RP306

ACCURATE MEASUREMENT OF SMALL ELECTRIC CHARGES BY A NULL METHOD [1]

By Lauriston S. Taylor

ABSTRACT

In the use of an electrostatic system for measuring charges and currents, it is necessary to know the electrostatic capacity of the system. For small capacities the error in this measurement may easily be 1 per cent. There is here described a new method for calibrating a null system in such a manner that the capacity of the leads does not enter and which, therefore, permits a reduction in the calibration error to one-tenth. When a system is once calibrated in the manner described, any unknown capacity whatever may be added to the leads without affecting the measurement of the desired quantities. Expressions are given for the sensitivity of the system in terms of readily measured quantities. Applications to the measurements of current, charge and capacity are discussed.

CONTENTS

I. INTRODUCTION

When making accurate measurements of small charges or currents by electrostatic means one of the greatest difficulties encountered is that of correctly determining the capacity of the system. This is due largely to the inability, without taking elaborate precautions, to determine corrections for the capacity of the leads from the capacity bridge to the electrometer system. These errors have been discussed in detail by Rosa and Dorsey [2] who showed that for capacities of the order of 500 cm they might be as much as 1.0 per cent. This difficulty has been encountered in endeavoring to make accurate measurements of the ionization produced in air by X rays, in connection with the international unit of X-ray quantity—the roentgen.

Investigators have used several different means of measuring the small currents involved. The factors favoring the use of an electrostatic measuring system have been discussed and it has been shown [3] [4] that the null electrostatic method, in that it does not introduce field

[1] See footnote, Radiology, 16, p. 9; 1931.
[2] E. B. Rosa and N. E. Dorsey, B. S. Bull., 3, p. 433.
[3] G. Failla, Am. J. Roent., 21, p. 47; 1929.
[4] L. S. Taylor, B. S. Jour. Research, 2 (RP56), p. 771; 1929.

distortion in the ionization chamber, has an advantage over deflection methods. An additional difficulty in deflection electrostatic systems of small capacity arises from the motion of the leaf or fiber altering the capacity and thereby the calibration.

In null electrostatic measuring systems the potential of the electrometer and collecting system is maintained at some constant value—usually zero with respect to ground—by one of three methods. One method of accomplishing this is by communicating to the collector system an evaluated charge or current opposite in sign to that measured. Failla [5] and Jaeger [6] have devised sources of current for this purpose which serve as working standards. For absolute measurements this method is impractical because the working standard requires a calibration in kind. In the second method, the charge accumulated on the collector system is compensated by increasing its capacity to maintain the potential constant. Aside from the mechanical difficulties involved in a circuit of this kind, it has the disadvantage necessitating an elaborate calibration. In a third method, sometimes known as the Townsend method,[7] the communicated charge is entirely localized in a condenser of constant known capacity, but varied potential difference as determined by a voltmeter. In this, the greatest source of uncertainty is the voltmeter while in the second method it is the variable condenser. Since, in general, a variable condenser suitable for such purposes is a less dependable instrument than a voltmeter, the third method is from this standpoint distinctly preferable to the second.

II. THEORY OF METHOD

1. MEASUREMENT OF CHARGE

Given a circuit as represented in Figure 1, an unknown charge is communicated to the system containing an accurately known capacity C_1 and the unknown remainder C (which represents the capacity of the leads, ionization chamber, and electrometer, together with a calibrated variable capacity which will be described below). One plate of C is connected to earth, while the corresponding plate of C_1 is connected to a potentiometer circuit so as to bring it to any desired potential above or below that of the earth. As indicated by the deflection of the electroscope E this communicated charge raises the insulated side to a calibrated potential v relative to the earth as zero. Calibration of the electroscope is readily carried out by closing the key K and shifting the potentiometer contact to give any desired potential v which is read directly on the voltmeter. (The electroscope can at the same time be adjusted so that the deflection is at the most sensitive part of the scale.) With the electroscope at v, K is opened and an unknown charge Q communicated to the system through, say, the ionization chamber I. This requires the potentiometer contact to be shifted to V in order to bring back to, or maintain the potential of the system, at the calibrated value v, as indicated by the electroscope.

