	14022
5	112012	....		5	112331	12	14403	5	113221	12	14023
5	112013	11	10233	5	112332	11	10212	5	113223	11	10343
5	112014	12	14422	5	112334	....		5	113224	....	
5	112021	12	14423	5	112341	13	13342	5	113231	....	

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	113232	112	1011	5	114123	12	14144	5	120021	11	11411
5	113233	12	14024	5	114124	134	1311	5	120023	134	1442
5	113234	11	10344	5	114131	11	10421	5	120024	12	10002
5	113241	....		5	114132	....		5	120031	12	10003
5	113242	13	13404	5	114134	103	1113	5	120032	11	11412
5	113243	....		5	114141	....		5	120034	14	13331
5	113244	14	12021	5	114142	11	10422	5	120041	111	1131
5	113311	11	10301	5	114143	13	13011	5	120043	11	11413
5	113312	....		5	114144	14	12121	5	120112	142	1311
5	113313	133	1311	5	114212	12	14101	5	120113	12	10014
5	113314	13	13413	5	114213	11	10433	5	120114	11	11424
5	113321	....		5	114214	142	1242	5	120121	124	1014
5	113322	11	10302	5	114221	133	1322	5	120123	13	14321
5	113323	....		5	114223	13	13021	5	120124	14	13341
5	113324	....		5	114224	11	10434	5	120131	11	11421
5	113331	111	1021	5	114231	12	14103	5	120132	12	10011
5	113332	....		5	114232	103	1114	5	120134	....	
5	113334	12	14032	5	114233	....		5	120141	....	
5	113341	12	14033	5	114234	14	12131	5	120142	11	11422
5	113342	....		5	114241	11	10431	5	120143	....	
5	113343	13	13411	5	114242	....		5	120212	....	
5	113411	12	14043	5	114243	12	14104	5	120213	11	11433
5	113412	....		5	114312	11	10442	5	120214	12	10022
5	113413	134	1302	5	114313	12	14114	5	120221	12	10023
5	113414	124	1411	5	114314	....		5	120223	14	13302
5	113421	11	10311	5	114321	....		5	120224	11	11434
5	113423	12	14044	5	114323	11	10443	5	120231	134	1444
5	113424	14	12041	5	114324	13	13033	5	120232	13	14334
5	113431	13	13422	5	114331	....		5	120234	....	
5	113432	11	10312	5	114332	12	14111	5	120241	11	11431
5	113434	....		5	114334	11	10444	5	120242	....	
5	113441	133	1312	5	114341	141	1201	5	120243	....	
5	113442	12	14041	5	114342	14	12143	5	120312	11	11442
5	113443	11	10313	5	114343	....		5	120313	14	13312
5	114012	....		5	114411	11	10401	5	120314	124	1011
5	114013	13	13001	5	114412	13	13044	5	120321	....	
5	114014	....		5	114413	123	1431	5	120323	11	11443
5	114021	11	10411	5	114414	12	14122	5	120324	111	1134
5	114023	141	1203	5	114421	12	14123	5	120331	14	13314
5	114024	....		5	114423	14	12102	5	120332	....	
5	114031	14	12114	5	114424	....		5	120334	11	11444
5	114032	11	10412	5	114431	....		5	120341	12	10033
5	114034	12	14132	5	114432	14	12103	5	120342	123	1024
5	114041	12	14133	5	114434	....		5	120343	....	
5	114042	13	13004	5	114441	13	13042	5	120412	14	13323
5	114043	11	10413	5	114442	....		5	120413	13	14301
5	114112	14	12123	5	114443	142	1244	5	120414	103	1223
5	114113	111	1033	5	120012	12	10001	5	120421	13	14302
5	114114	11	10424	5	120013	....		5	120423	12	10044
5	114121	14	12124	5	120014	13	14313	5	120424	....	

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	120431	....		5	121413	....		5	122342	11	11122
5	120432	....		5	121414	12	10122	5	122343	14	13032
5	120434	13	14303	5	121421	12	10123	5	122412	12	10201
5	120441	....		5	121423	11	11043	5	122413	11	11133
5	120442	12	10041	5	121424	....		5	122414	....	
5	120443	141	1323	5	121431	13	14422	5	122421	....	
5	121012	....		5	121432	....		5	122423	....	
