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This report on electromagnetics covers the contributions made by workers in the United States during the period 1960–1963. The headings of its component parts and their authors are listed below.

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Electromagnetics Introduction

K. M. Siegel*

Professor Keller was originally chosen to lead the single-body scattering and diffraction effort, but because he spent the latter part of the period in Brazil he was not able to make a contribution to this report. The final reports on rough surfaces by Ament and on coherence theory by Zucker are not yet available at this writing, but should be available at least as supplementary material at the General Assembly in Japan.

1. The Progress Report on Antennas, 1960 to 1962, has proven to be an impressive document. It points out new results obtained on log periodic antennas, on conical and multi-element spirals, and on the utilization of small antennas as elements of large arrays. It describes a significant expansion of effort in electrical scanning and array techniques, pointing out the relative decline in favor of ferrite phase shifters and frequency variation techniques for the scanning of arrays. Those devices and techniques have been replaced by the utilization of multiple beams and digital arrays which offer more versatility with little or no increase in complexity. Significant advances have been made in the utilization of nonuniform arrays which have fewer elements than uniform arrays without sacrificing the gain of the main lobe. It is quite clear that an increasing amount of future effort will go into nonuniform arrays, especially for the purpose of reducing side lobes. It is also clear that low-noise antenna systems have made significant advances.

This time period has been marked by the first significant publications in the United States literature on synthetic arrays, and the use of these arrays, especially in view of their application to ground mapping, definitely represents a major advance in antenna theory. The large buildup of information on this subject during this period, however, is deceptive since much of the work was done many years ago but is now reaching the open literature for the first time. Of course, the principle of synthetic arrays has been familiar in radio astronomy for some time, but the new application of this principle to ground mapping by airborne radar has made it possible to obtain absolute azimuth resolution without sacrificing gain. It is clear that absolute azimuth resolution will allow radar techniques to compete for the first time with more conventional techniques associated with certain portions of the optical spectrum.

2. Statistical problems in electromagnetics have received an increasing amount of theoretical attention, especially in the field of partial coherence and the construction of possible distribution functions to model rough surfaces mathematically.

The utilization of statistical methods to interpret scattering data from the moon has not as yet effected a prediction of the lunar return for any case not previously measured. Of course, with enough arbitrary parameters present in the statistical theory one can almost predict the results of all previous experiments. One of the first tests of the utility of statistical theories of radar scattering should come from experimental work on the scattering of electromagnetic waves by planets. It can be expected that the next three-year period will see a multiplication of the effort in this area by a big factor in comparison to that of the last three-year period. Critical analyses have been made which increase our understanding of the effect of turbulence and other random fluctuating media on electromagnetic fields.

New approximations have been introduced in the studies of propagation in random media and diffraction from rough surfaces. The use of impedance approximations for such problems has been found justified and improved results have been obtained. However, many of the approximate solutions involving impedance boundary conditions with different numerical techniques have in fact yielded apparently different answers, and it is hoped that during the next three years a better understanding will come into existence of which approximate numerical methods are the correct ones under various impedance boundary condition situations. It is, for example, clear that when one type of approximation is used a roughening of a surface always increases the radar cross section of the surface. Yet when a different approximation is used this is not the case for an instantaneous solution, and only if one takes an average over many different instantaneous solutions does it become the case. It is for this reason, among others, that confusion exists not only in connection with the utility of particular methods but in the arguments presented by different authors on results obtained presumable for the same models.

The work of partial coherence has been aimed primarily at the region of optical wavelengths although it is beginning to find application to other portions of the spectrum, especially in radio astronomy. It is clear that if appropriate data processing techniques are used after measurements are made both of the amplitude and the phase of signals, a significant increase in information on such quantities as the size of objects being measured and the distance between random sources will result. The theory of partial coherence should find further application in optical analog computing devices, soon to be instrumented to supplement, and, in some cases, replace digital computer techniques when large quantities of information must be processed.

3. The work on radiation has pretty well followed the predicted patterns. The theory of guided waves especially has developed as expected. Much attention has been given to leaky and surface wave phenomena. Periodic structures and structures loaded with anisotropic media have been investigated

*Conductron Corporation, Ann Arbor, Mich.

further. Military and space research requirements apparently have stimulated many investigations connected with the propagation and generation of electromagnetic waves in plasmas. Studies and applications have been made of special features involved in the propagation of waves through a plasma, such as the possibility of obtaining a dielectric constant with a negative real part. Much work has been devoted to the propagation of electromagnetic waves in lossy media because of an interest in communications between underground or underwater terminals. It is quite clear that these efforts will continue as long as people are discussing hardened sites, hardened communications systems, and as long as people are worried about nuclear attack.

4. In the field of classical diffraction and scattering, considerable efforts have been made involving passive and active reflectors and especially their applications to satellites. This is recognized in the present report by the inclusion of a paper of J. R. Burke of NASA who discusses passive communication satellites and the Project West Ford launches of 1961 and 1962. The general field of passive and active reflectors has been covered in a paper originally submitted in two separate versions by J. Kaiser of the Institute of Defense Analysis and I. Kay of Conductron Corporation. These have been combined into one paper of eliminate duplication.

The many-body problem has continued to receive attention and this work is summarized within.

In the field of diffraction and scattering we find that the usual problems are receiving ever increasing attention, and workers are obtaining new approximate solutions by old methods and old solutions by new methods. The writer predicted the last time that this report was written that there would be a considerable effort made for the resonance region during the years 1960 to 1963. However, few new results yet exist for the resonance region, and it becomes apparent that the love for the asymptotic theories is an ever consuming one. Better and more accurate answers continue to accumulate for the region we already know a considerable amount about, but people are not generating methods for use in the region where little is really known. There is a ray of hope, however, in that more people are becoming interested in obtaining values for the currents on a body, and in fact this interest has led to the investigation of a new kind of problem during the period. This problem is based on introducing an artificial boundary condition to simulate such structures as fine wire gratings: a requirement that boundary currents be unidirectional. This procedure is similar to one previously used to analyze traveling wave tubes.

The electromagnetic theory field is growing in the United States. The volume of information is increasing. The harder problems, however, are still not really being attacked; only in antenna theory can we see that really significant advances have been reported as compared with the last period.

I acknowledge with appreciation the assistance of Irvin Kay in helping me assemble and summarize section 6.3.

Part 1. Progress in Antennas 1960–1962

V. H. Rumsey,[1] R. C. Hansen,[2] and A. Ksienski [3]

Progress in the United States on antennas during the triennium 1960–62 is reported under three categories: frequency independent and small antennas; arrays and electronic scanning, near fields, aperture antennas; data processing and synthetic aperture antennas. The survey is based on the definitive papers with a bibliography of supporting literature. All important U.S. journals have been covered.

A number of new log periodic antennas have been invented consisting of TEM biaxial or coaxial lines which are loaded with monopole or dipole antennas. Conical and multielement spiral antennas have been developed giving circular polarization in all directions and an interesting variety of amplitude and phase patterns. Solutions of Maxwell's equations have been worked out for idealized spiral and periodic antenna types which show how the effective aperture is controlled by the geometry as well as predicting some unexpected and unverified effects.

A more precise and thorough investigation of the characteristics of small antennas for use as elements of large arrays has been carried out.

The ferrite phase shifter and frequency scanned arrays have declined in popularity, because multiple beam and digital arrays offer more versatility with little or no increase in complexity. Effort in the latter areas will probably expand, especially on distributed transmitter arrays, and on receiving arrays with an amplifier/processor at each element. Digital array control may be the ultimate for multiple targets and target discrimination. Nonuniform spacing of elements has come of age, but very few of the design techniques produce low sidelobes; a conspicuous exception is the space tapering approach where 20 db has been achieved. A theoretical analysis and synthesis technique is badly needed to allow synthesis of low sidelobe and other practical designs. Automatic steering systems for arraying large dishes, for automatic angle return, for atmospheric scintillation compensation, and for beaming large amounts of RF power represent one of the most exciting developments. Much work remains to be done in this area.

The analytical understanding of radiation and near fields, of interest particularly for power transfer and high power focusing, has improved markedly. Promise for millimeter, submillimeter, and laser applications is offered by the beam waveguide.

Low noise antenna systems have made significant advances, especially in hardware techniques. Use of the Gassegrain geometry and of feed shaping has produced antenna temperatures well below 20 °K. These systems will continue to be important for space and satellite communications. Several different types of large radio astronomy instruments have come to fruition: the fixed cylindrical parabola with scanning linear array feed; the standing parabola with tiltable flat plane reflector; a Christianson array of dishes, in a cross form. The unfortunate cancellation of the Sugar Grove 600-ft dish project is a loss to the science of space communications.

The last three years saw an increasing interest in unconventional techniques for satisfying the demands imposed on antenna systems. The synthetic array is an excellent example of what can be achieved if an antenna system is optimized for a particular application, i.e., ground mapping by means of a moving vehicle. This type of specialized solution will become more common as the conversion of electromagnetic (or acoustic) signals into required data is considered as a single continuous process which is to be optimized jointly for the desired final objective. The generalization of the analytical treatments to include at once all dimensions of interest and the inclusion of processing as part of the antenna proper are symptomatic of this development. The concepts of feedback and self-adaptiveness will become familiar in antenna design, and as the characteristics of various data processing arrays are better assessed they will provide the versatility of solutions needed for the ever increasing variety of objects.

U.S. National Committee reports on antennas for the previous two triennia have been published and may be of interest: Bickmore, R. W., and R. C. Hansen (Nov.–Dec. 1960), J. Res. NBS **64D** (Radio Prop.), No. 6, 731–741; Cottony, H. V., et al. (Jan. 1959), IRE Trans. Antennas Propagation, **AP-7,** No. 1, 87–98.

1.1. Frequency Independent Antennas

Since the ideal of frequency independent antennas was first mooted in 1954 at the University of Illinois and successfully demonstrated there in 1956, much progress has been made in the understanding and in the development of these antennas. They can be classed as spiral types or log periodic types. Roughly speaking the distinction is that the former give patterns which, while they remain constant, rotate around the spiral axis with frequency, while the latter give patterns which change but repeat when the logarithm of the frequency is increased by a fixed number. In practice the rotation in the former and the variation within a period in the latter are negligible effects, so that the patterns and impedances are frequency independent over a band which is limited at the high end by the size of the feeding transmission line and at the low end by the size of the antenna. Bandwidths of 20:1 were obtained in the early models and these do not appear to have been exceeded since then presumably because there are no transmitters and receivers which could make use of such wide bandwidths. In this connection it is worth noting

[1] University of California, Berkeley, Calif.
[2] Aerospace Corporation, Los Angeles, Calif.
[3] Hughes Aircraft Company, Culver City, Calif.

that a beautiful solution to the problem of designing a very wide band balanced to unbalanced transformer has recently been published [Duncan and Minerva, 1960].

Many valuable forms of log periodic antennas have been invented during the last three years such as the log periodic dipole [Isbell, 1960; Mayes and Carrel, 1961] and monopole [Berry and Ore, 1961; Greiser and Mayes, 1961] arrays, the tapered ladder which is fed by capacitive coupling to a transmission line [Wickersham, et al., 1961] and the zigzag which is fed directly from a parallel wire transmission line [Carr, 1961]. Unlike the original log periodic antennas these all take the form of a conventional TEM biaxial or coaxial line which is log periodically loaded with radiating elements. The successful development of these inventions has been based on simple ideas combined with much practical experiment. Although no high gain forms have been found, gains of up to 17 db (± ?) have been reported for the dipole array type.

There have also been some interesting measurements on the current distribution and field in the vicinity of typical log periodic antennas of the earlier inclined zigzag type [Bell et al., 1960]. They show the existence of a nonradiating wave traveling outward from the terminals which gets converted in the "active region" to a radiating wave traveling inward with the orthogonal polarization. A typical characteristic of the active region is that the currents in adjacent elements of the zigzag are 90° out of phase.

There has also been a sizeable effort on the theory of log periodic antennas, although no solution of Maxwell's equations for even the most rudimentary log periodic structure has been discovered. The dipole array type has been analyzed by ignoring all but a finite number of dipoles in the active region, and so a useful body of design information has been worked out [Carrel, 1961]. Another approach assumes that the solution is approximately obtained by substituting an expanding amplitude and period into the solution for a simply periodic structure, a reasonable approximation for slowly expanding log periodic antennas [Mayes et al., 1961; Oliner, 1963; Rumsey, 1963]. This admits the application of the well developed periodic structure theory particularly the Brillouin diagram methods. A solution of Maxwell's equations has been obtained for the modes of a plane sheet of sinusoidal wires and, despite its far-fetched connection with log periodic antennas, shows remarkable agreement with amplitude and phase measurements of currents on log periodic zigzags. More general arguments based on the properties of periodic structures without regard to Maxwell's equations have contributed significantly to the understanding of log periodic structures [DuHamel, 1963; Mittra, 1962]. In this way, it is seen that the attenuation of current with distance from the center of a log periodic antenna sets in when the propagation constant of the periodic structure becomes complex. At present there is a vigorous controversy on whether the attenuation effect is comparable to the slow wave stop band effect or the fast wave radiating effect associated with open periodic structures. It does appear that this approach gives a good idea of the current distribution in the active region, in much the same way that uniform transmission line theory gives a good idea of the current on straight wire antennas.

It was recognized at the outset when frequency independent antennas were first bruited that not all log periodic structures could be truncated; i.e., the current was not certain to be attenuated. This still remains a fundamental question but the periodic structure method is getting very close to an answer. Another important question still more or less unresolved is how to minimize the variation of pattern within a period.

Turning now to the spiral types of frequency independent antenna, here also there have been several practical and theoretical discoveries in the last three years. Beginning with the theoretical side we note that Maxwell's equations have been solved for an anisotropic sheet of equiangular spiral wires [Cheo et al., 1961]. This is a far cry from the practical two element antennas, having instead an infinite number of filamentary elements, but it does represent many of the essential features of the antenna and the theoretical results agree amazingly well with pattern and polarization measurements. The solution shows that for large r the magnetic field at the antenna, i.e., surface current density, decreases as r^{-2}. The reduction in current with r depends markedly on the spiral curvature, but much less markedly on the distance along the spiral: typically the current is reduced to 0.1 of the input current when $r=0.3\lambda$ and the spiral radius expands by a factor of two per turn. The polarization is circular in all directions; it is a fact equally as amazing as the frequency independent pattern, that this appears to hold also for the measured radiation from many plane and conical spiral antennas.

One of the surprising results of the theoretical solution is this. When the excitation is such as to produce an outward current wave at the input, the current for large r consists of an inward wave, but the pattern and current amplitude are the same as for the reverse excitation. Measurements of the phase in the distant field of two element antennas always correspond to the solution with an inward current wave near the input, which may thus be called the natural sense of the antenna. Attempts to drive four and six element antennas in the unnatural sense have produced some remarkable results [Dyson et al., 1961; Sussman, 1961]. Measurements of the continuous variation of phase around the axis showed that near the input only the unnatural mode was present but in the radiation field the direction of circumferential phase velocity was reversed and consisted again of only one mode, namely the lowest harmonic which would fit the given (discrete) set of phases at the input terminals [Deschamps, 1959]. An input variation of 2π thus gets converted to $2\pi(1-n)$ in the radiation field of an n element antenna. Why this is so is a most interesting and unresolved question. A variety of patterns has been obtained from four and six element antennas, some of which give excellent omnidirectional coverage in the horizontal plane with maximum gain and circular polarization.

Summary

A number of new log periodic antennas have been invented consisting of TEM biaxial or coaxial lines which are loaded with monopole or dipole antennas. Conical and multielement spiral antennas have been developed giving circular polarization in all directions and an interesting variety of amplitude and phase patterns. Solutions of Maxwell's equations have been worked out for idealized spiral and periodic antenna types which show how the effective aperture is controlled by the geometry as well as predicting some unexpected and unverified effects.

Small Antennas

A considerable part of the effort on small antennas has gone into more precise evaluation of their effect on the performance of arrays. The cause of this appears to be the fact that sidelobes nominally designed to be 40, 50, ∞ db down rarely turn out to be more than 30 db down. Of the many obvious causes for this, mutual impedance is probably dominant. Theoretically, it can be taken into account, but the practical difficulties of doing so are formidable. For an infinite uniformly spaced array of identical elements, so energized as to launch a plane wave in a single direction, the analysis can evidently be reduced to consideration of a unit cell of the structure. In this way the variation of impedance with direction of the beam has been worked out [Edelberg and Oliner, 1960a and 1960b]. Obviously this does not apply near the edge of a finite array. This edge effect has been thoroughly explored, and extensive quantitative results have been obtained, both theoretically [Carter, 1960] and practically [Kurtz, et al., 1961]. Application of these results to precise low sidelobe design has not yet been reported but should give much improved performance.

Better or alternative methods of exciting slots in waveguides have been the subject of several contributions, such as the use of position and orientation of the slot to get independent control of amplitude and phase [Maxum, 1960], the use of asymmetrical irises to control the excitation of colinear slots [Dudley, 1961; Tang, 1960] and the variation of slot length [Rutz, 1960]. The use of islands as natural slot antennas for VLF has also been studied, with rather negative results [Morgan, 1960; Staras, 1963]. Chen and King [1961] have developed a dipole electromagnetically coupled to a two wire line; this is the two wire line analog of the waveguide slot.

The question of what really is the current distribution on antennas of assorted shapes and sizes persists with pristine fascination, the cylindrical antenna retaining its preeminence. It still stands as an elusive mathematical problem to which numerous solutions have been worked out, but none of which unfortunately is exactly correct: hence the intractable endurance of the problem. Some interesting new results for the current density near the input of a cylindrical antenna have been discovered [Chen et al., 1962; Duncan, 1962; Harrison et al., 1961; Hasserjian et al., 1962].

Finally, let us note a practical application for supergain arrays [Schildknecht, 1962]. The objection to these antennas is their extremely low efficiency, but for reception at VLF this may be tolerable because of the high background noise, so, if the frequency and excitation can be maintained with sufficient precision, we may have an excuse to try out this extraordinary aspect of array theory.

Summary

A more precise and thorough investigation of the characteristics of small antennas for use as elements of large arrays has been carried out.

References

Bell, R. L., C. T. Elfving, and R. E. Franks (Sept. 1960), Near-field measurements on a logarithmically periodic antenna, IRE Trans. Ant. Prop. **AP–8,** 559–566.

Berry, D. G., and F. R. Ore (1961), Log periodic monopole array, IRE Conv. Record, pt. 1, 76–85.

Carr, J. W. (Mar. 1961), Some variations in log periodic antenna structures, IRE Trans. Ant. Prop. **AP–9,** 229.

Carrel, R. (1961), The design of log periodic dipole antennas, IRE Conv. Record, pt. 1, 61–75.

Carter, P. S., Jr., (May 1960), Mutual impedance effects in large beam scanning arrays, IRE Trans. Ant. Prop. **AP–8,** 276–285.

Chen, K. M., and R. W. P. King (Sept. 1961), Dipole antennas coupled electromagnetically to a two-wire transmission line, IRE Trans. Ant. Prop. **AP–9,** 425.

Chen, Yung Ming, and Joseph B. Keller (Jan.–Feb. 1962). Current on and input impedance of a cylindrical antenna, J. Res. NBS **66D** (Radio Prop.) No. 1, 15–22.

Cheo, B. R.-S, V. H. Rumsey, and W. J. Welch (Nov. 1961), A solution to the frequency-independent antenna problem, IRE Trans. Ant. Prop. **AP–9,** 527–534.

Deschamps, G. A. (Dec. 1959), Impedance properties of complementary multiterminal planar structures, IRE Trans. Ant. Prop. **AP–7,** Special issue, S371–S378.

Dudley, D. G., Jr. (July 1961), An iris-excited slot radiator in the narrow wall of rectangular waveguide, IRE Trans. Ant. Prop. **AP–9,** 361–364.

DuHamel, R. H. (1963), Log periodic antennas and circuits, In Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, Proceedings, pp. 43–46 (Pergamon Press, Oxford, London).

Duncan, J. W., and V. P. Minerva, (Feb. 1960), L00:1, Balun transformer, Proc. IRE **84,** 156–164.

Duncan, R. H. (Mar.–Apr. 1962), Theory of the infinite cylindrical antenna including the feedpoint singularity in antenna current, J. Res. NBS **66D** (Radio Prop.) No. 2, 181–187.

Dyson, J. D., and P. E. Mayes (July 1961), New circularly-polarized frequency-independent antennas with conical beam or omnidirectional patterns, IRE Trans. Ant. Prop. **AP–9,** 334–341.

Edelberg, S., and A. A. Oliner (May 1960a), Mutual coupling effects in large antenna arrays: Part I. Slot arrays, IRE Trans. Ant. Prop. **AP–8,** 286–297.

Edelberg, S., and A. A. Oliner (July 1960b), Mutual coupling effects in large antenna arrays. Part II. Compensation effects, IRE Trans. Ant. Prop. **AP–8,** 360–367.

Greiser, J. W., and P. E. Mayes (1961), Vertically polarized log periodic zig zag antennas, Proc. National Electronic Conference **17,** 193–204.

Harrison, C. W., Jr., and R. W. P. King (Mar. 1961), Folded dipoles and loops, IRE Trans. Ant. Prop. **AP–9,** 171–186.

Hasserjian, Gerard, and A. Ishimaru (May–June 1962), Currents induced on the surface of a conducting circular cylinder by a slot, J. Res. NBS **66D** (Radio Prop.) No. 3, 335–366.
Isbell, D. E. (May 1960), Log periodic dipole arrays, IRE Trans. Ant. Prop. **AP–8,** 260–267.
Kurtz, L. A., R. S. Elliott, S. Wehn, and W. L. Flock (Sept. 1961), Mutual coupling effects in scanning dipole arrays, IRE Trans. Ant. Prop. **AP–9,** 433–443.
Maxum, B. J. (July 1960), Resonant slots with independent control of amplitude and phase, IRE Trans. Ant. Prop. **AP–8,** 384–389.
Mayes, P. E., and R. L. Carrel (1961), Log periodic resonant V arrays, IRE Wescon Record.
Mayes, P. E., G. A. Deschamps, and W. T. Patton (May 1961), Backward wave radiation from periodic structures and application to the design of frequency independent antennas, Proc. IRE **49,** No. 5, 962–963.
Mittra, R. (1962), Log periodic circuit analysis (abstract), IRE Conv. Record, pt. 1., p. 43.
Morgan, M. G. (Sept. 1960), An island as a natural very-low-frequency transmitting antenna, IRE Trans. Ant. Prop. **AP–8,** 528–530.
Oliner, A. A. (1963), The role of leaky waves in electromagnetic phenomena, In Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, pp. 172–179 (Pergamon Press, Oxford, London).
Rumsey, V. H. (1963), On the theory of frequency independent antennas, In Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, pp. 39–42 (Pergamon Press, Oxford, London).
Rutz, E. M. (July 1960), Resonant antenna array with tilted beam, IRE Trans. Ant. Prop. **AP–8,** 435–436.
Schildknecht, R. O. (1962), Superdirective antenna arrays for improved VLF reception (abstract), IRE Conv. Record, pt. 1, 164.
Staras, H. (1963), An analysis of a natural VLF slot antenna, In Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, 261–264 (Pergamon Press, Oxford, London).
Sussman, R. (1961), The equiangular spiral antenna (Masters Thesis), University of California, Berkeley, Calif.
Tang, Raymond (Jan. 1960), A slot with variable coupling and its application to a linear array, IRE Trans. Ant. Prop. **AP–8,** 97–101.
Wickersham, A. F. Jr., R. E. Franks, and R. L. Bell (Jan. 1961), Further developments in tapered ladder antennas, Proc. IRE **49,** 378.

Additional References

Barbano, N. (1960), Phase center distributions of spiral antennas, IRE Wescon Conv. Record, pt. 1, 123–130.
Curtis, W. L. (May 1960), Spiral antennas, IRE Trans. Ant. Prop. **AP–8,** 298–306.
Dyson, John D. (Jan.–Feb. 1962), A survey of the very wide band and frequency independent antennas-1945 to the present, J. Res. NBS **66D** (Radio Prop.) No. 1, 1–6.
Gillard, C. W., and R. E. Franks (Feb. 1961), Frequency independent antennas. Several new and undeveloped ideas, Microwave J. **4,** 67.
Kay, A. F., and A. J. Simmons (July 1960), Mutual coupling of shunt slots, IRE Trans. Ant. Prop. **AP–8,** 389–400.
Pulfer, J. K. (Mar. 1961), Dispersive properties of broadband antennas, Proc. IRE **49,** 644.
Shimizu, J. K. (Mar. 1961), Octave bandwidth feed horn for paraboloid, IRE Trans. Ant. Prop. **AP–9,** 223–224.
Wait, J. R. (1963), The theory of an antenna over an inhomogeneous ground plane, pp. 1079–1097, Electromagnetic Theory and Antennas, ed. by E. C. Jordan (Pergamon Press, Oxford).
Wait, J. R. (Feb. 1963), A note on *E*-field and *H*-field losses for ground-based antennas, Proc. IEEE **51,** No. 2, p. 366.
Wait, J. R., and L. C. Walters (Apr. 15, 1963), Influence of a sector ground screen on the field of a vertical antenna, NBS Mono. No. 60.

1.2. Arrays and Electronic Scanning

The triennial period has seen the appearance in the literature of papers describing several advanced electronic scanning systems which have been under investigation for several years. In particular, the multiple beam array has received considerable attention. Shelton and Kelleher [1961] showed that independent beams can be formed by a lossless passive and matched network only for sin x/x patterns or combinations thereof. Beam orthogonality was shown by Allen [1961, and White 1962] to be independent of antenna structure, and to impose a restrictive relation between beam crossover level and sidelobe level. These restrictions can be overcome by use of lossy or active combining networks. Other papers include a matrix approach using cross coupling factors [Stein, 1962; Kahn and Kurss, 1962]. Hardware realizations include a trapezoidal assembly of waveguide couplers (due to Blass) with one set of guides connected to elements and the other set connected to receivers [Blass, 1960], and a similar HF system using tapped delay lines [Brueckmann et al., 1962]. A more sophisticated multiple beam matrix system is the Butler array [Butler and Lowe, 1961; Delaney, 1962], which is a combining network consisting of rows of hybrids with appropriate fixed phase shifters between rows. The Butler array requires fewer hybrids or couplers to produce N beams from N elements than do matrix arrays; n levels of couplers are used where $n = \log_2 N$. Effects of random errors in this multiple beam system have been investigated by Kiss [1962]. Another scheme uses a digital approach, limiting sampling and combining the element signals [Gore, 1962]. The limiting and sampling process introduces new sidelobes, but their effect can be negated by proper element spacing. The high speed of the digital operations allows essentially simultaneous multiple beam performance. In addition, control over sidelobes and shaped beams is nearly unlimited. Other scanning techniques utilize IF heterodyning [Cottony and Wilson, 1961; Pels and Liang, 1962], and bridge circuits employing variable capacitance diodes [Vogt, 1962]. Very little has appeared on the formerly popular ferrite phase shifter array [Roush and Wiltse, 1961]. The frequency scanned array has been analyzed by Ishimaru and Tuan [1962] using ω-β-diagrams; this approach gives insight into the interrelationships among the main lobes.

General studies of electronic scanning effects and systems have been prepared by Von Aulock [1960], Ogg [1961], and Shnitkin [1961]. Some applicable material appears in a book by Jasik [1962]. A related topic is the gain or effective aperture versus scan angle. An approximate value has been obtained [King and Thomas, 1960; Thomas, 1962], and has been extended to cases where polarization or impedance are not matched [Tai, 1961]. Allen [1962] has measured and calculated mutual impedance effects under the assumption that the element to element mutual impedance or element feed current do not change with scan but that the effects of scan are artificially attributed to changes in element pattern and directivity. Regardless of the conceptual approach, the results are that array performance is insensitive to height of elements above a ground plane, and sensitive to element spacing. The physical parameters may be chosen to minimize the effects of scan changes. For scan angles other than broadside, isolation of the order of the sidelobe level between transmitted and received signals is provided by the fact that the transmitted and received phase fronts are at supplementary angles [Rubin and Rabinowitz, 1962].

Tracking accuracy has been analyzed from several viewpoints. Nester [1962; Leichter, 1960], and others have related excitation errors to tracking accuracy. Brennan [1961; Kirkpatrick, 1962], treated a linear array with individual noisy amplifiers for both amplitude and phase comparison systems. His results extend those of Manasse [1960] who derived the tracking accuracy (in terms of beamwidth) as a function of S/N and system time-bandwidth product.

A new class of automatic steering arrays has appeared of which the simplest example is the Van Atta array [Sharp and Diab, 1960; Bauer, 1961] and subsequently others [Bahret, 1961; Wanselow, 1962] proposed passively modulating telemetry on such an array (in a satellite) thereby affording space communications with economy of vehicle power. The modulation removes a serious objection to the Van Atta array: the pattern nulls produced by interference between scattered and reradiated energy. Active Van Atta arrays for communications satellites were studied by Hansen [1961a, 1961b] who showed that their advantage is for those situations where narrow beamwidths can be utilized; i.e., for transmission back to the interrogation point, or for spacecraft at distances greater than 20,000 nmi.

Turning now attention to pattern synthesis, equal sidelobe line source synthesis has been extended to the circular aperture by Taylor [1960]. Here, as in the case of the line source, a function is constructed with the proper asymptotic behavior and with equal level sidelobes out to a specified point. Tables of this "approximate space factor" for circular apertures are available [Hansen, 1960]. Cheng and Ma [1961] have treated the linear array as a sampled data system in which the pattern is the Z-transform of the array distribution envelope. Using this approach, equal sidelobe distributions were rederived [Cheng and Ma, 1961]. Ksienski derived maximally flat and ripple top synthesis techniques for sector-type beams, using filter synthesis ideas. However, the array polynomial has no poles so that the maximally flat synthesis is more difficult than for the Butterworth filter where poles are used to produce a steep dropoff [Ksienski, 1960]. Other synthesis work includes a least squares fit with computer solution [Hoffman, 1961] and a shaped beam technique with coefficients adjusted for the number of terms used [Bricout, 1960]. Harrington [1960] extended the work of Chu on limitations of antennas, specifically on bandwidth and Q as a function of excitation and size. Using spherical wave functions he showed that any antenna with gain much greater than that obtained with uniform excitation has a narrow bandwidth, and if lossy materials are utilized, also has very low efficiency. The curves presented show conclusively that for large antennas supergain is impractical. Hansen [1960] considered large antenna designs where sidelobes are equal level out to a specified point. For a sufficiently narrow main beam, the energy in the sidelobes may be comparable to that in the main beam, producing a limitation on gain. A sin x/x envelope will always obviate this problem; he shows where the sidelobe taper must start for a given aperture to avoid gain loss. Finally, the gain values for Tschebycheff arrays of Brown and Sharp's tables (to 40 elements) have been extended to 120 elements [Stegen, 1960].

Nonuniformly spaced arrays have aroused much interest of late, although the initial work was done by Unz in 1955. The objectives of nonuniform spacing are to reduce grating lobes, and to use fewer elements for a given beamwidth. Random removal, symmetrical about the center, has been studied by Maher and Cheng [1962]; however, most efforts have been devoted to finding an analytical or psuedorandom approach that will optimize parameters. Several techniques have been tried including perturbations from uniform spacing [Harrington, 1961] numerical integration (quadratures) such as trapezoidal rule to produce nonuniform spacing [Bruce and Unz, 1962a and 1962b; Lo, 1962; Maffett, 1962], and computer search [Andreasen, 1962]. A variety of analytic distributions such as log spacing have been tried with little success [King et al., 1960]. The nonuniform spacing is equivalent to an amplitude taper [Willey, 1962], and an equivalent uniformly spaced array may be defined [Sandler, 1960]. Ishimaru [1962] finds an equivalent line source using a sum of finite Fourier transforms, which for the uniformly spaced array reduces to a sum of sin x/x patterns. Each sin x/x represents a main beam. This latter approach provides some insight into the nonuniform phenomenon. For 5-element arrays, a solution has been obtained which for a given sidelobe level yields a shorter array than the Dolph-Tschebycheff array for the same sidelobe level [Brown, 1962]. However, the beamwidth is correspondingly greater in the 5-element nonuniform array. Lastly, Allen [1962] has observed that the nonuniformity of the mutual coupling in a nonuniform array will have a profound effect on the array illumination.

Summary

The ferrite phase shifter and frequency scanned arrays have declined in popularity, because multiple beam and digital arrays offer more versatility with little or no increase in complexity. Effort in the latter areas will probably expand, especially on distributed transmitter arrays, and on receiving arrays with an amplifier/processor at each element. Digital array control may be the ultimate for multiple targets and target discrimination. Nonuniform spacing of elements has come of age, but very few of the design techniques produce low sidelobes; a conspicuous exception is the space tapering approach where 20 db has been achieved. A theoretical analysis and synthesis technique is badly needed to allow synthesis of low sidelobe and other practical designs. Automatic steering systems for arraying large dishes, for automatic angle return, for atmospheric scintillation compensation, and for beaming large amounts of RF power represent one of the most exciting developments. Much work remains to be done in this area.

Near Fields

A significant advance in near field theory was made by Goubau and Schwering [1961; Christian and Goubau, 1961a] with the invention of the iris beam waveguide. At a point in the radiating near field region, each mode of the field can be reconstructed to the original aperture value by a phase transformation, thus the field reiterates. The phase transformer is a convex lens which retards the beam center with respect to the outside; i.e., it refocuses the beam. Thus the structure is a periodic beam waveguide, capable of carrying power with low loss, expecially for beams large in wavelength. The phase transformers may be approximated by irises: experimental measurements [Christian and Goubau, 1961b] and higher mode analyses [Beyer and Scheibe, 1962] have disclosed a loss per iris below 0.1 db for spacing of 1/20 of D^2/λ. Closely related to the beam waveguide is the work of Kay [1960] on the optimum phase and amplitude distribution for maximum power transfer between two large apertures. In addition to the phase curvature needed, as in focusing for power transfer [Bickmore, 1960] some amplitude changes are required for short separations. Jull [1962] has shown that measured sidelobes are generally higher than expected for antenna separations less than $2D^2/\lambda$, but that for certain combinations of distance and diameter measured, sidelobes may be lower. Near-field calculations by Hu previously available only in report form, have now been published [Ming-Kuei Hu, 1960 and 1961]. Hu uses the small angle Fresnel approximation for circular apertures of $(1-\rho^{2n})$ distribution and obtains the field in terms of a pair of Lommel functions of two variables. Plots of field amplitude and phase are given for a number of fractions of D^2/λ. These may be compared with the general Fresnel results of Hansen and Bailin [1959]. Recently published Russian tables [Dekanosideze, 1960] of these Lommel functions will extend the usefulness of Hu's work. Other near field studies are on focused apertures [Sherman, 1962].

Summary

The analytical understanding of radiation and near fields, of interest particularly for power transfer and high power focusing, has improved markedly. Promise for millimeter, submillimeter, and laser applications is offered by the beam waveguide.

Aperture Antennas

The Cassegrain reflector system as a large antenna has found considerable use as a low noise temperature device and for applications requiring large or complex feeds. Hannan [1961] and others [Wilkinson and Appelbaum, 1961; White and DeSize, 1961; Martin and Schwartzman, 1960] have analyzed the Cassegrain antenna and have derived the minimum blockage criterion. Minimum blockage occurs when the feed and subreflector shadows are equal. Monopulse arrays have been designed in a Dolph-Tschebycheff sense with element values obtained by simple transformation from a Dolph array [Price and Hyneman, 1960]. Wheeler [1962] has made a semiquantitative analysis of monopulse and defocused feeds by considering an infinite Gaussian aperture distribution. Even with defocusing (for acquisition or for wide targets) the pattern is also Gaussian, and the difference pattern is the derivative or Rayleigh pattern. The maximum gain realizable in the difference pattern has been shown to be -2.15 db below sum gain, for a rectangular aperture [Hannan, 1961a; Kinsey, 1962]. Multihorn feeds with more than four horns offer advantages in optimizing difference pattern performance, and have been investigated by Hannan [1961b and 1961c; Hannan and Loth, 1961] and Ricardi [1961]. A two-dish phase-amplitude monopulse system has also been developed [Hausz and Zachary, 1962].

The geodesic or "tin hat" Luneberg lens has been studied analytically and experimentally by Johnson [1962] and Walter [1962]. The coma corrected zoned reflector proposed and analyzed by di Francia [1961] has been studied further both theoretically [Dasgupta and Lo, 1961] and experimentally. Such a reflector allowed scanning [Provencher, 1960] by feed movement over about ± 10 beamwidths compared to ± 4 for a solid parabola. A stepped parabola has also been used in a satellite radiometer to reduce feed heating [Richter, 1962]. The steps were flat and cylindrical instead of conical (as in the zoned reflector) and were closely spaced at the operating wavelength. Beam scanning of a parabola by feed movement has engendered additional work [Sandler, 1960; Lo, 1960]. Work has continued [Altshuler, 1962] on line source feeds for scanning spherical reflectors, such as the 1000 ft bowl under construction at Arecibo, Puerto Rico. Several millimeter wavelength parabolas have been successfully produced by the liquid spin casting process [Dawson, 1962]. An X–Y or elevation-on-elevation mount has been developed for tracking near zenith [Rolinski et al., 1962]. An important development in wide-band dish feeds for telemetry has been a conical lobing dual log spiral, with the two spirals symmetrically disposed about the lobing axis. The two spirals are decoupled through use of opposite sense circular polarization, and cover a 4:1 band [Jasik et al., 1961]. Low noise antennas have been of great practical interest for space communications [Pauling-Toth, 1962; Livingston, 1961]; the hog horn antenna developed by Crawford [1961] and others and used with

TELSTAR probably represents the highest achievement in sidelobe and backlobe control. Schuster [1962] and Potter [1962] have achieved an antenna noise temperature of 15 °K at 2.4 Gc/s in an 85 ft dish using a shaped beam horn feed. The latter consists of a horn in ground plane, with a corrugated surface wave annulus on the outer portion of the ground plane.

Radio astronomy instruments continue to occupy much engineering effort. Kraus and Ko [1961] have described a 360 ft standing parabola with ground plane and tilting reflector, which has just been completed. Swenson and Lo [1961] describe a 600 ft× 400 ft fixed parabolic cylinder with nonuniformly spaced scanning feed array. Bracewell [1961] has described a Christianson array of dishes built as a cross interferometer. Finally, Reber [1961] has written on the history of the cross antenna.

Summary

Low noise antenna systems have made significant advances, especially in hardware techniques. Use of the Cassegrain geometry and of feed shaping has produced antenna temperatures well below 20 °K. These systems will continue to be important for space and satellite communications. Several different types of large radio astronomy instruments have come to fruition: the fixed cylindrical parabola with scanning linear array feed; the standing parabola with tiltable flat plane reflector; a Christianson array of dishes, in a cross form. The unfortunate cancellation of the Sugar Grove 600 ft dish project is a loss to the science of space communications.

References

Allen, J. L. (July 1961), A theoretical limitation on the formation of lossless multiple beams in linear arrays, IRE Trans. Ant. Prop. **AP–9,** 350–352.

Allen, J. L. (Sept. 1962), Gain and impedance variation in scanned dipole arrays, IRE Trans. Ant. Prop. **AP–10,** 566–572.

Allen, J. L. and W. P. Delaney (Nov. 1962), On the effect of mutual coupling on unequally-spaced dipole arrays, IRE Trans. Ant. Prop. **AP–10,** (Communications), 784.

Altshuler, E. E. (Mar. 1962), Primary pattern measurements of a line-source feed for a spherical reflector, IRE Trans. Ant. Prop. **AP–10,** 214.

Andreasen, M. G. (Mar. 1962), Linear arrays with variable inter-element spacings, IRE Trans. Ant. Prop. **AP–10,** 137–143.

Bahret, W. F. (Nov. 1961), Technique for amplitude modulating a Van Atta radar reflector, Proc. IRE **49,** 1692.

Bauer, L. H. (Mar. 1961), Technique for amplitude modulating a Van Atta radar reflector, Proc. IRE **49,** 634.

Beyer, J. B. and E. H. Scheibe (May 1962), Higher modes in guided electromagnetic wave beams, IRE Trans. Ant. Prop. **AP–10,** 349.

Bickmore, R. W. (Mar. 1960), Power transmission via radio waves, Proc. IRE **48,** 366.

Blass, J. (1960), Multidirectional antenna—a new approach to stacked beams, IRE Conv. Record pt. 1, 48–50.

Bracewell, R. N. and G. Swarup (Jan. 1961), The stanford microwave spectroheliograph antenna, a microsteradian pencil beam interferometer, IRE Trans. Ant. Prop. **AP–9,** 22–30.

Brennan, L. E. (May 1961), Angular accuracy of a phased array radar, IRE Trans. Ant. Prop. **AP–9,** 268–275.

Bricout, P. A. (July 1960), Pattern synthesis using weighted functions, IRE Trans. Ant. Prop. **AP–8,** 441–444.

Brown, F. W. (Sept. 1962), Note on nonuniformly-spaced arrays, IRE Trans. Ant. Prop. **AP–10,** 639.

Bruce, J. D. and H. Unz (Feb. 1962a), Broadband nonuniformly-spaced arrays, Proc. IRE **50,** 228.

Bruce, J. D. and H. Unz (Oct. 1962b), Mechanical quadratures to synthesize nonuniformly-spaced antenna arrays, Proc. IRE **50,** 2128(L).

Brueckmann, H., J. R. Gruber, and C. A. Bramble (1962), ISCAN—Inertialess steerable communication antenna, IRE Conv. Record, pt. 1, 152–163.

Butler, J. and R. Lowe (Apr. 1961), Beam forming matrix simplifies design of electronically scanned antennas, electronic design, 170–173.

Cheng, D. K. and M. T. Ma (May 1960), A new mathematical approach for linear array analysis, IRE Trans. Ant. Prop. **AP–8,** 255–259.

Cheng, D. K. and M. T. Ma (1961), A critical study of linear arrays with equal side lobes, IRE Conv. Record, pt. 1, 110–122.

Cottony, H. V. and A. C. Wilson (Jan.–Feb. 1961), A high-resolution rapid-scan antenna, J. Res. NBS **65D** (Radio Prop.) No. 1, 101–110.

Crawford, A. B., D. C. Hogg, and L. E. Hunt (July 1961), A horn-reflector antenna for space communication, Bell System Tech. J. **40,** 1095–1116.

Christian, J. R. and G. Goubau (May 1961a), Experimental studies on a beam waveguide for millimeter waves, IRE Trans. Ant. Prop. **AP–9,** 256–262.

Christian, J. R. and G. Goubau (Nov. 1961b), Some measurements on an iris beam waveguide, Proc. IRE **49,** 1679.

Dasgupta, S. and Y. T. Lo (Mar. 1961), A study of the coma-corrected zoned mirror by diffraction theory, IRE Trans. Ant. Prop. **AP–9,** 130–139.

Dawson, J. W. (June 1962), 28-ft liquid-spun radio reflector for millimeter wavelengths, Proc. IRE **50,** 1541.

Dekanosidze, E. N. (1960), Tables of Lommel's functions of two variables (Pergamon Press translation, New York and London).

Delaney, W. P. (Apr. 1962), An RF multiple beam-forming technique, IRE Trans. Military Electron. **MIL–6,** 179–186.

Gore, W. C. (Nov. 1962), Nonlinear signal processing in receiving arrays, IRE Trans. Ant. Prop. **AP–10,** 656–662.

Goubau, G. and F. Schwering (May 1961), On the guided propagation of electromagnetic wave beams, IRE Trans. Ant. Prop. **AP–9,** 248–255.

Hannan, P. W. (Mar. 1961a), Microwave antennas derived from the Cassegrain telescope, IRE Trans. Ant. Prop. **AP–9,** 140–153.

Hannan, P. W. (May 1961b), Maximum gain in monopulse difference mode, IRE Trans. Ant. Prop. **AP–9,** 314–315.

Hannan, P. W. (Sept. 1961c), Optimum feeds for all three modes of a monopulse antenna I: theory, IRE Trans. Ant. Prop. **AP–9,** 444–454.

Hannan, P. W. (Sept. 1961d), Optimum feeds for all three modes of a monopulse antenna II: practice, IRE Trans. Ant. Prop. **AP–9,** 454–461.

Hannan, P. W. and P. A. Loth (1961), A monopulse antenna having independent optimization of the sum and difference modes, IRE Conv. Record, pt. 1, 57–60.

Hansen, R. C. (Jan. 1960a), Tables of Taylor distributions for circular aperture antennas, IRE Trans. Ant. Prop. **AP–8,** 23–26.

Hansen, R. C. (Sept. 1960b), Gain limitations of large antennas, IRE Trans. Ant. Prop. **AP–8,** 490–495.

Hansen, R. C. (Mar. 1961a), Gain limitations of large antennas, IRE Trans. Ant. Prop. **AP–9,** 192.

Hansen, R. C. (June 1961b), Communications satellites using arrays, Proc. IRE **49,** 1066–1074.

Hansen, R. C. (Aug. 1961c), Correction to communications satellites using arrays, Proc. IRE **49,** 1340.

Hansen, R. C. and L. L. Bailin (Dec. 1959), A new method of near-field analysis, IRE Trans. Ant. Prop. **AP–7,** S458–S467.

Harrington, R. F. (Jan.–Feb. 1960), Effect of antenna size on gain, bandwidth, and efficiency, J. Res. NBS **64D** (Radio Prop) No. 1, 1–12.

Harrington, R. F. (Mar. 1961), Sidelobe reduction by nonuniform element spacing, IRE Trans. Ant. Prop. **AP–9,** 187–192.

Hausz, W. and R. A. Zachary (Apr. 1962), Phase-amplitude monopulse system, IRE Trans. Military Electron. **MIL–6,** 140–146.

Hoffman, M. (Jan. 1961), the utility of the array pattern matrix for linear array computations, IRE Trans. Ant. Prop. **AP–9,** 97–100.

Ishimaru, A. (Nov. 1962), Theory of unequally-spaced arrays, IRE Trans. Ant. Prop. **AP–19,** 691.

Ishimaru, A. and H. S. Tuan (Mar. 1962), Theory of frequency scanning of antennas, IRE Trans. Ant. Prop. **AP–10,** 144–150.

Jasik, H. (1962), Antenna engineering handbook (McGraw-Hill Book Co., Inc., New York, N.Y.).

Jasik, H., A. D. Bresler, and A. Kampinsky (Dec. 1961), A wideband conical scan antenna feed system, Microwave J. **4,** 97.

Johnson, R. C. (Aug. 1962), The geodesic luneberg lens, Microwave J. **5,** No. 8, 76–85.

Jull, E. V. (July 1962), An investigation of near-field radiation patterns measured with large antennas, IRE Trans. Ant. Prop. **AP–10,** 363–368.

Kahn, W. K. and H. Kurss (Jan. 1962), The uniqueness of the lossless feed network for a multibeam array, IRE Trans. Ant. Prop. **AP–10,** 100–101.

Kay, A. F. (Nov. 1960), Near-field gain of aperture antennas, IRE Trans. Ant. Prop. **AP–8,** 586–593.

King, D. D., R. F. Packard, and R. K. Thomas (July 1960), Unequally-spaced broad-band antenna arrays, IRE Trans. Ant. Prop. **AP–8,** 380–384.

King, M. J. and R. K. Thomas (Sept. 1960), Gain of large scanned arrays, IRE Trans. Ant. Prop. **AP–8,** 635–636.

Kinsey, R. R. (May 1962), Monopulse difference slope and gain standards, IRE Trans. Ant. Prop. **AP–10,** 343–344.

Kirkpatrick, G. M. (May 1962), A relationship between slope functions for array and aperture monopulse antennas IRE Trans. Ant. Prop. **AP–10,** 350.

Kiss, M. J. (Nov. 1962), Effects of random errors on the performance of a linear butler array, IRE Trans. Ant. Prop. **AP–10,** 768.

Kraus, J. D., R. T. Nash, and H. C. Ko (Jan. 1961), Some characteristics of the Ohio State University 360-foot radio telescope, IRE Trans. Ant. Prop. **AP–9,** 4–8.

Ksienski, A. (Sept. 1960), Maximally flat and quasi-smooth sector beams, IRE Trans. Ant. Prop. **AP–8,** 476–484.

Leighter, M. (May 1960), Beam pointing errors of long line sources, IRE Trans. Ant. Prop. **AP–8,** 268–275.

Livingston, M. L. (Sept. 1961), The effect of antenna char, acteristics on antenna noise temperature and system SNR-IRE Trans. Space Electron. Telemetry **SET–7,** 71–79.

Lo, Y. T. (May 1960), On the beam deviation factor of a parabolic reflector, IRE Trans. Ant. Prop. **AP–8,** 347–349.

Lo, Y. T. (1962), A spacing weighted antenna array, IRE Conv. Record, pt. 1, 191.

Maffett, A. L. (Mar. 1962), Array factors with nonuniform spacing parameter, IRE Trans. Ant. Prop. **AP–10,** 131–136.

Maher, T. M. and D. K. Cheng (1962), On random removal of radiators from large linear arrays, IRE Conv. Record, pt. 1, 183–190.

Manasee, R. (Jan. 1960), Maximum angular accuracy of tracking a radio star by lobe comparison, IRE Trans. Ant. Prop. **AP–8,** 50–56.

Martin, R. W. and L. Schwartzman (1960), A monopulse cassegrainian antenna, IRE Conv. Record, pt. 1, 96–102.

Ming-Kuei Hu (May 1960), Fresnel region field distributions of circular aperture antennas, IRE Trans. Ant. Prop. **AP–8,** 344–346.

Ming-Kuei Hu (March–April 1961), Fresnel region fields of circular aperture antennas, J. Res. NBS **65D** (Radio Prop.) No. 2, 137–149.

Nester, W. H. (May 1962), A study of tracking accuracy in monopulse phased arrays, IRE Trans. Ant. Prop. **AP–10,** 237–245.

Ogg, F. C. (Apr. 1961), Steerable array radars, IRE Trans. Military Electron. **MIL–5,** 80–94.

Pauliny-Toth, I. I. K., I. R. Shakeshaft, and R. Wielebinski (Dec. 1962), The use of a paraboloidal reflector of small focal ratio as a low-noise antenna system, Proc. IRE **50,** No. 12, 2483.

Pels, E. and W. Liang (Jan. 1962), A method of array steering by means of phase control through heterodyning, IRE Trans. Ant. Prop. **AP–10,** 100.

Potter, P. (June 1962), The application of the cassegrainian principle to ground antennas for space communications, IRE Trans. Space Electron. Telemetry **SET–8,** 154–158.

Price, O. R. and R. F. Hyneman (Nov. 1960), Distribution functions for monopulse antenna difference patterns, IRE Trans. Ant. Prop. **AP–8,** 567–576.

Provencher, J. A. (May 1960), Experimental study of a diffraction reflector, IRE Trans. Ant. Prop. **AP–8,** 331–336.

Reber, G. (Feb. 1961), History of the cross antenna, Proc. IRE 49, 528.

Ricardi, L. J. and L. Niro (1961), Design of a twelve-horn monopulse feed, IRE Conv. Record, pt. 1, 49–56.

Richter, E. W. (May 1962), Stepped parabolic antennas, Microwave J. **5,** 95–98.

Rolinske, A. J. et al. (June 1962), The *X–Y* antenna mount for data acquisition from satellites, IRE Trans. Space Electron. Telemetry **SET–8,** 159–163.

Ronchi, L., V. Russo, and G. Toraldo Di Francia (Jan. 1961), Stepped cylindrical antennas for radio astronomy, IRE Trans. Ant. Prop. **AP–9,** 68–74.

Roush, R. G. and J. C. Wiltse (Jan. 1961), Electronically steerable *S*-band array, IRE Trans. Ant. Prop. **AP–9,** 107–109.

Rubin, M. D. and S. J. Rabinowitz (Nov. 1962), Transmitter-receiver isolation in a CW array, IRE Trans. Ant. Prop. **AP–10,** 781 (Communications).

Rudduck, R. C. and C. H. Walter (July 1962), A general analysis of geodesic Luneberg lenses, IRE Trans. Ant. Prop. **AP–10,** 444–449.

Sandler, S. S. (July 1960a), Paraboloidal reflector patterns for off-axis feed, IRE Trans. Ant. Prop. **AP–8,** 368–379.

Sandler, S. S. (Sept. 1960b), Some equivalences between equally and unequally spaced arrays, IRE Trans. Ant. Prop. **AP–8,** 496–500.

Schuster, D., C. T. Stelzried, and G. S. Levy (May 1962), The determination of noise temperatures of large paraboloidal antennas, IRE Trans. Ant. Prop. **AP–10,** 286–291.

Sharp, E. D. and M. A. Diab (July 1960), Van Atta reflector array, IRE Trans. Ant. Prop. **AP–8,** 436–438.

Shelton, J. P. and K. S. Kelleher (Mar. 1961), Multiple beams from linear arrays, IRE Trans. Ant. Prop. **AP–9,** 154–161-

Sherman, J. W. III (July 1962), Properties of focused apertures in the fresnel region, IRE Trans. Ant. Prop. **AP–10,** 399–407.

Shnitkin, H. (Jan. 1961), Survey of electronically scanned antennas, Microwave J. Part I **3,** 67–72.

Stegan, R. J. (Nov. 1960), Gain of Tchebycheff arrays, IRE Trans. Ant. Prop. **AP–8,** 629–631.

Stein, S. (Sept. 1962), On cross coupling in multiple-beam antennas, IRE Trans. Ant. Prop. **AP–10,** 548–557.

Swenson, G. W., Jr., and Y. T. Lo (Jan. 1961), The Univ. of Illinois Radio Telescope, IRE Trans. Ant. Prop. **AP–9,** 9–16.

Tai, C. T. (Mar. 1961), On the definition of the effective aperture of antennas, IRE Trans. Ant. Prop. **AP–9,** 224–225.

Taylor, T. T. (Jan. 1960), Design of circular apertures for narrow beamwidth and low sidelobes, IRE Trans. Ant. Prop. **AP–8,** 17–22.

Thomas, R. K. (Mar. 1962), Gain of scanned arrays vs element pattern, IRE Trans. Ant. Prop. **AP–10,** 212.

Vogt, G. (Mar. 1962), An electronic method for steering the beam and polarization of HF antennas, IRE Trans. Ant. Prop. **AP–10,** 193–200.

Wanselow, R. D. (Nov. 1962), A proposed high gain wide angle coverage, passive, modulated re-radiator, IRE Trans. Ant. Prop. **AP–10,** 785 (Communications).

Wheeler, H. A. (Sept. 1962), Antenna beam patterns which retain shape with defocusing, IRE Trans. Ant. Prop. **AP–10,** 573–580.

White, W. D. and L. K. DeSize (July 1961), Focal length of a cassegrain reflector, IRE Trans. Ant. Prop. **AP–9,** 412.