Since v is unchanged by this operation the charge on the part of the system of capacity C remains unchanged, hence the entire charge Q

[5] G. Failla, Radiology, **15**, p. 437; 1930.
[6] R. Jaeger, Strahlentherapie, **33**, p. 542; 1929.
[7] J. S. Townsend, Phil. Mag., **6**, p. 598; 1903.

is accumulated on C_1. Its magnitude is readily obtained from the measured values; that is

$$Q = C_1\ (v - V) \tag{1}$$

2. CALIBRATION OF CONDENSER

The difficult part of this method lies in the accurate calibration of C_1. To present an adequate method of calibration is the prime object of the present communication.

C is here made up, in part, by an accurately calibrated variable condenser connected into the system as dotted in the figure. The total capacity in situ of this condenser is not accurately known because the leads, etc., introduce an uncertainty. However, its capacity differences are very accurately known.

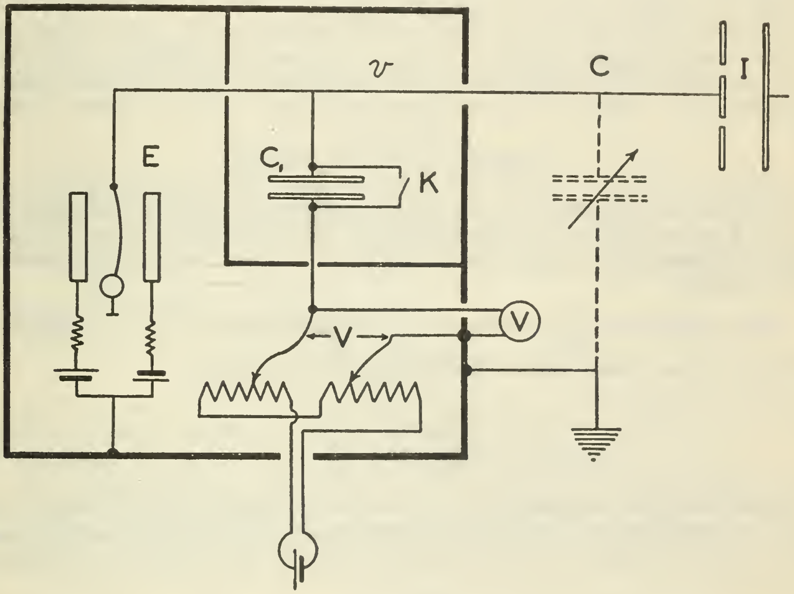

FIGURE 1.—*Null electrostatic system for measuring small charges or currents*

The part within the heavy lines is referred to as the "isolated system" and is so constructed that all capacities remain fixed.

Starting now with the system uncharged (that is, when V is zero, K is first closed then opened again), the potentiometer contact is shifted to a value V such that the electroscope indicates accurately any convenient potential v. Since no charge has been communicated to the system by this operation the induced charge $(-Q)$ on the unknown capacity C_1 must be exactly equal and opposite to that (Q) on the unknown remainder of capacity C.

Since

$$-Q = C_1\ (v - V)$$

and

$$Q = Cv$$

it follows that

$$C_1(V - v) = Cv \tag{2}$$

The experiment is now repeated with C changed by an accurately known increment to a new value C', and V, consequently to a new value V'.

In this case

$$C_1(V'-v)=C'v \tag{3}$$

Subtracting equation (3) from equation (2), C_1, the desired quantity, is obtained in terms of the accurately known magnitude $(C-C')$, namely

$$C_1=(C-C')\frac{v}{V-V'} \tag{4}$$

from which expression all extraneous capacity, such as the ionization chamber, electroscope, and leads are wholly eliminated. Since the capacity of the system varies with the deflection of the electrometer, the same deflection must be used throughout any one calibration. After having determined C_1, other capacities of unknown value may be inserted in the collector system without in any way affecting its calibration. This is of great practical importance, as shown later.