5	121014	....		5	121434	11	11044	5	122424	11	11134
5	121023	....		5	121441	....		5	122431	12	10203
5	121024	13	14433	5	121442	134	1403	5	122432	124	1033
5	121031	....		5	121443	14	13432	5	122441	11	11131
5	121032	13	14434	5	122012	11	11142	5	122443	12	10204
5	121041	12	10133	5	122014	14	13001	5	123012	133	1434
5	121043	....		5	122023	11	11143	5	123014	....	
5	121112	13	14444	5	122024	134	1411	5	123023	12	10344
5	121113	133	1411	5	122031	141	1341	5	123024	11	11234
5	121114	14	13401	5	122032	12	10211	5	123031	13	14122
5	121121	11	11011	5	122041	14	13004	5	123032	141	1302
5	121123	12	10144	5	122043	....		5	123041	11	11231
5	121124	142	1322	5	122112	....		5	123043	111	1113
5	121131	....		5	122113	14	13012	5	123112	11	11242
5	121132	11	11012	5	122114	12	10222	5	123114	....	
5	121134	102	1242	5	122123	....		5	123123	11	11243
5	121141	13	14442	5	122124	....		5	123124	12	10302
5	121142	12	10141	5	122131	14	13014	5	123131	12	10303
5	121143	11	11013	5	122132	....		5	123132	13	14134
5	121212	12	10101	5	122134	123	1043	5	123134	11	11244
5	121213	13	14401	5	122141	....		5	123141	....	
5	121214	11	11024	5	122142	....		5	123142	....	
5	121221	13	14402	5	122143	13	14011	5	123143	12	10304
5	121223	....		5	122212	14	13023	5	123212	13	14144
5	121224	12	10102	5	122213	141	1343	5	123213	12	10314
5	121231	11	11021	5	122214	....		5	123214	141	1304
5	121232	....		5	122221	11	11111	5	123223	133	1431
5	121234	13	14403	5	122223	13	14021	5	123224	....	
5	121241	133	1412	5	122224	....		5	123231	....	
5	121242	11	11022	5	122231	13	14022	5	123232	12	10311
5	121243	12	10104	5	122232	11	11112	5	123234	14	13131
5	121312	14	13423	5	122234	12	10232	5	123241	13	14142
5	121314	13	14413	5	122241	12	10233	5	123243	14	13132
5	121321	14	13424	5	122243	11	11113	5	123312	112	1101
5	121323	111	1143	5	122312	....		5	123314	12	10322
5	121324	11	11034	5	122314	11	11124	5	123321	11	11211
5	121331	123	1032	5	122321	103	1242	5	123323	102	1214
5	121332	12	10111	5	122323	12	10244	5	123324	14	13141
5	121334	....		5	122324	13	14033	5	123331	....	
5	121341	11	11031	5	122331	11	11121	5	123332	11	11212
5	121342	....		5	122332	13	14034	5	123341	....	
5	121343	13	14411	5	122334	14	13031	5	123342	13	14104
5	121412	11	11042	5	122341	....		5	123343	11	11213

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	123412	....		5	124441	134	1434	5	131132	13	10134
5	123414	11	11224	5	124443	11	11313	5	131141	12	11433
5	123421	....		5	130012	....		5	131143	124	1102
5	123423	14	13102	5	130014	11	12324	5	131212	13	10144
5	123424	112	1102	5	130023	12	11344	5	131214	134	1021
5	123431	11	11221	5	130024	14	14441	5	131223	12	11444
5	123432	14	13103	5	130031	11	12321	5	131224	11	12434
5	123441	12	10333	5	130032	133	1024	5	131231	....	
5	123443	13	14111	5	130041	123	1102	5	131232	102	1341
5	124012	13	14244	5	130112	12	11301	5	131241	11	12431
5	124014	11	11324	5	130114	13	10013	5	131243	....	
5	124023	....		5	130123	14	14402	5	131312	11	12442
5	124024	....		5	130124	11	12334	5	131314	112	1222
5	124031	11	11321	5	130131	12	11303	5	131323	11	12443
5	124032	103	1214	5	130132	14	14403	5	131324	12	11402
5	124041	13	14242	5	130141	11	12331	5	131331	12	11403
5	124043	....		5	130143	12	11304	5	131332	....	
5	124112	123	1014	5	130212	11	12342	5	131341	....	