716–797—64——7

White, W. D. (July 1962), Pattern limitations in multiple-beam antennas, IRE Trans. Ant. Prop. **AP–10,** 430–435.
Wilhelm Von Aulock, H. (Oct. 1960), Properties of phased arrays, Proc. IRE **48,** 1715–1727.
Wilkinson, E. J. and A. J. Appelbaum (Jan. 1961), Cassegrain systems, IRE Trans. Ant. Prop. **AP–9,** 119–120.
Willey, R. E. (July 1962), Space tapering of linear and planar arrays, IRE Trans. Ant. Prop. **AP–10,** 369–376.

1.3. Data Processing and Synthetic Aperture Antennas

A steadily increasing number of applications require extreme angular accuracies. These may be specified in terms of pointing accuracy for single target situations, such as in satellite tracking and communication. Other applications may require maximum resolution without high gain, such as in radio astronomy where integration time may offset possible loss in gain. In some applications, however, such as ground mapping, one desires both high resolution and gain. In the conventional antenna, such as a lens, reflector, or linear array, gain, resolution and pointing accuracy are rigidly connected and there is no way of optimizing one aspect of the performance at the expense of others. Thus, the conventional approach could solve these problems only in terms of very large apertures and/or very high frequencies, with the accompanying tolerance problems. In the last decade several unconventional solutions were obtained by various researchers; these new techniques may be termed collectively data processing antennas. These techniques indeed exceed performance of the conventional antenna, but most of them, e.g., nonlinear antennas, do so in regard to a specific characteristic while sacrificing others. Certain techniques introduce a new parameter such as time or bandwidth and thus avoid major parameter tradeoffs.

Contributions in the field which appeared in the literature up to 1960 were reported in the last URSI report. A relatively detailed description of these techniques is also available in the literature [Ksienski, 1961a and 1961b]. During the last three years, a significant portion of the research activity, theoretical and experimental, has been concentrated in the evaluation of these techniques under various operating conditions. Several new techniques also appeared which were particularly applicable to radar and communication, and most significantly, the synthetic array radar appeared for the first time in the open literature 10 years after its inception. The continuing effort to extract more information from the signal arriving at the antenna led to the analytical investigations which attempt to determine the ultimate capability of the processing arrays as well as that of the conventional antenna. Information theory, detection theory, parameter estimation, and other techniques which were successfully applied in the past to communication and radar systems are being applied to the antenna in an attempt to include the angular variables with the range variable and formulate a more general set of performance criteria and to optimize the antenna performance as a part of the overall system.

A rather interesting and useful concept introduced into the antenna system is that of adaptiveness. Thus, the antenna must be capable of changing some of its parameters in response to the signal and/or the environment. This idea was incorporated in a self-focusing antenna array [Breese et al., 1961; Schrader, 1962], where each individual element of the array continually adjusts the phase shifter (or delay line) so as to combine its signal contribution coherently with those of the other elements. This scheme provides optimum signal reception for a given number of antenna elements independent of their relative locations and spacing (except for mutual effects), and can be used to compensate for atmospheric irregularities. The signal reception is optimum, due not only to the fact that all signals add in phase, but that the detection process for the array as a whole is phase coherent. This antenna system is excellent for communication purposes where little interference is present; however, it is not capable of discriminating against other sources (or returns) which operate at the same frequency as the communicating source. Further, signal components within the phase lock loop pass bands may cause spurious performance. This technique was experimentally tested with a two-element array [Breese and Sferrozza, 1962] with performance close to the predicted. The self-focusing technique is a good example of what can be achieved if one aspect of an antenna array is optimized at the expense of others. Here, resolution was ignored in favor of gain or SNR.

The general problem of optimum excitation of a linear array which has often been investigated in the past has received further attention. Myers [1960a and 1960b] investigated experimentally the image quality produced by antennas of various excitations and found that the uniform illumination is close to the optimum in terms of the ease with which both a mechanical as well as a human observer can recognize the image. An information theoretic study [Young and Ksienski, 1961] of the same problems reached similar conclusions, although under the condition of zero image noise, which is equivalent to a noiseless receiver, the information rate was found to be unaffected by illumination. These results are similar to those obtained by Linfoot [1955] for optical images. One may approach the problem of optimizing an antenna in terms of separability of returns from two closely spaced targets. Thus, Woodward's ambiguity function [Woodward, 1953] may be generalized to include angular variables. This was carried out by Urkowitz et al. [1962] who also computed the generalized resolution constants in four dimensions: range, range rate, and azimuth and elevation angles. It should be pointed out, however, that the ambiguity function per se is only a measure of resolution and does not predict at what angle of separation (or range separation) two targets can be resolved. A concept which is most useful in evaluating the imaging properties of antennas is that of spatial frequency, which is equivalent to the temporal frequency [Bracewell and Roberts, 1954] in networks.

For the important question of what is the limit of the information obtainable from a finite aperture, the answer would be given (except for *noise*) in terms of the spatial frequency bandwidth of the antenna. In accordance with Bracewell [1961], this bandwidth corresponds to the size of the antenna in wavelengths, with zero response beyond it. It was shown, however, that no such bound or cutoff [Lo, 1961; Walter, 1961; Ksienski, 1962; Chisholm, 1962] exists, as evidenced for example by supergain [Spitz, 1959]. For large antennas such as are used in radio astronomy, Bracewell's conclusion is practically correct, but for small antennas it may not be. The response of nonlinear arrays can also be more conveniently evaluated in terms of the spatial frequence response [Young and Ksienski, 1961; Bracewell, 1961]. Multiplication of the outputs of two antennas convolves their transforms and widens the overall bandwidth, but this does not necessarily imply that more information can be passed through this filter than was possible before multiplication. This depends on the coherence properties of the object. Thus, for radio astronomy, where the signals arriving from different portions of the sky are almost totally uncorrelated, the spatial pass band does increase, as shown by Bracewell [1961]. For partially coherent or completely coherent signals the results are far more complex and resolution improvement is limited. The improvement in resolution when obtained will still often involve loss in gain and SNR. Because of the great importance of the signal characteristics on product arrays, a large amount of work was done in analyzing the performance of these arrays for various signal conditions. Drane and Parrent [1962] computed the response of a single product array in terms of the mutual coherence function of the object distribution. The coherence function represents the correlation between the signals emanating from two points on the object separated by arbitrary distances in time and space. The results of the paper show that on the incoherent limit, such as in radio astronomy, the single product array is appropriate for mapping extended sources, since it is linear in power. Linder [1961] investigated the response of single product arrays to both perfectly coherent (monochromatic) and incoherent signals and computed the effect of integration time on the response in the case of single product array to uncorrelated targets. Mays and Cheng [1960] computed the effect of varying the RC constant of an averaging filter following a single product array. Linder, as well as others [Pedinoff and Ksienski, 1962], conclude that multiple product arrays are of limited value for even completely incoherent extended sources or multiple target situation. The response of various single product arrays to two perfectly coherent in-phase targets was considered in some detail by Pedinoff and Ksienski [1962], and it was shown that even for the restricted in-phase condition, the responses sometimes bear little resemblance to the true target distribution, except for one particular array configuration. But even this configuration results in distortions when arbitary relative phases between targets are assumed [MacPhie, Pedinoff, and Ksienski, 1962]. These distortions, however, do occur in linear arrays as well when targets of arbitrary relative phase and amplitude are assumed. A detailed study of the effect of varying phase and amplitude in linear and several single-product arrays indicates that the average boresight error is about the same in most arrays considered [Ksienski et al., 1962a and 1962b]. A rather interesting experimental study by Welsby [1961] shows that at least for sonar application, a multiplicative array has resolution capability definitely superior to that of a linear array.

Experimental work on the response of a product array to a single target was performed by Gabriel [1961] with results agreeing well with theory. A multiple-product array was experimentally investigated by Davenport and Drane [1961] and by Band [1961]. The response that was obtained to a single point target agreed with theoretical predictions. A numerical and experimental study of the response of several single-product arrays to two partially correlated objects was carried out [Ksienski et al., 1962a and 1962b] with results indicating that improved resolution is achieved by the product arrays, as compared to a uniformly illuminated square law detected linear array, for practically all correlation levels from zero to unity.

A subject of considerable importance to the product array performance is the correlation of the background noise, e.g., clutter. Freeman [1960] investigated the systematic error in direction finding by means of correlation arrays when the background noise is partially coherent. This noise coherence also limits the useful integration time for the array; thus, the relation of the maximum useful averaging time to the noise coherence is obtained. The effect of mechanical versus electronic scanning of the antenna elements and/or the array on the output SNR of a correlation array was investigated by Jacobson and Talham [1960]. In another paper [Jacobson and Talham, 1961] these authors consider the effect of scanning errors on correlation array output SNR. Jacobson [1962] also investigated the effect of beam steering by both time and space translation on the output of product arrays due to background noise correlation.

In the product arrays discussed above, the illumination of the various elements was kept uniform. One may try, however, to modify that illumination in order to further improve the resolution of the processing array. Jacobson [1961] suggested the use of two adjacent linear arrays whose elements are so weighted that when the outputs are multiplied and averaged, one obtains a Tchebycheff pattern. Price [1960] proposed a scheme which is essentially equivalent to a single product and produces a Tchebycheff pattern corresponding to a linear array twice the physical size of the processed array. Mattingly [1960] suggested a nonreciprocal scheme applicable to two-way patterns only. The transmit and receive patterns differ from each other such as to form a product which yields a Tchebycheff pattern of the order of $2n$ instead of obtaining a two-way pattern corresponding to an nth order squared. This process is completely linear, and its main disadvantage compared to reciprocal patterns is some loss in gain. MacPhie [1962] combined the two-way mode of operation with a multiplicative process on receive to obtain a Tchebycheff of order $4n$, compared to a square law detected reciprocal array whose response corresponds to T_n^4.

In the above discussion, an implied constraint on the antenna was its physical size; thus, one by assumption would sample only the energy falling in the area occupied by the antenna. If that constraint is removed, a completely new field is provided—that of synthetic arrays.

A synthetic array, in the present context, simulates the performance of a conventional antenna array by sequentially moving a single element of the array to the various positions which would be simultaneously occupied by the elements of the conventional array.

There are 2 main areas to which the synthetic array principle has been applied. One field is radio astronomy, where Ryle and Hewish [1960] successfully synthesized both one-and two-dimensional apertures. In this case, the sources are essentially incoherent and the synthetic aperture is passive, i.e., operates in the receive mode alone. The second is in reconnaissance or ground mapping, in which the successive returns of an active radar on a moving vehicle were coherently combined. The resolutions obtained substantially exceeded that obtained from the same radar operating in the conventional mode. There is little doubt that this technique, wherever applicable, such as in ground mapping application, yields results far more spectacular than those obtainable from other data processing antennas and does not involve serious tradeoffs in important performance characteristics such as SNR. The great merits of the technique are probably the main reason for the 10-year delay in their appearance in the published literature. In fact, some of the material on signal processing is still classified.

The possibility of obtaining synthetic apertures was recognized at least as early as 1951 [Sherwin et al., 1962], and in 1952 the principle was experimentally demonstrated, with actual high resolution maps being obtained in 1953. The resolution was improved by a factor of 10 beyond the one obtainable in the conventional mode of operation. Most of the work since that time involved the development of data processing equipment for the implementation of synthetic arrays. One of the most important requirements is the preservation of phase over the whole integration period or over the total synthetic array length. Various sources contribute to deviations from the exact phase required for optimum processing. These include equipment instability, erratic vehicle movements, propagation distortions due to the medium, and Fresnel zone effects because of the extreme length of the synthetic aperture. Some of these effects may be compensated by appropriate processing, while others, particularly the random phase errors (except for those caused by vehicle motion which can be compensated), will degrade the synthetic array performance in various ways [Greene and Moller, 1962]. The most significant phase deviation is caused by the Fresnel zone effect and requires a focusing process to prevent destructive interference for arrays beyond a certain length. In fact, the various synthetic arrays are divided into two broad classes, the focused and unfocused arrays. The focused arrays require much more complicated processing systems than the unfocused ones, particularly since the focusing function is range dependent. The resolution obtained, however, when focusing is performed is theoretically unlimited (up to the uncertainty limit) and even practically far in excess of that of the unfocused arrays. An excellent paper presenting the patterns produced by both focused and unfocused synthetic arrays is by R. C. Heimiller [1962]. General expressions are derived, and numerous performance curves are given for various ranges, frequencies of operation, and array lengths. A somewhat simplified comparison between the resolution of conventional and synthetic arrays is presented by Cutrona and Hall [1962]. The discussion and results given in this article as well as the one by Cutrona et al. [1961], are mostly qualitative, but helpful in understanding the principles involved in a synthetic array. The synthetic array performance may be described, as was done above, in terms of an interference pattern produced by the coherent addition of various returns. An alternate explanation of the performance of a synthetic array is in terms of the Doppler characteristics of the return signal. It can be seen quite easily that the return from a target which at a given instant is located exactly on a normal to the flight path of the radar will have at that instant no velocity component towards the radar and thus will experience no Doppler shift. A target which is in the same range at the same interval but at a slightly different angle to the flight path will return a Doppler shifted signal. The ability to resolve the two returns depends on the smallest separation bandwidth detectable by the system, which in turn is inversely proportioual to the integration time allowed. Thus, the Doppler resolution is directly proportional to the total synthetic array length which coincides with the considerations based on interference patterns. A paper discussing various representations of the synthetic array process is by H. L. McCord [1962]. The early development of the concepts and the experimental demonstrations including the resulting high resolution maps are presented by Sherwin et al., [1962]. Also, the signal processing used and some of the actual equipment are described both by Sherwin et al., [1962] and Cutrona et al., [1961]. The article by Sherwin is very helpful in understanding the implementation of some of the original processing technique. As mentioned earlier, uncompensated phase errors may be expected to present the major limiting factor in the resolution capability of the synthetic array.

This problem is investigated by Green and Moller [1962], and various effects of random phase errors of prescribed statistical characteristics are presented graphically. The effects computed include degradation of gain, beam width and sidelobe structure, and beam-pointing scanning effects. The effect of random tropospheric phase errors on synthetic array is considered by Rondinelli and Zeoli [1962]. Both theoretical and experimental results are presented.

An interesting application of the synthetic array principle is in the achievement of simultaneous scanning. If the elements of a fixed linear array are successively excited, simulating a traveling source, the Doppler effects discussed above are produced, i.e., one can distinguish the returns from different angular locations on the basis of their varying frequency characteristics. This technique was successfully implemented both in the electromagnetic field (1961) and in acoustics (1962). An important shortcoming of the system is its low gain, but it also provides advantages over other scanning methods [Hewish and Shanks, 1962; Kummer et al., 1962].

Summary

The last three years saw an increasing interest in unconventional techniques for satisfying the demands imposed on antenna systems. The synthetic array is an excellent example of what can be achieved if an antenna system is optimized for a particular application, i.e., ground mapping by means of a moving vehicle. This type of specialized solution will become more common as the conversion of electromagnetic (or acoustic) signals into required data is considered as a single continuous process which is to be optimized jointly for the desired final objective. The generalization of the analytical treatments to include at once all dimensions of interest and the inclusion of processing as part of the antenna proper are symptomatic of this development. The concepts of feedback and self-adaptiveness will become familiar in antenna design, and as the characteristics of various data processing arrays are better assessed they will provide the versatility of solutions needed for the ever increasing variety of objectives.

References

Band, H. E. (Sept. 1961), Non-linear techniques in antenna theory, Pickard and Burns, Inc., Final Rept. on Contract No. AF 19(604)–7275.

Bracewell, R. N. (Jan. 1961), Interferometry and the spectral sensitivity island diagram, IRE Trans. Ant. Prop. **AP–9,** 59–67.

Bracewell, R. N., and J. D. Roberts (Dec. 1954), Aerial smoothing in radio astronomy, Australian J. Phys. **7,** No. 4, 615–640.

Breese, M., et al. (Dec. 1961), Phase-locked loops for electrically scanned antenna arrays, IRE Trans. Space Electron. Telemetry **7,** 95–100.

Breese, M., and P. J. Sferrozza (1962), A self-focusing antenna, IRE Wescon Conv. Record **6,** paper 12.2.

Chisholm, R. M. (1963), Transfer functions and the resolving power of radio telescopes, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 1281–1284 (Pergamon Press, New York and London).

Cutrona, L. J., and G. O. Hall (Apr. 1962), A comparison of techniques for achieving fine azimuth resolution, IRE Trans. Military Electron. **MIL–6,** 119–122.

Cutrona, L. J., W. E. Vivian, E. N. Leith, and G. O. Hall (Apr. 1961), A high resolution radar combat-surveillance system, IRE Trans. Military Electron. **MIL–5,** 127–131.

Davenport, L. C., and C. J. Drane (1961), Correlation antennas with non-uniformly spaced elements for incoherent sources, IRE Wescon Conv. Record **5,** paper 11.2.

Drane, C. J., and G. B. Parrent, Jr., (Mar. 1962), On the mapping of extended sources with non-linear correlation antennas, IRE Trans. Ant. Prop. **AP–10,** 126–130.

Freeman, J. J. (Aug. 1960), A systematic error in underwater acoustic direction finding, J. Acoust. Soc. Am. **32,** 1025–1027.

Gabriel, W. F. (Oct. 1961), Pattern measurements on an experimental compound interferometer antenna, Stanford Res. Inst. SR 1 on Contract AF 19(604)–8059.

Greene, C. A., and R. T. Moller (Apr. 1962), The effect of normally distributed random phase errors on synthetic array gain patterns, IRE Trans. Military Electron. **MIL–6,** 130–140.

Heimiller, R. C. (Apr. 1962), Theory and evaluation of gain patterns of synthetic arrays, IRE Trans. Military Electron. **MIL–6,** 122–130.

Hewish, A., and H. E. Shanks (Jan. 1962), Time-modulated antennas and the technique of aperture synthesis, IRE Trans. Ant. Prop. **AP–10,** 97.

Jacobson, M. J. (Aug. 1961), Optimum envelope resolution in an array correlator, J. Acoust. Soc. Am. **33,** No. 8, 1055–1060.

Jacobson, M. J. (July 1962), Space-time correlation in spherical and circular noise fields, J. Acoust. Soc. Am. **34,** No. 7, 971–978.

Jacobson, M. J., and R. J. Talham (July 1960), Angular deviation in directional receiver correlation, J. Acoust. Soc. Am. **32,** No. 7, 810–820.

Jacobson, M. J., and R. J. Talham (Apr. 1961), Comparison analysis of four directional receiver correlators, J. Acoust. Soc. Am. **33,** No. 4, 518–526.

Ksienski, A. (Oct. 1961a), Signal processing antennas, part I, Microwave J. **IV,** 77.

Ksienski, A. (Nov. 1961b), Signal processing antennas, part II, Microwave J. **IV,** 87.

Ksienski, A. (1963), Spatial frequency characteristics of finite aperture antennas, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 1249–1265 (Pergamon Press, New York and London).

Ksienski, A., et al. (June 1962a), Very high resolution techniques, Hughes Aircraft Co. Rept. No. 1, Contract DA–36–(039)–SC–90772.

Ksienski, A., et al. (Dec. 1962b), Very high resolution techniques, Hughes Aircraft Co. Rept. No. 2, Contract DA–36–(039)–SC–90772.

Kummer, W. H., et al. (Jan. 1962), Time domain antenna techniques, Hughes Aircraft Co. Tech. Rept, Contract AF 30(602)–2410.

Linder, Isham W. (May–June 1961), Resolution characteristics of correlation arrays, J. Res. NBS **65D** (Radio Prop.) No. 3, 245–252.

Linfoot, E. H. (Oct. 1955), Information theory and optical images, J. Opt. Soc. Am. **45,** No. 10, 808–819.

Lo, Y. T. (Oct. 1961), On the theoretical limitation of a radio telescope in determining the sky temperature distribution, J. Appl. Phys. **32,** No. 10, 2052.

MacPhie, R. H. (Dec. 1962), Optimum cross correlation radar system, Proc. IRE **50,**[4] 2508.

MacPhie, R. H., M. E. Pedinoff, and A. A. Ksienski (Sept. 1962), Comments on multiple target response of data-processing systems, IRE Trans. Ant. Prop. **AP–10,** 642.

Mattingly, R. L. (Apr. 1960), Nonreciprocal radar antennas, Proc. IRE **48,** 795–796.

Mays, B. R., and D. K. Cheng (June 1960), Output signal to noise, IRE Nat. Convention on Military Electronics.

McCord, H. L. (Apr. 1962), The equivalence among three approaches to deriving synthetic array patterns, IRE Trans. Military Electronics **MIL–6,** 116–119.

Meyers, J. J. (Jan. 1960a), Antenna image quality evaluation, Part I. By an optical simulation method, IRE Trans. Ant. Prop. **AP–8,** 78–82.
Meyers, J. J. (Jan. 1960b), Antenna image quality evaluation, Part II. By a mechanical observer, IRE Trans. Ant. Prop. **AP–8,** 83–87.
Pedinoff, M. E., and A. A. Ksienski (Mar. 1962), Multiple target response of data-processing antennas, IRE Trans. Ant. Prop. **AP–10,** 112–125.
Pigott, M. T., and D. C. Whitmarsh, and J. L. Brown, Jr., (Mar. 1962), Scanned line hydrophone method for determining angle of arrival of sound in water, J. Acoust. Soc. Am. **34,** No. 3, 319–328.
Price, O. R. (June 1960), Reduction of sidelobe level and beamwidth for receiving antennas, Proc. IRE **48,** 1177.
Rondinelli, L. A., and G. W. Zeoli (Mar. 1962), Evaluation of the effect of random tropospheric propagation phase errors on synthetic array performance, Hughes Aircraft Co. Tech. Memo 699.
Ryle, M., and A. Hewish (1960), The synthesis of large radio telescopes, Monthly Notices of the Royal Astronomical Soc. **12,** No. 3, 220–230.
Schrader, J. H. (June 1962), Receiving system design for the arraying of independently steerable antennas, IRE Trans. Space Electron. Telemetry **8,** 148–153.
Shanks, H. E. (Mar. 1961), A new technique for electronic scanning, IRE Trans. Ant. Prop. **AP–9,** 162–170.
Sherwin, C. W., J. P. Ruina, and R. D. Rawcliffe (April 1962), Some early developments in synthetic aperture radar systems, IRE, Trans. Military Electron. **MIL–6,** 111–115.
Spitz, E. (June 1959), Supergain in volumetric antennas, Tech. Rept. No. AFCRC–TR–59–194.
Urkowitz, H., et al. (Oct. 1962), Generalized resolution in radar systems, Proc. IRE **50,**[3] 2093–2106.
Walter, H. (1961), On basic analogies and principal differences between optical and electronic information, Prog. in Optics **I,** 157–209.
Welsby, V. G. (July 1961), Multiplicative receiving arrays, J. British IRE **I,** No. 1, 5–12.
Woodward, P. M. (1953), Probability and information theory with application to radar (Pergamon Press, London).
Young, G. O., and A. A. Ksienski (April 1961), Signal and data processing antennas, IRE Trans. Military Electron. **MIL–5,** 94–102.

Part 2. Statistical Problems in Electromagnetics

2.1. Rough Surface

W. S. Ament [1]

The archtypal, rough surface is that of a tideless sea. The sea receives plane-wave illumination as from a distant elevated radar at grazing angle θ, at a surface point observed from above by a distant receiving antenna at grazing angle θ', the relative azimuth and the polarizations being left implicit. A description of the surface is given in terms of appropriate statistics, the simplest being the rms surface height H above the mean (sea level) plane, a length L characteristic of the dominant ocean wavelengths, time-lagged correlations of surface heights, slope skewness due to surface winds, and other second-order statistics. Basic problems are to calculate, as averages over the surface statistics, (a) the interferometrically measurable specular reflection coefficient $R=R(\theta)$ with phase referred to the mean plane, (b) the closely related surface boundary conditions effective for normal modes propagating over the surface, (c) the (dimensionless) scattering parameter $\sigma(\theta, \theta')$, or bistatic radar area per unit area of mean plane, which goes into the radar equation for predicting mean received power, (d) radar and geometric parameters and data-processing methods permitting best inferences of oceanographically interesting data. This is also the problem in radar terrain surveillance and planetary radar design.

Ideally, one would solve Maxwell's equations for each conformation of the surface, from the appropriate field or power flux at the receiver, and average over the statistics controlling the conformations. The intractable initial problem leads one to deal with simplified model surfaces and to use various methods of premature averaging. The chief models are:

(1) The actual surface is taken as perfectly conducting and the statistics are simplified.

(2) The mean plane is assumed perfectly conducting, with particular objects or "bosses" randomly distributed over the plane to resemble waves.

(3) The mean plane is the mathematical locus of randomly distributed objects, this two-dimensional random array being isolated in space. A major subclass of this is (3a), in which the objects are semi-infinite parallel structures, such as wires or parallel plates with ends or edges in the mean plane.

(4) The mean plane bounds a half-space filled with a statistically described multiply scattering medium. Major advances with this model are described elsewhere in this report.

In 2, 3, 4 the scattering parameters of the individual objects are assumed known and the problem is one of properly accounting for interactions before or during the eventual averaging. Some phenomenological models highly useful for organizing and explaining radar data in terms of oceanographic surface parameters are discussed in the Commission II report.

Dimensionless numbers important in the problem are kH, kL, H/L, $Ra=kH\sin\theta$, $Z\equiv\sin^2\theta/kL$, $Z'\equiv\sin\theta\sin\theta'/kL$, where $k=2\pi/$(radar wavelength in air). When both kH and kL are small, quasi-static methods apply. This is the case for estimating losses in rough waveguides. When Ra is definable and small, the surfaces satisfies the "Rayleigh criterion" for smoothness, and specular reflection often dominates the scattered energy. Radar resolution is high and the scattering problem difficult for values of Z, Z' near unity; here diffraction over one wave crest becomes important in the field illuminating the next crest. The useful range of phenomenological models is sometimes extended to regions of small Ra by considering a wave reflected from the mean plane or from the locally smoothed surface as part of a coherent illuminating field, but this practice breaks down for $Z\sim 1$. This review deals principally with the theoretically interesting and intractable problems in which the excitations and the scattering are controlled by interactions among surface elements.

Methods

One can (in principle) write exact integral equations with Green's function kernels for particular conformations of model surfaces, and variational expressions [Meecham, 1956] of the Schwinger type for scattering into particular antennas, but the taking of averages poses unsolved difficulties. A variational scheme retaining its efficacy for optimum field or power estimates throughout the necessary averaging would form a major advance. One is driven to a self-consistent calculation: a parameterized position-dependent excitation is assumed for all parts of the surface. This establishes the scattered field, in the neighborhood of a surface point p, as a superposition of scatterings by individual objects or parts of the surface. This superposition is then averaged and added to the originally incident field to give the average excitation about p. The parameters of the beginning excitation's representation are then adjusted so that the represented and calculated excitations are identical. (Recent improvements in this self-consistent method are outlined in the section on multiple scattering. Wiener-Hopf methods are useful here with models 3a and 4.)

Having estimated a plausible excitation of, and therefore the scattering from, an object at p, the problem becomes one of averaging the scattering to get σ and R. With no nonreciprocal elements in the surface, reciprocity theorems apply: imagine a dyadic Green's function G appropriate to a particular *fixed* surface with the object or portion at p removed.

[1] Electromagnetic Research Corporation, College Park, Md.

With this G one can show that the partial contribution ApB of an object at p to a field at B due to a radar at A is the same as BpA, the partial field measured on interchanging transmitter and receiver. It follows that this "local reciprocity" is true for *average* fields and powers, and that the scattering from a surface point partakes of interactions of the same type and importance as the excitation.

General criteria for evaluating proposed solutions to rough surface problems in "statistical reciprocity": Reversing incident and scattering directions should produce no change in σ or in R, for surfaces built of reciprocal elements. A more general criterion is energy conservation, which demands that the incident minus the specularly reflected power, per unit area of the mean plane, be accounted for in the total randomly scattered power implied in σ, plus that calculated as absorbed in the surface under the unit area. A third criterion, applying immediately to model 1 and to 3a and 4 when the objects have finite losses, is that coherent and random power fluxes die out with distance beneath the mean plane. This is so obvious with model 1 that it should be examined for suggestions as to calculation methods.

The second law of thermodynamics is violated unless $\sigma(\theta, \theta')$ falls off as θ' or faster for $\theta' \rightarrow 0$; solutions satisfying reciprocity and within some measure of meeting the energy criterion would appear necessarily to have the correct behavior here [Ament, 1956]. Next, fields and currents estimated or assumed on parts of the surface should satisfy locally applicable criteria of diffraction theory such as Meixner's [1949] edge condition and Raisbeck's passivity criterion [Richards, 1959, p. 97]. (For model 3a and $Z \sim 1$, the edge condition should have a central importance.) Finally, extrapolations to nature must agree with observations; some of the empirical data are found in the Commission II report.

The foregoing discussion is concerned with averages taken over regions large compared with L^2; the more detailed averages necessary for the radar resolution, and for time-dependence backscatter, seem to have been treated only phenomenologicaly.

We turn to some conjectures and unsolved problems (beyond the methodological ones implied above) that should be within grasp. Calculations for versions of model 1, 3a, and 4 yield $R(\theta) \rightarrow -1$ and the factorization $\sigma \sim f(\theta)f(\theta')$ in the small θ, θ' limit. Plausibility arguments suggest that this "grazing factorization" is universally true when $R(\theta) \rightarrow -1$, but can one prove it? Chandrasekhar's exact albedo calculations [1960] for dilute versions of model 4 "factor" differently, but, for these, $R \equiv 0$. Next, assume an ocean with dielectric constant fractionally less than that of the air above, permeability being unity everywhere. For dead calms, reflection at the mean plane is then total for θ less than some small critical angle θ_0. Assuming rounded waves of large kL, kH, and kx (rms radius of curvature), what σ do we get for $\theta_0 \leq \theta \leq 2\theta_0$, knowing that double reflections can take place, both with total reflection by ray theory? This problem is basic for wave-layer theories of tropospheric scatter and there are ionospheric equivalents.

We turn to specific papers. Ament [1960] introduces the foregoing reciprocity concepts and raises the problem of "grazing factorization." For a tilted parallel-plate version of model 3a, he uses a self-consistent method to find approximate currents in the plates, and local reciprocity to find a reciprocity-satisfying σ obeying "grazing factorization." But the edge conditions and energy conservation are not satisfied, and R changes on reversal of incident and specular direction. For a model 4 surface with isotropic scatterers and scalar waves, the method yields better results which compare informatively with Chandrasekhar's exact albedo calculations [1960]. His final paragraph, arguing qualitatively that grazing radar backscatter should always be independent of polarization, uses an assumption suggesting that radar resolution is also independent of polarization; this is contrary to observations with oceanographic radars.

Marsh [1961a, b] writes the scattered field as a superposition of upgoing plane waves of various real horizontal components of propagation vector, and asks that the superposition cancel the downgoing plane incident wave at each point of the model 1 surface, which is specified by a spectrum and rms surface height H. He expands in powers of H, finds a process, based on Fourier transform theory and expressed in the form of iterated operators, for obtaining coefficients of H^n in the scattered-wave spectrum. Putting the results back together in a formal expression involving operators, he has a result amenable to machine methods for finding specular and random field components to arbitrary preassigned accuracy. The results satisfy reciprocity, energy conservation, and grazing factorization [private communication]. While the validity of neglecting downgoing components of the scattered field between waves is suspect [Lippmann, 1953], the neglect may be rectifiable; meanwhile the paper appears the first to develop the full power of transform methods in rough surface problems.

The only known rough surface model for which all scattered waves are everywhere upgoing is one having an impedance boundary condition which varies randomly over the mean plane. Hessel [1960a, b] and Oliner have treated a sinusoidally varying reactive impedance boundary condition and obtain exact results in full detail. Their study is directed at traveling-wave antenna applications so they do not consider the random case. Here one would have the difficulty of assigning statistics so that the surface is everywhere electromagnetically passive, i.e., satisfies Raisbeck's criterion everywhere.

Senior [1960a] discusses an impedance boundary condition effective at imperfect and certain rough surfaces in terms of the impedance ratio η of local impedance of a medium to the free-space impedance. He shows that for plane and slowly curving boundaries between free space and a medium of high refractive index, having small relative variations of dielectric constant only, the average impedance boundary is best described in terms of an average η. Senior's second paper [1960b] treats a model 1 surface of small-scale isotropic roughness. His main result is a local 2×2 impedance matrix connecting the components of E and H tangential to the mean plane. The matrix, expressed in terms of an average η, diagonalizes when preferred axes are chosen; the results seem not unlike those of Biot [1957, 1958a, 1958b, 1960] for a small-scale model 2 surface. But the merit of Senior's form is that it applies to curved surfaces. To estimate the effect of small-scale roughness on backscatter from large spheres, Hiatt, Senior, and Weston [1960] average Senior's matrix boundary condition over all incidence angles and obtain a scalar impedance boundary condition expressed in terms of an effective η: $\eta \sim ikH^2/L$. From this condition they estimate the coherent backscatter field through terms linear in η. They compare the obsolute-squared result with measured backscattered powers, but give no theoretical estimate of the nonspecular scattered components in their measured backscatter or elsewhere.

Although introducing no new diffraction theoretic ideas, Chen and Peake [1961] develop the basic concepts in the closely related problem of the angular spectrum of thermal radiation from a rough ground surface of temperature T which is illuminated by prescribed thermal sources distributed over the sky. Insofar as the surface elements are lossy, absorbed radiation must reappear in the form of heat radiation, and the problem is correspondingly more complex than the diffraction-theoretic one of accounting for the remnants of the incident radiation only.

Twersky [1962a, b, c, d] gives for models 2, 3, and 4 several formalisms for self-consistent calculations of the specular reflection coefficient R and the mean coherent excitation of general objects of the surface. Treating the resulting scattered waves as traveling in object-free regions, he finds energy conserved, but his σ's appear valid only at high angles. His R's appear as the natural consequence of his self-consistent approach and are conjectured to be automatically correct when the scattering by the individual objects is correctly calculated.

Twersky's papers and earlier summary [1960] list most of the known earlier papers in the rough surface field. Other bibliographies have been prepared by Bachynski [1959], Hagn et al. [1961], Lysanov [1958], Marsh [1962], and Wolff [1960].

References

Ament, W. S. (1956), Forward and backscattering from certain rough surfaces, IRE Trans, Ant. Prop. **AP–4,** 369.

Ament, W. S. (1960), Reciprocity and scattering by certain rough surfaces, IRE Trans, Ant. Prop. **AP–8,** 167.

Bachynski, M. P. (1959), Microwave propagation over rough surfaces, RCA Rev. **20,** 308.

Beard, C. I. (1961), Coherent and incoherent scattering of microwaves from the ocean, IRE Trans. Ant. Prop. **AP–9,** 470.

Biot, M. A. (1957), Reflection on a rough surface from an acoustic point source, J. Acoust. Soc. Am. **29,** 1193.

Biot, M. A. (1958a), On the reflection of acoustic waves on a rough surface, J. Acoust. Soc. Am. **30,** 479.

Biot, M. A. (1958b), On the reflection of electromagnetic waves on a rough surface, J. Appl. Phys. **29,** 998.

Biot, M. A. (1957), Some new aspects of the reflection of electromagnetic waves on a rough surface, J. Appl. Phys. **28,** 1455.

Bracewell, R. N. (1961), Tolerance theory of large antennas IRE Trans. Ant. Prop. **AP–9,** 49.

Bracewell, R. N. (1960), Antenna tolerance theory, Statistical Methods of Radio Wave Propagation, 179 (Pergamon Press, N.Y.)

Chandrasekhar, S. (1960) Radiative Transfer, (Dover Publications, N.Y.)

Chen, S.N. C., and Peake, W. H. (1961), Apparent temperatures of smooth and rough terrain, IRE Trans. Ant. Prop. **AP–9,** 567.

Cosgriff, R. L., Peake, W. H., and Taylor, R. C. (1960), Terrain scattering properties for sensor system design, Terrain handbook, Ohio State University Engineering College, Columbus, Ohio.

Daniels, Fred B. (May–Aug. 1961), A theory of radar reflection from the moon and planets, Geophys. Res. **66,**[2] 1781.

Edison, A. R. (1960), Radar terrain return measured at near-vertical incidence, IRE Trans. Ant. Prop. **AP–8,** 246.

Gossard, E. E. (1962), The reflection of microwaves by a refractive layer perturbed by waves, IRE Trans. Ant. Prop. **AP–10,** 317.

Hagn, George H., Nielson, Donald H., and Smith, Frances H. (June 1961), Back-scatter literature survey, Stanford Research Institute Project No. 3311.

Hayre, H. S. (1961), Radar scattering cross section applied to moon return, Proc. IRE **49,** 1433.

Hayre, H. S., and Moore, R. K. (1961), Theoretical scattering coefficient for near vertical incidence from contour maps, J. Res. NBS **65D** (Radio Prop.), No. 5, 427.

Hayre, H. S. (1961), Scatter theories and their applications to lunar radar return, University of New Mexico PR–33, p. 1, Albuquerque, N.M.

Hayre, H. S. (1962), Roughness and radar return, Proc. IRE **50,** 223.

Hessel, Alexander (1960), Guiding and scattering by sinusoidally-modulated reactance surfaces, AFCRL–130, PIBMRI 825–60, p. 1.

Hessel, A., and Oliner, A. A. (1960), Wave propagation in a medium with a moving sinusoidal disturbance, AFCRL–102, PIBMRI RP–880–60, p. 1.

Hiatt, R. E., Senior, T. B. A., and Weston, V. H. (1960), A study of surface roughness and its effect on the backscattering cross section of spheres, Proc. IRE **48,** 2008.

Lippmann, B. A. (1953), Note on the theory of gratings, J. Opt. Soc. Am. **43,** No. 5, 408.

Katzin, M., Wolff, E. A., Katzin, J. C. (1960), Investigation of ground clutter and ground scattering, AFCRC–TR–60–127, p. 1.

Lysanov, Iu, P. (1958), Theory of the scattering of waves at periodically uneven surfaces, Sov. Phys. Acoust. **4,** 1.

Marsh, H. W. (1961a), Exact solution of wave scattering by irregular surfaces, J. Acoust. Soc. Am. **33,** 330.

Marsh, H. W., Schulkin, M., and Kneale, S. G. (1961b), Scattering of underwater sound by the sea surface, J. Acoust. Soc. Am. **33,** 334.

Marsh, H. W., and Schulkin, M. (1962), Shallow-water transmission, J. Acoust Soc. Am. **34,** 863.

Marsh, H. W. (1962), Sea surface statistics deduced from underwater sound measurements, preprint Avco Corporation, New London, Conn.

Marsh, H. W. (1962), Non-specular scattering of underwater sound by the sea surface, Preprint, Avco Corporation, New London, Conn.

Marsh, H. W. (1962), Sound reflection and scattering from the sea surface, Preprint, Avco Corporation, New London, Conn.

Marsh, H. W. (Sept. 1962), Underwater sound reverberation—a survey report, Avco Corporation, New London, Conn. (available from Code 468, Office of Naval Research).

Meecham, W. C. (1956), Variational method for the calculation of the distribution of energy reflected from a periodic surface, I., J. Appl. Phys. **27,** 361.

Meixner, J. (Sept. 1949), The edge condition in the theory of electromagnetic waves at perfectly conducting plane screens, Ann. Physik **6,** 2.

Oliner, A. A., and Hessel, A. (Spec. Supplement 1959), Guide waves on sinusoidally modulated reactance surfaces, IRE Trans. Ant. Prop. **AP–7,** S201.

716–797—64——8

Richards, P. I. (1959), Manual of mathematical physics (Pergamon Press, N.Y.)
Senior, T. B. A. (1960a), Impedance boundary conditions for imperfectly conducting surfaces, Appl. Sci. Res. Sect. B **8,** 418.
Senior, T. B. A. (1960b), Impedance boundary conditions for statistically rough surfaces, Appl. Sci. Res. Sect. B **8,** 437.
Twersky, V. (1960), On multiple scattering of waves, J. Res. NBS **64D** (Radio Prop.), No. 6, 715.
Twersky, V. (1962a), Multiple scattering by arbitrary configurations in three dimensions, J. Math. Phys. **3,** 83.
Twersky, V. (1962b), Multiple scattering of waves and optical phenomena, J. Opt. Soc. Am. **52,** 145.
Twersky, V. (1962c), On a general class of scattering problems, J. Math. Phys. **3,** 716.
Twersky, V. (1962d), On scattering by random distributions, I. Free-space scatterer formalism; II.; Two-space scatterer formalism, J. Math. Phys. **3,** 700.
Wolff, E. A. (1960), A review of theories and measurements of radar ground return, AFCRC–TN–60–173, Report No. CRC 5182–2.

2.2. Electromagnetic Wave Propagation in a Random Medium*

W. C. Hoffman*

1. Introduction

The phenomenon of wave propagation through a random medium has, historically speaking, two main origins. One stems from statistical mechanics, and accordingly, as classical or quantum models are assumed, is capable in principle at least, of explaining all electromagnetic phenomena in a material medium. In practice this approach tends to be limited to one single aspect of "randomness," viz, configurations of randomly placed scatterers, and to the determination of one single parameter, usually the average value of some field quantity associated with the incident plus scattered fields. Only this approach seems to be adequate for analyzing resonance phenomena, such as fluorescence, Doppler broadening, resonance absorption, anomalous dispersion, and the like.

The other approach, essentially macroscopic in character, emerged from the problem of scattering of acoustic and electromagnetic radiation by turbulent media. The constitutive parameters of the medium are not derived from first principles but are taken as given, i.e., to constitute a stochastic process [1] of some sort. In this context, therefore, the electromagnetic field is a functional of the stochastic process representing the random refractive index of the medium. The theoretical problem thus has both a probabilistic as well as an electromagnetic character. In much of the past work the probabilistic side has often languished in comparison to the electromagnetic. A smooth though random variation of the medium is an essential characteristic of this model.

The coverage of this portion of the report is restricted to the macroscopic model, the so-called "perturbed continuum" problem [Bremmer, 1963] [2], while the electromagnetic phenomena associated with random configurations of scatterers are described in the section on multiple scattering by J. E. Burke and V. Twersky. By the same token, only problems involving the field within an unbounded medium are taken up here, the companion subject of scattering from random surfaces being discussed by W. S. Ament in another section of this report. Essentially then this part of the report deals with the theory of wave propagation in still, cold, random, continuous media at radio frequencies not greater than 3 Gc/s, and the contributions discussed below are by and large restricted to those made during the last triennium. The greater part of the work prior to the current triennium has been referenced in the summary "On Multiple Scattering of Waves," prepared by V. Twersky [1960], for the USA National Committee report to the XIII General Assembly, to which the interested reader is referred.

Section two of this report deals with the mathematical theory of electromagnetic waves in a random continuum. This is followed by sections on ray-tracing in a random medium, transport equations of various types, and what has come to be known as turbulent scatter. Reflection and transmission problems associated with random stacks of slabs have received considerable attention of late and an assessment is made of these developments under the same head as transport processes. Finally, we take up certain mathematical and statistical results which may have significance for the analysis of *r–f* radiation in random media.

2. Theory of Wave Propagation in a Random Continuum

Nearly all treatments of wave propagation in a random continuum have assumed harmonic time dependence, so that the reduced wave equation $\nabla^2 U + k^2(\mathbf{x}) U(\mathbf{x}) = 0$ can be employed rather than the full wave equation in $\mathbf{x}$ and t. This condition will be taken to apply in the sequel except where specific mention is made to the contrary. As pointed out by Silver [1963], this assumption requires that the phenomenon be essentially monochromatic, and such things as Doppler diffusion of frequency in hot, energetic media are thereby excluded. In general, use of the reduced wave equation appears valid whenever the time variation in the properties of the medium is much slower than the propagation time of the wave. A medium which is swirling just as rapidly as the wave is traveling will clearly have frequency shifts which invalidate the monochromaticity assumption. The probabilistic nature of a slowly swirling medium can be treated either through the time dependence of the stochastic process or by limiting consideration to realizations which are sufficiently far apart in time to be independent. However, as pointed out by Silver [1963], an ergodic theorem for relating time averages to expectations (ensemble averages) is required in either case.

Perhaps the most important single work on wave propagation in a random medium that has appeared during the last triennium is that due to J. B. Keller [1962]. The first portion of Keller's paper is noteworthy for a very lucid discussion of the problem and its context. He subsequently covers ray-tracing in a randomly inhomogeneous medium, and finally the exterior problem for the wave equation in a random medium. The latter portion of Keller's analysis is based on perturbation theory and contains many new and important results for the field and its first and second moments. The principal result is that to terms of the 3d order in the perturbation parameter

*Boeing Scientific Research Laboratories, Seattle, Wash.

[1] A stochastic process is a one parameter family of random variables, the parameter being time (as in random noise phenomena) or points in space or space-time, etc.

[2] The debt that the present writer owes to H. Bremmer for his definitive report on "Scattering by a Perturbed Continuum" will be clear to anyone who has read the latter.

ϵ the mean value of the field propagates as if in a medium of effective complex refractive index defined by

$$n^*=\left\{1+\epsilon^2<\mu^2>(1-2ik\int_0^\infty (e^{2ikr}-1)N(r)dr\right\}^{1/2}$$

where $\epsilon\mu(\mathbf{x})$ is the random component of the refractive index $n(\mathbf{x})$ and $N(|\mathbf{x}-\mathbf{x}'|)$ is the correlation function $<\mu(\mathbf{x})\mu(\mathbf{x}')>/<\mu^2>$.

Another treatment of the problem of radiation in a randomly inhomogeneous medium has been given by W. C. Meecham [1961]. Through appropriate use of Green's functions Meecham was able to determine the field generated by a point source in a medium having small point to point fluctuations of dielectric constant. Energy is found to be transferred from the average field to the fluctuation field at a rate dependent upon the low frequency components of the random fluctuation of the medium, and it is shown that the phase velocity of the average solution is decreased. The method is applied to the case of a medium fluctuation governed by a Gaussian correlation function. The average solution thus formed is in accord with the result of Foldy and Lax for the random configuration problem. For the one-dimensional case the formal procedures of the Foldy-Lax method have been put on a rigorous basis by J. Bazer [1959], who derived sufficient conditions for the approximation of the solution of the random configuration problem by the corresponding solution for a continuous distribution of scattering material. These approaches are also closely related to the work of Twersky [1962], discussed elsewhere in the Commission 6 report under the head of multiple scattering.

A rigorous probabilistic treatment of the electromagnetic field in a randomly inhomogeneous medium has been given by Hoffman [1959], based upon proper spectral representations for the stochastic process representing the refractive index. A generalized 3-dimensional Riccati equation is obtained and used to relate spectral representations of field and gradient of refractive index. On the other hand the reduced inhomogeneous wave equation is shown to be equivalent to an inhomogeneous Fredholm integral equation. The latter can be solved by a mean-square convergent Neumann series for the case of a Gaussian $n^2(\mathbf{x})$ provided the Neumann series for the nonrandom medium converges in the ordinary way. Thus multiple scattering of all orders can be taken into account in this formulation.

A theoretical treatment of the effect of medium irregularities of the order of a wavelength in size upon the field of an oscillating dipole has been given by E. C. Barrows [1963]. Barrows' technique consists in first expressing the spatial fluctuatious in permittivity as a Fourier integral, introducing the result into the Green's dyadic solution of the vector wave equation, and evaluating the result to within the Born approximation. The probabilistic nature of the Fourier integral representation is not mentioned, although an attempt is made to introduce "randomness" by means of a time average correlation matrix for the electric field itself.

3. Ray Paths in Random Media

A substantial portion of Keller's paper mentioned above [Keller, 1962] is devoted to determination of ray paths in a medium containing small random inhomogeneities. After determining the perturbation equations which govern such a ray path, Keller obtains formulas (valid for not too large path lengths) for the average ray position, equivalent path length, mean-square distance between the endpoints of a ray, the mean-square transverse displacement, and the ray diffusion coefficient. He then treats the propagation of rays as a Markov process starting from the appropriate Fokker-Planck equation and obtains results which agree with those of the perturbation equation approach within their mutual ranges of validity. Phase and amplitude fluctuations are then studied by means of normal congruences of rays and the method of characteristics, and the respective means and variances are calculated.

W. S. Ament [1960] has analyzed the effect of multiple scattering upon tropospheric forward scatter signals in terms of ray tracing through successive "blobs." An analog computer specifically designed for ray-tracing was employed and perturbations consisting of delta-function voltages of random amplitudes and zero means were introduced with random spacing. Ament proposed several problems of fundamental importance that bear directly, in the further treatment of the multiple scattering approach, upon the neglected phase coherence aspect of "blob" scattering.

4. Transport Equations for Field Quantities in a Random Medium

Although electromagnetic waves must obey a vector wave equation, certain quantities associated with a randomly scattered field, notably the probability density function for the ray position after n scatterings and the spectral density function for the scattered energy, must obey a diffusion equation. This approach assumes that the scattered electromagnetic field constitutes a stationary (strict sense) Markov process. The probability density function must therefore satisfy a Fokker-Planck differential equation (or alternatively, in integral form, the Chapman-Kolmogorov equation). In the United States this approach to the problem of scattering by a random continuum has been carried forward by D. S. Bugnolo [1960a, 1960b, 1961a]. The basic theory of Bugnolo's approach is set forth mainly in the first of the above references, in which he develops a transport equation for the spectral density function of a multiply scattered electromagnetic field and gives a detailed solution for the case of dielectric noise. The result is applied by Bugnolo [1960a and 1960b] to forward scatter in the troposphere and to laying down a statistical criterion for the validity of the usual single-scattering hypothesis. The spectral width increases with distance in the transport formulation, whereas this realistic feature is lacking in the usual single scattering theories of forward scatter. Bugnolo [1961a] has also employed a transport equation for the photon density function to analyze the effects of ionospheric multiple scattering upon radio star scintillation.

The method of "invariant embedding" [Bellman and Kalaba, 1960; 1961; Ueno, 1961] constitutes another aspect of the approach via transport equations. In this method functional equations are obtained for the reflection and transmission coefficients of a plane wave incident normally upon one or more slabs whose refractive indexes are random functions of distance from the interface. The functional equations in the single slab case turn out to be the well-known Riccati equations for the reflection and transmission coefficients of an inhomogeneous transmission line. Reflection from a stratified slab, each stratum having a different random wave number, has also been considered, and recurrence relations for the transmission and reflection coefficients obtained. Some attempt has been made to give a probabilistic cast to the results, but no real advance has yet been accomplished for wave propagation problems. The method has enjoyed notable success with respect to such transport phenomena as neutron scattering and radiative transfer in stellar atmospheres, but seems inherently incapable of taking into account diffraction effects or non-plane-parallel geometries.

Significant results with respect to the one-dimensional scattering problem for a random stack of dielectric slabs have been achieved by Kay and Silverman [1958]. These authors obtain a Neumann series for the mean-square transmission and reflection coefficients resulting from multiple scattering. An upper bound for the strength of scattering of any order from the random medium is used to show that these Neumann series converge much more rapidly, for a large number of slabs, than would be expected on the basis of non-random scattering. On the other hand the randomness of the scattering medium does not effectively reduce multiple scattering when the order of the scattering exceeds the number of slabs.

5. Turbulent Scatter

Interest in the problem of electromagnetic radiation in a randomly inhomogeneous continuum has been motivated principally by the phenomenon of turbulent forward scatter. It is not our intention to review here the theory of turbulent forward scatter. This subject properly belongs to the domain of Commissions 2 and 3, and several excellent review articles are in existence [Wheelon, 1959; Bremmer, 1963]. However, those aspects of turbulent forward scatter theory which are relevant to areas of interest to Commission 6 will be briefly surveyed here.

Three theoretical approaches to the problem of the turbulent forward scatter have been pursued: (i) extensions of the Booker-Gordon single scattering theory, (ii) partial reflection from atmospheric "platelets", and (iii) diffusion theories for the multiple scattering of signals. The latter have been discussed under sec. 4 above, and the only U.S. contributions to the second category have been made by Friis, Crawford, and Hogg [Bremmer, 1963], prior to the triennium covered in this report. There remains category (i): generalizations of the classical single scattering theory. Contributions in this area by Barrows [1963] and Wheelon [1959] have already been noted. In addition, Bowhill [1961a] has studied the case of a medium with inhomogeneities having different scales along three space axes. In this event the effect of the medium is *not* equivalent to a succession of thin phase-changing screens, which are independent, probabilistically speaking, of one another. The diffractive changes as the wave passes from one inhomogeneity to one arbitrarily nearby correlated with it renders invalid such an extension of the theory of emerging angular power spectrum. In a companion paper Bowhill [1961b] discusses the effect of a "random shallow phase screen" upon the transmission of radio waves. Bowhill's analysis applies only to a stationary Gaussian process. The results are expressed in terms of spatial spectra and correlation functions for the emergent wave. The effects of anisotropy of the diffracting screen and oblique incidence of the waves are also included in the analysis.

Yeh [1962] has investigated the propagation of spherical waves through a medium containing anistropic (elongated) inhomogeneities. The correlation function of the refractive index is taken to be a Gaussian function of position with ellipsoidal symmetry. A perturbation analysis applied to the scalar reduced wave equation then leads to first order expressions for the mean-square deviations and correlation functions of the phase and logarithmic amplitude.

A sort of turbulent scatter which depends upon the velocity distribution functions of the electrons and ions in a plasma has received much consideration recently in connection with "incoherent scatter" of high-power radio signals in the ionosphere. A discussion of American contributions to this field may be found in the section of the Commission 6 report devoted to plasma propagation.

6. Miscellaneous Theoretical and Statistical Complements

In a comprehensive discussion of the subject of scattering by a randomly inhomogeneous medium Silver [1963] brought out many of the difficulties which beset an adequate general theory. To generalize single particle scattering to a random configuration, a solution of the many body problem must be available and certain questions having to do with the ergodic hypothesis must be settled. If the medium is turbulent, it may well not be in the thermodynamic equilibrium, and this feature poses an additional complication for theories based on statistical mechanics. If on the other hand a randomly fluctuating continuum is assumed, then an approach to a generally valid theory of constitutive parameters is required, e.g., in such media as stellar atmospheres. Silver advocates an approach to problems of this second type through the polarization vector $\mathbf{P}$ and the probabilistic properties of the Fourier amplitudes representing the distribution of scattering elements. It is also noted that the assumption of monochromatic time dependence usually made requires that no Doppler diffusion of frequency take place. An ergodic theorem, relating temporal behavior to the ensemble of stationary configurations, thus seems required. Such a theorem would probably involve a hypothesis that time variations of the constitutive parameters are of a much lower order than the propagation time of the wave. Alternatively, from a theoretical standpoint, one could restrict attention to independent realizations, that is, thinking of the medium as a continuously swirling fluid, one would "freeze" the medium at times far enough apart so that the resulting configurations (realizations) of refractive index are independent in the probability sense.

