RP316

THE LATERAL CHROMATIC ABERRATION OF APOCHROMATIC MICROSCOPE SYSTEMS

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ABSTRACT

Methods are described for the measurement of the lateral chromatic aberration and distortion of microscope objectives and eyepieces. Measurements have been made on 20 apochromatic microscope objectives of four different makes, and on 10 compensating eyepieces of two makes. The results show that the compensating eyepieces, in general, do not entirely compensate the chromatic aberration of the objectives, and that the complete microscope system is usually undercorrected. The bearing of this on the performance of the system in photomicrographic work by a 3-color process and in the direct visual observation of objects is considered in detail.

CONTENTS

I. INTRODUCTION

For the apochromatic microscope objective the images produced by light from different parts of the spectrum differ in size. As a consequence, at all points other than the center of the field, the images do not register, the lack of registry increasing with increase of distance from the center of the field. Such a defect is termed lateral chromatic aberration. This aberration is designedly left in the objective because, by this means, a more perfect correction of spherical aberration for the entire range of the visible spectrum may be secured. The lateral chromatic aberration in the image, produced by the objective, is then compensated by the eyepiece which introduces a lateral chromatic aberration of the opposite sense, thus giving origin to the designation "compensating eyepiece." For the objectives of shorter focal length and greater numerical aperture the limitations of the optical materials available prescribe the amount of lateral chromatic aberration present. For the objectives of longer focal length and smaller aperture ratio, the lateral chromatic aberration, although it could be more perfectly corrected without detriment to the correction for spherical aberration, is made the same in amount as that for the objectives of greater numerical aperture in order that all of a series may be used interchangeably with a single series of compensating eyepieces.

For very precise work it may then become a question whether or not the lateral chromatic aberration is held sufficiently constant for the different focal lengths to give the desired freedom from color and also whether or not the lateral chromatic aberrations of objective and eyepiece are sufficiently standardized to permit the interchangeable use of an objective of one manufacturer with an eyepiece manufactured by another. The present investigation is the result of a question of this character which arose in connection with the making of photomicrographs for 3-color reproduction at the U. S. Army Medical Museum. In such work three photographs are taken successively through Eastman photographic filters, A, B, and C, which correspond approximately to wave lengths 620, 550, and 450 mμ, respectively, and from these three photographs the three plates are made for producing the impressions in three different colors which form the final color print. Such work constitutes a more severe test of the color correction of a system than does the more conventional use of the microscope, since a very slight lack of register in the color print may be detected and is considered objectionable.

It should be mentioned that this use does not strictly accord with the conditions for which the microscope system is designed. When the microscope is used for the direct visual examination of an object, the image formed by the eyepiece is virtual, whereas when used for photographic purposes the image is real. The difference arising from this variation in use is probably not important because in both cases the image distance is large in comparison with the focal length of the eyepiece.

