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Voltage Ratio Measurements With a Transformer Capacitance Bridge*

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A bridge having inductively-coupled ratio arms, designed for the calibration of capacitors, is applicable to the accurate measurement of voltage ratio and phase angle of a-c voltage dividers at audiofrequencies. The ability to measure quickly the ratios of certain capacitors in the bridge circuit, and the excellent inherent accuracy of the inductively-coupled ratio arms in the bridge, combine to permit the measurement of ratio of voltage dividers by a method independent of absolute determinations of any of the electrical units. This paper describes equipment now available and procedures developed at the National Bureau of Standards for the accurate calibration of voltage dividers at audiofrequencies by this method.

1. Introduction

Equipment commonly used for the establishment of known voltage ratios includes volt boxes, resistive voltage dividers, attenuators, and inductive voltage dividers. With the exception of inductive voltage dividers, these devices are generally constructed of resistive elements, which, if used on alternating current at higher audiofrequencies, cannot be relied upon to the accuracy with which they can be calibrated on direct current, because of the deleterious effects of uncompensated inductance and capacitance.

Variable inductive voltage dividers are now widely used as ratio arms of bridges for the calibration of resistors, capacitors, and inductors as well as other ratio devices throughout a large part of the audiofrequency spectrum. The inherent stability and accuracy of inductively coupled ratio arms, together with excellent resolution of ratio, have challenged the capabilities of equipment and techniques formerly used to measure voltage ratios.

The search for better accuracy in the calibration of inductive voltage dividers has led to the development of several methods [1, 2, 3] for determining corrections to the nominal readings. Another method, capable of determining these corrections with an uncertainty less than $\pm 0.000\ 000\ 2$, has resulted from an investigation of possible other applications of apparatus intended for the accurate measurement of capacitance.

2. Capacitance Bridge

The bridge circuit that is used for the voltage ratio measurements has been described [4], and consists of accurate inductively-coupled ratio arms having very low effective series impedances, a group of three-terminal air capacitors, and a conductance balancing circuit. These components and the unknown capacitor, C_X, are shown schematically in figure 1. Each of the capacitors, C_1, C_2, ..., C_8, has one electrode connected to the detector and the other electrode connected separately to the rotors of eight switches. Each switch has 12 positions, the fixed contacts being connected to 12 taps on the winding of the inductively-coupled ratio arms. The position of the contact arm of each switch is numbered according to its connection to the ratio arms, −1, 0, 1, 2, 3, . . . 10, and each switch is identified by the capacitance-per-step that it controls. The corrected reading of the dials associated with these switches represents the capacitance required to balance the bridge and is denoted by C_A. A similar arrangement of switches associated with an auxiliary inductive voltage divider and a resistance-capacitance network provides control over the conductance balance. If C_X and G_X are the capacitance and the shunt conductance of the unknown, G_A is the conductance associated with the capacitance complex C_A, and G_B is the reading of the conductance balance controls on the bridge, then

$$C_X = C_A \tag{1}$$

and

$$G_X = G_A + G_B \tag{2}$$

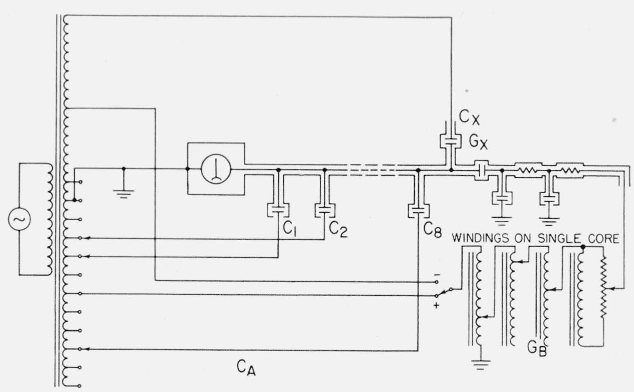

FIGURE 1. *Schematic of a transformer capacitance and conductance bridge used for the accurate measurement of direct capacitance.*

*Contribution from the Radio Standards Laboratory, National Bureau of Standards, Boulder, Colo.

A self-consistent calibration of the capacitance decades in the bridge can be accomplished quickly by an internal step-up method. The accuracy of the bridge for capacitance calibration work is limited by the accuracy of the standard capacitor to which the self-consistent calibration is referred. This does not detract from the ability of the bridge to measure the ratio of the capacitances of two nearly equal capacitors to better than one-tenth part per million.

