RP324

A MULTIPLE MANOMETER AND PISTON GAGES FOR PRECISION MEASUREMENTS

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ABSTRACT

The paper describes a 15-atmosphere differential mercury manometer and a group of piston gages constructed and maintained at the National Bureau of Standards in connection with the determination of the thermodynamic properties of substances.

The differential mercury manometer consists of five U tubes connected in series. Oil, alcohol, and water have been used at different times to transmit the pressure from one U tube to the next, water being the most satisfactory. The temperature of the manometer is measured with a mercury-in-glass thermometer, the bulb of which has the same length as the U tubes. The manometer was found to be accurate within 1 part in 10,000.

Each piston gage consists of a hardened steel piston and cylinder mounted vertically in a cast-iron base in such a manner that the piston and its load may be rotated continuously. The design is such as to permit applying the weights above the piston and yet avoid eccentric loading.

The procedure for calculating pressures from observations with these instruments is described, and the corrections to be applied are discussed.

The piston gages were calibrated as absolute instruments by means of measurements of piston diameter and width of crevice and as secondary instruments by direct comparison with the mercury manometer. Slight irregularities in the forms of pistons and cylinders impair the accuracy of the absolute calibration. Secular changes in the steel necessitate recalibration from time to time if the accuracy of which the gages are capable, 1 part in 10,000, is to be maintained.

CONTENTS

I. INTRODUCTION

A multiple column mercury manometer for measuring pressure differences up to 15 bars,[1] and a group of piston gages for measuring pressures up to 100 bars, have been constructed and maintained at the National Bureau of Standards in connection with the determination of the thermodynamic properties of substances.

[1] The term "bar" is used in this paper in accordance with the usage now internationally accepted; that is, to indicate a pressure of 1,000,000 dynes per square centimeter or about 0.96784 normal atmospheres. The term microbar is used to indicate a pressure of 1 dyne per square centimeter.

These gages have been repeatedly intercompared and the technique of their operation developed, the purpose being twofold: (1) To increase the accuracy of the measurement of pressure as one of the thermodynamic variables, (2) to investigate the reliability of the piston gage as a primary standard instrument.

II. DESCRIPTION OF MANOMETER

The 15-bar manometer in its present form, as shown in Figure 1, is the result of a gradual development extending over a period of years. The original design, as developed by Dr. Edgar Buckingham, provided for an instrument for routine laboratory use, made so that

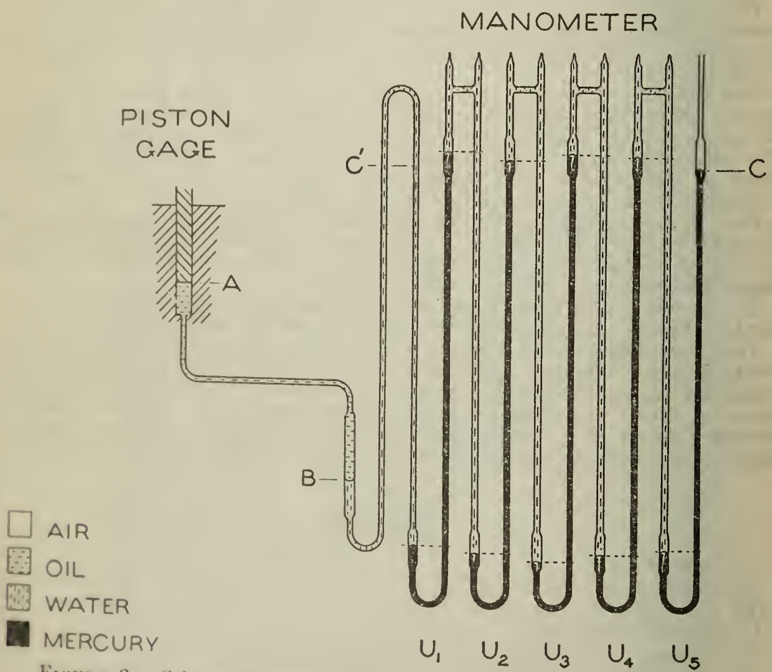

FIGURE 2.—*Schematic diagram of manometer connected to piston gage*

any pressure difference between zero and 15 bars could be measured. When it was found later that pressures as low as 1.2 bars could be measured much more conveniently and with equal accuracy with a piston gage, the manometer became merely a standard for the calibration of the piston gages. When used for this purpose, measurements at intervals of three bars were ample, and the design of the manometer was greatly simplified by the omission of glass-to-metal joints, valves, etc. These changes served to eliminate many points at which leakage could occur, removed possible sources of fouling of the mercury, simplified the manipulation, and increased the dependability without sacrificing anything except the possibility of measuring every pressure within its range.

Liquid with a low or moderate vapor pressure was chosen in preference to gas for transmitting the pressure from one U tube to the

FIGURE 1.—*Photograph of manometer and piston gage*

next, because the use of liquid offers less danger from flying glass in case of rupture, and simplifies the adjustment of the manometer. Oil, specially purified absolute alcohol, and distilled water have been used for this purpose. The use of oil proved unsatisfactory because of its action upon mercury; the latter became badly fouled and even emulsified. The use of alcohol was also unsatisfactory; the mercury became slightly fouled, and occasional redeterminations of the density of the alcohol were necessary, due to the absorption of moisture. The use of water, possible only after the omission of certain metallic parts, has given satisfactory results; the positions of the mercury surfaces were definite to within 0.05 mm.

As may be seen in the schematic diagram (fig. 2) the manometer consists of five glass U tubes about 2.5 m long connected in series. The instrument was designed in this manner so that it could be housed in a single room. The glass tubing is mounted in a rigid steel frame, each U tube being supported near the top by a clamp lined with soft rubber, and kept in alignment by loosely fitting clamps near the middle and the bottom. The glass tubing has an inside diameter of about 3 mm and a wall thickness of about 1 mm, except at places where readings are to be taken. At such places there are sections of tubing about 10 cm long having an inside diameter of about 10 mm and a wall thickness of about 2.5 mm. One of these sections is located near the bottom of the left-hand branch of each U tube and another near the top of the right-hand branch, as illustrated in Figure 2.

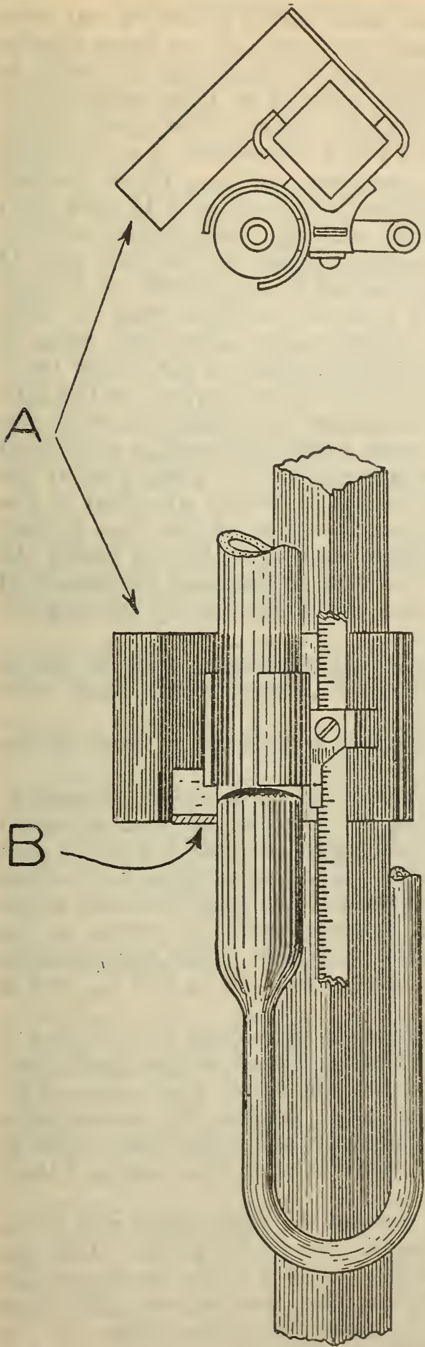

Figure 3.—*Reading device for manometer*

(a) Housing for electric lamp.
(b) Illuminated background.

Figure 3 illustrates one of the reading devices for the manometer.

Each of these consists of (1) a brass collar which reaches about two-thirds of the way around the manometer tube, (2) an index rigidly fixed to the collar in such manner that the mark on the index is opposite the lower edge of the collar, and (3) an illuminated white background located below and behind the collar. The whole device slides on a vertical, square, steel bar which serves also as a mounting for the clamps supporting the manometer tubes. These reading devices are located at the top and the bottom of each U tube with the collars surrounding the sections of large glass tubing already mentioned. The index of each reading device on the first four U tubes bears upon a 10 cm scale graduated in millimeters on a short steel tape. These steel tapes are attached at both ends to their respective supporting rods. The upper scales have their zeros at the lower ends; the lower scales have their zeros at the upper ends. The indices of both the upper and lower reading devices on the fifth U tube bear upon a 2.8 m scale graduated in millimeters with the zero at the lower end. The tape upon which this scale is graduated extends over the full height of the manometer; it is supported at the upper end and kept under tension by a 2 kg weight attached at the lower end. Figure 3 shows only a short section of the glass tubing, of the divided scale, and of the square supporting bar on which the reading device slides; it omits for the sake of clearness the means of attaching the scale to its support and the mechanism for moving the reading device.

The manometer is connected to metal apparatus, such as piston gages, by means of soldered glass-metal joints which have been described elsewhere.[2] [3]

For convenience in operation, there is a platform in front of the manometer about 1.5 m above the floor.

The mean temperature of the manometer is measured by a mercury-in-glass thermometer made specially for the purpose and mounted vertically about 6 cm behind the manometer, so that it may be seen between the second and third U tubes. The thermometer bulb which is about 2.5 m long and made from tubing of the same diameter as the smaller tubing in the manometer, is placed with its center at the same elevation as the center of the U tubes. The thermometer scale, about 20 cm long, covers a range from 10° to 36° C., and is divided into intervals of 0.2° C.

The thermometer has been calibrated several times by inclosing it in a pipe through which a stream of water flowed at practically constant temperature. The temperature of the water was measured by standard thermometers before and after passing the termometer under test. The instrument has also been calibrated once while in place behind the manometer by comparison with four standard thermometers hanging beside it.

During all observations, the air around the manometer was circulated by a fan located on the opposite side of the room. This fan withdrew air from near the floor and returned it to the room near the ceiling. With this fan in operation the temperature difference between the top and bottom of the manometer was about 0.6° or 0.7° C., and the temperature variation with time was usually not more than 0.1° or 0.2° per hour.

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[2] McKelvy and Taylor, J. Am. Chem. Soc. **42**, p. 1364; 1920.
[3] Meyers, J. Am. Chem. Soc. **45**, p. 2135; 1923.

III. OPERATION OF THE MANOMETER

With the present design in which all the U tubes have approximately the same capacity and the mercury in each is brought to approximately the same level, it is necessary to vary the amount of the pressure-transmitting liquid only at the high-pressure end of the manometer.

In adjusting the reading devices, the brass collar was lowered until only a faint streak of light from the illuminated background could be seen between the collar and the meniscus. The positions of the indices on the fixed scales were then read with the aid of a magnifying glass. The same observer read both ends of the manometer for any one measurement, to avoid the errors due to systematic difference in the adjustments made by different observers.

To determine the elevations of the zeros of the short scales with reference to the zero of the 2.8 m tape, a plane surface was leveled at a suitable height in front of the lower end of the manometer, the first four lower reading devices were lowered until the brass collars touched the plane surface. The positions of their indices were then read upon the short scales, and that of an index rigidly attached to the plane surface was read on the 2.8 m tape. The plane surface was then leveled at a suitable height in front of the upper end of the manometer and the procedure repeated.

From these readings the effective elevations of the zeros of the short scales above that of the long tape can be directly obtained when the vertical distance from the plane surface to the index attached to it is known, the effective elevation of a zero being that which when suitably combined with the reading of the index gives the actual elevation of the bottom of the associated collar. It differs from the true elevation by the amount by which the index is displaced from the the plane of the lower edge of its associated collar, and its use automatically eliminates the effect of such displacement.

But in each set of readings (upper or lower), the differences in the effective elevation of the zeros of the several short scales are given directly by the differences in the readings of their indices, entirely irrespective of the vertical distance from the plane surface to the index attached to it. Likewise, the effective vertical distance between the upper and the lower zero of any pair of short scales is given by the difference in the readings of the index on the long scale, suitably modified by the readings of the appropriate indices on the short ones. Again the distance from the plane surface to its index is a matter of indifference, and need not be known. It may be easily shown that the only place in which a knowledge of the distance from the plane surface to its index need be known is in the computation of the height of the water column in the manometer. This distance can easily be measured with sufficient accuracy for that purpose.

The effect of errors in the location of the indices of the reading devices for the fifth tube, was determined by using them to measure the positions of the mercury menisci in the two branches of a short open U tube, which temporarily replaced the fifth manometer tube. By interchanging the positions of the branches of this short U tube, any error due to a consistent difference in the capillary forces in the two branches was eliminated.

IV. DESCRIPTION OF PISTON GAGES

The first 100-bar piston gage, as originally constructed, was designed by Drs. H. C. Dickinson and M. S. Van Dusen. Later, a motor-driven device for rotating the piston was added, and a 10-bar gage and another 100-bar gage constructed, both with a motor-driven rotating device.

