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ABSORPTION AND SCATTERING BY SOUND-ABSORBENT CYLINDERS

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ABSTRACT

The absorption and scattering of a plane wave of sound by an infinitely long circular cylinder whose axis is perpendicular to the direction of propagation of the wave are calculated. The surface of the cylinder is assumed to have a known normal acoustic impedance. The calculations take account of diffraction effects. Absorption measurements were made on long cylinders placed in a reverberation room, where the incident wave directions are at all angles to the axes of the cylinders, and were compared with the calculated values. In order to make the comparison, the reverberation-room statistics appropriate for cylinders (which are different from the statistics for flat patches of absorbent material) are developed and applied. The theory predicts, and measurements confirm, that absorbent cylinders can have coefficients of absorption greater than unity. Fairly good agreement between the calculated and measured coefficients is found. The reverberation-room statistics appropriate for spherical absorbers are also developed, although no measurements were made on spheres.

CONTENTS

I. INTRODUCTION

Acoustic materials, which are used to reduce noise and control reverberation in rooms, are usually applied directly to walls and ceilings. Recently a manufacturer has developed an acoustic treatment that consists of the installation of sound-absorbent material in the shape of long circular cylinders located at a distance from the walls. Rooms in which it is not possible to apply acoustic materials to the walls and ceilings can thus be treated acoustically.

In order to extend the absolute measurement of sound-absorption coefficients, it is important to study both theoretically and experimentally the absorption of cylinders having a known normal acoustic impedance. Owing to diffraction effects, the absorption is an entirely different function of frequency than is the absorption of a flat surface of the same material having an area equal to the peripheral area. Also, sound waves can approach a cylinder from all directions, whereas acoustic material on a flat wall can be approached by sound waves through a solid angle of only 2π. This means that the reverberation-room statistics for a cylindrical absorber will be different from those for a flat patch of absorbent material on the wall.

II. WAVE FIELD SCATTERED BY AN ABSORBENT CYLINDER

Consider the absorption and scattering of a plane single-frequency wave by an infinitely long circular cylinder of sound-absorbent material whose axis is assumed to be perpendicular to the direction of propagation of the wave. The ratio of sound pressure at a point on the surface to the component of particle velocity normal to the surface is assumed to have a constant value, the normal impedance Z, independent of location of the point on the surface. This is equivalent to assuming that the acoustic effects of motion of a point on the surface are transmitted to other points on the surface only through the surrounding air and not through the body of the cylinder.

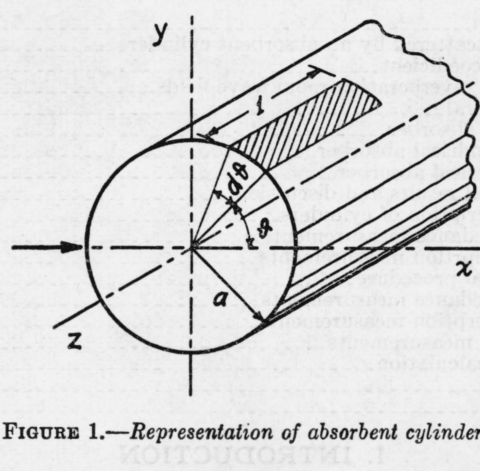

FIGURE 1.—*Representation of absorbent cylinder.*

If the distributions of sound pressure and particle velocity are independent of z (incident wave fronts parallel to the z-axis, which coincides with the axis of the cylinder, as shown in fig. 1), what is the scattered wave field?

Let k be the generalized wave number of the incident plane wave, and let ω be the generalized angular velocity. Then the velocity potential is

$$\Phi_i=\phi_0 e^{i(\omega t-kx)}=\phi_0 e^{i(\omega t-kr\cos\vartheta)}$$
$$=\phi_0\left[J_0(kr)+2\sum_{\mu=1}^{\infty}(-i)^\mu J_\mu(kr)\cos\mu\vartheta\right]e^{i\omega t} \qquad (1)$$

and there is no loss of generality in supposing $\phi_0=1$. This well-known expansion of Φ_i in a series of Bessel functions $J_\mu(kr)$ is convergent for all values of r and ϑ. The introduction of Bessel functions facilitates satisfaction of the boundary conditions at the surface of the absorbent cylinder.