3. Voltage Divider Calibration

In figure 2 a generalized voltage divider having an output-to-input-voltage ratio denoted by A is shown connected to the bridge and to a three-terminal capacitor, C. The voltage divider reduces the voltage applied to one electrode of the three-terminal capacitor by a factor A times the voltage that would be applied if the capacitor were connected directly to the upper extremity of the bridge transformer. In general, the voltage at the adjustable tap of a voltage divider will not be exactly in phase with the voltage applied to its input terminals, and the difference in phase can be expressed as a phase angle. A very small phase angle affects the conductance balance to a significant extent but has only a second order effect on the capacitance balance. When the bridge is balanced, the readings of the bridge, corrected for internal errors, yield values for the apparent capacitance and conductance, C_X and G_X, connected to the terminals of the bridge.

The capacitance balance of the bridge is a measurement of the ratio of the in-phase (real) components of the complex voltages, and the conductance balance is a measurement of the quadrature (imaginary) component. Because the phase angles involved are very small (see fig. 3) no appreciable error arises from considering that the ratio of the magnitudes of the voltages, $A=A_N[(1+\alpha)^2+\beta^2]^{\frac{1}{2}}$, equals the ratio of the in-phase components, $A_N(1+\alpha)$. Also, no significant error arises by considering that the phase angle, γ, equals β in the equation $\mathbf{A}=\mathbf{A}_N(1+\alpha+j\beta)$. Because of the somewhat independent nature of the capacitance and the conductance balance, they will be considered separately.

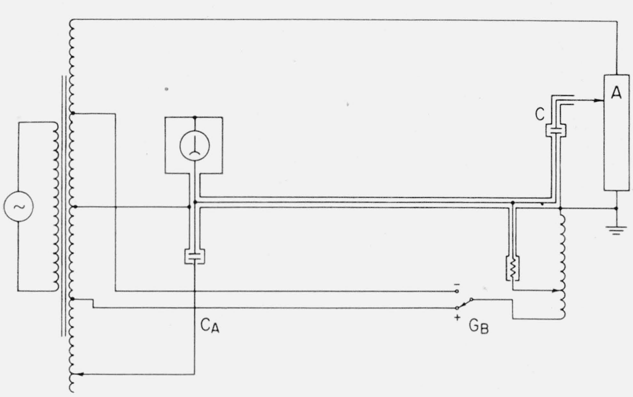

FIGURE 2. *Simplified schematic of a transformer capacitance bridge used to measure the complex voltage ratio of voltage dividers.*

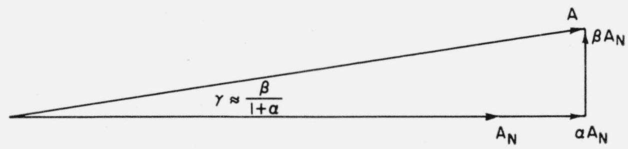

FIGURE 3. *The phasor diagram defines the true ratio,* **A**, *and phase angle,* γ, *in terms of the nominal ratio,* $\mathbf{A}_N$, *the ratio error,* α, *and the quadrature component,* β.

The phase angle and ratio errors are grossly exaggerated for clarity.

4. Voltage Ratio Measurement

The voltage ratio of a generalized divider is indicated primarily by the capacitance balance of the bridge. The reduced voltage applied to capacitor C is equivalent to the full voltage applied to the measured capacitance, C_X. In figure 4 it is evident that the relationship between C_X and C is

$$C_X=AC. \tag{3}$$

Therefore, the scalar voltage ratio of the generalized voltage divider is

$$A=\frac{C_X}{C}. \tag{4}$$

The resolution of the capacitance balance of the bridge often approaches 1 part in 10^8; hence, excellent accuracy in ratio measurements is easily achieved if care is taken to eliminate the effect of environmental disturbances such as changes of temperature.

The actual ratio measured by this procedure may differ, both in magnitude and phase angle, from the ratio that would be obtained if no current were withdrawn from the divider at the adjustable tap. The

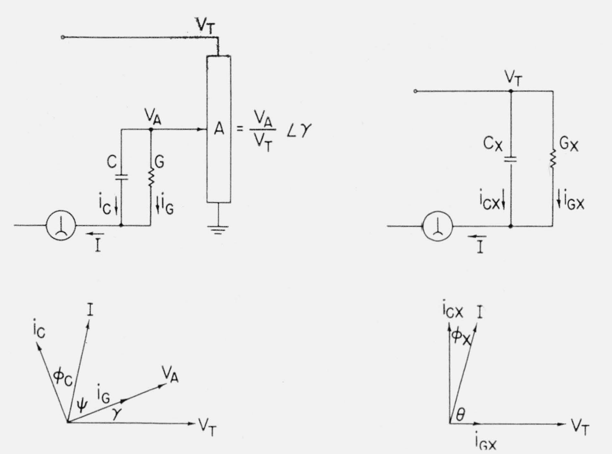

FIGURE 4. *The equivalent circuit and phasor diagram on the right represents the capacitance and parallel conductance equivalent to the actual circuit with voltage divider shown on the left.*