3. SENSITIVITY OF SYSTEM

Although, as seen in equation (1), the quantity of the charge measured does not depend on the magnitude of the distributed capacity C, yet the sensitivity of the measuring system must obviously decrease as C increases.

The sensitivity $\frac{\Delta s}{\Delta Q}$ of this type of null circuit can be expressed in terms of the sensitivity of the integral parts; that is

$$\frac{\Delta s}{\Delta Q}=\frac{\Delta s}{\Delta v}\cdot\frac{\Delta v}{\Delta Q} \tag{5}$$

where Δs is the increment in scale divisions of the electrometer deflection for a given increment ΔQ of the imparted charge; and at the same time Δv is the increment in volts corresponding to the deflection increment Δs.

The electrometer sensitivity, $\frac{\Delta s}{\Delta v}$, is a magnitude which is obtainable separately, is in no way dependent upon the remainder of the system, and may be considered as known. As to the other factor, $\frac{\Delta v}{\Delta Q}$, the sensitivity of the compensating system, we have (from the general law $Q=CV$) $\Delta Q=C\Delta V$. Any error Δv in the adjustment of the potential v introduces an error ΔQ, which is given in this case by

$$\Delta Q=(C_1+C)\Delta v \tag{6}$$

That is, the error in the determined magnitude Q is proportional to the total capacity of the system.

The uncertainty, Δv, may readily be obtained by comparing the electroscope independently with an adequate voltmeter; the value of C_1 is supposed to be accurately known; leaving C, which includes all capacity of the circuit external to C_1 to be evaluated in terms of observed magnitudes.

From equation (2)

$$\frac{C_1+C}{C_1}=\frac{V}{v} \tag{7}$$

hence equation (6) may be rewritten

$$\Delta Q=C_1\frac{V}{v}\Delta v \tag{8}$$

which in turn, since from equation (7) $\frac{V}{v}$ is a constant (say k) of the system as given, may be written

$$\Delta Q=C_1 k\Delta v$$

The sensitivity of the compensating system is then

$$\frac{\Delta v}{\Delta Q}=\frac{1}{C_1 k} \tag{9}$$

in which l/k is a *sensitivity factor* of the system as given. Since C_1 is constant and known from (4) we have now expressed the sensitivity $\frac{\Delta v}{\Delta Q}$ without C being explicitly involved. The working sensitivity of equation (5) then becomes

$$\frac{\Delta s}{\Delta Q}=\frac{\Delta s}{\Delta v}\cdot\frac{1}{C_1 k} \tag{10}$$

which, since k increases with the stray capacity C, shows that the sensitivity decreases as the stray capacity increases.

By considering all capacities outside the heavy lines of Figure 1 to be removed from the system, we may determine a value of the sensitivity factor k_o which is characteristic of the isolated part. Thus

$$k_o=\frac{V_o}{v_o} \tag{11}$$

serves in practice as a ready control of the system.

4. METHOD OF OPERATION

Referring to equation (4), only two capacities whose difference is accurately known are required for the calibration of C_1. If, however, a variable capacity C having a number of accurately known capacity differences be used for the calibration, the following graphical analysis which gives equal weight to all observations shows readily how to obtain the magnitude of any of the capacities in the system, including C_1.

Remembering that $\frac{V}{v}=k$, we have from equation (7)

$$C=C_1 k-C_1 \tag{12}$$

showing a linear relationship between C and k. Here it is seen that C_1 may be obtained either from the slope or from the C intercept; also that $k=1$ when $C=0$; and finally that for $k=k_o$, C takes on a derived value which will later be found serviceable in the analysis.

In plotting C against k, as in Figure 2, the capacity differences for various settings of C being accurately known while the absolute capacity is yet undetermined, we are unable to assign C its correct coordinate position at the start but merely a place, say a. Then, for a C scale, may be chosen coordinates representing *capacity differences* employed without regard to the location $C=0$.

