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Incoherent Scattering by Free Electrons as a Technique For Studying the Ionosphere and Exosphere: Some Observations and Theoretical Considerations [1]

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Incoherent scattering by the free electrons of the ionosphere has been suggested as a technique for measuring the electron density profile both below and above the *F* region maximum. This paper reports observations which confirm the existence of the incoherent scatter and show that its intensity is essentially the predicted value. The observed Doppler broadening is considerably smaller than originally predicted. In the second part of the paper, an explanation for the reduced Doppler broadening is offered. The scatter is explained as arising from statistical fluctuations of electron density, the distribution of which is controlled by the positive ions.

Introduction

The ability of a random distribution of free electrons to exhibit weak scattering of radio waves has been cited by Gordon [1] as a possible means of measuring electron densities and temperatures continuously to heights of several thousand kilometers above the earth's surface. Gordon's prediction is based upon the assumption that the free electrons may be considered to scatter mutually independently. His development is essentially a combination of the Thompson scattering property of the free electron with Lord Rayleigh's theory of the scattering of light by the particles of a gas [2].

Use of such incoherent scatter to study the ionized regions of the outer atmosphere should have the great virtues of theoretical simplicity, and the ability to observe a complete vertical cross section of electron density and other parameters continuously for long periods of time with ground-based equipment. With these considerations as motivation, and having available a large high-power pulse transmitter, the National Bureau of Standards undertook experiments to test the existence of the predicted scatter and to verify its utility. Results of the first preliminary observation were reported by the writer in a short communication [3]. Since then, more sensitive observations have been made that are described in this paper, along with a theoretical interpretation. Much the same material was originally circulated among a limited number of workers in this field as an unpublished report of the National Bureau of Standards [16].

Part I of the paper is devoted to the experimental observations. The characteristics and special features of the equipment are outlined. Observations in which the incoherent scatter may be identified are illustrated using original oscillograph records. The incoherent scattered power is found to be within a factor of two of the value predicted using Gordon's approach. Representative electron density-versus-height profiles are given for two of the oscillograms, and shown to agree reasonably well with profiles for the lower part of the *F* region obtained from conventional ionograms. The possibility that any one of several other well-known propagation modes might have resulted in ambiguous observations is examined and rejected.

In spite of the apparent verification of Gordon's prediction for echo *power*, a distinct departure is found in terms of the observed echo *spectrum*. The spectral broadening of the scattered echoes is found to be considerably less than the Doppler broadening predicted using Gordon's approach. While a spectral width of approximately 100 kc/s might have been expected for *F* region echoes, the observation is that the transmitted bandwidth of about 9 kc/s must be broadened only slightly by the scatter process.

In *Part II* we propose that the scatter should be regarded as arising from random fluctuations of *electron density*, rather than from electrons scattering individually. In this way we develop a simple explanation for the slight echo broadening, based upon Coulomb interactions between ions and electrons.

In the past the density fluctuation approach has been used successfully to predict the intensity of scatter from aggregates of gas particles even in cases where the distributions of particle density were not entirely random. However, little attention appears to have been paid to the question of spectral broaden-

[1] Contribution from Central Radio Propagation Laboratory, National Bureau of Standards, Boulder, Colo.

ing using density fluctuations. It is generally assumed that when plasma oscillations can be neglected, as in the case of high frequency "reflections" from the ionosphere, the spectrum of scatter from aggregates of free particles must be directly proportional to the distribution of line-of-sight velocities of the several particles. We offer a simple illustration to show that this proportionality may not necessarily hold true when there is a degree of mutual correlation among the positions of the particles. We argue that such correlation is automatically introduced if there are many particles within a zone whose depth corresponds to approximately 180 degrees of phase or less. This zone defines a preferred "scale" among the density fluctuations, and the rate at which irregularities of this scale change from one state of fluctuation to another provides a means of estimating the spectral breadth of the scatter.

Obviously the picture of density fluctuations breaks down when the individual scattering particles are so sparsely distributed that only one can be found in a volume large relative to the characteristic scale. Such a case is found with radar "chaff," where there is every indication that the chaff particles contribute to the scatter independently. Evidently some criterion must be established to determine when it is better to consider the scatter as arising from density fluctuations, and when from independent particles. Two possible "independence criteria" are examined—one based on mean spacing between neighboring particles, the other based on mean-free-path as proposed by Gordon. The present experiment offers an opportunity to distinguish between these criteria.

It is shown that, apart from a proportionality factor close to unity, the spectral width of radiation scattered from a randomly distributed gas is the same, using either density fluctuations or independent particles. From the experimental results, we draw the conclusion that the distribution of free electrons in the ionosphere is not entirely random (even within a volume small enough that the mean density gradient can be regarded as negligible).

We examine the effect of Coulomb interactions between ions and electrons on the fluctuations of electron density. For scales of irregularities large compared with the Debye shielding distance, λ_D, we find that the fluctuations of ion density and those of the electrons are strongly correlated. The *magnitude* of the fluctuations is close to that of a neutral gas whose density is equal to the electron density. Because the ions have so much more momentum, the *rate of change* from one state of fluctuation to another is controlled almost exclusively by the ions. We therefore estimate, for example, that a spectral broadening of the order of 500 c/s would be correct for echoes from the F region at our 41-Mc/s operating frequency. Since the results of recent experiments indicate that this estimate is correct, we infer that the density fluctuation approach is correct in the F region. This suggests that the correct independence criterion may be based upon the mean spacing between neighboring particles.

The predicted effects of the earth's magnetic field on the observations are discussed briefly. It is pointed out that these effects may permit one to identify the ionic constituents and to measure the magnetic field strength in the earth's outer atmosphere.

Part I. Experimental Observations

1. Equipment

Except for the dimensions involved, the equipment is in most respects a conventional radar directed vertically. Operating parameters are summarized in table 1.

TABLE 1. *Parameters of radar equipment used*

Operating frequency	40.92 Mc/s
Peak pulse power	(4 to 6)$\times 10^6$ w or 1×10^6 w (see text)
Pulse duration	3 to 150×10^{-6} sec available 120×10^{-6} sec for all data shown in this paper
Average power	4×10^4 w maximum
Pulse repetition frequency	Flexible within above limitations. 25 to 40 p.p.s. used for most work shown in this paper (see text)
Receiver noise (power) bandwidth	9, 13, or 35 kc/s
Receiver noise factor	2
Signal/noise improvement by averaging	Up to approximate factor 100 (20 db) (see text)
Antenna cross section	116 by 140 m (1.6×10^4m^2)
Antenna polarization	Linear north-south
Antenna gain, efficiency, etc.	See text
Location	NBS Long Branch, Illinois transmitting station—WWI Latitude 40.2° N. Longitude 90.0° W.

The transmitter is a newly built unit of a hitherto untried design. Not all malfunctions in its operation had been removed at the time the experiments here reported were carried out. Absolute measurements of the incoherent scatter intensity made during February 1959 (described in detail later in the paper) make it clear that the transmitter was generating roughly 1-Mw peak power at the operating frequency, rather than 4 to 6 Mw, during the work reported in reference 3. The difficulties responsible for this malfunctioning have been corrected, but a new problem appeared before the February tests. Due to damage from a severe frost, the transmitter could only be operated using its driver amplifier with an output power of 1.0 Mw. This power was carefully calibrated using a probe technique for comparison with the known output of a lower power amplifier. Approximately 1 db of attenuation was experienced in the temporary connection of the driver amplifier to the antenna, and this is reported below as part of the total loss of antenna efficiency.

The receiver is a communications unit of conventional design, modified to permit pulse operation.