effect of loading on the voltage ratio of resistive and inductive voltage dividers is considered in sections 7 and 8 of this paper. In general, this effect may be expressed as a magnitude error and a phase angle error. Corrections, μ and ρ, to the measured ratio, **A**, may be defined by the following equation:

$$\mathbf{A}_0=\mathbf{A}(1+\mu+j\rho) \tag{5}$$

where $\mathbf{A}_0$ is the voltage ratio when no load is connected.

5. Phase Angle Measurement

The conductance balance of the bridge is related to the phase angle associated with the voltage divider as well as to the conductances of capacitors C and C_A (fig. 2). The relationship among these parameters is such that the phase angle of the divider may be calculated from the difference between certain conductance balances of the bridge. In this paper the phase angle, γ, of the voltage divider is defined as the angle between the phasors V_A and V_T, where V_A is the voltage at the adjustable tap and V_T is the voltage at one extremity of the divider, both referred to a common point at the other extremity unless otherwise noted. The phase angle is considered positive if V_A leads V_T and negative if V_A lags V_T. These phasors and the circuits to which they refer are depicted in figure 4. The right half of the figure represents the equivalent circuit, as measured by the bridge, of the more complex actual circuit shown on the left. It is evident that

$$\theta=\frac{\pi}{2}-\phi_x, \qquad \psi=\frac{\pi}{2}-\phi_c \tag{6}$$

and

$$\theta=\psi+\gamma. \tag{7}$$

Then

$$\gamma=\theta-\psi=\phi_c-\phi_x, \tag{8}$$

and at any frequency, f, if ϕ_c and ϕ_x are very small angles, it can be shown that

$$\phi_c=\frac{G}{C}T, \tag{9}$$

and

$$\phi_x=\frac{G_X}{C_X}T \tag{10}$$

where $T=\frac{1}{2\pi f}$. Combining eqs. (8), (9), and (10) yields

$$\gamma=T\left(\frac{G}{C}-\frac{G_x}{C_x}\right). \tag{11}$$

It has been found convenient to express the phase angle of voltage dividers in fractions of a radian. If T is expressed in microseconds, G and G_X in picomhos, C and C_X in picofarads, eq (11) yields γ in microradians.

The phase angle correction for loading is significant and is discussed in section 7.

6. Three-Terminal and Four-Terminal Dividers

The excellent resolution obtained by this method led to the consideration of errors resulting from the impedance in the leads and connectors that are used to connect voltage dividers to other circuit elements, as well as the errors resulting from undesirable losses within the ratio apparatus itself.

In order that the calibration of ratio devices be useful, the conditions under which the equipment is used must be duplicated at the time of calibration, or corrections must be made for any significant differences. One source of significant difference is the manner in which connections are made to the input and output terminals. Many ratio devices, both resistive and inductive, are constructed with four terminals, so that a source may be connected to two "input" terminals, and a load connected to the two "output" terminals. Often one input terminal and one output terminal are connected together internally by a wire of low, but not infinitesimal, impedance. Nevertheless, such a device may be described as "four-terminal," and the internal branch-point may be considered the lower extremity of the output of the divider.

On the other hand, one of the common terminals may be ignored and the divider connected to a circuit as a three-terminal device. Current in the wires between the terminals and the internal branch-points produces voltage drops that can cause a three-terminal calibration to differ significantly from a four-terminal calibration. In figure 5 the input voltage, E_I, of the divider is the potential difference between the input terminals 3 and 1. If v_n represents the magnitude of the in-phase component[1] of potential at terminal n relative to terminal 0,

$$E_I'=v_3-v_1. \tag{12}$$

The output voltage, E_H, of the divider, considered as a four-terminal network, is

$$E_H'=v_4-v_2, \tag{13}$$

and the output voltage, E_T, of the divider, considered as a three-terminal network, is

$$E_T'=v_4-v_1. \tag{14}$$

Now certain ratios of the in-phase component of potential, relative to the 0 terminal, may be defined as follows (see fig. 6):

$$A_1=\frac{v_1}{v_5},\ A_2=\frac{v_2}{v_5},\ A_3=\frac{v_3}{v_5},\ \text{and}\ A_4=\frac{v_4}{v_5}, \tag{15}$$

from which

$$v_1=A_1v_5,\ v_2=A_2v_5,\ v_3=A_3v_5,\ \text{and}\ v_4=A_4v_5. \tag{16}$$

[1] The term "in-phase component" and the prime notation here denotes components in phase with the reference voltage phasor, E_5 (or reference voltage ratio phasor, A_5). As indicated previously, these in-phase components are very nearly equal to the absolute values because the quadrature components are comparatively very small.