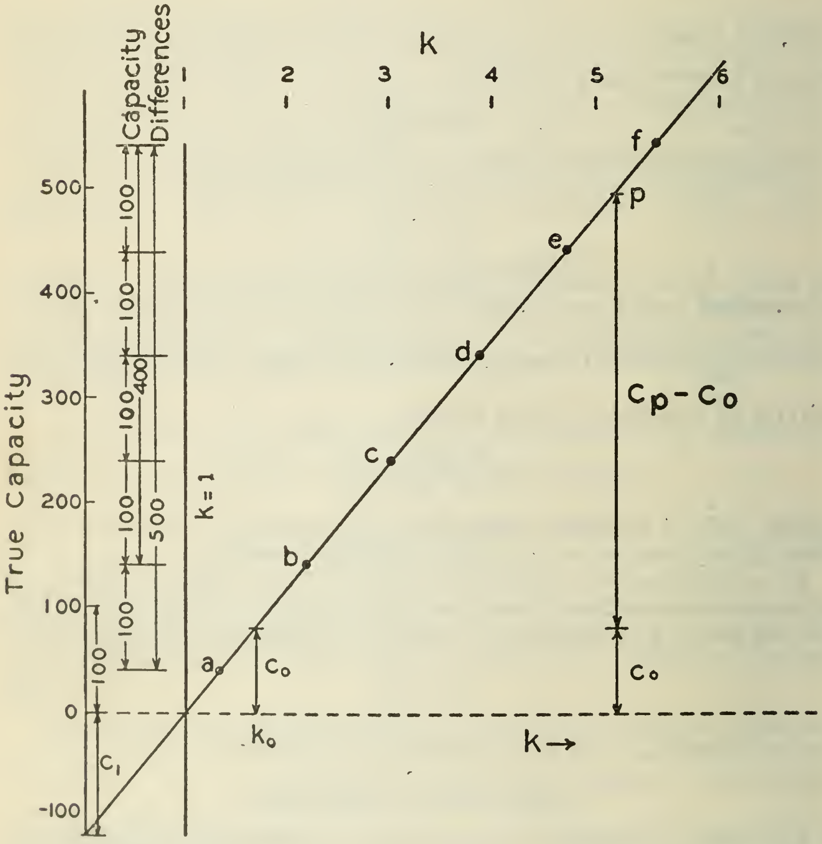

FIGURE 2.—*Schematic method of evaluating fixed capacity C_1*

The next point b corresponding to a known capacity change from a is then plotted with reference to a, etc. A straight line drawn through the plotted points must also pass through the point $k=1$, $C=0$ and thus locate the origin. Having determined the position $C=0$ it is now possible to read off the graph the value of the other capacities in the circuit. For example, the isolated system has a determined sensitivity factor k_o so that referring to (12) the corresponding ordinate C_o gives what will be called the stray capacity of the isolated system. Similarly, the difference between the ordinate