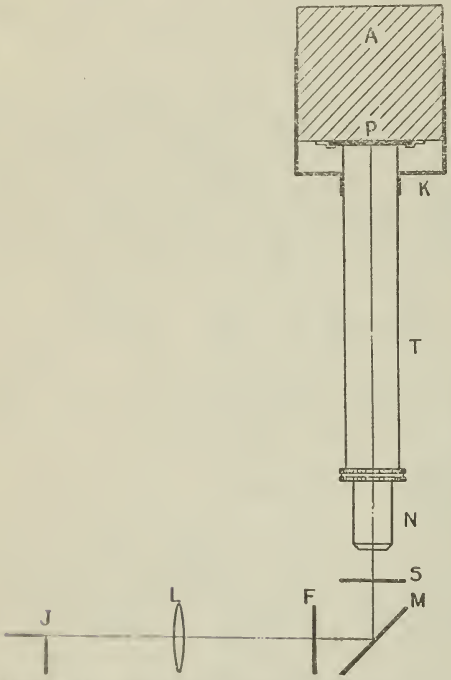

FIGURE 1.—*Arrangement for testing the objectives*

II. CHROMATIC ABERRATION OF THE OBJECTIVES

The method of test applied to the objective is illustrated in Figure 1. A microscope stand (not indicated in the figure) was adjusted to bring the tube *T* in a vertical position and the objective to be tested was mounted in the usual position at *N*. On the stage of the microscope was placed the object *S*, which was a glass slide carrying a series of parallel lines, spaced 0.1 mm ruled in silver by means of a diamond point. The arc at *J*, the condenser lens *L*, and the mirror *M*, with the Eastman A, B, and C filters, which were mounted in turn at *F*, formed the illuminating system. The photographic plate *P*, approximately 25 mm square, rested on the upper end of the draw tube which was adjusted to bring the plate in the image plane corresponding to a tube length of 160 mm. Stray light from the room was excluded by the brass cup *K*, mounted on the upper end of the microscope tube, and by the wooden plug *A*, to which the plate was loosely attached in order to facilitate its removal from the brass cup after the exposure.

For each objective three negatives were obtained, one corresponding to each of the three filters. The spacing of the lines in the image, as recorded on the negative, was measured on a comparator and the magnification obtained by dividing the distance between two lines on the negative by the distance between the two homologous lines on the object slide. This comparison of distances was made for each pair of adjacent lines recorded on the plate. If there were an appreciable amount of distortion the values of the magnification thus obtained for any one lens would have changed systematically, either increasing or decreasing, as pairs of lines were selected farther from the center of the field. In none of the objectives tested was such a progressive change found, and it is concluded that the distortion present is substantially less than that corresponding to a variation in magnification of 0.1 per cent for the different parts of a field 18 mm in diameter.

Table 1 gives the results of measurements which have been made on 20 objectives of four different makes.

TABLE 1.—*Magnification ratios of objectives tested*

Reference No.	Make	Focal length (mm)	Magnification, B filter	Magnification, B filter divided by magnification, A filter	Magnification, C filter divided by magnification, A filter
1	I	4	33.9	1.005	1.014
2	I	8	16.4	1.005	1.014
3	I	8	16.6	1.005	1.012
4	I	16	8.2	1.005	1.014
5	I	16	7.8	1.006	1.015
6	I	Average		1.005	1.014
7	II	4	39.2	1.006	1.015
8	II	4	37.0	1.005	1.016
9	II	8	16.5	1.006	1.017
10	II	8	16.8	1.005	1.016
11	II	16	8.3	1.004	1.013
12	II	16	8.2	1.006	1.014
13	II	16	8.1	1.005	1.013
14	II	Average		1.005	1.015
15	III	3	45.5	1.003	1.017
16	III	4	35.9	1.007	1.019
17	III	8	17.4	1.004	1.012
18	III	16	8.1	1.006	1.020
19	III	16	9.0	1.006	1.018
20	III	Average		1.005	1.017
21	IV	2	57.7	1.000	1.007
22	IV	4	36.0	1.005	1.013
23	IV	16	8.7	1.005	1.013
24	IV	Average		1.003	1.011

The fourth column gives the magnification for the B filter, the fifth the ratio of magnification for B filter to the magnification for A filter, and the sixth column the ratio of magnification for C filter to magnification for A filter. If there were no primary lateral chromatic aberration the A and C images would be of the same size and the ratios in the sixth column would be 1. Reference to these ratios shows that the A image is smaller than the B image which, in turn, is smaller than the C image. The image of a white point source not on the axis of the objective will, therefore, be a short spectrum, placed radially in the field with the red end turned toward the center. If one of these apochromatic objectives were used to produce an image 250 mm in diameter with no compensation introduced by the eyepiece, then for the A and C images of a point at the edge of the picture the failure to register would be of the order of 2 mm. This illustrates the importance of compensation and indicates why the prominent colored fringes are observed when the apochromat is used with an eyepiece which is not compensating. Reference to the average values for the different makes shows that III and IV differ most in the character of compensation which their objectives offer. Again, using a projected image 250 mm in diameter as an illustration, an eyepiece which compensated an objective of Group III perfectly, might, if used with an objective of Group IV, give A and C images which fail to register at the edge of the picture by 0.8 mm.