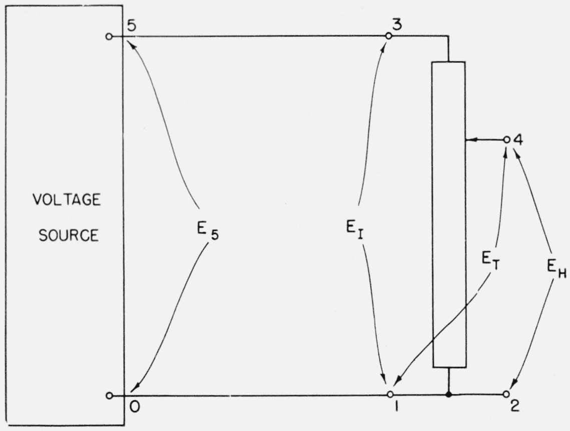

FIGURE 5. *A generalized four-terminal voltage divider connected to a source of voltage.*

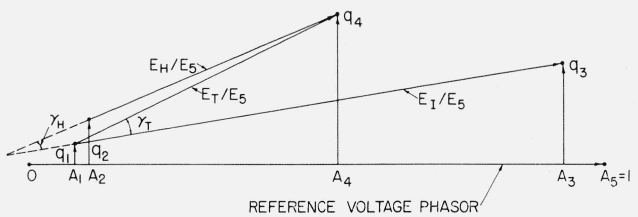

FIGURE 6. *Phasor relationships of the voltages shown in figure 5.* A_1, A_2, A_3, and A_4 *are the in-phase components of the voltage-ratio phasors along the reference voltage phasor.*

E here denotes the magnitude of the phasor $\mathbf{E}_5$.
The angles have been grossly exaggerated for clarity.

It is apparent that the in-phase component of voltage ratio for the four-terminal network is

$$A_H=\frac{E'_H}{E'_I}=\frac{v_4-v_2}{v_3-v_1}=\frac{A_4-A_2}{A_3-A_1}, \tag{17}$$

and the corresponding ratio for the three-terminal network is

$$A_T=\frac{E'_T}{E'_I}=\frac{v_4-v_1}{v_3-v_1}=\frac{A_4-A_1}{A_3-A_1}. \tag{18}$$

These equations show that the desired ratios, A_H or A_T, may be computed from measurements of certain other elementary ratios. It can be shown that A_T and A_H are related by the equation

$$A_T=A_H+\delta, \tag{19}$$

where

$$\delta=\frac{A_2-A_1}{A_3-A_1}. \tag{20}$$

In the calibration of the decade ratio apparatus now in use as a standard at NBS Electronic Calibration Center it was unnecessary to measure A_1, A_2, and A_3 more than once because these ratios are primarily dependent upon impedances which remain essentially constant during the calibration. The ratio A_4 was measured for each dial setting of interest, and the ratio A_H or A_T, or both, was computed according to eqs (17), (18), or (19). It is evident from eq (20) that δ in eq (19) is independent of ratio setting A_4. In the calibration of other decade dividers, δ may not be truly independent of A_4. A redistribution of internal stray impedances or large external loading impedance, as A_4 is varied, may cause a significant change in A_1, A_2, A_3, and it may be necessary to measure these for each value of A_4.

Turning now to the consideration of phase angle measurements, it will be convenient to express the phase angle, γ, in terms of the in-phase and quadrature components of the complex voltage ratio. Here γ is intended to represent the angle between the output-voltage phasor and the input-voltage phasor, even though the separate phasors may not have a common origin (as in the four-terminal network described).

The quadrature components of the complex voltage ratio at the terminals 1, 2, 3, and 4, are denoted by q_1, q_2, q_3, and q_4, as shown in figure 6. At each point of measurement the quadrature component measured from the reference voltage phasor can be obtained from $q=A\gamma$ where γ is determined in accordance with eq (11). The phase angle for the four-terminal network is the angle between E_H and E_I

$$\gamma_H=\frac{q_4-q_2}{A_4-A_2}-\frac{q_3-q_1}{A_3-A_1}, \tag{21}$$

and since A_1 and A_2 are very small, and A_3 is approximately 1, eq (21) can be simplified for practical computations, giving

$$\gamma_H=\frac{q_4-q_2}{A_4}-(q_3-q_1). \tag{22}$$

Similarly, the phase angle for the three-terminal network is the angle between E_T and E_I

$$\gamma_T=\frac{q_4-q_1}{A_4}-(q_3-q_1), \tag{23}$$

hence

$$\gamma_T=\gamma_H+\frac{q_2-q_1}{A_4}. \tag{24}$$

The eqs (22), (23), and (24) provide a useful means by which the phase angles for the four-terminal and three-terminal networks may be computed from the quadrature components of the complex voltage ratio.