The application of the ergodic theorem to problems of wave propagation through a random time-varying medium has been considered by T. J. Skinner [1961]. He showed that the time-varying medium can be replaced in the solution of the scalar problem by a time independent ensemble provided at the boundary of the time varying medium the scalar field $V(\mathbf{x},t)=A(\mathbf{x},t)\exp\{i\phi(\mathbf{x},t)+2\pi i\sigma t\}$ is such that

(i) $A(\mathbf{x},t)$ and $\phi(\mathbf{x},t)$ are ergodic;
(ii) The absolute value of any frequency component of either $A(\mathbf{x},t)\sin\phi(\mathbf{x},t)$ or $A(\mathbf{x},t)\cos\phi(\mathbf{x},t)$ is less than σ, the mean frequency of the source;
(iii) $V(\mathbf{x},t)$ is quasi-monochromatic.

D. S. Bugnolo [1961b] has given a heuristic discussion of the question of stationariness (in the probability sense) when the dielectric constant is a stochastic process in both space and time. He concludes in the tropospheric case that the permittivity must of necessity be a nonstationary process and proposes a redefinition of the expected value of the permittivity which would make the corresponding homogeneous vector wave equation consistent with the purely deterministic problem for an inhomogeneous atmosphere.

C. I. Beard [1961] in a clever experimental simulation of the idealized conditions of statistical mechanics has shown that in the "mid-field" region the phase quadrature components of the incoherent scattered field are *not* Rayleigh distributed.

Samuels and Eringen [1959/60] have studied the behavior of the nth order linear differential equation with random coefficients and find a phenomenon of mean-square instability under certain conditions. The problem of the dynamics of first order linear and nonlinear oscillators with random coefficient has been exhaustively studied by Kraichnan [1961] by means of an auxiliary set of random coupling coefficients. Bourret [1961] has investigated the angular response of a linear array to a scalar signal from a point source when the medium is slightly randomly inhomogeneous.

Significant statistical contributions (in the sense of statistical inference) to the propagation of electromagnetic waves in random media have been made by S. S. Siddiqui [1960, 1961, 1962]. M. J. Beran [1960] has derived the equations governing the propagation of the mth order correlation function for a scalar wave field, and studied the propagation of the second order correlation function between parallel planes in free space.

7. References

Ament, W. S. (1960), Modification of a ray-tracer for Monte Carlo prediction of multiple scattered fields, Statistical Methods in Radio Wave Propagation, ed. W. C. Hoffman, 184–196 (Pergamon Press, New York and Longdon).

Barrows, E. C. (1963), Correlation matrix for radio waves in the troposphere, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, Pt. 2, 743–746 (Pergamon Press, New York and London).

Bazer, J. (1959), Multiple scattering in one dimension, New York Univ. Math. Sci. Rept. EM–122.

Beard, C. I. (1961), Statistics of phase quadrature components of microwave field transmitted through a random medium, Sylvania Electronic Def. Labs. Tech. Memo., EDL–M402.

Beard, C. I. (Nov. 1963), Statistics of phase quadrature components of microwave field transmitted through a random medium, IEEE Trans. Ant. Prop. **AP–11.**

Bellman, R., and R. Kalaba (1960), Invariant imbedding and wave propagation in stochastic media, Electromagnetic Wave Propagation, ed. Desirant and Michaels, 243–252 (Academic Press, London).

Bellman, R., and R. Kalaba (Sept. 1961), Wave branching processes and invariant imbedding, Proc. U.S. Nat. Acad. Sci. **47,** No. 9, 1507–1509.

Beran, M. (1960), On the propagation of random radiation in free space, Statistical Methods in Radio Wave Propagation, ed. W. C. Hoffman, 93–98 (Pergamon Press, New York and London).

Bourret, R. C. (1961), Directivity of a linear array in a random transmission medium, J. Acoust. Soc. Am. **33,** No. 12, 1793–1797.

Bowhill, S. A. (Jan. 1961a), The scattering of radio waves by an extended randomly refracting medium, J. Atmospheric Terrest. Phys. **20,** 9–18.

Bowhill, S. A. (1961b), Statistics of a radio wave diffracted by a random ionosphere, J. Res. NBS **65D** (Radio Prop.), No. 3, 275–292.

Bremmer, H. (1963), Scattering by a perturbed continuum, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 665–699 (Pergamon Press, New York and London).

Bugnolo, D. S. (Mar. 1960a), On the question of multiple scattering in the troposphere, J. Geophys. Res. **65,** No. 3, 879–884.

Bugnolo, D. S. (July 1960b), Transport equation for the spectral density of a multiple-scattered electromagnetic field, J. Appl. Phys. **31,** No. 7, 1176–1182.

Bugnolo, D. S. (Jan. 1961a), Radio star scintillation and multiple scattering in the ionosphere, Trans. IRE Ant. Prop. **AP–9,** No. 1, 89–96.

Bugnolo, D. S. (Mar. 1961b), Stochastic processes and beyond-the-horizon propagation, Trans. IRE Ant. Prop. **AP–9,** 226–228.

Hoffman, W. C. (Dec. 1959), The electromagnetic field in a randomly inhomogeneous mediun, IRE Trans. Ant. Prop. **AP–7** (Spec. Supp.), S301–306.

Kay, I., and R. A. Silverman (1958), Multiple scattering by a random stack of dielectric slabs, Nuovo Cimento (10) **9** (Supp.), 626–45.

Keller, J. B. (1962), Wave propagation in random media, Proc. Symposia in Applied Mathematics, Hydrodynamic Instability **13** (Am. Math. Soc. New York, N.Y.).

Kraichnan, R. H. (Jan.–Feb. 1961), Dynamics of nonlinear stochastic systems, J. Math. Phys. **2,** No. 1, 124–148.

Meecham, W. C. (1961), On radiation in a randomly inhomogeneous medium, Space Tech. Labs. Rept. BSD–TR–61–36.

Samuels, J. C., and A. C. Eringen (1959–1960), On stochastic linear systems, J. Math, Phys. **38,** 83–103.

Siddiqui, M. M. (1960), The components of power appearing in the harmonic analysis of a stationary process, Statistical Methods in Radio Wave Propagation, ed. W. C. Hoffman, 112–116 (Pergamon Press, New York and London).

Siddiqui, M. M. (April–June 1961), Some properties of the empirical distribution function of a random process, J. Res. NBS **65B** (Math. and Math. Physics), No. 2, 117–127.

Siddiqui, M. M. (Mar.–April 1962), Some problems connected with Rayleigh distributions, J. Res. NBS **66D** (Radio Prop.), No. 2, 167–174.

Silver, S. S. (1963), Delineations and methodology in the subject of scattering by a statistically inhomogeneous medium, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 661–663 (Pergamon Press, New York and London).

Skinner, T. J. (Nov. 1961), Ergodic theorem in the solution of the scalar wave equation with statistical boundary conditions, J. Opt. Soc. Am. **51**, No. 11, 1246–1251.

Twersky, V. (Nov.–Dec. 1960), On multiple scattering of waves, J. Res. NBS **64D** (Radio Prop.), No. 6, 715–730.

Twersky, V. (July–Aug. 1962a), On scattering of waves by random distributions, I. Free-space scatterer formalism, J. Math. Phys. **3**, No. 4, 700–715.

Twersky, V. (July–Aug. 1962b), On scattering of waves by random distributions, II. Two-space scatterer formalism, J. Math. Phys. **3**, No. 4, 724–734.

Ueno, S. (April 1961), Stochastic equations in radiative transfer by invariant imbedding method, J. Math. Anal. Appl. **2**, No. 2, 217–222.

Wheelon, A. D. (Sept.–Oct. 1959), Radio-wave scattering by tropospheric irregularities, J. Res. NBS **63D** (Radio Prop.), No. 2, 205–233.

Yeh, K. C. (Sept.–Oct. 1962), Propagation of spherical waves through an ionosphere containing anisotropic irregularities, J. Res. NBS **66D** (Radio Prop.), No. 5, 621–636.

2.3. Partially Coherent Electromagnetic Fields

Francis J. Zucker [1]

This summary report begins where the previous URSI report on coherence theory [Parrent, 1960] left off. Activity during the past three years has been brisk. A conference held in 1960 [University of Rochester, 1961; O'Neill and Bradley, 1961] paid much attention to partial coherence, and the Proceedings of the 1962 URSI Copenhagen Symposium on Electromagnetic Theory and Antennas (Pergamon Press, Oxford, 1963) will contain six papers in the field. The proceedings of two 1963 meetings, the Third International Conference on Quantum Electronics (held in Paris) and the Symposium on Optical Masers (held at the Polytechnic Institute of Brooklyn) should be watched for articles on laser coherence and on the connection between coherence and quantum mechanics.

The basic reference is still Wolf [Born and Wolf, 1959]. An introduction to the present state of the art was written by Zucker [1963], and two comprehensive surveys [Mandel, 1963a; Gamo, 1963b] cover recent developments. A book on statistical optics [O'Neill, 1963] devotes two chapters to coherence topics, and another book, now in press [Beran and Parrent, 1963b] promises a detailed treatment of the entire subject.

We first consider applications of partial coherence theory to problems in microwaves and optics.

Aperture diffraction. The diffraction patterns of infinite slots [Parrent and Skinner, 1961] and circular apertures [Shore, 1963] have been calculated for a continuous range of aperture size to coherence interval ratios; as might be expected, the pattern approaches that of a dipole as the coherence interval becomes small compared to the aperture diameter. Schell [1961] pointed out that the partially coherent power pattern is given by the convolution of the coherent pattern with the Fourier transform of the degree of coherence, which suggests rapid graphical methods for antenna and aperture pattern determinations. The polarization aspects of partially coherent diffraction were examined by Karczewski and Wolf [1963], with the result that approximate evaluations known to give almost identical results for monochromatic radiation turned out to differ considerably in the polychromatic case.

Propagation in random media. By the ergodic hypothesis, it should be possible to establish an equivalence between the aperture pattern in a clear atmosphere for a partially coherent signal, and the aperture pattern in a turbid atmosphere for a coherent signal, provided the statistics of the partially coherent signal and of the turbid atmosphere are identical; the conditions under which this equivalence holds, however, have not been rigorously established. A formal attack on the propagation of partially coherent radiation through a turbid atmosphere (i.e., randomness in signal as well as in medium) was made by Parrent, Shore, and Skinner [1962], and specific applications by Skinner are in the offing.

Partial polarization. The original work in optics on the relation between degree of polarization, Stokes vector, and coherency matrix [Born and Wolf, 1959; Parrent and Roman, 1960] has been capably reviewed by Marathay [O'Neill, 1963], and adapted for antenna usage by Ko [1961a, b, c; 1962]. The response of an elliptically polarized antenna to quasi-monochromatic, partially polarized radio waves from an extended source has been worked out in detail. Ko clearly shows the advantages of this language in the design and evaluation of radioastronomical polarization measurements. The effect of partial polarization on the effective antenna aperture has also been considered [Tai, 1961; Ko, 1962].

Imaging. The optical theory of image formation has been generalized to include the effects of partial coherence [Parrent, 1961], which in turn has suggested a re-examination of the concept of resolution [Parrent and Rojak, 1963]. As a most useful application in the microwave field, it has been shown [Drane and Parrent, 1962] that data-processing antennas of the Drane-Covington type map extended incoherent sources (such as the radio sky) linearly in power, even though they employ nonlinear elements. Because of the finite aperture size, a sampling theorem originally due to Gabor [Miyamoto 1960, 1961] can be applied to the object and image, leading to a formulation of imaging in terms of finite matrices or equivalently, because of the finite number of degrees of freedom, to a formulation in terms of entropy transformations [Gamo, 1960; O'Neill and Asakura, 1961; O'Neill, 1963; Gamo 1963b].

The coherence aspects of two important non-Gaussian radiators have been investigated, the *blackbody* and the *maser* (and laser). An initial discussion of the blackbody by Bourret [1960] was given a broader coherence-language base by Kano and Wolf [1962] and Mehta [1963], and then related to its quantum mechanical background by Sarfatt [1963]. That the spatial coherence of maser light can be explained in terms of the general property of incoherent radiation becoming progressively more coherent as it passes through a periodic medium was discovered by Wolf [1963]. That maser radiation differs widely from thermal light is well known [Mandel, 1961a; Smith and Williams, 1962]; it is therefore a significant advance that at the Third International Conference on Quantum Electronics (Paris, 1963), Mandel was able to present a probability distribution for the radiation of a well-stabilized single-mode maser, from which the complete coherence properties should now be deducible.

The concept of coherence itself has been clarified and extended in several directions.

Clarifications of coherence. Skinner [1961] showed that arbitrarily narrow-band (but not monochro-

[1] Air Force Cambridge Research Laboratories, Bedford, Mass.

matic) radiation can be very incoherent, and Mandel and Wolf [1961b] demonstrated the converse, that to be coherent, radiation does not necessarily have to be very narrow-band. If the coherence function can be factorized into a purely space-dependent and a purely time-dependent term, Mandel [1961b] refers to the radiation as "spectrally pure"; he shows that light from an extended incoherent source has this property. The limiting cases of complete coherence and incoherence have been carefully analyzed, with the result that neither condition is physically realizable [Parrent, 1963; Beran and Parrent, 1963a, 1963b]. In preparation for the treatment of non-Gaussian sources, the concepts of bandwidth and coherence time have been carefully re-examined [Mandel and Wolf, 1962].

Generalizations of coherence. To describe the coherence properties of electromagnetic fields, the coherence function of scalar fields (or the coherency matrix of partially polarized plane waves) must now be replaced by three-by-three electric and magnetic coherency tensors [Roman and Wolf, 1960; Roman, 1961b; Karczewski, 1963]. The physical meaning of some of the differential equations connecting these tensors is still obscure. Source terms can be included in the field equations [Beran and Parrent, 1962; Roman, 1961a], but attempts at extending the equations to nonstationary ensembles [Kano, 1962] have not yet succeeded.

Transient interference. The limitation of the coherence concept to infinite time averages has inhibited its application to all cases in which time-variable interference effects might occur. The discussion on this point has been brisk [Golay, 1961; Bracewell, 1962; Neugebauer, 1962; Ready, 1962]. Mandel [1962b] contributed the fundamental observation that in the case of Gaussian sources, transient interference can be handled if the concept of "photon degeneracy" (the number of photons falling on a coherence area in the coherence time) is added to that of coherence, while for non-Gaussian sources, higher order correlation functions will have to be taken into account as well.

So far, only second-order correlation (i.e., coherence) effects have been discussed; we now turn to fourth-order effects, i.e., *intensity correlations.* It has been known for some time (Hanbury Brown and Twiss experiments) that intensity correlations can yield information about the spatial and spectral intensity distributions of sources; Wolf showed that they also yield polarization information [Wolf, 1960; Mandel and Wolf, 1961a; Mandel, 1963c]. Compared with the traditional (Michelson) second-order method, intensity correlation has the advantage of relative independence from atmospheric scintillations, but the disadvantage of yielding only the magnitude, not the phase of the coherence function; unless the intensity, spectral, or polarization distributions are known to be symmetric, they cannot therefore be fully determined. Gamo has devised an ingenious optical method for the direct measurement of the phase of the coherence function by injecting a coherent background; the method works best with intense and narrowband sources [Gamo, 1961a, 1961b, 1963a]. More attention has to date been paid to this method in optics [Mandel, 1963a] than in microwaves; the physics of optical square-law detectors and photon-coincidence counters has therefore been carefully investigated [Mandel, 1960; Mandel and Wolf, 1961a; Mandel 1963a, 1963c].

In intensity-correlation *spectroscopy* [Mandel, 1963b], Wolf showed that analytic properties of the coherence function often lead to a full determination of the spectral profile, even when Gamo's phase determination is not available [Wolf, 1962]. Forrester [1961] proposed an optical superheterodyne technique for improving the signal-to-noise ratio, with which Javan et al. [1961] then measured laser spectra. Spectral modulation in superposed, coherently modulated light beams, first discussed by Alford and Gold, has been extended to coherent but unmodulated beams by Givens [1961, 1962], and was shown by Mandel [1962a] to be a fourth-order correlation effect, like that of Hanbury Brown and Twiss.

For a *complete stochastic description* of electromagnetic radiation, the total probability distribution must of course be specified. In the Gaussian case, the probability distribution depends only on the phase coherence and on the degree of polarization [Mandel, 1963c], but in general all higher-order probability densities, and thus correlations, must be known [Beran and Parrent, 1963b]. Wolf showed at the Symposium on Optical Masers (Polytechnic Institute of Brooklyn, 1963) that a quantum-mechanical transcription by means of the correspondence principle of the higher-order correlation leads to the quantum-theoretical coherence concept already formulated by Glauber [1963; Mandel and Wolf, 1963]. The question as to the relation between coherence theory and quantum mechanics, raised some time ago by Purcell, Dicke, and Fano (see for example University of Rochester [1961]), was carefully examined by Sudarsham at the Brooklyn Symposium; it appears that, by working with the analytic signal and admitting negative as well as positive probability distributions, one obtains a stochastic theory of coherence which, at least for all linear effects, is isomorphic with the quantized description of radiation in terms of the density matrix (see also Sudarshan [1963]).

References

Beran, M. and G. B. Parrent (1962), On the equations governing the coherence of the radiation emitted by random charge and current distributions, J. Opt. Soc. Am. **52,** 98.

Beran, M. and G. B. Parrent (1963a), The mutual coherence of incoherent radiation, Nuovo Cimento **27,** 1049.

Beran, M. and G. B. Parrent (1936b), Theory of partial coherence (Prentice-Hall, New Jersey).

Born, M. and E. Wolf (1959), Principles of Optics, Ch. 10 (Pergamon Press, London and New York).

Bourret, R. C. (1960), Coherence properties of blackbody radiation, Nuovo Cimento **18,** 347.

Bracewell, R. N. (1962), Defining the coherence of a signal, Proc. IRE **50,** 214.

Drane, C. J. and G. B. Parrent (1962), On the mapping of extended sources with nonlinear correlation antennas, IRE Trans., Ant. Prop. **AP–10,** 126.

Fano, U. (1961), Quantum theory of interference effects in the mixing of light from phase-independent sources, An. J. Phys. **29,** 539.

Forrester, A. T. (1961), Photoelectric mixing as a spectroscopic tool, J. Opt. Soc. Am. **51,** 253.

Gamo, H. (1960), An aspect of information theory in optics, IRE 1960 International Convention Record, pt. 4, 189.

Gamo, H. (1961a), On the intensity interferometer with coherence background, Advances in Quantum Electronics, 252 (Columbia University Press, New York).

Gamo, H. (1961b), An intensity interferometer for partially polarized and partially coherent beams, J. Opt. Soc. Am. **51,** 1474.

Gamo, H. (1963a), Phase determination of coherence functions by the intensity interferometer, Copenhagen Symposium on Electromagnetic Theory and Antennas, June 1962, ed. E. C. Jordan, pt. 2, 801–810 (Pergamon Press, Oxford).

Gamo, H. (1963b), A matrix theory of partial coherence, Progress in Optics **3** (North-Holland Publishing Co., Amsterdam, in press).

Givens, M. P. (1962), Photoelectric detection of interference between two light beams having a large path difference, J. Opt. Soc. Am. **51,** 1030 (1961); also J. Opt. Soc. Am. **52,** 225.

Glauber, R. (1963), Photon correlations, Phys. Rev. Letters **10,** 84.

Golay, M. J. E. (1961), Note on coherence vs narrow bandedness in regenerative oscillators, masers, lasers, etc., Proc. IRE **49,** 958.

Javan, A., W. R. Bennett, Jr., and D. R. Herriott (1961), Population inversion and continuous optical maser oscillation in a gas discharge containing a He-Ne mixture, Phys. Rev. Letters **6,** 106–110.

Kano, Y. (1962), Equations governing the propagation of second-order correlations in non-stationary electromagnetic fields, Nuovo Cimento **23,** 328.

Kano, Y., and E. Wolf (1962), Temporal coherence of black body radiation, Proc. Phys. Soc. (London) **80,** 1273.

Karczewski, B. (1963), Degree of coherence of the electromagnetic field, Physics Letters **5,** 191.

Karczewski, B. and E. Wolf (1963), Polarization properties of the electromagnetic field diffracted from an aperture, Copenhagen Symposium on Electromagnetic Theory and Antennas, June 1962, ed. E. C. Jordan, Pt. 2, 797–799 (Pergamon Press, Oxford).

Ko, H. C. (1961a), Theoretical techniques for handling partially polarized radio waves, with special reference to antennas, Proc. IRE **49,** 1446.

Ko, H. C. (1961b), The use of the statistical matrix and the Stokes vector in formulating the effective aperture of antennas, IRE Trans. Ant. Prop. **AP–9,** 581.

Ko, H.C. (1961c), On the response of a radio antenna to complex radio waves, Proc. Nat. Electron. Conf. **17,** 500.

Ko, H. C. (1962), On the reception of quasi-monochromatic, partially polarized radio waves, Proc. IRE **50,** 1950.

Mandel, L. (1960), Detection of interference effects with incoherent light beams by photon counting, J. Opt. Soc. Am. **50,** 1131.

Mandel, L. (1961a), Photon degeneracy in light from optical maser and other sources, J. Opt. Soc. Am. **51,** 797.

Mandel, L. (1961b), Concept of cross-spectral purity in coherence theory, J. Opt. Soc. Am. **51,** 1342.

Mandel, L. (1962a), Interference and the Alford and Gold effect, J. Opt. Soc. Am. **52,** 1335.

Mandel, L. (1962b), Transient coherence in optics, J. Opt. Soc. Am. **52,** 1407.

Mandel, L. (1963), Fluctuations of light beams, Progress in Optics **2** (North-Holland Publishing Co., Amsterdam).

Mandel, L. (1963b), The determination of spectral density distributions from measurements of radiation fluctuations, Copenhagen Symposium on Electromagnetic Theory and Antennas, June 1962, ed. E. C. Jordan, Pt. 2, 811–817 (Pergamon Press, Oxford).

Mandel, L. (1963c), Intensity fluctuations of partially polarized light, Proc. Phys. Soc. (London), in press.

Mandel, L. and E. Wolf (1961a), Correlation in the fluctuating outputs from two square-law detectors illuminated by light of any state of coherence and polarization, Phys. Rev. **124,** 1696.

Mandel, L. and E. Wolf (1961b), Some properties of coherent light, J. Opt. Soc. Am. **51,** 815.

Mandel, L. and E. Wolf (1962), The measures of bandwidth and coherence time in optics, Proc. Phys. Soc. (London) **80,** 894.

Mandel, L. and E. Wolf (1963), Photon correlations, Phys. Rev. Letters **10,** 276.

Mehta, C. L. (1963), Coherence-time and effective bandwidth of blackbody radiation, Nuovo Cimento **28,** 401.

Miyamoto, K. (1961), On Gabor's expansion theorem, J. Opt. Soc. Am. **50,** 856 (1960); also J. Opt. Soc. Am. **51,** 910.

Neugebauer, H. E. J. (1962), Coherence time of a maser, J. Opt. Soc. Am. **52,** 470.

O'Neill, E. L. (1963), Introduction to Statistical Optics (Addison-Wesley, Boston).

O'Neill, E. L. and T. Asakura (1961), Optical image formation in terms of entropy transformations, J. Phys. Soc. Japan **16,** 301.

O'Neill, E. L. and L. C. Bradley (1961), Coherence properties of electromagnetic radiation, Physics Today **14,** No. 6, 28.

Parrent, G. B. (1960), A bibliography on coherence theory, J. Research NBS **64D** (Radio Prop.) 742.

Parrent, G. B. (1961), Imaging of extended polychromatic sources and generalized transfer functions, J. Op. Soc. Am. **51,** 143.

Parrent, G. B. (1963), Some fundamental questions of coherence theory, Copenhagen Symposium on Electromagnetic Theory and Antennas, June 1962, ed. E. C. Jordan Pt. 2, 785–786 (Pergamon Press, Oxford).

Parrent, G. B., and F. Rojak (1963), Effects of coherence on resolution, J. Opt. Soc. Am. **53** (in press).

Parrent, G. B. and P. Roman (1960), On the matrix formulation of the theory of partial polarization in terms of observables, Nuovo Cimento **15,** 370.

Parrent, G. B., R. A. Shore, and T. J. Skinner (1962), On the mutual coherence function in an inhomogeneous medium, J. Math. Phys. **3,** 678.

Parrent, G. B., and T. J. Skinner (1961), Diffraction of partially coherent light by a plane aperture, Optica Acta **8,** 93.

Ready, J. F. (1962), Time coherence in ruby lasers, Proc. IRE **50,** 1695.

Roman, P., (1961a), Correlation theory of stationary electromagnetic fields. Part III—The presence of random sources, Nuovo Cimento **20,** 759.

Roman, P. (1961b), Correlation theory of stationary electromagnetic fields. Part IV—Second order conservation laws, Nuovo Cimento **22,** 1005.

Roman, P. and E. Wolf (1960), Correlation theory of stationary electromagnetic fields, part I: the basic field equations; part II: conservation laws, Nuovo Cimento **17,** 462 and 477.

Sarfatt, J. (1963), Quantum mechanical correlation theory of electromagnetic fields, Nuovo Cimento **27,** 1119.

Schell, A. C. (1961), Doctoral dissertation, Massachusetts Institute of Technology.

Shore, R. A. (1963), Partially coherent diffraction by a circular aperture, Copenhagen Symposium on Electromagnetic Theory and Antennas, June 1962, ed. E. C. Jordan, pt. 2, 787–795 (Pergamon Press, Oxford).

Skinner, T. J. (1961), Incoherent source with an arbitrarily narrow power spectrum, J. Opt. Soc. Am. **51,** 909.

Smith, A. W., and G. W. Williams (1962), On the detection of maser signlas by photoelectric mixing, J. Opt. Soc. Am. **52,** 337.

Sudarshan, E. C. G. (1963), Equivalence of semiclassical and quantum mechanical descriptions of statistical light beams, Phys. Rev. Letters **10,** 277.

Tai, C. T. (1961), On the definition of the effective aperture of antennas, IRE Trans. Ant. Prop. **AP–9,** 224.

University of Rochester (1961), Report on Conference on Coherence Properties of Electromagnetic Radiation, AFOSR–583, Report No. 5 under Contract AF49(638)–602, University of Rochester, N.Y.

Wolf, E. (1960), Correlation between photons in partially polarized light beams, Proc. Phys. Soc. (London) **76,** 424.

Wolf, E. (1962), Is a complete determination of the energy spectrum of light possible from measurements of the degree of coherence?, Proc. Phys. Soc. (London) **80,** 1269.

Wolf, E. (1963), Spatial coherence of resonant modes in a maser interferometer, Physics Letters **3,** 166.

Zucker, F. J. (1963), Introduction to partially coherent electromagnetic waves, Proc. 1962 Copenhagen Symposium on Electromagnetic Theory and Antennas (Pergamon Press, Oxford).

Part 3. Radiation

3.1. Electromagnetic Fields in Lossy Media

James R. Wait [1]

In recent years the propagation of electromagnetic waves in dissipative or lossy media has received considerable attention. This work has been motivated by a need to communicate between underground or underwater terminals. The general problem has been discussed by Ghose [1960a] and Hansen [1963] in two valuable papers.

One proposed mode of propagation is based on the existence of a zone with very low conductivity within the crust of the earth [Wait, 1954; Wheeler, 1961; de Bettencourt and Carolan, 1963]. According to G. V. Keller [1963], it is reasonable to expect that such a zone exists, since the pressures at depth tend to reduce the pore space in which water may exist.

Unfortunately, no measurements have been reported in the literature which confirm the existence of low-conductivity layers in the earth's crust. Some of the first experiments indicating low attenuation [Barrett, 1949, 1952] have been open to serious criticism [Pritchett, 1952; Wait, 1954]. It was actually suggested by Barrett that conventional electromagnetic theory was not valid for geological conductors. On the other hand, Pritchett's experiments were carried out under ideal conditions, and it was assured that the signal traversed the medium and was not conducted along pipes, wires, and other buried conductors; his measured attenuations were of the order of 1 db per foot at 1 Mc/s and are in good accord with plane wave theory. The more recent experiments of Saran and Held [1960] also confirm the classical exponential depth attenuation predicted by simple skin-depth theory; they lowered a rope-supported receiving antenna into a fresh-water lake about 100 miles from a VLF transmitter. Other related experiments have been carried out by Harmon [1961], de Bettencourt and Frazier [1963], and Ghose [1960b].

Scale-model measurements of the fields of a horizontal magnetic dipole immersed in a conducting fluid were carried out by Kraichman [1960], who found good agreement between theory and experiment. Similar measurements have been carried out recently by Blair [1963], employing horizontal electric dipoles immersed in the solution.

Theoretical work on fields within lossy media is very extensive. In this category we may include groundwave propagation with the antenna above or below the air/ground interface. A number of papers dealing with this topic have appeared [Gerks, 1962; Wait, 1961a; Walters and Johler, 1962; Wait and Walters, 1963]. In some of these, attention has been given to mixed path conditions such as occur in propagation from land to sea.

Theoretical treatments dealing with dipoles submerged in a conducting half-space have been particularly numerous. Wait [1961b] has given a unified derivation for all distance regions, the terminal antennas being above or below the interface. The derivation made direct use of his earlier published work in the period 1951–1954. Moore and Blair [1961] have treated the problem as an extension of Sommerfeld's classical solutions. Their results, which are based partly on Moore's [1951] thesis, cover near, intermediate, and far zones but omit the quasi-static region.

Biggs [1962] has considered the radiation field in air from a submerged horizontal antenna. The final results appear to be identical to those of Norton [1937], apart from the expected exponential depth factor. Ghose [1960a] has also considered the buried horizontal dipole antenna and, at the same time, he has given some useful design information. Two papers by Durrani [1962 a and b] treat essentially the same problem and contain some new information. The fields in a three-part lossy media have been considered by Anderson [1962]; this is the seawater-air-ionosphere problem with the source dipole located in the seawater medium.

It is expected that a very comprehensive book on the subject of dipoles in the presence of a half-space will be published soon by Professor A. Baños of the University of California in Los Angeles. This will be based in part on his earlier (1953) unpublished reports coauthored with J. P. Wesley. In fact, most of the published work mentioned above is contained implicitly in these two monumental reports.

The radiation from pulse-excited dipoles in homogeneous conducting media has received some attention. Solutions of this type for unbounded media were given by Anderson and Moore [1960], Galejs [1960], Zisk [1960], and Burrows [1962]. The corresponding transient solutions for dipoles immersed in a half-space were carried out in an extensive paper by Wait [1960], who showed how pulse waveforms are distorted when they propagate into the half-space. Some extensions were published by Mijnarends [1962], who treated dipoles excited by step-function currents. In the results given by Anderson and Moore, Galejs, Zisk, and Burrows mentioned above, displacement currents were neglected. Solutions taking these into account have been given by Wait [1960] and Burrows [1963]. Their effect is important when dealing with poorly conducting media and at small times in the transient response.

Propagation of groundwave pulses over the surface of the earth was reviewed by Johler [1962]. Certain

[1] National Bureau of Standards, Boulder, Colo.

spects of early published results on this subject were discussed by Wait [1962a] in a short note.

Two recent survey papers which contain material on fields in lossy media were published by Logan [1962] and Wait [1962b]. Both of these papers deal with methods for computing diffraction fields for large convex lossy surfaces.

In many problems, involving the influence of the earth on electromagnetic fields, it is permissible to utilize the concept of surface impedance. This approach has been utilized extensively in the study of magnetotelluric fields [Wait, 1962c; Price, 1962; Jackson, Wait, and Walters, 1962].

Galejs [1962a] has published an interesting paper dealing with scattering from a conducting sphere which is embedded in a conducting half-space. The induced electric and magnetic dipole moments depend on the incident surface wave and the interactions with the interface. The fields which are scattered from the sphere into the lossless medium are determined from known solutions of this dipole problem [Wait, 1961b]. Galejs [1962b] has also considered the excitation of slots above a lossy dielectric half-space. This problem has application to the design of peninsula and island antennas. The latter problem has also been considered by Staras [1962].

Scattering from lossy coated objects in free space has been investigated in a thorough manner by Hiatt, Siegel, and Weil [1960a and b]. For electrically small spheres, it is shown that the coating can have little effect on the Rayleigh cross sections. They also found that scattering from large objects had a strong forward lobe which was actually enhanced by the coating. This was true even for coatings which are "radar absorbers."

The input resistances of dipoles in the presence of a conducting half-space have been calculated by Vogler [1963]. Similar results have been given by Wait [1962d], who gave his results in graphical form. The latter author also considered the extension to multiple boundaries and to anisotropic lossy media.

A survey of electromagnetic waves in stratified lossy media has been published by Wait [1962e] in a recent text. Other related topics contained in this book are phase integral methods for dissipative media, modes in waveguides with lossy walls, propagation along a curved (lossy) surface, connections between modes and rays in lossy media, influence of curvature in terrestrial waveguides, etc.

References

Anderson, W. L. (Jan.–Feb. 1962), Fields of electric dipoles in sea water, J. Res. NBS **66D** (Radio Prop.), 63–72.

Anderson, W. L., and R. K. Moore (1960), Frequency spectra of transient electromagnetic pulses in a conducting medium, IRE Trans. Ant. Prop. **AP–8,** 603.

Barrett, W. M. (1949), Salt mine tests prove earth penetration by radio waves, World Petroleum **20,** 62–63.

Barrett, W. M. (1952), Note on the radio-transmission demonstrations at Grand Saline, Texas, Geophys. **17,** 544–549.

Biggs, A. W. (1962), Radiation fields from a horizontal dipole in a semi-infinite medium, IRE Trans. Ant. Prop. **AP–10,** 358–362.

Blair, W. E. (1963), Experimental verification of dipole radiation in a conducting half space (to be published in Special PGAP Issue on Sub-surface Propagation).

Burrows, C. R. (1962), D.C. signaling in conducting media, IRE Trans. Ant. Prop. **AP–10,** 328–334.

Burrows, C. R. (1963), Transient response in an imperfect dielectric (to be published in Special PGAP Issue on Sub-surface Propagation).

de Bettencourt, J. T., and J. Carolan (1963), Radio propagation in deep strata along rock-overburden interface (to be published in Special PGAP Issue on Sub-surface Propagation).

de Bettencourt, J. T., and J. W. Frazier (1963), Measurement of rock electrical characteristics by depth attenuation rates in drill holes (to be published in Special PGAP Issue on Sub-surface Propagation).

Durrani, S. H. (1962a), Air-to-undersea communication, Proc. IRE **50,** 96–97.

Durrani, S. H. (1962b), Air-to-undersea communication with electric dipoles, IRE Trans. Ant. Prop. **AP–10,** No. 5, 524–528.

Galejs, J. (1960), Impluse excitation of a conducting medium, IRE Trans. Ant. Prop. **AP–8,** 227–228.

Galejs, J. (Sept.-Oct. 1962a), Scattering from a conducting sphere embedded in a semi-infinite dissipative medium, J. Res. NBS **66D** (Radio Prop.), 607–612.

Galejs, J. (1962b), Excitation of slots in a conducting screen above a lossy dielectric half space, IRE Trans. Ant. Prop. **AP–10,** 435–443.

Gerks, I. H. (1962), Use of a high-speed computer for ground wave calculations, IRE Trans. Ant. Prop. **AP–10,** 292–299.

Ghose, R. N. (1960a), The radiator to medium coupling in an underground communication system, Proc. Nat. Elect. Conf., 279–282.

Ghose, R. N. (1960b), The long range sub-surface communication system, Proc. Sixth IRE Nat. Comm. Symp., 110–120.

Hansen, R. C. (1963), Radiation and reception with buried and submerged antennas (to be published in Special PGAP Issue on Sub-surface Propagation).

Harmon, G. J. (1961), Radio wave propagation through the earth's rock strata—a new medium of communication, IRE Globecom Conv. Rec., 31–34.

Hiatt, R. E., K. M. Siegel, and H. Weil (1960a), The ineffectiveness of absorbing coatings on conducting objects illuminated by long wavelength radar, Proc. IRE **48,** No. 9, 1636–1642.

Hiatt, R. E., K. M. Siegel, and H. Weil (1960b), Forward scattering by coated objects illuminated by short wavelength radar, Proc. IRE **48,** No. 9, 1630–1635.

Jackson, C. M., J. R. Wait, and L. C. Walters (1962), Numerical results for the surface impedance of a stratified conductor, NBS Tech. Note 143.

Johler, J. R. (1962), Propagation of the low frequency radio signal, Proc. IRE **50,** 404–427.

Keller, G. V. (1963), Electrical properties in the deep crust (to be published in Special PGAP Issue on Sub-surface Propagation).

Kraichman, M. B. (Jan.–Feb. 1960), Basic experimental studies of the magnetic field for electromagnetic sources immersed in a semi-infinite conducting medium, J. Res. NBS **64D** (Radio Prop.), 22–25.

Logan, N. A. (1962), A mathematical model for diffraction by convex surfaces, in Electromagnetic Waves (ed. R. E. Langer), Univ. of Wisconsin Press, 139–180.

Mijnarends, P. E. (1962), Propagation of electromagnetic step functions over conducting media, J. Appl. Phys. **33,** 2556–2564.

Moore, R. K., and W. E. Blair (Nov.–Dec. 1961), Dipole radiation in a conducting half-space, J. Res. NBS **65D** (Radio Prop.), 547–563.

Norton, K. A. (1937), The propagation of radio waves over the surface of the earth and in the upper atmosphere, Proc. IRE **25,** No. 9, 1203–1236.

Price, A. T. (1962), The theory of magneto-telluric methods when the source field is considered, J. Geophys. Res. **67,** 1907–1908.

Pritchett, W. C. (1952), Attenuation of radio waves through the earth, Geophys. **17,** 193–214.

Saran, G. S., and G. Held (Sept.–Oct. 1960), Field strength measurements in fresh water, J. Res. NBS **64D** (Radio Prop.), 435–437.

Staras, H. (1962), An analysis of a natural VLF slot antenna (paper presented at Symposium on Electromagnetic Theory and Antennas, Copenhagen).
Vogler, L. (1963), Point-to-point communication on the moon, J. Res. NBS **67D** (Radio Prop.) No. 1, 5–21.
Wait, J. R. (1954), On anomalous propagation of radio waves in earth strata, Geophys. **19,** 342–343.
Wait, J. R. (1960), Propagation of electromagnetic pulses in a homogeneous conducting earth, App. Sci. Res. **B8,** 213–253.
Wait, J. R. (July–Aug. 1961a), On the theory of mixed-path ground wave propagation on a spherical earth, J. Res. NBS **64D** (Radio Prop.), No. 4, 401–410.
Wait, J. R. (1961b), The electromagnetic fields of a horizontal dipole in the presence of a conducting half-space, Can. J. Phys. **39,** 1017–1028.
Wait, J. R. (1962a), A note on the propagation of electromagnetic pulses over the earth's surface, Can. J. Phys. **40,** 1264–1268.
Wait, J. R. (1962b), The propagation of electromagnetic waves along the earth's surface, in Electromagnetic Waves (ed. R. E. Langer), Univ. of Wisconsin Press, 243–290.
Wait, J. R. (Sept.–Oct. 1962c), Theory of magneto-telluric fields, J. Res. NBS **66D** (Radio Prop.), No. 5, 509–542.
Wait, J. R. (Sept.–Oct. 1962d), Possible influence of the ionosphere on the impedance of a ground-based antenna, J. Res. NBS **66D** (Radio Prop.), No. 5, 563–569.
Wait, J. R. (1962e), Electromagnetic Waves in Stratified Media, Pergamon Press, Oxford.
Wait, J. R., and L. C. Walters (1963), Curves for ground wave propagation over mixed land and sea paths, IEEE Trans. Ant. Prop. PGAP (in press).
Walters, L. C., and J. R. Johler (Jan.–Feb. 1962), On the diffraction of spherical radio waves by a finitely conducting spherical earth, J. Res. NBS **66D** (Radio Prop.), 101–106.
Wheeler, H. A. (1961), Radio wave propagation in the earth's crust, J. Res. NBS **65D** (Radio Prop.), 189–191.
Zisk, S. H. (1960), Electromagnetic transients in conducting media, IRE Trans. Ant. Prop. **AP–8,** 229–230.

Additional Related References

Deschamps, G. A. (1962), Impedance of an antenna in a conducting medium, IRE Trans. Ant. Prop. **AP–10,** No. 5, 648–650.
Dobrott, D., and A. Ishimaru (Jan.–Feb. 1961), East-west effect on VLF mode transmission across the earth's magnetic field, J. Res. NBS **65D** (Radio Prop.), No. 1, 47–52.
Guy, A. W., and G. Hasserjian (1961), Design criteria for buried antennas, Boeing Airplane Co., document No. D2–7760.
Harrison, C. W. Jr., and R. W. P. King (1962), On the impedance of a base-driven vertical antenna with a radial ground system (IRE Trans. Ant. Prop. **AP–10,** No. 5, 640–642.
Ishimaru, A. (1962), The effect of unidirectional surface wave along a perfectly conducting plane on the radiation from a plasma sheath (The Second Symposium on Plasma Sheath—to be published by Plenum Press).
Ishimaru, A. (1962), Unidirectional waves in anisotropic media (paper presented at the Symposium on Electromagnetic Theory and Antennas, Copenhagen).
Kane, J. (1962), The surface wave antenna as a boundary value problem (paper presented at the Symposium on Electromagnetic Theory and Antennas, Copenhagen).
Kownacki, S. (1962), A method of investigating air-to-subsurface VLF propagation, IRE Trans. Ant. Prop. **AP–10,** No. 5, 650–651.
Kraichman, M. B. (July–Aug. 1962), Impedance of a circular loop in an infinite conducting medium, J. Res. NBS **66D** (Radio Prop.), No. 4, 499–503.
Polk, C., and F. Fitchen (May–June 1962), Schumann resonances of the earth-ionosphere cavity—extremely low frequency reception at Kingston, Rhode Island, J. Res. NBS **66D** (Radio Prop.), No. 3, 313–318.
Tyras, G., A. Ishimaru and H. M. Swarm (1962), Lateral waves on air-magnetoplasma interfaces (paper presented at the Symposium on Electromagnetic Theory and Antennas, Copenhagen).
Vanwormhoudt, M., and H. A. Haus (1962), Thermal noise in linear, lossy, electromagnetic media, J. Appl. Phys. **33,** 2572–2577.
Wait, J. R. (1956), Mixed path ground wave propagation—1. Short distances, J. Res. NBS **57,** No. 1, 1–15, RP2687.
Wait, J. R. (1960), The electromagnetic fields of a dipole in the presence of a thin plasma sheet, Appl. Sci. Res. **B8,** 397–417.
Wait, J. R. (1960), On the excitation of electromagnetic surface waves on a curved surface, IRE Trans. Ant. Prop. **AP–8,** No. 4, 445–448.
Wait, J. R. (1961), On the impedance of long wire suspended over the ground, Proc. IRE **49,** No. 10, 1576.
Wait, J. R. (Apr.–June 1961), Some boundary value problems involving plasma media, J. Res. NBS **65B,** No. 2, 137–150.
Wait, J. R. (1962), Effective impedance of a wire grid parallel to the earth's surface, IRE Trans. Ant. Prop. **AP–10,** No. 5, 538–542.
Wait, J. R. (Jan.–Feb. 1962), On the propagation of VLF and ELF radio waves when the ionosphere is not sharply bounded, J. Res. NBS **66D** (Radio Prop.), No. 1, 53–61.
Wait, J. R., and A. M. Conda (1960), On the computation of diffraction fields for grazing angles, in Electromagnetic Wave Propagation, Academic Press, New York.
Wait, J. R., and J. Householder (1957), Mixed path ground wave propagation—2. Larger distances, J. Res. NBS **59,** No. 1, 19–26, RP2770.
Wheeler, H. A. (1958), Fundamental limitations of a small VLF antenna for submarines, IRE Trans. Ant. Prop. **AP–6,** 123–125.
Wheeler, H. A. (Jan.–Feb. 1961), Useful radiation from an underground antenna, J. Res. NBS **65D** (Radio Prop.), 89–91.

3.2. Antennas in Lossy Media

C. T. Tai [1]

Since this is the first time that U.S. Commission VI has selected the above title for a specific report, some early studies will be mentioned, mainly to compile a useful bibliography. This report will cover principally problems concerning the characteristics of antennas in a lossy medium of either infinite or finite extent. In regard to the nomenclature, one normally would interpret a "lossy medium" either as one with ohmic loss due to conduction, such as salt water, or as one ionized with collision loss, such as the ionosphere. However, we shall not adhere to such a strict interpretation; problems dealing with antennas in an ideal plasma without collision loss, or in a lossless anisotropic medium, will also be mentioned.

During the preparation of this report many authors have been kind enough to supply material describing their work. Unfortunately, a large amount of the data quoted by some authors is contained either in unofficial company memoranda or technical reports to which this reviewer does not have access. It is therefore unavoidable that some valuable information may have been overlooked.

Early Studies

The early investigations by Campbell [1923], Carson [1926], and Foster [1931] concerning the effects of earth conduction on transmission systems represent the typical problems that attracted attention during the thirties. Much of this work is well documented in the excellent book by E. D. Sunde [1948], who also made an outstanding contribution in this field. The theoretical technique used by these authors follows closely the method originated by A. Sommerfeld [1909] in formulating the dipole and earth problem. No particular emphasis, however, was placed on radiating power and antenna characteristics in these early studies. The radiating power of a dipole in a lossy medium was touched upon by Sommerfeld [Frank and von Mises, 1935] in formulating a reciprocity theorem for two dipoles in a lossy medium. The master, however, did not define clearly the radiating power in this situation. The fact that an infinitesimal dipole in direct contact with a lossy medium cannot maintain a finite amount of power was later pointed out by Sommerfeld and Renner [1942]. In the latter part of the forties, several groups in the United States were also engaged in research on antennas in lossy media. The main interest at that time was stimulated by problems connected with submarine antennas.

The author of this report, under the direction of Professor R. W. P. King at Harvard, wrote two technical reports on this subject [Tai, 1947, 1949]. By investigating the power relationship pertaining to an infinitesimal dipole immersed in a lossy medium, he confirmed the conclusions of Sommerfeld and Renner, namely that the input power is infinite An insulated dipole was then introduced as a more realistic model and Sommerfeld's reciprocity theorem was modified accordingly, based upon this model. The radiation field of a spherically stratified dipole was studied by J. B. Keller and H. B. Keller [1951]. J. R. Wait [1957] derived an expression for the radiation resistance of an insulated circular loop. In the case of antennas of finite dimensions, the theory of cylindrical antennas based upon the integral equation method and the theory of biconical antennas were formally extended to lossy media [Tai, 1949]. Except for the case of an insulated biconical antenna, no numerical calculations were made according to these formulations owing to a lack, at the time, of adequate tables of functions, particularly the sine- and cosine-integrals and the spherical Bessel functions with complex arguments. The behavior of submarine antennas was also carefully studied by Moore at that time. The results of his investigation were contained in his Cornell dissertation [Moore, 1951]. A part of his work deals with antennas in a lossy medium of infinite extent. The major portion of his dissertation is concerned with the effectiveness of wave propagation across an air-water boundary with an antenna placed in the water. His work describes the fundamental difference between the characteristics of an antenna in air and in a lossy medium [Moore, 1962]. In the mid fifties, not many activities were reported in this field, except for one basic work by H. A. Wheeler [1958] on small antennas where the importance of the dissipation in the neighborhood of an antenna placed in a lossy medium is emphasized. In recent years, the advancement of satellites brought with it a number of problems relating to the satellite induced plasma and hence a renewed activity in this field.

Recent Studies

A paper surveying the recent studies on the topic which Moore investigated previously was given by Hansen [1962] at the Copenhagen Symposium. Hansen introduced several useful merit factors in analyzing the efficiency of transmission. The characteristics of low-frequency subsurface radiating structures have been investigated in great detail, both theoretically and experimentally, by Guy and Hasserjian [1962]. Their theoretical results are presented in the form of generalized curves which may be used to design an optimum antenna for any given set of uncontrollable variables such as ground conductivity and frequency.

Extensive research has been conducted by King and his associates on cylindrical antennas in a conducting medium [King, 1962; King and Harrison, 1960; King, Harrison, and Denton, 1961; Iizuka

[1] Ohio State University, Columbus, Ohio.

and King, 1962a, b; Gooch, Harrison, King, and Wu, 1962]. The experimental work by Iizuka and King is particularly interesting as it gives a very comprehensive picture of the current distribution on the dipole. The measured data are in fair agreement with the theoretical results obtained by King and Harrison [1960]. Larson [1962] has obtained some experimental data on the input impedance of a plasma imbedded dipole over a wide range of collision and operating frequencies. It is found that the theory of King and Harrison for short and half-wavelength dipoles is generally followed, and fair agreement is obtained with a Langmuir probe on the values of electron density.

The input impedance of a circular loop in a dissipative medium has been computed by Kraichman [1962] and by Chen [1962]. The latter is an extension of Storer's work [1955] on the impedance of a thin-wire loop antenna in air. Numerical values have been compiled by Chen and Tai [1962a] for the input impedance of thin biconical antennas in a dissipative medium. The same two authors [1962b] have also computed the radiation patterns of some linear arrays in a lossy medium. The relationship between the impedance function of a class of antennas defined in a dissipative medium and in a lossless medium is pointed out by Deschamps [1962]. For media with moderate losses, lengthy computations can be avoided by using a graphical extrapolation based upon the existing data obtained for a lossless medium. Polk [1959] has studied the resonance and supergain effects in small ferromagnetically or dielectrically loaded biconical antennas. The effect of dielectric loading of electric dipole antennas has also been investigated by Galejs [1962]. The results show that Wheeler's general conclusion is not applicable to some cases.

In a number of technical reports, Katzin and his associates [1957, 1958, 1959, 1960] have investigated the radiation pattern and impedances of several radiating systems under the influence of a plasma sheath.

Since the original formulation by Bunkin [1957] on the problem of radiation in an anisotropic medium, some advancements have been made [Arbel, 1960; Kogelnik, 1960; Kuehl, 1961; Mittra and Deschamps, 1962; Wu, 1962]. The work of Arbel was particularly elegant. It supplied many details on the power relationship and the radiation patterns of electric dipoles in such a medium. Kogelnik and Kuehl have evaluated the radiation resistance of electric dipoles in a magnetoionic medium for the case of a very high gyromagnetic frequency. Mittra and Deschamps have studied the near-zone field. Wu has supplied some details about the far-zone fields due to magnetic dipoles. Very little is yet known about the input impedance of an antenna placed in such a medium. Cohen [1962] has obtained the radiation resistance of a current element in a plasma medium which takes into consideration the acoustic source as well as the electromagnetic source.

Upon the completion of this report, we were informed of an extensive research program conducted at Raytheon Company on antennas and fields in rock strata. The details are contained in a report entitled "Studies in Deep Strata Radio Communications" by J. T. deBettencourt, D. A. Hedlund, R. A. Sutcliffe, L. Ames, J. F. Frazier, and A. Orange, issued by Raytheon Company, Norwood, Mass. [Oct. 1962].

References

Arbel, E. (1960), Radiation from a point source in an anisotropic medium, Res. Rept. PIBMRI-861-60, Polytechnic Institute of Brooklyn.

Bunkin, F. V. (1957), On radiation in anisotropic media, J. Exp. Theoret. Phys. (USSR) **32,** 338.

Campbell, G. A. (1923), Mutual impedances of grounded circuits, Bell System Tech. J. **2,** 1.

Carson, J. R. (1926), Wave propagation in overhead wires with ground return, Bell System Tech. J. **5,** 539.

Chen, C. C. (1962), The small loop antenna immersed in a dissipative medium, Cruft Lab. Tech. Rept. No. 369, Harvard University.

Chen, S., and C. T. Tai (1962a), The input impedance of thin biconical antennas immersed in a dissipative or an ionized medium, Antenna Lab. Tech. Rept. 1021–14, Ohio State University.

Chen, S., and C. T. Tai (1962b), The radiation of an antenna array immersed in a dissipative medium, a paper presented at the URSI meeting in Ottawa, Canada.

Cohen, M. (1962), Radiation in plasma III. Metal boundaries, Phys. Rev. **126,** 398.

Deschamps, G. A. (1962), Impedance of an antenna in a conducting medium, IRE Trans. Ant. Prop. **AP–10,** 648.

Foster, R. M. (1931), Mutual impedances of grounded wires lying on the surface of the earth, Bell System Tech. J. **10,** 408.

Frank, P., und R. von Mises (1935), Differential und integral Gleichungen der Mechanik und Physik (Vieweg, Brunswich).

Galejs, J. (1962), Dielectric loading of electric dipole antennas, J. Res. NBS **66D** (Radio Prop.), No. 5, 557.

Gooch, D. W., C. W. Harrison, Jr., R. W. P. King, and T. T. Wu (1962), Impedances and admittances of long antennas in air and in dissipative media with tables of the functions $\sqrt{1 \pm ip}$, Cruft Lab. Tech. Rept. No. 353, Harvard University.

Guy, A. W., and G. Hasserjian (Oct. 1962), Low frequency subsurface radiating structures (a paper presented at the University of Illinois Antenna Symposium).

Hansen, R. C. (1963), Radiation and reception with buried and submerged antennas, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 1173–1177 (Pergamon Press, New York and London).

Iizuka, K. (1962), An experimental study of the insulated dipole antenna immersed in a conducting medium, Scientific Rept. No. 3, Gordon McKay Lab., Harvard University.

Iizuka, K., and R. W. P. King (1962a), The dipole antenna immersed in a homogeneous conducting medium, Trans. IRE Ant. Prop. **AP–10,** No. 4, 384–392.

Iizuka, K., and R. W. P. King (1962b), An experimental study of the half-wave dipole antenna immersed in a stratified conducting medium, Trans. IRE Ant. Prop. **AP–10,** No. 4, 393–399.

Katzin, M. (1957), Radiation of an electric dipole into an ionized sheath, Electromagnetic Res. Corp. Rept. 60175–7.

Katzin, M., J. W. Marini, B. Y. C. Koo (1960), Effects of re-entry plasma sheath on antenna characteristics, Electromagnetic Res. Corp. Rept. 61527–3.

Keller, J. B., and H. B. Keller (1951), A point source in spherically symmetric media, Math. Rev. **12,** 224.

King, R., and C. W. Harrison (1960), Half-wave cylindrical antenna in a dissipative medium, J. Res. NBS **64D** (Radio Prop.), No. 4, 365–380.

King, R., C. W. Harrison, Jr., and D. H. Denton, Jr. (1961), The electrically short antenna as a probe for measuring free electron densities and collision frequencies in an ionized region, J. Res. NBS **65D** (Radio Prop.), No. 4, 371–384.