A preamplifier and converter provide low noise features not available in the commercial receiver. Bandwidths suitable for pulse work were obtained by loading the IF transformers. The switching arrangement of the receiver then permitted the 9-kc/s band to have the characteristic of 6 cascaded L-C circuits. The 35-kc/s band had a flat top, with skirts roughly the same as those of the 9-kc/s band. The bandwidths quoted were measured by comparing and equating the output of the receiver for calibrated inputs from a noise diode source, then from a cw standard signal generator, taking the noise statistics and detector operation into account. The bandwidths, between frequencies at which the response is down 6 db from the maximum, differ only slightly from the noise bandwidths.

Averaging of the receiver output was used to improve the quality of the scatter measurements. The averaging device, or "integrator," is built around a multiple-deck mercury-jet commutator. Each of 240 independent capacitors is connected to the receiver output through a large resistor only during an assigned time interval following the transmitter pulse. Thus the voltage appearing across each capacitor represents the RC running average of receiver output corresponding to a specific range interval—usually 15 to 25 km wide. The vertical d-c amplifier of an oscilloscope having high input impedance is also connected to the capacitors in sequence. The presentation on the oscilloscope is therefore a histogram representing the running average of many conventional A-'scope sweeps. The limitations of the device are such that a maximum improvement in effective signal-to-noise ratio of about 20 db (a factor of 100) can be obtained. Several different time constants were used in taking the photographs presented in this report, in each case the signal-to-noise ratio improvement being somewhat less than 20 db.

The antenna is a simple broadside array of 1,024 half-wave dipoles located 0.16 wavelength above ground. The dipoles are situated in approximately a square array and fed in phase through open wire transmission lines. By attaching the half-wave elements to the transmission line at one wavelength intervals, transposition of the lines is avoided. A total of 32 elements are connected effectively in parallel and fed at the center of each such arrangement. Those elements located at alternate half wavelength intervals are fed by the adjacent transmission line. There are a total of 32 such transmission lines spaced at half wavelength intervals. The neighboring combinations are then connected by pairing successively through equal length feeders until the whole array is connected to one pair of terminals. The reflection coefficient of the ground has been increased and stabilized by placing copper wires on the ground, parallel to the dipoles. The T-R and A-T-R switches are built in open wire line, using open tungsten spark gaps in the conventional configuration.

By a fortunate coincidence, a very suitable celestial radio source is available as a check on antenna performance. The station is located at latitude 40.2° while the radio source known as Cygnus-A is at 40.5° [4]. For practical purposes Cygnus-A can therefore be considered as passing through the center of the main beam of the antenna. A number of total power records of the Cygnus source as it transits the beam were taken as a means of measuring the east-west antenna pattern and calibrating the gain. One such record is shown in figure 1.

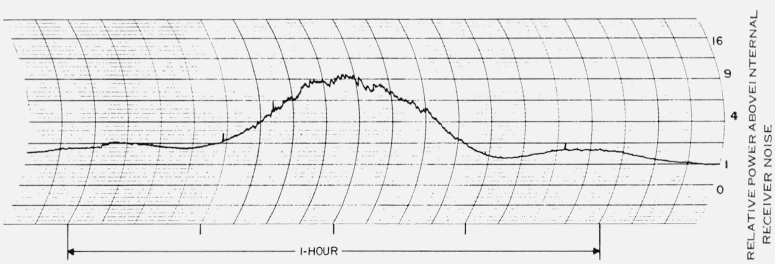

FIGURE 1. *Example of the kind of total-power record obtained using the large dipole-array during transit of the radio source Cygnus-A.*

Although scintillations were often quite severe, several passes were obtained when scintillations were less than 10 percent of the total power received. Hence, a good estimate of the maximum power could be made. As may be seen from figure 1, the antenna exhibits the behavior expected in the main lobe and lowest order side lobes for a uniformly illuminated aperture. The measured beam width between half-power points is 3.1°, and between first nulls is 7.2°, agreeing with the values expected for an aperture 16 wavelengths wide [5]. It may be noted that there appears to be little or no broadening of the trace of Cygnus due to the broad angular component known as Cygnus-X. Using the best available measurements of the flux of power from Cygnus-A, those of H. W. Wells [6], it is estimated that 1.75×10^{-18} w per c/s should be available at the antenna terminals at the peak of the Cygnus trace, assuming perfect efficiency on one polarization. It was found tnat an equivalent noise diode current of 110 ma through 50 ohms gives a comparable deflection. Since this current generates an input power of approximately 4.4×10^{-19} w per c/s, the antenna is estimated to be 25 percent efficient or to have a degradation in gain of about 6 db. The accuracy of the measurements is certainly not better than 1 db. The loss in efficiency may be explained by the following three factors: 1 db. is lost in coaxial lines and matching networks connecting the T–R system to the receiver input terminal (or transmitter output terminal). Perhaps half of the remainder is represented by resistance losses in the poorly screened ground, and in the transmission lines, matching networks, etc. The remaining efficiency is presumably lost through spurious radiation from portions of the open-wire transmission lines on which matching was not attempted, portions of wires and guys only partially removed from the circuit by quarter-wavelength shorted stubs, etc. This radiation must appear as energy in sidelobes randomly distributed over the hemisphere illuminated by the antenna.

2. Scatter Observations Identified

The kind of observations obtainable using the conventional "A′scope" display is illustrated in figure 2. Cosmic noise is represented by the constant "grass" level, peaking at about one-third of the vertical scale. The apparent rise in the grass level, peaking broadly at about 350 km, is due to the incoherent scatter echo. The sharp peak at about 80 km appears to arise from the same kind of scatter responsible for at least part of oblique path VHF scatter propagation [7]. Further observations of this D region scatter have been made and it is hoped will be discussed in a separate paper.

The kind of record obtainable using an A′scope presentation of the "integrator" output is illustrated in figure 3, a direct photograph of the oscilloscope-screen. The cosmic noise level in this case is just under the first horizontal line above the lower border of the photograph. In this record, and others on the same presentation shown below, the broad features of the electron-density profile of the F region of the ionosphere may be clearly seen. Note that while the E region electron density must be close to one-tenth the F region maximum, the echo intensity varies only slightly between the heights of 100 and 350 km. This results from the decay of system power sensitivity with the inverse square of the height.

Before positively identifying the observed echoes as incoherent scatter, one must reject the possibility that they might arise from sidelobe response to some other known mode of propagation. Overlapping meteor echoes observed via the sidelobes are an obvious possibility. Figure 4 illustrates the appearance of a typical meteor echo. The sharp spike in the midst of the broader but weaker rise in the grass level is a meteor echo which lasted a fraction of a second compared with the 4-sec exposure time. Figure 5 shows the integrator presentation for a time close to 0600 hours local time. As would be expected for the F region, the main features of the broad trace are considerably reduced in amplitude compared with the midday record of figure 3. Between 100- and 200-km range, however, there appears to be stronger echo intensity with a much rougher distribution, due to the greatly increased meteor activity typical of times near 0600. Adjacent integrator intervals show intensities which vary randomly in an uncorrelated fashion. Rejection of meteor echoes as a serious contributing factor therefore seems justified, if allowance is made for their occasional appearance on the records, particularly at ranges less than 700 km.