The identification mark and diameter of each piston and cylinder, as well as the identification mark of each cast-iron base, are given in Table 1. Each base was made for the piston and cylinder opposite its number, but either of the 100-bar bases may be used with any one of the cylinders A, B, or C.

Table 1.—*Identification of pistons and cylinders*

100-BAR GAGES

Piston			Cylinder			Cast-iron base No.
Mark	Material	Diameter	Mark	Material	Diameter	
		mm			*mm*	
1	Drill rod (carbon steel)	11.297	[1] A		11.300	12083
2	Sanderson's special steel	11.318	A_1		11.322	
3	Drill rod (carbon steel)	11.261	B		11.264	30213
4	Chrome steel	11.269	C	Chrome steel	11.273	
10-BAR GAGE						
5		35.675	D		35.683	30778

[1] Before lapping to fit piston 2.

Note.—Since the parts are to some extent interchangeable, the gages will be identified in this paper by giving the identification mark of the piston, cylinder, and base in the order named; for example, 3–B–30213.

The 10-bar gage is illustrated partly in section in Figures 4 and 5; it is practically the same as the 100-bar gage with the exception of its larger piston and cylinder and slightly larger oil pump. A scale of centimeters is given along with the drawings to show the size of the gages.

The steel cylinder is held rigidly in the cast-iron base by a threaded ring (No. *26* in figs. 4 and 5). This cylinder is composed of two parts, a hardened steel bushing (*22*) and a soft steel jacket (*25*). The outside of the bushing and the inside of the jacket are ground to a force fit over the upper two-thirds of their length, but have about 0.2 mm clearance over the lower third. This part, as may be seen in Figure 4, is somewhat larger in diameter but due to the reduced scale the clearance is not visible. The lower end of the cylinder is closed by screwing on a steel cap (*27*). By this construction a shoulder is formed on the bushing which prevents that part from being displaced upward by the pressure, and the bushing is separated from the jacket at the place where strains are likely to occur in the latter due to screwing on the cap. The clearance between the lower portion of the bushing and jacket has been made small so that an inappreciable amount of air is entrapped when the gage is being filled.

The length of bearing surface between the piston and cylinder in each gage is about 7 cm.

All the pistons and cylinders are of steel, the kind of steel being stated, in so far as known, in Table 1. Piston No. 2 was furnace annealed and the others quenched and then drawn to about 300° C. The pistons and cylinders were ground to approximately the correct diameter and finished by lapping to uniform diameters, such that they fit together with very little clearance and yet do not seize when stationary.

The load, applied to the gage by placing weights upon the cap (*1*), is transmitted through a steel shaft (*5*) to a steel ball (*29*) near the bottom of the hollow piston.

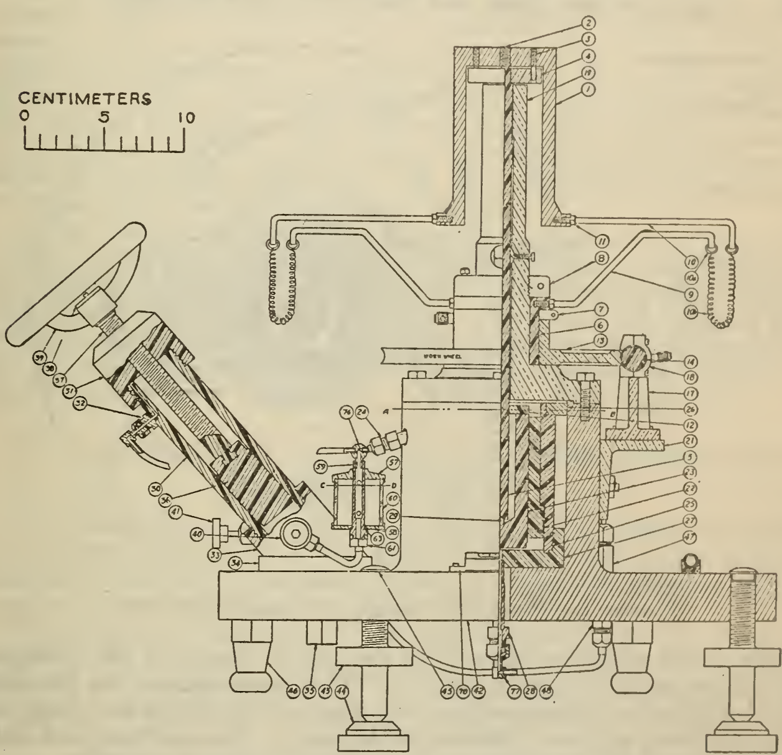

FIGURE 4.—*Elevation of piston gage, shown partly in section*

The temperature of each gage is measured by a mercury-in-glass thermometer (not shown in fig. 4 or 5) inserted in a hole in the cast-iron base in such manner that the thermometer bulb is at the same elevation as the center of the cylinder. The pressure to be measured is applied at the valve (*47*).

Frictional forces retarding the vertical motion of the piston are reduced by rotating the piston continuously in one direction. The rotation is produced by a small motor driving a worm gear (*13* and *14* in fig. 4). The adjustable friction clutch (*7*) drives a steel sleeve which carries a pair of arms (*9*). These arms are in turn connected through helical springs to another pair of arms mounted on the cap (*1*) in such manner that the ends of the two pairs are at the same height when the piston is about midway between the limits of its vertical travel, in other words, at its normal position. The location of the piston may be determined by observation of the relative position of a mark on the shaft (*5*) and the point of a set screw mounted in the pedestal (*19*), both being visible through the sight hole shown in Figure 4 just below the cap (*1*). In certain pressure measurements where it was desired to observe the position of the piston very accurately, a miscroscope was used. The piston is at its normal position when the mark and point are at the same level. Two pins in the cap (*1*) which engage with two holes in the disk (*4*), and projections on the shaft (*5*) which engage with notches in the upper end of the piston, cause the cap, shaft, and piston to rotate as a unit.

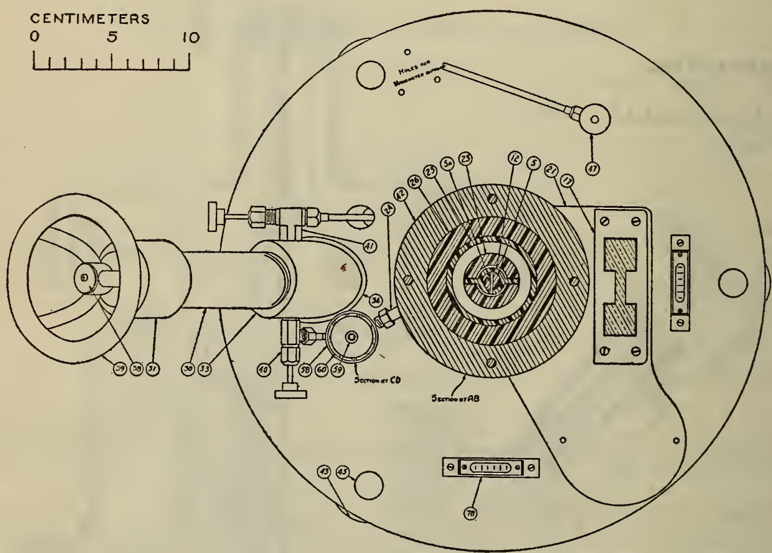

FIGURE 5.—*Plan of piston gage, shown partly in section*

The pump which supplies oil to the gage is shown at the left-hand side of the gage in Figures 4 and 5. The pump plunger for the gage first built has a leather packing; the plungers for the newer gages are of steel fitted to steel cylinders by lapping. The leakage past both the pump and the gage pistons is returned to the oil reservoir.

A short auxiliary mercury manometer is mounted on the cast-iron base. The purpose of this manometer is to separate the oil in the gage from the fluid, the pressure of which is to be measured. When the diameters of the manometer tubes are small, observation of the position of the mercury is a sensitive means of determining lack or presence of equilibrium. When it is unnecessary to use a separating device, or is more convenient to use a manometer mounted on other apparatus, the manometer and mounting on the gage may be removed.

V. OPERATION OF PISTON GAGES

When a pressure is to be measured, the gage is leveled, the approximate required weight is placed on the gage, oil is pumped in until the piston is at its normal position, the piston is set in rotation, the valve leading to the pressure to be measured is temporarily opened slightly, the velocity with which the piston moves up or down noted, and the weight adjusted accordingly. Failure to keep this valve closed causes no damage to the gage, since the piston travel is confined to narrow limits by stops; it may, however, cause the spilling of liquid from manometers connected to the gage if these manometers are of small capacity.

After the gage has been adjusted and the valve leading to the pressure has been opened wide, time is allowed for equilibrium to be attained. This period of time depends on the individual gage as well as on the conditions of observation and the precision required. For example, when the manometer is connected to a piston gage, the departure from equilibrium is reduced to half value in about one-half minute and becomes too small to be detected in 5 to 10 minutes. When two piston gages are connected with only oil between them, the ratio of the displacement from equilibrium to restoring force is much greater, and 15 or 20 minutes are required for the pistons to attain an equilibrium position. The time required for measurement of vapor pressures is equally long or longer.[4]

In the comparison of piston gages with the manometer, the observations were divided into two groups, approximately equal in number, the division being made with regard to the direction in which equilibrium was approached. One group was preceded by a downward displacement of the piston from the equilibrium position, the other by an upward displacement, time being allowed for return to equilibrium in both cases.

VI. CALCULATION OF PRESSURES AND RECORD OF OBSERVATIONS

The description of the calculation of pressures is clarified somewhat if the pressure developed by the manometer and piston gage is analyzed in detail. For this purpose a schematic diagram of the manometer and piston gage is given in Figure 2.

The pressure at the point C' in the left arm of U_1 and at the same height as the mercury surface in the right arm of U_5 at C, is equal to the sum of the following: (1) The pressure At_c of the atmosphere at C; (2) the difference in pressure (p_{cap}) due to the combined capillary forces of the mercury-air meniscus at C and all the mercury-water menisci in the manometer; and (3) the excess (E) of the pressure exerted by a mercury column of height (H), equal to the sum of the differences in level of the menisci bounding the several mercury columns, over that exerted by a water column of the same height.

The last item is easily visualized if one considers the path which starts at C (fig. 2) and follows down the right side of U_5 to the short horizontal dotted line, thence along the dotted line and up the left side of U_5 to the upper dotted line, thence along that dotted line, down and so on to C'. Each dotted line intersects two tubes at points of equal pressure, the downward part of the path corresponds to an increase in pressure equal to the head of mercury in a column of height (H), and the upward part corresponds to a decrease in pressure equal to the head of water in a column of the same vertical

[4] Cragoe, Meyers, and Taylor, B. S. Sci. Papers, **16**, p. 1; 1920. Sci. Paper No. 369.

height, since the path starts and ends at the same level, and consequently the downward and upward paths must be equal.

The pressure at A is equal to that at C' plus that of a water column of height BC minus that of an oil column of height BA, the amount by which the level of A exceeds that of B. If the densities of mercury, oil, and water are denoted, respectively, by ρ_{Hg} ρ_o and ρ_w then the pressure at A is

$$P_A = At_c + (p_{cap}) + \rho_{Hg}\, gH - \rho_w\, {}_gH + \rho_w\, {}_g\, BC - \rho_o\, {}_g\, BA$$
$$= At_c + (p_{cap}) + {}_g\{(\rho_{Hg} - \rho_w)\, H + \rho_w\, BC - \rho_o\, BA\} \quad (1)$$

This pressure acts upward at A upon the piston of the pressure gage.

If the base of the piston were a horizontal plane, and if there were no leakage of oil past it, then when equilibrium exists this pressure (P_A) would equal the ratio of the force acting downward through the piston, to the area of the base of the piston. That force is equal to the atmospheric pressure (At_A) at the level A times the area of the base of the piston, plus the weight (W) of the piston and the load it carries, corrected for the buoyancy of the atmosphere. Hence

$$P_A = At_c + (p_{cap}) + g\{(\rho_{Hg} - \rho_w)H + \rho_w BC - \rho_o h_o\} = At_A + \frac{Wg}{AREA} \quad (2)$$

or

$$AREA = Wg/[(At_c - At_A) + (p_{cap}) + {}_g\{(\rho_{Hg} - \rho_w)H + \rho_w BC - \rho_o h_o\}] \quad (3)$$
$$= Wg/(P_A - At_A)$$

In the actual gages there is always some leakage of oil past the piston. This results in a lifting force on the piston, due to the viscous drag of the oil. This force is very closely proportional to the product of pressure times the cross-sectional area of the crevice between the piston and cylinder, and consequently its presence is equivalent to a slight increase in the radius of the piston. The area defined by this greater radius is called the effective area of the piston; it is the area that is defined by equation (3). The relation between the effective area and the actual area is considered later.

The observations made with the manometer during a comparison with a piston gage and the computation of pressure from these observations are given in Table 2.