King, R. W. P. (1962), Dipoles in dissipative medium, Symposium on Electromagnetic Waves (University of Wisconsin Press, Madison, Wisconsin).
Kogelnik, H. (1960), On electromagnetic radiation in magneto-ionic medium, J. Res. NBS **64D** (Radio Prop.) No. 5, 515–523.
Kraichman, M. B. (1962), Impedance of a circular loop in an infinite conducting medium, J. Res. NBS **66D** (Radio Prop.) No. 4, 499–503.
Kuehl, H. H. (1961), Electromagnetic radiation from a dipole in an anisotropic plasma, Tech. Rept. 79–203, Dept. of Electrical Engineering, University of Southern California.
Larson, R. W. (April 1962), Plasma imbedded antennas, a paper presented at the Second Plasma Sheath Symposium, Boston, Mass.
Marini, J. W., B. Y. C. Koo, and J. K. Buckley (1959), Calculation of the effect of an ionized sheath on the admittance of an insulated spherical slot antenna, Electromagnetic Res. Corp. Rept. 61527–1.
Marini, J. W., B. Y. C. Koo, and M. Katzin, Radiation and impedance of antennas surrounded by an ionized sheath, Electromagnetic Res. Corp. Rept. 603–1–6.
Moore, R. K. (1951), The theory of radio communication between submerged submarines, Ph. D. Thesis, Cornell University.
Moore, R. K. (1963), Effects of a surrounding conducting medium on antenna analysis, IEEE Trans. Ant. Prop. **AP–11,** No. 3, 216–225.
Polk, Charles (1959), Resonance and supergain effects in small ferromagnetically or dielectrically loaded biconical antennas, IRE Trans. Ant. Prop. **AP–7,** S414.
Sommerfeld, A. (1909), Uber die Ausbreitung der Wellen in der drahtlosen Telegraphie, Ann. Phys. **28,** 665.
Sommerfeld, A., und F. Renner (1942), Strahlungsenergie und Erbaksorption bei Dipolantennen, Ann. Phys. **41,** 1.
Storer, J. E. (1955), Impedance of thin-wire loop antenna, Cruft Lab. Tech. Rept. No. 212, Harvard University.
Sunde, E. D. (1948), Earth conduction effects in transmission systems (D. Van Nostrand Co., New York).
Tai, C. T. (1947), Radiation in a Hertzian dipole immersed in a dissipative medium, Cruft Lab. Tech. Rept. No. 21, Harvard University.
Tai, C. T. (1949), On radiation and radiating systems in the presence of a dissipative medium, Cruft Lab. Tech. Rept. No. 77, Harvard University.
Wait, J. R. (1957), Insulated loop antenna immersed in a conducting medium, J. Res. NBS **59,** 133.
Sheeler, H. A. (1958), Fundamental limitations of a small VLF antenna for submarines, IRE Trans. Ant. Prop. **AP–6,** 123.
Wu, C. P. (1962), A study of the radiation from elementary dipoles in a magneto-ionic medium, Ph. D. Dissertation. Ohio State University.

3.3. Inhomogeneous Media and Guided Waves

R. E. Collin [1] and A. A. Oliner [2]

This section is concerned with progress during the past three years in those aspects of guided waves which are relevant to radiation phenomena. The topics chosen for review are waves along interfaces, including surface (trapped) waves and leaky waves, the excitation of such waves due to sources and to geometric discontinuities, propagation phenomena in unbounded periodic media, and guiding and radiation effects on periodically-modulated surfaces.

Waves Along Interfaces

Electromagnetic waves guided by interfaces may possess purely real wavenumbers, in which case they are completely bound to the interface and are called surface or trapped waves, or their wavenumbers may be complex. In the latter case, such waves may be associated with anisotropic media within closed waveguides, or may be found under various circumstances on open-boundary structures; in open regions, such complex waves are also called leaky waves, and may be either spectral or nonspectral. A systematic discussion of these various wave types in open regions, including the different kinds of leaky waves, their spectral or nonspectral character, and the treatment of surface waves as a special case of complex waves, has been presented by Tamir and Oliner [1962a]. A corollary paper [Tamir and Oliner, 1962b] treats the manner in which radiation patterns are influenced by the presence of these waves. The discussion below considers the surface waves and the complex waves separately.

Surface (Trapped) Waves

Despite the fact that the basic features of surface waves are well known, and that a considerable literature already exists, a surprising amount of activity still continues on this topic. Most of this activity is associated with the investigation of the electromagnetic properties of new structures, including those consisting of anisotropic ferrites, dielectrics with transverse variation, or plasmas with negative dielectric constant.

Two excellent reviews of surface wave structures and their properties have appeared during this period. We should mention first the notable chapter, written by Zucker [1961], which appears in the Antenna Engineering Handbook, edited by Jasik. Zucker [1961] presents a thorough survey of the properties of surface waves, methods of measuring these properties, characteristics of the most important structures which support these waves, and the use of these structures as antennas including design principles for single antennas and arrays. The second review, by Harvey [1960], includes descriptions of various types of surface wave structures and their properties, and also considers multiple media, effects of curvature, and questions of launching. Both reviews also present extensive bibliographies. A paper with a general tone, by Lengyel and Mitzner [1961], examines the conditions under which surface waves of the type which decrease exponentially on both sides of an interface can be guided by a single plane interface, but does not treat any specific structures.

A new phenomenon has also been noted in recent years that a *backward* surface wave can exist on an interface in a uniform structure. Furthermore, this backward wave is a true mode and not a space harmonic as occurs in periodic structures. The existence of such a backward wave has been observed previously in anisotropic media, such as ferrites or plasmas in the presence of magnetic fields, and on isotropic plasma columns possessing certain inhomogeneities in their cross section. More recently, Trivelpiece, Ignatius, and Holscher [1961] have proposed the consideration of backward waves on a magnetized ferrite rod for use in a backward wave oscillator at millimeter wavelengths, thereby eliminating the need for a delicate periodic structure at these wavelengths. They have computed the properties of the wave by using a quasi-static approximation, and have experimentally verified the computed wave characteristics. Thompson [1961] points out the distinction between body and surface backward waves, and indicates that while the calculations of Trivelpiece et al. [1961] apply to the body modes, their experiments relate to the surface waves. A reply [1962] agrees with these criticisms but states that further measurements will be taken under different conditions to finally resolve the question.

Backward surface waves were also found to exist on isotropic uniform plasma slabs [Tamir and Oliner, 1961; Oliner and Tamir, 1962] or plasma-coated cylinders [Paik, 1962] in the range of negative plasma dielectric constant, ϵ_p. It was also noted [Tamir and Oliner, 1961; Oliner and Tamir 1962] that while forward surface waves on plasmas may be present only when ϵ_p is negative and $|\epsilon_p|<1$, these backward surface waves occur essentially in the range $0<|\epsilon_p|<1$. The properties of forward surface waves on plasma cylinders were also investigated [Rusch, 1962].

Hessel, Marcuvitz, and Shmoys [1962] have examined the properties of forward surface waves on an interface between air and a compressible plasma. The compressibility of the plasma permits the presence of acoustic modes which are coupled to the electromagnetic modes at the interface. The resulting surface wave includes both the acoustic and electromagnetic contributions, and differs from the purely electromagnetic solution only at the higher frequencies, where the former wave does not exhibit the customary high-frequency cutoff.

A number of studies have been conducted on structures which may be regarded as variants of

[1] Case Institute of Technology, Cleveland, Ohio.
[2] Polytechnic Institute of Brooklyn, Brooklyn, N.Y.

structures with known properties. The V-line [Diament et al., 1961] has been devised, and the properties of the line have been explored theoretically and experimentally; the V-line is a variant of the dielectric image line, but the paper also considers modifications of the V-line which could be of practical interest. A thorough theoretical and experimental study [Cohn, 1960] has also been made of the dielectric-loaded through line, which may be viewed as a bisected H-guide. The properties of surface waves on symmetrical three-layer dielectric sandwiches have been examined theoretically [Richmond, 1960] and studies were conducted [Schlesinger et al., 1960] of higher-order hybrid modes on dielectric cylinders. The propagation characteristics of elliptical dielectric waveguides have also been investigated [King et al., 1960; Yeh, 1962] theoretically and experimentally, with the motivation being the possible use of this guiding structure at millimeter wavelengths since it was noted experimentally that the attenuation of a circular dielectric rod is reduced considerably when the rod is flattened.

Several other investigations of the properties of structures which can support surface waves have been conducted with specific applications in mind. With the object of obtaining a transmission means with low attenuation at millimeter and submillimeter wavelengths, detailed studies [Sobel et al., 1961; King et al., 1962] were conducted by Wiltse and associates of the properties of various types of single-wire lines at these frequencies. Among the lines examined were uncoated and dielectric-coated metal wires of cylindrical cross section, and uncoated wires of elliptical cross section. The properties of dielectric cylinders encased in a dielectric cladding of different dielectric constant were studied by Snitzer [1963] to understand phenomena associated with fiber optics. These properties have been examined in considerable detail, including those of higher modes, small differences between the dielectric constants of the cladding and the core, and mode coupling between neighboring fibers. Snitzer [1961] has also proposed the use of such optical fibers as a mode selector in optical masers.

The application of surface wave structures to Luneberg lenses has been proposed by Walter [1960]; he considers as possible structures a dielectric sheet, a bed of metal posts, and a plate filled with holes, all over a ground plane, derives their indices of refraction, and applies the results to the design of several types of Luneberg lenses. Lichtenberg and Woodyard [1962] have shown that a waveguide composed of an air gap between two semi-infinite plasma regions can support a mode with an attenuation constant significantly lower than that of parallel plate waveguide with the usual metallic walls, even when plasma collision losses are taken into account. The mode is a fast wave with a sinusoidal field distribution in the air region, and an exponential decay into the plasma regions. Yen [1961] has employed the coupling between surface waves as a means of determining the radiation pattern from a broadside array of endfire antennas.

Studies have been made of surface waves guided by structures with a transverse variation in the properties of the structure. Vigants and Schlesinger [1962] present an approximate method, employing the programming of a computer, for the determination of the propagation characteristics of surface waves on dielectric cylinders with arbitrary radial variation in the dielectric constant. Hirsch and Shmoys [1962] examine the modifications in the properties of a surface wave guided by an air-plasma interface when the single-step interface is replaced by a double-step transition.

The properties of surface waves supported by anisotropic sheets consisting of ideally closely spaced wires have also been examined. Rumsey [1961] has developed a new way of solving Maxwell's equations which is ideally suited to such unidirectional anisotropic sheets; his solution leads to circularly polarized surface waves along the sheet. His method has also been applied by Cheo, Rumsey, and Welch [1961] to a sheet with spiral anisotropy as an explanation for the behavior of frequency-independent equiangular spiral antennas. The solution is highly successful and yields radiation patterns which agree closely with measured results on two- and four-element antennas. Karal and Karp [1963], and Seshadri [1962], consider a field solution which includes a surface wave propagating along a plane unidirectionally conducting screen; Seshadri's phrasing of the problem is more general in that for his case the waves travel obliquely across the wires, but the approach employed in both cases by these authors differs from Rumsey's [1961].

The surface waves supported by a ferrite slab in a rectangular waveguide with a transverse d-c magnetic field were examined by Bresler [1960] in an attempt to resolve the so-called "thermodynamic paradox." He found that one-way transmission was indeed obtained when the ferrite slab is located against one wall, but that bidirectional propagation always occurs when the slab is removed from the wall, no matter how small this distance. The paradox is therefore to be resolved by choosing as the correct solution that which is obtained by removing the slab a finite distance from the wall and then reducing this distance to zero as a limit, rather than choosing the distance to be zero initially. Ishimaru [1963] also treated this question, but disagreed with Bresler's [1960] approach; he preferred instead, Seidel's concept of "intrinsic loss," and presented arguments in its defense. Ishimaru [1962] has also demonstrated that a unidirectional surface wave can exist along a perfectly conducting plane which bounds a semi-infinite plasma with a d-c magnetic field.

The last type of guided wave to be mentioned here is not strictly a surface wave, but is the so-called wave beam, recently devised for application at millimeter and submillimeter wavelengths. These wave beams have been treated theoretically by Goubau and Schwering [1961], Beyer and Scheibe [1962], and by Goubau [1963]; experimental data were presented by Christian and Goubau [1961].

These wave beams are reiterated and guided by reconstituting the cross-sectional phase distribution at certain regular intervals by means of lenses; the beams are shown to satisfy orthogonality relations like mores in a waveguide.

Complex Waves, Including Leaky Waves

The chapter by Zucker [1961], referred to above in connection with surface waves, contains an equally fine review of the common structures which support leaky waves and the characteristics of these structures, the design of leaky wave antennas, and an extensive bibliography. A paper by Oliner [1963] discusses the distinction between spectral and nonspectral leaky waves and reviews the role played by leaky waves in a variety of electromagnetic phenomena in open regions. Among these phenomena are radiation from plasma layers, radiation from periodically-modulated surfaces, log periodic antennas, C̊erenkov radiation, and Wood's anomalies. The properties of leaky waves guided by slits in rectangular waveguide are well known. Goldstone and Oliner [1961] have derived the characteristics of such waves on slitted cylinders, for both circular electric and circular magnetic excitation, by the use of radial transmission line relations and a perturbation technique. The effects of coupling on the leaky wave properties due to the presence of a neighboring slit on a plane and on a cylinder have been investigated theoretically by Nishida [1960a, 1960b].

A surface wave on a plane interface is purely bound, and propagates without radiation; on a curved surface, however, it is known that leakage is associated with the wave. The quantitative influence of curvature on the attenuation constant of a surface wave was considered by Wait [1960], who treated the waves excited by a vertical electric dipole on an inductively layered sphere. He disagrees with a previous paper by Barlow, and corroborates the original conclusions of Elliott. In reply, Barlow [1960] indicates that the low attenuation values which follow from Elliott's original analysis are not in agreement with some preliminary experiments, and that the question will be explored further. Logan and Yee [1962] have derived a simplified mathematical expression which permits one to obtain more easily the propagation properties of surfaces of the type treated by Wait [1960].

Some studies have been conducted which demonstrate the presence of leaky waves on plasma slabs. Tamir and Oliner [1962] showed that E–type leaky waves are very strongly excited on isotropic plasma slabs in the range for which the plasma dielectric constant is positive. A study by Ishimaru [1962], referred to above, showed that a unidirectional surface wave can be supported in a semi-infinite plasma by a metal plate in the presence of a d-c magnetic field. When the plasma height becomes finite, this surface wave is transformed into a unidirectional leaky wave. In both of these studies, the leaky wave is shown to influence strongly the radiation pattern.

Complex waves of the spectral type also exist; they are characterized by the facts that they occur in degenerate pairs and do not carry any real power. The presence of such modes in anisotropic ferrite regions has been recognized previously; a demonstration of their existence based on the asymptotic form of the characteristic equations has been given by Tai [1960]. Tamir and Oliner [1961] have found that such complex waves are also present on isotropic plasma slabs in the range for which the plasma dielectric constant is negative.

Excitation of Waves on Open Boundary Structures

In the previous section various structures capable of supporting surface and leaky waves were discussed. In practice the ultimate usefulness of such structures for antenna applications depends on the relative ease and efficiency of launching or exciting the desired wave types. It is this particular application that often motivates the analysis of wave excitation on surface wave structures. The problem is essentially solved once the fields excited by a unit source of electric (or magnetic) current has been found since other sources may be considered to be a suitable weighted distribution of unit strength sources. Interest in problems of this type has continued during the past three years despite the fact that the basic techniques of analysis were developed earlier, together with results for a number of specific structures. The following section presents a survey of the problems in this category that have been investigated in the period since January 1960.

A second class of problems concerned with surface-wave excitation are those in which surface waves and a radiation field are excited by a surface wave impinging on a discontinuity along the surface wave structure, or by a combination of a source and a discontinuity. For the most part only those problems that can be classified as two-part boundary value problems have received extensive treatment, the obvious reason being that these can be solved by the Wiener-Hopf technique. Studies of specific structures lend insight into the mechanism of radiation from surface-wave end-fire antennas, and hence antenna applications are again a strong motivating factor for work in this area. Needless to say, these two-part boundary value problems with excitation by an incident surface wave or line source are much more intricate to solve, from a mathematical point of view, than those pertaining to excitation of waves on a uniform unbounded structure. The section on Discontinuity Effects discusses the progress achieved in this area over the past three-year period.

Not all structures that have been analyzed are necessarily appropriate structures for antenna applications, for example, a plasma slab. Nevertheless, it is of considerable interest to examine the types of waves that may be excited on these general open boundary structures because of the overall insight into radiation and diffraction phenomena that is gained. Discussion of these is included along with those that have potential application in the antenna field since there is no reason to differentiate open boundary structures on the basis of possible applications. Mathematically they fall into the same category.

Source Excitation

a. Uniform Planar Structures With Line Sources

The canonical problem consists of a line source located above a reactance plane or a slab of material (dielectric, ferrite, plasma, etc.). A solution in the form of an integral is found for the field component parallel to the line source in terms of transmission in the direction perpendicular to the slab. The poles of the integrand determine the surface and leaky waves that may be excited; the integrand also has one or more branch cuts which yield the radiation field. The integral is usually evaluated by deforming the integration contour into a steepest descent contour. During this deformation some of the poles may be crossed, in which case part of the solution consists of discrete modes. The remainder of the solution is determined by an asymptotic evaluation of the integral along the steepest descent contour to yield results valid for the far zone radiation field.

The above basic procedure was applied by Cohn, Cassedy, and Kott [1960] to determine the TE waves excited on a dielectric slab above a ground plane. These authors present a number of computed curves for the power radiation patterns and also for the bidirectional launching efficiency for the dominant TE_{20} surface mode. A range of values for slab thickness and line source position is considered. Experimental data were obtained by placing two conducting planes perpendicular to the slab and a small distance apart; the fields were then not disturbed, since the source is an electric current. The agreement with theory was excellent and verified that as much as 97 percent of the total power could be excited as a surface wave.

In a second paper by two of the same authors, Cassedy and Cohn [1961], an experimental check of the importance of the leaky wave as a partial, but dominant, representation of the near zone field at the guiding surface was made. Using the same dielectric-loaded trough waveguide as in the earlier paper, it was possible to demonstrate, by suitable amplitude and phase measurements of the total field near the guiding surface, that the leaky wave can be used to respresent a major portion of the total radiation field (not including the surface wave) near the surface.

The case of surface wave excitation by a magnetic line source in the presence of two parallel, but separated, dielectric slabs has been studied in great detail by Angulo and Chang [1961].

A problem, somewhat related to the above, where the dielectric slab is replaced by a plasma slab and the source is a magnetic or electric current line source located on the ground plane on which the plasma slab rests was treated by Tamir [1961] and Tamir and Oliner [1961, 1962b]. The plasma is characterized as a medium with a dielectric constant which may lie between zero and unity or be negative. Tamir [1961] has explored this geometry in great detail, and has found that for the electric line source no surface waves or spectral complex waves are present, but only a large number of nonspectral leaky waves. For the magnetic line source case, many interesting results were obtained, and some of these were reported in the previous section. In the negative dielectric constant range, surface waves [Tamir and Oliner, 1961; Oliner and Tamir, 1962] of the backward and forward type, and spectral complex waves [Tamir and Oliner, 1961], are present. When the dielectric constant is positive, only leaky waves are present, but it is significant that the near field is then dominated by the presence of a single leaky wave [Tamir and Oliner, 1962b].

The importance of this last result is that the radiation field may then be determined by a Kirchhoff-Huygens integration over the slab surface using the leaky wave field as the source distribution. This is in actual fact the procedure used in calculating the radiation field from a leaky wave antenna. Radiation patterns computed in this manner exhibit one or more sharp peaks at angles closely related to the leaky wave angles. These patterns agree very well with those computed directly from saddle point evaluations. The analysis by Tamir and Oliner [1962b] provides strong analytical justification for this approach to evaluating leaky wave antenna radiation patterns.

Along the same lines, an analysis of a specific idealized model of a leaky wave antenna (an inductive grid structure) was given by Collin [1962]. It was demonstrated that for this particular structure the use of the leaky wave field as the aperture distribution yields a radiation pattern in excellent agreement with a more rigorously computed pattern (the saddle point evaluation). Further considerations regarding the relation between leaky wave poles and radiation patterns have been given by Hessel [1962].

In the structures discussed above, the impedance presented to the incident waves at the plane of the slab was isotropic. The surface impedance can instead be made anisotropic in various ways. For example, this plane may be replaced by the unidirectionally conducting plane [Karal and Karp, 1963; Seshadri, 1962] discussed in the section on Surface Waves. We saw there that along such a plane a surface wave mode of propagation is possible. The launching of this surface wave mode by an electric line source has been examined, and it was found [Seshadri, 1962] that excitation efficiencies exceeding 90 percent were possible when the line source is located close to the surface of the screen.

A situation that could be expected to have features in common with the general anisotropic impedance surface mentioned above is that of a plasma slab with an applied magnetostatic field. In the general case, TE and TM modes are coupled together by the anisotropy, but if the line source is directed parallel to the applied magnetostatic field, the TE and TM modes are not coupled together. This problem is therefore considerably easier to analyze. Two independent investigations Ishimaru [1963], Seshadri, [1962b] of the excitation of waves in a grounded plasma half-space, or on a grounded plasma slab, by a magnetic line source oriented parallel with the magnetostatic field have been carried out. Seshadri [1962b] considered a half-space filled with plasma and bounded by a conducting plane. Ishimaru [1962] considered the case of several plasma layers bounded at the bottom by a conducting plane. An interesting feature of the solution (discussed briefly under the Surface Waves section) is the existence, for certain plasma parameters, of a surface wave guided along the perfectly conducting plane. In an isotropic medium such a surface wave does not exist. The presence of an anisotropic medium makes it possible for the magnetic field to satisfy a boundary condition of the type

$$\frac{\partial H_y}{\partial x}+\lambda H_y=0$$

at the conducting plane. This boundary condition is of the same type as that encountered for a reactive surface and it thus predicts the existence of a surface wave. A further property of the surface wave is that it is unidirectional, i.e., it propagates in one direction only at right angles to the source. Seshadri [1962b] also examined the nature of the radiation field and the efficiency with which the surface wave could be excited. In the slab problem analyzed by Ishimaru [1962], a much wider range of plasma parameters was considered. In the opaque region under appropriate conditions it was found that the surface wave pole was, in actual fact, a leaky wave pole and furthermore that it was located very near to the real axis. This resulted in a sharply peaked radiation pattern in accord with the usual behavior of leaky wave antennas. In the unbounded plasma the surface wave is a fast wave, as in a waveguide. The presence of an upper boundary perturbs this surface wave into a leaky wave that radiates strongly at a particular angle to the surface.

Another modification of the basic problem discussed at the beginning of this section occurs in the case of a compressible plasma slab. Using a fluid model of a plasma, Hessel, Marcuvitz, and Shmoys [1962] have demonstrated that the field in a compressible plasma can be decomposed into two uncoupled sets which are called optical and plasma modes. In the presence of a finite-thickness slab or an air-plasma interface these modes become coupled together through the boundary conditions at the surface. The formal solution for the fields excited by a magnetic line source located above the interface of such a compressible plasma medium and air was given. The surface wave portion of the solution was discussed in the section on Surface Waves. A still different modification of the canonical geometry occurs in the problem solved by Kornhauser and Keller [1963] that of the field due to a point source in a stratified medium with a refractive index that varies as $n=n_0\ (1-a^2z^2)^{1/2}$, which is a case of duct propagation. The field solution is obtained rigorously in terms of tabulated functions, and the ray solution has also a particularly simple form.

b. Cylindrical Structures With Line Sources

Although the majority of work pertaining to surface wave excitation has been for planar structures, some attention has also been given to other geometries, in particular, cylindrical structures.

The efficiency of launching the dominant HE_{11} (dipole) mode on a circular dielectric rod by a magnetic ring source and that of launching the TM_{01} (Sommerfeld-Goubau mode) wave on a finitely conducting circular rod by a similar ring source was investigated by Cohn and King [1962]. For the HE_{11} mode the source had a sinusoidal angular amplitude variation, while for the TM_{01} mode there was no azimuthal variation. Launching efficiencies of 80 percent or more were demonstrated.

As was mentioned above in the section on Complex Waves, if the surface which supports a surface wave is curved in the direction of propagation, the surface wave mode is perturbed into a complex mode that loses some energy by radiation. For surfaces with large radii of curvature the concept of a surface wave mode is still a useful one and enables certain properties of the curved surface to be described in a satisfactory physical way. A corrugated cylinder, with waves excited such that they propagate along the angular coordinate, is an example of a curved surface along which a perturbed surface wave mode may be excited. The resonance excitation of a corrugated cylinder excited by an axial electric line source was studied by Cullen [1960]. The cases of an axial magnetic line source and axial slots and dipoles were considered by Wait and Conda [1960 and 1961]. The radiation field was found to have an angular dependence factor which contains a denominator that can be made almost zero by an appropriate choice of surface impedance. For such a choice, the power in a given mode may become several hundred times greater than that in any of the other modes. Consequently, the radiation pattern will vary essentially as cos $n\phi$, if the nth mode is the "resonant" one.

Although simple line sources and dipole sources are generally considered when studying surface wave excitation, these are not always the most useful sources because of limitations such as narrow bandwidth, high Q, etc. Chu and Kilcoyne have, instead, used a helix to excite a dielectric rod antenna in an efficient manner [Chu and Kilcoyne, 1961]. In a short note by Ravid-Weissberg, the use of distributed slots along a waveguide was shown to also provide efficient surface wave excitation [Ravid-Weissberg, 1961].

c. Surface Waves on Wedges and Cones

Although planar and cylindrical structures have been most frequently the type of structure considered in surface wave studies, other geometries such as wedges and cones have not by any means been bypassed. Simple solutions for these structures, under the assumption of an impedance type boundary condition, are possible only if the surface impedance varies linearly. If the surface impedance does not have this type of variation then, although the wave equation is separable, the boundary conditions are not possible and mode coupling takes place. The excitation of surface waves on wedges and cones having such a linearly varying surface impedance has been systematically examined in detail by Felsen [1960]. Other work pertaining to surface waves on wedges in the presence of discontinuities is discussed in the next section.

Discontinuity Effects

a. Planar Structures

The most obvious planar discontinuity problem that can be posed and which is amenable to solution is that classified as a two-part boundary value problem. Such problems have a rather long history as far as fundamental analytical techniques go. In recent years, the well-known Wiener-Hopf method for the solution of such problems has been applied to obtain solutions to a number of specific surface wave structures where the guiding surface is either semi-infinite or has different electrical properties in the two regions $z < 0$ and $z > 0$. Excitation is either by an incident surface wave, a line source, or waveguide aperture. These solutions are of interest in connection with surface wave antennas which may, at least for a first approximation, be treated as a semi-infinite surface waveguide with uniform properties. The basic study in this area was published a few years ago by A. F. Kay; an outstanding feature of his solution is that, in spite of the nonelementary functions required to describe the field structure, the expressions for radiated power involve only elementary functions and are readily evaluated.

The case of a reactive half-plane in the region $z < 0$ and a perfectly conducting half-plane in the $z > 0$ region excited by a magnetic line source at the junction was solved by Kane [1960]. His analysis is very similar to that in Kay's work. The power radiated as a surface wave was shown to reach values of 80 percent or more as the surface reactance was increased. Above 80 percent the rate of increase of surface wave power with reactance was small, however.

The radiation field at the reactive surface is zero to at least the first order in $(k_0 r)^{-1}$, where r is the distance from the source. This principle, referred to as the Karp-Karal lemma by Kane, permits the interaction of two or more sufficiently spaced discontinuities on the reactive surface to be evaluated as in a conventional waveguide with single mode propagation. The reason is that along the surface the radiation field arising from each discontinuity does not interact, to order $(k_0 r)^{-1}$, with the adjacent discontinuities. Utilizing this lemma, Kane applied the results of the analysis of the half-plane problem to obtain some approximate results for a surface wave antenna of finite length [Kane, 1962a, 1962b, 1963]. The effect of antenna length, reactance, etc., on gain was determined.

A two-part boundary value problem involving unidirectional conducting screens was treated by Sheshadri [1962b]. Three cases were considered, involving respectively a semi-infinite unidirectional conducting screen, a semi-infinite unidirectional conducting screen and a perfectly conducting half-plane, and finally two different unidirectional conducting screens. The analysis was again very similar to that in the problem considered earlier by Kay.

Excitation of surface waves on a reactive plane by a wave incident in a parallel-plate transmission line was treated by Wenger [1962]. By choosing the extension of the reactive surface into the parallel-plate line properly, the coupling to the surface wave can be maximized. The author has delayed the publication of the formal analysis in a journal until numerical results can be computed. The solution is similar in several respects to that of Kay and others, although in the present problem no simple expression for radiated power seems possible.

b. Wedges

It was pointed out earlier that the general wedge problem with mixed boundary conditions is readily solved only if the surface reactance is linearly varying. However, for the special case of a right-angle wedge it is possible to find a solution when the surface impedance is a constant. The trick used here is the introduction of an auxiliary function that is a linear combination of the wave function being sought and its cartesian derivatives. The auxiliary function satisfies simple Neuman or Dirichlet boundary conditions and, hence, can be determined in a straightforward manner.

By using the above transformation a number of authors examined the problem of excitation of surface waves on right angled wedges and the radiation produced when a surface wave is incident along one face of the wedge [Chu et al., 1962; Karal et al., 1961]. This work is an extension of earlier work by Karal and Karp dealing with surface wave propagation along a wedge.

c. Approximate Methods

Many of the open boundary structures of interest for surface wave antenna applications are not amenable to rigorous solution. For such structures approximate methods of analysis must be devised. Many of the rigorous solutions for idealized structures are of great value for testing the accuracy of approximate solutions. This is one reason why it is worthwhile to obtain solutions for idealized structures even though at times it might appear that such problems are only mathematical exercises.

For surface wave problems involving dielectric slabs, Jones [1961] has demonstrated that, by replacing the dielectric slab by a generalized mixed linear boundary condition involving a linear combination of the wave function and its normal and tangential derivatives (a technique first introduced by Karp and Kane), approximate solutions may be obtained that agree quite well with those special cases for which rigorous solutions are available.

The technique may thus be extended with reasonable confidence to slab problems for which exact solutions cannot be found, e.g., a semi-infinite slab.

Another approximate theory has been developed by Keller and Karal [1960]. This is a geometrical optics or ray theory and yields excellent results for the high-frequency solutions. The theory enables the amplitude of the excited surface wave to be found. When applied to the problems of a line source and wedge with a variable impedance surface, and a line source in the presence of a circular cylinder, the results were found to agree exactly with the high-frequency asymptotic values of the exact solutions.

Oliner and Tamir [1962b] have used the fact that the near field on a plasma slab is dominated by a single leaky wave to determine the influence of different terminations on the plasma slab. The radiation patterns corresponding to two different types of discontinuities were evaluated numerically for various lengths between the source and the discontinuity. It was found that for short lengths, about one or two wavelengths, the effect of the discontinuity is significant, while for distances as large as twenty wavelengths the influence is negligible. This dependence on length is readily explainable from the properties of the leaky wave, since for large distances most of the power has already been radiated away before the termination is reached. It should be noted that this dependence on length would have been very different if a surface wave, rather than a leaky wave, were involved.

Felsen has applied the results of his study of surface waves on wedges with linearly varying surface reactance [Felsen, 1960a] to the study of a model for a surface wave antenna [Felsen, 1960b]. The structure considered consisted of a constant reactance plane extending over $-\infty < x < a$, a section with reactance decreasing as x^{-1} over $a < x < b$, and a plane with zero reactance from $x = b$ to infinity. Estimates of the effects of taper impedance and length on gain, beamwidth and sidelobe level, were obtained.

Periodic Structures

This section is concerned with propagation in unbounded periodic media, and in guiding and scattering effects associated with periodically modulated surfaces. The discussion is restricted to those phenomena and those specific structures related to radiation or to propagation in open regions. It does not include, for example, slow wave structures in closed waveguides, parametric effects due to time modulation, periodicity introduced by nonlinearities induced by high-power incident waves, or mutual coupling effects in two-dimensional antenna arrays. It is assumed that such treatments will appear elsewhere.

a. Unbounded Structures

Activity in the area of periodic structures of the unbounded type has not been particularly great in recent years. General properties of periodic structures within closed boundary waveguides and also unbounded structures in free space are well known for the most part. On the other hand, only a relatively few reasonably accurate solutions for specific structures are available. Also, a general investigation into the anisotropic properties of periodic structures, and in particular, periodic media with other than cubic or tetragonal lattice structures has not been adequately carried out to date. Recent work on periodic media, directed in part towards the latter-mentioned aspects of the problem, is briefly reviewed below.

The well-known periodic medium consisting of thin circular disks arranged in a tetragonal lattice has been analyzed many years ago by using the dual of Bethe's small aperture theory to compute the shunt susceptance of a single plane of disks. This shunt susceptance may then be used in a conventional analysis for loaded transmission lines to obtain the propagation constant for the unbounded medium. Unfortunately, as shown by recent measurements [Gardner, 1960], this approach yields accurate results only for disks with $k_0 a$ less than about 0.4, where $k_0 = 2\pi/\lambda_0$ and a is the disk radius. Eggimann [1961] has improved the original calculations by extending the Bouwkamp power series expansion to obtain higher order expressions for the electric and magnetic dipole moments, as well as higher order moments, induced in a disk by an arbitrary incident field. Results accurate to order $(k_0 a)^3$ were obtained for the dipole moments.

In an earlier attempt to improve on the results based on Bethe's theory and static interaction fields between disks, Collin and Eggimann [1962] evaluated the dynamic interaction constants for a planar array of disks. However, the use of these dynamic field interaction constants, together with zero-order expressions for the dipole moments, yields results that are even further away from the experimental values. This is perhaps not too surprising since the dynamic interaction constants are accurate to much higher order in $k_0 a$ than the Bethe expressions for the dipole moments, and hence the use of other than static interaction fields is not justified without first obtaining more accurate expressions for the dipole moments.

By using third-order expressions for the dipole moments, dynamic interaction constants computed for interacting dipole fields (not plane waves), and quadripole radiation terms, it is found that computed results for the shunt susceptance agrees well with the measured results for $k_0 a$ up to one [Eggimann and Collin, 1962]. Beyond this point the analysis is too involved to be practical.

A problem which is the dual of that of a planar array of disks is a conducting plane with a doubly periodic array of similar apertures. Such a screen comprising rectangular apertures has been considered by Kieburtz and Ishimaru [1962]. An approximate solution based on an integral equation formulation was obtained. Numerical results for the aperture field and scattering cross section for $k_0 d$ up to 2 were computed, where d is the width of the square aperture. The scattering cross section shows a resonance effect for $k_0 d \approx 1.5$, at which point the transmission through the screen is a maximum. These authors have also obtained the equivalent circuit parameters and scattering coefficients [Kieburtz and Ishimaru, 1961] for such a periodically apertured screen by means of a variational expression which they reformulated to include higher mode propagation.

Another classical periodic medium that has been reexamined is the two-dimensional strip medium. This medium exhibits pronounced anisotropic effects. The early analysis of this medium treated each plane of strips as a capacitive grating which could be considered to load a transmission line periodically with shunt capacitive susceptances. As long as there is only dominant mode interaction between successive planes of strips good results are obtained. The extension to higher mode interaction is cumbersome. By considering a different modal expansion, Kolettis and Collin [1961] were able to apply the results of the Carlson-Heins theory for the parallel-plate interface problem to overcome the problem of close spacing between adjacent gratings. A combination of the two approaches gave theoretical results in excellent agreement with measured values for a wide range of spacings. The analysis has also been extended to the case of strips arranged in a nonorthogonal lattice pattern [Case Institute of Technology, 1962]. No extensive numerical results have been computed to date, however.

Kurss [1961] has examined the formal solution for eigenwaves in a periodic structure composed of scatterers in a lattice defined by translational vectors. The same author also formulated the transmission line approach so as to be applicable to semiconductor band calculations [Kurss, 1961]. There are other lattice structures besides those defined by a translation. In particular, a periodic medium can be built up from planar arrays of scatterers with adjacent planes being obtained by a combination of rotation and translation. The properties of such structures are presently being investigated by H. Haskal at Case Institute of Technology. To date it has been established that the modes for such media are elliptically polarized waves.

The scattering of waves at the interface between free space and an infinite array of thin dielectric sheets has been treated by Collin [1960]. The dielectric sheets were placed by infinitely thin polarization current sheets, and a rigorous solution was obtained for the scattering by the use of bilateral Laplace transforms. Numerical results for some special cases compared favorably with an approximate solution obtained previously via the Rayleigh-Ritz method.

b. Periodically Modulated Surfaces

The studies during this period on the guiding and radiation associated with periodically modulated surfaces continue to be motivated primarily by antenna applications. The problems posed fall into two groups. The first, motivated by an earlier paper by Thomas and Zucker, considers the relation between the radiation pattern and the modulated amplitude and phase distribution over an equivalent aperture. The second problem, following an earlier paper by Oliner and Hessel, involves the relation between either the radiation field or the aperture amplitude and phase distribution and physical structures which are modulated periodically.

The former of these problems was considered by Ishimaru and Bernard [1962], who presented a closed form of analysis rather than the series type of solution discussed by Thomas and Zucker. The radiation pattern is expressed as the product of an array factor and the radiation from one complete cycle; for sinusoidal modulation, Anger functions are involved. The case of constant amplitude, but with sinusoidal phase modulation, was discussed in detail and was applied to a modulated slow wave structure.

Bolljahn [1961] presented an exact synthesis procedure for the design of a surface which would support a prescribed group of surface waves simultaneously. The surface not only requires a periodic variation of surface impedance along its length, but it would itself have to be modulated. Some general properties of the surfaces and the composite waves they support were discussed, and it was indicated that the method is applicable only to corrugated surfaces and not to dielectric slabs because the analysis postulates that no energy flow occurs across this surface.

Ishimaru and Tuan [1962] extended the earlier analysis of Oliner and Hessel to discuss the frequency scanning of antennas in terms of k versus β diagrams. Modifications were introduced to make the results applicable to antennas with a finite number of elements, and the case of Tchebycheff arrays was treated in detail. Further investigations at the Polytechnic Institute of Brooklyn on radiation from periodically-modulated surfaces were described by Oliner [1963]. The "mode-coupling" regions of the k versus β diagrams were examined in detail, particularly in the leaky wave range. It was shown that at broadside radiation, the periodic structure approach predicts a sharp drop in radiated power and severe reflections, in agreement with experimental evidence; other interesting effects were found to occur at the backward and forward endfire positions. The propagation characteristics of waves on sinusoidally modulated corrugated rods were considered by Wang and Kornhauser [1962]. They measured the guide wavelengths on two types of modulated rods, and performed an analysis based on the Mathieu equation. The measured and theoretical values agreed rather well with each other, but both measurement and theory were confined to the range in which the guided waves were purely bound.

The scattering of electromagnetic waves by a sinusoidally varying air-dielectric interface in the limit of small wavelength has been investigated by Jacobson [1962]. He applies the Luneberg-Kline analysis to a general air-dielectric interface, and then treats in detail the case of a sinusoidal interface. The solution is valid only in the small wavelength range, and has been used to evaluate quantitatively the accuracy of the geometric optics approximation for certain parameter values. In the range considered, geometric optics serves as an excellent approximation.

A new theory of Wood's anomalies on periodic structures has been developed by Hessel and Oliner [1963]. Wood's anomalies are resonance effects which occur in the scattering of plane waves by periodic structures; the effects have been known for many years, and the new contribution lies in the recognition of their relation to leaky waves. This theory based on leaky waves is more complete than the previous, multiple-scattering theory, and also permits the prediction of resonance effects in other scattering phenomena. Wood's anomaly resonances were also found by Li and Oliner [1963] on a structure composed of a strip grating backed by a metal plate; the behavior of these resonances was explained readily by the above-mentioned leaky wave theory [1963]. The multiple-scattering theory of Wood's anomalies has been developed by Twersky and others in a series of earlier papers; this approach was applied in detail by Twersky [1962] to the case of an infinite grating of circular cylinders. A discussion of other features of his paper and other papers concerned with multiple-scattering is presented elsewhere in this report.

The properties of periodic structures are being found useful in an explanation of the operation of log-periodic antennas. Such antennas are members of the class of frequency-independent antennas and are characterized by a linear variation in the element spacing and element length along the structure. If this variation is slow, any given small section of the structure may be regarded as locally constant. Mayes, Deschamps, and Patton [1961] were among the first to appreciate this viewpoint, and to propose the application of periodic structure theory to this class of antennas. A recent report [Mayes et al., 1962] by these authors applies these viewpoints in detail to a variety of such antennas, including a helix-fed monopole array, the bifilar zigzag antenna, and the log-periodic dipole array. Radiation patterns are presented and some of the features of operation are explained in terms of k versus β diagrams. Such diagrams have also been employed by Oliner [1962, 1963] in an explanation of some of the symmetry requirements associated with proper operation of these antennas. Mayes and Ingerson [1962] have taken extensive near field measurements on a periodic analog of the log-periodic dipole array in order to achieve a better understanding ot these structures. A study of logarithmically periodic circuits, including the effect of loss which is introduced to represent radiation resistance, has been conducted by Mittra [1962] and Jones and Mittra [1962]. Log-periodic circuits have also been investigated by Du Hamel [1963], who has considered the general properties of such transmission lines. Rumsey [1963] has analyzed an idealization of a zigzag antenna consisting of an anisotropic sheet which is perfectly conducting only along a sinusoidal path. He has found the dispersion characteristics of the structure and has demonstrated various similarities between the operation of this structure and that of the corresponding antenna. Gans [1962] has continued this study by considering the behavior of a pair of such anisotropic sheets.

References

Angulo, C. M., and W. S. T. Chang (Mar. 1961), The launching of surface waves by a magnetic line source, Proc. IEE **108,** Pt. C, 187.

Barlow, H. M. (July 1960), Note on "The excitation of electromagnetic surface waves on a curved surface," IRE Trans. Ant. Prop. **AP–8,** 449.

Beyer, J. B., and E. H. Scheibe (May 1962), Higher modes in guided electromagnetic wave beams, IRE Trans. Ant. Prop. **AP–10,** 349.

Bolljahn, J. T. (May 1961), Synthesis of modulated corrugated surface wave structures, IRE Trans. Ant. Prop. **AP–9,** 236.

Bresler, A. D. (Jan. 1960), On the TE_{no} modes of a ferrite slab loaded rectangular waveguide and the associated thermodynamic paradox, IRE Trans. Microwave Theory Tech. **MTT–8,** 81.

Case Institute of Technology (Oct. 1962), Study of control of aperture fields, Final Rept. issued on Contract AF 19(604)–3887, Cleveland, Ohio.

Cassedy, E. S., and M. Cohn (May 1961), On the existence of leaky waves due to a line source above a grounded dielectric slab, IRE Trans. Microwave Theory Tech. **MTT–9,** 243.

Cheo, B. R., V. H. Rumsey, and W. J. Welch (Nov. 1961), A solution to the frequency-independent antenna problem, IRE Trans. Ant. Prop. **AP–9,** 527.

Christian, J. R., and G. Goubau (May 1961), Experimental studies on a beam waveguide for millimeter waves, IRE Trans. Ant. Prop. **AP–9,** 256.

Chu, T., and N. R. Kilcoyne (July 1961), The excitation of a dielectric-rod antenna by a helix, IRE Trans. Ant. Prop. **AP–9,** 416.

Chu, T. S., R. J. Kouyoumjian, F. C. Karal, and S. N. Karp (Nov. 1962), The diffraction of surface waves by a terminated structure in the form of a right-angle bend, IRE Trans. Ant. Prop. **AP–10,** 679.

Cohn, M. (July 1960), TE modes of the dielectric loaded trough line, IRE Trans. Microwave Theory Tech. **MTT–8,** 449.

Cohn, M., E. S. Cassedy, and M. A. Kott (Sept. 1960), TE mode excitation on dielectric loaded parallel plane and trough waveguides, IRE Trans. Microwave Theory Tech. **MTT–8,** 545.

Cohn, M., and M. J. King (Mar. 1962), Selected surface wave excitation studies, Electronic Commun., Inc., Timonium, Md., Sci. Rept. No. 3.

Collin, R. E. (Jan. 1960), Scattering by an array of thin dielectric sheets, IRE Trans. Ant. Prop. **AP–8,** 62.

Collin, R. E. (Sept. 1962), Analytical solution for a leaky-wave antenna, IRE Trans. Ant. Prop. **AP–10,** 561.

Collin, R. E., and W. H. Eggimann (Mar. 1961), Dynamic interaction fields in a two-dimensional lattice, IRE Trans. Microwave Theory Tech. **MTT–9,** 110.

Cullen, A. L. (Jan.–Feb. 1960), Surface-wave resonance effect in a reactive cylindrical structure excited by an axial line source, J. Res. NBS **64D** (Radio Prop.), 13.

Diament, P., S. P. Schlesinger, and A. Vigants (July 1961), A dielectric surface wave structure: the V-line, IRE Trans. Microwave Theory Tech. **MTT–9,** 332.

DuHamel, R. H. (1963), Log periodic antennas and circuits, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 1031–1050 (Pergamon Press, New York and London).

Eggiman, W. H. (Sept. 1961), Higher order evaluation of electromagnetic diffraction by circular disks, IRE Trans. Microwave Theory Tech. **MTT–9,** 408.

Eggiman W. H., and R. E. Collin (1962), Electromagnetic diffraction by a planar array of circular disks, Paper presented at the 1962 PGMTT National Symposium; to be published in IRE Trans. Microwave Theory Tech.

Felsen, L. B. (Apr. 1960a), On the electromagnetic properties of wedges and cones with a linearly varying surface impedance, Polytechnic Institute of Brooklyn, Rept. No. 736–59.

716–797—64——9

Felsen, L. B. (Nov. 1960b), Radiation from a tapered surface wave antenna, IRE Trans. Ant. Prop. **AP–8,** 577.
Gans, M. J. (Oct. 1962), Waves supported by two parallel sinusoidally anisotropic planes, Fall URSI–IRE Meeting, Ottawa, Canada, Oct. 1962.
Gardner, R. A. (Apr. 1960), Shunt susceptance of planar arrays of conducting disks, Case Institute of Technology, Cleveland, Ohio, Sci. Rept. No. 13.
Goldstone, L. L., and A. A. Oliner (May 1961), Leaky-wave antennas, II. Circular waveguides, IRE Trans. Ant. Prop. **AP–9,** 280.
Goubau, G. (1963), Optical relations for coherent wave beams, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 907–918 (Pergamon Press, New York and London).
Goubau, G., and F. Schwering (May 1961), On the guided propagation of electromagnetic wave beams, IRE Trans. Ant. Prop. **AP–9,** 248.
Harvey, A. F. (Jan. 1960), Periodic and guiding structures at microwave frequencies, IRE Trans. Microwave Theory, Tech. **MTT–8,** 30.
Hessel, A. (Sept. 1962), On the influence of complex poles on the radiation pattern of leaky-wave antennas, IRE Trans. Ant. Prop. **AP–10,** 646.
Hessel, A., N. Marcuvitz, and J. Shmoys (Jan. 1962), Scattering and guided waves at an interface between air and a compressible plasma, IRE Trans. Ant. Prop. **AP–10,** 48.
Hessel, A., and A. A. Oliner (1963), Wood's anaomalies and leaky waves, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 887–890 (Pergamon Press, New York and London).
Hirsch, P., and J. Shmoys (Oct. 1962), Surface waves on plasma slabs, Polytechnic Institute of Brooklyn, Rept. No. 1093–62.
Ishimaru, A. (Apr. 1962), The effect of a unidirectional surface wave along a perfectly conducting plane on the radiation from a plasma sheath, Second Symposium on the Plasma Sheath, Boston, Mass., Apr. 10–12, 1962.
Ishimaru, A. (1963), Unidirectional waves in anisotropic media, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C., Jordan, pt. 1, 591–601 (Pergamon Press, New York and London).
Ishimaru, A., and G. D. Bernard (Mar. 1962), Closed form analysis for the radiation pattern of the modulated antenna, IRE Trans. Ant. Prop. **AP–10,** 151.
Ishimaru, A., and H.-S. Tuan (Mar. 1962), Theory of frequency scanning of antennas, IRE Trans. Ant. Prop. **AP–10,** 144.
Jacobson, A. D. (Nov. 1962), Luneberg-Kline analysis of scattering from a sinusoidal dielectric interface, IRE Trans. Ant. Prop. **AP–10,** 715.
Jones, K. E., and R. Mittra (Oct. 1962), A survey of continuously scaled and log-periodic transmission lines with application to antennas problems, Fall URSI–IRE Meeting, Ottawa, Canada, Oct. 15–17, 1962.
Jones, W. R. (Oct. 1961), An approximate approach to problems involving surface waves, Hughes Aircraft Co., Fullerton, Calif., Rept. No. FL 61–60.
Kane, J. (Sept. 1960), The efficiency of launching surface waves on a reactive half-plane by an arbitrary antenna, IRE Trans. Ant. Prop. **AP–8,** 500.
Kane, J. (Mar. 1962a), Optimum parameters for a class of surface wave antennas, University of Rhode Island, Sci. Rept. No. 7983/1.
Kane, J. (Nov. 1962b), A mathematical analysis of a surface wave antenna of finite length, University of Rhode Island, Sci. Rept. No. 7983/1.
Kane, J. (1963), A surface wave antenna as a boundary value problem, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 891–894 (Pergamon Press, New York and London).
Karal, F. C., S. N. Karp, T. S. Chu, and R. G. Kouyoumjian (Feb. 1961), Scattering of a surface wave by a discontinuity in the surface reactance on a right-angled wedge, Commun. Pure Appl. Math. **14,** 35.
Keller, J. B., and F. C. Karal (June 1960), Surface wave excitation and propagation, J. Appl. Phys. **31,** 1039.
Kieburtz, R. B., and A. Ishimaru (Nov. 1961), Scattering by a periodically apertured conducting screen, IRE Trans. Ant. Prop. **AP–9,** 506.
Kieburtz, R. B., and A. Ishimaru (Nov. 1962), Aperture fields of an array of rectangular apertures, IRE Trans. Ant. Prop. **AP–10,** 663.
King, M. J., and J. C. Wiltse (Aug. 1960), Surface-wave propagation on a dielectric rod of elliptical cross-section, Electronic Commun., Inc., Timonium, Md. Sci. Rept. No. 1.
King, M. J., and J. C. Wiltse (May 1962), Surface-wave propagation on coated or uncoated metal wires at millimeter wavelengths, IRE Trans. Ant. Prop. **AP–10,** 246.
Kolettis, N. J., and R. E. Collin (Sept. 1961), Anisotropic properties of strip-type artificial dielectric, IRE Trans. Microwave Theory Tech. **MTT–9,** 436.
Kornhauser, E. T., and G. S. Heller (1963), A soluble problem in duct propagation, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 981–990 (Pergamon Press, New York and London).
Kurss, H. (Feb. 1961a), Transmission line formulation for semi-conductors, Polytechnic Institute of Brooklyn, Rept. No. 885–60.
Kurss, H. (July 1961b), Eigenwaves on a lattice of small scatterers, Polytechnic Institute of Brooklyn Rept. No. 936–61.
Lengyel, B. A., and K. M. Mitzner (Sept. 1961), Electromagnetic surface waves on a plane interface, J. Appl. Phys. **32,** 1758.
Li, R., and A. A. Oliner (June 1961), Scattering resonances on a fast-wave structure, Polytechnic Institute of Brooklyn Rept. PB 1156–63, 887–890.
Lichtenberg, A. J., and J. R. Woodyard (June 1962), Plasma waveguides as low-loss structures, J. Appl. Phys. **33,** 1976.
Logan, N. A., and K. S. Yee (Jan. 1962), A simple expression for the propagation constants associated with a reactive convex surface, IRE Trans. Ant. Prop. **AP–10,** 103.
Mayes, P. E., G. A. Deschamps, and W. T. Patton (May 1961), Backward wave radiation from periodic structures and application to the design of frequency-independent antennas, Proc. IRE **49,** 962.
Mayes, P. E., G. A. Deschamps, and W. T. Patton (Dec. 1962), Backward-wave radiation from periodic structures and application to the design of frequency-independent antennas, Univ. of Illinois, Tech. Rept. No. 60.
Mayes, P. E., and P. G. Interson (Oct. 1962), Near-field measurements on backfire periodic dipole arrays, Fall URSI–IRE Meeting, Ottawa, Canada, Oct. 15–17, 1962.
Mittra, R. (July 1962), Theoretical study of a class of logarithmically periodic circuits, Univ. of Illinois, Tech. Rept. No. 59.
Nishida, S. (May 1960a), Coupled leaky waveguides, I. Two parallel slits in a plane, IRE Trans. Ant. Prop. **AP–8,** 323.
Nishida, S. (July 1960b), Coupled leaky waveguides, II. Two parallel slits in a cylinder, IRE Trans. Ant. Prop. **AP–8,** 354.
Oliner, A. A. (Oct. 1962), A dispersion curve approach to the analysis of log-periodic antennas, Fall URSI–IRE Meeting, Ottawa, Canada.
Oliner, A. A. (1963), The role of leaky waves in electromagnetic phenomena, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 837–856 (Pergamon Press, New York and London).
Oliner, A. A., and T. Tamir (Jan. 1962a), Backward waves on isotropic plasma slabs, J. Appl. Phys. **33,** 231.
Oliner, A. A., and T. Tamir (Apr. 1962b), Radiation from semi-infinite slot-excited plasma sheath configurations, Second Symposium on the Plasma Sheath, Boston, Mass., Apr. 10–12, 1962.
Paik, S. F. (Apr. 1962), A backward wave in plasma waveguide, Proc. IRE **50,** 462.
Ravid-Weissberg, E. (Nov. 1961), On the efficiency of excitation of surface waves by distributed coupling, IRE Trans. Microwave Theory Tech. **MTT–9,** 573.
Richmond, J. H. (Sept. 1960), Surface waves on symmetrical three-layer sandwiches, IRE Trans. Microwave Theory Tech. **MMT–8,** 572.
Rumsey, V. H. (Sept. 1961), A new way of solving Maxwell's equations, IRE Trans. Ant. Prop. **AP–9,** 461.
Rumsey, V. H. (1963), On the theory of frequency inde-

pendent antennas, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 1011–1030 (Pergamon Press, New York and London).
Rusch, W. V. T. (Mar. 1962), Propagation constants of surface waves on a plasma-clad cylinder, IRE Trans. Ant. Prop. **AP–10,** 213.
Schlesinger, S. P., P. Diament, and A. Vigants (Mar. 1960), On higher order hybrid modes of dielectric cylinders, IRE Trans. Microwave Theory Tech. **MTT–8,** 252.
Seshadri, S. R. (July 1962a), Excitation of surface waves on a unidirectionally conducting screen, IRE Trans. Microwave Theory Tech. **MTT–10,** 279.
Seshadri, S. R. (Sept. 1962b), Scattering of surface waves by discontinuities on a unidirectionally conducting screen, IRE Trans. Microwave Theory Tech. **MTT–10,** 367.
Snitzer, E. (Jan. 1961), Proposed fiber cavities for optical masers, J. Appl. Phys. **32,** 36.
Snitzer, E. (1963), Optical dielectric waveguides, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 903–905 (Pergamon Press, New York and London).
Sobel, F., F. L. Wentworth, and J. C. Wiltse (Nov. 1961), Quasi-optical surface waveguide and other components for the 100-300 gc region, IRE Trans. Microwave Theory Tech. **MTT–9,** 512.
Spector, J., and A. W. Trivelpiece (June 1962), Backward waves in longitudinally magnetized ferrite rods, J. Appl. Phys. **33,** 2142.
Tai, C. T. (Jan. 1960), Evanescent modes in a partially-filled gyromagnetic rectangular waveguide, J. Appl. Phys. **31,** 220.
Tamir, T. (Sept. 1961), The electromagnetic field of a source-excited plasma slab, Polytechnic Institute of Brooklyn, Rept. No. 932–61.
Tamir, T., and A. A. Oliner (Dec. 1961), The spectrum of electromagnetic waves guided by a plasma layer, Polytechnic Institute of Brooklyn, Rept. No. 970–61; also Proc. IRE, in press.
Tamir, T., and A. A. Oliner (Jan. 1962), The influence of complex waves on the radiation field of a slot-excited plasma layer, IRE Trans. Ant. Prop. **AP–10,** 55.
Tamir, T., and A. A. Oliner (Mar. 1962a), Guided complex waves, Part I. Fields at an interface, Polytechnic Institute of Brooklyn, Rept. No. 1008–62, also Proc. IEE, Pt. B, in press.
Tamir, T., and A. A. Iliner (Apr. 1962b), Guided complex waves, Part II. Relation to radiation patterns, Polytechnic Institute of Brooklyn, Rept. No. 1009–62; also Proc. IEE, Pt. B, in press.
Thompson, G. H. B. (Nov. 1961), Backward waves in longitudinally magnetized ferrite rods, J. Appl. Phys. **32,** 2491.
Trivelpiece, A. W., A. Ignatius, and P. C. Holscher (Feb. 1961), Backward waves in longitudinally magnetized ferrite rods, J. Appl. Phys. **32,** 259.
Twersky, V. (Nov. 1962), On the scattering of waves by the infinite grating of circular cylinders, IRE Trans Ant. Prop. **AP–10,** 737.
Vigants, A., and S. P. Schlesinger (Sept. 1962), Surface waves on radially inhomogeneous cylinders, IRE Trans. Microwave Theory Tech. **MTT–10,** 375.
Wait, J. R. (July 1960), On the excitation of electromagnetic surface waves on a curved surface, IRE Trans. Ant. Prop. **AP–8,** 445.
Wait, J. R., and A. M. Conda (Sept. 1960), The resonance excitation of a corrugated-cylinder antenna, Proc. IEE **107,** Pt. C, 362.
Wait, J. R., and A. M. Conda (July 1961), Resonance characteristics of a corrugated-cylinder excited by a magnetic dipole, IRE Trans. Ant. Prop. **AP–9,** 330.
Walter, C. H. (Sept. 1960), Surface wave Luneberg lens antennas, IRE Trans. Ant. Prop. **AP–8,** 509.
Wang, C. C., and E. T. Kornhauser (May 1962), Propagation on modulated corrugated rods, IRE Trans., Microwave Theory Tech. **MTT–10,** 161.
Wenger, N. (June 1962), Launching a surface wave on a reactive plane by a parallel plate waveguide, M.Sc. Thesis, Case Institute of Technology.
Yeh, C. (Nov. 1962), Elliptical dielectric waveguides, J. Appl. Phys. **33,** 3235.
Zucker, F. J. (1961), Surface- and leaky-wave antennas, Antenna Engineering Handbook, ed. H. Jasik, ch. 16 (McGraw-Hill Book Co., New York, N.Y.).