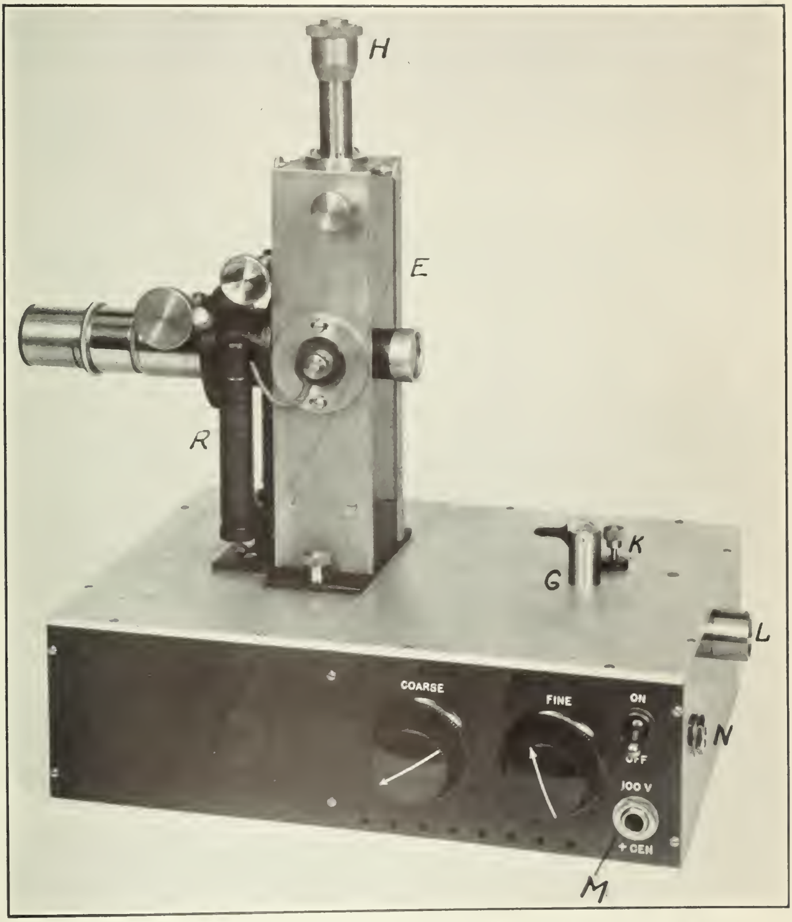

FIGURE 3.—*Photograph of "isolated system" as used in the study*

C_o and that C_p at any other point P on the curve gives the corresponding capacity of the external system.

Obviously such a system is applicable to the accurate measurement of any inserted capacity; for example, $(C_p - C_o)$. We will consider C as made up of the stray capacity C_o of the isolated system and the total external capacity C_{ext}. Then equation (12) becomes

$$C_1 k = C_1 + C_o + C_{ext} \tag{13}$$

In this C_1 is determined by equation (4), C_{ext} is removed, and C_o is determined from the relation

$$C_1 k_o = C_1 + C_o \tag{14}$$

Combining these

$$C_{ext} = C_1 (k - k_o) \tag{15}$$

For any given measuring system, C_1 and C_o and k_o remain fixed so that the measurement of any capacity, such as C_{ext}, requires but a determination of k with the external capacity inserted.

III. EXPERIMENTAL STUDY

1. DESCRIPTION OF SYSTEM AS USED

We will now examine a specific null circuit represented in general by the diagram in Figure 1 and for the isolated part in Figure 3. In this latter, E is a string electrometer of the general Edelmann type in which the deflection of a very fine platinum (Wollaston) wire, under an adjustable tension between the charged knife-edges, is observed by a suitable microscope. The potential on these knife-edges is supplied, through very high protective resistances R, by two 22.5 volt batteries. For any given potential on the knife-edges, the voltage sensitivity $\frac{\Delta s}{\Delta v}$ of the electrometer is adjusted by varying the tension on the fiber by the micrometer head, H. The sensitivity of this particular instrument can be readily changed from 100 divisions per volt to 0.01 division per volt.

The resistances of the compensating potentiometer are operated by the knobs "coarse" and "fine" on the front of the aluminum box container. Potentiometer, charging batteries, compensating condenser C_1 and the necessary leads are all contained in this box, the electrometer circuit being led out through an amber bushing L. All parts are carefully shielded electrostatically. G is a grounding key and K a switch to short circuit C_1 (for the purpose of calibrating the electrometer). Leads to the potentiometer battery and the voltmeter are in the form of jack and plug at M and N, respectively.

TABLE 1

VARIABLE CONDENSER ALONE

Nominal capacity C	k	Average deviation from mean	Deviation from mean
1	2	3	4
			Per cent
[1] 0.0	1.057	0.0015	0.14
43.8	1.120	.0016	.14
97.6	1.185	.0018	.15
175.9	1.279	.0008	.06
254.5	1.378	.0021	.15
313.5	1.470	.0014	.09
412.0	1.569	.0018	.12
490.6	1.666	.0009	.05
		Average---	.11
VARIABLE CONDENSER + RUBBER CABLE			
[2] 215.2	1.968	0.008	0.36
313.5	2.075	.005	.24
412.0	2.232	.006	.32
		Average---	.31
VARIABLE CONDENSER, HIGH VOLTAGE			
117.1	1.207	0.005	0.41
195.6	1.303	.003	.23
274.1	1.397	.002	.14
352.8	1.495	.005	.33
431.7	1.590	.004	.25
		Average---	.27

[1] Variable condenser actually removed and L covered.
[2] Capacity of variable condenser only. Full value of C is this plus the unknown capacity of the cable.