Table 2.—*Record of pressure measurements with the manometer*

[Observer C. H. M. Computed by C. H. M. Date, March 2, 1928]

1	Observed temperature °C	21.90	22.00	22.15	22.15	22.20	22.30	22.30	22.30	22.35
2	Corrected temperature do	21.90	22.00	22.15	22.15	22.20	22.30	22.30	22.30	22.35
3	Density of oil	.890	.890	.890	.890	.890	.890	.890	.890	.890
4	Density of water	.9978	.9978	.9978	.9978	.9978	.9977	.9977	.9977	.9977
5	Density (Hg minus H_2O)	12.5434	12.5432	12.5429	12.5429	12.5427	12.5425	12.5425	12.5425	12.5425
6	U 1 right cm	4.585	4.585	4.590	4.580	4.590	4.735	4.735	4.735	4.730
7	U 1 left do	5.970	5.980	5.980	5.980	5.985	6.115	6.115	6.130	6.130
8	U 2 R do	7.560	7.565	7.570	7.570	7.570	7.710	7.715	7.715	7.715
9	U 2 L do	4.440	4.440	4.440	4.435	4.440	4.575	4.585	4.585	4.585
10	U 3 R do	7.425	7.430	7.430	7.425	7.430	7.600	7.600	7.600	7.600
11	U 3 L do	6.725	6.725	6.730	6.730	6.725	6.870	6.865	6.875	6.870
12	U 4 R do	6.390	6.395	6.410	6.400	6.405	6.550	6.555	6.550	6.555
13	U 4 L do	7.595	7.590	7.600	7.595	7.595	7.740	7.740	7.740	7.745
14	U 5 R do	275.635	275.635	275.660	275.645	275.640	275.835	275.840	275.835	275.835
15	Tape correction do	.095	.097	.100	.100	.100	.102	.102	.102	.103
16	Compressibility correction +.007; air column −0.201 cm	−.014	−.014	−.014	−.014	−.014	−.014	−.014	−.014	−.014
17	Distance between zeros do	937.060	937.060	937.060	937.060	937.060	937.060	937.060	937.060	937.060
18	Subtotal do	1,263.466	1,263.488	1,263.556	1,263.506	1,263.526	1,264.878	1,264.898	1,264.913	1,264.914
19	U 5 L do	−31.695	−31.705	−31.690	−31.690	−31.695	−31.500	−31.500	−31.500	−31.500
20	(Hg minus H_2O) do	1,231.771	1,231.783	1,231.866	1,231.816	1,231.831	1,233.378	1,233.398	1,233.413	1,233.414
21	Water do	226.8	226.8	226.8	226.8	226.8	227.1	227.1	227.1	227.1
22	Oil do	−12.6	−12.6	−12.6	−12.6	−12.6	−12.6	−12.5	−12.5	−12.5
23	Density×cm (Hg minus H_2O) gram weight/cm²	15,450.6	15,450.5	15,451.2	15,450.5	15,450.5	15,469.6	15,469.9	15,470.1	15,470.1
24	Water do	226.3	226.3	226.3	226.3	226.3	226.6	226.6	226.6	226.6
25	Oil do	−11.2	−11.2	−11.2	−11.2	−11.2	−11.2	−11.1	−11.1	−11.1
26	A (fig. 2) do	15,665.7	15,665.6	15,666.3	15,665.6	15,665.6	15,685.0	15,685.4	15,685.6	15,685.6
27	Time	12.20	1.30	1.47	2	2.35	2.55	3.25	3.50	4.10

In this table, each column represents a separate measurement of pressure, which was made simultaneously with a measurement of the same pressure recorded in the corresponding column of Table 4. Such symbols as U 2 R indicate the meniscus of which the associated numbers are the scale readings; U 2 indicates the particular U tube (fig. 2), and the letter R (or L) the branch of the tube right (or left). Item 16 is the sum of the correction for the compressibility of the mercury, oil, and water and of that for the weight of an air column of a height equal to the vertical distance between C and A. (Fig. 2.) Item 17 is the sum of the vertical distances between the zeros of the short scales on each of the first four U tubes. Item 20 is equal to the quantity H of equation (3) increased by $(At_c - At_a)$ expressed in centimeters of a hypothetical liquid of density equal to $(\rho_{Hg} - \rho_w)$. Item 21 is the height of the water column BC (fig. 2, equation (3)), and item (22) is the height of the oil column (AB fig. 2, h_o equation (3)). The tape reading of the oil-water meniscus is recorded in Table 4. When item (26), the sum of items (23), (24), and (25) is multiplied by the value of g at this laboratory and the product is increased by p_{cap}, the gage pressure at A (fig. 2) is obtained in dynes per cm^2. The value of p_{cap} is exceedingly small if the tubes are tapped before readings are taken. In the present work it has been assumed to be negligible. The other items of the table are self-explanatory.

TABLE 3.—*Record of pressure measurements with the piston gage*

Observer C. H. M. Computed by C. H. M. Gage 4-C-12083, renown engine oil, 6 r. p. m. clockwise. Date Mar. 2, 1928

Observed temperature .... °C	22.7	23.4	24.0	24.0	24.0	24.0	24.0	24.2	24.2
Corrected temperature .... do	21.8	22.5	23.1	23.1	23.1	23.1	23.1	23.3	23.3
Effective height gage .... cm	61.37	61.37	61.37	61.37	61.37	61.37	61.37	61.37	61.37
Oil-water meniscus .... do	48.8	48.8	48.8	48.8	48.8	48.7	48.7	48.7	48.7
Cap and piston .... g	1,234.0	1,234.0	1,234.0	1,234.0	1,234.0	1,234.0	1,234.0	1,234.0	1,234.0
Weights .... do	10,000.0	10,000.0	10,000.0	10,000.0	10,000.0	10,000.0	10,000.0	10,000.0	10,000.0
	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0
	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0	2,000.0
	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
						20.0	20.0	20.0	20.0
Sum of weights .... do	15,634.0	15,634.0	15,634.0	15,634.0	15,634.0	15,654.0	15,654.0	15,654.0	15,654.0
Buoyancy correction .... do	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2
Corrected sum .... do	15,631.8	15,631.8	15,631.8	15,631.8	15,631.8	15,651.8	15,651.8	15,651.8	15,651.8
Area at observed temperature .... cm^2	.99784	.99784	.99780	.99784	.99784	.99788	.99786	.99785	.99785
Temperature correction .... do	−.00004	−.00006	−.00007	−.00007	−.00007	−.00007	−.00007	−.00007	−.00007
Area at 20° C .... do	.99780	.99778	.99773	.99777	.99777	.99781	.99779	.99778	.99778
Time	12.20	1.30	1.47	2	2.35	2.55	3.20	3.40	4.05
Pressure approaching equilibrium	(+)	(−)	(+)	[1]	(−)	(+)	(−)	(+)	(−)

[1] Piston not disturbed.

Table 3 is the corresponding record for the piston gage. In this table the items "effective height of gage" and "oil-water meniscus" are, respectively, the elevations in centimeters, of the effective bottom of the piston (*A*, fig. 2) and of the oil-water meniscus (*B*, fig. 2) above the zero of the 2.8 m scale. The several values given for the "area at observed temperature" are obtained by dividing each of the several numbers in the preceding line (corrected sum of the weights) by the corresponding number in item 26 of Table 2; this area is the effective area of the piston at the existing temperature of the gage. In deducing from this the area at 20° C., it was assumed that the coefficient of linear expansion of the piston and cylinder is 0.0000115 per °C. A + in the last line of the table indicates that the piston rose to equilibrium; a − that it descended. The other items are self-explanatory.

In using the piston gage to measure pressure, the absolute pressure at *A* (fig. 2) is calculated from

$$P_A = At_A + \frac{Wg}{a} \tag{4}$$

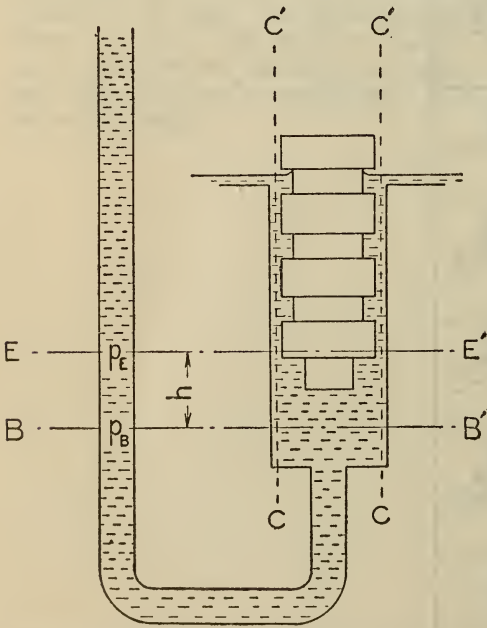

Figure 6.—*Diagram showing the effective elevation of an irregular piston*

where At_A is the pressure of the atmosphere at *A* (fig. 2), *W* is the sum of the weights, including the weight of the piston, etc., corrected for the buoyancy of the atmosphere, *a* is the effective area of the piston as determined from the comparison with the manometer, and *g* is the acceleration due to gravity (980.091 in this laboratory).[5] The pressure at any other point, such as *B*, differs from that at *A* by an amount depending on the difference in level of *A* and *B*, and on the density of the fluid or fluids in the connecting tubing.

If there were no leakage of oil past the piston, and if its base were a horizontal plane, then the area *a* in equation (4) would be the actual cross-sectional area of the piston, and the level of *A* (fig. 2), at which the pressure is given by equation (4), would be the level of the base of the piston. For convenience in construction, and also for the protection of the lapped surfaces, each of the pistons used in the gages described here has near its base a short section whose diameter is less than that of the main body of the piston. Each piston also has a center hole in its base. Further, there is always some leakage of oil past the piston. It is still possible, however, to use the expression (4) to calculate the pressure at some height, called the effective height of the gage. This may be shown as follows: Figure 6 illustrates

[5] B. S. Bull., 8, p. 363; 1911. (Sci. Paper No. 171).

a gage whose piston, for the sake of generality, has not only the smaller section at the bottom, but also grooves at several places along its length. It is assumed that the grooves are deep enough so that there is no appreciable pressure drop due to the flow of the oil across them. Consider the cylindrical surface of radius r, represented by the dotted lines CC' (fig. 6), located where there is no shear in the oil film. The material outside this cylinder does not exert any vertical force on the material inside the cylinder. It follows that the pressure in the oil at any level B is given by

$$P_B = At_B + \frac{Wg + (\rho_o - \rho')\,vg}{\pi r^2}$$

where v is the volume of oil above B and inside the cylinder CC' and ρ' the density of air. The pressure in the connecting tube at any level E at a height h above B is

$$P_E = At_B + \frac{W\,g + (\rho_o - \rho')\,v\,g}{\pi r^2} - \rho_o\,g\,h$$

$$= At_E + \frac{W\,g + (\rho_o - \rho')\,v\,g - \pi r^2\,(\rho_o - \rho')\,g\,h}{\pi r^2}$$

now if E be so chosen that

$$h = \frac{v}{\pi r^2}$$

then the pressure at E is given by

$$P_E = At_E + \frac{W\,g}{\pi r^2}$$

which is of the same form as equation (4). This is equivalent to choosing the level E so that the volume of oil within the cylinder CC' and above E is equal to the volume of that part of the piston which lies below E. The area πr^2 is called the effective area of the gage, and E is called the effective elevation of the piston. If the pressure (measured by the manometer) at any other level were used to compute the effective area by means of the formula

$$a = \frac{W\,g}{P - At}$$

the value found would not be constant, but would be a function of the pressure.

The weights have been calibrated about once every two years; the corrections used were either taken directly from a calibration or from an interpolation between calibrations. The first weights were made of cast iron and nickel plated. These gradually increased in weight, the increase being about 3 g in 10 years for the 20 kg weights and proportionately smaller for the smaller weights. More recently 20 kg nickel-plated steel weights and smaller nickel-plated brass weights were constructed and adjusted to within 0.1 or 0.2 g of the correct value. The changes in these 20 kg weights, with two exceptions, were 0.3 g or less in two years. The smaller nickel-plated brass weights have undergone no appreciable changes.

VII. CALIBRATION OF PISTON GAGES

The calibration of a piston gage consists essentially in the determination of the effective area of the piston. This may be accomplished in two ways, namely: (1) The gage may be calibrated as a secondary gage by comparison with a mercury manometer, and (2) the gage may be calibrated as a primary instrument by measurement of various dimensions of the gage.

1. CALIBRATION AS A SECONDARY INSTRUMENT BY COMPARISON WITH A MANOMETER

The results of comparisons up to 15 bars, of individual piston gages with the manometer are presented in Table 4, the table being arranged so that the measurements with each gage form a group. The notation used to represent the gages is indicated in Table 1.