3.4. Radiation From Plasmas

L. B. Felsen [1]

This part of the report deals with the radiation from applied electromagnetic sources in the presence of bounded or unbounded plasmas; the sources may be either stationary in space and oscillate in time (as in the antenna problem), or they may comprise moving charges (as in Cerenkov radiation). A distributed source may be regarded as a superposition of vector point sources, whence the problem of radiation from the latter is fundamental and has received some detailed attention during the past 3 years. Since explicit closed-form solutions for point-source radiation problems can be found only for a few selected cases, it is generally necessary to synthesize the point-source field as a superposition of wave fields which individually satisfy the source-free field equations. A knowledge of the propagation characteristics of plane waves (or their equivalent) in the medium is therefore essential to the study of radiation from localized source distributions. Because of the functional complexity of the formal solutions resulting after this superposition, detailed investigations of radiation problems have been carried out so far only for the simplest characterizations of the plasma medium. In most of the results to be reported below, temperature and ionic effects have been neglected and the plasma is assumed to be describable solely in terms of an equivalent tensor or scalar dielectric constant, depending on whether or not an external magnetic field is present. This assumption has reasonable validity for certain types of plasmas and externally located sources, but is bound to be inaccurate for sources in direct contact with the plasma since pressure effects, and the associated acoustic waves, may then not be ignored. These latter phenomena have been included in several studies. Because of the complexity of wave couplings in inhomogeneous anisotropic regions, the analysis of radiation problems has been limited primarily to homogeneous media or to inhomogeneous media simulated by piecewise constant layers.

While the representability of a plasma via the above-sketched simplified models is open to question, the study of electromagnetic radiation phenomena for such "artificial" media can shed considerable insight on some of the effects to be anticipated under more general and more realistic conditions.

A. Stationary Sources in Isotropic Plasmas

1. Incompressible Medium

A number of contributions have appeared on the radiation from time-harmonic electromagnetic sources in a bounded isotropic plasma characterized by the equivalent relative dielectric constant

$$\epsilon=1-\frac{\omega_p^2}{\omega(\omega+i\nu)},\ (e^{-i\omega t}\text{ dependence})$$

where ω_p is the (angular) plasma frequency, ω the applied frequency, and ν the electron-ion collision frequency. Most of the formal results are identical with those obtained in previous studies of ordinary dielectrics but the structure of the radiated fields may differ appreciably from the ordinary case since the real part of ϵ is less than unity and may be negative. An important difference arises, for example, from the possibility of supporting a true undamped surface wave on a plane interface between two media having real dielectric constants ϵ and ϵ_0, respectively, provided that $\epsilon<-\epsilon_0$ and that the wave has an electric field component normal to the interface (E mode). As a result, the spectrum of waves supported by a plasma layer may differ significantly from that of an ordinary dielectric layer [Tamir and Oliner, 1961; Oliner and Tamir, 1962a] and the excitation by localized sources of interface surface waves or of other wave types peculiar to a medium with negative dielectric constant is of interest. These aspects have been studied by Tamir and Oliner [1962] in connection with the problem of radiation from a line source of magnetic current covered by a homogeneous plasma slab, and they emphasize the role played by the leaky waves in determining some of the prominent features of the radiation field exterior to the slab region. Some related investigations have been made by Larson [1962] and Yee [1962]. Yee [1961] has also dealt with the coupling between two narrow slots covered by a thick plasma layer with $0<\epsilon<1$, and he presents curves for the input and mutual admittance of a slot as a function of slot separation. Calculations of the fields radiated through a spherical plasma shell by a small electric or magnetic antenna have been carried out by Raemer [1962], who has likewise considered the perturbation in antenna impedance caused by the plasma shell, and by Marini [1961], and by Katzin and Koo [1962]. The excitation of interface surface waves by a ring source of magnetic currents concentric with a homogeneous cylindrical plasma column was treated by Samaddar [1962a], who has also considered the surface wave problem for a plasma column with continuous radial variation of dielectric constant [1962b]. Hasserjian and Higgins [1962] have made calculations of the fields excited by a slot on a large circular cylinder surrounded by a concentric plasma layer. Methods of simulating plasmas by artificial media have been discussed by Rotman [1962]. A formal initial value solution of the time dependent Maxwell field equations has been given by Gerwin [1962].

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[1] Polytechnic Institute of Brooklyn, Brooklyn, N.Y.

2. Compressible Medium

In a more sophisticated description of the plasma, the assumption of incompressibility is dropped and an isotropic pressure gradient of the electron gas is taken into account. In this instance, an electromagnetic source will excite both electromagnetic and acoustic waves; the relative strength of excitation of these waves is a matter of some concern since calculations based on the incompressible model are likely to be inaccurate under conditions which favor the strong excitation of acoustic waves. Comparison with experimental measurements has shown that the observed impedance of an antenna surrounded by a plasma cannot sometimes be satisfactorily explained without taking into account the compressibility of the medium [Whale, 1962]. The radiation from an oscillating electric current element in an infinite, homogeneous, collisionless, compressible electron plasma has been calculated by Hessel and Shmoys [1961]. They obtain a simple closed form solution for the electromagnetic and acoustic fields and find that the ratio of power in the acoustic wave to that in the electromagnetic wave is proportional to $(c/a)^3(\omega_p/\omega)^2$, where c and a are the electromagnetic and acoustic wave speeds, respectively, in the same gas without electronic charge. A similar calculation for a quarter wavelength dipole was made by Cohen [1962b], who also found that the acoustic contribution to the radiation resistance predominates, especially when $\omega \approx \omega_p$. Since (c/a) is usually a large quantity, these results show that compressional effects do play a role, although the assumed model (which neglects nonlinear effects, collisions, motions of heavy particles, Debye shielding of the antenna) is likely to be unrealistic very near the source where strong field gradients exist. Cohen [1962b] has also considered the boundary condition at a perfect conductor in a compressible plasma and has made calculations of the positive sheath required to assure that no net current flows into the boundary.

Calculations have also been made of the radiation from currents on a sphere embedded in the plasma, or from a dipole located at the center of a spherical void in the plasma [Hessel and Shmoys, 1961]. If the sphere radius is small compared to both the electromagnetic and acoustic wavelengths, the power ratio is essentially that given above while a large-radius sphere reduces the acoustic wave excitation substantially; hence, as noted previously, acoustic wave effects are expected to be pronounced when the plasma extends into the induction field of the source. Cohen [1962a] has dealt with the general source problem, discussed the boundary conditions at discontinuities in the medium, and formulated equivalence theorems for the acoustic and electromagnetic fields. He has applied these results to the calculation of scattering by a tenuous spherical inhomogeneity in the plasma.

The excitation of a compressible plasma half-space by a magnetic line current placed in the exterior medium has been considered by Hessel, Marcuvitz, and Shmoys [1962]. They find that the interface can support a surface wave, and that the energy in an electromagnetic plane wave incident almost normally on the plasma half space can be converted entirely into the acoustic wave if ω is slightly higher than ω_p. The efficiency of conversion is extremely sensitive to the angle of incidence and to the wave frequency, and the effect is not expected to be significant if the line source is far enough above the interface.

B. Stationary Sources in Anisotropic Plasmas

1. Incompressible Medium

a. Unbounded Region

If an incompressible plasma is subjected to an external steady magnetic field H_0 oriented along the z-axis, the electromagnetic properties of the medium are describable by a normalized dielectric tensor

$$\underset{\sim}{\epsilon}=\begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix},$$

where ϵ_z has the form previously given and ϵ_1, ϵ_2 are functions of ω, ω_p, ν, and ω_c, with ω_c representing the electron-cyclotron frequency. Even in the collisionless case $\nu=0$, with ϵ_1, e_2, ϵ_z real, the (ordinary and extraordinary) plane waves capable of propagating in this anisotropic medium may be separated into some eight categories distinguished by the values of the ratios ω/ω_p and ω/ω_c; see Allis [1961], Haskell and Holt [1961], Deschamps and Weeks [1962], for recent contributions in this area, whence the detailed study of the radiation from localized sources embedded in such a medium poses considerable difficulties. Two principal approaches have been employed to analyze the radiation from current elements in an infinite, homogeneous medium described by the above dielectric tensor: the first is based on the formulation of the dipole field in terms of a spectrum of three-dimensional plane waves ("cavity" mode approach) while the second utilizes a two-dimensional representation in a plane perpendicular to a preferred space direction ("guided" mode approach). The first procedure, employed first by Bunkin [1957] in the U.S.S.R. and subsequently in the United States by Kogelnik [1960], Meecham [1961], Ford [1961], Kuehl [1961, 1962], Mitra and Deschamps [1962], leads naturally to a triple Fourier integral representation while the second, used by Abraham [1953], Arbel [1960], Arbel and Felsen [1962a], and by Wu [1962], involves double Fourier integrals. The cavity mode procedure is well suited to the calculation of radiated power in an infinite medium [Kogelnik, 1960] and also permits the representation of the dyadic Green's function in an appealing symmetrical form; it has recently been employed by Chow [1962] to study radiation in media having both a tensor permeability and a tensor dielectric constant. The guided wave formalism is easily adapted to the analysis of stratified anisotropic media comprising piecewise constant layers whose boundary is perpendicular to the symmetry axis [Arbel, 1960; Arbel and Felsen, 1962a], and is also more convenient for an asymptotic evaluation of the radiation fields since it involves double, rather than triple, integrals.

An important consideration for radiation problems in unbounded anisotropic regions is the phrasing of a radiation condition. While the requirement of bounded fields at infinity assures the proper formulation when losses are present, it is necessary in a lossless medium to replace the usual "outgoing wave" condition by an "outgoing energy" condition [Bresler, 1959]. This is not a trivial distinction, since the directions of phase and energy propagation in an anisotropic medium generally do not coincide. Arbel and Felsen [1962a] have shown how the plane wave dispersion (refractive index) curves for the medium can be used in imposing the radiation condition, and that each of the constituent propagating plane waves must carry energy away from a localized source.

Asymptotic calculations of the radiation fields have been made for various possible values of the plasma parameters ω_c/ω and ω_p/ω and for different orientations of the dipole source [Arbel, 1960; Arbel and Felsen, 1962b; Meecham, 1961; Ford, 1961; Kuehl, 1961; Wu, 1962]. Because of the anisotropy, the detailed field structure depends markedly on the plasma parameters and on the angular location of the observation point with respect to the magnetic field direction. For a restricted range of observation angles in certain plasmas, the field may comprise as many as four spherical waves which propagate with different phase velocities and individually carry energy in the direction from the source to the observation point; for other conditions, no propagation is possible. The number and intensity of these wave contributions can be predicted from the refractive index curves or their equivalent. In plasmas having a refractive index which becomes singular along certain directions, the fields exhibit strong singularities and the total radiated power is infinite (yielding an infinite radiation resistance for the dipole antenna) [Kogelnik, 1960]. These nonphysical singularities disappear if the discontinuous source function is replaced by a distributed source whose strength goes to zero gradually, or if loss is included. Enhanced, but bounded, field strengths are also encountered when the refractive index curves possess inflection points [Arbel and Felsen, 1962b].

If the applied magnetic field is infinite ($\omega_c=\infty$), $\epsilon_2=0$ in (2) and the dielectric tensor is diagonal (uniaxial medium). For this special case, solutions for the point source or line source excitation problem can be obtained in closed form [Arbel, 1960; Kuehl, 1961; Mittra, 1962; Arbel and Felsen, 1962c; Tuan and Seshadri, 1962]. Arbel and Felsen [1962b] have shown how these closed form solutions can be used to study the previously mentioned field singularities when ω_c is finite. Mittra and Deschamps [1962] have expressed the dyadic Green's function for finite ω_c in terms of a modification of the closed form results for $\omega_c=\infty$ plus a correction term; since the closed form expression is dominant in the near field of the source, this formulation appears attractive for antenna impedance calculations. A variational approach to the antenna impedance problem has been presented by Ament, Katzin, and Katzin [1962].

Some general observations about radiation problems in transposed anisotropic media have been made by Tai [1961] and by Villeneuve [1961].

b. Bounded Regions

Problems of radiation in the presence of confined anisotropic plasmas have also received attention. Ishimaru [1962] and Seshadri [1962a] have considered a magnetic line current in a plasma bounded by a perfectly conducting plane, with the external magnetic field H_0 directed parallel to the line source. They note the asymmetry of the radiation pattern, and find that there may exist a unidirectional surface wave which is launched only to one side of the source and decays away from the bounding plane; the asymmetry is reversed upon a reversal of the direction of H_0. Seshadri has calculated the efficiency of excitation of the surface wave as a function of the source location. Ishimaru has also dealt with a magnetic-line-source-excited plasma slab of finite width, as have Hodara and Cohn [1962a] and Shore and Meltz [1962]; in the latter analysis, the direction of H_0 is perpendicular to that of the line current. Via the two-dimensional Fourier transform procedure, Hodara [1962] has also discussed, and presented numerical data for, the radiation from a finite slot in a ground plane covered by a lossy anisotropic plasma slab but has omitted any consideration of surface or interface waves which may be excited. Wu [1962] has performed similar calculations for the radiation from an electric dipole source located inside or outside the slab. Some of the two-dimensional surface waves capable of propagating in an anisotropic plasma slab have been investigated by Hodara and Cohn [1962b]. For a slab of negligible thickness, Wait [1960a] has studied the possible surface wave solutions and has also calculated (1960b) their influence on the fields radiated by vertical magnetic or electric dipoles. Imposition of the previously mentioned radiation condition in the plasma can be circumvented for the slab problem, in view of the finite width of the anisotropic region; the radiation condition in the isotropic half-space is the conventional one.

There have been several studies of radiation from a plasma half-space. Arbel [1960] has dealt with the case where H_0 is perpendicular to the interface, and where an electric dipole parallel to H_0 is located either inside or outside the anisotropic region. He has made an asymptotic calculation of the far field and obtained an interpretation in terms of geometrical rays which are reflected, refracted and coupled at the interface, but he did not consider possible additional contributions arising from surface waves or from lateral (or head) waves. The latter, intimately connected with the phenomenon of critical refraction, are excited by an incident wave which is refracted into the exterior region in a direction parallel to the interface, and sheds energy back into the first medium by the reverse process. Analytically, the lateral waves usually arise from branch cut integral contributions which enter into the asymptotic field expressions. Their effects were studied by Tyras, Ishimaru, and Swarm [1962] for the case of a weak external magnetic field H_0 directed parallel to the interface, with a magnetic dipole source parallel to H_0. They find a type of lateral wave which arises from the interface coupling of the ordinary and extraordinary rays and seems to have no counterpart in isotropic media. Felsen [1962a, b] has analyzed radiation from a uniaxially anisotropic half-space ($\omega_c=\infty$), with the magnetic field oriented arbitrarily with respect to the interface. For $\omega<\omega_p$, he finds that the lateral waves constitute the dominant field contribution in those geometrical shadow regions of the plasma which cannot be reached by the waves radiated directly from the source or by waves reflected from the interface. The lateral waves in strongly anisotropic plasmas may therefore be of greater importance than in isotropic regions where they generally constitute a second-order effect.

Samaddar [1961] has obtained modal expansions for the fields excited by a ring source concentric with a shielded cylindrical column with gyrotropic dielectric constant and (or) permeability, and he has calculated the dispersion characteristics for the plasma case, with emphasis on possible slow surface waves. Wait [1961] has given the harmonic series solution for the field excited by a line source in the presence of a radially stratified plasma cylinder subjected to an axial magnetic field.

2. Compressible Medium

The radiation from a magnetic line source in an infinite, homogeneous, compressible plasma subjected to a steady magnetic field H_0 parallel to the source direction has been investigated by Seshadri [1962b]. He obtains a closed form solution for the radiated field in terms of electromagnetic and acoustic wave contributions, presents their dispersion characteristics, calculates the power coupled into each, and finds that the power in the electromagnetic wave is always smaller than when the source is located in free space.

C. Radiation From Moving Sources

An electric charge moving along a straight line with constant speed v produces electromagnetic radiation in a medium which can support waves having phase speeds smaller than v (Cerenkov effect). In a plasma without an externally impressed magnetic field, the dielectric constant ϵ is less than that of vacuum whence Cerenkov radiation cannot occur. However, a moving charge may excite acoustic oscillations provided that $v>a$, where a is the acoustic wave speed in the electron gas. Calculations to this effect were carried out by Abele (1961) and Cohen [1961]. Abele calculated the electromagnetic and acoustic fields, as well as the spectral distribution of radiated energy, for a line charge moving either in an infinite compressible plasma, or exterior and parallel to the surface of a plasma half-space. Cohen carried out a similar treatment for a point charge in an infinite compressible plasma. Tuan and Seshadri [1962] have calculated the radiation from a moving line charge in a uniaxially anisotropic medium ($\omega_c=\infty$) and have obtained expressions for the spectral distribution of the radiated energy; see also Johnson [1962]. In contrast to the case of an isotropic medium, a radially expanding spherically symmetric shell of charge may radiate in an anisotropic plasma. For a high-velocity burst, Ford [1961] has calculated the radiation fields and spectral distribution of energy in the low-frequency range $\omega<<\omega_c$, ω_p, and he finds that the radiation is confined within a narrow cone about the direction of the external magnetic field.

D. Diffraction by Objects in Anisotropic Plasmas

The study of diffraction by objects embedded in an incompressible isotropic plasma characterized by the aforementioned dielectric constant proceeds as for ordinary dielectrics and has therefore received little attention per se. However, the effects of anisotropy cause considerable modification and have stimulated some interest. While diffraction problems in a tensor dielectric medium are generally exceedingly complicated, there exist special cases which result in substantial simplification. For the uniaxial plasma ($H_0=\infty$, $\epsilon_2=0$ above), it can be shown that the analysis of diffraction by perfectly conducting cylindrical scatterers of arbitrary cross section can be simply related to the corresponding scattering problem in an isotropic medium, provided that the cylinder axis is parallel to H_0 [Arbel and Felsen, 1962c; Friedman, 1962]. Thus, formal solutions for such two-dimensional and three-dimensional diffraction problems involving a perfectly conducting half-plane, wedge, circular cylinder, etc., can be found with comparative ease; the real task is the meaningful interpretation of these solutions (or of their asymptotic approximations) so as to highlight in the broadest possible manner the effect of medium anisotropy upon the diffraction field. If H_0 in a uniaxially anisotropic medium is perpendicular to the cylinder axis, two-dimensional diffraction problems (independent of the axial coordinate) can likewise be related to equivalent problems in an isotropic region [Felsen, 1962c].

Simplifications also obtain for certain axially independent problems in a plasma with finite H_0 directed parallel to the cylinder axis. Problems of excitation by a line source of electric current are completely equivalent to the isotropic case; calculation of the fields excited by a magnetic line current is more involved but can be performed by the separation of variables method for the case of the circular cylinder [Samaddar, 1962c].

E. Summary and Prognosis

Theoretical activities during the past three years have clarified many of the effects to be expected when time-harmonic stationary sources or moving charges radiate from or near unbounded or bounded isotropic and anisotropic plasmas. To render these initial investigations tractable, the plasma has been represented by a highly idealized model, and the results are of questionable validity when applied to certain actual plasma configurations. It may therefore be expected that studies of radiation from stationary or moving sources during the next triennium will be concerned with more realistic models allowing for compressibility and motion of heavy particles, and that attention will also be given to the calculation of the impedance characteristics of variously shaped radiators of finite size in an infinite, homogeneous, anisotropic plasma. Because of the complicating effects of anisotropy even for the simple tensor medium noted above, the influence of interfaces or inhomogeneities on the fields radiated by a prescribed source distribution still remains to be clarified, especially as regards the excitation of surface waves or other types of interface waves. Some of the many experimental programs may be expected to come to fruition during the next three years and supply the theoretician with badly needed physical data to substantiate his choice of a given plasma model.

New and difficult mathematical boundary value problems have also arisen in connection with the determination of the radiation or diffraction pattern of a source in the presence of variously shaped objects embedded in an anisotropic medium characterized by a dielectric tensor as above. To gain an understanding of these phenomena for mathematically tractable situations, certain idealized shapes and medium properties have been considered, involving perfectly conducting cylindrical objects oriented with their axes parallel or perpendicular to the external magnetic field H_0. Much work still remains to be done even in this category, to say nothing of the case wherein H_0 is inclined with respect to the obstacle axis. It is desirable to represent the asymptotic evaluation of the formal solution in a manner which highlights and localizes the physical scattering contributions from such structural effects as surface curvature, edges, etc., when the object is large compared to the wavelength, or from its volume and orientation when the object is small. Much of the effort in diffraction theory in anisotropic media is thus likely to be devoted to (a) obtaining exact solutions and asymptotic approximations for scattering by simple shapes, and (b) developing the extension to anisotropic regions of such techniques as Rayleigh scattering approximations for long wavelengths or Keller's geometrical theory of diffraction for short wavelengths.

References

Abele, M. (1961), Radiation in a plasma from a uniformly moving distribution of electric charge, Proc. Symposium on Electromagnetics and Fluid Dynamics of Gaseous Plasma (Polytechnic Press of the Polytechnic Institute of Brooklyn, Brooklyn, N.Y.).

Abraham, L. G. (Aug. 1953), Extensions of the magneto-ionic theory to radio wave propagation in the ionosphere, including antenna radiation and plane wave scattering, Rept. No. 13, School of Elect. Eng., Cornell University, Ithaca, N.Y.

Allis, W. P. (Jan. 1961), Propagation of waves in a plasma in a magnetic field, IRE Trans. Microwave Theory Tech. **MTT–9,** 79–82.

Ament, W. S., J. C. Katzin, and M. Katzin (Oct. 1962), Impedance of a thin, center-driven cylindrical dipole in a homogeneous magnetosphere, paper presented at URSI Fall Meeting, Ottawa, Canada.

Arbel, E. (Nov. 1960), Radiation from a point source in an anisotropic medium, Rept. PIBMRI–861–60, Microwave Res. Institute, Polytechnic Institute of Brooklyn, Brooklyn, N.Y.

Arbel, E., and L. B. Felsen (1962a), Radiation from sources in plane stratified anisotropic plasma media, I, URSI Symposium on Electromagnetic Theory and Antennas (Pergamon Press, New York and London).

Arbel, E., and L. B. Felsen (1962b), Radiation from sources in plane stratified anisotropic plasma media, II, URSI Symposium on Electromagnetic Theory and Antennas (Pergamon Press, New York and London).

Arbel, E., and L. B. Felsen (Jan. 1962c), On electromagnetic Green's functions for uniaxially anisotropic regions, Rept. PIBMRI–985–61, Microwave Res. Institute, Polytechnic Institute of Brooklyn, Brooklyn, N.Y.

Bachynski, M. P., T. W. Johnston, and I. P. Shkarofsky (Jan. 1961), Universal representation of electromagnetic parameters in presence of D.C. magnetic fields, Proc. IRE **49,** 354–355.

Bresler, A. D. (1959), The far fields excited by a point source in a passive dissipationless anisotropic uniform waveguide, IRE Trans. Microwave Theory Tech. **MTT–7,** 282–287.

Bunkin, F. V. (1957), J. Exp. Theor. Phys. (USSR).

Chow, Y. (1962), A note on radiation in a gyro-electric-magnetic medium—an extension of Bunkin's calculation, IRE Trans. Ant. Prop. **AP–10,** 464–469.

Cohen, M. H. (1961), Radiation in a plasma, I. Cerenkov effect, Phys. Rev. **123,** 711–721.

Cohen, M. H. (1962a), Radiation in a plasma, II. Equivalent sources, Phys. Rev. **126,** 389–397.

Cohen, M. H. (1962b), Radiation in a plasma, III. Metal boundaries, Phys. Rev. **126,** 398–404.

Deschamps, G. A., and W. L. Weeks (May 1962), Charts for computing the refractive indexes of a magneto-ionic medium, IRE Trans. Ant. Prop. **AP–10,** 305.

Felsen, L. B. (May 1962a), Lateral waves on an anisotropic plasma interface, IRE Trans. Ant. Prop. **AP–10,** 347–349.

Felsen, L. B. (Aug. 1962b), Radiation from a uniaxially anisotropic half-space, Rept. PIBMRI–1058–62, Microwave Res. Institute, Polytechnic Institute of Brooklyn, Brooklyn, N.Y.

Felsen, L. B. (Aug. 1962c), On diffraction by objects in a uniaxially anisotropic medium, Rept. PIBMRI–1073–62, Microwave Res. Institute, Polytechnic Institute of Brooklyn, Brooklyn, N.Y.

Flock, W. L., and R. S. Elliott (Jan. 1962), The radiation pattern of a microwave horn and a plasma layer, IRE Trans. Ant. Prop **AP–10,** 65.

Ford, G. W. (1961), Electromagnetic radiation from a source in a plasma, Ann. Phys. **16,** No. 2, 185–200.

Friedman, B. (1962), Propagation in a non-homogeneous medium, Electromagnetic Waves, edited by R. E. Langer (Univ. of Wisconsin Press, Madison, Wis.).

Galejs, J. (Nov. 1961), ELF waves in the presence of exponential ionospheric conductivity profiles, IRE Trans. Ant. Prop. **AP–9,** 554.

Garbacz, R. J. (1962), Electromagnetic scattering from radially inhomogeneous spheres, Proc. IRE **50,** 1837.

Gerwin, R. A. (1962), Initial value solution of Maxwell's equations in cold plasma, Ann. Phys. **30,** No. 10, 711–715.

Harley, T. B., and G. Tyras (Dec. 1961), Transmission of E.M. waves through an ionized layer in the presence of a strong magnetic field, Proc. IRE **49,** 1822–1824.

Harris, J. H. (April 1962), Radiation through cylindrical plasma sheaths, paper presented at Second Symposium on the Plasma Sheath (sponsored by Electronic Res. Directorate, AFCRL, OAR), Boston, Mass.

Haskell, R. E., and E. H. Holt (Oct. 1961), Representation of propagation parameters for the plasma in a magnetic field, Proc. IRE **49,** 1584.

Hasserjian, G., and T. P. Higgins (Oct. 1962), Fields of a curved plasma layer excited by a slot, paper presented at URSI-IRE meeting, Ottawa, Canada.
Hershberger, W. D. (1961), Radiation field and Q of a resonant cylindrical plasma column, J. Phys. Fluids **4,** 740–742.
Hessel, A., N. Marcuvitz, and J. Shmoys (1962), Scattering and guided waves at an interface between air and a compressible plasma, IRE Trans. Ant. Prop. **AP–10,** 48–54.
Hessel, A., and J. Shmoys (1961), Excitation of plasma waves by a dipole in a homogeneous isotropic plasma, Proc. of the Symposium on Electromagnetics and Fluid Dynamics of Gaseous Plasma (Polytechnic Press of the Polytechnic Institute of Brooklyn, Brooklyn, N.Y.).
Hodara, H. (1961), The use of magnetic fields in the elimination of the re-entry radio blackout, Proc. IRE **49,** 1825–1830.
Hodara, H. (1962), Elimination of re-entry radio blackout, Proc. IRE **50,** 2108–2110.
Hodara, H., and G. I. Cohn (July 1962a), Wave propagation in magneto-plasma slabs, IRE Trans. Ant. Prop. **AP–10,** No. 4, 452–459.
Hodara, H., and G. I. Cohn (Sept. 1962b), Radiation from a gyro-plasma coated magnetic line source, IRE Trans. Ant. Prop. **AP–10,** 581.
Ishimaru, A. (Apr. 1962), The effect of a unidirectional surface wave along a perfectly conducting plane on the radiation from a plasma sheath, paper presented at the Second Symposium on the Plasma Sheath (sponsored by the Electronic Res. Directorate, AFCRL, OAR), Boston Mass. To be published in the Proc. of the Symposium.
Johnson, P. S. (1962), Cerenkov radiation spectra for a cold magnetoactive plasma, J. Phys. Fluids **5,** 118–119.
Katzin, M., and B. Y. C. Koo (Apr. 1962), Effects of re-entry plasma sheath on antenna characteristics, paper presented at Second Symposium on the Plasma Sheath (sponsored by the Electronic Res. Directorate, AFCRL, OAR), Boston, Mass.
Kogelnik, H. (1960), On electromagnetic radiation in magneto-ionic media, J. Res. NBS **64D** (Radio Prop.) No. 5, 515–523.
Kuehl, H. H. (1961), Electromagnetic radiation from an electric dipole in a cold anisotropic plasma, Rept. 79–203, Dept. of Electrical Eng., University of Southern California.
Kuehl, H. H. (1962), Electromagnetic radiation from an electric dipole in a cold anisotropic plasma, J. Phys. Fluids **5,** 1095–1105.
Larson, R. W. (Apr. 1962), Plasma imbedded antenna characteristics, paper presented at Second Symposium on the Plasma Sheath (sponsored by the Electronic Res. Directorate, AFCRL, OAR), Boston, Mass.
Marini, J. W. (1961), Radiation and admittance of a slotted-sphere antenna surrounded by a plasma sheath, Planetary Space Sci. **6,** 116–122.
Meecham, W. C. (1961), Source and reflection problems in magneto-ionic media, J. Phys. Fluids **4,** 1517–1524.
Miller III, G. F. (1962), Propagation of E. M. waves in a non-uniformly ionized medium, J. Phys. Fluids **5,** 899–907.
Mittra, R. (Jan. 1962), Solution of Maxwell's equations in a magneto-ionic medium with sources, Sci. Rept. No. 4, Antenna Lab., Dept. of Elect. Eng., University of Illinois, Urbana, Ill.
Mittra, R., and G. Deschamps (1963), Field solution for a dipole in an anisotropic medium, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 495–512 (Pergamon Press, New York and London).
Oliner, A. A., and T. Tamir (1962a), Backward waves on isotropic plasma slabs, J. Appl. Phys. **33,** 231–233.
Oliner, A. A., and T. Tamir (Apr. 1962b), Radiation from semi-infinite slot-excited plasma sheath configurations, presented at Second Symposium on Plasma Sheath, Boston, Mass.
Paik, S. F. (1962), A backward wave in plasma waveguide, Proc. IRE **50,** 462–463.
Raemer, H. R. (Jan. 1962), Radiation from linear electric or magnetic antennas surrounded by a spherical plasma shell, IRE Trans. Ant. Prop. **AP–10,** 69.
Rothman, H. S., and T. Morita (Apr. 1962), Transmission through an ionized medium in the presence of a strong magnetic field, paper presented at Second Symposium on the Plasma Sheath (sponsored by the Electronic Res. Directorate, AFCRL, OAR), Boston, Mass.
Rotman, W. (Jan. 1962), Plasma simulation by artificial dielectrics and parallel plate media, IRE Trans. Ant. Prop. **AP–10,** 82.
Rusch, W. V. T. (May 1962), Propagation constants of surface waves on a plasma-clad cylinder, IRE Trans. Ant. Prop. **AP–10,** 213–214.
Samaddar, S. N. (1961), Wave propagation in an anisotropic column with ring source excitation, Rept. 4386–I–T, Dept. of Elect. Eng., Radiation Lab., University of Michigan, Ann Arbor, Mich.
Samaddar, S. N. (1962a), Slow wave propagation along the boundary of two coaxial anisotropic media, J. Electron. Control **12,** No. 5, 353–368.
Samaddar, S. N. (1962b), Wave propagation in a cylindrical waveguide containing inhomogeneous plasma involving a turning point, Tech. Rept. ESRN–31, Raytheon Co., Wayland, Mass.
Samaddar, S. N. (1962c), Two-dimensional diffraction in anisotropic media, IRE Trans. Ant. Prop. **AP–10,** 621.
Seshadri, S. (May 1962a), Excitation of surface waves on a perfectly conducting screen covered with anisotropic plasma, Tech. Rept. No. 366, Cruft Lab., Harvard University.
Seshadri, S. (July 1962b), Excitation of plasma waves in an unbounded homogeneous plasma by a line source, Tech. Rept. No. 373, Cruft Lab., Harvard University.
Shore, R., and G. Meltz (Jan. 1962), Anisotropic plasma-covered magnetic line source, IRE Trans. Ant. Prop. **AP–10,** 78.
Tai, C. T. (1961), On the transposed radiating systems in an anisotropic medium, IRE Trans. Ant. Prop. **AP–9,** 502.
Tamir, T., and A. A. Oliner (1961), The spectrum of electromagnetic waves guided by a plasma layer, Rept. PIBMRI–970–61, Microwave Res. Institute, Polytechnic Institute of Brooklyn, Brooklyn, N.Y.
Tamir, T., and A. A. Oliner (1962), The influence of complex waves on the radiation field of a slot-excited plasma layer, IRE Trans. Ant. Prop. **AP–10,** 55–65.
Tamir, T., and A. A. Oliner (Feb. 1963), The spectrum of electromagnetic waves guided by a plasma layer, Proc. IEEE **51,** No. 3, 317–332.
Taylor, L. S. (1961), RF reflectance of plasma sheaths, Proc. IRE **49,** 1831–1836.
Tuan, H. S., and S. R. Seshadri (Aug. 1962), Radiation from a line source in a uniaxially anisotropic plasma, Tech. Rept. 375, Cruft Lab., Harvard University.
Tyras, G., A. Ichimaru, and H. M. Swarm (1963), Lateral waves on air-magnetoplasm interfaces, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 517–535 (Pergamon Press, New York and London).
Villeneuve, A. T. (Mar. 1961), Green's function techniques for inhomogeneous anisotropic media, IRE Trans. Microwave Theory Tech. **MTT–9,** 197–198.
Wait, J. R. (1960a), Propagation of electromagnetic surface waves along a thin plasma sheet, Can. J. Phys. **38,** 1586–1594.
Wait, J. R. (1960b), The electromagnetic fields of a dipole in the presence of a thin plasma sheet, Appl. Sci. Res. **B8,** 397–417.
Wait, J. R. (Apr.-June 1961), Some boundary value problems involving plasma media, J. Res. NBS **65B** (Math. and Math. Phys.), No. 2, 137–150.
Whale, H. A. (July 1962), The impedance of an electrically short antenna in the ionosphere, presented at Internat. Conf. on the Ionosphere, London.
Wu, C. P. (Aug. 1962), A study on the radiation from elementary dipoles in a magneto-ionic medium, Rept. 1021–20, Dept. of Elect. Eng., Ohio State Univ.
Yee, J. S. (1961), Mutual coupling of two thin infinitely-long slots located on a perfectly conducting plane in the presence of a uniform plasma layer, Proc. IRE **49,** 1837–1845.
Yee, J. S. (Apr. 1962), Voltage breakdown of a traveling-wave slotted waveguide array with and without a plasma environment, paper presented at Second Symposium on the Plasma Sheath (sponsored by the Electronic Res. Directorate, AFCRL, OAR), Boston, Mass.
Yildiz, M., and U. Ingard (July 1962), The radiation from moving sources in a homogeneons plasma, presented at Internat. Conf. on the Ionosphere, London.

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3.5. Electromagnetic Wave Propagation in Inhomogeneous Plasmas and/or Magnetoplasmas

W. C. Hoffman [1, 2]

In addition to the kind of inhomogeneity usually associated with electromagnetic wave propagation, viz, that of spatially varying permittivity, inhomogeneities of an entirely different nature can arise out of the fact that plasmas are characterized by a finite temperature distribution. For example, interactions resulting from noncentral peaks in the velocity distribution function can modulate or grow waves in an otherwise uniform plasma. "Incoherent scatter" is another case in point (of which we shall have more to say later). Another such situation is that wherein a periodic variation of permittivity, arising from a plasma oscillation, causes an incident electromagnetic wave to be scattered [Rosen, 1960]. Other such interactions can occur and generate actual r–f emission by the plasma. Since we are restricted to cold stationary plasmas, such situations will, however, be in the minority. The propagation of electromagnetic waves in plasmas whose complex permittivity exhibits a spatial variation will certainly be the case of greatest importance. We first of all consider the situation where no ambient magnetostatic field is present.

Many investigations of the transmission characteristics of an inhomogeneous plasma have been motivated by the "plasma sheath" surrounding a reentry body in the upper atmosphere. Klein, Greyber, King, and Brueckner [1961] have investigated plasma sheath interaction with incident microwave radiation at both normal and oblique incidence over a wide range of the parameters of interest (electron density, wavelength, and collision frequency). At normal incidence absorption dominates reflection. For oblique incidence, the absorption was determined in the WKB approximation and found to have a steady value below 45° but to drop off rapidly for larger angles of incidence. The electromagnetics of the plasma sheath problem have also been investigated by Sisco and Fiskin [1959, 1961]. Miller [1962] has given a theory, based on a set of pseudo-plane wave functions, for electromagnetic wave propagation through an exponentially varying two-dimensional plasma sheath over an infinite plane conductor. The case of a plane TE wave in a stratified medium whose ionization density varies as $\exp(z/z_0)$ has been considered by Taylor [1961]. Taylor [1962] has also investigated the effect of an irregular variation of the degree of ionization of a slightly ionized gas, and finds, in distinction to Budden, that such irregularities give rise to an "anti-Lorentz term" which in general would hinder rather than aid whistler propagation through the lower ionosphere. The resonance that occurs as a result of the interaction of a high frequency electromagnetic signal with electrons trapped in the potential troughs of an ion wave has been analyzed by J. E. Drummond [1961b]. Drummond obtains an approximate expression, based on a nonlinear ion wave theory, for the high-frequency conduction current density, and is thus able to predict the existence of an "ion wave window" for VHF electromagnetic waves in a dense plasma ($\omega_p^2 >> \omega^2$), the nature and location of the window being controlled by the frequency and amplitude of an ion wave in the plasma. The interaction of low-frequency electromagnetic waves with a semi-infinite plasma has been investigated by Turcotte and Schubert [1961] from the standpoint of coupling between electromagnetic and acoustic modes. An interesting result has been obtained by Ichimaru, Pines, and Rostoker [1962] in connection with a theoretical analysis of plasma wave instabilities due to drift of electrons relative to ions. They find that such instabilities are accompanied by a strongly enhanced scattering of electromagnetic waves similar to critical opalescence in a gas.

Electromagnetic wave propagation in a magnetoplasma with spatially varying conductivity tensor unites the analytical difficulties of anisotropy and inhomogeneity, and accordingly there has been correspondingly relatively less progress in this area. A significant advance in the application of the matrizant solution of the first-order vector wave equation to propagation through an inhomogeneous magnetoplasma has been made by H. B. Keller and J. B. Keller [1962]. The propagation of electromagnetic waves in an anisotropic stratified medium has also been studied by Hougardy and Saxon [1963], who showed that the wave equation can be reduced approximately to a scalar one-dimensional form, and cross coupling taken into account by iteration methods. A penetrating study of the absorption and reflection of electromagnetic and plasma waves through a plasma whose density and/or magnetic field are slowly varying functions of position has been made by Stix [1960]. Hoffman [1962] has formulated the case of an axially symmetric magnetoplasma as a vector integral equation, which can be approximated by a set of scalar integral equations having as solutions mean square convergent Neumann series. The exterior problem for a perfectly conducting sphere in an inhomogeneous magnetoplasma has also been considered by Hoffman [1963a, b] in terms of an asymptotic form of the vector wave equation in dipolar coordinates as well as a vector integral equation involving the Green's dyadic for a sphere in free space. A formal analysis in terms of the Green's dyadic for an inhomogeneous anisotropic medium has also been given by Villeneuve [1961].

The subject of "incoherent scatter" from irregularities in electron density has received much attention during the last triennium. Since this

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[1] Boeing Scientific Research Laboratories, Seattle, Wash.

[2] The author wishes to acknowledge the helpful suggestions of Dr. James E. Drummond in the preparation of this section.

phenomenon can be regarded as the result of a sort of fine-structure inhomogeneity in the plasma, its theoretical aspects may legitimately be discussed here, a complete discussion of the overall phenomenon being reserved for the Commission 4 report. The physical model involves radar scattering from local random fluctuations in plasma density, the spectral width being that corresponding to the ion velocities [Bowles, 1961]. American contributions to the mathematical theory of such "incoherent scatter" have been made by Bowles, Ochs, and Green [1962], Buneman [1961, 1962], W. E. Drummond [1962], Renau [1960], Renau, Camnitz, and Flood [1961], Renau [1962], Rosenbluth and Rostoker [1962], and Salpeter [1960a, 1960b, 1961a, 1961b]. The theoretical approach has involved either a generalized Nyquist theorem or solution of the Boltzmann equation, although extensions to nonequilibrium plasmas have been carried out by Buneman [1961], Rosenbluth and Rostoker [1962], and Renau [1962]. Analyses of the case where an ambient magnetic field is present have been carried out by Buneman [1961], extending a previous detailed study by Hagfors [1961], and by Salpeter [1961a, 1961b]. The effect of an ambient magnetic field is to cause discrete spectral peaks to appear at multiples of the ion gyrofrequencies whenever the angle between the ambient magnetic field and the direction of propagation becomes close to 90°.

Under certain circumstances plasma oscillations or other wave motions within a plasma can generate electromagnetic radiation. For instance, the theory of excitation of electromagnetic radiation by gradient coupling in plasmas, first analyzed by Field [1956], has been refined and extended by Tidman and Boyd [1962], Tidman and Weiss [1960, 1961a], and Wetzel [1961, 1963]. The amount of electromagnetic radiation produced by nonlinear coupling between large amplitude transverse and longitudinal plasma oscillations has been calculated by Tidman and Weiss [1961b]. Harris and Simon [1960] have shown that incoherent radiation from a plasma cannot be calculated on the basis of the Vlasov equation alone. Kino [1960] has developed a traveling wave parametric amplifier theory employing a plasma (or magnetoplasma) as the nonlinear propagating structure, and some refinements of the theory for small diameter plasma columns have been given by Paik [1962].

Electromagnetic Wave Propagation in Homogeneous Magnetoplasmas

In this part of the report we consider the interaction of an electromagnetic wave with a uniform plasma containing an ambient magnetic field. However, phenomena involving energetic particles (such as synchrotron radiation) and transport processes (such as electron beams and hydromagnetic waves) are specifically excluded.

Generalizations of the classical conductivity tensor in the direction of finite temperature effects will be discussed first of all. Sen and Wyller [1960] have solved the Boltzmann equation for a uniform Lorentz gas in an ambient magnetic field by expanding the velocity distribution function according to the Chapman-Enskog method and thus arrived at a generalized Appleton-Hartree formula. The effects of velocity dependence upon the collison frequency between electrons and neutral air molecules are fully incorporated in the Sen and Wyller analysis. A similar study for the solar atmosphere (proton-electron and electron-electron collisions) has been carried out by Wyller [1961]. The wave normal surfaces associated with a finite temperature Appleton-Hartree formula have been considered by Allis [1961] and Papa and Allis [1961], who discuss not only the two electromagnetic waves but also a third, plasma wave whose velocity is of the order of the sound velocity (the "magnetoacoustic wave"). The conductivity of a partially ionized gas in the presence of an ambient magnetic field has also been analyzed by Drummond, Gerwin, and Springer [1961] and Kelly [1960]. An analysis of the polarization of electromagnetic waves propagating in a magnetoplasma has been carried out by Haskell and Holt [1961].

A number of books dealing at least in part with the electromagnetic phenomena associated with magnetoplasmas have appeared since those referenced in the last USA National Committee report on plasmas [Smullin, 1960]. The American publications include the volumes edited by Clauser [1960] and J. E. Drummond [1961] and the excellent treatise by Rose and Clark [1961].

Electromagnetic wave theory for homogeneous magnetoplasmas has received considerable attention during the last triennium. Some of this work has already been described above under generalizations of the Appleton-Hartree formula [Papa and Allis, 1961]. Finite-temperature effects associated with longitudinal propagation in a magnetoplasma have also been investigated by Pradhan [1957], Narasinga Rao, Verdeyen, and Goldstein [1961], Scarf [1962], and Willett [1962]. Interesting effects are found to occur in the vicinity of electron-cyclotron resonance. The effects of strong magnetic fields upon plasma propagation have been investigated by Poeverlein [1961], who found, for one propagation mode, perfect guidance along the lines of force and, for the other, isotropic propagation. Harley and Tyras [1961] and Rothman and Morita [1961] have also studied the effects of an intense magnetic field. Chow [1962] has extended Bunkin's analysis of radiation in a magnetoplasma to a gyro-electro-magnetic medium and shown that the Sommerfeld radiation condition carries over. Kogelnik [1960] and Meecham [1961] have given dyadic Green's theorems appropriate to radiation in uniform magnetoplasmas. Wait [1961] has obtained exact solutions for some boundary value problems involving a two-dimensional magnetoplasma, and determined the reflection coefficients of such a stratified plasma in planar and cylindrical geometries.

Buchsbaum, Mower, and Brown [1960] have investigated the cavity modes of oscillation of a confined magnetoplasma. Hodara and Cohn [1962] have obtained dispersion equations for propagation of electromagnetic waves within an infinitely long plasma slab containing an axial magnetic field.

The interaction between electromagnetic waves and magnetoplasmas is usually analyzed by means of Maxwell's equations plus the Boltzmann equation. Solutions of the Boltzmann equation as such are therefore of interest. American contributions in the area have been made by Pradhan [1957], who solved the initial value problem ($E(z,0)$ given) for longitudinal propagation and Scarf [1962], who treated the corresponding boundary value problem ($E(0,t)$ given). The latter analysis was actually directed toward the Landau damping of transverse electromagnetic wave packets and admits the possibility of determining the temperature of energetic electrons in the exosphere through the proper use of whistler data [Liemohn and Scarf, 1962]. The electron distribution function of a weakly ionized magnetoplasma in an arbitrary time-dependent electric field has been determined by Zmindzinas and Wu [1961].

We now turn to a consideration of mechanisms for radiofrequency emission from cool stationary magnetoplasmas. (Bremsstrahlung, Cerenkov radiation, multiple stream interaction, and the like are thus specifically excluded.) Hirshfield and Brown [1961] have studied the thermal power that can be emitted according to Kirchhoff's radiation law from a magnetoplasma of radiation temperature T_r, and give curves for absorption and emission via cyclotron radiation. Wyld [1960] has shown that electromagnetic energy can be radiated from the surface bounding a finite magnetoplasma whenever plasma oscillations exist in the latter. He calculates this thermal radiation in the limit of small ratio of gyrofrequency to plasma frequency, and finds it is less than the synchrotron radiation by orders of magnitude.

The nonlinear interactions of an electromagnetic wave incident normally upon a plasma slab in a transverse magnetic field have been studied by Whitmer and Barrett [1961, 1962]. The dependent variables in the Boltzmann equation (including the nonlinear terms) and the (coupled) Maxwell equations are expanded in a Fourier series in time, and the resulting differential equations for each harmonic of the electric field solved for plane-wave propagation. In the first paper, the properties of the coupled field at the second harmonic frequency are discussed in terms of the following parameters: ambient magnetic field, electron density, electron-neutral particle collision frequency, field strength of the incident wave, and the thickness of the plasma layer. In the second paper the dependence of the third and fourth harmonics on the same set of parameters is analyzed, and the conversion loss obtained for the hth harmonic. A general equation is given which enables one to take account of interactions with all higher and lower harmonics in computing the electric field at the hth harmonic. Coherent nonlinearities excited in a magnetoplasma by intense radiation, and the energy conversion from the intense wave to a harmonic wave or the parametric increase of another, weak wave present have also been discussed by Eastman [1961]. J. E. Drummond [1961a, ch. 11] has analyzed the generation of harmonics and subharmonics at multiples of the cyclotron frequency by intense microwave illumination of a magnetoplasma.

References

Allis, W. P. (Jan. 1961), Propagation of waves in a plasma in a magnetic field, IRE Trans. Microwave Theory Tech. **MTT-9,** No. 1, 79–82.

Bowles, K. L. (1961), Incoherent scattering by free electrons as a technique for studying the ionosphere and exosphere: some observations and theoretical considerations, J. Res. NBS **65D** (Radio Prop.), No. 1, 1–14.

Bowles, K. L., G. R. Ochs, and J. L. Green (July–Aug. 1962), On the absolute intensity of incoherent scatter echoes from the ionosphere, J. Res. NBS **66D** (Radio Prop.), No. 4, 395–407.

Buchsbaum, S. J., L. Mower, and S. C. Brown (Sept.–Oct. 1960), Interaction between cold plasmas and guided electromagnetic waves, Phys. Fluids **3,** No. 5, 806–819.

Buneman, O. (June 1961), Fluctuations in a multicomponent plasma, J. Geophys. Res. **66,** No. 6, 1978–1979.

Buneman, O. (May 1962), Scattering of radiation by the fluctuations in a non-equilibrium plasma, J. Geophys. Res. **67,** No. 5, 2050–2053.

Chow, Y. (July 1962), A note on radiation in a gyro-electric-magnetic medium—an extension of Bunkin's calculation, IRE Trans. Ant. Prop. **AP-10,** No. 4, 464–469.

Clauser, F. H. (1960), ed., Symposium of Plasma Dynamics (Addison-Wesley).

Drummond, J. E. (1961a), ed., Plasma Physics (McGraw-Hill Book Co., Inc., New York, N.Y.).

Drummond, J. E. (1961b), High frequency propagation across an ion plasma wave, in Proc. Fifth Internat. Conf. on Ionization Phenomena in Gases, Munich, 1961, 517–527 (North Holland Publ. Co., Amsterdam).

Drummond, J. E., R. Gerwin, and B. Springer (1960), The concept of conductivity, J. Nucl. Energy (C)-Plasma Phys. **2,** 98–108.

Drummond, W. E. (Sept. 1962), Microwave scattering from unstable plasma waves, Phys. Fluids **5,** No. 9, 1133–1134.

Eastman, L. F. (1961), Coherent non-linearities in a magneto-ionic medium (unpublished memorandum), Abstract in Program of USA URSI Meeting, Spring 1961, 18–19.

Field, G. B. (Nov. 1956), Radiation by plasma oscillations, Astrophys. J. **124,** No. 3, 555–570.

Hagfors, T. (July 1961), Density fluctuations in a plasma in a magnetic field, with applications to the ionosphere, J. Geophys. Res. **66,** No. 7, 1699–1712.

Harley, T. P., and G. Tyras (Dec. 1961), Transmission of electromagnetic waves through an ionized layer in the presence of a strong magnetic field, Proc. IRE **49,** No. 12, 1822–1824.

Harris, E. G., and A. Simon (Mar.–Apr. 1960), Coherent and incoherent radiation from a plasma, Phys. Fluids **3,** No. 2, 255–258.

Haskell, R. E., and E. H. Holt (Oct. 1961), Study of electromagnetic wave polarization in magnetoplasmas by a matrix method of crystal optics, Proc. IRE **49,** No. 10, 1584–1585.