5	124114	....		5	130214	103	1323	5	131412	123	1114
5	124123	....		5	130223	11	12343	5	131414	13	10113
5	124124	11	11334	5	130224	....		5	131423	142	1434
5	124131	133	1442	5	130231	13	10022	5	131424	14	14041
5	124132	....		5	130232	12	11311	5	131431	102	1343
5	124134	12	10432	5	130241	....		5	131432	12	11411
5	124141	11	11331	5	130243	133	1021	5	131441	....	
5	124143	14	13232	5	130312	14	14423	5	131443	13	10111
5	124212	11	11342	5	130314	12	11322	5	132014	133	1043
5	124213	112	1114	5	130323	....		5	132023	103	1341
5	124214	13	14213	5	130324	13	10033	5	132032	....	
5	124223	11	11343	5	130331	....		5	132041	13	10242
5	124224	14	13241	5	130332	13	10034	5	132112	14	14123
5	124231	....		5	130341	....		5	132114	142	1442
5	124232	....		5	130343	....		5	132123	....	
5	124241	102	1223	5	130412	13	10044	5	132124	....	
5	124243	13	14211	5	130414	....		5	132132	11	12012
5	124312	12	10401	5	130423	123	1101	5	132141	....	
5	124314	14	13201	5	130424	134	1011	5	132143	11	12013
5	124323	13	14221	5	130431	....		5	132214	11	12024
5	124324	12	10402	5	130432	11	12312	5	132223	141	1403
5	124331	12	10403	5	130441	12	11333	5	132224	103	1343
5	124332	14	13203	5	130443	11	12313	5	132231	11	12021
5	124341	14	13204	5	131014	12	11422	5	132232	....	
5	124343	12	10404	5	131023	13	10121	5	132241	....	
5	124412	....		5	131032	11	12412	5	132243	13	10211
5	124414	....		5	131041	14	14004	5	132312	111	1242
5	124421	11	11311	5	131112	....		5	132314	141	1404
5	124423	....		5	131114	11	12424	5	132323	13	10221
5	124424	13	14233	5	131123	....		5	132324	11	12034
5	124431	14	13214	5	131124	13	10133	5	132331	13	10222
5	124432	11	11312	5	131131	11	12421	5	132332	....	

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
5	132341	11	12031	5	134332	11	12212	5	142423	13	11421
5	132414	14	14101	5	134341	....		5	142441	12	12333
5	132423	11	12043	5	134414	11	12224	5	143114	13	11013
5	132431	133	1042	5	134423	13	10421	5	143132	11	13012
5	132432	13	10234	5	134431	11	12221	5	143214	11	13024
5	132441	14	14104	5	134432	....		5	143223	12	12444
5	132443	....		5	134441	102	1323	5	143332	13	11034
5	133014	13	10313	5	140014	14	10001	5	143341	11	13031
5	133023	11	12143	5	140023	14	10002	5	143441	13	11042
5	133032	....		5	140041	11	13231	5	144114	14	10401
5	133041	12	11133	5	140114	....		5	144223	....	
5	133112	....		5	140123	11	13243	5	144441	11	13131
5	133114	....		5	140132	111	1312	6	1234132	13	141134
5	133123	12	11144	5	140141	....		6	1234134	11	112244
5	133124	....		5	140214	13	11213	6	1234141	....	
5	133132	....		5	140223	....		6	1234143	12	103304
5	133141	....		5	140232	....		6	1234212	13	141144
5	133214	....		5	140241	134	1134	6	1234213	12	103314
5	133223	112	1244	5	140314	133	1143	6	1234214	111	11124
5	133224	12	11102	5	140323	13	11221	6	1234223	....	
5	133232	11	12112	5	140332	11	13212	6	1234224	1322	1442
5	133241	....		5	140341	....		6	1234231	1312	1403
5	133243	11	12113	5	140414	11	13224	6	1234232	12	103311
5	133312	13	10344	5	140423	124	1222	6	1234241	13	141142
5	133314	11	12124	5	140432	13	11234	6	1234243	1304	1412
5	133323	14	14202	5	140441	....		6	1234312	....	
5	133331	11	12121	5	141114	12	12222	6	1234314	12	103322
5	133332	12	11111	5	141123	13	11321	6	1234321	11	112211
5	133341	13	10342	5	141132	....		6	1234323	....	
5	133414	12	11122	5	141141	11	13331	6	1234324	141	13014
5	133423	134	1042	5	141214	....		6	1234331	....	
5	133431	14	14214	5	141223	11	13343	6	1234332	11	112212
5	133432	....		5	141232	13	11334	6	1234341	....	