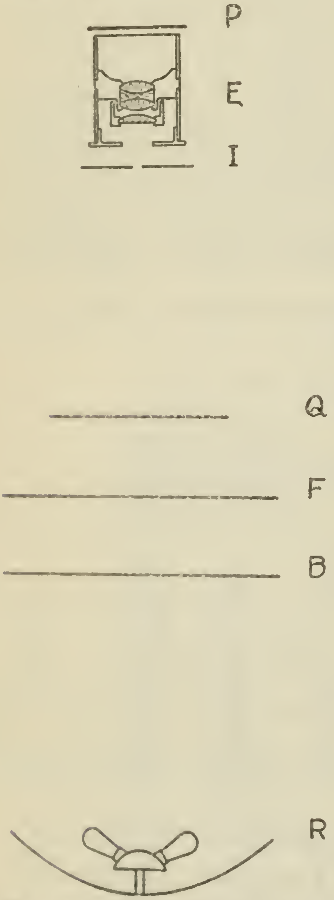

Figure 2.—*Arrangement for testing the eyepieces*

III. CHROMATIC ABERRATION OF THE EYEPIECES

The arrangement for testing an eyepiece is shown in Figure 2. The source of light was a diffusing screen of opal glass at *B*, illuminated by a cluster of incandescent lamps near the focus of a concave reflector, *R*, built up from small pieces of plane mirrors mounted in a frame. This illuminating system is of the type commonly marketed by photographic supply houses for illuminating negatives which are to be enlarged without the use of a condensing lens. The object at *Q* was a 10-inch strip of glass coated with silver bearing ruled parallel lines spaced 4 mm. The filters were of gelatine, 8 by 10 inches, mounted loosely between two sheets of glass and placed at *F* between the object and the diffusing screen of the illuminating system. The eyepiece to be tested was at *E* with the eye end toward *Q*.

All of the eyepieces were positive, a characteristic which is essential for this method of test. The optical system of each eyepiece was removed as a unit and mounted in a shorter tube in order that a small photographic plate might be conveniently placed in the image plane. The diaphragm at *I* was 0.8 mm in diameter and was placed, with reference to the eyepiece, in the plane occupied by the exit pupil of the eyepiece when it is mounted in a microscope. The use of this diaphragm is quite necessary, as otherwise a much larger beam of light would be transmitted by the eyepiece in this testing apparatus than in normal use, the position of the pupil points would be quite different, and the distortion and lateral chromatic aberration would be greatly changed. The image is received on the photographic plate at *P*. The distance from the eyelens to the object *Q* was 26 cm and the half angle of the field of view on the eye side of the eyepiece was approximately 17°. With the arrangement of parts as indicated it will be recognized that the course of the rays corresponds to that when the eyepiece is used for projection purposes with the screen distant 26 cm, except that the course of the rays is reversed and object and image are interchanged. This arrangement involving a reversal in the direction of travel of the light was adopted because it was more convenient to measure the small negatives thus secured. As in the case of the objectives, filters *A*, *B*, and *C* were used, and the magnification determined by comparing the distances between homologous pairs of adjacent lines on the object and negative. The values referred to as magnifications will be the ratios of lengths at Q to lengths at *P*, thus corresponding to the magnification when light traverses the eyepiece in the usual direction. In subsequent references the terms, object and image, will refer to the planes at *P* and Q, respectively; that is, they refer to object and image as the eyepiece is commonly used and not to the condition of this test in which the direction of travel of the light is reversed.

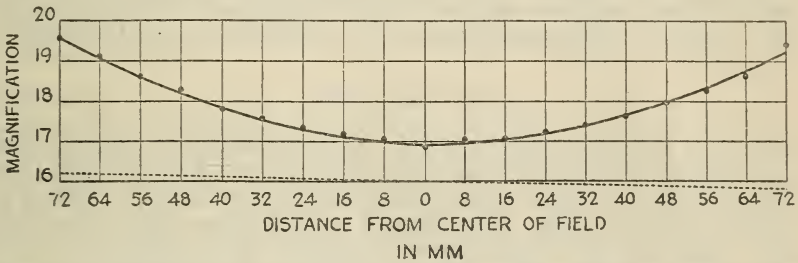

Figure 3.—*Variation of magnifications with distance from the center of the field for a typical eyepiece*