Hirshfield, J. L., and S. C. Brown (May 1, 1961), Incoherent microwave radiation from a plasma in a magnetic field, Phys. Rev. (2) **122,** No. 3, 719–725.

Hodara, H., and G. I. Cohn (July 1962), Wave propagation in magneto-plasma slabs, IRE Trans. Ant. Prop. **AP-10,** No. 4, 452–458.

Hoffman, W. C. (Jan.–Feb. 1962), An approximate full wave solution for low frequency electromagnetic waves in an unbounded magneto-ionic medium, J. Res. NBS **66D** (Radio Prop.), No. 1, 107–111.

Hoffman, W. C. (1963a), The magnetospheric wave equation in dipolar coordinates, in Proc. Conf. on the Ionosphere, London, July 1962, Phys. Soc. (London).

Hoffman, W. C. (1963b), Vector integral equations for the electric field in an inhomogeneous magneto-ionic medium, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 359–363 (Pergamon Press, New York and London).

Hougardy, R. W., and D. S. Saxon (1963), Propagation of electromagnetic waves in an anisotropic stratified medium, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 635–639 (Pergamon Press, New York and London).

Ichimaru, S., D. Pines, and N. Rostoker (Mar. 15, 1962), Observation of critical fluctuations associated with plasma-wave instabilities, Phys. Rev. Letters **8,** No. 6, 231–233.

Keller, H. B., and J. B. Keller (June 1962), Exponential-like solutions of systems of linear ordinary differential equations, J. Soc. Ind. Appl. Math. **10,** No. 2, 246–259.

Kelly, D. C. (July 1, 1960), Microwave conductivity of a plasma in a magnetic field, Phys. Rev. **119,** No. 1, 27–39.

Kino, G. S. (Aug. 1960), Parametric amplifier theory for plasmas and electron beams, J. Appl. Phys. **31,** No. 8, 1449–1458.

Klein, M. M., H. D. Greyber, J. I. F. King, and K. A. Brueckner (June 1961), Interaction of a non-uniform plasma with microwave radiation, Planet. Space Sci. **6,** 105–115.

Kogelnik, H. (Sept.–Oct. 1960), On electromagnetic radiation in a magneto-ionic media, J. Res. NBS **64D** (Radio Prop.), No. 5, 515–523.

Liemohn, H. B., and F. L. Scarf (Oct. 1962), Whistler attenuation by electrons with an $E^{-2.5}$ distribution, J. Geophys. Res. **67,** No. 11, 4163–4168.

Meecham, W. C. (Dec. 1961), Source and reflection problems in magneto-ionic media, Phys. Fluids **4,** No. 12, 1517–1524.

Miller III, G. F. (Aug. 1962), Propagation of electromagnetic waves in a nonuniformly ionized medium, Phys. Fluids **5,** No. 8, 899–907.

Narasinga Rao, K. V., J. T. Verdeyen, and L. Goldstein, Interaction of microwaves in gaseous plasmas immersed in magnetic fields, Proc. IRE **49,** No. 12, 1877–1889.

Paik, S. F. (Mar. 1962), A note on the parametric amplifier theory for plasmas, J. Appl. Phys. **33,** No. 3, 1017–1018.

Papa, R. J., and W. P. Allis (June 1961), Waves in a plasma in a magnetic field, Planet. Space Sci. **6,** 100–104.

Poeverlein, H. (April 1961), Propagation of electromagnetic waves in a plasma with strong magnetic field, Phys. Fluids **4,** No. 4, 397–405.

Pradhan, T. (1957), Plasma oscillations in a steady magnetic field: circularly polarized electromagnetic modes, Phys. Rev. (2) **107,** 1222–1227.

Renau, J. (Nov. 1960), Scattering of electromagnetic waves from a nondegenerate ionized gas, J. Geophys. Res. **65,** No. 11, 3631–3640.

Renau, J. (Aug. 1962), The cross section for scattering of electromagnetic waves from an ionized gas in thermal nonequilibrium, J. Geophys. Res. **67,** No. 9, 3624–3626.

Renau, J., H. Camnitz, and W. Flood (Sept. 1961), The spectrum and the total intensity of electromagnetic waves scattered from an ionized gas in thermal equilibrium in the present of a static quasi-uniform magnetic field, J. Geophys. Res. **66,** No. 9, 2703–2732.

Rose, D. J., and M. Clark, Jr. (1961), Plasmas and Controlled Fusion (John Wiley & Sons Inc., New York, N.Y.).

Rosen, P. (1960), Scattering of electromagnetic waves by longitudinal plasma waves, Phys. Fluids **3,** 416–417.

Rosenbluth, M. N., and N. Rostoker (July 1962), Scattering of electromagnetic waves by a nonequilibrium plasma, Phys. Fluids **5,** No. 7, 776–788.

Rothman, H., and T. Morita (May 1961), Transmission through an ionized medium in the presence of a strong magnetic field, Stanford Res. Inst. Tech. Rept. 74, Stanford University.

Salpeter, E. E. (1960a), Scattering of radio waves by electrons above the ionosphere, J. Geophys. Res. **65,** No. 6, 1851–1852.

Salpeter, E. E. (1960b), Electron density fluctuations in a plasma, Phys. Rev. **120,** No. 5, 1528–1535.

Salpeter, E. E. (1961a), Effect of the magnetic field in ionospheric backscatter, J. Geophys. Res. **66,** No. 3, 982–984.

Salpeter, E. E. (1961b), Plasma density fluctuations in a magnetic field, Phys. Rev. **122,** No. 6, 1663–1674.

Scarf, F. L. (Jan. 1962), Landau damping and the attenuation of whistlers, Phys. Fluids **5,** No. 1, 6–13.

Sen, H. K., and A. A. Wyller (Dec. 1960), Generalization of the Appleton-Hartree magneto-ionic formula, J. Geophys. Res. **65,** No. 12, 3931–3950.

Sisco, W. B., and J. M. Fiskin (July 1959), Effect of relatively strong fields on the propagation of electromagnetic waves through a hypersonically produced plasma, IRE Trans. Ant. Prop. **AP–7,** 240–244.

Sisco, W. B., and J. M. Fiskin (June 1961), Basic hypersonic plasma data of equilibrium air for electromagnetic and other requirements, Planet. Space Sci. **6,** 47–73.

Smullin, L. (Nov.–Dec. 1960), Interaction between plasmas and electromagnetic fields, J. Res. NBS **64D** (Radio Prop.), No. 6, 766–767.

Stix, T. H. (1960), Absorption of plasma waves, Phys. Fluids **3,** 19–32.

Taylor, L. S. (Sept. 1961), Electromagnetic propagation in an exponential ionization density, IRE Trans. Ant. Prop. **AP–9,** No. 5, 483–487.

Taylor, L. S. (Sept. 1962), The index of refraction of an irregular ionized gas, J. Geophys. Res. **67,** No. 10, 3843–3849.

Tidman, D. A., and J. M. Boyd (Feb. 1962), Radiation by plasma oscillations incident on a density discontinuity, Phys. Fluids **5,** No. 2, 213–218.

Tidman, D. A., and G. H. Weiss (1961a), Radio emission by plasma oscillations in nonuniform plasmas, Phys. Fluids **4,** 703–710.

Tidman, D. A., and G. H. Weiss (1961b), Radiation by a large amplitude plasma oscillation, Phys. Fluids **4,** 866–868.

Turcotte, D. L., and G. Schubert (1961), Interaction of low-frequency electromagnetic waves with a plasma, Phys. Fluids **4,** 1156–1161.

Villeneuve, A. T. (Mar. 1961), Green's function techniques for inhomogeneous anisotropic media, IRE Trans. Microwave Theory Tech. **MTT–9,** No. 2, 197–198.

Wait, J. R. (Apr.–June 1961), Some boundary value problems involving plasma media, J. Res. NBS **65B** (Math. and Math. Phys.), No. 2, 137–150.

Wetzel, L. (Feb. 1961), Wave interaction in plasma inhomogeneities, J. Appl. Phys. **32,** No. 2, 327–328.

Wetzel, L. (1963), Harmonic excitation in and reradiation from non-uniform ionized regions, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 365–374 (Pergamon Press, New York and London).

Whitmer, R. F., and E. B. Barrett (Feb. 1961), Nonlinear interaction of an electromagnetic wave with a plasma layer in the presence of a static magnetic field, Part I. Theory of harmonic generation, Phys. Rev. **121,** 661–668.

Whitmer, R. F., and E. B. Barrett (Mar. 1, 1962), Nonlinear interaction of an electromagnetic wave with a plasma layer in the presence of a static magnetic field, Part II. Higher harmonics and a nonlinear propagation theory, Phys. Rev. **125,** No. 5, 1478–1484.

Willett, J. E. (Mar. 1962), Effects of electron random motion on microwave propagation through a plasma parallel to a magnetic field, J. Appl. Phys. **33,** No. 3, 898–906.

Wyld, H. W., Jr. (May–June 1960), Radiation by plasma oscillations in a bounded plasma in a magnetic field, Phys. Fluids **3,** No. 3, 408–415.

Wyller, A. A. (Nov. 1961), Magneto-ionic theory for a fully ionized hydrogen plasma, Astrophys. J. **134,** No. 3, 718–723.

Zmindzinas, J. S., and C. S. Wu (June 15, 1961), Electron distribution function of a weakly ionized gas in a magnetic field and a time-dependent electric field, California Institute Tech. Jet. Prop. Lab. Rept. 32–102.

Part 4. Classical Diffraction and Scattering

4.1. Diffraction and Scattering

L. B. Felsen [1] and V. H. Weston [2]

A. Introduction and Summary

Work on electromagnetic diffraction and scattering since the last General Assembly has proceeded essentially along the lines discussed in the previous report, with the great majority of contributions concerned with high-frequency effects. We distinguish again between exact solutions for specially shaped objects (canonical problems) and approximate methods applicable to arbitrary shapes. In the first category a variety of new problems has been solved, and more accurate asymptotic solutions of known problems have been derived. No new general techniques of analysis of high-frequency scattering have appeared; Keller's geometrical theory of diffraction and improved versions of Fock's theory together seem capable of interpreting quasi-optic diffraction phenomena, and additional evidence has been presented to support their validity. Interest has been stimulated to achieve a similar general formulation for diffraction at low frequencies and several contributions have appeared in this area. Little progress has been made in bridging the analytical gap between the low-frequency and high-frequency asymptotic expansions for general scatterers although one or two ideas have been presented on this important topic.

Though not exhaustive, the references cited in this review are believed to be quite representative of American activity in electromagnetic diffraction and scattering, and are taken from recognized scientific journals only. An exception has been made in including papers appearing (in summary form) in the preprints of the Symposium on Electromagnetic Theory and Antennas, held in Copenhagen, Denmark, during June 1962. Since the Proceedings of the Symposium will appear in book form it did not seem out of place to anticipate this publication. Multiple scattering and scattering in anisotropic media has been specifically excluded; these topics are covered in other reports.

The authors express their appreciation to J. B. Keller who collaborated in the early phases of preparation of this summary but was unable, due to other commitments, to participate further.

B. High-Frequency Diffraction

1. Canonical Problems

a. Constant impedance wedge: A number of the previously reported studies involving wedges with constant impedance boundary conditions have been further extended. Karal and Karp have given additional applications of their formulation of diffraction problems involving right-angle wedges which have reactive surfaces on one or two sides, with major interest centering around the excitation of surface waves. The cases considered include plane wave diffraction (Karal and Karp, 1962b) and excitation by a line dipole at the vertex of the wedge (Karp and Karal, 1960). Theoretical aspects of diffraction of a surface wave incident on the apex from one of the reactive wedge faces were investigated by Karal, Karp, Chu, and Kouyoumjian [1961], while numerical and experimental data are given by Chu, Kouyoumjian, Karal, and Karp [1962]. Radiation pattern computations and measurements in the last paper are compared with approximate calculations based on the Kirchhoff method and good agreement was noted. The results are of interest in connection with the radiation from finite surface wave antennas.

As regards the now familiar problem of diffraction by a lossy half-plane or by the related configuration of two coplanar half-planes with different surface impedance, Bazer and Karp [1962] have published results pertaining to coastal refraction which were obtained by them several years ago, while Marcinkowski [1961] has presented numerical data for the scattering from a strongly absorbing surface which he has compared with, and found to resemble, the scattered fields derived for a Sommerfeld-type black screen.

b. Infinite cylinder: Investigations of high-frequency diffraction by cylinders and spheres have aimed primarily at improving the asymptotic representations for perfect conductivity; at assessing the influence of imperfect conductivity and of surface coatings; and at providing precise numerical results computed from the exact, but slowly convergent harmonic series, especially in the resonance region where neither the quasi-optic nor the low-frequency approximations are very accurate. As regards the perfectly conducting case, Hasserjian and Ishimaru

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[1] Department of Electrophysics, Polytechnic Institute of Brooklyn.
[2] Conductron Corporation, Ann Arbor, Mich.

[1962] have obtained solutions for the surface currents excited by an arbitrarily oriented small slot and by axial and transverse half wavelength slots. The results are expressed in terms of those for an infinite plane sheet times a correction factor accounting for curvature; the latter is given in terms of optical ray path coordinates, thereby suggesting the applicability of the results to other convex shapes. Verification by experiment shows that the formulas hold for observation points even very close to the slots. Logan et al., [1962] have presented more accurate expressions than previously available for the zeros in the order ν of Hankel functions having a fixed argument, and for the pattern function associated with the radiated field from a circumferential slot on a circular cylinder. The effect of spherically symmetric waves incident on infinite cylinders has been treated by Brick [1961] who examines the formal and asymptotic connection between the currents induced by a normally or obliquely incident plane wave and by point source excitation, and by Brysk [1960] who has investigated the radar cross section. Numerical calculations of the equatorial plane radiation pattern of a circumferential half-wavelength slot in the interval $1 \leq ka \leq 30$ have been presented by Knop and Batista [1961], while the radiation pattern of a radial dipole in the low-frequency range $0.05 < (a/\lambda) < 0.5$ has been computed by Levis [1960]. Here, $k = 2\pi/\lambda$ denotes the free-space wave number, and "a" the cylinder radius.

As regards high-frequency scattering from dielectric coated cylinders, Kodis [1961] has developed expressions for the diffracted field produced by a line source either inside or outside the coating. The field in the region containing the source is expressed as a sum of two terms, the first corresponding to the result that would be obtained if the outer surface were also perfectly conducting. For plane wave scattering, perturbation calculations are carried out for the three limiting cases of thin dielectric, low refractive index, and very small surface curvature. The special case of backscattering has also been considered [Kodis, 1962], and the high-frequency asymptotic results are given in terms of the directly reflected rays and the rays which suffer internal reflections in the dielectric coating before leaving the outer surface in the backscattered direction. Since grazing rays are neglected, these results are best for coatings of moderate thickness. High-frequency formulas for possibly lossy coatings have also been presented by Helstrom [1962]. Numerical calculations from the harmonic (Mie) series expressions have been carried out for plane wave scattering by two concentric circular cylinders composed of different material [Kerker and Matijevic, 1961b]; for plane wave backscattering when a dielectric shell is spaced a resonant distance from the surface of a large, perfectly conducting cylinder (Plonus, 1960); for the equatorial plane radiation pattern due to a circumferential half-wavelength slot in a perfectly conducting cylinder surrounded by various adjacent lossy layers [Knop, 1961]; and for the radiation pattern of an axial magnetic dipole in the presence of a cylinder having shallow axial grooves [Wait and Conda, 1961]. The harmonic series expansion for the scattering of a normally incident plane wave by a magnetized ferrite cylinder was obtained by Eggiman [1960a] for the case where the incident electrical vector and the d-c magnetic field are parallel to the axis. Numerical calculations for thin cylinders were given.

c. Sphere: The geometrical optics cross section of a large perfectly conducting sphere was recomputed, but incorrectly, by Vea et al. [1960], and their error was pointed out by Kennaugh et al. [1960] and by Logan [1960]. In addition, Logan [1960] has given terms in the geometric optical scattered field expansion up to the order of $(ka)^{-4}$, where "a" is the radius of the sphere, thus extend previous knowledge in the field. Kazarinoff and Senior [1962] discuss the failure of simple creeping wave theory as regards the representation of the minor component of the surface field diffracted into the shadow region. However, as pointed out by Logan (to be published in the IRE Transactions PGAP), this phenomenon has previously been noted and corrected in the Russian literature. A technique proposed by T. T. Wu, in which it was possible through the use of boundary layer theory to obtain higher order terms in the series expansion for the field in the shadow region of a circular cylinder, was extended by Tang [1961] to the sphere problem. He derived the first two terms in this series for observation points in the shadow region, including the vicinity surrounding the axial caustic, and the boundary layer near the surface of the sphere. Weston [1961] deals with the near zone backscattered field produced by a plane wave incident on a large perfectly conducting sphere; the geometrical optics field is computed by using two terms of the order of $(ka)^{-2}$, thereby making this result valid for any location of the observation point.

Nonperfectly conducting and coated spheres have also been treated. Hiatt et al. [1960], discuss fine-grained random roughness on the surface of a large perfectly conducting sphere via an impedance boundary condition. The cross section is computed and compared with measurements, and it is found that the result is within 0.1 db of that for a smooth sphere, for roughness depths up to 0.01λ. Weston and Hemenger [1962] have derived the geometrical optics and creeping wave expressions for the high-frequency backscattered field produced by a sphere coated with a thin layer of material of complex permittivity and permeability. Plonus [1961] uses the classical series expansion to derive a formal solution and calculate numerical values for the diffraction of a plane wave by a perfectly conducting sphere with a concentric shell, see also Garbaez [1962].

High-frequency scalar scattering from a penetrable sphere has been treated by Rubinow [1961]. By the use of the Poisson sum formula, he expresses the exact series for the scattering amplitude in integral form and decomposes it into two sets of integrals. One set is evaluated in terms of residues and leads to the diffracted field contribution, while the other set is evaluated by the method of stationary phase and gives rise to the geometric optical field, which includes the bow term. The resulting expressions fail in the forward and backward directions, but have also been modified to include these cases [Rubinow, 1962]. Numerical tables of scattering coefficients of light have been computed for concentric dielectric spheres [Kerker et al., 1962]; for single dielectric spheres [Meehan and Hugus, 1961, and Kerker and Matijevic, 1961]; for partially absorbing spheres [Deirmendjian et al., 1961]; and for colored spheres [Chromey, 1960]. Penndorf [1962b] obtains an approximate formula for the forward scattering of light by a spherical aerosol, for particles whose radius is greater than λ. In addition, approximations are obtained for the scattering and extinction coefficients for small aerosols ($r<\lambda$) [1962c]. He also discusses the forward, sideward, and backward scattering patterns based on data for spheres with refractive index $n=1.33$ [1962a].

Nonhomogeneous transparent bodies have been treated via the Born, and associated, approximations [Schiff, 1962]. In particular, nonhomogeneous spheres or cylinders were studied by Albini [1962] and Albini and Nagelberg [1962], respectively, and general nonspherical particles by Greenberg [1960].

d. Noncircular cylinder and spheroid: The principal purpose of studying high-frequency diffraction by noncircular cylinders and by spheroids is to obtain rigorous asymptotic field representations for objects with nonconstant surface curvature. Such results may then be compared with predictions derived from approximate theories applicable to bodies of arbitrary shape, thereby furnishing important tests of the validity of the general methods. A radial eigenfunction solution for the line source Green's function for the elliptic cylinder was given by Levy [1960] and was evaluated by him in the asymptotic limit where all radii of curvature are large relative to the incident wavelength. The resulting expression confirmed predictions made by Keller's geometrical theory of diffraction. A similar confirmation was obtained by Clemmow and Weston [1961] for the field scattered from a slightly noncircular cylinder having smooth shallow periodic corrugations around the periphery; this nonseparable boundary value problem was analyzed by a perturbation method. Measurements of currents on perfectly conducting elliptic cylinders [Wetzel and Brick, 1960] verify the validity of the Fock approximation near the shadow boundary even down to $kR_0=2.2$ where R_0 is the radius of curvature at the tip; the agreement is not good in the deep shadow since the Fock approximation contains as a parameter the distance along the tangent ray rather than the (correct) distance along the surface.

Levy and Keller [1960] employed a method of analysis similar to that mentioned above for the elliptic cylinder to show that the high-frequency diffracted field for a spheroid agrees with that calculated by the geometrical theory of diffraction if all radii of curvature are large compared to the wavelength. They then show that for high frequencies the electromagnetic backscattered fields are derivable from those for the two scalar cases, a result which is expected to hold generally for nose-on incidence on perfectly conducting bodies of revolution. The exact solution for a thin oblate spheroid illuminated by a spherically symmetric wave centered on the axis was treated by Goodrich et al., [1962]. They derive a high-frequency expression for the field on the surface in the parameter range $ka>>1$, $(kR_0)^{1/2}<<1$, where $R_0=b^2/a$ is the (smaller) radius of curvature at the edge, and "a" and "b" are the semimajor and semiminor axes, respectively. It is shown that the reflection coefficient at the edge is proportional to the square root of kR_0, the same as for the thin elliptic cylinder. The asymptotic treatment also holds for the limiting case of the disk.

e. Elliptic cone: One of the most interesting canonical bodies, the elliptic cone, has been treated by Kraus and Levine [1961]. By the method of separation of variables in a "uniformized" version of the sphero-conal coordinate system, they have constructed scalar Green's functions which satisfy either the Neumann or the Dirichlet boundary conditions on the cone surface. The solution involves simply periodic and nonperiodic Lamé functions as well as Bessel functions. Since a plane angular sector is a degenerate surface, the problem of diffraction by such a sector is included as a special case. These solutions can serve as a starting point for the determination of tip conditions (this result is to be published by Levine) and the diffraction coefficients for a vertex needed in the geometrical theory of diffraction.

f. Slit and circular aperture: By an application of the integral equation procedure described in the last report, Sheshadri and Wu have calculated the high-frequency scattering of electromagnetic plane waves by a circular aperture [Seshadri and Wu, 1960b] and by an infinite slit [Seshadri and Wu, 1960a] in an infinite, plane, perfectly conducting screen. The results for the ratio of the transmission cross section to the geometric-optical cross section are given to $0[(ka)^{-5/2}]$ for the aperture and to $0[(ka)^{-11/2}]$ for the slit, with "a" representing the aperture radius and slit width, respectively. In the slit problem, emphasis is placed on the case of grazing incidence. A novel method of solving the problem of diffraction by an infinite, perfectly conducting strip was presented by Timman and Kleinmann at the 13th URSI General Assembly. Via their procedure, the line source Green's function for a strip is represented in terms of a complex double integral with variable integration limits—a complication compensated for in part by a simple kernel involving Bessel functions. Because of the involved form of the solution it has been difficult to verify that all the required boundary conditions are satisfied. Kleinmann [1962] has now proceeded to investigate the limiting case of plane wave incidence.

A comparison between measured field distributions induced in a slit by an obliquely incident plane wave and between exact values calculated from the Mathieu function series has been given by Hsu [1960], who has noted satisfactory agreement for slit widths ranging from 1.27 to 3.5 wavelengths. Macrakis [1960] has carried out a comparison between measured values of the plane wave backscattering cross section of a strip and calculated results obtained from the Mathieu function series, the high-frequency asymptotic solution of Levine [1959], and the variational method. A quasi-optic approximation for the slit (or strip) problem was employed by Plonsey [1961] who represents the currents induced by a cylindrical wave incident on a strip in terms of their geometric-optical value which is corrected for edge effects by the inclusion of equivalent edge line currents. Measurements in a parallel plane setup (with the electric field parallel to the edge) showed reasonable agreement with calculated values for a strip width as small as 1.6 wavelengths. As is to be expected, the effects of the edge currents are most pronounced when the wave arrives near glancing incidence.

g. Unidirectional screen: A relatively new class of diffraction problems, involving unidirectionally conducting surfaces, has received detailed attention. Such an anisotropic surface constrains electric currents to flow along a specified rectilinear direction and is an idealized model of an array of closely spaced insulated thin wires. The electric field tangential to the preferred direction on the screen is required to vanish while the perpendicular electric and the tangential magnetic field components pass unchanged through the surface. The first problem of this kind was considered by Toraldo di Francia [1956] who evaluated the low-frequency scattering cross section of a unidirectionally conducting disk. Karp then showed that the problem of diffraction by a unidirectionally conducting half plane could be solved by an application of the Wiener-Hopf integral equation technique, and this recognition has stimulated the adaptation of integral equation procedures employed previously for smooth screens to the study of a variety of discontinuity problems in unidirectionally conducting surfaces. These surfaces have the additional interesting property of being able to support both a true surface wave which decays exponentially, and a TEM-type (multiwire) mode which propagates along the wire direction and decays algebraically, with distance from the surface.

The problem of plane wave diffraction by a slit in a plane screen having an arbitrary direction of conductivity with respect to the slit edges was treated by Seshadri who solved the integral equation in the high-frequency limit [1961]. He also obtained the low-frequency approximation for the plane wave scattering from a strip [1960]. In the former case, the solution is given up to terms of $0[(ka)^{-5/2}]$ while in the latter, terms up to $0[(ka)^2]$ are included, where $(2a)$ is the slit width and k is the free space wavenumber. Seshadri has also considered the efficiency of excitation of the surface wave by a line source of electric current (1962b), and the diffraction of an incident surface wave by the edge of a unidirectional semi-infinite screen, a planar junction of two different unidirectional semi-infinite screens, and a planar junction between a unidirectional and a perfectly conducting half plane [1962a]. The excitation of the TEM-type wave by a dipole source was treated by Karal and Karp [1962a].

h. Variable impedance surfaces: While the high-frequency diffraction by such diverse structural effects as slow surface curvature and abrupt (edge) discontinuities on a perfectly conducting scatterer is reasonably well understood at present, new phenomena may arise when the surface properties exhibit variation. For gradual changes (over a length interval equal to the local wavelength), the quasi-optic concepts of "local" diffraction may still be expected to apply; for rapid changes, however, the rate of variation of the surface properties in the vicinity of the quasi-optic diffraction point may be expected to exert an influence on the scattered field. The effects of surface impedance changes near an edge discontinuity were investigated by Felsen [1961] who considered diffraction by a wedge with a linearly varying surface impedance and exhibited the dependence of the edge diffracted field on the rate of impedance variation. Variations on a curved surface were investigated by Felsen and Marcinkowski [1962] and Marcinkowski and Felsen [1962a, 1962b] via the model of a circular cylinder with a sinusoidally changing surface impedance around the periphery. For rapid oscillations they find that the reflected field in the illuminated region can be interpreted in terms of a spectrum of diffracted rays which are analogous to those encountered for a planar grating, while slowly changing surface conditions permit the combination of diffracted rays into a single specularly reflected ray having a reflection coefficient determined by the surface impedance at the reflection point. Since the latter condition forms the basis of the geometric-optical approximation, criteria can be derived which delimit the validity of the geometric-optical procedure for constructing the reflected field from a gently curved object having a variable surface impedance.

i. Finite cylinder: The very difficult problem of diffraction by a capped cylinder of finite length has also received attention. Kuehl [1961b] employs an approximate procedure suitable when the cylinder is thin and many wavelengths long, with the source not located near the ends. He first obtains the induced currents on the infinitely long cylinder and then calculates the radiation from these (unchanged) currents when the cylinder is truncated, thereby neglecting reflections from the ends. Asymptotic evaluations of the radiation pattern, carried out for a radial electric dipole located near the cylinder surface, agree reasonably well with experimental measurements. Kieburtz [1962] has used an essentially rigorous integral equation and function-theoretic method wherein multiple reflections between the ends are taken into account, with due regard for the edge condition at the ends of the cylinder. He has utilized his results to obtain the scattering cross section of thin cylindrical wires. Cassedy and Fainberg [1960a] have calculated the backscattering from thin lossy cylinders of finite length by the variational method, using zeroth and first-order trial functions. They find, and have confirmed by experiments on copper and platinum wires, that loss in the wires reduces the backscattering cross section but leaves the resonant length virtually unaffected.

2. Asymptotic Theories

As anticipated, most of the effort on general aspects of high-frequency diffraction has centered around the improvement and refinement, and around further applications, of the asymptotic theories of Fock and Keller which were discussed in the previous Assembly Report. Keller [1962a] studied the initial value problem for the time-independent wave equation in a region exterior to a polygonal cylinder and has deduced the asymptotic behavior of the time-harmonic solution from the nature of the singularities in the time-dependent solution. The result obtained for the leading term in the time-harmonic asymptotic series in inverse powers of k agrees with that predicted by Keller's geometrical theory of diffraction, thereby providing a justification of the theory when applied to determine the initial term in the high-frequency asymptotic expansion for scattering by polygonal cylindrical structures. The verification for the higher order terms remains to be carried out.[3] The geometrical theory of diffraction has also been applied by Keller [1960, 1961] to the determination of the plane wave backscattering from a finite cone, and by Keller and Magiros [1961] to the study of the diffracted fields arising from a cylindrically tipped half-plane. In the former case, where no exact solution is available, agreement was confirmed by comparison with experiment. In the latter case, comparison was made with values computed from the exact series solution and good agreement was found for $ka>2$, where a is the cylinder radius. Another application by Keller and Karal [1960] has dealt with the excitation of surface waves by a line source above a reactive surface. In this instance, complex rays are required and the geometrical theory so formulated has been checked against certain special configurations for which exact solutions are available.

Buchal and Keller [1960] have employed boundary layer techniques to deal with corrections to the geometrical optics field and to the field computed from Keller's diffracted ray theory in the vicinity of caustics, edges, and shadow boundaries where these fields possess either singularities or discontinuities. The actual fields are finite and continuous but undergo rapid transitions in thin layers termed boundary layers about these regions. Specifically treated are the problems of the field in the vicinity of a two-dimensional smooth, convex caustic and also the special case where this caustic degenerates into a line; the field in the vicinity of the edge of an aperture in a plane screen; and the field in the vicinity of a shadow boundary. W. P. Brown [1962] also discusses the boundary layer approach and estimates the boundary layer thickness (the distance from the caustic) to be the order of $(ka/2)^{-2/3}\,a$, where "a" is the radius of curvature of the caustic.

Refinements of the Fock theory, dealing primarily with the description of the diffraction field near the shadow boundary on a smooth convex object, have likewise been carried out. For convex cylindrical objects, Logan and Yee [1962a] have performed extensive computations of what has commonly been referred to as the Fock function, and they have achieved a uniform formulation which is valid in the illuminated, shadow, and transition regions. Thus, high-frequency diffraction phenomena arising from smooth convex objects can be characterized in terms of a single function, thereby providing a significant unification of the theory. Logan and Yee have also shown how this scattering function transforms into the solution predicted by the geometrical theory of diffraction whence their results can be employed to correct that theory in transition regions. Related phenomena have been investigated by Rubinow and Keller [1961] who studied the shift of the geometric-optical shadow boundary on an opaque circular cylinder when (λ/a) is small but finite. Both zero and finite values of surface impedance were assumed, and the shadow boundary shift was related to the correction in the geometric-optical scattering cross section. The results are formulated in a manner which suggests their applicability also to other convex two-dimensional and three-dimensional shapes.

While the preceding considerations have dealt with surfaces whose shapes are describable by analytic functions, additional diffraction effects arise when the surface contours are nonanalytic. The influence of a discontinuity in surface curvature was investigated by Weston [1962] who employed an integral equation procedure to evaluate the high-frequency currents induced by an incident plane wave on a perfectly conducting cylindrical surface comprised of two different but smoothly joined parabolic half cylinders. He finds that the discontinuity in curvature launches diffracted (creeping) waves on the surface which are smaller by $0(1/k)$ than those originating from a discontinuity in slope (i.e., an edge). It is again suggestive that the excitation coefficient for these creeping waves can also be applied to other cylindrical shapes having a discontinuity in surface curvature.

Various other contributions, listed in section 1, have also led to a formulation of asymptotic results which may be expected to apply to certain classes of scatterers of relatively arbitrary shape.

From an analysis based on the theorem which relates the scattering cross section and the forward scattered field amplitude, some general remarks have been made concerning the effects of lossy coatings on large scatterers [Hiatt et al., 1960a]. It was concluded, and verified experimentally from measurements on spheres, spheroids and finite cones, that the coatings have a negligible effect on forward scattering but an appreciable effect on backscattering, see also Ryerson [1962] and Seigel and Hiatt [1961].

3. Experimental Aspects

While experimental phases associated with various investigations are included at other appropriate

[3] A recent summary of the geometrical theory of diffraction is contained in Keller [1962b].

places in this report, we list here several contributions which deal with experimental techniques in the measurement of diffraction fields. Plonsey [1962] has dealt with the measurement of surface current by an electric probe and has paid attention to the error introduced by the interaction between the probe and the obstacle. Strait and Cheng [1962] have used an auxiliary scatterer for the accurate experimental determination of RF magnetic fields. Greenberg et al., [1961] have discussed a microwave technique which has been developed for obtaining total cross sections and angular distributions arising from the scattering of electromagnetic radiation by nonspherical particles whose size is of the order of a wavelength. They compare experiments and approximate theoretical results for prolate spheroids and finite cylinders. C. I. Beard et al., [1962] deal with the error inherent in the use of "midfield" experimental results to obtain the far zone forward scattering by almost transparent spheres with radii large compared to the wavelength. They show theoretically that by not being in the true "far-zone," a major error occurs in the phase of the forward-scattered field.

C. Low-Frequency Diffraction

As in the case of high-frequency diffraction, though on a considerably smaller scale, studies of scattering of low-frequency waves can be divided into those dealing with exact solutions for special shapes, and with general methods applied to arbitrary shapes. In the first category, the celebrated problem of diffraction by a circular disk (or aperture) has been subjected to increasingly detailed and sophisticated study and as been shown to be susceptible to solution by a variety of methods, thereby being reminiscent of the simpler two-dimensional half-plane problem. In most of the recent formulations, one seeks to specify the field on the disk implicitly by a Fredholm integration equation of the second kind which can then be solved by power series expansion in terms of the small parameter k.

Heins and MacCamy [1960] have calculated the first few terms in the low-frequency asymptotic expansion for the scalar field on a disk excited by an obliquely incident plane wave, while Bazer and Hochstadt [1962] have performed a similar calculation for the aperture and have also given the scattered far fields and the transmission cross section. Expressions for the higher-order terms in the induced electric and magnetic dipole moments, and in the electromagnetic scattered far fields of a small disk and aperture have been presented by Eggimann [1960b, 1961], and the results have been employed by Eggimann and Collin [1962] to deal with the scattering from a planar array of circular disks.

The scalar problem of diffraction of a plane wave by a prolate spheroid is considered by Senior [1960a]. He expands the exact solution in ascending powers of ka up to and including terms of $0(ka)^6$, where "a" is the interfocal distance, and also considers the particular cases of the sphere and disk. The unidirectional conducting infinite strip has been treated by Seshadri [1960] who formulates the problem in terms of an integral equation and obtains expressions for the far zone fields and the first two terms in the series expansion of the total scattering cross section.

While it is suggestive that certain novel integral equation methods applied to the disk can also be employed to solve for the diffraction by rotationally symmetric scatterers of arbitrary shape [Heins, 1962a, b], no explicit results for other objects have as yet been obtained. In general terms, it is desired to infer the low-frequency scattering properties of an object from a knowledge of the zero-frequency (static) solution. An approximate procedure has been utilized by Siegel [1962] and by Senior, Darling, and Hiatt [1962], who seek to determine the unknown coefficients in the dynamic scattered field expansion by matching to the known static solution on the smallest ficticious spherical surface which completely encloses the arbitrarily shaped scatterer. Siegel has applied this method to the scattering from a cone-sphere combination, the static solution for which was obtained by Darling. Karp [1962b] seeks to bridge the gap between low-frequency and high-frequency asymptotic expansions by proceeding from a formulation involving a Fredholm integral equation of the second kind which is, however, not solved by iteration (thereby yielding the customary low-frequency expansion) but rather by the method of Fredholm determinants which converges for all real frequencies.

Some new electrostatic results have also been found. Van Bladel and Mei [1962] calculated the capacitance of a metallic cube, using a variational principle. The electrostatic dipole moment of a dielectric cube introduced into a uniform electrostatic field is evaluated by Edwards and Van Bladel [1961] for a range of dielectric constants between 1 and 1000 by numerically solving an appropriate surface integral equation.

The effect of absorbing coatings on perfectly conducting objects of various shapes has been investigated theoretically and experimentally by Hiatt et al. [1960], with the conclusion that the Rayleigh scattering cross section is not markedly affected by the coating.

D. Resonance Region

No new techniques are in evidence for the calculation of the scattering by objects having characteristic lengths of the order of a wavelength. Available results are usually obtained by numerical evaluation of series solutions for special shapes, and have been referred to at other appropriate places in this report.

E. Pulse Solutions

Gardner and Keller [1962] have treated a pulsed vertical dipole situated in the plane interface between two semi-infinite homogeneous dielectric media. They first find explicitly the two-dimensional field due to a line dipole by using the conical flow transformation and then solve the three-dimensional problem via a representation of the field of a point source as a superposition of line sources. They show that the field on the interface agrees with that obtained previously by Pekeris and Alterman. Ting [1960] generalizes the results of Keller and Blank for the diffraction of a plane Heaviside pulse by a wedge to include arbitrary two-dimensional pulses. He also considers diffraction of an arbitrary pulse by a cone.

An approximate method has been used to calculate the scattering of a periodic train of short pulses by a sphere [Kennaugh, 1961] and by a finite cone capped with a spherical segment [Kennaugh and Moffatt, 1962], and certain properties of the continuous wave scattering are deduced from the pulse solution [Kennaugh, 1962]. The results for the cone sphere are compared with those of Keller [1960], see also Siegel [1963a, b].

F. Some General Results

Uniqueness and existence theorems involved in the diffraction of electromagnetic waves by a metallic obstacle have been considered by Wilcox [1962]. C. S. Morawetz [1961] has shown that if a disturbance which is initially confined to a finite region propagates in free space according to the wave equation and is reflected from a star-shaped body, the rate of decay with time t at a fixed point in space is at least like $t^{-1/2}$. Marcuvitz [1962] has discussed the use of abstract operator methods in the formulation and formal solution of electromagnetic diffraction problems.

In a paper by Felsen and Karp [1960], an exact equivalence is established between certain ring-source excited three-dimensional, and line source excited two-dimensional problems, both in the same medium. The former class includes perfectly conducting bodies of revolution superimposed upon the edge of a perfectly conducting half plane, excited by a ring source with variation $\exp\left(\pm i\frac{\varphi}{2}\right)$, where φ is the azimuthal variable. The second class involves line source excitation of perfectly conducting cylindrical objects.

In connection with two-dimensional diffraction problems, Karp [1961] has shown that for a two-dimensional outgoing wave function regular in a region outside the circle $r>a$ and satisfying the time-harmonic wave equation, there exists a convergent expansion in the domain $r>a$. This expansion can be written as a linear combination of the Hankel functions of order of zero and one, the coefficients of which are expanded in inverse powers of r. Additional conditions on the analytic properties of the coefficients are proved, and a relationship between the leading terms of these coefficients and the complex radiation pattern is shown. Karp [1962a] has also considered the inverse two-dimensional scattering problem and has given some new properties of the plane wave scattering amplitude function. If the far-field pattern function for a scatterer with Dirichlet boundary conditions has the form $f(\theta, \theta_0)=f(\theta-\theta_0)$, where θ_0 is the angle of incidence of the plane wave and θ is the observation angle at infinity, then it can be shown that the scatterer is a circular cylinder.

Welch [1960] uses Rumsey's reaction concept to derive certain variational expressions for electromagnetic waves scattered from perfect conductors. This is later extended to imperfectly conducting media through the use of advanced as well as retarded solutions [Welch, 1961]. Senior has reviewed the derivation of the Leontovich impedance boundary condition [1962].

References

Albini, F. A. (1962), Scattering of a plane wave by an inhomogeneous sphere under the Born approximation, J. Appl. Phys. **33,** No. 10, 3032–3036.

Albini, F. A., and E. R. Nagelberg (1962), Scattering of a plane wave by an infinite inhomogeneous dielectric cylinder—an application of the Born approximation, J. Appl. Phys. **33,** No. 5, 1706–1713.

Bazer, J., and A. Brown (1959), Diffraction of scalar waves by a circular aperture, IRE Trans. Ant. Prop. **AP–7,** S12–S20.

Bazer, J., and H. Hochstadt (1962), Diffraction of scalar waves by a circular aperture. II, Comm. Pure and Appl. Math. **15,** No. 1, 1–33.

Bazer, J., and S. N. Karp (1962), Propagation of plane electromagnetic waves past a shoreline, J. Res. NBS **66D** (Radio Prop.), No. 3, 319–334.

Beard, C. I., T. H. Kays, and V. Twersky (1962), Mid-field forward scattering, J. Appl. Phys. **33,** No. 9, 2851–2867.

Borowitz, S. (1962), Application of Feynman's method of integrals over paths to diffraction problems, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962.

Brick, D. B. (1961), Current distributions on cylinders excited by spherical electromagnetic waves, IRE Trans. Ant. Prop. **AP–9,** No. 3, 315–317.

Brown, Jr., W. P. (1963), The transition from far-field to near-field geometrical optics: An estimate of the boundary layer thickness in diffraction problems, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 125 (Pergamon Press, New York and London).

Brysk, H. (1960), The radar cross section of a semi-infinite body, Can. J. Phys. **38,** 48–56.

Buchal, R. N., and J. B. Keller (1960), Boundary layer problems in diffraction theory, Comm. Pure and Appl. Math. **13,** No. 1, 85–114.

Burke, J. E., L. Mower, and V. Twersky (1962), Elementary results for scattering by large cones, J. Opt. Soc. Am. **52,** No. 10, 1093–1107.

Cassedy, E. S., and J. Fainberg (1960a), Back scattering cross sections of cylindrical wires of finite conductivity, IRE Trans. Ant. Prop. **AP–8,** No. 1, 1–7.

Cassedy, E. S., and J. Fainberg (1960b), Total electromagnetic cross sections of imperfectly conducting cylinders, J. Appl. Phys. **31,** No. 4, 739–740.

Chromey, F. C. (1960), Evaluation of Mie equations for colored spheres, J. Opt. Soc. Am. **50,** No. 7, 730–737.

Chu, T. S., R. G. Kouyoumjian, F. C. Karal, and S. N. Karp (1962), The diffraction of surface waves by a terminated structure in the form of a right-angle bend, IRE Trans. Ant. Prop. **AP–10,** No. 6, 679–686.

Clemmow, P. C., and V. H. Weston (1961), Diffraction of a plane wave by an almost circular cylinder, Proc. Roy. Soc. (London) **A264,** 246–248.

Collin, R. E. (1960), Scattering by an infinite array of thin dielectric sheets, IRE Trans. Ant. Prop. **AP–8,** No. 1, 62–67.

Copeland, J. R. (1960), Radar target classification by polarization properties, Proc. IRE **48,** No. 7, 1290–1296.

Deirmendjian, D., R. Classen, and W. Viezee (1961), Mie scattering with complex index of refraction, J. Opt. Soc. Am. **51,** No. 6, 620–633.

Deschamps, G. A. (1963), Scattering diagrams in electromagnetic theory, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, Ed. E. C. Jordan, pt. 1, 235–251 (Pergamon Press, New York and London).
Edwards, T. W., and J. Van Bladel (1961), Electrostatic dipole moment of a dielectric cube, Appl. Sci. Res., Sec. **B9**, No. 2, 151–155.
Eggimann, W. H. (1960a), Scattering of a plane wave by a ferrite cylinder at normal incidence, IRE Trans., Microwave Theory Tech. **MTT–8,** No. 4, 440.
Eggimann, W. H. (1960b), Higher-order evaluation of dipole moments of a small circular disk, IRE Trans., Microwave Theory Tech. **MTT–8,** No. 5, 573.
Eggimann, W. H. (1961), Higher-order evaluation of electromagnetic diffraction by circular disks, IRE Trans., Microwave Theory Tech. **MTT–9,** No. 5, 408–418.
Eggimann, W. H., and R. E. Collin (1962), Electromagnetic diffraction by a planar array of circular disks, IRE Trans., Microwave Theory Tech. **MTT–10,** No. 6, 528–535.
Felsen, L. B., and S. N. Karp (1960), Relation between a class of two-dimensional and three-dimensional diffraction problems, IRE Trans., Ant. Prop. **AP–8,** No. 4, 407–414.
Felsen, L. B., and C. J. Marcinkowski (1962), Diffraction by a cylinder with a variable surface impedance, Proc. Roy. Soc. (London) **A267,** 329–350.
Felsen, L. B. (1961), High-frequency diffraction by a wedge with linearly varying surface impedance, Appl. Sci. Res. Sec. B **9,** No. 3, 170–188.
Garbacz, R. J., and M. A. Plonus (1962), Comments on "Diffraction of a plane wave by a perfectly conducting sphere with a concentric shell," IRE Trans., Antennas Propagation **AP–10,** No. 3, 345–347.
Garbacz, R. J., and D. L. Moffatt (1961), An experimental study of bistatic scattering from some small, absorber-coated, metal shapes, Proc. IRE **49,** No. 7, 1184–1192.
Gardner, C. S., and J. B. Keller (1962), The field of a pulsed dipole in an interface, Comm. Pure Appl. Math. **15,** No. 2, 99–108.
Goodrich, R. F., N. D. Kazarinoff, and V. H. Weston (1963), Scalar diffraction by a thin, oblate spheroid, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 27–38 (Pergamon Press, New York and London).
Greenberg, J. M. (1960), Scattering by nonspherical particles, J. Appl. Phys. **31,** No. 1, 82–84.
Greenberg, J. M., L. Libelo, A. Lind, and R. T. Want (1963), Scattering by nonspherical particles whose size is of the order of the wavelength, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 81–92 (Pergamon Press, New York and London).
Greenberg, J. M., N. E. Pedersen, and J. C. Pedersen (1961), Microwave analog to the scattering of light by nonspherical particles, J. Appl. Phys. **32,** No. 2, 233–242.
Harrington, R. F. (1962), Small resonant scatterers and their use for field measurements, IRE Trans. Microwave Theory Tech. **MTT–10,** No. 3, 165–174.
Hasserjian, G., and A. Ishimaru (1962), Excitation of a conducting cylindrical surface of large radius of curvature, IRE Trans., Ant. Prop. **AP–10,** No. 3, 264–273.
Heins, A. E. (1960), Some remarks on "Diffraction of scalar waves by a circular aperture," IRE Trans. Ant. Prop. **AP–8,** No. 6, 633.
Heins, A. E. (1962), Function-theoretic aspects of diffraction theory, Electromagnetic Waves, ed. R. E. Langer, 99–108 (University of Wisconsin Press, Madison).
Heins, A. E. (1963), Some recent developments in scattering and diffraction theory, Symposium on Electromagnetic theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 185–196 (Pergamon Press, New York and London).
Heins, A. E. ,and R. C. MacCamy (1960), On the scattering of waves by a disk, J. Appl. Math. Phys. **XI,** Fasc. 4, 249–264.
Helstrom, C. W. (1963), Scattering from a cylinder coated with a dielectric material, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 133–144 (Pergamon Press, New York and London).
Hiatt, R. E., K. M. Siegel, and H. Weil (1960), Forward scattering by coated objects illuminated by short wavelength radar, Proc. IRE **48,** No. 9, 1630–1635.
Hiatt, R. E., K. M. Siegel, and H. Wiel (1960), The ineffectiveness of absorbing coatings on conducting objects illuminated by long wavelength radar, Proc. IRE **48,** No. 9, 1636–1642.
Hiatt, R. E., T. B. A. Senior, and V. H. Weston (1960), A study of surface roughness and its effect on the backscattering cross section of spheres, Proc. IRE **48,** No. 12, 2008–2016.
Hsu, Hwei-Piao (1960), Aperture fields in the diffraction by a slit, J. Appl. Phys. **31,** No. 10, 1742–1746.
Karal, Jr., F. C. and S. N. Karp (1962), Diffraction of a plane wave by a right angled-wedge which sustains surface waves on one face, Quart. Appl. Math. **XX,** No. 2, 97–106.
Karal, Jr., F. C., and S. N. Karp (1963), Propagation of electromagnetic waves along unidirectionally conducting screens, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 2, 967–980 (Pergamon Press, New York and London).
Karal Jr., F. C., S. N. Karp, Ta-Shing Chu, and R. G. Kouyoumjian (1961), Scattering of a surface wave by a discontinuity in the surface reactance on a right-angled wedge, Comm. Pure and Appl. Math. **14,** No. 1, 35–48.
Karp, S. N. (1960), Two-dimensional Green's function for a right-angled wedge under an impedance boundary condition, Comm. Pure and Appl. Math. **13,** No. 2, 203–216.
Karp, S. N. (1961), A convergent "far-field" expansion for two-dimensional radiation functions, Comm. Pure and Appl. Math. **14,** No. 3, 427–434.
Karp, S. N. (1962), Far field amplitudes and inverse diffraction theory, Electromagnetic Waves, 291–300, ed. R. E. Langer (The University of Wisconsin Press; Madison).
Karp, S. N. (1963), Systematic improvement of quasistatic calculations, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 205–207 (Pergamon Press, New York and London).
Karp, S. N., and F. C. Karal, Jr. (1960), Vertex excited surface waves on one face of a right-angled wedge, Quart. Appl. Math. **XVIII,** No. 3, 235–243 (1960).
Kazarinoff, N. D., and T. B. A. Senior (1962), A failure of creeping-wave theory, IRE Trans. Antennas Propagation **AP–10,** No. 5, 634–638
Keller, J. B. (1960), Backscattering from a finite cone, IRE Trans., Antennas Propagation **AP–8,** No. 2, 175–183.
Keller, J. B. (1961), Backscattering from a finite cone—comparison of theory and experiment, IRE Trans., Ant. Prop. **AP–9,** No. 4, 411–412.
Keller, J. B. (1962a), Diffraction by polygonal cylinders, Electromagnetic Waves, ed. R. E. Langer, 129–137 (The University of Wisconsin Press, Madison).
Keller, J. B. (1962b), Geometrical theory of diffraction J. Opt. Soc. Am. **52,** No. 2, 116–130.
Keller, J. B., and F. C. Karal, Jr. (1960), Surface wave excitation and propagation, J. Appl. Phys. **31,** No. 6, 1039–1046.
Keller, J. B., and D. G. Magiros (1961), Diffraction by a semi-infinite screen with a rounded end, Comm. Pure Appl. Math. **14,** No. 3, 457–471.
Kennaugh, E. M. (1961), The scattering of short electromagnetic pulses by a conducting sphere, Proc. IRE **49,** No. 1, 380.
Kennaugh, E. M. (1963), Scattering and diffraction of transient plane electromagnetic waves, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 93–95 (Pergamon Press, New York and London).
Kennaugh, E. M., and D. L. Moffatt (1962), On the axial echo area of the cone-sphere shape, Proc. IRE **50,** No. 2, 199.
Kennaugh, E. M., S. P. Morgan, and H. Weil (1960), Scattering by a spherical satellite, Proc. IRE, **48,** No. 10, 1781.
Kerker, M., J. P. Kratohvil, and E. Matijevic (1962), Light scattering functions for concentric shapes. Total scattering coefficients, $m_1=2.1050$, $m_2=1.4821$, J. Opt. Soc. Am. **52,** No. 5, 551–561.
Kerker, M., and E. Matijevic (1961a), Mie scattering functions for refractive index of 2.105, J. Opt. Soc. Am. **51,** No. 1, 87–92.

Kerker, M., and E. Matijevic (1961b), Scattering of electromagnetic waves from concentric infinite cylinders, J. Opt. Soc. Am. **51,** No. 5, 506–508.

Kieburtz, R. B. (1963), Scattering by a finite cylinder, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 145–156 (Pergamon Press, New York and London).

Kleinman, R. E. (1963), Plane wave diffraction by a strip, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 97–103 (Pergamon Press, New York and London).

Kline, M. (1962), Electromagnetic theory and geometrical optics, Electromagnetic Waves, ed. R. E. Langer 3–33 (The University of Wisconsin Press, Madison).

Kline, M. (1962), The status of asymptotic series solution of electromagnetic problems, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 25 to 30, 1962.

Knop, C. M. (1961), The radiation fields from a circumferential slot on a metal cyclinder coated with a lossy dielectric, IRE Trans., Ant. Prop. **AP–9,** No. 6, 535–545.

Knop, C. M., and A. R. Battista (1961), Calculated equatorial plane radiation patterns produced by a circumferential slot on a cylinder, IRE Trans., Ant. Prop. **AP–9,** No. 5, 498–499.

Kodis, R. D. (1961), On the theory of diffraction by a composite cylinder, J. Res. NBS **65D** (Radio Prop.), No. 1, 19–33.

Kodis, R. D. (1963), The radar cross section of a conducting cylinder with dielectric sleeve at the optical limit, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 127–132 (Pergamon Press, New York and London).

Kraus, L. and L. Levine (1961), Diffraction by an elliptic cone, Comm. Pure Appl. Math. **14,** 1, 49–68.

Kuehl, H. H. (1961a), Measurements on the asymmetrically excited prolate spheroidal antenna, IRE Trans. Ant. Prop. **AP–9,** 1, 105–107.

Kuehl, H. H. (1961b), Radiation from a radial electric dipole near a long finite circular cylinder, IRE Trans. Ant. Prop. **AP–9,** 6, 546–553.

Levine, H. (1959), Diffraction by an infinite slit, J. Appl. Phys. **30,** No. 11, 1673–1682.

Levis, C. A. (1960), Patterns of a radial dipole on an infinite circular cylinder: numerical values, IRE Trans. Ant. Prop. **AP–8,** No. 2, 218–222.

Levy, B. R. (1960), Diffraction by an elliptic cylinder, J. Math. Mech. **9,** No. 2, 147–165.

Levy, B. R., and J. B. Keller (1960), Diffraction by a spheroid, Can. J. Phys. **38,** 128–144.

Logan, N. A. (1960), Scattering properties of large spheres, Proc. IRE **48,** No. 10, 1782.

Logan, N. A., and K. S. Yee (1962a), A mathematical model for diffraction by convex surfaces, Electromagnetic Waves, ed. R. E. Langer, 139–180 (The University of Wisconsin Press, Madison).

Logan, N. A., and K. S. Yee (1962b), A simple expression for propagation constants associated with a reactive convex surface, IRE Trans. Ant. Prop. **AP–10,** No. 1, 103.

Logan, N. A., R. L. Mason, and K. S. Yee (1962), Equatorial plane radiation fields produced by a circumferential slot on a large circular cylinder, IRE Trans. Ant. Prop. **AP–10,** No. 3, 345–347.

McDonald, K. L. (1962), Diffraction of spherical scalar and vector waves at axial points of a circular aperture and disk, J. Appl. Phys. **33,** No. 6, 1944–1949.