Long-range F layer-propagated ground-backscatter occasionally produces strong interfering echoes in the system. Examples of such echoes are shown in figures 6a and 6b, where slightly more than one complete radar cycle is displayed. A second transmitted pulse occurs in 6a and 6b at about 5,000-km indicated range. Figure 6a is a record obtained in the customary manner using the large broadside antenna both for transmitting and receiving. Figure 6b was obtained by substituting, for reception, a simple half-wave dipole antenna located one-half wavelength above ground. Since the illumination of the scattering medium was the same for both examples, it is clear that the response of the large antenna at low radiation angles approximates the response of the dipole within a few decibels. Figures 6c and 6d were taken using expanded range and intensity scales for making the same comparison. Figure 6c, taken with the large antenna used for both transmitting and receiving, shows the usual F region profile. Figure 6d, taken with the large antenna used for transmitting, and the dipole for receiving, shows no sign of the F region profile. By utilizing this comparison, and by taking continual care to adjust the spacing between successive transmitter pulses, it has been possible to detect and eliminate any contribution to the interesting part of the records due to long-range backscatter. A similar comparison of the dipole with the large antenna has permitted the rejection of the possibility of important echo contributions due to magnetic field alined irregularities, such as from the aurora.

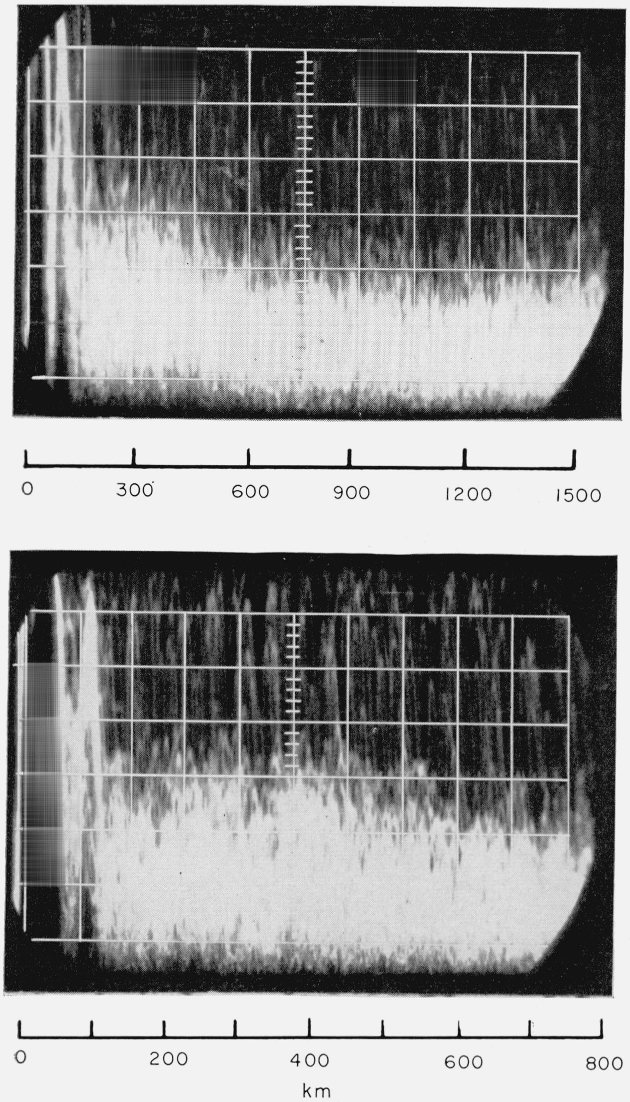

FIGURE 2. *A'scope photographs obtained using 4-sec exposure.*
Near midday, October 22, 1958.

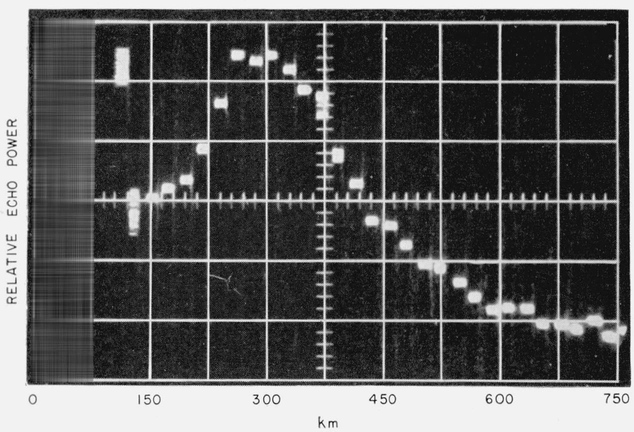

FIGURE 3. *A'scope photograph using the "integrator" display.*
1135 l.t., February 27, 1959.

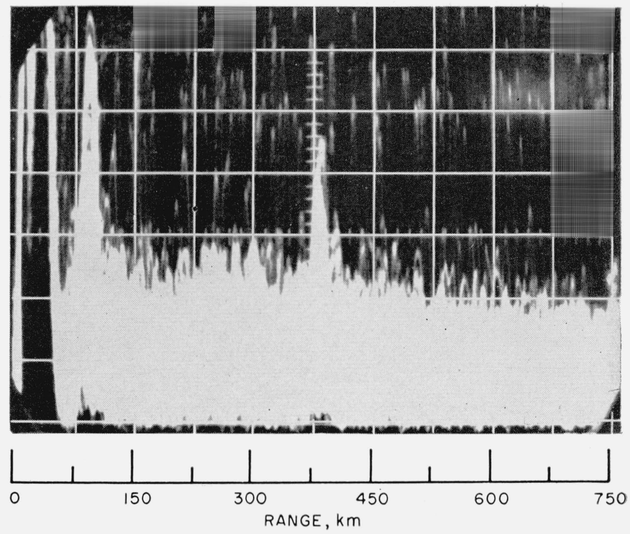

FIGURE 4. *A'scope photograph showing transient trace due to a meteor echo, in addition to incoherent scatter.*

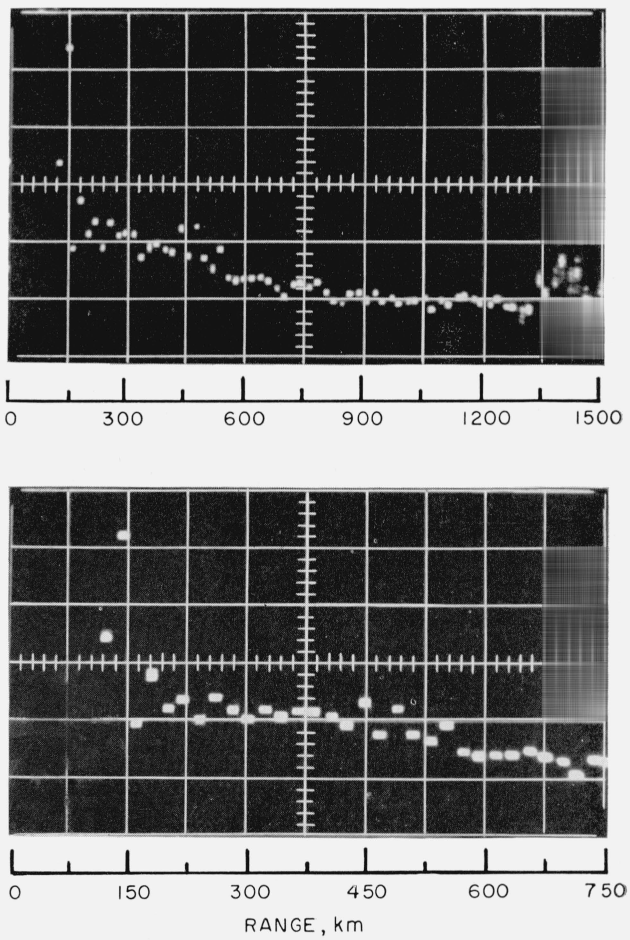

FIGURE 5. *Integrator A'scope photographs taken at about 0600 l.t., February 28, 1959.*