7. Resistive Divider Calibration

Several sources of error accompany the measurement of ratio of resistive voltage dividers by the method described in this paper. The small voltage drops in the wires connecting the voltage divider to the bridge constitute a source of error for which correction may be made. The resistance in the leads in series with the resistive voltage divider causes a voltage drop affecting the measurement of ratio. This error, if not unduly large, can be eliminated by the procedure described in section 6.

Resistive dividers having low resistance, when connected to the bridge transformer as in figure 2, can burden the transformer to an intolerable extent. If loading on the secondary of the bridge transformer imposes a limitation on the accuracy of the measurement, the transformer may be energized by connecting the a-c source across that part of the secondary in parallel with the resistive divider rather than to the primary winding. When this connection is made, the primary of the bridge transformer is ignored and the secondary serves as an inductive voltage divider, in which the loading from interturn capacitance and leakage inductance becomes a source of error, but of smaller magnitude. Thus, the corrections for the errors in ratio of the transformer would not be expected to be the same as those discussed in section 9, and may be considerably larger. The calibration of resistive voltage dividers by this method has not progressed to the extent that accuracy of measurement was restricted by errors in the transformer. It is more convenient to calibrate resistive voltage dividers by comparison with inductive voltage dividers which have been calibrated by the method described in this paper.

Another source of significant error arises from the loading on the resistive voltage divider, predominantly by the capacitance to ground, C_g, from the adjustable tap. Resistive voltage dividers having high resistance are particularly susceptible to loading of this sort. The current from this capacitance in the resistive voltage divider causes ratio and phase angle errors at the adjustable tap of the divider that can be expressed by the equation

$$\mathbf{A}_0=\mathbf{A}\left[1-(A-A^2)\frac{X_L}{X_{Cg}}+j(A-A^2)\frac{R}{X_{Cg}}\right] \quad (25)$$

where $\mathbf{A}_0$ is the ratio with no load connected to the output of the divider, $\mathbf{A}$ is the voltage ratio in the presence of loading, X_{Cg} is the reactance of the capacitance C_g, and $R+jX_L=Z$, where Z is the total impedance of the voltage divider. In resistive voltage dividers it is expected that $X_L<<R$, so that the most prominent correction term is the last one as shown in eq (25). Thus, the effect of capacitive loading on the real component of the complex ratio is generally negligible, while the effect on the phase angle may be significant. The correction to the phase angle measurement is dependent upon the ratio being measured and is the greatest when $A=0.5$.

It must be noted that eq (25) is applicable only to a resistive divider circuit having a uniformly distributed inductance. Loading errors of decade voltage dividers having the Kelvin-Varley slide are represented only approximately by eq (25). The treatment of loading errors for other, more complex, resistive voltage dividing circuits requires detailed analysis and will not be discussed here. The voltage division associated with resistive attenuators is particularly susceptible to loading, and for this reason attenuators are not usually considered suitable for use as standards of voltage ratio in the audiofrequencies.

For simple resistive voltage dividers it is evident from eq (25) that the corrections, μ and ρ, defined in eq (5) are

$$\mu=-\omega^2(A-A^2)C_gL \quad (26)$$

and

$$\rho=\omega(A-A^2)C_gR. \quad (27)$$

Corrections can be made to reduce these errors, leaving only second order errors to contribute to the overall uncertainty of measurement. In practical measurement work the small second order errors (resulting from the use of certain approximations in the derivations of the above equation) are generally extremely small relative to the accuracy limit imposed by the resolution and stability of the resistive divider being calibrated.

If a reasonably accurate measurement of phase angle is desired, it is evident from eq (27) that C_g should be small in order to reduce to a minimum the quadrature error component for which correction must be made. A practical limit to the reduction of C_g is imposed by the necessity of obtaining sufficient resolution in the ratio measurements as indicated by eq (4), in which the direct capacitance, C, is a part of the total capacitive load, C_g, on the divider.