TABLE 4.—*Comparison of individual piston gages with the manometer*

Date	Number of observations	Pressure (gage)	Effective area at 20° C.	Effective diameter at 20° C.	Average deviation from mean area	Remarks
1	2	3	4	5	6	7
MEASUREMENTS WITH GAGE 1–A						
		Bars	*cm²*	*cm*	*cm²*	
June, 1918	20	9.2–15.3	1.00294	1.13004	0.00022	
May, 1919	66	4.0–14.6	1.00311	1.13013	.00019	
November, 1919	44	15.1	1.00269	1.12990	.00010	
July, 1924	22	15.4	1.00266	1.12988	.00003	
August, 1925	16	15.3	1.00268	1.12989	.00005	
MEASUREMENTS WITH GAGE 2–A						
Mar. 25, 1926	8	15.2	1.00649	1.13203	0.00005	Base 30213.
Mar. 27, 1926	5	15.2	1.00652	1.13205	.00004	Base 30213.
Apr. 12, 1926						Lapped piston, change in weight indicates a decrease of 0.00006 cm. in piston diameter.
May 12, 1926	10	15.0	1.00650	1.13204	.00006	Base 12083.
May 14, 1926	7	15.2	1.00654	1.13206	.00005	Base 12083.
May 26, 1926						Lapped piston, change in weight indicates a decrease of 0.00001 cm. in piston diameter.
June 2, 1926	10	15.3	1.00650	1.13204	.00003	Base 12083.
June 25, 1926	4	15.3	1.00645	1.13201	.00002	Base 30213.
June 28, 1926	3	15.3	1.00650	1.13204	.00004	Base 30213, motor removed 25 cm.
June 29, 1926	4	15.3	1.00651	1.13205	.00001	Base 30213, motor removed 25 cm.
July 1, 1926	4	15.3	1.00654	1.13206	.00002	Base 30213, motor removed 25 cm.
Do	4	15.3	1.00648	1.13203	.00002	Base 30213, motor replaced.
Aug. 4, 1927	5	15.4	1.00643	1.13200	.00003	Base 12083.
Aug. 6, 1927	8	15.3	1.00641	1.13199	.00002	Base 12083.
MEASUREMENTS WITH GAGE 3–B						
Aug. 11 and 12, 1925	16	15.3	0.99632	1.12630	0.00008	Base 30213.
Mar. 16, 1926	12	15.3	.99629	1.12628	.00003	Base 30213.
Mar. 13, 1928	4	15.4	.99612	1.12619	.00001	Base 12083.
May 16, 1928	4	15.2	.99610	1.12618	.00001	Base 12083.
MEASUREMENTS WITH GAGE 3–C						
June 2, 8, and 10, 1927.	25	15.2	0.99676	1.12655	0.00003	Base 30213.

TABLE 4.—*Comparison of individual piston gages with the manometer*—Continued

MEASUREMENTS WITH GAGE 4-C

Date	Number of observations	Pressure (gage)	Effective area at 20° C.	Effective diameter at 20° C.	Average deviation from mean area	Remarks
1	2	3	4	5	6	7
		Bars	*cm²*	*cm*	*cm²*	
Mar. 9, 1926	9	15.4	0.9972	1.1267	0.002	Piston sticking badly, lapped September, 1927.
Mar. 2, 1928	9	15.4	.99777	1.12712	.00001	Base 12083.

MEASUREMENTS WITH 10-BAR GAGE

Date	Number of observations	Pressure (gage)	Effective area at 20° C.	Effective diameter at 20° C.	Average deviation from mean area	Remarks
Dec. 11 and 12, 1925	19	3.1	9.9977	3.5679	0.0004	Base 30778.
Dec. 16 and 17, 1925	20	6.2	9.9981	3.5679	.0003	Base 30778.
Dec. 21 and 23, 1925	16	9.3	9.9982	3.5679	.0004	Base 30778.
May 23, 1928	10	9.1	9.9976	3.5678	.0002	Base 30778.

For pressures higher than about 15 bars, a method described by Holborn and Schultze was used.[6] In this method two piston gages were connected to the manometer, one to the high-pressure end, the other to the low-pressure end. Observations were made with the first of these piston gages balanced against the combined pressure exerted by the manometer and the second piston gage, the manometer being adjusted to exert a pressure of approximately 15 bars and the second piston gage adjusted to exert its minimum pressure of 1.5 bars. The load on each gage was then increased by steps of 15 kg and the observations repeated at each pressure until the maximum pressure of 75 bars was reached. This procedure was then repeated with the two piston gages interchanged. The bases were not interchanged. The observations thus obtained provided data for calculating the effective areas of both gages at pressures up to 75 bars, the limit set by the strength of the glass in the manomter. The results of these comparisons are given in Table 5 where the gage connected to the low-pressure end of the manometer is referred to as the first piston gage, and that connected to the high-pressure end as the second gage. The value used in column 4 for the effective area at both 1.5 and 16.3 bars was the mean of values taken from column 8 and from Table 4 for the same gage at approximately 15 bars. The effective areas of both gages at about 32 bars, given in column 8, were calculated from these data and then used for calculation with observations at the next higher pressure.

[6] Ann. Physik. (ser. 4), 47, p. 1089; 1915.

TABLE 5.—*Comparison between two piston gages and the manometer*

Date	Number of observations	First piston gage			Second piston gage			
		Gage No.	Effective area at 20° C.	Pressure (gage)	Gage No.	Pressure (gage)	Calculated effective area, 20° C.	Average deviation from mean
1	2	3	4	5	6	7	8	9
			cm²	*Bars*		*Bars*	*cm²*	*cm²*
Apr. 13, 1928	5	4-C-30213	0.99776	1.5	3-B-12083	16.7	0.99606	0.00006
Apr. 14, 1928	2	4-C-30213	.99776	1.5	3-B-12083	16.7	.99609	0
Apr. 16, 1928	2	4-C-30213	.99776	1.5	3-B-12083	16.7	.99608	.00003
Apr. 17, 1928	2	4-C-30213	.99776	1.5	3-B-12083	16.7	.99606	.00003
Apr. 18, 1928	5	4-C-30213	.99776	16.3	3-B-12083	31.4	.99608	.00005
Apr. 19, 1928	5	4-C-30213	.99773	30.0	3-B-12083	45.0	.99606	.00004
	5	4-C-30212	.99773	45.8	3-B-12083	60.8	.99605	.00003
Apr. 27, 1928	5	3-B-30213	.99608	1.5	4-C-12083	16.5	.99778	.00005
Apr. 28, 1928	6	3-B-30213	.99608	1.5	4-C-12083	16.5	.99773	.00002
	2	3-B-30213	.99608	16.3	4-C-12083	31.2	.99773	.00001
Apr. 30, 1928	4	3-B-30213	.99608	16.3	4-C-12083	31.3	.99773	.00001
	4	3-B-30213	.99608	31.1	4-C-12083	46.0	.99773	.00001
May 2, 1928	6	3-B-30213	.99606	45.8	4-C-12083	60.7	.99771	.00002
May 3, 1928	1	3-B-30213	.99606	60.6	4-C-12083	75.4	.99771	----------
May 4, 1928	3	3-B-30213	.99606	60.6	4-C-12083	75.4	.99771	.00001
May 7, 1928	4	4-C-30213	.99771	60.5	3-B-12083	75.5	.99605	.00004
May 8, 1928	4	4-C-30213	.99776	1.5	3-B-12083	16.4	.99609	.00002
May 10, 1928	4	4-C-30213	.99776	1.5	3-B-12083	16.5	.99610	.00003
May 11, 1928	4	4-C-30213	.99776	1.5	3-B-12083	16.5	.99608	.00002

The precision obtained for the comparisons at the high pressures was about the same as at 15 bars; that is, a few parts in 100,000.

Table 6 presents the results of direct comparisons of piston gages, and provides a check on some of the data in the two preceding tables. The contents of the first six columns need no explanation. The observed ratio of the areas of the two gages corrected to the same temperature is given in column 7. The same ratio calculated from data in the preceding tables is given in column 8, and the number of the table from which the data were taken is given in column 9. In the last five observations the speed of rotation of the pistons was purposely varied over a wide range in order to ascertain whether the effective areas of the gages were altered by changing the speed of rotation, as is suggested by the work of Wagner.[7] No such effect is indicated.

[7] Wagner, Ann. Physik. (ser. 4), 15, p. 906; 1904.

TABLE 6.—*Comparison of piston gages*

Date	First gage		Second gage		Pressure (gage)	Observed ratio of areas 1st/2d	Calculated ratio	Source of calculated ratio
	No.	Revolutions per minute	No.	Revolutions per minute				
1	2	3	4	5	6	7	8	9
					B			*Table No.*
Mar. 31, 1926	2-A-30213		10 bar		9.06	0.100660		
1925 to 1926	2-A		10 bar		9.06		0.100670	4
Mar. 7, 1928	3-B-30213	18cc	4-C-12083	6c	21.2	.99832		
Mar. 9, 1928	3-B-30213	18cc	4-C-12083	6c	21.2	.99838		
Mar. 10, 1928	3-B-30213	18cc	4-C-12083	6c	21.2	.99836		
March, 1928	3-B		4-C		21.2		.99833	5
Mar. 12, 1928	3-B-30213	10cc	4-C-12083	4c	40.8	.99835		
	3-B-30213	10cc	4-C-12083	4c	40.8	.99837		
March, 1928	3-B		4-C		40.8		.99833	5
Mar. 12, 1928	3-B-30213	10cc	4-C-12083	4c	60.4	.99836		
	3-B-30213	10cc	4-C-12083	4c	60.4	.99834		
March, 1928	3-B		4-C		60.4		.99833	5
Mar. 13, 1928	3-B-30213	10cc	4-C-12083	4c	80.1	.99835		
	3-B-30213	10cc	4-C-12083	4c	80.1	.99835		
March, 1928	3-B		4-C		80.1		.99833	5
May 25, 1928	4-C-30213	7cc	10 bar	5.5c	9.04	.099807		
	4-C-30213	7cc	10 bar	5.5c	9.04	.099799		
	4-C-30213	7cc	10 bar	5.5c	9.09	.099806		
	4-C-30213	7cc	10 bar	5.5c	9.09	.099797		
	4-C-30213	7cc	10 bar	5.5c	9.11	.099797		
	4-C-30213	7cc	10 bar	5.5c	9.11	.099806		
	4-C-30213	7cc	10 bar	5.5c	9.12	.099799		
	4-C-30213	7cc	10 bar	5.5c	9.12	.099807		
May, 1928							.099801	4
Jan. 4, 1929 [1]	4-C-12083	8c	10 bar	2.4cc	8.17	.099799		
	4-C-12083	4c	10 bar	6.2cc	8.17	.099800		
	4-C-12083	3.2c	10 bar	2.9cc	8.17	.099801		
	4-C-12083	4c	10 bar	4.1cc	8.17	.099800		
	4-C-12083	18c	10 bar	4.1cc	8.17	.099800		

NOTE.—"c" represents clockwise rotation, "cc" represents counterclockwise.

[1] All observations given for this date were made with piston 4-C-12083 moving upward toward equilibrium.

Some authors have stated a preference for oscillatory rotation of the piston, instead of continuous rotation, and have claimed that the former method maintains the piston more nearly in a central position in the cylinder.[8]

For gages with pistons sticking at a definite azimuth, the use of oscillatory rotation enables the observer to adjust the piston to such azimuth as to reduce sticking to a minimum. In the present investigation the torque required for rotating the pistons at speeds of four or more revolutions per minute was proportional to the speed, a condition indicating that there was a complete film of oil around the piston, and that the latter was fairly well centered. A comparison of the results obtained in this investigation with those published in the various papers referred to in the Appendix shows that equally precise pressure measurements have been made with both types of gages.

In one instance [9] it has been reported that the effective area of piston gages changed with the pressure by as much as 2 parts in 1,000

[8] F. G. Keyes and Jane Dewey, J. Opt. Soc. of Am. 14, p. 491; 1927.
[9] Crommelin and Smid, Leiden Comm. No. 146c; 1915.

for pressures up to 100 bars, but calculation of the strains caused by pressures in that range indicate that there should be no such large change in any gage having metal parts of reasonable thickness. The data of the present investigation show no appreciable changes of this nature up to the highest pressure observed, 75 bars. The slight variations which do appear in the table are no greater than the experimental error.

2. CALIBRATION AS A PRIMARY INSTRUMENT, BY MEASUREMENT OF VARIOUS DIMENSIONS

Considerable effort was devoted to the calibration of the piston gages as primary, or absolute instruments. As stated in the preceding section, only when there is no leakage of oil past the piston is the area used for calculating the pressure the same as the cross-sectional area of the piston. Such a condition is not practicable, since its attainment requires a piston and cylinder in the form of true geometric cylinders of revolution having the same diameter. In actual gages the true geometric form is approximated as nearly as possible and the cylinder made slightly larger than the piston, the choice of sizes being a compromise in which the difference must be large enough to prevent the sticking caused by deviations from true form and yet small enough to avoid excessive leakage of fluid past the piston. In such gages the effective area is an average of the cross-sectional area of a surface lying somewhere between the surfaces of the piston and cylinder. The process of lapping is likely to leave the piston small at the ends, although in these gages this irregularity extends only over a short distance as illustrated for piston No. 1 in Figure 7. Similarly, the ends of the cylinders are likely to be large. These irregularities cause a tapering crevice. Secular changes in the steel may cause the piston or cylinder to become elliptical or the axis of either to become curved. Even though pistons and cylinders of true geometric form are attained, the slightest eccentricity in the application of the load will result in a torque which will throw the axis of the piston out of parallel with that of the cylinder.

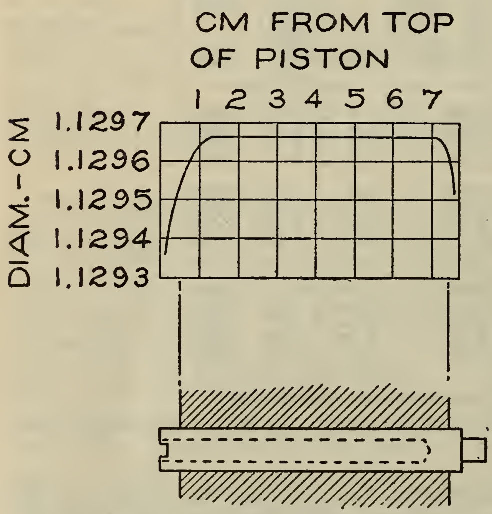

FIGURE 7.—*Contour of piston No. 1*

These possible causes of irregularity are so numerous that a combination of circumferential and axial variations in the width of crevice is to be expected. An approximate determination of the effective area can be made from a measurement of the average diameters of the piston and cylinder or from a measurement of the average piston diameter and the width of crevice between the piston and cylinder.