Besides the incident wave there is a wave scattered by the cylinder. A function which satisfies the wave equation and which represents an outgoing wave, as can be seen by its asymptotic expansion for large r, is $[J_\mu(kr)-iY_\mu(kr)]e^{i\omega t}\cos\mu\vartheta$. The function $Y_\mu(kr)$ is the other solution of Bessel's differential equation, as defined in Whittaker and Watson's "Modern Analysis." This function is sometimes denoted by $N_\mu(kr)$. The entire expression $J_\mu(kr)-iY_\mu(kr)$ is Hankel's function $H_\mu^{(2)}(kr)$.

Hence a general expression for the velocity potential of the scattered wave is

$$\Phi_s=\sum_{\mu=0}^{\infty} b_\mu(J_\mu-iY_\mu)e^{i\omega t}\cos\mu\vartheta. \qquad (2)$$

The b_μ's are constants to be determined by the boundary conditions at the surface of the cylinder, and the Bessel functions J_μ and Y_μ are understood to be functions of kr. Terms involving $\sin\mu\vartheta$ do not appear, as can be seen from considerations of symmetry.

The total velocity potential due to both incident and scattered waves is $\Phi=\Phi_i+\Phi_s$. The total sound pressure at any point is $p=-\rho(\partial\Phi/\partial t)$, ($\rho$=density of air) and the radial velocity component, which is normal to the surface of the cylinder, is $v_r=(\partial\Phi/\partial r)$. The boundary condition at the surface of the cylinder is

$$p=-Zv_r,\ (r=a), \qquad (3)$$

where the constant Z is the usual normal impedance. The negative sign must be used because the positive direction of v_r is *out* of the absorbent surface ,whereas it is customary to measure the impedance Z with the positive direction for particle velocity *into* the absorbent surface.

Calculation of p and v_r from the total velocity potential Φ and substitution into eq 3 yield the following equations for determination of the b_μ's:

$$\left.\begin{aligned}&-\frac{Z}{\rho c}[J_0'+b_0(J_0'-iY_0')]+i[J_0+b_0(J_0-iY_0)]=0\\&-\frac{Z}{\rho c}[2(-i)^\mu J_\mu'+b_\mu(J_\mu'-iY_\mu')]+i[2(-i)^\mu J_\mu+b_\mu(J_\mu-iY_\mu)]=0\\&\mu=1,\ 2,\ 3,\ \ldots,\end{aligned}\right\} \qquad (4)$$

in which it is understood that $J_\mu = J_\mu(ka)$, $J'_\mu = \frac{dJ_\mu(ka)}{d(ka)}$, and similarly for J_0, J'_0, Y_0, Y'_0, Y_μ, Y'_μ. The quantity c is the velocity of sound. After the b_μ's are computed from eq 4, substitution into eq 2 will yield the final expression for the velocity potential of the wave scattered by the absorbent cylinder. The scattered wave potential has possible applicatons in the design of dead rooms and reverberation rooms.

III. ABSORPTION COEFFICIENT

The total time-averaged acoustic power, P, absorbed by unit length of the cylinder from the incident plane wave can be readily computed with the help of the b_μ's given in eq 4.

Let p and v_r, both functions of ϑ, be the sound pressure and the normal component of particle velocity, respectively, at a point on the surface of the cylinder (see fig. 1). Then the force due to the sound field on the shaded strip shown in figure 1 is $-apd\vartheta$. In complex notation,

$$p = |p|e^{i(\omega t+\phi_1)},\ v_r = |v_r|e^{i(\omega t+\phi_2)}, \tag{5}$$

where ϕ_1 and ϕ_2 are phase angles. Therefore, the time-averaged power, dP, of the force $-apd\vartheta$ causing the velocity v_r is

$$dP = -\tfrac{1}{2}|p|\ |v_r|a\cos(\phi_1 - \phi_2)d\vartheta. \tag{6}$$

By hypothesis, $p = -Zv_r$, and if $Z = |Z|e^{i\delta}$, then $\phi_1 - \phi_2 = \pi + \delta$. Substitution into eq 6 and integration yield for the total time-averaged power absorbed by unit length of the cylinder

$$P = \frac{a\cos\delta}{2|Z|}\int_0^{2\pi}|p|^2 d\vartheta. \tag{7}$$

The Fourier series for p can be written

$$p = \sum_{\mu=0}^{\infty} a_\mu\, e^{i\omega t}\cos\mu\vartheta, \tag{8}$$

in which the $a\mu$'s are, in general, complex numbers.