2. CALIBRATION

The capacity used for the calibration of C_1 was a precision variable condenser of well-known make; the voltmeter was of the laboratory standard type; the capacity differences and voltmeter were calibrated with an accuracy of one-tenth per cent by the electrical division of this bureau.

The accuracy of the results obtainable under working conditions is best brought out graphically. The factor k (equation (9)) may be obtained from the slope of the curve V plotted against v—a series of which curves are given in Figure 4 for several different settings of the variable condenser C.

The first part of Table 1 gives results of a number of determinations of k for different nominal values of the external capacity C, in which the variable condenser was connected to the terminal L by a well insulated and shielded conductor of unknown capacity. The average deviation of five determinations of each k is seen to be within the one-tenth per cent accuracy of the measuring instruments.

Plotting (circles, fig. 5, Curve I) these values of k as abscissae and the corresponding variable condenser values as ordinates, it is found that all of the points lie close on a straight line. According to (12) the slope $\frac{\Delta C}{\Delta k}$, which is obtainable in terms of the accurate differences in capacity, gives C_1 as 809.2 ± 0.4 $\mu\mu f$.

Having removed the condenser C and covered the amber bushing L with a cap, the sensitivity factor k_o for the electrometer-condenser system alone was found to be 1.057 ± 0.0005.

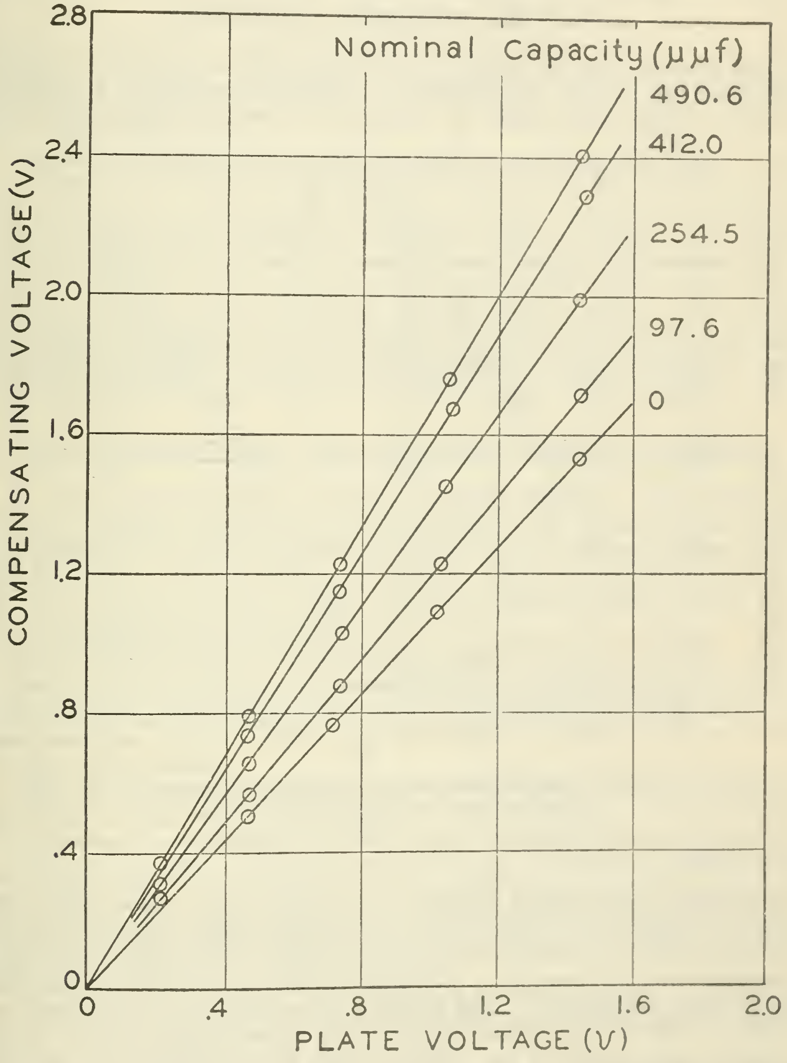

FIGURE 4.—*Curves showing the evaluation of the sensitivity factor k over a range of potentials*