In the event that the correction for loading, as described here, is inconvenient or creates intolerable uncertainties, a method is available for reducing the loading to an imperceptible level. A capacitance, $C'=C_g\frac{A}{1-A}$, may be connected from the output terminal of the voltage divider to the upper input terminal (see fig. 2). The capacitances, C' and C_g, then form a capacitive voltage divider having approximately the same ratio, A, as the divider being calibrated, and the loading effect is much reduced.

8. Inductive Divider Calibration

In general, with the method described, less difficulty is encountered in the calibration of inductive voltage dividers than resistive voltage dividers. The close coupling between the sections of inductive voltage dividers results in relatively low effective series output impedance which minimizes the effects of external loading as well as internal variations of impedance. The voltage drop resulting from current in the effective series impedance, Z_s, shown in figure 7 produces an actual voltage ratio, $\mathbf{A}$, differing in magnitude and phase angle from the voltage ratio, $\mathbf{A}_0$, without load as indicated by the equation

$$\mathbf{A}_0=\mathbf{A}\left(1-\frac{X_{Ls}}{X_{Cg}}+j\frac{R_s}{X_{Cg}}\right). \quad (28)$$

It must be noted that the effective series impedance generally is not constant, but varies with the setting of the divider. Correction for loading errors usually can be made by measuring the actual voltage ratio and phase angle at the terminals with two known

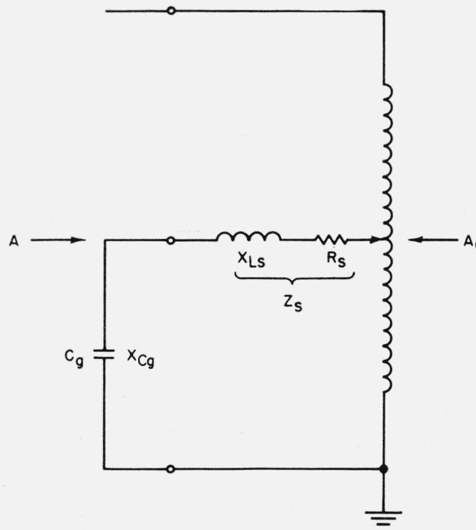

FIGURE 7. *The actual voltage ratio,* **A**, *at the terminals of an inductive voltage divider differs from the ratio,* $\mathbf{A}_0$, *as a result of loading by the capacitance,* C_g, *on the effective series output impedance of the divider,* Z_s.

capacitive loads and extrapolating to determine the voltage ratio and phase angle at zero load.

Comparison of eqs (5) and (28) shows that for inductive voltage dividers

$$\mu = -\omega^2 C_g L_s \tag{29}$$

and

$$\rho = \omega C_g R_s. \tag{30}$$

At a frequency of 1,000 c/s, if $C_g = 200$ pf, $L_s = 100$ μh, and $R_s = 10$ ohms, μ is approximately 0.8×10^{-6}, and ρ is approximately 12×10^{-6}. The method described in section 7 for eliminating these corrections is applicable here also, although the extrapolation to zero load as described above has been found convienient.

9. Calibration Techniques and Accuracy

In general, the calibration of adjustable voltage dividers consists of determinations of voltage ratio and phase angle corresponding to certain nominal ratio settings when no load is connected to the output terminals. The expression for voltage ratio for the four-terminal network given by eq (17) can be modified according to eq (5) to yield the voltage ratio under no-load conditions

$$A_{H0} = \frac{A_4 - A_2}{A_3 - A_1}(1+\mu). \tag{31}$$

From eqs (5) and (18) the voltage ratio for the three-terminal network is

$$A_{T0} = \frac{A_4 - A_1}{A_3 - A_1}(1+\mu). \tag{32}$$

The phase angle for the four-terminal network, from eqs (5) and (22), is

$$\gamma_{H0} = \frac{q_4 - q_2}{A_4} - (q_3 - q_1) + \rho, \tag{33}$$

and, from eqs (5) and (23), the phase angle for the three-terminal network is

$$\gamma_{T0} = \frac{q_4 - q_1}{A_4} - (q_3 - q_1) + \rho. \tag{34}$$

The loading corrections, μ and ρ, are usually very small and often can be computed on the basis of measurements involving no great accuracy. The ratios A_1, A_2, and A_3 need be measured only once in the calibration of a voltage divider if the input impedance of the divider remains constant when the divider ratio is changed.

The accuracy of the measurement of voltage ratio and phase angle by this method is dependent upon a relatively few systematic errors and a number of random errors. A satisfactory appraisal of the propagated errors must allow for more or less complete cancellation or certain errors that, if considered independently, are of rather large magnitude. Errors in the values of the capacitors are of this nature, since the ratio of the capacitances and conductances of two capacitors can be measured with much better accuracy by substitution methods than the accuracy with which any one capacitor can be measured.