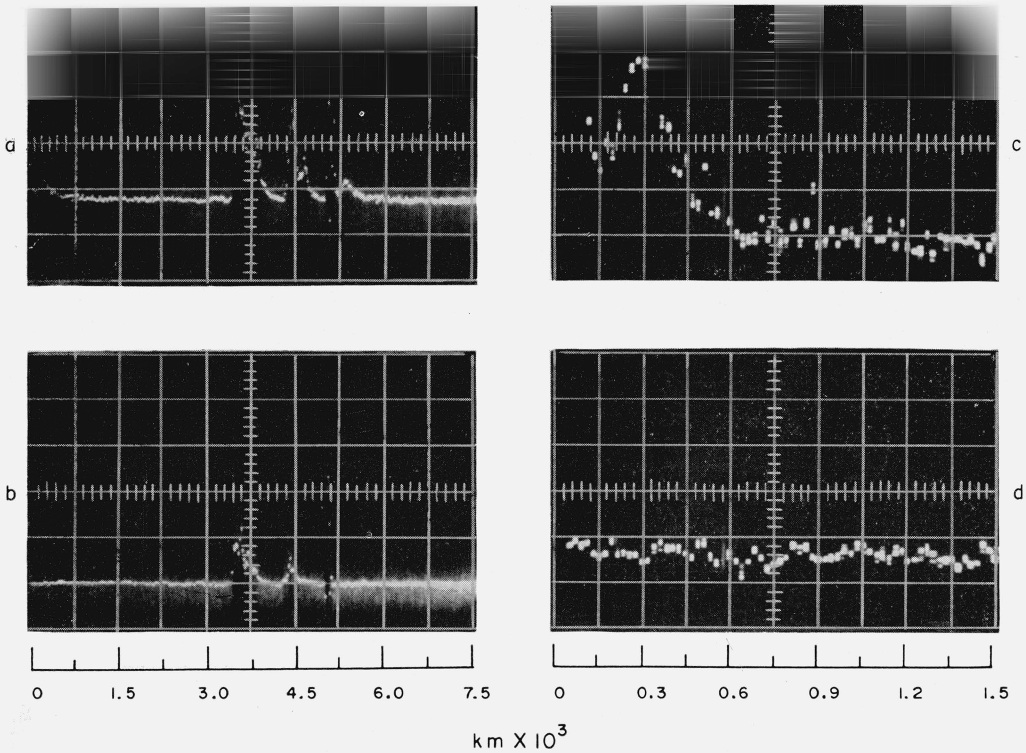

FIGURE 6. *Long-range backscatter records obtained using: (a) Large array, (b) single dipole—for reception. Short range, high gain records using: (c) Large array, single dipole—for reception.*

While it is possible that other interfering modes of propagation might appear, a convincing final argument for the identification of the echoes as incoherent scatter is provided by a comparison with known F region characteristics. Assuming the height of the F region maximum electron density to remain roughly constant, the maximum scatter intensity should be proportional to the square of the $F2$ region critical frequency observed on a conventional vertical incidence sweep frequency sounder. Unfortunately, such a sounder had not been installed at the Long Branch, Illinois station at the time of this experiment. The scatter records were compared with critical frequencies observed on the sweep frequency sounder at Fort Belvoir, which is located at about the same latitude. Critical frequencies were estimated from the scatter records by normalizing the system sensitivity such that at one particular hour the maximum indicated electron density corresponded with the Fort Belvoir critical frequency for that hour. The Fort Belvoir critical frequencies were then found to lie within one-half megacycle of the values calculated from the scatter records, on the several days in February 1959 when comparisons were made. Figure 7 is a replot of figure 3 to account for the height variation of sensitivity, and should be proportional to the electron density versus height profile. The dotted curves superimposed on this profile are computed from the Fort Monmouth and Boulder ionograms for the same day and local time using the NBS

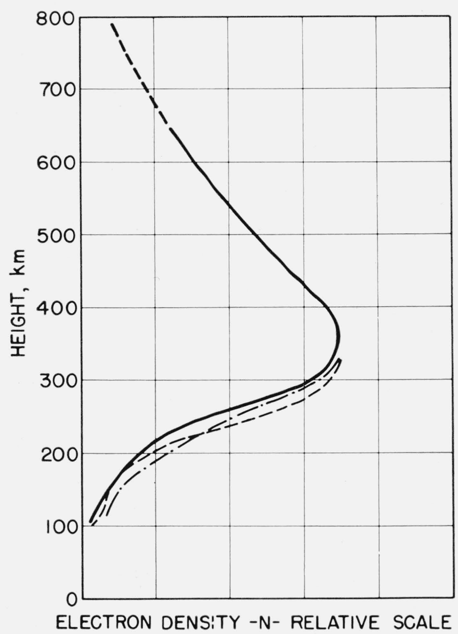

FIGURE 7. *Electron density profile obtained from A'scope display of figure 3.*

Dotted curves are "true height" profiles computed from Fort Monmouth and Boulder ionograms normalized to the same f_0F2.

"true height" technique [8]. A second pair of observations, made in the same way, are shown in figures 8 and 9, respectively. This pair, made shortly after sundown, shows the rapid decay of ionization below F max compared with figures 3 and 7. Although neither the agreement of the scatter profile with the ionograms, nor the ionograms with each other, is exact, the results exhibit a convincing similarity. A more precise comparison has since been made utilizing a conventional sounder at the Long Branch station [26].

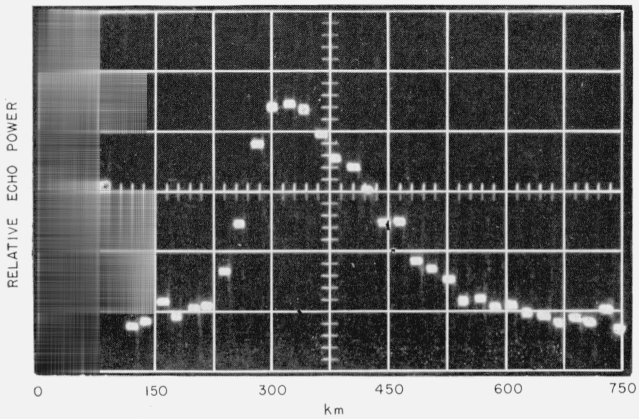

FIGURE 8. *Integrator A'scope photograph taken at 1940 l.t., February 27, 1959.*

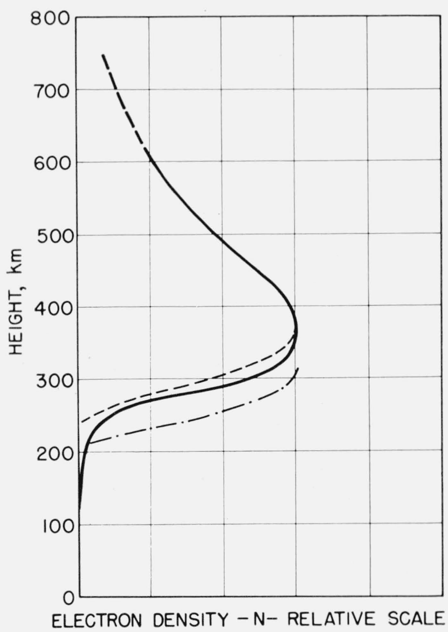

FIGURE 9. *Electron density profile obtained from A'scope display of figure 8.*

Dotted curves are "true height" profiles computed from Fort Monmouth and Boulder ionograms, normalized to the same f_0F2.

3. Comparison of Experimental Observations With Theoretical Predictions

Comparison of the observed scatter power with the values predicted theoretically using Gordon's method [1] is possible, using the system parameters as outlined above. Magneto-ionic effects in the ionosphere, which produce Faraday rotation of polarization [9], degrade the system sensitivity by 3 db. This may be seen as follows: Somewhat over 50 complete rotations of polarization would be expected for a 41-Mc/s wave passing twice through the entire daytime ionosphere. Thus more than one-half revolution can be expected over any given interval of 18-km height corresponding to the transmitted pulse length. The net effect is similar to a randomization of polarization so that on reception the linearly polarized antenna can extract only half of the energy in the incident scattered wave.