For the objectives the magnification obtained for pairs of lines at different distances from the center of the field showed no systematic variation, but, in contrast with this, for the eyepieces the magnification increases as the place of measurement recedes from the center of the field, thus indicating the presence of a considerable amount of positive distortion. Figure 3 illustrates a typical set of observations in which the value of the magnification as observed is 16.90 at the center of the field and 19.60 at the edge. If it be assumed that only distortion of the third order is present, the change in magnification will vary as the square of the distance from the center of the field, and the points located in Figure 3 should lie on a parabola. Accordingly, for each set of observations, such as is shown in Figure 3 (except as noted below), a parabola has been drawn which fits the observed points satisfactorily and the ordinate of the vertex of this parabola has been taken as the value of the axial magnification. The choice of parabolas was determined by first drawing a family of parabolas of different curvatures on transparent paper and superposing the parabolas in turn upon the plotted observations.[1] Two eyepieces were found which gave curves to which parabolas could not be fitted satisfactorily. One showed such excessive distortion that it is believed to have been incorrectly assembled or otherwise defective. The other showed an unusually small amount of distortion, the third and higher orders compensating each other at a point near the edge of the field. For all the other eyepieces tested the observed points could be fitted very closely by a parabola. In such cases, it is evident that the ordinate at the vertex of the parabola gives a better value of the axial magnification than if the measured value at the center of the field were taken because the measurement to the vertex of the parabola gives weight to all the observed points.

After the parabola which satisfactorily fits the observations has been selected, one has not only determined a value of the axial magnification, but there is also obtained a measure of the distortion. The value M' of the magnification for any point in the field may be expressed in the form

$$M' = M + Ar^2_0 \tag{1}$$

where M is the axial magnification, r_0 is the distance of the point from the center of the field in the object space and A is the parameter corresponding to the parabola which has been selected as fitting the observed points. Then, if there were no distortion, and the magnification were M, the distance from the center of the field to the point in the image conjugate to r_0 would be Mr_0. With distortion present, this distance is

$$\int_0^{r_0} (M + Ar^2)dr = Mr_0 + \frac{1}{3} Ar^3_0 \tag{2}$$

and the linear distortion; that is, the displacement arising from the distortion, measured in the image plane, is

$$\frac{1}{3} Ar^3_0 = \frac{1}{3} \frac{Ar^2_0}{M} Mr_0 \tag{3}$$

for the point r_0. If r_0 denotes a point at the extreme edge of the field, Mr_0 is the diameter of field in the image plane when the aberrations are neglected and Ar^2_0 is the difference in the ordinates, at center and edge of field, for the parabola which has been drawn to fit the observed points.

Table 2 gives the results of tests on 10 eyepieces of two different makes.

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[1] The dotted line is perpendicular to the axis of the parabola. It is oblique with respect to the coordinate axes because of a slight lack of parallelism of object and photographic plate. The effect of such tilts was considered when measuring the distortions.

TABLE 2.—*Axial magnification ratios and distortion constants of eyepieces tested*

Reference No.	Make	Nominal magnification	Magnification B filter	Magnification B filter divided by magnification A filter	Magnification C filter divided by magnification A filter	Linear distortion, 12° from axis	Angular distortion 12° from axis	Excess of angular distortion for C image over that for A image, 12° from axis
						mm	*Minutes*	*Minutes*
1	I	20	20.4	0.996	0.990	0.87	11.0	−1.5
2	I	15	15.3	1.003	.999	.44	5.6	−1.8
3	I	15	15.2	.997	.988	.73	9.3	−3.5
4	I		16.9	.998	1.000	1.32	16.8	−3.4
5	I		12.1	.997	.980			
6	I	Average		.998	.991	.84	10.6	−2.6
7	II	25	25.9	.995	.985	1.03	13.1	−2.0
8	II	25	25.2	.997	.986	1.06	13.5	−2.0
9	II	25	25.4	.998	.984	.79	10.0	−.9
10	II	15	15.5	1.000	.982	1.00	12.7	−1.6
11	II	15	14.9	1.000	.995			
12	II	Average		.998	.996	.97	12.3	−1.6

NOTE.—For eyepiece No. 5 the third and higher order aberrations were so well balanced that the magnification at the edge of field was substantially the same as at the axis. It would, therefore, be misleading to give a measure of distortion based on third order analysis. For practical purposes it may be considered to be distortion free.

For eyepiece No. 11 the aberration of orders higher than third are so great that values based on third order analysis are not given.