The precision of the measurements can be enhanced by reducing the effect of temperature changes in the bridge components. In the work described, the temperature of the bridge was regulated by forced warm air. The 100 pf capacitor, C, was temperature compensated, and the design of the enclosures of this capacitor, as well as those within the bridge, incorporated a large amount of thermal lagging. The drift of these capacitors with time was a source of error that was reduced by arranging the sequence of work so that the time between critical measurements was minimum. It may be possible to further reduce the effects of drift by a timed program of sandwiched measurements, although this procedure has not yet been necessary. The capacitance ratio, C/C_A, can be determined most accurately if C is nominally equal to C_1, or C_2, or . . . C_8, shown in figure 1.

The total estimated maximum error in the establishment of known ratios by the method described in this paper was derived by adding, without regard to sign, the estimated maximum error associated with each step in the measurement and estimated possible residual systematic errors. This method of expressing the accuracy of the measurements was chosen because most of the elementary errors are not completely independent. A number of tests were made to determine the magnitude of errors in the ratios of the secondary taps on the transformer in the capacitance bridge. Although several methods designed to detect ratio errors (departure from linearity) were used, no errors were found that exceeded twice the propagated uncertainties attributed to resolution of the bridge during these measurements. These methods are discussed below. In general, no error in ratio was found that exceeded 0.000 000 06, and the uncertainty of these errors was about ±0.000 000 03. Accordingly, the estimated uncertainty from this source was considered to be ±0.03 ppm of ratio when the largest capacitor in use (C_1, C_2, . . . or C_8, in fig. 1) spans the entire lower half of the secondary. This estimated uncertainty then increases to ±0.3 ppm of ratio when the largest capacitor in use spans only 0.1 of the lower half of the secondary. The resolution of the equipment, as used for the determination of the ratio A_4, was equivalent to a ratio of ±0.000 000 01, i.e., ±0.01 microvolt output voltage per volt input. The estimated maximum uncertainty from this cause was conservatively estimated as equivalent to a ratio of ±0.000 000 02. A similar uncertainty exists in the measurement of A_1 or A_2. In the measurement of lower ratios these two sources of uncertainty predominate, limiting accuracy to within ±0.000 000 04 at a ratio of 0.001. The determination of capacitance ratio can be accomplished with an estimated uncertainty of ±0.03 ppm of ratio for measurements of voltage ratios from 1 to 0.1; ±0.2 ppm of ratio for ratios from 0.1 to 0.01 wherein two steps are required; and ±2 ppm of ratio for ratios from 0.01 to 0.001 wherein three steps are required. The determination of A_3-A_1, as it enters into the computation of ratio, can be accomplished with a relatively small uncertainty of ±0.02 ppm of ratio. The sum of these uncertainties, without regard to sign, and expressed as an additive ratio, is shown in table 1 as the estimated maximum uncertainty in ratio at a frequency of 1,000 c/s.

TABLE 1. *Estimated accuracy at 1,000 cycles per second*

Nominal ratio	Estimated maximum uncertainty	
	Ratio	Phase angle
		Microradians
1	±0.000 000 12	±0.5
0.5	±.000 000 10	±2.3
.1	±.000 000 08	±4
.05	±.000 000 06	±3
.01	±.000 000 06	±7
.005	±.000 000 05	±13
.001	±.000 000 04	±46