Wagner [10] determined the average diameter of gage cylinders from their length and the amount of mercury required to fill them. This method was not used in the present investigation, since it appeared that a measurement of crevice width offered equally accurate results, with less experimental difficulty. Three experimental methods were used for determining the width of crevice, the first involving a measurement of the oil leakage past the piston, the other two involving the use of the gage as a torsional viscometer. Before entering into experimental details and presenting the observed data, the fundamental relations between the dimensions of the piston and cylinder, and the sizes of the observed quantities will be discussed in the following order: (1) A formula for calculating the effective radius will be developed. The effective width of crevice—that is, the amount to be added to the diameter of the piston to obtain the diameter corresponding to the effective area—will be shown to differ by a negligible amount from the difference between the radii of the piston and cylinder, which are assumed to be coaxial cylinders of revolution; (2) formulas for calculating the difference between the radii of the piston and cylinder from data obtained by using the gage as a viscometer will be developed, the assumptions in regard to the piston and cylinder being the same as before; (3) a discussion will be given of the errors involved in the application of these formulas to a gage in which the crevice tapers along the axis but is uniform at any given cross section; (4) a similar discussion will be given for a gage in which the crevice is uniform along an element of the piston, but varies around a given cross section; and (5) the upward frictional force caused by the gradual descent of the piston in displacing the oil leakage will be discussed.

In these discussions the following notation will be used:

μ = the viscosity, in poises, of the fluid used in the gage.
Q = rate of leakage of fluid, in cm^3 per second.
v = velocity at any given point, in cm per second.
L = length of crevice, in cm.
r_1 = radius of piston, in cm.
r_2 = radius of cylinder, in cm.
$r_m = \frac{r_1 + r_2}{2}$.
R = effective radius, in cm.
p = excess, over the atmospheric pressure, of the pressure acting upward at the base of the piston (dynes per cm^2).
f = load on gage, in dynes.
T = torque, in dyne cm.
ω = angular velocity, in radians per second.
n = speed of rotation, in revolutions per second.
I = moment of inertia of rotating system (g cm^2).
t = time, in seconds.

1. The effective radius may be calculated as follows: Consider a hypothetical cylinder coaxial with the piston and steel cylinder, which may have a radius r anywhere between the limits r_1 and r_2. The forces acting on this cylinder will be the downward force f due to the load, the upward force $p\pi r^2$ due to the pressure, and the force due

[10] Ann. Physik. (ser. 4), 15, p. 906; 1904.

to the viscous drag of the oil which is $2\pi r L\mu \frac{dv}{dr}$, streamlineflowbeing assumed. For equilibrium conditions the sum of these forces is equal to zero; that is

$$f-p\pi r^2-2\pi r L\mu \frac{dv}{dr}=0 \tag{5}$$

Integration of the above equation gives

$$f \log_e r-\frac{p\pi r^2}{2}-2\pi L\mu v=C \tag{6}$$

where C is a constant of integration.

Since the velocity of the fluid must be zero at the surface of the piston and of the cylinder, $v=0$ for $r=r_1$ and $r=r_2$.

Substitution of these limiting conditions in equation (6) gives

$$C=\frac{p\pi\ (r_2^2 \log_e r_1-r_1^2 \log_e r_2)}{2 \log_e \frac{r_2}{r_1}} \tag{7}$$

and

$$f=\frac{p\pi\ (r_2^2-r_1^2)}{2 \log_e \frac{r_2}{r_1}} \tag{8}$$

Since the effective area is defined as the load divided by the pressure, the effective radius deduced from equation (8) is

$$R=\sqrt{\frac{f}{p\pi}}=\sqrt{\frac{r_2^2-r_1^2}{2 \log_e \frac{r_2}{r_1}}} \tag{9}$$

The only assumptions made in deducing this value for the effective radius are that the piston is a true geometric cylinder of circular cross section inside of and coaxial with a hollow cylinder of similar form, and that the flow is stream line. The latter assumption is justified by the fact that the velocity is much smaller than that which Buckingham[11] has shown to be necessary for the production of turbulence.

For comparison with the values of effective radius used by some other authors, that deduced here may be converted into another form by substituting

$$\frac{r_2}{r_1}=1+x$$

and the formula

$$\log\ (1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\cdot\cdot\cdot\cdot$$

in equation (9).

[11] Engineering, 115, p. 225; 1923.

The cube term in the series obtained is zero and the higher powers are quite negligible.

Whence

$$R=\sqrt{r_2 r_1+\frac{(r_2-r_1)^2}{6}-\dots} \tag{9a}$$

Klein [12] has considered the effective area to be the mean of the areas of the piston and cylinder. The effective radius thus defined is

$$R'=\sqrt{\frac{r_2^2+r_1^2}{2}}=\sqrt{r_2 r_1+\frac{(r_2-r_1)^2}{2}} \tag{10}$$

Some authors have used the mean of r_2 and r_1; that is,

$$R''=\frac{r_2+r_1}{2}=\sqrt{r_2 r_1+\frac{(r_2-r_1)^2}{4}} \tag{11}$$

Since in piston gages the difference between r_2 and r_1 is less than one-thousandth part of these radii, it is easily seen that the three values of R defined by equations (9) or (9a), (10), and (11) will differ by less than 1 part in 1,000,000. The value derived in this paper (equation (9)), which is exactly correct for the conditions assumed, is smaller than either of the other two values, but all three are so nearly equal that the difference is only of academic interest.

If the effective width of crevice W_1 is defined as the amount to be added to the piston diameter to obtain the diameter corresponding to the effective area, then

$$W_1=2\ (R-r_1)=2\left[\sqrt{\frac{r_2^2-r_1^2}{2\log_e\frac{r_2}{r_1}}}-r_1\right] \tag{12}$$

From the preceding discussion in regard to R, it is easily deduced that the effective width W_1 is negligibly smaller than the true width $(r_2-r_1)=W$.

2a. The difference between the radii of the piston and cylinder may be calculated from the rate of leakage past the piston, by substitution of equations (6), (7), and (8) in the equation

$$Q=\int_{r_1}^{r_2} v\,2\pi r\,dr$$

and performing the integration. After several algebraic transformations one obtains

$$Q=\frac{\pi p r_1^4}{8\ L\mu}\left[\left(\frac{r_2^4}{r_1^4}-1\right)-\frac{\left(\frac{r_2^2}{r_1^2}-1\right)^2}{\log_e\frac{r_2}{r_1}}\right] \tag{13}$$

This equation is also derived in H. Lamb's Hydrodynamics, fifth edition, page 555.

[12] Berlin T. H. Diss., 1909. George Klein, Untersuchung und Kritik von Hochdruckmessern.

By following the same procedure as used in reducing equation (9) to the form (9a), equation (13) may be converted into the form

$$Q=\frac{\pi p r_1^4}{8L\mu}\left[\frac{4}{3}x^3+\frac{2}{3}x^4+\frac{x^5}{45}+\cdot\cdot\cdot\cdot\cdot\right]$$

where, as before

$$x=\frac{r_2}{r_1}-1=\frac{W}{r_1}$$

or

$$Q=\frac{p\pi r_1(r_2-r_1)^3}{6L\mu}\left[1+\frac{W}{2r_1}+\frac{\left(\frac{W}{r_1}\right)^2}{60}+\cdot\cdot\cdot\right] \tag{14}$$

Since this equation is to be used in calculating (r_2-r_1), a quantity which enters as a small correction to the piston diameter, and since for piston gages in general, $\frac{W}{r_1}$ has a value less than 0.001, all terms of the series except the first may be neglected. Whence, denoting the difference in radii calculated in this manner by W_2, we obtain equation (15) which was used for calculating the width of crevice from the observed rate of oil leakage past the piston.

$$W_2=r_2-r_1=\sqrt[3]{\frac{6L\mu Q}{p\pi r_1}} \tag{15}$$

The terms that were found to be negligible in the series of equation (14) correct for the curvature of the crevice, and equation (15) represents exactly the relation between the width of crevice between two infinite plane surfaces and the leakage through a portion of this crevice $2\pi r_1$ in width.

2b. The width of crevice may be calculated from measurements of the torque required to rotate the piston, as follows:

The torque is represented by the equation

$$T=-2\pi r^3 L\mu\frac{d\omega}{dr}$$

where ω is the angular velocity in the lubricant at the radius r. Whence

$$\int_{r_2}^{r_1}\frac{dr}{r^3}=-\int_0^{\omega_1}\frac{2\pi L\mu d\omega}{T}$$

ω_1 being the angular velocity of the piston. Integrating and substituting $2\pi n$ for ω_1 gives

$$\frac{4\pi^2 L\mu n}{T}=(r_2-r_1)\frac{(r_2+r_1)}{2\,r_1^2 r_2^2} \tag{16}$$

If we represent the width of crevice thus calculated by W_3 then,

$$W_3=r_2-r_1=\frac{4\pi^2 L\mu n r_m^3}{T}\left(1-\frac{W^2}{2r_m^2}+\frac{W^4}{16r_m^4}\right)$$

The last two terms of the polynomial are negligible, whence

$$W_3=\frac{4\pi^2L\mu nr_m^3}{T}$$

For the small widths of crevice occurring in piston gages it is amply accurate to substitute in this equation r_1 in place of r_m whence

$$W_3=\frac{4\pi^2L\mu nr_1^3}{T} \tag{17}$$

which is the form used in this investigation for calculating crevice widths from torque measurements at constant speed.

2c. It was found that for constant speeds above a certain minimum the torque measured was proportional to the speed. If this is assumed to be true also for variable speeds then, when the piston and weights are kept rotating only by the inertia of the system, the torque exerted on the cylinder by the piston is

$$T=a\omega=-I\frac{d\omega}{dt} \tag{18}$$

where a is a constant, I is the moment of inertia of the rotating system, and ω is its angular velocity at any time t. Integration of equation (18) gives

$$\omega=\frac{d\theta}{dt}=\omega_1 e^{-\frac{a}{I}(t_2-t_1)}$$

where ω_1 is the angular velocity at the time t_1 and θ is the angular position or azimuth of the rotating system at any time t. A second integration gives

$$\theta_2-\theta_1=-\frac{I\omega_1}{a}\left\{e^{-\frac{a}{I}(t_2-t_1)}-1\right\} \tag{19}$$

The determination of θ_1, θ_2, and θ_3 corresponding to the times t_1, t_2, t_3 chosen such that $t_3-t_2=t_2-t_1$ makes it possible to express a as an explicit function of the variables, thus

$$a=\frac{I}{t_2-t_1}\log_e\frac{\theta_3-\theta_2}{\theta_2-\theta_1} \tag{20}$$

But equations (17) and (18) lead to

$$a=\frac{2\pi\ L\ \mu r_1^3}{W_3}$$

Therefore

$$W_3=\frac{2\pi\ L\ \mu r_1^3\ (t_2-t_1)}{I\log_e\frac{\theta_3-\theta_2}{\theta_2-\theta_1}} \tag{21}$$

which is the equation used to calculate the width of crevice from observations of time and corresponding angular position of the rotating system spinning free of any driving mechanism.

The preceding formulas were derived for a crevice of uniform width, but, as already stated, a combination of axial and circumferential variations in the width of crevice is to be expected. For simplicity, each of the two types of irregularity will be considered separately, and the consequent errors involved in the application of the preceding formulas will be estimated.

3. Errors involved in the application of the formulas to a gage in which the crevice tapers along the axis. For simplicity, it is assumed that the piston is a geometrical cylinder of circular cross section and that the steel cylinder forms a conical frustrum of circular cross section as illustrated in Figure 8. The same notation will be used as

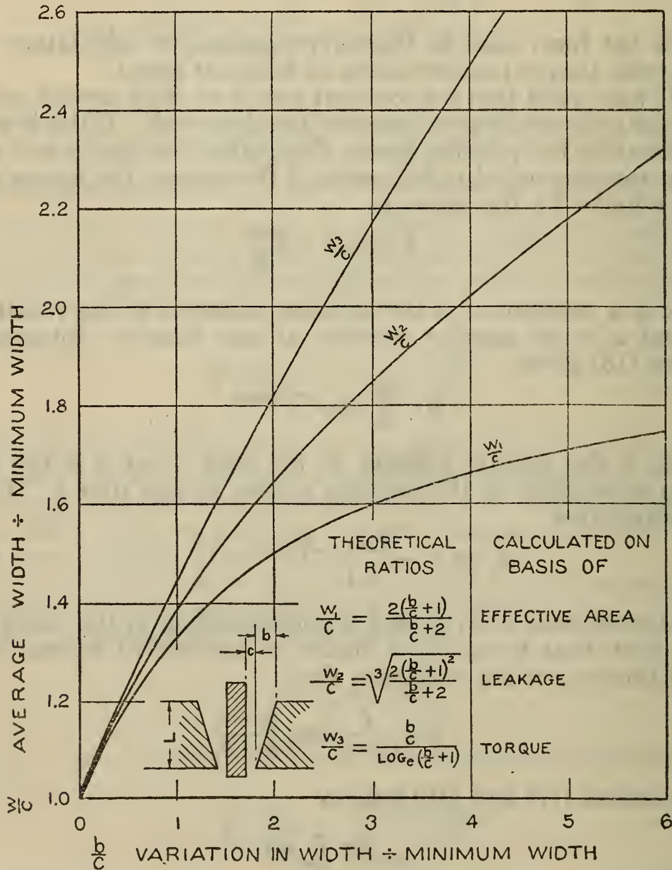

FIGURE 8.—*Effect of taper on the width of crevice as calculated by various methods*

before, and, in addition, x will represent the width of crevice at any distance y from the narrow end of the cylinder. Then

$$x=\frac{by}{L}+c \tag{22}$$

where c is the width of crevice at the narrowest place (see fig. 8), and b is the change in width of crevice from the narrowest to the widest part.