Based on all these considerations, the peak scatter power at the receiver terminals corresponding to an $F2$ region critical frequency of 12.8 Mc/s (2×10^6 electrons/cm^3) should be 3.5×10^{-16} w. The height of the $F2$ region maximum electron density is here assumed to be 330 km, in good agreement with the observed height. The observed scatter power at the receiver terminals, corresponding with the $F2$ region maximum, was 4.3×10^{-16} w when the critical frequency was about 13 Mc/s. This must be considered as excellent agreement between experiment and theory, inasmuch as the overall accuracy of the computation cannot be better than plus or minus 3 db. For receiver bandwidths of both 9 and 35 kc/s the observed scatter power was the same.

In spite of the agreement of the observed echo intensity with the prediction, one major difference is apparent from the equality of the echo strength for 9- and 35-kc/s bandwidths. Doppler broadening of the echo due to independent scattering from free electrons should be about 100 kc/s at F region heights. The F region temperature is here assumed to be approximately 1500 degrees Kelvin [10]. Since the 9-kc/s bandwidth apparently admits as much echo power as the 35-kc/s bandwidth, the echo must be no broader than about 9 kc/s. The spectrum of the received energy must be at least 9 kc/s wide since that is the spectral width of the transmitted energy. One must therefore conclude that the spectral broadening due to the scattering process must be considerably less than 9 kc/s, otherwise the received spectrum would be considerably wider. To further test this conclusion, observations were made with the receiver slightly detuned from the transmitted frequency. One such sequence is shown in figure 10. Figure 10a was obtained in the same manner as the records shown above with the receiver correctly tuned. Figures 10b and 10c were obtained with the receiver tuned 15 kc/s above and 15 kc/s below the transmitter frequency, respectively. The latter two records show no sign of an echo from the F region, leading to the conclusion that the bandwidth of the echo power must be considerably less than half the receiver pass band of 9 kc/s. Since the average ve-

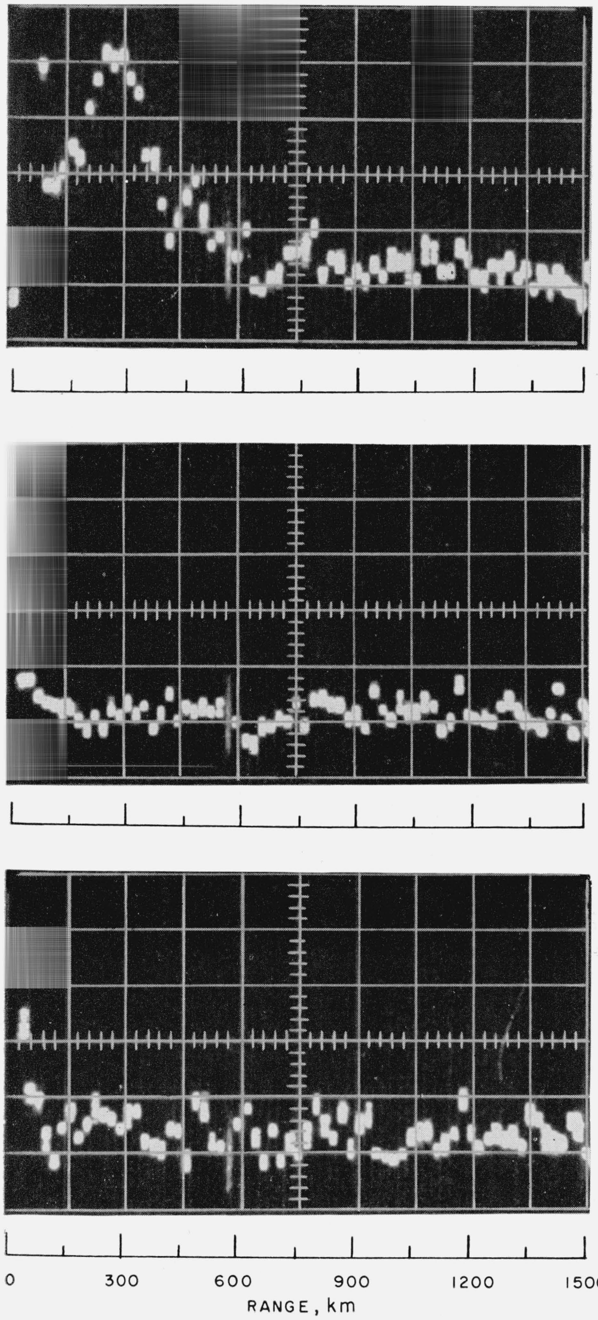

FIGURE 10. *Integrator A'scope photographs taken: (a) With receiver tuned to transmitted frequency, (b) receiver tuned 15 kc/s high, (c) receiver tuned 15 kc/s low.*

locity of the electrons is expected to be proportional to the square root of their kinetic temperature, the 1500 degree estimate for the F region would have to be high by a factor of more than 100 to account for these observations on the simple theory.

Other measures of ionospheric temperature appear to be consistent with the 1500 degree estimate well within one order of magnitude. The second part of this paper is a discussion of the observed echo bandwidth.

Recently several theoretical papers have appeared which predict that the intensity of the scatter from the ionosphere should be half that predicted by Gordon's method [11, 12, 13, 14, 15]. The reduced scatter power predicted by these theories is within the experimental accuracy of the present experiment and no disagreement can be inferred. Another experiment has also been performed by Pineo and others [17] at 440 Mc/s, and results very similar to ours have been obtained. Their measurement of the spectrum of the echoes was much more precise than possible with our equipment, and is discussed further in Part II, 3.

Part II. Modified Theory

1. Independent Particles Versus Density Fluctuations

There is a considerable body of literature on the subject of scattering of electromagnetic radiation by small particles. Following the early work of Lord Rayleigh [2], attention has been given to the criterion by which individual particles can be considered to scatter *independently* or, using Rayleigh's term, in *arbitrary phase.* The word "independence" is used in this paper in the statistical sense and implies a lack of statistical correlation.

For particle densities greater than those for which such independence is correct, calculation of the scatter properties has been found to depend upon an understanding and description of the statistical fluctuations of particle density. The reader is referred to Rocard [18] for a comprehensive summary and bibliography of the early developments in this field. A better statistical description as well as a bibliography of more recent work are given by De Boer [19]. Using the De Boer approach, the writer suggests the following arguments as a basis for a modified theory of the incoherent scatter.

The density fluctuation theory begins with a statistical description of the spatial distribution of the scattering particles. When *neutral* particles are separated by many particle diameters so that interparticle forces are small, a random distribution of locations is established. (This is strictly true, of course, only when the gas has no strong gradient of density in a distance comparable with the scale size under consideration.) The statistical fluctuations of particle density create a region containing weak irregularities of refractive index which are capable of scattering radio waves. *When the particle distribution can be considered random over all scale sizes comparable with the wavelength of the incident radiation, De Boer* [19] *finds the scattered intensity to be equal to that of the simple independent scattering case.* De Boer implies that the scattered *intensity* would not depend upon the scale size chosen for the irregularities. On the other hand, we shall see that a specification of scale size is important to an understanding of the *spectral broadening* of the scattered radiation.

On the other hand, when the spatial distribution of the electrons is such that the phases of their respective contributions cannot be considered mutually independent, the simple Doppler profile of the previous paragraph is no longer correct. This conclusion is supported by the fact that the observed frequency broadening of the incoherent scatter echo is much smaller than that implied by the above Doppler profile when any reasonable kinetic temperature is assumed for the ionosphere. (As in the previous section, we have assumed that the temperature in the F region is of the order of 1500 degrees Kelvin.) Assuming that the correct explanation of the discrepancy is to be found in terms of a collective description of the spatial distribution of the electrons —the fluctuation in density of the electrons due to the Brownian motion—an estimate of Doppler broadening which is consistent with the observations is obtained below.