The estimated maximum uncertainties listed in table 1 do not indicate the actual error in any particular measurement. It is to be expected that actual errors are much smaller than the estimated maximum uncertainties. A separate experiment was conducted to obtain a quantitative appreciation of the actual error in the measured voltage ratio assigned to particular voltage dividers. This utilized two decade voltage dividers, one of which was carefully calibrated as a three-terminal divider by the method described above. This calibrated divider then was used as a standard for the calibration of the highest decade of the other divider by a comparison method in which the input terminals were connected in parallel and the output terminals connected to a phase-sensitive detector. The two dividers formed an a-c bridge in which the true voltage ratios were equal under conditions of balance. The connections to the input terminals of the divider under test were reversed, and a second calibration of the highest decade was obtained. In the first comparison, the readings of the standard and the test dividers were matched, i.e., (0.9, 0.9), (0.8, 0.8), . . ., (0.1, 0.1,), while in the latter comparison, the readings were related in the manner (0.9, 0.1), (0.8, 0.2), . . ., (0.1, 0.9). Additive corrections to each step of the highest decade were thus obtained, based upon two different ratios of the standard divider. The difference between corresponding additive corrections to the nominal ratio in no case exceeded 0.000 000 04, and generally the corrections agreed within 0.000 000 02. Better agreement than this could hardly be expected, because the precision of the initial calibration of the standard was estimated as ±0.000 000 02. Although corrections to the divider under test at the ratio 0.5 can be obtained accurately by the reversed-comparison method independently of the voltage ratios assigned to the standard voltage divider, it should be understood that this method does not reveal residual errors in the calibrated standard that are symmetrical about the ratio 0.5. An investigation to discover such symmetry in the capacitance bridge was conducted by a step-up method in which a fixed capacitance was repeatedly added to one side of the bridge and the differences in readings of the capacitance complex in the lower side of the bridge were noted. Although this investigation suffered from a small drift in the capacitors, and a timed sequence of readings was used to eliminate the effects of this drift, there was no evidence of errors of symmetry greater than 0.000 000 06. The uncertainty of these errors was ±0.000 000 03, which was the limit of uncertainty in the investigation, primarily a result of uncontrolled drift. In this test for symmetry, the differences between 0.1 sections of the lower half of the secondary were adjusted for drift in the capacitor, and added cumulatively to obtain the total error at each tap. The results were averaged over several tests and then adjusted to eliminate the effect of nonsymmetrical errors. The measured, algebraically additive, errors of symmetry were found to have magnitudes of approximately 0.04, 0.06, 0.04, and 0.02 $\times 10^{-6}$ (all with an uncertainty of $\pm 0.03 \times 10^{-6}$), corresponding to the ratios 0.1 and 0.9, 0.2 and 0.8, 0.3 and 0.7, and 0.4 and 0.6, respectively.

The accuracy of position of the centertap on the secondary of the transformer was determined by connecting a 7-decade inductive voltage divider across the entire secondary and adjusting the inductive voltage divider to obtain a balanced condition. Then the connections from the divider to the transformer secondary were interchanged, and the inductive voltage divider again adjusted to obtain balance. Half the difference in readings of the inductive voltage divider is indicative of the error in position of the centertap. In the experiment, the interpolated readings were equal, with a precision of ±0. 000 000-01. Hence the centertap is believed to be within ±0. 000 000 01 of true center.

Although great accuracy usually is not required in the measurement of the phase angle, the capacitance bridge method presented in this paper is capable of providing quantitave phase angle measurements on inductive voltage dividers. A similar analysis was made of the errors contributing to the uncertainty of the phase angle. A simple method was developed to eliminate the effect on phase angle of conductance in the capacitors. This is accomplished by measuring the difference in conductances between the external capacitor, C, and the capacitor in use within the bridge. It is this difference of conductance that enters into the computation of phase angle. The difference can be measured with an accuracy within 0.3 percent of the difference. In the work described, the differences in conductances were measured quite simply at the times the capacitors were compared. At ratios near 1, the largest uncertainty in phase angle is contributed by the transformer. It is believed that this error is no larger than ±0.3 microradian when the largest capacitor in use spans the entire lower half of the secondary, and ±3 microradians when the largest capacitor in use spans only 0.1 of the lower half of the secondary. The largest uncertainties in phase angle at low ratio measurements are contributed by a series of two conductance balances, each contributing about $\pm 0.02 \times 10^{-6}$ uncertainty to the measurement of the quadrature component, part of which is systematic, and another conductance balance that introduces about $\pm 0.003 \times 10^{-6}$ uncertainty, although there is some cancellation of the previously mentioned systematic error. At a ratio of 0.001, the sum of these uncertainties in the quadrature component converted to phase angle is about ±46 microrodians. The overall maximum estimated error at a frequency of 1,000 c/s is given in table 1.

10. Conclusion

The method described has been employed for the very accurate calibration of a few decade inductive voltage dividers that, in turn, serve as standards for the more economical calibration of other dividers by a comparison method.

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11. References

[1] W. C. Sze, Trans. AIEE, pt. I, **76,** 444 (1957).
[2] C. B. Pinckney, Trans, AIEE, pt. I, **78,** 182 (1959).
[3] N. E. Morrison, Electro Scientific Industries, AIEE Summer and Pacific General Meeting, Paper 59–997 (1959).
[4] M. C. McGregor, J. F. Hersh, R. D. Cutkosky, F. K. Harris, and F. R. Kotter, Trans. IRE on Instrumentation, **I–7,** 253 (1958).
[5] R. D. Cutkosky and J. Q. Shields, Trans. IRE on Instrumentation, **I–9,** 243 (1960).

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