The values in the first six columns of Table 2 are similar to those tabulated for the objectives in the corresponding columns of Table 1. In the seventh column, the linear value of the distortion, under the conditions of the test, is given for a point 12° from the center of the field in the image space. In all cases the distortion is positive; that is, the image of a point is farther from the center of the field than its distortion-free position. These data for distortion are based upon measurements of the B image. For all eyepieces tested, the distortion for the C image is the same as for the B image, and the distortion for the A image is greater than for the B or C images. This excess of distortion for the A image over that for the B or C image is a chromatic difference in distortion which modifies the lateral chromatic aberration, its importance increasing as points farther from the center of the field are considered. The values tabulated in column 9 are for a point 12° from the center of the field and, therefore, correspond directly with the values of column 7. It may at first seem questionable to obtain, as has been done in Table 2, an average value for the distortion of the eyepiece of a given make by the process of averaging distortion constants for eyepieces differing in focal length. When, however, it is remembered that these eyepieces are designed to be used interchangeably with different objectives, it becomes apparent that this procedure is legitimate.

In the eighth column of Table 2, the angular values of the distortion for the B image at a point 12° from the center of the field are tabulated. Although these values are derived from measurements corresponding to a real image 260 mm. from the exit pupil, the angular value of the distortion remains approximately constant for any object distance, and this value, therefore, affords a means for determining the linear distortion for object distances differing from those of this test.

IV. CHROMATIC ABERRATION OF THE COMPLETE OPTICAL SYSTEM

The product of the magnification of the objective by the magnification of the eyepiece gives the total magnification for the optical system and similarly the product of the two magnification ratios gives the corresponding magnification ratio for the complete system. If the ratio magnification, filter C, divided by magnification, filter A, is 1, and if there is no chromatic variation in distortion, the A and C images will be of the same size, the registry of the two images will be perfect at all points of the field and there will be no primary lateral chromatic aberration for the effective wave lengths of filters A and C. If this condition is not realized the departure of this ratio from 1, multiplied by the radius of the field in the image space, expressed in either linear or angular units, will give, in the corresponding units, the distance between the A and C images of a point at the edge of the field when the chromatic variation in distortion is ignored. These values for a point 12° from the center of the field are tabulated in Table 3, column 7. Column 8 gives for the same point, the relative displacement of the A and C images which arises from the chromatic variation in distortion of the eyepiece. These latter values are taken directly from Table 2, column 9. By adding these two displacements for each combination, the angualr values of the lateral chromatic aberration for the complete system are obtained. Table 3 gives results of this character for several different combinations of the objectives and eyepieces which have been tested.

TABLE 3.—*Axial magnification ratios and lateral chromatic aberration of selected combinations of objectives and eyepieces*

Reference No. of combination	No. of objective (see Table 1)	Make	No. of eyepiece (see Table 2)	Make	Magnification, C filter divided by magnification, A filter, for complete system	Angular lateral chromatic aberration arising from difference in magnification	Angular chromatic difference in distortion	Lateral angular chromatic aberration at point 12° from center of field
						Minutes	*Minutes*	*Minutes*
1	6	I	6	I	1.005	+3.6	−2.6	+1.0
2	14	II	6	I	1.006	+4.3	−2.6	+1.7
3	20	III	6	I	1.008	+5.8	−2.6	+3.2
4	24	IV	6	I	1.002	+1.4	−2.6	−1.2
5	6	I	12	II	1.000	0.0	−1.6	−1.6
6	14	II	12	II	1.001	+ .7	−1.6	− .9
7	20	III	12	II	1.003	+2.2	−1.6	+ .6
8	24	IV	12	II	.997	−2.2	−1.6	−3.8
9	6	I	2	I	1.013	+9.4	−1.8	+7.6
10	6	I	5	I	.994	−4.3	0	−4.3
11	6	I	7	II	.999	−.7	−2.0	−2.7
12	6	I	10	II	.996	−2.9	−1.6	−4.5

Reference to Tables 1 and 2 indicates that the variation in the magnification ratios is greater for the eyepieces than for the objectives. The first four combinations listed in Table 3 show the effect of combining an average objective of each make listed in Table 1 with the average eyepiece of the make designated I in Table 2. The angular values of the lateral chromatic aberration resulting from the combined effects of the chromatic differences in magnification and distortion are tabulated in the last column of Table 3. These values vary from −1.2 to +3.2 minutes. The tabulated values are for a point 12° from the axis in a real image plane distant 260 mm from the exit pupil and a positive value indicates that the image of a point formed by light transmitted by the blue (C) filter lies farther from the center of the field than does the corresponding image formed by light transmitted by the red (A) filter.