This curve might be given its coordinate position on a C–k plot by extending it to the point $k=1$; hence, also, the ordinate value $C=0$—located as seen at −48 on the variable condenser scale. The value of the internal lead capacity C_o may be read from the curve at the point

$k=1.057$, thus giving $C_o=46\ \mu\mu f$. Likewise the value of C_p, at the point P for example, is found to be 460 $\mu\mu f$.

These capacity values may be calculated more accurately from equations (14) and (15). Using equation 14 for $k_o=1.057$ we find $C_1+C_o=855.3\ \mu\mu f$ where, since $C_1=809.2\ \mu\mu f$, we have $C_o=46.1\ \mu\mu f$. The capacity at C_p, for which $k=1.569$, is obtained by substituting the values of k and k_o in equation (15) giving $C_p=459.8\ \mu\mu f$.

3. FACTORS AFFECTING SENSITIVITY

The second part of Table 1 gives values of k for three nominal values of the variable condenser, to which was connected a shielded, rubber, single-wire cable of unknown capacity, with the shield

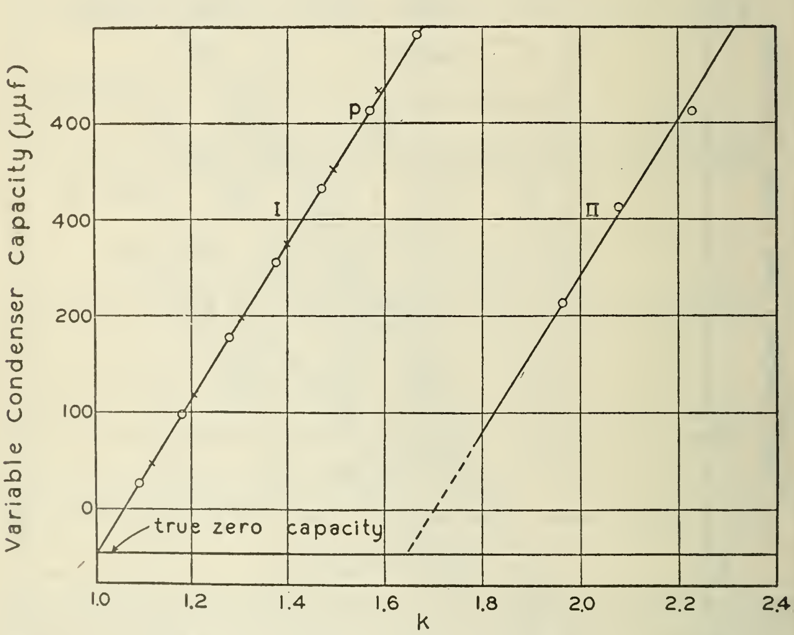

FIGURE 5.—*Curves showing the evaluation of the fixed capacity C_1 by means of an accurate variable external capacity*

Curve I (circles) using low voltage, (crosses) using high voltage. Curve II using a rubber cable in the external system.

grounded to serve as a condenser having a poor dielectric. The fourth column of this part compared with the part above shows that the precision of measurement was not so good—the average deviation changing to 0.31 from 0.11 per cent which, as already stated, was the error of calibration. The greatest error (0.31 per cent), however, is due to the experimental difficulty in determining k; for, due to dielectric absorption in the rubber cable, it was difficult to fix with accuracy upon the V necessary to apply to C_1 to induce a definite potential v 0 on the electrometer.[8]

Reference to the corresponding Curve II in Figure 5 shows the same effect in that the plotted points determined with the cable in the sys-

[8] This absorption is not operative when using, as in current measuring, the system at $v=o$.

tem do not fall closely on the straight line drawn parallel to Curve I. The superiority of conditions when all questionable external capacity is removed while calibrating C_1[9] is obvious—a conclusion supported also by the fact that, when the cable was replaced by a highly insulated condenser of about the same capacity, the points fell well along a straight line, the slope of which represented satisfactorily the capacity as checked by other means. Furthermore, the average deviation from the mean for the values of k made with the well insulated unknown was 0.12 per cent as against 0.31 per cent for the cable. This small average deviation, being again of the order of the error of the instrument calibration, indicates that the larger deviation (0.31 per cent) for the rubber cable was due to absorption and not to the magnitude of C.