For the sake of convenience, the order in which the formulas were discussed in the preceding section will be changed, and the leakage for the piston and cylinder just described will be calculated first.

From equations (15) and (22) we have for any infinitesimal length of the piston

$$\left\{\frac{by}{L}+c\right\}^3=-\frac{6\ \mu Q}{\pi r_1}\frac{dy}{dp} \tag{23}$$

Integration gives

$$p=\frac{6\ \mu Q}{\pi r_1}\int_{y=0}^{y=L}\frac{dy}{\left\{\frac{by}{L}+c\right\}^3}=\frac{3\ \mu Q\ L}{\pi b\ r_1}\left\{\frac{1}{c^2}-\frac{1}{(b+c)^2}\right\} \tag{24}$$

If, as was done in this investigation, equation (15) is used to calculate W_2 then

$$p=\frac{6\ \mu Q\ L}{W_2^3\ \pi r_1}$$

Substitution of this value of p in equation (24) gives

$$\frac{W_2}{c}=\sqrt[3]{\frac{2\ (b/c+1)^2}{b/c+2}} \tag{25}$$

which is the relation between the average width of crevice W_2, calculated from the measurement of oil leakage, and the actual dimensions of the crevice.

To deduce the relation between the average effective width and the dimensions shown in Figure 8 it will be assumed that the effective area for each element of length dy is the mean of the corresponding areas of the piston and cylinder. Then the element of force due to the drop in pressure along the element dy is

$$dF=\pi r_1^2\frac{dp}{dy}dy-\pi r_m x\frac{dp}{dy}dy \tag{26}$$

By replacing r_m with the approximate value r_1, and by substituting in (26) the value of $\frac{dp}{dy}$ given by equation (23) and the value of Q given by equation (24) we obtain

$$F=\pi r_1^2 p+2\pi p r_1\int_{y=0}^{y=L}\frac{1}{\frac{1}{c^2}-\frac{1}{(b+c)^2}}\frac{b}{Lx^2}dy$$

therefore

$$F=\pi r_1^2 p+2\pi p r_1\left\{\frac{1}{\frac{1}{c}+\frac{1}{(b+c)}}\right\}$$

The effective area of the gage minus the area of the piston is

$$\frac{F}{p}-\pi r_1^2=\frac{2\pi r_1}{\frac{1}{c}+\frac{1}{b+c}}=S$$

The effective width of the crevice, or the amount to be added to the piston diameter for calculating the effective area, is then approximately

$$W_1 = \frac{2S}{2\pi r_1} = \frac{2}{\frac{1}{c} + \frac{1}{b+c}}$$

or

$$\frac{W_1}{c} = \frac{2(b/c+1)}{b/c+2} \tag{27}$$

The relation between the average width, calculated from torque measurements and the dimensions b and c, is deduced by a method similar to that used for the average width calculated from leakage measurements.

The torque contributed by the element of length dy is (from equation (17))

$$dT = \frac{4\pi^2 r_1^3 n\mu dy}{\frac{by}{L} + c}$$

Integration of this differential equation and substitution in equation (17) gives

$$\frac{W_3}{c} = \frac{b/c}{\log_e\left\{\frac{b}{c} + 1\right\}} \tag{28}$$

Values of the three ratios just derived have been calculated for various values of $\frac{b}{c}$, and the data presented graphically in Figure 8. This figure shows that for a tapering crevice the measurement of the actual effective area and piston diameter should give the smallest width, the measurement of leakage a larger value, and the measurement of the torque the largest value of the three.

4. The effect of departures of either piston or cylinder from true circular form may be illustrated by assuming a piston which forms a true cylinder of revolution surrounded by a composite cylinder made up of two or more coaxial segments having different radii. Application of equations (12), (15), and (17) to a specific problem shows that when such irregularities occur, measurement of effective area and piston diameter should give an intermediate value, the measurement of leakage a larger value, while the torque should give the smallest value for the width of crevice.

For a given sectional area, irregularities in the form of piston or cylinder cause less change in the effective width than in the widths determined from oil leakage and torque measurements. In fact, the effective width is essentially unchanged by such irregularities if the dimensions are such as occur in precise piston gages. Michels [13] reached the same conclusion. The equation for the leakage of a

[13] Ann. d. Phys. (ser. 4), **73**, p. 603; 1924.

viscous fluid between a piston and cylinder which are geometric cylinders of revolution and have parallel axes, is

$$Q=\frac{p\pi r_m W_1^3}{6\mu L}\left(1+\frac{3e^2}{2W_1^2}\right)$$

where e is the distance between the axes. This equation, which was derived by Becker,[14] also shows that the leakage is increased by circumferential irregularities.

5. In the paper just referred to, Michels discussed the upward frictional force caused by the gradual sinking of the piston as it displaces the oil which leaks from the gage, and expressed this force in terms of the observed velocity of descent and the torque required to rotate the piston. If the only leakage is past the piston, it is easily shown that this frictional force is negligible in the present investigation.

Let V_1 be the downward velocity of the piston, and f_1 the corresponding force, then substitution in equation (6) of $-V_1$ instead of zero for the velocity at the radius r_1 gives a value for f exceeding that in equation (8) by the amount

$$f_1=\frac{2LV_1\pi\mu}{\log_e\frac{r_2}{r_1}}$$

or approximately

$$f_1=\frac{2LV_1r_1\pi\mu}{W}$$

The corresponding proportional error in the measured pressure is

$$\frac{f_1}{p\pi r_1^2}=\frac{2V_1L\mu}{pWr_1} \tag{29}$$

If the only leakage is past the piston

$$V_1=\frac{Q}{\pi r_1^2}$$

but from equation (15)

$$Q=\frac{\pi\ p\ W_2^3\ r_1}{6\ L\ \mu}$$

Substitution of these values in equation (29) gives, since W_2 and W are interchangeable without appreciable error

$$\frac{f_1}{p\ \pi\ r_1^2}=\frac{W^2}{3\ r_1^2} \tag{30}$$

which for the gages used is less than 1 part in 10,000,000.

When leakage other than that past the piston is encountered, the percentage error in pressure measurement will be proportional to the total leakage as may be deduced from equation (29).

[14] Forsch. Arb. Ingenieurw., 48, p. 1; 1907.

In the earlier pressure measurements, leakage from valves, etc., was considerably greater than that past the piston, but as time progressed extraneous leakage has been reduced, until recently it is much smaller than that past the piston. It seems fairly safe to say that the error from this source has never been more than 1 part in 100,000, since even this small error would correspond to a leakage 100 times as great as that past the piston, a rate which would have constituted a nuisance, due to the frequency with which the oil in the gage would have had to be replenished.

This discussion leads to the following conclusions: (1) For a coaxial piston and cylinder having the form of true geometric cylinders of revolution, the effective width of crevice, that is, the amount to be added to the piston diameter to obtain the effective diameter, is approximately the difference in radii of the piston and cylinder. (2) For such a piston and cylinder the width of crevice calculated from leakage or torque measurements is practically the same as the effective width. (3) Any irregularity in the crevice will cause the width calculated from the leakage measurements to be somewhat larger than the effective width. If the irregularity is axial, the width calculated from the torque measurements is the largest of the three; if circumferential, it is the smallest. The width defined by the effective area is essentially independent of circumferential variations in the width of crevice and varies with axial variations as indicated by equation (27). (4) The error due to the sinking of the piston in displacing oil leakage is entirely negligible.

These conclusions form a basis for the critical examination of the experimental data which will be presented. The methods used for measuring the width of crevice are as follows:

(1) Measurement of the rate of oil leakage; (2) rotation of the piston at a constant speed and use of a calibrated spring to measure the torque required; and (3) initial rotation of the piston and weights by hand, and determination of the retarding torque from observations of the speed, the deceleration, and the moment of inertia of the rotating system.

1. For the measurement of oil leakage a small amount of weighed absorbent cotton was placed around the upper end of the piston in an annular space previously wiped dry, the load was applied to the piston, and the piston rotated at the usual speed for a period of 10 minutes to an hour, the time being chosen so that the amount of leakage was usually between 0.05 and 0.2 g. The cotton was again weighed and the weight of the oil obtained as the difference of the two observed weights.

2. The torque required to rotate the piston at a constant speed was determined by replacing the regular driving spring by a calibrated spring of suitable strength and, by means of a scale and pointer, observing the stretch of the spring while the piston was rotated. Since the torque was not uniform, due in part to irregular motor speed, it was necessary to take the mean of readings at several different azimuths of the piston. The observations with gage 2–A were further complicated by the fact that the torque passed through a maximum as the piston turned through a certain azimuth, and, furthermore, this maximum was much larger every second revolution than for alternate revolutions. The observations which indicated such anomalous behavior were confirmed by observations of similar minima

in the electrical resistance of the oil film between the piston and the cylinder. The difficulties arising from variable motor speed led to the use of a third method for measuring the width of crevice.

3. The frictional torque which retarded the rotating system at a decelerating speed was measured by disconnecting the driving springs, giving the piston and weights an initial spin by hand and repeatedly observing the time and the angular position of the rotating system until the rotation ceased. For convenience in using equation (21) it is desirable to make these observations at equal time intervals, but to make the experimental procedure easier, the time was observed at which consecutive revolutions or groups of revolutions were completed as well as the time and angular position at which the rotation ceased. These data were plotted with time and revolutions as coordinates, a smooth curve was drawn through the points, and the values of the angular positions given by this curve at equal time intervals were tabulated. Then the following were calculated: (1) Differences between the consecutive tabulated values, (2) the quotients obtained by dividing each difference by the following difference, and (3) the Napierian logarithms of these quotients. These logarithms were used in equation (21) to calculate a series of values for the width of crevice, a mean of these values being considered as a single observation. The values of width obtained near the end of the spin were smaller than those obtained for the higher speeds and were neglected in calculating the mean.

Table 7 contains the results obtained by these three methods. The results for each gage are grouped together. Each of the values given is the mean of several individual observations. With the exceptions stated in footnotes to the table, an oil with a viscosity of about 0.5 poise was used to obtain these results.

TABLE 7.—*Measurements of the average width of crevice between piston and cylinder*

GAGE 1-A

Date and gage No.	Method [1]	Load on gage (in kilograms)					
		1	21	41	61	81	101
1	2	3	4	5	6	7	8
		μ	μ	μ	μ	μ	μ
March, 1920	2	0.7	--------	0.7	--------	--------	--------
	[2] 1	--------	--------	1.0	--------	--------	--------
July-August, 1923	1	--------	2.2	2.4	2.5	2.6	2.6
August, 1925	1	--------	2.2	--------	2.4	2.4	2.5

GAGE 2-A

Date and gage No.	Method [1]	1	21	41	61	81	101
February, 1926	1	--------	1.4	1.3	1.2	1.1	1.1
April-May, 1926	1	--------	--------	1.4	1.3	1.3	--------
	2	--------	1.3	[3] 1.8	.8	.6	--------
June, 1926	1	--------	1.7	1.7	1.6	1.5	--------
August, 1926	3	--------	.8	.7	.5	.8	--------
August, 1927	1	--------	2.0	--------	--------	1.8	--------
	3	--------	.8	.9	.9	.9	--------

[1] Three methods were used: (1) Measurement of oil leakage, (2) measurement of torque by means of a calibrated spring, (3) measurement of torque by deceleration of the rotating system.

[2] Piston lapped twice in 1922.

[3] The mean of only three observations. If one observation is discarded, the mean is 1.4.

Table 7.—*Measurements of the average width of crevice between piston and cylinder*—Continued

GAGE 3–B

Date and gage No.	Method	Load on gage (in kilograms) 1	21	41	61	81	101
1	2	3	4	5	6	7	8
		µ	µ	µ	µ	µ	µ
August, 1925	1	----	1.1	1.0	1.0	1.0	1.0
March, 1926	1	----	1.0	----	----	1.0	----
August, 1927	1	----	----	.9	----	.9	----
	3	----	.8	.8	.8	.8	----
April, 1928	3	----	.7	----	.6	----	----
	1	----	.8	.8	.8	----	----

GAGE 3–C

Date and gage No.	Method	1	21	41	61	81	101
March, 1926	1	----	4.3	----	----	----	----
July, 1927	3	----	[4] 5.3	----	----	----	----
	1	----	[5] 5.2	5.0	----	----	----

GAGE 4–C

Date and gage No.	Method	1	21	41	61	81	101
February, 1926	1	----	----	1.0	1.0	----	----
March, 1926	1	----	1.0	----	----	1.0	----
July, 1927	1	----	[6] 1.9	1.3	----	----	----
March, 1928	3	----	1.4	----	1.0	----	----
	1	----	2.2	----	2.1	----	----

10–BAR GAGE

Date and gage No.	Method	1	21	41	61	81	101
1925	1	----	4.2	----	4.7	----	----
May, 1928	1	----	----	4.2	----	4.4	----
	3	----	4.3	3.3	3.5	3.7	----

[4] Obtained with cylinder stock. The value obtained with renown engine oil was about 3.0.
[5] Obtained with both renown engine oil (viscosity at room temperature about 0.5 poise), and cylinder stock (viscosity about 11 poises).
[6] Piston sticking badly at this date. Piston was lapped in September, 1927.