Similarly combinations Nos. 5 to 8, inclusive, embody the results obtained when the average objective of each make listed in Table 1 is combined in turn with the average eyepiece of the make designated II in Table 2. For these combinations the average lateral chromatic aberration varies from −3.8 to +0.6 minutes. Combinations Nos. 9 to 12, inclusive, are obtained by combining the average objective of make I listed in Table 1 with individual eyepieces of which each was selected because in one or more features it manifests an extreme variation from the normal characteristics of the group.

The results in the final column of Table 3 are expressed in angular units and they may, therefore, be expected to give a good approximation to the lateral chromatic aberration present, either when the image is real and projected as in photomicrography or when the image is virtual as when a microscope is used for the visual examination of an object. The significance of the results, however, are somewhat different for these two cases and it is advantageous to consider them separately.

The photographic application will be considered first. If the film which receives the image is 260 mm from the exit pupil, as in the present test, one minute of arc corresponds to approximately 0.1 mm linear displacement in the plane of the screen. With this projection distance, therefore, if one of the optical combinations considered above is used to make color separation negatives for producing photoengraved plates from which the red, yellow, and blue impressions which form a color print are made, it follows that the impressions made by the three plates at a point 55 mm (12°) from the center of the field will fail to register by from 0.1 to 0.8 mm. accordingly as the different systems listed in Table 3 are employed. This lack of registry is much more harmful in the color print than in a virtual image. In the color print there are three images in contrasting colors and they differ in size by finite amounts, whereas, in the virtual image, if the illumination is by white light, there is an infinity of images varying in a continuous manner in size and color. Furthermore the virtual image, under all conditions of viewing, has approximately the same angular dimensions, whereas, the color print is apt to be brought close to the observer's eye for careful scrutiny with consequent angular enlargement. These considerations indicate that much greater freedom from lateral chromatic aberration is necessary for 3-color photomicrography than for the usual use of a microscope.

The eyepieces of this test are not of the type specially designed for photomicrographic work and the results obtained indicate that one can not expect satisfactory results from the commercial product except when the eyepiece and objective are specially selected for that purpose. Special types of eyepieces are made for projection purposes. They should not only be expected to give a better performance as regards freedom from chromatic aberration, but should also be more nearly free from distortion. This test shows that considerable distortion is present in the ordinary compensating eyepiece and this is probably made necessary by the condition that the eye distance be relatively large, a requirement which need not be imposed on the projection eyepiece.

If the eyepieces are to be used in the usual manner for visual observations the angular values of the lateral chromatic aberrations which are tabulated in the final column of Table 3 need not be converted into linear units. In view of the fact that the angular limit of resolution for the eye is one minute one would expect the systems with large amounts of chromatic aberration to show excessive color fringes. The prominence of these color fringes, however, may be very much lessened by the compensating chromatic aberration which results from an eccentric placement of the entrance pupil of the observer's eye with respect to the exit pupil of the microscope. Conrady[2] has called attention to this chromatic aberration of the eye which is unconsciously introduced by the observer in order to improve the definition of the image. If the eccentricity between the pupil of the eye and that of the microscope is 1 mm there are introduced approximately three minutes of compensating chromatic aberration which is negative and therefore tends to correct an undercorrected system. Reference to Table 3 indicates that under suitable conditions it is possible to correct a large portion of the chromatic aberration of any of the undercorrected systems in this manner although it should not be understood that this method of compensation makes a fairly complete elimination of chromatic aberration in the microscope unneccessary. It rather signifies that such chromatic aberration as remains in the microscope system should be positive rather than negative. Reference to Table 2 indicates that the average values of the magnification constants for eyepieces of makes I and II correspond closely to the desired values as indicated by the magnification constants of the objectives of corresponding makes listed in Table 1, although in each case individual eyepieces show rather large discrepancies. Furthermore, for these two makes greater uniformity is shown by the objectives than by the eyepieces.

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[2] Applied Optics and Optical Design, 1, p. 510. Published by Oxford University Press; London.