Macrakis, M. S. (1960), Theoretical and experimental study of the backscattering cross section of an infinite ribbon, J. Appl. Phys. **31,** No. 12, 2261–2266.

Marcinkowski, C. J., (1961), Some diffraction patterns of an absorbing half-plane, Appl. Sci. Res., Sec. B **9,** No. 3, 189–198.

Marcinkowski, C. J., and L. B. Felsen (1962a), On the geometrical optics of curved surfaces with periodic impedance properties, J. Res. NBS **66D** (Radio Prop), No. 6, 699–705.

Marcinkowski, C. J., and L. B. Felsen (1962b), On the limitations of geometrical optics solutions for curved surfaces with variable impedance properties, J. Res. NBS **66D** (Radio Prop.), No. 6, 707–712.

Marcuvitz, N. (1962), Abstract operator methods in electromagnetic diffraction, Electromagnetic Waves, ed. R. E. Langer, 109–128 (The University of Wisconsin Press, Madison).

Meehan, J. E., and Z. Z. Hugus (1961), Light scattering functions for spherical particles with $m=1.65$ (0.05) 1.85, J. Opt. Soc. Am. **51,** 3, 260.

Morawetz, C. S. (1961), The decay of solutions of the exterior initial boundary value problem for the wave equation, Comm. Pure Appl. Math. **14,** 3, 561–568.

Odeh, F. M. (1961), Principles of limiting absorption and limiting amplitude in scattering theory II. The wave equation in an inhomogeneous medium, J. Math. Phys. **2,** 800.

Penndorf, R. B. (1962a), Angular Mie scattering, J. Opt. Soc. Am. **52,** 4, 402–407.

Penndorf, R. B. (1962b), Approximation formula for forward scattering, J. Opt. Soc. Am. **52,** 7, 797–800.

Penndorf, R. B. (1962c), Scattering and extinction coefficients for small absorbing and nonabsorbing aerosols, J. Opt. Soc. Am. **52,** 8, 896–904.

Plonsey, R. (1961), Diffraction by a slit, IRE Trans. Ant. Prop. **AP–9,** 217–219.

Plonsey, R. (1962), Surface current measurements with an electric probe, IRE Trans. Microwave Theory Tech. **MTT–10,** 3, 214–217.

Plonus, M. A. (1960), Backscattering from a conducting cylinder with a surrounding shell, Can. J. Phys. **38,** 12, 1665–1676.

Plonus, M. A. (1961), Diffraction of a plane wave by a perfectly conducting sphere with a concentric shell, IRE Trans, Ant. Prop. **AP–9,** 6, 573–576.

Rogers, C. C., J. K. Schlinder, and F. V. Schultz (1963), The scattering of a plane electromagnetic wave by a finite cone, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 67–80 (Pergamon Press, New York and London).

Rubinow, S. I. (1961), Scattering from a penetrable sphere at short wavelengths, Ann. Phys. **14,** No. 1, 305–332.

Rubinow, S. L. (1963), Forward and backward scattering from a penetrable sphere at short wavelengths, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, No. 123 (Pergamon Press, New York and London).

Rubinow, S. I., and J. B. Keller (1961), Shift of the shadow boundary and scattering cross section of an opaque object, J. Appl. Phys. **32,** No. 5, 814–820.

Ryerson, J. C. (1962), Scatterer echo area enhancement, Proc. IRE **50,** No. 9, 1979–1980.

Schiff, L. I. (1962), Scattering of waves and particles by inhomogeneous regions, J. Opt. Soc. Am. **52,** No. 2, 140–144.

Schmitt, H. J., and D. L. Sengupta (1960), On the reflection of electromagnetic waves from a medium excited by acoustic waves, J. Appl. Phys. **31,** No. 2, 439–440.

Senior, T. B. A. (1960a), Scalar diffraction by a prolate spheroid at low frequencies, Can. J. Phys. **38,** No. 12, 1632–1641.

Senior, T. B. A. (1960b), Impedance boundary conditions for imperfectly conducting surfaces, Appl. Sci. Res., Sec. B **8,** 418–436.

Senior, T. B. A. (1960c) Impedance boundary conditions for statistically rough surfaces, Appl. Sci. Res., Sec. B **8,** 437–462.

Senior, T. B. A. (1960d), A note on Hansen's vector wave functions, Can. J. Phys. **38,** 1702–1705.

Senior, T. B. A. (1962), A note on impedance boundary conditions, Can. J. Phys. **40,** No. 5, 663–665.

Senior, T. B. A., D. A. Darling, and R. E. Hiatt (1962), Low frequency expansion for scattering by separable and nonseparable bodies, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 25 to 30, 1962.

Seshadri, S. R. (1960), Scattering by a narrow unidirectionally conducting infinite strip, Can. J. Phys. **38,** No. 12, 1623–1631.

Seshadri, S. R. (1961), Diffraction of a plane wave by an infinite slit in a unidirectionally conducting screen, IRE Trans. Ant. Prop. **AP–9,** No. 2, 199–207.

Seshadri, S. R. (1962a), Scattering of surface waves by discontinuities of a unidirectionally conducting screen, IRE Trans. Microwave Theory Tech. **MTT-10,** No. 5, 367–375.

Seshadri, S. R. (1962b), Excitation of surface waves on an unidirectionally conducting screen, IRE Trans. Microwave Theory Tech. **MTT-10,** No. 4, 279–286.

Seshadri, S. R., and T. T. Wu (1960a), High-frequency diffraction of plane waves by an infinite slit for grazing incidence, IRE Trans. Ant. Prop. **AP-8,** No. 1, 37–42.

Seshadri, S. R., and T. T. Wu (1960b), High-frequency diffraction of electromagnetic waves by a circular aperture in an infinite plane conducting screen, IRE Trans. Ant. Prop. **AP-8,** No. 1, 27–36.

Siegel, K. M. (1962), The quasi-static radar cross sections of complex bodies of radiation, electromagnetic waves, ed. R. E. Langer, 181–197 (The University of Wisconsin Press, Madison).

Siegel, K. M. (1963a), Low-frequency radar cross section computations, Proc. IRE **51,** No. 1, 232–233.

Siegel, K. M. (1963b), Radar cross section of a cone sphere, Proc. IRE **51,** No. 1, 231–232.

Siegel, K. M., and R. E. Hiatt (also J. Fisch) (1961), Discussion of "Forward Scattering by Coated Objects Illuminated by Short Wavelength Radar," Proc. IRE **49,** No. 12, 1951.

Strait, B. J., and D. K. Cheng (1962), Measurement of microwave magnetic diffraction fields, IRE Trans. Ant. Prop. **AP-10,** No. 5, 643–645.

Tang, C. L. (1961), High-frequency diffraction by a sphere, Can. J. Phys. **39,** 1486–1494.

Ting, Lu (1960), On the diffraction of an arbitrary pulse by a wedge or a cone, Quart. Appl. Math. **XVIII,** No. 1, 89–92.

Toraldo di Francia, G. (1956), Electromagnetic cross section of a small circular disk with unidirectional conductivity, Nuovo Cim., Series X **3,** 1276–1284.

Van Bladel, J., and K. Mei (1962), On the capacitance of a cube, Appl. Sci. Res., Sec. B **9,** Nos. 4–5, 267–270.

Vea, T. H., J. B. Day, and R. T. Smith (1960), The use of a passive spherical satellite for communication and propagation experiments, Proc. IRE **48,** No. 4, 620–624.

Wait, J. R., and A. M. Conda (1961), Resonance characteristics of a corrugated cylinder excited by a magnetic dipole, IRE Trans. Ant. Prop. **AP-9,** No. 4, 330–333.

Wait, J. R., and C. M. Knop (1962), Comments on "Calculated Equatorial Plane Radiation patterns Produced by a Circumferential Slot on a Cylinder," IRE Trans. Ant. Prop. **AP-10,** No. 2, 211.

Welch, W. J. (1960), Reciprocity theorems for electromagnetic fields whose time dependence is arbitrary, IRE Trans. Ant. Prop. **AP-8,** No. 1, 68–73.

Welch, W. J. (1961), Comment on "Reciprocity Theorems for Electromagnetic Fields Whose Time Dependence is Arbitrary," IRE Trans. Ant. Prop. **AP-9,** No. 1, 114–115.

Weston, V. H. (1961), Near-zone back-scattering from large spheres, Appl. Sci. Res., Sec. B **9,** 107–116.

Weston, V. H. (1962), The effect of a discontinuity in curvature in high-frequency scattering, IRE Trans. Ant. Prop. **AP-10,** No. 6, 775–780.

Weston, V. H., and R. Hemenger (1962), High-frequency scattering from a coated sphere, J. Res. NBS **66D** (Radio Prop.), No. 5, 613–619.

Wetzel, L., and D. B. Brick, (1960), An experimental investigation of the Fock approximation for conducting cylinders, IRE Trans. Ant. Prop. **AP-8,** No. 6.

Wilcox, C. H. (1962), The mathematical foundations of diffraction theory, Electromagnetic Waves, ed. R. E. Langer, 65–97 (The University of Wisconsin Press, Madison).

Wolf, E., E. Marchand, and K. Miyamoto (1963), A boundary wave theory of diffraction at an aperture, Symposium on Electromagnetic Theory and Antennas, Copenhagen, June 1962, ed. E. C. Jordan, pt. 1, 109–111 (Pergamon Press, New York and London).

Additional Reference:

Logan, N. A. (1962), Early history of Mie solution, J. Opt. Soc. Am. **26,** No. 3, 342–343.

4.2. On Scattering of Waves by Many Bodies*

J. E. Burke and V. Twersky [1]

1. Purpose

This report surveys some of the recent analytical work on multiple scattering of waves by fixed configurations and by random distributions of many scatterers. Only papers concerned with discrete scatterer formalisms for the reduced wave equations are discussed explicitly, but representative papers of other formalisms are cited in order to indicate the relevance of other approaches.[2] Since the analogous previous report to URSI [Twersky, 1960a] contains historical and introductory discussions to the several categories of problems of interest, the present report will be relatively brief. Citations to specific papers and reports will be restricted more or less to those published during the last three years. Work available in research report form for the last survey will be cited again if since published in Journals. Some previous citations will also be mentioned to facilitate discussion of recent generalizations; however, these will be indicated by an asterisk and not listed in the present references (e.g., 1959* means look in previous survey for particulars).

2. Fixed Configurations of N-Scatterers

2.1. General Procedures

Most generally, the problems of present interest correspond to solutions (Ψ) of reduced wave equations consisting of a source term (φ) plus several scattered terms subject to appropriate boundary conditions (in a general sense) at the surfaces of the scatterers, and governed by different wave equations outside and inside the scatterers. The methods of primary interest to the multiple scattering problem are those which are essentially independent of the specific scatterers involved; using such methods, we need not apply boundary conditions explicitly and need not rework parts of the problem corresponding to isolated scatterer problems. Thus, assuming that the scattered waves (u_s) for the component scatterers in isolation are known, one seeks to embody them in functional equations for the multiple-scattered fields which make the effects of the configuration explicit.

For concreteness, we consider a source $\varphi=e^{i\mathbf{k}\cdot\mathbf{r}}$ exciting scatterers located at $\mathbf{b}_s(s=1,2,\ldots N)$. We write the total scattered wave as $\mathcal{U}(k\mathbf{r})=\Sigma U_s(k|\mathbf{r}-\mathbf{b}_s|)e^{i\mathbf{k}\cdot\mathbf{b}_s}$, where U_s, the multiple-scattered field of body s, reduces to its known single-scattered value u_s as the other bodies recede to infinity. We seek to express $\mathcal{U}$ and U_s in terms of u_s, or in terms of auxiliary known functions. The most useful auxiliaries are the "scattering amplitudes" defined through the far-field form $V\sim\mathcal{H}(kr)F(\mathbf{o},\mathbf{i})$; here V stands for $\mathcal{U}$, U_s, or u_s; $\mathcal{H}(kr)$ equals $e^{ik|x|}$, the asymptotic form of $H_0^{(1)}(kr)$, or $h_0^{(1)}(kr)$ in one, two, or three dimensions; and F (with $\mathbf{o}$ and $\mathbf{i}$ equaling unit vectors in the directions of observation and incidence) stands for the corresponding scattering amplitudes $\mathcal{G}$, G_s, or g_s. (We use a scalar formalism for simplicity but the extension to vector problems is straightforward.)

We also consider the associated "scattering coefficients" $\mathcal{A}(u)$, $A(i)$ and $a(i)$ which arise in the series representations of V and F in terms of special functions, e.g., in the Fourier series form $g_s(\theta_0,\theta_i)=\sum a_{ns}(\theta_i)e^{in\theta_0}$. As in the previous report, the systems of integral equations $G(g)$ or algebraic equations $A(a)$ obtained from a multiple-scattering analysis will be labeled "self-consistent" equations, and their usual iterated forms in powers of g or of a will be called "successive-scattering" series.

The earlier general multiple scattering formalisms were restricted to configurations of nonidentical scatterers for which the isolated scatterer problems could be solved by separations of variables. Thus working in spherical coordinates, Kasterin [1897*] developed the required addition theorems to treat spheres in terms of Legendre-Bessel series, and Zaviska [1913*] and Ignatowsky [1914a, b*] did the same for circular cylinders in terms of Fourier-Bessel series. Using essentially similar procedures, Row [1955*] and Twersky [1952–54*, 1962g] considered configurations of circular cylinders, and Urick and Ament [1949*], Ament [1959], Morse [1956*], Korringa [1947], Segall [1957], Huang and Yang [1957], Eyges [1957, 1958], Waterman and Truell [1961], and Schick [1961], among others, considered configurations of spheres. The analogous expansion in terms of Mathieu functions was used by Saermark [1959a, b, c, 1960] who developed some of the required addition theorems in order to treat configurations of strips.

Essentially two kinds of analytical representations have been obtained from such special function expansion procedures: infinite sets of linear algebraic equations $A(a)$ for the multiple-scattered coefficients A_{ns} in terms of the known single-scattered coefficients A_{mt}; asymptotic forms (closed or series) for large separations (say $kb>>1$) for the multiple-scattered amplitude G_s in terms of the single-scattered values g_t. The equations $A(a)$ have led to iterated series expansions, to closed forms for scatterers small compared to wavelength, and they have also been treated variationally and numerically. The asymptotic forms are related to Zaviska's [1913*] finding that if two circular cylinders of arbitrary size are in each

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[1] Sylvania Electronic Defense Laboratories, Mountain View, Calif.
[2] The related URSI survey by W. C. Hoffman (on propagation in random media) discusses various papers which analyze distributions as perturbed continuous media, as well as other papers which use essentially particle scattering methods.

other's far fields, then the problem reduces essentially to that of one cylinder excited by two plane waves. This is the basis for the asymptotic expansions derived by Twersky [1952b*, 1953a*] by retaining only the largest terms involving kb in each order of successive scattering of $A(a)$, and for the closed forms he obtained by summing resulting geometric progressions.

Today, the separations of variables derivations of the algebraic equations $A(a)$ are only of academic interest. It has been shown (Twersky, 1962a, b) that such systems of equations can be obtained for nonseparable as well as separable shapes without knowing the addition theorems for the special functions of the system in which it is convenient to treat the problem. Similarly, the separations of variables derivation of the asymptotic forms in kb is superfluous. Thus Karp [1953*] obtained the leading terms for $G(g)$ in $1/kb$ for arbitrary scatterers, and Karp and Zitron [1961a, b; 1959*] considered subsequent terms of the expansion; for two arbitrary cylinders, they obtain the asymptotic expansion correct to order $(kb)^{-3/2}$, and for two arbitrary scatterers in three dimensions they obtain the convergent expansion correct to order $(kb)^{-2}$. Twersky [1962a] developed a procedure for obtaining the complete asymptotic expansion routinely for two-dimensional problems, and applied it for N scatterers to exhibit the terms correct to order $(kb)^{-5/2}$; he also derived a closed form in terms of a differential operator for two scatterers, which gave the complete asymptotic series. Similarly, for three dimensions, Twersky [1962b], exhibited the corresponding converging series for N scatterers to order $(kb)^{-3}$ and derived the closed form in terms of a differential operator for two scatterers; expanding the denominator of the closed form for this case generates the exact converging series in inverse powers of kb for the two scatterer problem.

Although the separations of variables procedure is redundant for the above purposes, the representation of the fields for nonseparable shapes as series of cylindrical functions, or spherical functions, etc., may be of interest for other purposes. On the one hand, such representations enable us to exploit directly for more complex shapes the explicit results previously obtained by special function methods for the "array part" of a problem involving elementary shapes; e.g., one of the procedures used by Burke and Twersky [1960*] for the grating of elliptic cylinders was based on expanding the G and g's as Fourier series (instead of Mathieu function series), and similarly, Twersky [1962g] used Fourier-Bessel series for arbitrary cylinders in order to exploit the corresponding computations for circular cylinders. In addition, the known characteristics of the special function series may obviate convergence questions raised by more general representations. (Of course, as far as the single-scattered amplitudes g and coefficients a involved in the representations of G and A are concerned, one uses the special function forms where convenient to treat distributions of the shapes in question; however, for such special computations these series representations for g and a need only be introduced after one has treated the array part of the problem.)

Most of the current work on multiple scattering problems is based on integral equations, or on the volume integral and surface integral representations for the scattered fields obtained by using Green's theorem. Thus we may use the symbolic form $V(\mathbf{r})=\{\mathcal{H}_0(k|\mathbf{r}-\mathbf{r}'|),\, V(\mathbf{r}')\}=c\int(\mathcal{H}_0\partial_n V - V\partial_n\mathcal{H}_0)dS$, where $c\mathcal{H}_0$ is the free-space Green's function; if $V=\mathcal{U}$ (or u_s), then the integral is over any surface that isolates the collection of scatterers (or the scatterer) from the observation point; if $V=U_s$, the surface also isolates scatterer s from the others. The corresponding scattering amplitudes equal $F(\mathbf{o},\mathbf{i})=\{e^{-ik\mathbf{o}\cdot\mathbf{r}_s'}, V(\mathbf{r}_s';\mathbf{i})\}$. Ignatowsky [1914a*] used essentially such forms to obtain integral equations for the current density on one cylinder of a grating, and related forms have been used by Schwarzschild [1902*], Lax [1951*, 1952*], Ekstein [1951*, 1953*], Karp [1953*], Storer and Sevick [1954*], Millar [1960*, 1961a, b], Schick [1961], and Twersky [1956*–1962].

Aside from their utility for specific problems, the integral representations and Green's function methods lead directly to various theorems that the functions of interest must satisfy. Thus $\mathcal{G}$ fulfills the same theorems as g (e.g., $\mathcal{G}(\mathbf{o},\mathbf{i})=\mathcal{G}(-\mathbf{i}-\mathbf{o})$, the energy theorem, etc. (see Twersky [1962a, b]), and this leads to relations between the G_s in the set. Of course, G_s does not in general fulfill relations as simple as those for g (i.e., the G_s relations take account of the other sources which excite scatterer s); however, relatively simple relations emerge for special distributions [Twersky, 1957*–1962].

To generalize the previous discussion of the separable shapes, Karp (1953*) uses essentially $U_s=\{H_0,\Psi\}$ at the surface of a scatterer, to obtain asymptotic representations for large kb. The primary feature of Karp's procedure for scatterers in each other's far fields is to use the fact that the contribution to the excitation of scatterer s arising from the scattered field of scatterer t is effectively a plane wave on arrival (i.e., $U_t\sim G_tH(k|\mathbf{b}_s-\mathbf{b}_t|\varphi)$; then, since the response of s to a plane wave φ is known, the response of s to all such far-field multiple-scattering excitations can be obtained by superposition. The self-consistent set of algebraic equations $G(g)$ so obtained can be solved in closed form for special configurations, or iterated to obtain the correct leading terms of series for G in terms of g and inverse powers of kb. To obtain higher order terms of the series, Karp and Zitron (1961a, b) derive additional terms of the expansion of U_t in plane waves and inverse powers of kb at scatterer s and use a successive-scattering procedure to treat the two-dimensional case to $(kb)^{-3/2}$ and the three-dimensional case to $(kb)^{-2}$.

To generalize the above, Twersky (1962a, b) uses the complete representation of the fields in terms of complex integrals of free-space plane waves, i.e., $V(\mathbf{r};\mathbf{i})=\int e^{ik\mathbf{p}\cdot\mathbf{r}}F(\mathbf{p},\mathbf{i})$, where $\mathbf{p}$ is a complex unit vector, and $\int$ represents integration over an appropriate complex range; in two dimensions, $\mathbf{p}\cdot\mathbf{r}=r\cos(\theta_0-\tau)$ and $\int=\int_c d\tau/\eta$ with c equal to the general path used by Sommerfeld for $H_0^{(1)}$; in three dimensions, $\int=\int_c\int_{c'}d\Omega(\mathbf{p})/2\eta$ is based on Weyl's representation for $h_0^{(1)}$. (The complex integral form holds at least for $\mathbf{r}$ outside the smallest sphere enclosing the scatterer.) The total scattered field may thus be written $\mathcal{U}=\Sigma\int G_s(\mathbf{p},\mathbf{i})$ exp $ik[(\mathbf{p}-\mathbf{i})\cdot b_s+\mathbf{p}\cdot\mathbf{r}]$ with $G_t(\mathbf{o},\mathbf{i})=g_t(\mathbf{o},\mathbf{i})+\Sigma^1\int g_t(\mathbf{o},\mathbf{p})G_s(\mathbf{p},\mathbf{i})$ $e^{ik}(\mathbf{p}-\mathbf{i})\cdot\mathbf{b}_{ts}$ following directly from the superposition principle applied to the plane wave expansion of $\Psi-U_t$; here Σ^1 means $s\neq t$, and $\mathbf{b}_{ts}=\mathbf{b}_t-\mathbf{b}_s=b_{ts}\hat{\mathbf{b}}_{ts}$. Earlier papers on bounded infinite distributions [Twersky, 1953–1959*] were based essentially on special cases of the above, and a summary of problems which can be treated by such representations is given elsewhere [Twersky, 1962c]. Millar [1961c, 1962] independently developed the general two-dimensional form $G(g)$ and applied it to the semi-infinite grating.

To obtain the expansions of G_t in inverse powers of kb, one transforms the complex set of integral equations directly to $G_t(\mathbf{o},\ \mathbf{i})=g_t(\mathbf{o},\ \mathbf{i})+\Sigma^1\mathcal{F}_{ts}g_t(\mathbf{o},\hat{\mathbf{b}}_{ts})G_s(\hat{\mathbf{b}}_{ts},\ \mathbf{i})$, where $\mathcal{F}_{ts}=\mathcal{H}(kb_{ts})e^{-i\mathbf{k}\cdot\mathbf{b}_{ts}}[D_{ts}+1]$ with D equal to a known series in inverse powers of kb_{st} whose coefficients involve derivatives with respect to angles. For three-dimensional problems the series is convergent; this follows from the fact that $h_n^{(1)}$ can be written as $h_0^{(1)}=\mathcal{H}$ times a polynomial in inverse powers of kb—an expansion used previously by Sommerfeld to obtain a series in inverse powers of kr for the single scattering problem. For two-dimensional problems the above multiple-scattering series based on the asymptotic form of $H_n^{(1)}$ is asymptotic; however, the exact expansion for this case can be obtained in terms of $e^{-i\mathbf{k}\cdot\mathbf{b}_{ts}}H_0^{(1)}(kb_{ts})L_{ts}$ and $e^{-i\mathbf{k}\cdot\mathbf{b}_{ts}}H_1^{(1)}(kb_{ts})M_{ts}$ where L and M (which are analogous to the above D) follow from the exact Lommel representation for $H_n^{(1)}(x)$ as $H_0^{(1)}$ times one polynomial in $1/x$ plus $H_1^{(1)}$ times another such polynomial. The exact series for $H_n^{(1)\prime}$ has recently been used by Karp (1961) to derive the corresponding exact expansion for the single scatterer problem in two dimensions.

The set of complex integral equations $G(g)$ also leads directly to various systems of algebraic equations for the scattering coefficients $A(a)$ for separable and nonseparable problems without requiring the addition theorems for the special functions used to represent the a's. Thus Twersky [1962a] substitutes Fourier series representations for G and g to obtain $A(a)$ in circular cylindrical coordinates (for noncircular and circular cylindrical scatterers), and substitutes angular Mathieu function series for G and g to obtain $A(a)$ in elliptic coordinates. Similarly, in three dimensions, Twersky (1962b) writes G and g as series of spherical harmonics and substitutes into the complex integrals $G(g)$ to obtain a system of algebraic equations $A(a)$ for not necessarily spherical scatterers.

2.2. One-Dimensional Problems

The above has stressed plane wave representations which are novel for two and three-dimensional problems. However, such representations are the usual ones for one-dimensional problems corresponding to scattering of plane waves by configurations of parallel infinite slabs, or for the analogous transmission line problems. Discrete and continuous distributions are considered by a variety of methods for various applications by Kay and Silverman [1958], Redheffer [1961, 1962], Bellman and Kalaba [1956, 1958, 1960], Brekhovskikh [1960], Bloembergen and Pershan [1962], Collin [1960], Ferraro [1962], Melzak [1962], Richmond [1962], Twersky [1962d], Wing [1962], Yen [1962], Baumeister [1962], Young [1962], Mendlowitz and Simpson [1962], Catalán [1962], Heavans [1960], and others.

2.3. Several Separable Scatterers in Two Dimensions

The results for two arbitrary cylinders and large separation to order $(kb)^{-3/2}$ obtained by Karp and Zitron [1961a] were also specialized by them to circular cylinders. The corresponding specialization for two elliptic cylinders was given by Zitron [1962]. The closed operational form for two cylinders and the expansions $g(a)$ (in terms of Fourier series and angular Mathieu functions series) given in Twersky [1962a] constitute the complete asymptotic expansions for large separations for pairs of circular cylinders and pairs of elliptic cylinders; the explicit forms of the terms for N scatterers and the associated series $g(a)$ for the separable scatterers give their expansions to $(kb)^{-5/2}$.

Millar [1960*] considers the simultaneous integral equations for a row of perfectly conducting cylinders, and (using a procedure analogous to Bouwkamp's for the single strip) reduces them to a system of linear algebraic equations for elliptic cylinders with significant dimensions small compared to wavelength ("narrow cylinders"). He gives the multiple scattering closed form for two elliptic monopoles, and considers the corrections to the single scattered values for elliptic monopole plus dipole. Twersky [1962a] gives the multiple scattering closed forms for arbitrary angles of incidence for two different narrow circular cylinders and for two different narrow elliptic cylinders for arbitrary boundary conditions; these results generalize those obtained originally by Zaviska [1913*] and Twersky [1952*] by separation of variables. The closed forms for two different monopoles, two different dipoles, and two different pairs of monopole plus dipole are derived, and shown to satisfy the general scattering theorems for the configuration. Illustrative graphs for the intensities and phases for a pair of monopoles and for a pair of dipoles (e.g., for **E** parallel and perpendicular respectively to a pair of narrow dielectric rods) are given in Twersky [1962c] as functions of the separation (kb). Computations are made with the closed forms using both exact and asymptotic forms of the functions of kb; the two sets of computations are indistinguishable for $kb>5$, and are close enough for practical purposes even down to $kb=2$.

Hansen [1959] uses integral equations and Bouwkamp's procedure to treat **H** parallel to several narrow slits in a perfectly conducting infinite plane. Two slits are considered in some detail, and the difference between approximate results for the limiting case when the two slits coalesce and the corresponding exact result for one slit of double width is shown to be small. Numerical results are given for several cases, e.g., the transmission coefficients for 1, 2, 4, and 6 slits are compared with Miles' [1949*] results for the infinite grating. The ratio of separation of centers to slit width (d) is 2.5, and the range $kd=0$ to 1 is considered.

Saermark [1959a, b, c, 1960] uses separation of variables in elliptic coordinates to treat multiple scattering by several coplanar perfectly conducting strips (**H** parallel, and angle of incidence arbitrary). The corresponding transmission coefficient versus scatterer separation for two slits in a plane is computed and compared with the result for one strip [1959a]. Similar results for the transmission coefficient for more than two strips [1959b] are compared with the results for the infinite grating.

Related work on antenna arrays includes papers by Wu [1961], Chen and King [1961], and Ishimaru [1962].

2.4. Several Separable Scatterers in Three Dimensions

Kumagai and Angelakos [1960] obtain approximations for back scattering of electromagnetic waves by several spheres large compared to wavelength by taking into account double reflections. Collins [1961] uses integral equations to consider diffraction by two rigid disks with a common axis of symmetry.

Karp and Zitron [1961b] give results to $(kb)^{-2}$ for two arbitrary scatterers at large separation. Twersky [1962b] gives a closed operational form for two arbitrary scatterers, and the forms of the terms to $(kb)^{-3}$ for N scatterers at large separations; these plus the spherical harmonic series for $g(a)$ constitute the analogous special results for spheres. He also gives closed form approximations for the multiple-scattered coefficients for arbitrary angles of incidence on two different monopoles, two different dipoles, and two different scatterers specified by monopole plus dipole terms.

In the wave mechanics literature, pairs of monopoles and dipoles have been considered by Brueckner [1953*], Watson [1953*], Drell and Verlet [1955*], and others. Schick [1961] compares the separation of variables procedure, Green's function procedure based on volume integrals, and the more abstract t-matrix formalism (Lippman and Schwinger) for scattering by two radially symmetric scattering regions. The separation of variables procedure has been discussed in some detail for wave mechanical scattering and eigenvalue problems by Eyges [1957, 1958], and related problems are considered by Huang and Yang [1957], and Rodberg [1957].

2.5. Periodic Structures

Millar [1961a, b] considers the integral equations for an infinite grating of perfectly conducting cylinders and rewrites the kernel as an asymptotic series plus an additional function in order to facilitate consideration of the "grating resonances" or "Wood anomalies." The modified equations this leads to are applied to obtain explicit expressions for narrow elliptic cylinders. Results are given for **E** and **H** parallel, both for the transmission grating and for the analogous reflection grating of protuberances on a ground plane. The behavior for values of the parameters near the resonances is emphasized, and both qualitative and numerical results are given for the two polarizations. Approximations for the finite grating are also discussed.

Twersky [1962g; 1958b*] treats the grating of circular cylinders for arbitrary angles of incidence and arbitrary boundary conditions by substituting Fourier series for the scattering amplitudes into the functional equation $G(g)$ for the general grating [Twersky, 1956a*, 1957b*]. The resulting algebraic equations $A(a)$ for the scattering coefficients (which he also derives by extending the separations of variables procedure introduced by Ignatowsky [1914*] for normal incidence) are used to construct various explicit approximations. For arbitrary angles of incidence, closed form approximations in terms of known results for one conducting or dielectric cylinder and certain series of elementary functions [Twersky, 1961; 1958c*] are given for monopoles, dipoles, and monopoles plus dipoles. Multiple scattering or coupling effects are most pronounced for closely spaced cylinders and for the Wood anomalies. It is shown that the low-frequency "packing effects" for **E** perpendicular to the axis merely increases the dipole moment of the isolated cylinder; in this range, the circular cylinder within the grating is equivalent to an isolated elliptic cylinder whose size and shape are independent of the angle of incidence. For both polarizations, explicit low-frequency closed forms are given which include all coupling effects up to poles of order 2^5. For the Wood anomalies, for narrow cylinders, the results for G in terms of $A(a)$ are applied for an evanescent mode "near grazing" and compared with the previous results for $G(g)$. Illustrative plots for the resonance situation for the reflection grating of protuberances on a ground plane are given in Twersky [1962c]; for three propagating modes, the intensities and phases versus kb (or angle of incidence) are compared; for five modes, the phase and intensity of one mode versus kb are compared for four different values of scatterer radius.

A selective survey of grating literature is given by Larsen [1962a]. Grids parallel to interfaces are considered by Wait [1958*, 1962] and Wait's results are applied numerically by Larsen [1962b]. Nondiscrete scatterer approaches to grating problems based on Rayleigh's [1907a*] Fourier series representation for the grating's profile, and other procedures are considered by Stroke [1960, 1961], LaCasce [1961], Bottema [1959], Namioka [1961], Ingard [1960], Marsh [1961], and others.

Kieburtz and Ishimaru [1961] apply a variational method to an infinite planar lattice of square apertures to consider scattering when several propagating modes exist; the equivalent susceptance is obtained from an eigenvalue of an energy operator. Numerical results for scattering coefficients versus wavelength are given for primary and higher order modes. Kieburtz and Ishimaru [1962] also consider the integral equation for the boundary value problem of the periodic lattice of apertures. For square apertures of side a and lattice constant d, they give numerical results for $kd=5$ and $ka=1$, 1.5, and 2. Additional analytical results are given by Kieburtz, Ishimaru, and Held [1961], and by Kieburtz [1961]. Collin and Eggimann [1961] consider scattering by a two-dimensional lattice of circular disks in a rectangular array and obtain closed form approximations plus correction terms for the range where the single disk can be specified by electric and magnetic dipoles. A related waveguide problem is considered by Kerns [1960].

Problems related to three-dimensional lattices of discrete scatterers are considered by Kohn and Rostoker [1954], Adler [1961], Korringa [1947], and Segall [1957].

3. Random Distributions of Discrete Scatters

A general introduction to this topic was given in the previous survey, with emphasis on papers by Rayleigh [1899*], Reiche [1916*], Foldy [1945*], Lax [1951*, 1952*] and Twersky [1950–1960*]. Starting with the multiple-scattering equations for the field of a specific configuration, one introduces a statistical ensemble of configurations and seeks the average coherent field $\langle\Psi\rangle$, the average total intensity $\langle|\Psi|^2\rangle$, and the corresponding average Poynting flux. Proceeding heuristically, one approximates averages of Ψ with two variables held fixed by the average for one fixed variable, or uses some equivalent approximation to obtain a determinate set of equations for $\langle\Psi\rangle$. These have been treated by several different procedures leading to explicit approximations for $\langle\Psi\rangle$ (or equivalently for the "bulk parameters" of the medium associated with coherent propagation) in terms of single-scattered amplitudes g.

Alternate procedures for the coherent effects in "random media" are considered by Keller [1960], Hoffman [1959*, 1960], Bremmer [1958*, 1962], Silver [1962], and others to be cited subsequently. For present purposes we note that Keller [1960] gives a critical review and comparison of heuristic and analytical procedures, Silver [1962] emphasizes the probabilistic restrictions implicit in current work, Bremmer [1962] gives a comprehensive review of the perturbed continuum literature, and Hoffman's book [1960] provides samples of the varied statistical problems in this subject.

The analytical expressions for $\langle|\Psi|^2\rangle-|\langle\Psi\rangle|^2$, and for the corresponding incoherent scattering I, are much more complex. These have been considered by Foldy [1945*] and by Lax [1951*], but relatively little has been done with them to get explicit results for specific problems. For special distribution, Twersky [1957*–1963] has used $\langle\Psi\rangle$ plus the energy theorem to obtain I explicity. Most writers ignore the coherent field and use essentially particle scattering procedures based on the work of Hopf [1934*] and Chandrasekar [1950*] to determine the incoherent scattering. Recent papers include those by Chu, Churchill, and Pang [1962], Grosjean [1962], and Sekera [1962].

3.1. Planar Distributions

Previous results for scattering by random planar distributions of arbitrary scatterers [Twersky, 1957a*] are specialized to elliptic cylinders by Burke and Twersky [1963]. Substituting the known scattering amplitude for an isolated perfectly conducting elliptic cylinder into the general results, they obtain the corresponding coherent field (intensity and phase) and differential scattering cross section (such that the total average intensity satisfies the required energy theorem). Both "random screens" of elliptic cylinders, and distributions of perfectly conducting elliptic protuberances on a ground plane, are treated for **E** perpendicular and parallel to the plane of incidence. For scatterers small and large compared to wavelength, the reflection coefficients, etc., for the two polarizations are plotted versus angle of incidence for various eccentricities ranging from perpendicular strips, through semicircles to the other limiting case of "flat strips." For low frequencies, the real and imaginary parts of the isolated scattering amplitudes required in the analysis are given to the eighth and sixth powers of frequency, respectively [Burke and Twersky, 1960]. Analogous results for spheres and circular cylinders are applied numerically by Twersky [1962c] to compare electromagnetic scattering by a "uniformly rough surface" of small perfectly conducting hemispheres on a ground plane and by the analogous "striated surface" of semicylinders. For both polarizations, for both distributions, the following functions are plotted for the full range of angle of incidence (normal to grazing): the coherent power reflection coefficients, the total phase of the coherent reflected field, and the "forward" and back scattered differential scattering cross sections per unit area. The graphs show "pseudo-Brewster" effects, etc.

Recent work on random screens and rough surfaces based on other approaches include papers by Ament [1960], Beard [1961], Beckmann [1961, 1962a, b], Bennett and Porteus [1961], Briggs [1961], Clay 1960], Davies [1954], Garrison, Murphy, and Potter [1960], Hagfors [1961], Hiatt, Senior, and Weston

[1960], LaCasce [1961], LaCasce, McCombe, and Thomas [1961], Longuet-Higgins [1960a, b, c], Marsh [1961], Marsh, Schulkin, and Kneale [1961], Senior [1960], Wait and Conda [1961], Winter [1962], Siegel and Senior [1962], Eby, Williams, Ryan, and Tamarkin [1960], and Mercier [1961].

3.2. Random Volume Distributions

O'Konski [1960] considers the bulk electrical properties of suspensions of ellipsoids (ionic polarization of macromolecules in polyelectrolytes). Mandel [1960] discusses the dielectric constant and dispersion in suspensions or oriented prolate spheroids. Amount [1959] used separations of variables and other procedures for obtaining the average coherent field and the corresponding bulk physical parameters of media composed of statistical distributions of spheres. He considers the field for a plane wave incident on a semi-infinite region of scatters, as well as the characteristics of a spherical region containing small spheres in suspension Approximations for spheres small compared to wavelength are given for problems such as the electromagnetic field in an artificial dielectric of dipole scatters, ferromagnetic exchange effects at optical frequencies, thermoelectric power of sintered mixtures frequency dependence of artificial dielectrics, sound waves in suspensions, and optical activity.

Waterman and Truell [1961] begin with Foldy's [1945*] formalism for the case of an electromagnetic wave normally incident on a semi-infinite half space of spheres and then use separations of variables in spherical coordinates to obtain an approximation for the propagation coefficient K in terms of the forward and back scattered values of the scattering amplitude of an isolated sphere. Essentially the same form had been obtained previously for the acoustic case of spheres by Urick and Ament [1949*], and had been generalized to arbitrary angle of incidence by Twersky [1953b*, 1958a*]. To generalize the results to arbitrary scatterers, Twersky [1958a*, 1962] starts with the ensemble average of the Green s function surface integral representation for arbitrary angle of incidence on a slab region of distribution. Approximating the multiple-scattered amplitude G in the kernel by a simple set of free-space scattering amplitudes leads to explicit results for K and for the bulk parameters (ϵ and μ) in terms of $g(\boldsymbol{k}',\boldsymbol{k})$ where $\boldsymbol{k}=k\boldsymbol{i}$, and $\boldsymbol{k}'$ is either $\boldsymbol{k}$ or its image in the slab face. (The corresponding approximation for the incoherent scattering is based on the energy theorem.) It is pointed out that such procedures are "not quite self-consistent," since one assumes initially that a fixed scatterer within the distribution is excited by a set of free-space fields of the form $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ (i.e., a set of "microscopic" waves in k-space), and then finds that the excitation consists of waves of the form $e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$ (the "macroscopic" field in K-space). Several limitations of the explicit results are discussed and standard procedures ("hole corrections," etc.) for continuing what is essentially the start of a perturbation procedure around the characteristics of free space are mentioned; as they stand, all such results are restricted to sparse distributions.

In order to obtain a more rapidly convergent representation for denser distributions than one based on $g(\mathbf{k}', \mathbf{k})$, and in order to develop a procedure that is fully self consistent on the macroscopic level, Twersky [1958d*, 1962e] introduces a generalized isolated scatterer amplitude $g(\mathbf{k}, \mathbf{K})$ corresponding to an obstacle excited in K-space but radiating into k-space. Approximating the field at a scatterer fixed with the distribution by the sum of the coherent field (waves of the form $e^{i\mathbf{K}\cdot\mathbf{r}}$) plus the field radiated by the fixed scatterer (a wave having propagation number k), Twersky [1959a*, 1962f] obtains functional equations for K, ϵ, and μ in terms of $g(\mathbf{k}, \mathbf{K})$. The equations are applied to small spheres of arbitrary ϵ' and μ' and the "Lorentz-Lorenz form" is obtained for each bulk parameter ϵ and μ; it is shown that ϵ and μ are independent of the direction of incidence and of the polarization of the incident field (i.e., the coherent field defines a unique Maxwellian medium). As a second approximation, the "loss terms" corresponding to incoherent scattering are included; these appear in the bulk ϵ and μ in the roles of electric and magnetic "conductivities." Another illustration considered is normal incidence on a slab of arbitrarily shaped scatterers large compared to wavelength and with parameters close to those of free space ("large tenuous scatterers"); for this case, the bulk parameters ϵ and μ equal the bulk index of refraction. Illustrative computations for the coherent phase and attenuation coefficients, and for the coherent and incoherent intensities versus the fractional volume for a "pure gas" are given in Twersky [1962a].

The above forms for K in terms of $g(\mathbf{k},' \mathbf{k})$ and $g(\mathbf{k}, \mathbf{K})$ are compared with that of Lax [1951*, 1952*] for K in terms of $g(\mathbf{K}, \mathbf{K})$ [Twersky, 1960c] and the differences in the explicit results for large tenuous scatterers are noted. All forms agree at low concentrations, but they diverge at higher concentrations: each formalism is essentially the start of a different perturbation procedure around the characteristics of free space. Twersky [1962h] gives an alternative, "more macroscopic," derivation for the forms ϵ and μ in terms of $g(\mathbf{k}, \mathbf{K})$; the forms follow on direct comparison of the integrals for the coherent transmitted and reflected fields in terms of the multiple-scattered amplitude G, with the usual Green's function representation for the fields transmitted and reflected by a uniform slab. He also compares the approximation for the incoherent scattering constructed to satisfy energy conservation (say I) with that obtained by inserting an exponential "shielding factor" in the volume integral for I to account for the "attenuation" that the incoherent scattering undergoes in leaving the slab. For large tenuous scatterers, it is shown that the "shielded form" is merely the first stage of a series of successive scatterings of the incoherent intensity, and that I is the corresponding closed form. Thus constructing I to satisfy the energy principle automatically takes into account all such orders of incoherent multiple scattering. Generalizations for distributions of nonidentical scatterers are derived, and it is shown that the results for such "mixtures" reduce consistently to those for identical scatterers if the volume available to a scatterer is taken as the original volume less the space taken up by the total number of scatterers.

In addition to the papers by Keller [1960], Silver [1962], Bremmer [1962], and Hoffman [1960] cited previously, recent papers using other methods to analyze physical problems in this field include those of Parrent, Shore, and Skinner [1962], Skinner [1961], Hoffman [1960], Butler [1962], Chernov [1960], Tatarski [1961], Yamada [1961], Ford and Meecham [1960], Bowhill [1961a,b], Budden [1959], Bugnolo [1960a,b; 1961a,b], Yeh [1962], Grosjean [1962], Chu, Churchill, and Pang [1962], Sekera [1962], Mulliken [1962], Beckmann [1962], Hessemer [1961], Kottler [1960], Isihara [1961], Kielich [1962], Melzak [1962], and Musgrave [1959].

3.3. Generalized Distributions

The previous sections were limited to either periodic distributions or to "gas-like" distributions completely specified by the one-particle distribution function, say p_s. For the periodic case, p_s is a series of delta functions; for the gas, p_s equals the number of scatterers divided by the available volume. More generally, it is of interest to consider distributions which give the results for the case as one limit of some parameters, and those for the periodic case as the other limit. Consistent generalizations of the full range from gas to crystal based on a two-particle distribution function $p_s f(|\mathbf{r}_s-\mathbf{r}_t|)$, where f is the "radial distribution" function, are at present limited to the one-dimensional case for which explicit enough forms for f exist; see discussion in Twersky [1960a,b], and work on planar distributions of cylinders by Zernicke and Prins [1927*] and by Twersky [1953a*, 1959b*]. Cole (1961) summarizes recent progress and reviews the literature of radial distribution functions.

Results characteristic of "quasi-random" and "quasi-periodic" distributions may also be obtained for a "two-phase" system in which some scatterers are considered to be in a "gas phase" and some in a "crystal phase." The scatterers in both phases contribute explicitly to the coherent field, but only those in the gas phase contribute explicitly to the incoherent scattering. Thus Twersky [1962i] considers a two-phase system of $N=N_r+N_x$ large tenuous scatterers such that N_r are in a gas phase and N_x are in a crystal phase, and obtains explicit results for the coherent and incoherent intensities, etc., in terms of fundamental scattering parameters. In order to investigate the behavior of the fields during a "compression process" in which the system starts as a pure gas and ends up as a pure crystal, he introduces a simple model for the change in relative populations as a function of total fractional volume (the fraction of space occupied by the N scatterers, say ω_0): as ω_0 increases from 0 to some maximum value (e.g., 0.74 for a face-centered cubic crystal of spheres), the ratio N_x/N (the fraction of crystal-type scatters) increases linearly to unity and the system changes from a gas to a crystal. Illustrative graphs of the fields for the two-phase system are compared with those for the pure gas for the full range of ω_0. He also applies the results for mixtures [Twersky, 1962h] to include a "multicomponent" gas phase consisting of differently oriented asymmetrical scatterers and different "clumps" (some scatterers stay single, some form doublets, triplets, etc.); in general the scatterers when clumped give more incoherent scattering than otherwise. The final expressions are simple enough in form to facilitate inverting measurements when seeking such "crystalizing functions" and "clumping functions" experimentally.

The theoretical results are compared in detail with parallel microwave experiments by Beard, Kays, and Twersky [1963] on a large scale dynamical model of a "compressible gas" of large styrofoam spheres. The theoretical model of a "multicomponent two-phase" system seems adequate to account for many of the observed phenomena. (Other large scale dynamical models of "compressible gasses" developed to further comparisons of theory and experiment under controlled conditions are described in Twersky [1962c].

3.4. Relations Between Scatterer Statistics and Signal Statistics

Under appropriate measuring conditions, the ensemble averages for configurations of discrete scatterers are equivalent to time-averaged measurements on a collection of scatterers in motion (i.e., time constants of the motion large compared to periodicity of incident wave and small compared to time interval for the measurement). The relevance of configuration averages to measured time averages can be determined in the course of actual experiments [Beard and Twersky, 1958*, 1960a*, 1960b*] and we may restrict considerations to situations where these are equivalent. Thus we equate time averages of the instantaneous signal $\mathcal{J}e^{i\tau}$ (where $\mathcal{J}=|\mathcal{J}|$ and τ is real) to configuration averages of the field $\psi=\varphi+\Sigma U_s$ where φ is the incident wave and U_s is the multiple-scattered wave of scatterer s. The configuration-averaged coherent field $<\psi>=\sqrt{C}e^{i\alpha}$ (where C is the "coherent intensity" and α is the "coherent phase") are related to time-averaged measurements through $C=|<\mathcal{J}e^{i\tau}>|^2$ and $\tan\ \alpha=<\mathcal{J}\sin\ \tau>/<\mathcal{J}\cos\ \tau>$. Equivalently, we may write $\mathcal{J}e^{i\tau}=\sqrt{C}e^{i\alpha}+\mathcal{G}e^{i\gamma}$ such that $<\mathcal{G}e^{i\gamma}>=0$. Similarly, the configuration averaged total intensity $T=<|\psi|^2>$ equals the time-averaged function $<\mathcal{J}^2>$. The corresponding incoherent intensity is

$$I=T-C=<|\psi|^2>-|<\psi>|^2=<\mathcal{J}^2>-|<\mathcal{J}e^{i\tau}>|^2=<\mathcal{G}^2>.$$

For many problems, we may use the approximation $I\approx\Sigma_s<|U_s|^2>$, i.e., I is approximately the sum of the average multiple scattering intensities of the component scatterers.

The papers mentioned in the previous sections were concerned with deriving expressions for C, α, and I in terms of fundamental scattering parameters; as indicated above, these can be compared with corresponding time-averaged measurements. However, additional time-averaged functions can be isolated in the course of experiments. Thus for the dynamical model of a gas [Beard and Twersky, 1958*], Beard [1963] has measured the variances of the phase quadrature components of the instantaneous field $<x^2>=<\mathcal{I}^2 \cos^2 \gamma>$ and $<y^2>=<\mathcal{I}^2 \sin^2 \gamma>$ (such that $<x^2>+<y^2>=<\mathcal{I}^2>=I$) and their covariance $<xy>=<\mathcal{I}^2 \cos \gamma \sin \gamma>$. For a Rayleigh distribution, we have $<xy>=0$ and $<x^2>=<y^2>=<\mathcal{I}^2>/2$. More generally, however, these second moments give the standard deviation and correlation coefficient for a bivariate description of the ensemble of signals. Because of this, and because the fundamental scattering parameters (size, shape, dielectric properties, etc.) were expected to enter differently in the second moments than they do in C, α, and I, it has been of interest to relate these moments to fundamental scattering theory.

The required relation was recently derived by Twersky (1962j). He introduced the "asymmetry function" $P=|P|e^{ip}$ such that $2P\varphi^2=<(\psi-<\psi>)^2> \approx \Sigma<U_s>^2$. In terms of I and P, the second moments of the ensemble of signals are given by $<x^2>=\frac{1}{2}I+|P| \cos p$, $<y^2>=\frac{1}{2}I-|P| \cos p$, and $<xy>=|P| \sin p$. Thus measurements of these second moments give I, $|P|$, and p, which together with C and α provide five independent functions for isolating scattering parameters from microwave measurements on distributions. Explicit approximations of P for the gas, two-phase system, etc., of large tenuous scatterers are given by Twersky [1962i].

In addition, the scattering function representation for P provides explicit relations for the leading terms of the moment expansions of other measurables. Thus for "small randomness" $<\mathcal{J}^2>-<\mathcal{J}>^2 \approx \frac{1}{2}I +|P| \cos (p-2\alpha)$, $<\mathcal{J}^2>-C\approx \frac{1}{2}I-|P| \cos (p-2\alpha)$, $<\tau>-\alpha \approx (|P|/C) \sin (p-2\alpha)$, $<\tau^2>-<\tau>^2 \approx [\frac{1}{2}I-|P| \cos (p-2\alpha)]/C$, etc.

3.5. Series Developments

The papers mentioned in the previous sections were based on more or less standard procedures, e.g., approximating an average with two variables held fixed by the average for one fixed variable, etc. The present section considers more analytical work. We begin by indicating the relationship between the heuristic and analytical procedures, and then summarize some recent results.

Consider the solution ψ for the scattering of a wave by a given configuration of N scatterers, say $\psi(\mathbf{1}, \mathbf{2}, \ldots \boldsymbol{N})$, where $\mathbf{1}$ stands for all significant properties of scatterer "one" (including its locations $\mathbf{r}$), etc. We introduce an ensemble of configurations in terms of an appropriate probability distribution function $W(\mathbf{1}, \mathbf{2}, \ldots \boldsymbol{N})$, and write the average of ψ, i.e., the coherent field as $<\psi>=\int \ldots \int \psi (\mathbf{1}, \mathbf{2}, \ldots \boldsymbol{N}) W(\mathbf{1}, \mathbf{2}, \ldots \boldsymbol{N}) \boldsymbol{d}/d\mathbf{2} \ldots d\boldsymbol{N}$. Similarly, we average the intensity $|\psi|^2$ over the ensemble, and write the average total intensity as $<|\psi|^2>=|<\psi>|^2+V$, where $|<\psi>|^2$ is the coherent intensity, and where the incoherent intensity V is the average absolute squared deviation of ψ from its mean value $<\psi>$. We may also consider the average of the corresponding Poynting vector, and the average of other scattering functions.

In general, we proceed by stating conditions on $\psi(\mathbf{1}, \mathbf{2}, \ldots \boldsymbol{N})$ to make the scattering problem for one configuration determinate, then introduce an ensemble of configurations in terms of $W(\mathbf{1}, \mathbf{2}, \ldots \boldsymbol{N})$, and then seek the expectation values $<\psi>$, $<|\psi|^2>$, etc. We have essentially two kinds of representations for ψ: "compact representations" in terms of the solutions of a set of simultaneous integral equations (say ψ_c) and "expanded" representations as series of orders of scattering (say ψ_e). If we work with ψ_e, then we obtain $<\psi>=<\psi_e>$, etc., by integration; the integrated forms of the terms of the series ψ_e may be sufficiently simpler than the originals and lead to closed or relatively compact forms for $<\psi_e>$ (see for example Twersky, 1953a*). More usually (practically all the papers cited in the previous sections) we average the compact representation ψ_c, proceed heuristically to construct a set of "average equations" that are simpler than the originals, and then solve these to obtain say $<\psi_c>\approx<\psi>$.

A more general discussion than the above, one that applies both to a random distribution of scatterers and to a randomly perturbed continuum, is given by Keller's [1960] parallel description of "honest" and "dishonest" methods; Keller uses both methods to derive results for scintillations of starlight arising from inhomogeneities in the atmosphere. The conditions for which $<\psi_c>$ approaches $<\psi_e>$ for a class of one-dimensional scattering problems have been determined by Bazer [1959*].

For three-dimensional distributions of discrete scatterers, the relationship of the heuristic c-forms to their corresponding e-forms is discussed by Twersky [1963]. The compact heuristic representations $<\psi_c>$ in the integral equation forms obtained by Foldy [1945*], Lax [1951*], and Twersky [1962d] differ from $<\psi_e>$ in that $<\psi_c>$ neglects terms corresponding to common path differences in a given order of scattering: each scatterer is "used only once" in a given order of scattering of $<\psi_c>$, so that, e.g., the back and forth interactions between a pair are neglected. Solving the two-body problems implicit in the general distribution, Twersky [1963] constructs an alternative heuristic compact form for $<\psi>$ to restore some of the missing processes ignored by $<\psi_c>$. Similarly, he extends the heuristic integral equation forms $<|\psi_c|^2>$ derived by Foldy and Lax by taking into account additional two-body processes. Comparison of $<|\psi_c|^2>$ with the average of the product of the exact series $<|\psi_e|^2>$ indicates that the heuristic forms $<|\psi_c|^2>$ are poorer approximations of $<|\psi_e|^2>$ than the $<\psi_c>$ are of $<\psi_e>$. In order to obtain a form of $<|\psi_c|^2>$ consistent with the approximation $<\psi_c>$, Twersky introduces into $<|\psi_e|^2>$ the same simplification that converts $<\psi_e>$ to $<\psi_c>$, i.e., each scatterer is "used only once" in a given chain of successive scatterings of the single-scattered value u. This simplification in the series $<|\psi_e|^2>$ leads directly to a relatively simple integral equation whose inhomogeneous term is $|<\psi_c>|^2$ and whose kernel involves a "new single scatterer" u_ϵ; the function u_ϵ satisfies the integral equation analogous to $<\psi_c>$ whose inhomogeneous term is u, i.e., u_ϵ is the corresponding "Green's function" (in the sense the term is used in physics) for an angle dependent source u. (For the case of monopole scatterers, Foldy [1945*] obtained two series representations for the kernel function in his integral equation for $<|\psi_c|^2>$; the initial term of one of these series corresponds to the new integral equation specialized to monopoles.)