In the measurements for the first and second parts of Table 1, the electrometer sensitivities were, respectively, about 13 and 17 divisions per volt; v was never greater than 1.5 volts, and, consequently, the external condenser offered no difficulties due to leakage. However, with v increased to 80 volts and the sensitivity decreased to about 0.3 division per volt, leakage became appreciable and k was subject to error as indicated in the third part of Table 1 where the average deviation was 0.27 per cent as against 0.11 per cent in the first part. The values obtained are plotted as crosses on Curve I (fig. 5). Such conditions should, of course, be avoided when determining C_1.

IV. APPLICATION TO CURRENT MEASUREMENTS

In the measurement of current by this method, a time measurement, of course, enters. In the particular case of measuring X-ray intensities, a shutter in the X-ray beam between tube and ionization chamber is so controlled that it remains open for a definite interval of time during which the charge is wholly compensated, holding $v=0$, by varying the potential on C.[10] [11]

In practice it is extremely advantageous to use a shielded rubber cable for connecting the electrometer to the source of current. As shown above, the presence of unknown lead capacities does not affect the measured value of the charge, although it does affect the accuracy of the measurement.

The effect of poor insulation may be eliminated by choosing such conditions that $v=0$ with respect to the shielding. This confines leakage to the condenser C_1, and it is a relatively simple matter to construct this fixed capacity so that leakage between its plates will be negligible. In the isolated part of the system here described, a 20-volt charge on the condenser leaked to ground at the rate of only 3 per cent in 8 hours; but, with a rubber cable attached, the same leakage occurred in about 10 minutes.

To obtain the desired sensitivity with any length of cable added to the system, the electrometer sensitivity may be changed in accord with k as brought out in equation (9).

To test the effect of a poor dielectric in the external condenser, two sets of intensity measurements were made of the same X-ray

[9] It is of interest to note that using equation (15), the capacity of the cable was found to be 522 $\mu\mu f$ as compared with 526 $\mu\mu f$ obtained with a capacity bridge calibrated with an accuracy of 1 per cent.

[10] D. L. Webster and A. E. Henning, Phys. Rev., 21, p. 301; 1923.

[11] L. S. Taylor, B. S. Jour. Research, 2 (RP56), p. 771; 1929.

beam, using a standard X-ray ionization chamber.[12] In the first set, the electrometer and ionization chamber were connected by an amber insulated conductor having a capacity of about 206 $\mu\mu$f and negligible leakage. In the second set, the connection was by the shielded rubber cable having a capacity of about 525 $\mu\mu$f. For this the electrometer sensitivity was adjusted to accord with the change in k, approximately double that of the first case. The respective average deviations from the mean of 10 observations for each of the two cases were found to be 0.33 and 0.32 per cent, respectively, the averages of the two runs differing by 0.13 per cent. These deviations from the mean were no more than should be expected from the X-ray intensity variations.

In a null circuit previously described by the author [13] the expression for the magnitude of the ionization current involved the total capacity C_T of the system instead of simply C_1 as above. The fact that the total capacity C_T can not be measured accurately (with a bridge, as previously done) renders the other method less satisfactory than the present one. Intensity measurements of the same X-ray beam by the two outfits showed a difference of nearly 1 per cent which is principally attributed to the error in the measurement of C_T. Compared with the average deviation above of one-third this magnitude for both the highly insulated and the cable connected systems, the superiority of the method here presented is evident.

WASHINGTON, December 30, 1930.

---

[12] L. S. Taylor and G. Singer, B. S. Jour. Research, 5 (RP211), p. 507; 1930.
[13] L. S. Taylor, B. S. Jour. Research, 2 (RP56), p. 771; 1929.