The precision with which the crevice width was determined from the measurement of oil leakage was rather high, the average deviation of individual observations from the mean being the largest for gage 2–A, namely, about 5 per cent. The precision of the torque measurements was not so good, the average deviation from the mean being about the same for all the gages, namely, 20 to 25 per cent. The accuracy of the torque measurements for the 10-bar gage with a load of 21 kg is poor, because the time of spin was short and observations had to be taken rapidly.

For some of the gages the width of crevice determined by leakage measurements appears to change consistently with the pressure. It is difficult to say whether this variation is significant, but slight variations in either direction are explainable as follows: (1) If the load is slightly eccentric, the angle between the axis of the piston and that of the cylinder may be expected to change with the load, thus varying the uniformity of the crevice; (2) if the piston and cylinder are lapped to fit more closely at the upper end than at the lower the application of pressure will cause the cylinder to stretch and the hollow piston to shrink, with a consequent increase in the average crevice width; and (3) if the piston and cylinder are lapped to fit more closely at the lower end than at the upper, the application of pressure will cause the reentrant part of the cylinder to shrink upon the solid portion of the piston with a consequent decrease in the average crevice width.

In Table 7 the values for crevice width obtained from torque measurements are, in general, appreciably less than those from leakage measurements. If the difference is caused by circumferential irregularities only, the effective width should be between the values given by the two methods; if there exists a combination of axial and circumferential irregularities, the effective width will be less than the value determined from leakage, but it may be either more or less than the value determined by torque measurements.

An attempt to determine the width of crevice by measurement of the electrical resistance across the oil film gave results much too small. The results indicated that either the voltage used (1 volt) had broken down the oil film, or that the piston was practically in contact with the cylinder at one or more spots.

It is now possible to compare the results obtained in the absolute calibration with those obtained by direct comparison with the mercury manometer. For this purpose, data from Tables 4, 5, and 7 have been correlated in Table 8, together with the values of piston diameter as measured with an interferometer. All values have been corrected to 20° C. In every case the upper of a pair of bracketed numbers was derived from measurements of leakage and the lower from those of torque. The values in column 3 have been chosen from Table 7 in such manner that they correspond chronologically as nearly as possible to the values in columns 2 and 6.

The importance of choosing observations which are approximately simultaneous is illustrated by the data on gage 3–C in 1927 and on gage 4–C in 1928. The two values for the diameter of the cylinder C which may be calculated from these data, with the assumption that the effective diameter is the mean of the piston and cylinder diameters, differ by over 3μ, indicating an increase in the diameter of cylinder C. This indication is confirmed by an analysis of the data in Tables 4, 5, and 7 for gage 4–C alone in the interval 1926 to 1928. Table 7 shows also that the diameter of piston No. 4 increased by 1μ in excess of the unknown amount removed by lapping, and that the diameter of piston No. 3 decreased by about 2μ.

Table 8 indicates that torque measurements give a more nearly correct value of the average crevice width than do the leakage measurements, but even they lead to a value that exceeds the effective by as much as 1.0μ, the average excess being 0.5μ. That this difference is not entirely due to errors in crevice measurements is shown by the fact that the effective diameter for gage 3–B in 1928 is actually less than the measured piston diameter. In this particular instance the discrepancy must be attributed to one or both of two sources: (1) Error in pressure measurement, (2) error in the measurement of the piston diameter with an interferometer.

TABLE 8.—*Comparison of different methods for calibrating gages*

GAGE 2-A

Gage number and date	Piston diameter	Crevice width	Column (2) plus column (3)	Area calculated from column (4)	Observed effective area	Column (5) minus column (6)	Observed effective diameter from column (6)	Column (4) minus column (8)
1	2	3	4	5	6	7	8	9
	cm	*cm*	*cm*	*cm* ²	*cm* ²	*cm* ²	*cm*	*cm*
February, 1926	1.13197							
		[1] 0.00014	1.13211	1.00662		0.00012		0.00007
April–May, 1926		.00013	1.13210	1.00660		.00010		.00006
March, 1926					1.00650		1.13204	
					1.00642		1.13200	
August, 1927		[1] .00020	1.13212	1.00664		.00022		.00012
		.00008	1.13200	1.00642		0		0
October, 1928	1.13192							
GAGE 3-B								
April, 1922	1.12642							
November, 1924	1.12643							
March, 1926	1.12622				0.99629		1.12629	
		[1] 0.00010	1.12632	0.99635		0.00006		0.00003
August, 1927		.00008	1.12630	.99632		.00003		.00001
March, 1928					.99612		1.12619	
April, 1928		[1] .00008	1.12630	.99632		.00020		.00011
		.00007	1.12629	.99630		.00018		.00010
October, 1928	1.12622							
GAGE 3-C								
June, 1927					0.99676		1.12655	
July, 1927		[1] 0.00052	1.12674	0.99710		0.00034		0.00019
		.00042	1.12664	.99692		.00016		.00009
October, 1928	1.12622							
GAGE 4-C								
March, 1926	[2] 1.12692				[3] 0.9972		1.1268	
					.99777		1.12712	
March, 1928		[1] 0.00022	1.12725	0.99800		0.00023		0.00013
		.00014	1.12717	.99786		.00009		.00005
October, 1928	1.12703							
	Mean difference using leakage measurements					.00020		.00011
	Mean difference using torque measurements					.00009		.00005

[1] The values in this row were obtained from leakage measurements and those in the next row from torque measurements.
[2] Piston lapped between the observation marked and the next observation.
[3] The piston was sticking badly. Piston was lapped September, 1927.

1. The estimated error which may occur in the use of the manometer is discussed in detail later, and is about the same as the discrepancy in area mentioned; that is, a little less than 1 part in 10,000, or less than 0.5 μ in the diameter of an 11 mm piston.

2. The measurement of piston diameter with an interferometer is capable of much greater accuracy than the irregularities in diameter permit. The total estimated error, including that due to any uncertainty in the temperature during measurement, is about 0.2μ for the

11 mm pistons (Nos. 1, 2, 3, and 4) and about 0.5 μ for the 35 mm piston (No. 5, 10-bar).

We may conclude that the use of leakage measurements in the absolute calibration gave a calculated effective area slightly too large, especially for gages with crevices over 1 or 2 μ in width, but that the use of torque measurements in the absolute calibration gave results which, for the gages observed, were about as accurate as those obtained from a comparison of the gages with the manometer described, the difference between the effective areas obtained by the absolute calibration and the comparison with the manometer being less than 2 parts in 10,000. For gages having larger and more irregular crevices a comparison with the manometer appears preferable to an attempt at absolute calibration, since for such gages the torque might be less accurately measurable. Furthermore, the direct comparison with the manometer gives a better indication of the accuracy and precision which may be expected of the gage in actual pressure measurements.

VIII. SOURCES OF ERROR IN MANOMETER AND GAGES

Sources of error arising from the use of the manometer are as follows:

1. *Graduation of scales and observation of reading devices.*—The error of graduation of the manometer scales does not exceed 0.04 mm. When oil was used as a pressure-transmitting liquid, the precision obtained with the manometer was limited by the inability of the observer to locate the mercury surfaces within about 0.5 mm. With the use of alcohol the uncertainty was reduced to 0.1 or 0.2 mm; with the use of water, to about 0.05 mm. In consequence, the combined error for the 10 water-mercury menisci will not exceed 1 mm, while the probable error will be about 0.3 mm, or 2 parts in 100,000.

2. *Error in temperature measurement.*—It is believed that the thermometer indicates the temperature of the manometer within 0.2° C., which corresponds to 4 parts in 100,000 in the pressure measurement.

3. *Capillary forces.*—For the observations taken in 1926 and in December, 1925, the heights of the mercury menisci were observed and correction was made for capillary forces, with the aid of certain data [15] and assumptions. The assumption that the correction for a mercury-water meniscus would be three-fourths of that for a mercury-air meniscus was made by inference from a paper by F. R. Watson [16] which gives the interfacial tension between mercury and water as about three-fourths of the surface tension of mercury.

The corrections for capillary forces rarely amounted to as much as 0.04 cm, positive corrections being about equal in number and amount to negative. Consequently, although the application of these corrections reduced the average deviation from the mean to about 0.8 of what it would have been if the corrections had not been applied, it did not appreciably change the mean values.

The observations made in 1927 and 1928 were not corrected in this manner, but the manometer tubes were tapped so as to disturb the mercury surfaces rather violently two or three times before readings were taken. The data in Table 4 show that the average deviation

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[15] Physikalisch-Chemische Tabellen, Landolt-Bornstein-Roth, 5th ed., p. 72.
[16] Phys. Rev., 12, p. 257; 1901.

from the mean values is about the same as for the observations in 1926, or that the tapping produced close reproducibility of form in all the mercury surfaces. In view of the uncertainty of capillary errors it is advisable to make observations in tubing of such size that the errors are negligible. The tubing of the 15-bar manometer fulfills this condition within the accuracy intended by the designer, but the accuracy required of this instrument has been considerably increased during its years of use.

Equations for estimating the diameter of tube corresponding to a capillary correction smaller than a specified amount have been developed mathematically by Laplace and by Lord Rayleigh.[17] The equations apply accurately only to narrow tubes or to those having diameters of 4 cm or more, but give an approximate estimate for intermediate sizes.

4. *Lack of equilibrium.*—The program of observation has been carried out in such manner that errors due to this source are probably negligible.

It is believed that the combined effect of these errors will not be greater than 1 part in 10,000, and that the manometer is accurate within that limit.

The use of the piston gages is accompanied by the following sources of error:

1. *Change in the dimensions of the piston or cylinder.*—Change in the dimensions of the steel in the gages has been the source of considerable trouble; not only has it caused a decrease of 2 or 3 parts in 10,000 in the effective area of gage 3–B, but it has necessitated a relapping of cylinder A, piston 4, and the piston of the 10-bar gage, to relieve sticking. The changes consisted of a bending of the axis of cylinder A, an increase in the diameter of piston 4, as well as its cylinder C, and a change from circular to elliptic cross section in the piston of the 10-bar gage.

2. *Sticking of the piston.*—No definite value can be assigned to the error due to this cause. In the procedure followed for calibration of the gages, any error of this nature would appear as a part of the variation in the results. The maximum variation of the data in Table 6 for a comparison between gages 3–B and 4–C is 6 parts in 100,000, while data from a comparison of 4–C with the manometer before the piston was lapped to relieve sticking showed variations as large as 2 parts in 1,000.

3. *Error in temperature measurement.*—The thermometers used at present probably measure the temperature of the gage within 0.3° C., which corresponds to an error of less than 1 part per 100,000. The accuracy of this temperature measurement could be improved easily should the need for such improvement arise.

4. *Magnetic forces.*—Data for gage 2–A given in Table 4 under date of June and July, 1926, show that the error due to the magnetic field of the motor is not greater than the precision of the results, about 3 parts in 100,000. The motor which rotates the piston in base 12,083 is farther removed from the other iron parts of the gage than the motor on base 30213. Consequently, the results with base 12083 and with the motor removed from its normal position in base 30213

[17] Scientific Papers by Lord Rayleigh, **6**, p. 350, published by Cambridge Univ. Press; or Proc. Roy. Soc. A, **42**, p. 184; 1915.

should be less subject to error than the results with the motor in its normal position on base 30213.

5. *Lack of equilibrium.*—Errors due to this cause have been made negligibly small by allotting sufficient time to each observation.

6. The vertical component of the force applied to rotate the piston may be neglected, since it is not over a few hundredths of a gram.

The most important source of error in measurements with the piston gage appears to be a secular change in the dimensions of steel, which may amount to 3 or 4 parts in 10,000 in the effective area. With calibrations sufficiently frequent to correct for this change, an accuracy better than 1 part in 10,000 may be attained.

IX. ACKNOWLEDGMENTS

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The authors are indebted to C. G. Peters, who made repeated measurements of the pistons with an interferometer, thereby assisting in the construction of pistons with uniform diameters and giving data on the diameters at various intervals after the gages had been constructed.

X. SUMMARY

The results of the work with the multiple U tube manometer and the group of piston gages maintained for the precise measurement of pressures may be summarized as follows:

Oil, alcohol, and water have been used successively to transmit the pressure between the manometer tubes. Of these, water has been found to be the most satisfactory.

The results emphasize the necessity of using tubing of such size at the mercury menisci that capillary forces are negligible.

The mercury manometer is capable of an accuracy within 1 part in 10,000. On account of secular changes in the dimensions of steel, the piston gages require occasional calibration when an accuracy within 3 or 4 parts in 10,000 is desired. The precision obtained for the direct comparison of two piston gages indicates that with the elimination of secular changes, or with sufficiently frequent comparison with a manometer of the requisite accuracy, these gages would be accurate within 3 or 4 parts in 100,000.

The rotational speed of the piston had no measurable effect upon the effective area of the gage.