5	133441	11	12131	5	141241	12	12233	6	1234342	13	141104
5	134014	....		5	141314	142	1042	6	1234343	11	112213
5	134023	....		5	141332	112	1311	6	1234412	....	
5	134032	12	11211	5	141341	13	11342	6	1234414	11	112224
5	134041	11	12231	5	141414	14	10101	6	1234421	1131	1141
5	134114	12	11222	5	141423	14	10102	6	1234423	133	14331
5	134123	11	12243	5	141432	11	13312	6	1234424	14	131041
5	134124	14	14341	5	141441	14	10104	6	1234431	11	112221
5	134132	....		5	142114	11	13424	6	1234432	1032	1201
5	134141	13	10442	5	142123	....		6	1234441	12	103333
5	134214	14	14301	5	142132	....		6	1234443	13	141111
5	134223	14	14302	5	142214	....		6	1240012	14	132223
5	134232	14	14303	5	142223	123	1201	6	1240014	....	
5	134241	12	11233	5	142241	11	13431	6	1240023	....	
5	134243	123	1141	5	142314	12	12322	6	1240024	11	113234
5	134314	13	10413	5	142332	14	10203	6	1240031	111	11221
5	134323	12	11244	5	142341	14	10204	6	1240032	12	104211

Polynomials, Modulo 5—*Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
6	1240041	11	113231	7	11211341	12	1443033	7	11212221	111	101011
6	1240043	142	13044	7	11211342	14	1240143	7	11212223	11	1024343
6	1240112	11	113242	7	11211343	11	1024213	7	11212224	12	1443102
6	1240114	12	104222	7	11211344	1113	10123	7	11212231	12	1443103
6	1240123	11	113243	7	11211411	12	1443043	7	11212232	142	122421
6	1240124	102	12212	7	11211412	13	1332044	7	11212233	13	1332141
6	1240131	123	10132	7	11211413	....		7	11212234	11	1024344
6	1240132	1311	1412	7	11211414	11	1024224	7	11212241	13	1332142
6	1240134	11	113244	7	11211421	....		7	11212242	14	1240243
6	1240141	13	142442	7	11211422	102	110411	7	11212243	12	1443104
6	1240143	14	132232	7	11211423	12	1443044	7	11212244	....	
6	1240212	....		7	11211424	112	100102	7	11212311	11	1024301
6	1240213	13	142401	7	11211431	11	1024221	8	114313414	134	1313321
6	1240214	....		7	11211432	14	1240103	8	114313421	14	12140324
6	1240223	1024	1222	7	11211433	13	1332041	8	114313423	12	14114044
6	1240224	14	132241	7	11211434	....		8	114313424	11	10442134
6	1240231	1101	1131	7	11211441	13	1332042	8	114313431	141	1201241
6	1240232	1312	1411	7	11211442	11	1024222	8	114313432	13	13032234
6	1240241	12	104233	7	11211443	1444	12042	8	114313434	1032	111312
6	1240243	124	10002	7	11212012	14	1240223	8	114313441	11	10442131
6	1240312	124	10003	7	11212013	133	130211	8	114313442	12	14114041
6	1240314	13	142413	7	11212014	11	1024324	8	114313443	....	
6	1240323	12	104244	7	11212021	14	1240224	8	114314012	....	
6	1240324	133	14413	7	11212023	13	1332121	8	114314013	11	10442233
6	1240331	1141	1141	7	11212024	141	123214	8	114314014	13	13032313
6	1240332	11	113212	7	11212031	11	1024321	8	114314021	111	1030311
6	1240341	14	132204	7	11212032	123	141204	8	114314023	103	1110341
6	1240343	11	113213	7	11212034	12	1443132	8	114314024	11	10442234
6	1240412	12	104201	7	11212041	12	1443133	8	114314031	....	
6	1240414	11	113224	7	11212042	11	1024322	8	114314032	123	1430204
6	1240421	141	13131	7	11212043	....		8	114314034	12	14114132
6	1240423	13	142421	7	11212112	1223	14214	8	114314041	11	10442231
6	1240424	12	104202	7	11212113	11	1024333	8	114314042	14	12140443
6	1240431	11	113221	7	11212114	....		8	114314043	13	13032311
6	1240432	1323	1404	7	11212121	1042	11303	8	114314112	11	10442242
6	1240441	....		7	11212122	....		8	114314113	124	1423232
6	1240443	12	104204	7	11212123	12	1443144	8	114314114	14	12140401
7	11211243	14	1240132	7	11212124	11	1024334	8	114314121	103	1110342
7	11211244	11	1024204	7	11212131	....		8	114314123	11	10442243
7	11211311	13	1332032	7	11212132	13	1332134	8	114314124	....	