A simple illustration will serve to show how a group of particles, having a Maxwellian distribution of thermal velocities, could under the proper conditions produce a resultant having little or no Doppler shift by comparison with the broadening which the particles would produce, were they independent. Figure 11a is an idealized sketch of a radar system intended for study of the ionosphere. The antenna beam is approximated by a uniform response within the sector, and zero response outside this range.

We now consider the relation between the concept of density fluctuations as scattering centers and the concept of individual electrons as scattering centers. In all cases of interest, the incident electromagnetic field is modified so slightly by scattering that as a first approximation the individual electrons must give rise only to single scattering of the incident energy. That is, each electron is considered to be coupled electromagnetically only to the radar system and not to other free electrons. Hence, a detailed summation of the contributions of all the free electrons individually would lead to the correct description of the scattered field. Such a summation would have to account for the phase of the contribution from each electron individually—an enormous computational problem. By taking advantage of ways in which large groups of electrons—and their individual contributions—exhibit collective behavior, it is possible to simplify the problem considerably. If the phase of the contribution from each free electron is uniformly distributed throughout a range of at least several times 2π radians and is statistically independent of the phase of the individual contribution from each other free electron, the approach used by Rayleigh [2] and Gordon [1] is correct. In this case the scatter due to each particle appears in the resultant signal at the Doppler-shifted frequency corresponding to its line of sight velocity, $f-f_0=2v/\lambda$. The spectrum of the scattered energy (arising from a monochromatic source) would have the functional form of the probability density function describing the number of electrons found at each line of sight velocity. Ordinarily this would reduce to the usual Gaussian, or Maxwellian [12], form

$$S(f)=S(0)\exp\left[-\left(\frac{f-f_0}{\frac{\pi^{\frac{1}{2}}\bar{v}}{\lambda}}\right)^2\right]$$

$$=S(0)\exp\left[-\left(\frac{f-f_0}{\frac{2}{\lambda}\left(\frac{2kT}{m_e}\right)^{\frac{1}{2}}}\right)^2\right], \qquad (1)$$

where $S(f)$ is the power spectrum function of the scattered energy in terms of the spectral frequency, f. The transmitted frequency is f_0, $\bar{v}$ is the mean speed of flight of the electrons, λ is the free space wavelength of the radio energy, T is the kinetic temperature, k is Boltzmann's constant, and m_e is the mass of the electron.

As shown by the phase vector, or phasor, diagrams in figure 11(b), the phase $P(A)$ of an echo arising from from a scatterer at a distance A from T is 360 degrees advanced relative to the phase $P(B)$ of the echo from another scatterer lying a distance $B=A+\lambda/2$ from T. In three dimensions A and B are the radii of the spherical surfaces of constant phase with centers at T.

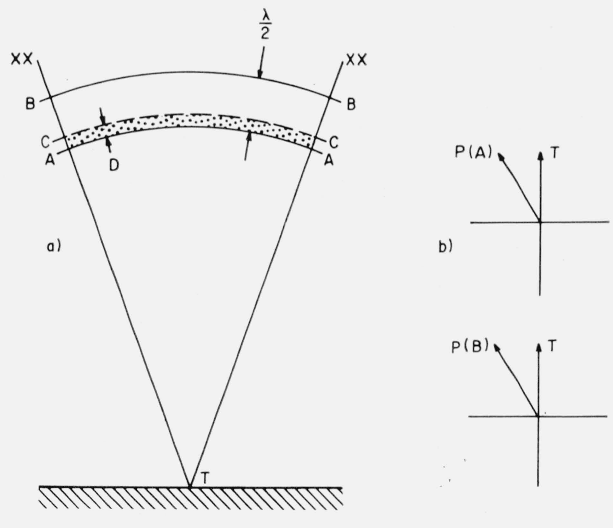

FIGURE 11. *Schematic sketch of radar system.*

Phasors are referred to the transmitted phase—T.

We now consider our group of particles to be initially located at points within the slab AA–CC which has thickness $d \ll \lambda/2$. The phasor resultant signal due to this group of particles is shown in figure 12 along with the individual components, for representative initial particle locations.

We allow enough time to pass that the particles have diffused into a slab of somewhat greater thickness than AA–CC. Figure 13 is a phasor diagram illustrating how the individual components and the

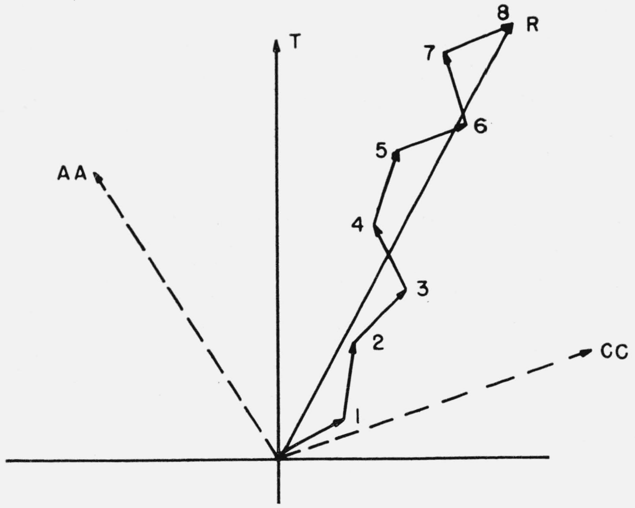

FIGURE 12. *Phasor diagram illustrating the initial distribution of contributions due to particles within the slab* AA-CC.

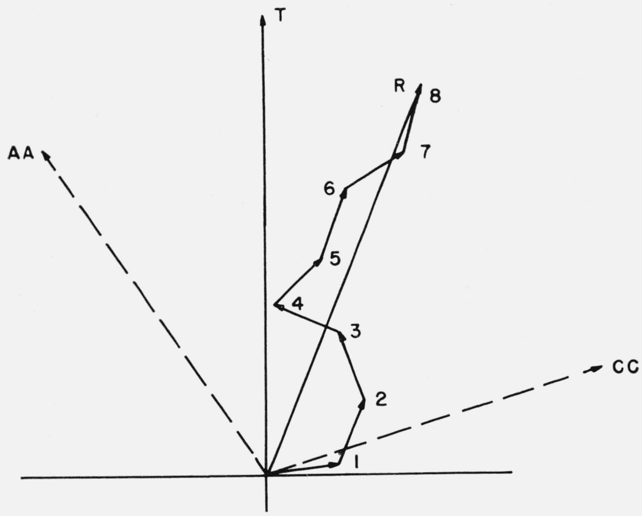

FIGURE 13. *Phasor diagram illustrating how the distribution of electron contributions of figure 12 may have changed after a short time has elapsed.*

resultant phasor might have changed during that time. We see that the phases of the individual components have rotated considerably in the new picture, but the phase and amplitude of the resultant have changed only slightly. Of course, the degree to which the resultant changes is a function of the distribution of rates of change of the components. In the simple illustration above, the reduced rate of change of phase of the resultant can be viewed as an average of the rates of change of the components. The reason for discussing rates of rotation of phasor components is, of course, that the shift of frequency of any component relative to the transmitted frequency is directly proportional to the rate of phase rotation of that component. In the simple illustration the phases of the components would rapidly become randomized as diffusion set in. Until this happened, the probability density function describing the equivalent spectrum of the resultant signal would be markedly reduced in width relative to the spectrum of the individual components. This simple illustration is analogous to the scatter by free electrons in an underdense meteor trail. The echo from a short length of such a trail is not Doppler broadened by thermal motions of the electrons.