The change in effective area with pressure, in the range 0 to 75 bars, was less than the precision of measurement, about 3 parts in 100,000, or about 1 part in 2,000,000 per bar. The effective radius is slightly smaller than either the mean of the radii of the piston and cylinder or the radius corresponding to the mean of the areas of piston and cylinder, but for all practical purposes the three values are equal.

The measurement of piston diameter with an interferometer and the determination of crevice width from measurements of oil leakage

or of torque, furnish data for the calibration of the piston gage as a primary instrument. For the gages described, such calibrations agree with the results of comparisons with the manometer within 2 or 3 parts in 10,000 of the effective area, the calibration by means of torque being the better, though not quite so reproducible as that by leakage. Direct comparison with the manometer is desirable, since it indicates also the accuracy and precision which may be expected of the gage in actual pressure measurements.

A list or references to papers on the subject of differential manometers and piston gages is given in the Appendix at the end of this paper. The titles of the papers have been omitted and brief abstracts of the contents have been given instead, since some of the titles do not indicate the presence of the data on this subject.

XI. APPENDIX

[A list of references to papers on the subject of differential manometers and piston gages]

1. Richard, Ann. Mines, **7**, p. 481; 1845. (A multiple tube manometer for pressures up to 7 atmospheres is described.)
2. V. Regnault, Relation des Experiences., **1**, p. 343; 1847. (A single column manometer for pressures up to 30 atmospheres is described.)
3. L. Seyss, Dingler's Polytech. J., **191**, p. 352; 1869. (A piston gage with two pistons, one inside the other, is described. The weights are so arranged that when the piston rises it picks up one weight at a time.)
4. Desgoffe, Dingler's Polytech. J., **202**, p. 393; 1871. (Illustrated description of a piston gage. The measured pressure forced a small piston downward against a larger piston which rested upon a diaphragm. The flexure of the diaphragm forced mercury upward in a glass tube. A calibrated scale behind this tube was graduated to read pressures directly.)
5. L. Cailletet, Compt. rend., **84**, p. 82; 1877. (A single column manometer for pressures up to 34 atmospheres is described.)
6. E. H. Amagat, Compt. rend., **87**, p. 432; 1878. (A single column manometer 300 m high at Lyon is described.)
7. L. Cailletet, Ann. chim. phys. (ser. 5), **19**, p. 386; 1880. (Description of a piston gage. The space between the piston and cylinder was not more than 0.01 mm. The leakage of water past the piston was stopped by goldbeater's skin. The piston was held down by a lever loaded with weights.)
8. M. Thiesen, Z. Instrumentenk., **1**, p. 114; 1881. (A proposed multiple tube manometer using water as a pressure transmitting fluid is described.)
9. E. Ruchholz, Dingler's Polytech. J., **247**, p. 21; 1883. (Illustrated description is given of a piston gage in which the weight pan was rigidly attached to the upper end of the piston. No packing around the piston is shown or mentioned.)
10. E. H. Amagat, Compt. rend., **103**, p. 429; 1886. (Description of a gage similar to that in reference No. 4, except for the omission of packing around the small piston and the diaphragm under the large piston with a closer fit between pistons and cylinders. Molasses was used in the high pressure cylinder and castor oil in the low pressure.)
11. P. G. Tait, Nature, **41**, p. 361; 1890. (A gage similar to the preceding one is briefly described.)
12. E. H. Amagat, Ann. chim. phys. (ser. 6), **29**, p. 70; 1893. (Illustrated description of gage similar to that in reference No. 10. The small piston and cylinder were of tempered steel, the large cylinder of bronze.)
13. Michael Altschul, Z. phys. Chem., **11**, p. 583; 1893. (A piston gage with design similar to that in reference No. 9 is briefly described. A steel cylinder and bronze piston were used.)
14. Physikalisch-Technische Reichsanstalt, Z. Instrumentenk., **13**, p. 125; 1893. (Announcement of completion of the manometer proposed by Thiesen, reference 8.)
15. S. W. Stratton, Phil. Mag., **38**, p. 160; 1894. (A short note proposing the use of a multiple tube manometer.)
16. Phys.-Tech. Reich., Z. Instrumentenk., **14**, p. 307; 1894. (Description of Stückrath's pressure balance with packing of goldbeater's skin, and results of comparison with manometer are given. The precision obtained was about 5 per cent.)
17. L. Cailletet, Compt. rend., **112**, p. 764; 1894. Z. compr. fluss. Gase Pressluft-Ind., **1**, p. 88; 1897. (A single column manometer 300 m high on the Eiffel Tower is described. The main column of mercury was inclosed in a steel tube to which glass branch tubes were attached through metal stopcocks at intervals of 3 m. The temperature of the manometer was calculated from the resistance of the telephone wire running parallel to it.)

18. H. F. Wiebe, Z. compr. fluss. Gase Pressluft-Ind., **1**, pp. 8, 25, 81, 101; 1897. (A series of papers gives, respectively (1) a brief history of manometers, and piston gages and an illustrated description of the multiple manometer at the Phys.-Tech. Reich.; (2) procedure for calculating pressures from observations of the manometer; (3) description of the Amagat gage invented by Gally-Cazalat and of the Stückrath pressure balance; (4) discussion of the precision of the Stückrath gage, about 2 parts in 1,000.)
19. H. Kamerlingh Onnes, Comm. Phys. Lab. Univ. Leiden No. 44; 1898. (The paper describes in detail a multiple tube manometer consisting of 15 U tubes for pressure differences up to 60 atmospheres and total pressures up to 100 atmospheres. A gas is used for transmitting the pressure.)
20. Jacobus, Engineering, **64**, p. 464; 1897, Z. Instrumentenk., **18**, p. 56; 1898. (A piston gage for pressures up to 1,000 atmospheres is described. The instrument was designed so that the load could be applied by means of platform scales or a testing machine.)
21. C. J. Schalkwijk, Comm. Phys. Lab. Univ. Leiden No. 70; 1901. (The various corrections to be applied and the accuracy of the manometer in reference No. 19 are discussed.)
22. Anonymous, Engineering, **75**, p. 31; 1903. (The paper gives an illustrated description of a piston gage at the National Physical Laboratory for pressures up to 10 tons per square inch. Comparison with a 50-foot manometer is mentioned.)
23. E. Wagner, Ann. Physik (ser. 4), **15**, p. 906; 1904. (Describes a Stückrath pressure balance and an Amagat gage. These gages were taken apart, the piston diameters measured, and the average cylinder diameters determined from the weight of mercury required to fill them. For the determination of the effective area the unmounted cylinders were attached to a manometer about 1.5 m high, and weights were piled on a cork disk rigidly attached to the upper end of the piston.)
24. L. Holborn and F. Henning, Ann. Physik (ser. 4), **26**, p. 833; 1908. (A part of a paper on the vapor pressure of water describes the 12 m single-column mercury manometer at the Phys.-Tech. Reich. The main column of mercury was inclosed in a steel tube to which glass branch tubes were attached through metal stopcocks at intervals of 2 m. The temperature of the manometer was determined from the resistance of a nickel wire parallel to the manometer and inclosed in the same housing. The corrections applied for the calculation of pressures are discussed.)
25. A. Martens, Z. Ver. deut. Ing., 51_2, p. 1184; 1907. (The paper discusses the effect of friction from packings upon the accuracy of the measurement of force with hydraulic presses. The effect was 7 to 23 per cent at 5 atmospheres and 1 or 2 per cent at 200 atmospheres. Curves showing the variation of the effect with pressure are given.)
26. P. W. Bridgman, Proc. Am. Acad. Arts Sci., **44**, p. 199; 1908–9; Phys. Rev., **28**, p. 145; 1909. (A piston gage with a packed $\frac{1}{16}$-inch hardened steel piston is described. The gage was sensitive to 2 kg per cm^2 at a pressure of 7,000 kg per cm^2.)
27. George Klein, Dissertation Kön. Tech. Hochschule Berlin September, 1908. A slightly abbreviated article is given in Z. Ver. deut. Ing., **54**, p. 791; 1910. (The paper gives a comparison with a manometer of several piston gages, including gages designed by Stückrath, Martens, Schaeffer and Budenberg. The author discusses sources of error of these gages and determines their sensitivity. Small holes were bored along a line parallel to the axis of an experimental cylinder and the pressure at these holes measured.)
28. A. Martens, Z. Ver. deut. Ing., **53**, p. 747; 1909. (The paper describes and illustrates various possible designs of gages as well as two gages constructed by Martens.)
29. P. P. Koch and E. Wagner, Ann. Physik (ser. 4), **31**, p. 31; 1910. (The paper describes a single-column manometer built in a 25 m tower at the University of Munich, and gives the results of a comparison of a gage similar to Amagat's with this manometer. The gage is sensitive to 0.004 atmospheres at 30 atmospheres.)
30. L. Holborn and A. Baumann, Ann. Physik, **31**, p. 945; 1910. (A portion of this paper on the vapor pressure of water gives an illustrated description of the piston gage used. The cylinder was of bronze. The steel piston was given an oscillatory rotation to avoid sticking.)

31. H. F. Wiebe, Z. Kompr. fluss. Gase Pressluft-Ind., **13**, p. 83; 1910. (A history of pressure gages and manometers is given. The piston gages built by Stückrath for the Phys.-Tech. Reich. are described. The older type was given an oscillatory rotation, while the newer type were rotated by a worm wheel. The cylinder and piston were made of 25 per cent nickel steel. Cup-shaped packings of goldbeater's skin were used. Precision of measurement was about 1 part in 1,000.)
32. L. Holborn and H. Schultze, Ann. Physik (ser. 4), **47**, p. 1089; 1915. (This portion of a paper on the isotherms of air, argon, and helium describes the method used to calibrate the two piston gages for pressures up to 100 atmospheres. One gage was connected to each end of a 12 m differential manometer described in reference 24 and a comparison made at pressure intervals of about 15 atmospheres. By making comparisons also with the gages interchanged the effective area of either gage at a given pressure could be used to calculate the effective area of the other at the next higher pressure.)
33. C. A. Crommelin and Miss E. I. Smid, Comm. Phys. Lab. Univ. Leiden No. 146c; 1915. (A comparison of a Schaeffer and Budenberg gage with a differential manometer at pressures up to 100 atmospheres is described. Two closed end manometers of 60 and 120 atmosphere capacity were compared with the differential manometer, and the piston gage was compared with these. The effective area was found to increase 2 parts per 1,000 up to 70 atmospheres and to decrease about the same amount from 70 to 100 atmospheres.)
34. F. G. Keyes and R. B. Brownlee, Thermodynamic Properties of Ammonia, p. 10, published by John Wiley & Sons. (A brief description of a piston gage and its calibration is given.)
35. F. G. Keyes and R. B. Brownlee, J. Am. Chem. Soc., **40**, p. 25; 1918. (A part of a paper on the vapor pressure of ammonia in which is a brief illustrated description of a piston gage and its calibration.)
36. L. Holborn, Z. Ver. deut. Ing., **67_1**, p. 188; 1923. (A brief illustrated description of the piston gage at Phys. Tech. Reich.)
37. A. Michels, Ann. Physik, **72**, p. 285; 1923. (The paper gives a theoretical discussion on the effect of rotation upon the sensitivity of piston gages as well as data obtained from observation of both the deceleration of the freely rotating piston and the electrical resistance of the oil film between the piston and cylinder. Both sets of data indicated that above a certain critical speed there was a continuous oil film between the piston and cylinder. An illustrated description is given of the gage used, which was of the Schaeffer and Budenberg type. The piston was rotated continuously in one direction.)
38. A. Michels, Ann. Physik, **73**, p. 577, 1924. (This paper describes the comparison of the gage mentioned in the preceding reference with a 9 m single-column mercury manometer. With the aid of a closed-end hydrogen manometer also compared with the differential manometer, the comparison was made at pressures up to 175 atmospheres. The effective area of the gage was found to be constant within the limits of experimental error, 1 part in 3,000. A theoretical discussion indicates that the effective area of the gage should not change more than 1 part in 4×10^6 per atmosphere.)
39. L. Holborn, Handbuch der Experimental Physik, **1**, Messmethoden, p. 63, published by Akademische Verlagsgesellschaft m. b. H., Leipzig; 1926. (A chapter of this book gives an illustrated description of several pressure gages at the Phys.-Tech. Reich.)
40. H. Ebert, Handbuch der Physik, **2**, chapter 8A, published by Julius Springer; 1926. (This chapter is devoted to the production and measurement of pressures. A history of the subject is given as well as a description of several manometers and gages, and discussion of their accuracy.)
41. F. G. Keyes, Mech. Eng., **49**, p. 163; 1927. (A short report on the calibration of the pressure gages at the Mass. Inst. of Tech. This subject is covered more in detail in the next reference.)
42. F. G. Keyes and Jane Dewey, J. Optical Soc. Am., **14**, p. 491; 1927. (The method used by Holborn and Schultze was followed to compare piston gages with a single column manometer at pressures up to 587 atmospheres. The effective area of the gage with the larger piston did not change more than 1 part in 40,000 for pressures up to 160 atmospheres; that of the gage with the smaller piston increased about a part in 11,000 when the pressure was increased from 0 to 587 atmospheres.)

43. E. P. Bartlett, H. L. Cupples, and T. H. Tremearne, J. Am. Chem. Soc., **50**, p. 1275; 1928. (A portion of a paper on the isotherms of nitrogen and hydrogen in which appears an illustrated description of two piston gages in use at the Bureau of Chemistry, Department of Agriculture.)

Washington, March 11, 1931.