7	11211312	112	100101	7	11212133	....		8	114314131	13	13032322
7	11211313	....		7	11212134	14	1240231	8	114314132	14	12140403
7	11211314	124	140011	7	11212141	11	1024331	8	114314134	11	10442244
7	11211321	11	1024211	7	11212142	12	1443141	8	114314141	14	12140404
7	11211322	12	1443031	7	11212143	14	1240232	8	114314142	12	14114141
7	11211323	....		7	11212144	....		8	114314143	1244	142442
7	11211324	13	1332033	7	11212211	1033	11402	8	114314212	12	14114101
7	11211331	1312	13203	7	11212212	11	1024342	8	114314213	14	12140412
7	11211332	11	1024212	7	11212213	112	100114	8	114314214	10123	11303
7	11211334	12	1443032	7	11212214	....		8	114314221	1341	131231

Polynomials, Modulo 5 – *Continued*

Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot	Deg	Poly	Fact	Quot
8	114314223	123	1430201	8	140144232	1222	124231	8	140201323	....	
8	114314224	12	14114102	8	140144241	11	13240431	8	140201332	12	12100111
8	114314231	12	14114103	8	140144314	102	1433332	8	140201341	1322	110343
8	114314232	13	13032334	8	140144323	11	13240443	8	140201414	12	12100122
8	114314233	11	10442203	8	140144332	....		8	140201423	124	1220222
8	114314234	....		8	140144341	....		8	140201432	....	
8	114314241	102	1121223	8	140144414	....		8	140201441	11	13200131
8	114314242	10102	11321	8	140144423	1032	142204	8	140202014	103	1420423
8	114314243	12	14114104	8	140144432	13	11204234	8	140202023	13	11212121
8	114314312	13	13032344	8	140144441	....		8	140202041	11	13200231
8	114314313	12	14114114	8	140200114	124	1220211	8	140202114	12	12100222
8	114314314	....		8	140200123	112	1301444	8	140202123	11	13200243
8	114314321	11	10442211	8	140200132	11	13200012	8	140202132	13	11212134
8	114314323	....		8	140200141	13	11212442	8	140202141	14	10022404
8	114314324	1201	141024	8	140200214	11	13200024	8	140202214	12021	12134
8	114314331	1322	133113	8	140200223	....		8	140202223	12312	12344
8	114314332	11	10442212	8	140200232	....		8	140202232	123	1230104
8	114314334	1134	101301	8	140200241	1214	120104	8	140202241	12	12100233
8	114314341	13	13032342	8	140200314	13	11212413	8	140202314	....	
8	114314342	....		8	140200323	14	10022202	8	140202323	12	12100244
8	114314343	11	10442213	8	140200332	14	10022203	8	140202332	11	13200212
8	114314411	14	12140434	8	140200341	11	13200031	8	140202341	....	
8	114314412	134	1313333	8	140200414	1011	144224	8	140202414	11	13200224
8	114314413	13	13032301	8	140200423	11	13200043	8	140202423	123	1230101
8	114314414	11	10442224	8	140200432	111	1313112	8	140202432	102	1434441
8	114314421	12	14114123	8	140200441	....		8	140202441	11222	13043
8	114314423	....		8	140201014	102	1434432	8	140203014	11	13200324
8	114314424	12021	14144	8	140201023	11	13200143	8	140203032	14	10022003
8	114314431	11	10442221	8	140201032	....		8	140203041	13	11212242
8	114314432	112	1021111	8	140201114	13	11212013	8	140203114	1311	111224
8	114314434	13	13032303	8	140201123	12	12100144	8	140203123	11113	13121
8	114314441	....		8	140201132	103	1420414	8	140203132	1014	144403
8	114314442	11	10442222	8	140201141	10313	14242	8	140203141	11	13200331
8	114314443	14	12140432	8	140201214	14	10022301	8	140203214	13	11212213
8	114320012	12	14110001	8	140201223	13	11212021	8	140203223	11	13200343
8	114320013	11	10443233	8	140201232	11	13200112	8	140203232	12	12100311
8	114320014	13102	13432	8	140201241	14	10022304				
8	140144223	11402	13314	8	140201314	11	13200124				

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