We now ignore, temporarily, the continuous motion of the electrons and consider the statistics of the signal scattered from a portion of the ionosphere in which the electrons are suddenly rendered motionless. We consider for two limiting cases the distribution of signal strength arising from a slab *AA–CC*, as in figure 11 above, on a number of successive such instants, or "realizations." In both cases we consider that the particles are randomly distributed within the slab, all positions having equal probability. When, as above, the thickness, d, of the slab is taken to be small relative to $\lambda/2$, or more specifically when $d \lesssim \lambda/4\pi$, the resultant *amplitude*, R, is almost equal to the arithmetic sum of the contributions from the individual particles. That is, the resultant echo power, $P \approx n^2 r_e$, where r_e is the scattered echo power arising from one electron and n is the total number of electrons within the slab. The phase of the resultant is somewhere between the phase corresponding to AA and that corresponding to CC, depending upon the distribution of particle density within the slab. The distribution of the resultant signal amplitude over a large number of realizations is simply proportional to the distribution of n. By virtue of their close spacing in phase, the particles in this case contribute to the resultant collectively, i.e., coherently.

The other limiting case is for $d \gg \lambda/2$. This is the two-dimensional random walk case considered by Rayleigh [2], in which $R_{\text{avg}} = nr_e$ and the resultant for any given realization is Rayleigh distributed.

$$P(R)\, dR = \frac{2}{nr_e^2} R \exp\left[-\frac{R^2}{nr_e^2}\right]$$

where $P(R)dR$ denotes the probability that R lies within the range R to $R+dR$. In this case the phase of the resultant is uniformly distributed throughout 2π radians, and without some detailed knowledge of the positions of the individual components the phase cannot be specified within that range. Judging from the result of the argument for a thin slab above, we can satisfactorily describe the properties of the signal from a thick slab by dividing it into components having $d \lesssim \lambda/4\pi$ i.e., knowledge of the position of every particle is not necessary. This is the import of the arguments given by Rocard [18] and De Boer [19] which were referred to above.

Evidently the transition from the collective behavior of the thin slab, to the incoherent behavior which describes the thick slab, is by no means sudden. As d increases from $\lambda/4\pi$ to larger values there is a transition region in which the coherent aspects of the thin slab are gradually lost. It should suffice to say that there should be some maximum d for a thin slab to exhibit essentially coherent behavior, and some minimum d for a thick slab to exhibit essentially incoherent behavior. We make the approximation that for some intermediate value of d, the electrons within each slab contribute coherently to the scatter, while the contribution from adjacent slabs is uncorrelated statistically in phase or in amplitude. This intermediate value of d will be termed the *scale* of the irregularities of electron density and designated by the letter L. We note, for the case of a radar, that L is of order $\lambda/4$.

A much more precise handling of these concepts is embodied in the theoretical work done in recent years with specific application to the problem of radio wave scattering from irregularities arising in ionospheric or tropospheric turbulence [20, 21, 22]. Here the intensity of the scatter is found to be proportional to the integral over the scattering volume of the spatial autocorrelation function of refractive index, taking phase into account. For a more precise account of this development the reader is referred to the papers cited. What is important to our present problem is that this integral is formally equal to the *spectrum* of the irregularities evaluated at the wavenumber k for the case of direct backscattering as observed by radar. The reciprocal of this wavenumber, $1/k$, is identified with the scale L multiplied by a constant close to unity.

As noted by Balser [13], the spectrum of scatter from a large volume containing independent scattering centers is simply the sum of the spectra associated with the individual centers. In other words, in our approximation the spectrum of incoherent scatter arising from a large volume of the ionosphere should have the form of the spectrum arising from any given slab of scale L (provided the mean characteristics of the volume are uniform throughout—temperature, density etc.). Since we have approximated the contents of such a slab as scattering coherently, the instantaneous field strength arising due to the slab is represented as proportional to the total electron content found at any instant within the confines of the slab.

2. Role of the Ions in Determining the Criterion for Independence

We now face the problem of deciding under what conditions the scattering is best considered to arise from independent particles, and under what conditions the representation using irregularities of particle density gives a more accurate result. We shall see that the ionosphere, being a partially ionized gas, provides a unique opportunity to answer this question.

Gordon [1] made the assumption that the characteristic scale L is meaningful only when the mean free path, l, of the scattering particles is small compared with L. He uses the converse of this assumption as the criterion for independent scattering from individual particles—i.e., when $L \ll l$. We shall refer to this as the "l criterion." An alternate and very different assumption is that the criterion could be set by the relationship of the scale L to the mean separation of neighboring particles. This says that when $L \gg n^{-1/3}$, one must use density fluctuations of scale L to calculate the scattering properties. Here $n^{-1/3}$ is the approximate mean separation between neighboring scattering particles and n is the number of particles per cubic centimeter. We shall refer to this as the "$n^{-1/3}$ criterion." We note that, in the F region and above, l is of order 1 km or greater, while a typical F region electron density of 10^6 per cm^3 corresponds to $n^{-1/3}=0.01$ cm. The scale L, for the experiments reported in the first part of this paper, is of order 2 m. Thus we have $n^{-1/3} \ll L \ll l$, i.e., both criteria are satisfied. However, since the independent particle approach suggests that a Doppler broadening much greater than actually observed should be found, we are led to inspect the consequences of the density fluctuation approach for the ionosphere.

We now offer a crude argument which demonstrates that the distribution of the free electrons cannot be entirely random. We consider the case of scattering from a randomly distributed gas of neutral particles having approximately the same total particle density as that of the F region of the ionosphere. Long-range forces among the particles are considered to be negligible so that only during collisions do the particles affect the motions of each other. Referring to figure 11 we consider the contents of a scattering slab AA–CC the thickness of which is equal to L. Since we have approximated that all particles located within this slab scatter coherently, the amplitude of the scattered contribution from the slab is proportional to its total content of particles. Beginning at a time t_0 we note that at a time $t_0+\tau$, the content of the slab has almost completely changed due to the line-of-sight motions of the particles. Those particles which had been within the slab at t_0 have almost all moved to new locations and have been replaced within AA–CC by other particles. The time τ is therefore given approximately by the time taken for the average particle to move a line-of-sight distance equal to L, or $\tau=L/\bar{v}_z$, where $\bar{v}_z$ is the root mean square line-of-sight velocity of the particles. Another way of saying this is that the content, or state of fluctuation, in the slab AA–CC loses *correlation* with its state at a previous time, if at least τ seconds have elapsed since that previous time. Using the well-known relation between the autocorrelation function and the power spectrum [23], we therefore estimate that the bandwidth of the scattering from the slab of thickness L is approximately $\Delta f=1/\tau=\bar{v}_z/L$. This corresponds to a bandwidth of $\Delta f=4/\lambda \times 0.96\bar{v}_z=0.96\bar{v}_z/L$ from eq (1), noting that $\bar{v}_z=1.085\bar{v}$ (see reference [12]), for the case of independent scattering from the particles. We see that the broadening expected for the two approaches is the same. Turning to the actual case of incoherent scatter from the F region of the ionosphere, we believe that the velocities of the free electrons are equivalent to the supposed kinetic temperature of about 1,500 degrees Kelvin. Since the observed frequency broadening of the scatter is much smaller than expected for a fully random distribution of electrons at this temperature, we look for an explanation in which some mutual correlation among the electron positions and velocities is found.