In order to exhibit the relationships between the heuristic and exact forms explicitly, Twersky [1963] restricts consideration to a very simple class of scattering problems: the radiation fields of the individual scatterers are significant only in the forward half-space, particularly near the forward scattering direction; the scatterers are located within a slab region of space on which a plane wave is normally incident; the scatterers are identical (or, equivalently, the averages over isolated scatterer parameters are identical), and their position statistics are those of a "rare gas." For the e-forms, he works in general with a finite number of scatterers (N) in a finite available volume (V); for the corresponding c-forms, N and V are infinite but their ratio is bounded. This illustrative problem is constructed around scatters which do not need to be taken into account more than once in each order of scattering. Thus, for this case, the heuristic form $<\psi_c>$ used explicitly by Foldy, Lax, Twersky, and used implicitly by practically all papers in the earlier physics literature is the correct limit of $<\psi_e>$ for $N\rightarrow\infty$, $V\rightarrow\infty$ [i.e., we have essentially $<\psi_c> =(1+f/N)^N\rightarrow e^f=<\psi_c>$]. However, the situation is quite different for $<|\psi|^2>$: the heuristic form constructed with the aid of the energy theorem [Twersky, 1962b] gives the correct value that $<|\psi_e|^2>$ approaches in the limit $N\rightarrow\infty$; the analog of Foldy's [1945] heuristic integral equation $<|\psi_c|^2>$ for which the kernel function is obtained in closed form for this simple illustration, is incorrect; the "$<\psi_c>$-consistent" heuristic integral equation for $<|\psi_c|^2>$ is identically the limit of $<|\psi_e|^2>$ for $N\rightarrow\infty$, and this is also the initial term of the analog of one of Foldy's two series.

References

Adler, S. (1961), A generalized Ewald method for lattice sums, Physica **27,** 1193–1201.

Ament, W. S. (1959), Wave propagation in suspensions, private communication.

Ament, W. S. (1960), Reciprocity and scattering by certain rough surfaces, IRE Trans. Ant. Prop. **AP–8,** 167–174.

Baumesiter, D. W. (1962), Methods of altering the characteristics of a multilayer stack, J. Opt. Soc. Am. **52,** 1149–1152.

Beard, C. I. (1961), Coherent and incoherent scattering of microwaves from the ocean, IRE Trans. Ant. Prop. **AP–9,** 470–483.

Beard, C. I. (1962), Statistics of phase quadrature components of microwave field transmitted through a random medium, IRE Trans. Ant. Prop. **AP–10,** 721–731.

Beard, C. I., T. H. Kays, and V. Twersky (1963), Experiment and theory for scattering by random distributions of spheres versus fractional volume, Report EDL–E74, Sylvania Electronic Defense Laboratories.

Beckmann, P. (1961), The scattering of waves by a periodic surface, Czechoslovak J. Phys. **11,** 863–870.

Beckmann, P. (1962a), Deviations from the Rayleigh distribution for a small and for a random number of interfering waves, Inst. of Radio Engineering and Electronics, Czech. Acad. of Sci. No. 25.

Beckmann, P. (1962b), Statistical distribution of the amplitude and phase of a multiply scattered field, J. Res. NBS **66D** (Radio Prop.), No. 3, 231–240.

Bellman, R. and R. Kalaba (1956), On the principle of invariant imbedding and propagation through inhomogeneous media, Proc. Nat. Acad. Sci. USA **42,** 629–632.

Bellman, R. and R. Kalaba (1958), Invariant imbedding; wave propagation and the WKB approximation, Proc. Nat. Acad. Sci. USA **44,** 317–319.

Bellman, R. and R. Kalaba (1960), Invariant imbedding and wave propagation in stochastic media, part of the book Electromagnetic Wave Propagation, pp. 243–252 (Academic Press, London).

Bennett, H. E., and J. O. Porteus (1961), Relation between surface roughness and specular reflectance at normal incidence, J. Opt. Soc. Am. **51,** 123–128.

Bloembergen, N., and P. S. Pershan (1962), Light waves at the boundary of non-linear media, Phys. Rev. **128,** 606–662.

Bottema, M. (1959), Scattering by imperfect gratings, The Johns Hopkins University Laboratory of Astrophysics and Physical Meteorology.

Bowhill, S. A. (1961a), Statistics of a radio wave diffracted by a random ionosphere, J. Res. NBS **65D** (Radio Prop.), No 3, 275–292.

Bowhill, S. A. (1961b), The scattering of radio waves by an extended randomly refracting medium, J. Atmos. Terrest. Phys. **20,** 9–18.

Brekhovskikh, L. M. (1960), Waves in layered media, translated by David Lieberman (Academic Press, New York).

Bremmer, H. (1963), Scattering by a perturbed continuum, URSI-Copenhagen Symposium on Electromagnetic Theory and Antennas (1962, 665–699, Pergamon Press, Oxford).

Briggs, B. H. (1961), Diffraction by an irregular screen of limited extent, Proc. Phys. Soc. **77,** 305–317.

Budden, K. G. (1959), Effect of small irregularities on the constitutive relations for the ionosphere, J. Res. NBS **63D** (Radio Prop.), No. 2, 135–149.

Bugnolo, D. S. (1960a), Transport equation for the spectral density of a multiple scattered electromagnetic field, J. Appl. Phys. **31,** 1176–1182.

Bugnolo, D. S. (1960b), On the question of multiple scattering in the troposphere. J. Geophys. Res. **65,** 879–884.

Bugnolo, D. S. (1961a), Radio star scintillation and multiple scattering in the ionosphere, IRE Trans. Ant. Prop. **AP–9,** 89–96.

Bugnolo, D. S. (1961b), Stochastic processes and beyond-the-horizon propagation, IRE Trans. Ant. Prop. **AP–9,** 226–228.

Burke, J. E., and V. Twersky (1960), On scattering of waves by an elliptic cylinder and by a semi-elliptical proturberance on a ground plane, Report EDL–M266, Sylvania Electronic Defense Laboratories.

Burke, J. E., and V. Twersky (1963), On scattering and reflection of waves by elliptically striated surfaces, Report EDL–E66, Sylvania Electronic Defense Laboratories.

Butler, W. L. (1962), Absorption of light by turbid materials, J. Opt. Soc. Am. **52,** 292–299.

Catalan, L. A. (1962), Some computed optical properties of antireflection coatings, J. Opt. Soc. Am. **52,** 437–440.

Chen, K. M., and R. W. P. King (1961), Dipole antennas coupled electrically to a two wire transmission line, IRE Trans. **AP–9,** 425–432.

Chernov, L. A. (1960), Wave propagation in a random

medium, translated by R. A. Silverman (McGraw-Hill Book Co. Inc., New York, N.Y.).
Chu, C. M., S. W. Churchill, and S. C. Pang (1962), A variable-order diffusion-type approximation for multiple scattering, Interdisciplinary Conference on Electromagnetic Scattering.
Clay, C. S. (1960), Fluctuations of sound reflected from the sea surface, J. Acoust. Soc. Am. **32,** 1547–1551.
Cole, G. H. A. (1961), Classical theory of the equilibrium liquid pair distribution part of the book Advances in Physics. **VII,** 224–251.
Collins, R. E. (1960), Scattering by an infinite array of thin dielectric sheets, IRE Trans. **AP–8,** 62–67.
Collins, R. E., and W. H. Eggimann (1961), Dynamic interaction fields in a two-dimensional lattice, IRE Trans. **MTT–9,** 110–115.
Collins, W. D. (1961), On the solution of some axisymmetric boundary value problems by means of integral equations. V. Some scalar diffraction problems for circular disks, Quart. J. Mech. Appl. Math. **14,** 101–117.
Davies, H. (1954), The reflection of electromagnetic waves from a rough surface, Proc. Inst. Elec. Engrs. **101,** 209.
Eby, R. E., A. O. Williams, R. P. Ryan, and P. Tamarkin (1960), Study of acoustic propagation in a two-layered model, J. Acoust. Soc. Am. **32,** 88–99.
Eyges, L. (1957), Some nonseparable boundary value problems and the many-body problem, Annals of Physics **2,** 101–128.
Eyges, L. (1958), Solutions of Schrödinger equation for a particle bound to more than one spherical potential, Phys. Rev. **111,** 683–689.
Ferraro, A. J. (1962), Multislab concept applied to radio-wave propagation in the ionosphere and its limit to the continuous ionosphere, J. Geo. Res. **67,** 3817–3821.
Ford, G. W., and W. C. Meecham (1960), Scattering of sound by isotropic turbulence of large Reynolds number, J. Acoust. Soc. Am. **32,** 1668–1672.
Garrison, G. R., S. R. Murphy, and D. S. Potter (1960), Measurements of the back-scattering of underwater sound from the sea surface, J. Acoust. Soc. Am. **32,** 104–111.
Grosjean, C. C. (1962), Recent progress in the development of a new approximate theory of multiple scattering, Interdisciplinary Conference on Electromagnetic Scattering.
Hagfors, Tor (1961), Some properties of radio waves reflected from the moon and their relation to the lunar surface, J. Geo. Res. **66,** 777–785.
Hansen, E. B. (1959), The diffraction of a plane wave through two or more slits in a plane screen, Appl. Sci. Res. Sec. **B8,** 73–83.
Heavans, O. S. (1960), Optical properties of thin films, Reports on Progress in Physics, **XXIII,** 1–65.
Hessemer, R. A. (1961), Scatter communications with radar chaff, IRE Trans. Ant. Prop. **AP–9,** 211–217.
Hiatt, R. E., T. B. A. Senior, and V. H. Weston (1960), A study of surface roughness and its effect on the backscattering cross section of spheres, Proc. IRE **48,** 2008–2016.
Hoffman, W. C. (Editor) (1960), Statistical methods in radio wave propagation, (Pergamon Press, N.Y.).
Huang, K., and C. N. Yang (1957), Quantum mechanical many-body problem with hard sphere interaction, Phys. Rev. **105,** 767–775.
Ingard, V. (1960), On the reflection of sound from a grating, J. Acoust. Soc. Am. **33,** 812–813.
Ishimaru, A. (1962), Theory of unequally-spaced arrays, IRE Trans. Ant. Prop. **AP–10,** 691–702.
Isihara, A. (1961), Theory of dielectric polarization in non-polar fluids, Physica **27,** 857–871.
Karp, S. N. (1961), A convergent "Farfield" expansion for two-dimensional radiation problems, Report EM–169, Inst. of Math. Sci., New York University.
Karp, S. N., and N. R. Zitron (1961a), Higher-order approximations in multiple scattering; I. Two-dimensional case, J. Math. Phys. **2,** 394–402.
Karp, S. N., and N. R. Zitron (1961b), Higher order approximations in multiple scattering; II. Three-dimensional case, J. Math. Phys. **2,** 402–406.
Kay, I., and R. A. Silverman (1958), Multiple scattering by a random stack of dielectric slabs, Report EM–113, Inst. of Math. Sci., New York University (1958); Nuovo Cimento, **9,** 626–645.
Keller, J. B. (1960), Wave propagation in random media, Research Report No. EM–164, Inst. Math. Sci., New York University.
Kerns, D. M. (1960), Half-round inductive obstacles in rectangular wave guides, J. Res. NBS **64B,** 113–130.
Kieburtz, R. B., A. Ishimaru, and G. Held (1961), Diffraction by a periodically apertured conducting screen, Dept. of Electrical Engineering, Univ. of Washington, Seattle, Wash., Tech. Report No. 48.
Kieburtz, R. B. (1961), Diffraction by a periodically apertured conducting screen, Ph. D. Dissertation, Univ. of Washington, Seattle, Wash.
Kieburtz, R. B., and A. Ishimaru (1961), Scattering by a periodically apertured conducting screen, IRE Trans. Ant. Prop. **AP–9,** 506–514.
Kieburtz, R. B., and A. Ishimaru (1962), Aperture fields of an array of rectangular apertures, IRE Trans. Ant. Prop. **AP–10,** 663–671.
Kielich, S. (1962), Molecular refraction of dense mixtures, Physica **28,** 1116–1123.
Kohn, W., and N. Rostoker (1954), Solution of the Schrödinger equation in periodic lattices with an application to metalic lithium, Phys. Rev. **94,** 1111–1120.
Korringa, J. (1947), On the calculation of the energy of a Bloch wave in a metal, Physica **13,** 392–400.
Kottler, F. (1960), Turbid media with plane-parallel surfaces. J. Opt. Soc. Am. **50,** 483–490.
Kumagai, N., and D. J. Angelakos (1960), Electromagnetic back scattering from multiple spheres, Institute of Engineering Research Report No. 116, University of California, Berkeley.
LaCasce, E. D., Jr. (1961), Some notes on the reflection of sound from a rigid corrugated surface, J. Acoust. Soc. Am. **33,** 1772–1777.
LaCasce, E. D., Jr., B. D. McCombe, and R. L. Thomas (1961), Measurements of sound reflection from a rigid corrugated surface, J. Acoust. Soc. Am. **33,** 1768–1771.
Larsen, Tove (1962a), A survey of the theory of wire grids, IRE Trans. **MTT–10,** 191–201.
Larsen, Tove (1962b), Numerical investigation of the equivalent impedance of a wire grid parallel to the interface between two media, J. Res. NBS **66D** (Radio Prop.), No. 1, 7–14.
Longuet-Higgins, M. S. (1960a), Reflection and refraction at a random moving surface. I. Pattern and paths of specular points, J. Opt. Soc. Am. **50,** 838–844.
Longuet-Higgins, M. S. (1960b), Reflection and refraction at a random moving surface. II. Number of specular points in a Gaussian surface, J. Opt. Soc. Am. **50,** 845–850.
Longuet-Higgins, M. S. (1960c), Reflection and refraction at a random moving surface. III. Frequency of twinkling in a Gaussian surface, J. Opt. Soc. Am. **50,** 851–856.
Mandel, M. (1960), The dielectric constant and Maxwell-Wagner dispersion of suspensions of oriented prolate spheroids, Physica **27,** 827–840.
Marsh, H. W. (1961), Exact solution of wave scattering by irregular surfaces, J. Acoust. Soc. Am. **33,** 330–333.
Marsh, H. W., M. Schulkin, and S. C. Kneale (1961), Scattering of underwater sound by the sea surface, J. Acoust. Soc. Am. **33,** 334–340.
Melzak, E. A. (1962), Scattering from random linear arrays with closest approach, Q. Appl. Math. **20,** 151–159.
Mendlowitz, H., and J. A. Simpson (1962), On the Theory of Diffraction Grating Interferometers, J. Opt. Soc. Am. **52,** 520–524.
Mercier, R. P. (1961), Diffraction by finite irregular objects, Proc. Phys. Soc. **77,** 318–327.
Millar, R. F., (1961a), Scattering by a grating I, Can. J. Phys. **39,** 81–103.
Millar, R. F. (1961b), Scattering by a grating II, Can. J. Phys. **39,** 104–118.
Millar, R. F. (1961c), Scattering by semi-infinite gratings, URSI Spring Meeting Washington, D.C.
Millar, R. F. (1962), Plane wave spectra in multiple scattering theory, URSI-Copenhagen Symposium.
Mullikin, T. W. (1962), Uniqueness problems in the mathe-

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matics of multiple scattering, Interdisciplinary Conference on Electromagnetic Scattering.
Musgrave, M. J. P. (1959), The propagation of elastic waves in crystals and other anisotropic media, Reports in Progress in Physics **XXII,** 74–96.
Namioka, T. (1961), Theory of the ellipsoidal concave grating I, II, J. Opt. Soc. Am. **51,** 4–14.
O'Konski, C. T. (1960), Electric properties of macromolecules. V. Theory of ionic polarization in polyelectrolytes, J. Phys. Chem. **64,** 605–619.
Parrent, George B., Jr. (1962), Robert A. Shore, and Thomas J. Skinner, On the mutual coherence function in an inhomogeneous medium, J. Math. Phys. **3,** 678–699.
Redheffer, R. (1961), Difference equations and functional equations in transmission-line theory, part of the book Modern Mathematics for the Engineer, 282–337 second series Ed. by E. F. Beckenbach (McGraw-Hill Book Co., Inc., New York, N.Y.).
Redheffer, R. (1962), On the relation of the transmission-line theory to scattering and transfer, J. Math. and Phys. **4,** 1–41.
Richmond, J. H. (1962), Transmission through inhomogeneous plane layers, IRE Trans. **AP–10,** 300–305.
Rodberg, L. S. (1957), The many-body problem and the Brueckner approximation, Annals of Physics **2,** 199–225.
Saermark, K. (1959a), Scattering of a plane monochromatic wave by a system of strips, Appl. Sci. Res. **B7,** 417–440.
Saermark, K. (1959b), Transmission coefficient for a system of parallel slits in a thin plane screen, Appl. Sci. Res. **B8,** 29–34.
Saermark, K. (1959c), Acoustic forces and torques on a system of strips, Appl. Sci. Res. **B8,** 13–28.
Saermark, K. (1960), On some two-dimensional diffraction problems, Dansk Videnskobs Forlag, Copenhagen.
Segall, B. (1957), Calculation of the band structure of "complex" crystals, Phys. Rev. **105,** 108–115.
Sekera, Z. (1962), Multiple scattering in media with anisotropic scattering, Interdisciplinary Conference on Electromagnetic Scattering.
Senior, T. B. A. (1960), Impedance boundary conditions in statistically rough surfaces, Appl. Sci. Res. **B8,** 437–462.
Schick, L. H. (1961), Multiple scattering analysis of the three-body scattering problem, Revs. Mod. Phys. **33,** 608–622.
Siegel, K. M., and T. B. A. Senior (1962), A lunar theory reasserted, J. Res. NBS **66D** (Radio Prop.), No. 3, 227–229.
Silver, S. (1962), Delineations and methodology in the subject of scattering by a statistically inhomogeneous medium, URSI–Copenhagen Symposium on Electromagnetic Theory and Antennas.
Skinner, Thomas J. (1961), Ergodic theorem in the solution of the scalar wave equation with statistical boundary conditions, J. Opt. Soc. Am. **51,** 1246–1251.
Stroke, G. W. (1960), Theoretical and experimental study on two aspects of diffraction by optical gratings, Revue d' Optique **39,** 291–396.
Stroke, G. W. (1961), Attainment of high resolution gratings by ruling under interferometric control, J. Opt. Soc. Am. **51,** 1321–1339.
Tatarski, V. I. (1961), Wave propagation in turbulent media, translated by R. S. Silverman (McGraw-Hill Book Co., Inc., New York, N.Y.)
Twersky, V. (1960a), On multiple scattering of waves, J. Res. NBS **64D** (Radio Prop.), No. 6, 715–730.
Twersky, V. (1960b), Electromagnetic waves, Physics Today **13,** 30–36.
Twersky, V. (1960c), Scattering processes in propagation, Report EDL–L17, Sylvania Electronic Defense Laboratories, 13th General Assembly of URSI, 1960.
Twersky, V. (1961), Elementary function representations of Schlömilch series, Archive for Rational Mechanics and Analysis **8,** 323–332.
Twersky, V. (1962a), On scattering of waves by two objects, part of the book Electromagnetic Waves, Ed. R. E. Langer, University of Wisconsin Press, 361–389.
Twersky, V. (1962b), Multiple scattering by arbitrary configurations in three dimensions, J. Math. Phys. **3,** 83–91.
Twersky, V. (1962c), Multiple scattering of waves and optical phenomena, J. Opt. Soc. Am. **52,** 145–171.
Twersky, V. (1962d), On scattering of waves by random distributions; I, free-space scatterer formalism, J. Math. Phys. **3,** 700–715.
Twersky, V. (1962e), On a general class of scattering problems, J. Math. Phys. **3,** 716–723.
Twersky, V. (1962f), On scattering of waves by random distributions; II, two-space scatterer formalism, J. Math. Phys. **3,** 724–734.
Twersky, V. (1962g), On scattering of waves by the infinite grating of circular cylinders, IRE Trans. Ant. Prop. **AP–10,** 737–765.
Twersky, V. (1962h), Scattering by random media, Report EDL–L21, Sylvania Electronic Defense Laboratories, URSI-Copenhagen Symposium on Electromagnetic Theory and Antennas 1962.
Twersky, V. (1962i), Scattering of waves by multi-component two-phase systems, Report EDL–E73, Sylvania Electronic Defense Laboratories, Interdisciplinary Conference on Electromagnetic Scattering 1962.
Twersky, V. (1962j), Signals, scatterers, and statistics, Report EDL–E72, Sylvania Electronic Defense Laboratories (1962j); to be published in the IRE Trans. Ant. Prop.
Twersky, V. (1963), On propagation in random media of discrete scatterers, Report EDL–L24; American Mathematical Society Symposium on Stochastic Processes in Mathematical Physics and Engineering.
Wait, J. R. (1962), Effective impedance of a wire grid parallel to the earth's surface, IRE Trans. Ant. Prop. (1961), **AP–10,** 538–542.
Wait, J. R., and A. M. Conda (1961), Resonance characteristics of a corrugated cylinder excited by a magnetic dipole, IRE Trans. Ant. Prop. **AP–9,** 330–333.
Waterman, P. C., and R. Truell (1961), Multiple scattering of waves, J. Math. Phys. **2,** 512–537.
Wing, G. M. (1962), Mathematical aspects of the problem of acoustic waves in a plane stratified medium, Q. Appl. Math. **19,** 309–319.
Winter, D. F. (1962), A theory of radar reflection from a rough moon, J. Res. NBS **66D** (Radio Prop.), No. 3, 215–226.
Wu, Tai-Tsun (1961), Theory of the dipole antenna and the two wire transmission line, J. Math. Phys. **2,** 550–574.
Yamada, R. (1961), Reflection of electromagnetic waves from a stratified inhomogeneity, IRE Trans. Amt. Prop. **AP–9,** 364–370.
Yeh, K. C. (1962), Propagation of spherical waves through an ionosphere containing anisotropic irregularities, J. Res. NBS **66D** (Radio Prop.), No. 5, 621–636.
Yen, J. L. (1962), Multiple scattering and wave propagation in periodic structures, IRE Trans. Ant. Prop. **AP–10,** 769–775.
Young, L. (1962), Prediction of absorption loss in multilayer interference filters, J. Opt. Soc. Am. **52,** 753–761.
Zitron, N. R. (1962), Multiple scattering of plane waves by two elliptic cylinders, report 316 of the Math. Res. Center, University of Wisconsin.

4.3. Passive Communications Satellites, Review—1960–1962

J. Russell Burke [1]

In the period from January 1, 1960 to January 1, 1963 Echo I, the first successful communications satellite, was launched on August 12, 1960, and successful communications experiments were conducted using the satellite. In addition, two other attempts were made to orbit passive communications satellites without resulting in successful communications experiments. Of these, one was the first Echo launch on May 13, 1960 in which orbit was not attained. The other was the West Ford launch of 1961.

The objectives of Project Echo were to demonstrate the feasibility of using large, inflatable spheres as communications reflectors, and to study the behavior of large, lightweight erectable structures in the space environment. Echo I is described in detail in Jaffe [1960]. Figure 1 shows the Echo structure inflated on the ground. The satellite was made of Mylar 0.0005 in. thick coated with vapor-deposited aluminum to provide radio reflectivity. The 100-ft. diam sphere weighed 136 lb. and was folded into a 26-in. diam container for launching. Two sublimating powders were sifted into the satellite (20 lb. of anthroquinone and 10 lb. of benzoic acid) to cause inflation of the satellite in the vacuum of space. Five free space ballistic tests of the balloon and inflation system were made using a solid fuel booster shown in figure 2 prior to orbital launch. These tests developed the packaging, ejection, and inflation techniques which resulted in the successful deployment of Echo I in orbit. The orbit achieved by Echo I was circular, 900 nautical miles in altitude, inclined 47° to the equator. Communications experiments were carried out via Echo I between ground stations at Holmdel, N.J., Goldstone, Calif., and Stump Neck, Md., and by independent experimenters located elsewhere in the United States and also in Europe. These experiments and their results have been described in detail [Project Echo, 1961; Beamer, 1961; Stevens and Victor, 1960]. In brief, two-way telephone transmissions of excellent quality were made via the satellite between the east and west coast of the United States. Voice and music were transmitted from Holmdel to Jodrell Bank, in England. Facsimile transmission via the satellite also took place. Many modulation techniques were employed.

No deviations from propagation theory were observed, and returned signal strength from Echo I was within 1 db of theoretical during the pressurized life of the satellite (14 days).

In addition to the communications experiments, Echo I has contributed significantly to understanding of the space environment and the causes of orbital perturbations; because of its large size and small relative weight, Echo I is very sensitive to the effect of atmospheric density and solar radiation pressure. Jaffe [1961]; Jastrow, and Bryant [1960] summarizes results of the Echo I experiments in this regard.

[1] National Aeronautics and Space Administration, Washington, D.C., 20546.

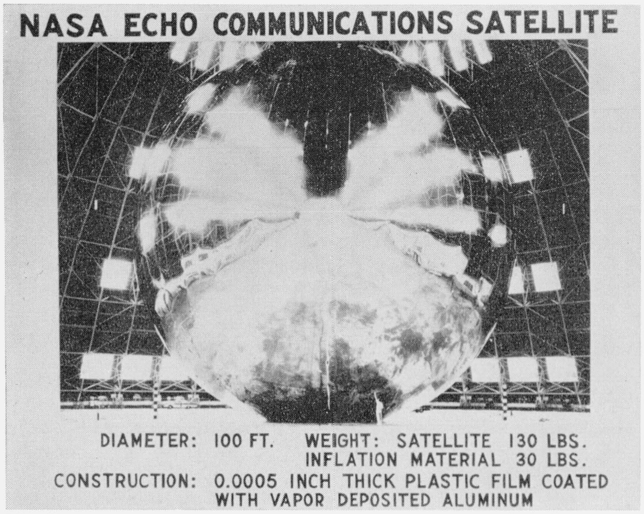

FIGURE 1.

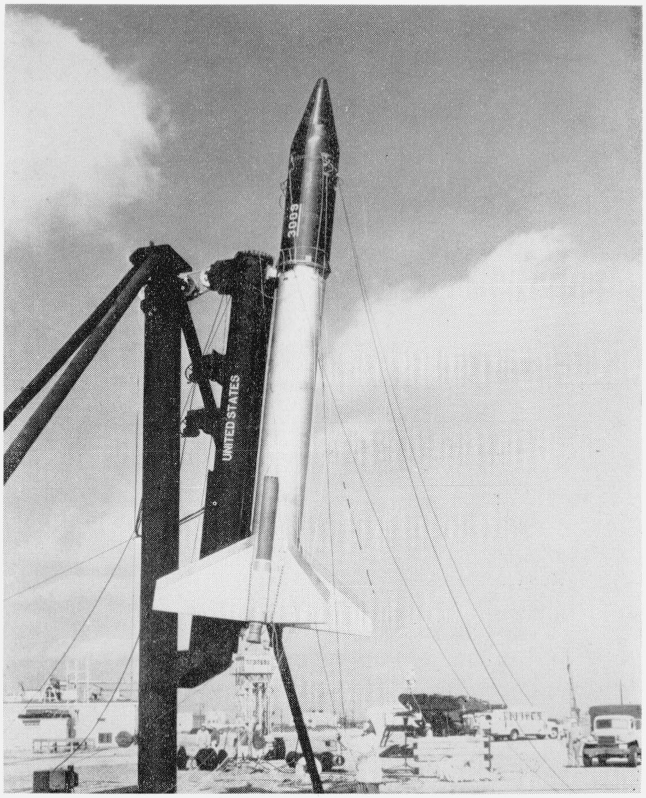

FIGURE 2.

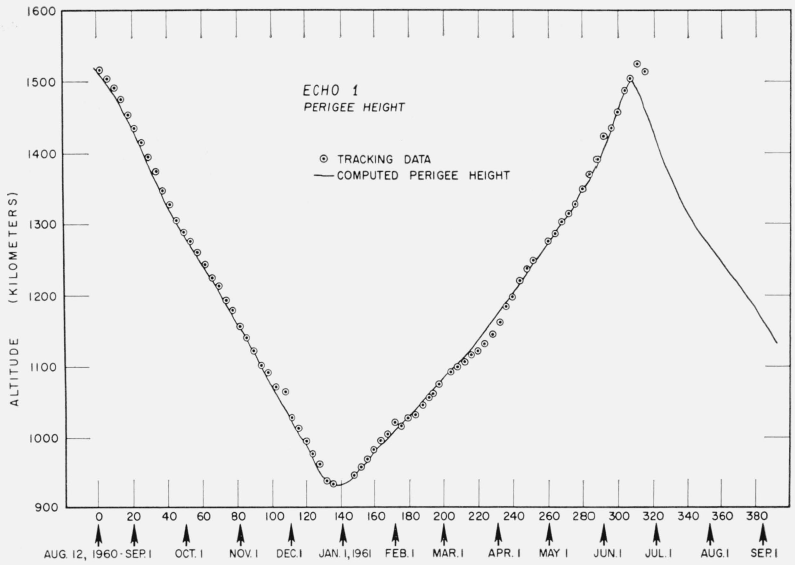

FIGURE 3.

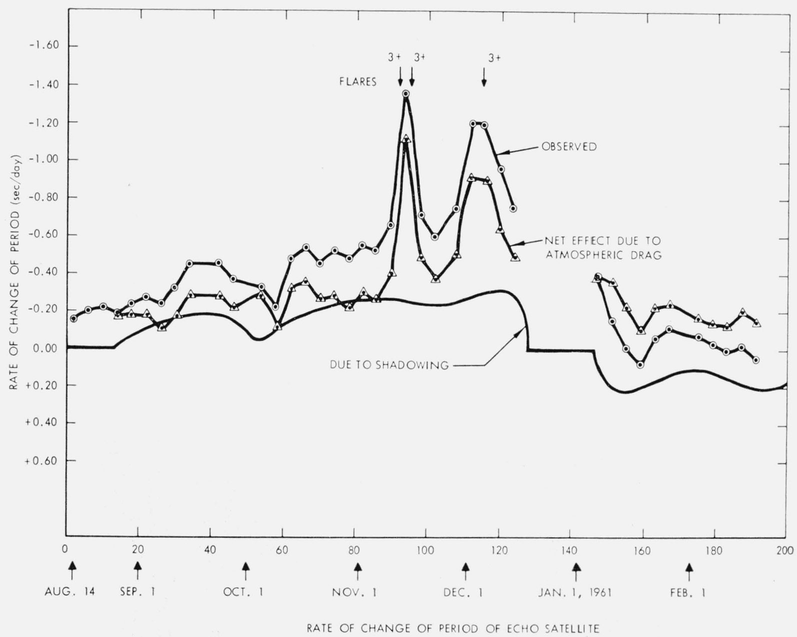

FIGURE 4.

Figures 3 and 4 respectively show the periodic variation of Echo I perigee height with time, due largely to solar pressure, during its first year in orbit, and the variation of period with time from which atmospheric density data have been derived.

Echo I is still in orbit, and although radar observations indicate a reduction in cross section of 3 to 6 db, as recently as May 1962, a television signal was transmitted from Massachusetts to California via the satellite by Lincoln Laboratory of MIT.

In Project West Ford, the objective is to orbit a belt of half-wave dipoles; in this case a volume of the belt illuminated by the ground transmitter would serve as the signal source for the ground receiver, rather than a single large scatterer such as Echo I. Details of the project are contained in [Morrow; 1960].

Echo I, although an excellent isotropic scatterer when pressurized, began to display scintillations after all of the pressurizing gas had escaped. These became larger with time and it has been deduced that either the structure is insufficiently rigid to resist the environmental forces or that the pattern reflecting the satellite's previous folded condition is gradually becoming enhanced by the plastic memory of the mylar. As a result, NASA has entered a

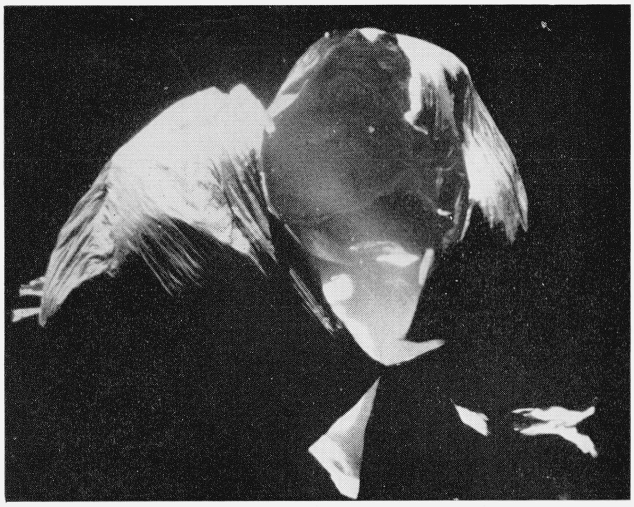

Figure 5.

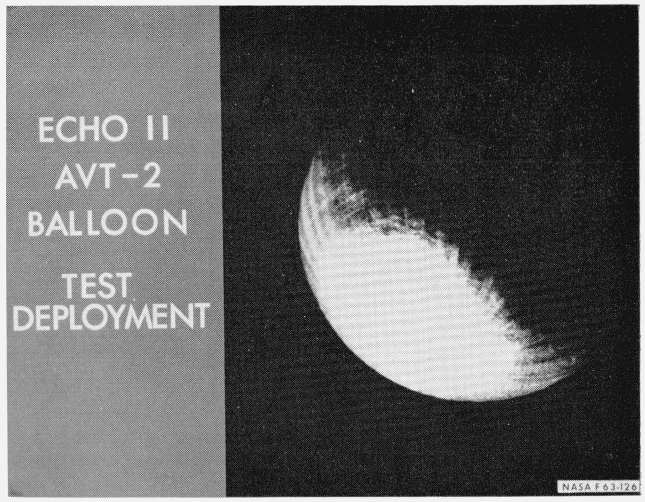

Figure 6.

second flight program for orbiting spherical passive satellites. In this one, Echo II, the satellite will be 135 ft in diameter (3 db larger in radar cross section than Echo I) and will be constructed of an aluminum-mylar-aluminum laminate which, although only about 50 percent thicker than the Echo I material will be almost one hundred times stiffer, resulting in a sphere sufficiently rigid to resist both the environment and the mylar's plastic memory. The material is twice as heavy as the Echo I material; as a result Echo II will be approximately four times as heavy as Echo I. A detailed description of Echo II is contained in Jaffe [1961]. The Echo II orbital launch is scheduled for 1963; during 1962 two suborbital launches were made to test the ejection, inflation and deployment systems. Because of the heavier weight of the structures, and because it was decided to photograph and televise the ejection and deployment from the booster, a launch vehicle of greater capability was required than for the Echo I suborbital tests, and a modified Thor DM–21 was employed. This gave a maximum altitude of 1500 km and a zero g, space vacuum condition for 20 min. The first test was conducted on January 15 and the deployment was unsuccessful due to excessive accelerations during the unfolding process. In the second test on July 18, a subliming material of reduced vapor pressure was employed (benzoic acid; acetamide had been used for the first test). Ejection, inflation and deployment were successful; however, radar observations showed scintillations as large as ± 12db for C-band which indicated that the final pressure reached was inadequate to produce an unwrinkled surface. Figures 5 and 6 show respectively the results of the first and second suborbital tests a few seconds after the inflation process commenced. A controlled inflation system is being developed in which the pressure source used for deployment is not that used for final stressing of the skin and hence deployment can take place slowly, the higher pressures being applied only after the sphere is fully deployed.

Although there has been only one successful communications satellite experiment during the period reviewed, the general area of passive communications satellites has been the source of many papers. Some of these present methods for comparing passive with active satellites [Campbell and Pollack, 1960]. However the area having the most interest has been demonstrated in methods for increasing the radio cross section of passive communications satellites without significant penalty in weight or size. Techniques for accomplishing this can be assigned to several categories. The first category involves some kind of predetermined orientation. An example of this is a spherical segment, vertically oriented which can be designed to have a modest gain, and still accept transmissions from all visible ground stations [Gillespie, 1960]. The second category involves satellites which do not require orientation. In one case, a rather modest gain can be obtained from a sphere if the surface is diffuse instead of specular, provided that the included angle between ground transmitter and receiver does not exceed 83.7° [Raabe, 1961]. Other cases include spheres studded with Luneberg lenses, corner reflectors, lens-reflector combinations, etc. [Stahler, and Johnson, 1961; Ryerson, 1960] or represent spheres which are themselves large Luneberg lenses which may be defocused to control angular coverage [Siegel et al., 1962]. Another case is the use of arrays such as the Van Atta array which are relatively insensitive to orientation, to reradiate received energy [Hansen, 1961]. Another category is represented by proposals to increase the size to weight ratio of spherical structures, and decrease the response to solar pressure through the use of openwork or mesh materials rather than a continuous web of conducting material [Bradley, 1962; Jastrow 1960].

References

Breamer, Clifford M. (May 1961), An experiment in radio propagation and communication using the passive reflecting satellite, Echo I, paper presented at the URSI spring meeting, Washington, D.C.

Bradley, W. E. (May, 1962), Orbiting passive electromagnetic reflectors at the spring meeting, URSI, Washington, D.C.

Campbell, D., and L. Pollack (1960), Active versus passive satellites for a multi-station network, IRE Convention Record, Part 5.

Gillespie, Warren, Jr. (Jan. 18–21, 1960), An earth-oriented communications satellite of the passive type, paper presented at the 6th National Meeting of the American Astronautical Society, New York, N.Y.

Hansen, R. C. (1961), Communications satellites using arrays, Proc. IRE **49,** 1061–1074.

Jaffe, J. (1960), Project "Echo", paper presented at International Telecommunications meeting, Mexico City, Mexico, April 19–May 6.

Jaffe, L. (May 1961a), Project Echo results, Astronautics.

Jaffe, L. (1961b), The NASA Communications Satellite Program, paper presented at the URSI Symposium of Communications Satellites, Paris, Sept. 18–22.

Jastrow, R., and R. W. Bryant (Oct. 1960), Variation in the orbit of the Echo satellite, J. Geophy. Res. **65,** No. 10, 3512–3513.

Morrow, W. E., Jr. (1960), Orbital scatter communications, paper presented at the XIIIth General Assembly of the URSI, London, Sept. 5–15.

Project Echo (July 1961), Complete issue devoted to Echo participation of Bell Telephone Laboratories, Bell System Tech. J. **XL,** No. 4, 975–1238.

Raabe, Herbert P. (May 22, 1961), Passive communication satellites with diffuse scattering characteristics, paper presented at the Fifth National Symposium on Global Communications, Chicago, Ill.

Rottmayer, E. (May 15, 1962), Wire film applications in space antennas, paper presented at National Aerospace Electronics Conference, Dayton, Ohio.

Ryerson, J. L. (April 1960), Passive satellite communications, Proc. IRE **48,** 613–619.

Siegel, K. M., R. Graham, and A. Wren (1962), Optimum spherically symmetric corner reflectors, paper presented at the Spring URSI meeting, Washington, D.C. April 30–May 3.

Stahler, Y. E., and A. L. Johnson (1961), Directive non-oriented reflectors as passive satellites in long distance communications, paper presented at National IAS–Ars Joint Meeting, Los Angeles, Calif., June 13–16.

Stevens, R., and W. K. Victor (Dec. 1960), The Goldstone Station Communications and Tracking System for Project Echo, Techn. Rept. No. 32–59, JPL California Institute of Technology, Pasadena.

4.4. Passive and Active Reflectors

J. Kaiser and I. Kay

In recent years considerable interest has been centered on reflectors in space for use as long distance communication relays between ground stations.

One of the earliest papers on this subject was by A. C. Clarke [1945] who in 1945 described a global communications system based on three uniformly spaced equatorial 24-hr orbit communication satellites.

Since that time the relative merits of passive versus active reflectors in space have been frequently discussed in the literature. Active reflectors, generally use active electronic elements and some form of electric power sources to amplify and repeat incoming signals whereas passive reflectors have no active elements or power sources.

As pointed out by a number of writers [Bartow et al., 1959; Bond et al., 1960; Jacoby, 1960; Kaiser, and Raisbeck, 1961; Pierce, 1959], the ground stations for communications systems based on active repeaters require generally much less transmitted power and antenna gain than those based on passive repeaters of equal weight, orbit altitude and communication capacity.

Active reflectors have been proposed for orbit altitudes from 1000 statute miles (s.m.) to 23,000 s.m., the choice of the particular altitude being governed by booster capabilities, communication system coverage, circuit availability, system economics, and applications. Passive reflectors for practical systems would generally be placed into the lower altitude orbits, unless they can be made extremely large [Bradley, 1961].

An interesting case is the moon, which is being used by the U.S. Navy for a 16-channel teletype communications system between Hawaii and Washington, D.C. The effective scattering cross section of the moon is found to be $9\times10^{11}\text{m}^2$, corresponding to a perfectly conducting sphere 670 miles in diameter [Jakes, 1961].

Experimental communication systems based on active and passive reflectors in space have been tested. Telstar and Relay are examples of active reflectors or repeaters. The moon and the Echo I balloon are examples of passive reflectors. The experimental performance of the Echo I balloon agreed with the predictions for a perfectly reflecting sphere of the same (100 ft) diameter.

Of the active reflector designs which have been considered, probably the most novel is due to Van Atta [Sharp, 1958; Sharpe et al., 1960; Van Atta, 1959]. The Van Atta array consists of antenna elements interconnected by transmission lines whose electrical lengths are so chosen that the phase distribution of a plane wave incident at an arbitrary angle will automatically recombine to generate a transmitted plane wave in the opposite direction. Modifications of the Van Atta array were considered by Bahret [1961; Bauer, 1961; Waneslow, 1962, and Rutz and Kramer, 1962]. An evaluation of the Van Atta array for communications was made by Hansen [1961] who concluded that its usefulness would be for long distance point-to-point transmission.

A number of workers have analyzed the possibility of enhancement of the signal return from passive reflectors in orbit. J. R. Pierce and C. C. Cutler [1959] list several forms of passive reflectors which may be useful for satellite radio relay, including balloons, saucers, corner reflectors, clustered polyhedrons, and other shapes. (See also M. G. Chatelain [1960].)

The prototype of the corner reflector, whose cross section is characterized by a pronounced dependence on wavelength, is the flat plate. At the other extreme, the perfectly reflecting sphere has been regarded as a standard for wide-angle reflectors since in the optical limit the sphere is an isotropic scatterer. Wide-angle reflectors such as the perfectly reflecting sphere are characterized by cross sections which are independent of wavelength when the wavelength is sufficiently short.

Passive reflectors having a relatively wide angle of return, whose cross section does, however, depend on frequency have been discussed by W. E. Morrow [1960, 1961]. A large number of dipoles randomly oriented in orbit would present a maximum realizable average scattering cross section $A_s=0.16\lambda^2$ where λ=wavelength.

Although the corner reflector does return a large amount of power in one direction, it would be necessary to provide attitude control in order to use it as a relay because of its highly directional reflection characteristics. On the other hand, while the perfectly reflecting sphere provides wide coverage its cross section is considered to be disappointingly small for its practical use as a satellite relay.

The attempts to surmount these difficulties also fall into two general categories. One attempt involves the use of a large number of corner reflectors either randomly placed or attached symmetrically to a large sphere so that the attitude control is no longer necessary [Bradley, 1961; Cutler, 1962; Harrington, 1962; Herriott, 1959; Morrow, 1961; Sinclair, 1959; Stahler and Johnson, 1960]. The other is to use some form of dielectric lens which orients the reflected rays in suitable directions [Bobey, 1959; Graham et al., 1961; Kay, 1959; Keyes, 1959] order to avoid the problem of attitude control in the latter type of reflector the lens is usually required to be spherically symmetric. It can be shown that there is a maximum cross section which can be obtained over a given angle for any spherically symmetric passive reflector. Thus the coverage requirement of a system must necessarily limit the return from such a reflector of a specified diameter.

Y. E. Stahler and A. L. Johnson [1960] have analyzed and experimented with reflectors having increased reflectivity. They have concluded that considerable improvement in directivity, above 13 db, can be obtained by placing reflecting elements or lenses combined with reflecting surfaces on spherical satellites. In the 1962 spring URSI meeting, R. Graham, K. M. Siegel, et al., [1961] discuss the optimum parameters for a spherically symmetric corner reflector. They obtained the best results for a solid spherically symmetric reflector, which is a Luneberg lens with a uniform coating of power reflection coefficient approximately equal to one-third and a power transmission coefficient of two-thirds. Such a lens was actually constructed and an experiment performed during the meeting to demonstrate the result. They estimated that theoretically for a lens the size of the Echo balloon the backscattering cross section at X-band is 60 db above that from Echo.

C. C. Cutler [1962] discusses some of the fundamental limitations on the possibility of obtaining useful directivity from a nonoriented, passive reflector in orbit. He derives a formula for the net advantage of a reflectivity enhanced scatterer such as a sphere covered with small reflecting elements over the reflectivity of a smooth sphere. The advantage is equal to $4\left(\frac{\sin\phi}{\theta}\right)^2$ where 2ϕ is the apex angle of a cone within the sphere which reflects the impinging waves and θ is the half-power beamwidth of the reflected beam. He shows by an example that for a satellite height of 4,000 miles and ground-station separation of 4,000 miles the gain is less than unity and at a height of 22,300 miles the gain is 8.5 db. He concludes that for the amount of gain obtained over that of the smooth sphere, the problems encountered with increased directivity scatters will probably negate their usefulness.

J. Kaiser and G. Raisbeck [1961] analyzed and evaluated passive communication satellite systems. These authors state that an important parameter for the evaluation of passive reflectors for communications is the effective area of the reflector which determines the power received from the ground transmitting station. It is noted that extremely large areas are required to receive sufficient power to compete favorably with active communication satellites having a radiated power of 1 w. W. L. Bradley [1961] discusses orbiting passive electromagnetic reflectors constructed of fine metallic wire mesh, which, if engineering problems of unfolding and erection could be solved, would allow large scattering cross section reflectors having relatively small weight.

A general conclusion which can be drawn from the various studies made on active and passive reflectors is that at the present time active reflectors seem to be the preferred relays for most communication systems, while at the same time there do exist special situations for which passive reflectors are probably more efficient.

Although the most dramatic application of passive and active reflectors seems at present to be in the field of long-distance communication systems, this report would not be complete without some mention of the many general purpose studies which have been made of radar enhancement devices. Most of these studies have been concerned with passive reflectors, and some very interesting results have been obtained. For example, D. Atlas has observed that a homogeneous dielectric sphere having a diameter less than 6.4 times the wavelength will, if it is capped at its rear pole by a small piece of metal, have a backscattering cross section in excess of that produced by a Luneberg lens reflector of the same diameter [Atlas, 1962].

Methods for designing reflectors having desired reflection properties were studied by S. T. Bobey [1959] L. Peters [1959a, 1959b] and Kennaugh and Bauemler [1958] of the Ohio State University Research Foundation. A synthesis procedure for inhomogeneous spherically symmetric radar enhancement lenses was prescribed by A. F. Kay [1959] who made a general analysis of this type of reflector.

Twersky [1961] discussed coherent and incoherent scattering by orbital distributions of scatterers. He derived mathematical expressions for scatterers distributed in regions of space bounded by various surfaces and illustrated the results with an example from geometrical optics.

In addition to such efforts many cross-section calculations and measurements were made for dielectric and metallic reflectors for particular shapes by other investigators. Some papers which describe work of this type are included in the reference list below although no specific mention of them has been made elsewhere in this report.

References

Atlas, D. (1962), Capped dielectric spheres as radar targets, ICRS, Clarkson College of Technology, Potsdam, New York, Aug. 13–15.

Bahret, W. P. (1961), Technique for amplitude modulating a Van Atta radar reflector, Proc. IRE **49**, p. 1692.

Bauer, L. H. (1961), Technique for amplitude modulating a Van Atta radar reflector, Proc. IRE **49**, p. 634.

Bartow, J. E., G. N. Krassner, R. C. Riehs (1959), Design considerations for space communications, IRE Nat. Convention Record, pt. 8, 154–166.

Bobey, S. T. (Aug. 15, 1959), The Luneberg reflector for bistatic radar enhancement, The Ohio State University Res. Found., Rept. 768–8, AF 33(616)–5078.

Bond, F. E., C. R. Cahn, and N. F. Meyer (1960), Interference and channel allocation problems associated with orbiting satellite communication relays, Proc. IRE **48**, 608–612.

Bradley, W. B. (1962), Orbiting passive electromagnetic reflectors constructed of fine metallic wire mesh, Nat. Acad. Soc., Nat. Res. Council USA, URSI Spring Meeting Prog., April 1961.

Chatelain, M. G. (1960), Anisotropic communication satellites, IRE Nat. Symposium on Global Communications, July 1960.

Clarke, A. C. (1945), Extraterrestrial relays, Wireless Work **51**, 305–308.

Cutler, C. C. (1962), Problems in non-oriented directional passive satellite repeaters, Am. Rocket Soc. J. **32**, 1400.

Graham, R., K. M. Siegel, et al. (1962), Optimum spherically symmetric corner reflectors, Nat. Acad. Soc., Nat. Res. Council USA, URSI Spring Meeting Prog., April 1961.

Handelsman, M. (1959), Performance equation for a "stationary" passive satellite relay (22,000 mile altitude) for communication, IRE Trans., Comm. Systems **CS–7**, 31–37.

Hansen, R. C. (1961), Communications satellites using arrays, Proc. IRE **49,** 1066–1074.
Harrington, R. F. (Jan. 1962), Small resonant scatterers and their use for field measurements, Syracuse University Res. Institute, Syracuse, New York, Rept. EE 492–620 ITB.
Herriott, D. R. (1961), Polyhedral satellites for more accurate measurement of orbit data of earth satellites, J. Opt. Soc. Am. **48,** 667.
Jacoby, D. L. (1960), Communication satellites, Proc. IRE **48,** 602–606.
Jakes, Jr., U. C. (1961), Participation of Bell Telephone Labs in Project ECHO and experimental results, Bell Systems Tech. J. **40,** 4.
Kaiser, J., C. Raisbeck (1962), Analysis and evaluation of passive communication satellite systems, Nat. Acad. Soc., Nat. Res. Council USA, URSI Spring Meeting Prog., April 1961.
Kay, A. P. (1959), Sphericlly symmetric lenses, IRE Trans., Antennas Propagation **AP–7,** 32–38.
Kennaugh, E. M., H. W. Bauemler (June 1958), Polarization properties of reactive wall corner reflectors, Rept. 768–3, Contract AF 33(616)–5078, The Ohio State University Research Foundation.
Keyes, J. E. (1959), The echoing area of a Luneberg lens reflector at K-band, Def. Res. Telecom. Est. (Canada) Rept. 47–03, ASTIA AD–302, 645.
Lichman, (Jan. 9, 1959), Passive relay communications, Hughes Aircraft Co., Rept. 4551–2, Culver, City Calif.
Lipsey, E. M. (1959), The theory of an omni-directional radar reflector, AF Inst. Tech., Rept. GA 558–7, ASTIA AD 16 1836, Wright-Patterson Air Force Base, Ohio.
MacLellan, D. C. (Oct. 1958), Some aspects of space satellites for space communications, Rec. Nat. Symposium on Extended Range and Space Communications, 20–24, George Washington University, Washington, D.C.
McCoy, C. T. (1959), Satellite communication, Proc. IRE **47,** 2019–2020.
Morrow, Jr., W. E. (1960), Orbital scatter communication, 13th Gen. Assembly URSI, London, England, Sept. 1960.
Morrow, Jr., W. E. (1961), Communication by orbiting dipoles, URSI Symposium on Space Communications Research, Paris, France, Sept. 1961.
Peters, Jr., L. (May 30, 1959a), A theory of corner reflectors as bistatic radar enhancement devices, Rept. 768–5, Contract AF 33(616)–5078, The Ohio State University Research Foundation.
Peters, Jr., L. (July 31, 1959b), Theory of spherical reflectors for bistatic radar enhancement, Rept. 768–6, Contract AF 33(616)–5078, The Ohio State University Research Foundation.
Pierce, J. R. (1955), Orbital radio relays, Jet Prop. **25,** 153–157.
Pierce, J. R., and C. C. Cutler (1959), Interplanetary communications, Advances in Space Science **1** (Academic Press, New York).
Pierce, J. R., and R. Kompfner (1959), Transoceanic communication by means of satellites, Proc. IRE **47,** 372–380.
Pollack, S., and D. Campbell (1960), Active vs. passive satellites for a multi-station network, IEE Int. Convention Record, pt. 5, 141–146.
Robertson, S.B. (1947), Targets for microwave navigation, Bell Systems Tech. J. **26,** 852–869.
Rutz, E. M., and E. Kramer (1962), Modulated array for space communications, Northeast Electronics Research and Engineering Meeting, Boston, Mass., Nov. 1962.
Ryerson, J. L. (1960), Passive satellite communications, Proc. IRE **43,** 613–619.
Ryerson, J. L. (1960), Scatter echo area enhancement, Proc. IRE **50,** 1919–1980.
Sharp, E. D. (1958), Properties of the Van Atta reflector array, RADCTR–58–33, Rome Air Development Center, Rome, New York.
Sharpe, E. C., and M. A. Diab (1960), Van Atta reflector array, IRE Trans. Ant. Prop. **AP–8,** 436–438.
Sinclair, A. R. (Dec. 1959), Reflection characteristics of artificial satellites constructed in the form of inflated polyhedrons, NASA Tech. Note D–174.
Stahler, Y. E., and A. L. Johnson (May 1960), Directive nonoriented reflectors as passive satellites in long-distance communications, Nat. Aeron. Electr. Conf. Proc., Dayton, Ohio, May 2–4, 1960, 257–265.
Twersky, V. (1962), Coherent and incoherent scattering by orbital distributions of scatterers, Nat. Acad. Sci., Nat. Res. Council USA, URSI Spring Meeting Prog., April 1961.
Van Atta, L. C. (Oct. 6, 1959), Electromagnetic reflector, U.S. Pat. No. 2,908,002.
Vea, L. H., J. B. Day, and R. T. Smith (1960), The use of a passive spherical satellite for communication and propagation experiments, Proc. IRE **48,** 620–624.
Wanselow, R. D. (1962), A proposed high gain wide angle coverage, passive, modulated re-radiator, IRE Trans. Ant. Prop. **AP–10,** 785.

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