We now explain the observed echo spectrum through the use of the density fluctuation approach. Insight regarding the distribution of the free electrons can be gained from a paper published by Pines and Bohm [24]. These authors considered the case of an electron gas embedded in a uniform smear of positive charge. Since the mobilities of electrons are so much greater than the mobilities of ions, this case is a good approximation to the situation in the ionosphere. Pines and Bohm found that the mean square intensity of fluctuations of electron density of scale size L is *reduced,* compared with that for the case of random motions, by a factor $(2\pi)^2\lambda^2_D/L^2$, for all $L\gg\lambda_D$. In this relation λ_D is the so-called Debye shielding distance, $\lambda_D=(kT/4\pi ne^2)^{1/2}$ and T is the temperature in degrees Kelvin, k is Boltzmann's constant, n is the electron density and e is the charge on the electron. Only for the condition $L\ll\lambda_D$ do the free electrons behave as though randomly distributed. Since L is of the order of one or more meters for the present and other proposed radar experiments, and λ_D is less than a few centimeters at all heights of interest, the *independent* random fluctuations in electron density of scale L must be viewed as negligible. The intensity of scatter arising from such fluctuations must also be negligible.

Similar considerations hold for the case of a gas of positive ions embedded in a uniform smear of negative charge. In both cases the mutual repulsion of charges of like sign causes the reduction of fluctuations. However, Spitzer [25] has shown that in a real plasma, the mobility of the electrons is so great as to cause their equilibrium distribution to balance the fluctuations of positive charge density nearly completely. Once the fluctuations of positive charge density are balanced by equal and opposite fluctuations of negative charge density, the ions may be distributed nearly randomly, as they would be distributed if they were neutral particles. The fluctuations of the free electrons from their equilibrium distribution are strongly reduced, in accordance with the argument of Pines and Bohm, inasmuch as the positive ion distribution departs only slightly from the approximation of a uniform smear of charge. Thus one is left with a picture of scattering from fluctuations of free electron density of scale L such that the *magnitude* of the scatter is the same as for a randomly distributed neutral gas, and its *rate of change* is appropriate to particles having the mean mass of the ions. The rate of fading must therefore be computed as if the positive ions, rather than the electrons, produced the scattered echo. This should not be taken to imply that the ions make a direct contribution to the scatter, since the free electrons have a considerably greater radar cross section.

For example, if we assume that the ionized constituent of the F layer is atomic oxygen, we estimate the Doppler broadening

$$\Delta f=\frac{v_z}{L}\approx 540 \text{ c/s}$$

for an operating frequency of 41 Mc/s and a temperature of 1,500 degrees Kelvin.

The result of the arguments given in this section is that the characteristics of incoherent scattering at VHF, for $n^{-1/3}\ll L$, if also $\lambda_D\ll L$, may be estimated by substituting for the ionosphere a fictitious gas of randomly distributed neutral particles with special characteristics. These particles would have the radar cross section of the free electron, and the mass (and consequent velocity distribution) of the ions. For $n^{-1/3}<L<\lambda_D$ the Doppler broadening should approach that of independent electron scatter. For both cases, the scattering is best described in terms of fluctuations of particle density. Only when $n^{-1/3}\gg L$ do we expect to find that the simple independent particle description of the scattering is correct.

3. The Echo Spectrum

In an informal report [16] and in the present paper, approximate calculations for the fading spectrum are presented. Two more rigorous theories have been developed by other workers [11, 12] which, although using different approaches, predict the same spectral distribution. This theoretical spectrum has been experimentally confirmed by Pineo et al., at 440 Mc/s [17]. Two other theoretical papers dealing primarily with the intensity of the scatter have also appeared [13, 14].

At 41 Mc/s, the bandwidth of the spectrum predicted by these authors is about 1,000 c/s for echoes from the F region. Recent statistical studies, of the fading of echoes observed on the equipment described in this paper, show that the bandwidth of 41 Mc/s echoes from the F region is certainly greater than about 600 c/s. This observation agrees with the results of Pineo et al. [17], and with the above mentioned theories.

The author's approximate calculation of spectral width differs from the precise theories by more than a factor of two. The difficulty apparently resulted from the assumption that echo contributions from adjacent volumes of scale L can be considered to be statistically independent. It is true that the fluctuations in density of such adjacent volumes are independently distributed. However the electrons, found in a given volume at one instant, move into adjacent volumes while the phase of the individual contribution of each electron various continuously. This continuous variation of phase implies a sort of long term correlation among many irregularities of scale L. The assumption basic to the approximate calculation was therefore not correct. None of the theoretical treatments mentioned above suffer from this difficulty. In spite of this difficulty with the author's treatment, the heuristic arguments of the foregoing sections offer an approximately correct explanation of the physical principles involved. The rate of rotation of the resultant phasor representing the contribution of the volume V must be related to the average of the rates of rotation of its component phasors. Since the individual components must have rates of rotation distributed in both advancing and receding senses relative to the reference, the average must be less than the mean absolute rate of phase rotation of the components.

4. Magnetic Field Effects

In the foregoing sections it has been assumed that the effects of the earth's magnetic field can be ignored. In regions where the mean free path is large compared with the wavelength this can only be true for propagation parallel with the lines of force of the earth's field, and for frequencies large compared with the gyrofrequency. The latter condition will always be satisfied in the earth's ionosphere and exosphere at practical operating frequencies. The quantity $\bar{v}$ used in section 2 is related to the Maxwellian distribution of particle velocities and to the components of that distribution in the line of sight. If the magnetic field does not seriously alter the Maxwellian components of velocity parallel to the field, then the results of the preceding sections will be approximately correct for a vertical incidence radar in temperate or arctic zones.

If the propagation is nearly perpendicular to the lines of force of the earth's field, magnetic control of the particles can be serious. The ions now execute paths with a circular component of motion the plane of which includes the line of sight. If L is large compared with the radius of this circular motion, the particles are restricted to paths nearly parallel to the surfaces of constant phase. In this case the fading spectrum should be calculated as in the case of short mean free path [11] using an equivalent coefficient of diffusion, D_m, at right angles to the magnetic field. When L is small compared with the gyroradius, there should be little effect on the main spectrum of fading.

An interesting case arises when L is of the same order of magnitude as the average gyroradius of the ions. All ions of the same mass must have the same gyrofrequency within a volume of scale L. The rate of migration of the center of rotation of any given particle out of the volume, L thick and bounded by surfaces of constant phase, is given approximately by

$$\frac{\bar{v}}{\left[\left(\frac{R\lambda}{2}\right)^{1/2}\right]}$$

where R is the range to the volume in meters and the expression in brackets is the length of the first Fresnel Zone. The fading due to this component of motion would be only a few cycles per second at an operating frequency of 50 Mc/s. The gyrofrequency of the largest ion expected, O_2^+, is of the order of 10 c/s at F region heights over the magnetic equator. The gyrofrequency for protons, likely to be encountered in the exosphere, is of the order of 250 c/s. Thus each ion may be expected to execute a minimum of several turns during the period taken for its center of rotation to move a small fraction of 360° phase from the radar station. We may therefore expect that the scatter echo will exhibit a tendency toward periodicity having a frequency equal to the ion gyrofrequency. This periodicity should be easily measurable using autocorrelation analysis of sequences of pulse echoes.

From the approximate frequency of the periodic component one may infer the mass of gyrating ions. If several ionic constituents are present, several periodic components should appear, the relative power in each being proportional to the number of ions of the appropriate constituent. In this way the scatter radar can be used as a sort of mass spectrometer to determine the variation of ionic constituents with height. Once the ionic constituents have been identified, precise determination of the gyrofrequency should provide a measure of the profile of the magnetic field intensity as a function of height. At the same time, the thermal broadening of the echo spectrum will be only slightly reduced by the magnetic field when L is of the order of magnitude of the gyroradius. Knowing the mass of the ionic constituents, it should be possible to make a precise determination of temperature. A more refined theory to take the magnetic effects into account is under development.

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