Substitution of eq 8 into eq 7 yields

$$P = \frac{\pi a\cos\delta}{|Z|}\left\{|a_0|^2 + \tfrac{1}{2}\sum_{\mu=1}^{\infty}|a_\mu|^2\right\} \tag{9}$$

The a_μ's are found by computing p from the total velocity potential.

$$\left.\begin{aligned} &a_o = -i\omega\rho[J_o + b_o(J_o - iY_o)] \\ &a_\mu = -i\omega\rho[2(-i)^\mu J_\mu + b_\mu(J_\mu - iY_\mu)] \\ &\mu = 1, 2, 3, \ldots \end{aligned}\right\}, \tag{10}$$

in which, just as in eq 4, it is understood that $J_\mu = J_\mu(ka)$, etc. Substitution of the values of b_μ obtained from eq 4 into these equations gives

$$\left.\begin{aligned} |a_0|^2 &= \frac{4|Z|^2}{\pi^2 a^2} \cdot \frac{1}{\left| -\frac{Z}{\rho c}(J_0' - iY_0') + i(J_o - iY_o) \right|^2} \\ |a_\mu|^2 &= \frac{4|Z|^2}{\pi^2 a^2} \cdot \frac{4}{\left| -\frac{Z}{\rho c}(J_\mu' - iY_\mu') + i(J_\mu - iY_\mu) \right|^2} \end{aligned}\right\} \tag{11}$$

Finally, a definition of absorption coefficient is needed. The time-averaged acoustic power transmitted by the incident plane wave through unit area is $P_0 = \omega\rho k/2$. The power absorbed per unit peripheral area of the cylinder is $P/2\pi a$. We define the absorption coefficient α_p for a plane wave incident normally to the axis by

$$\alpha_p = \frac{P}{2\pi a P_0}. \tag{12}$$

It is shown in section IV that the random incidence effective absorption coefficient for a cylinder measured in a reverberation room is greater than α_p by a factor of π if the Sabine reverberation formula is used to compute absorption coefficient from measured decay times. We define the reverberation-room absorption coefficient α_r by

$$\alpha_r = \pi \alpha_p = \frac{P}{2aP_0} \tag{13}$$

Both α_p and α_r are expressed in sabins per unit peripheral area.

The final expression for the reverberation room absorption coefficient is

$$\alpha_r = \frac{4}{\pi} \frac{|Z| \cos\delta}{\rho c} \left(\frac{1}{ka} \right)^2 \left\{ \frac{1}{\left| -\frac{Z}{\rho c}(J_0' - iY_0') + i(J_o - iY_o) \right|^2} + \sum_{\mu=1}^{\infty} \frac{2}{\left| -\frac{Z}{\rho c}(J_\mu' - iY_\mu') + i(J_\mu - iY_\mu) \right|^2} \right\}. \tag{14}$$

With known values of Z and suitable tables of Bessel functions, one can compute the absorption coefficient of a cylinder by means of this equation.

IV. STATISTICS OF REVERBERATION ROOM WAVE FIELDS

1. GENERAL

An absorbent cylinder placed at a distance from the walls of a reverberation room is approached by sound waves from all directions, whereas a flat patch of absorbent material on the wall of a reverberation room can be approached by sound waves coming through a solid angle of only 2π. This will give rise to a difference in the reverberation-room statistics, by means of which coefficients of sound absorption are deduced from measurements of decay time. In order to emphasize the differences, the statistics for a flat patch of absorbent material will be briefly given. A technic of calculation in which plane waves are handled statistically is introduced in order to include diffraction effects in a logical way. This technic is different from those used by Jäger [1] [1] and Buckingham [2]. In addition, the statistics for cylinders and spheres will be developed.

Throughout the discussion, the following assumptions will hold: (1) The wave motion in the reverberation room of volume V is completely random. At every point within the room, plane waves of all directions and phases will have passed by after a sufficiently long time (short, however, in comparison with the reverberation time) has elapsed. The plane wave in any given cone of directions, of solid angle $d\Omega$, is assumed to have the same average intensity as every other plane wave. As a corollary, the energy density, W, is the same at every point within the room. (2) In the statistical calculations, plane waves incident on the absorber are associated with elements of area, $R^2d\Omega$, located on a spherical surface of integration whose center is at the absorber. The time-averaged intensity of an incident plane wave associated with such an element of area is assumed to be directly proportional to the solid angle $d\Omega$ subtended by the area at the absorber. (3) At any given instant, there might be a number of plane waves incident on the sound absorber. The rate of absorption of energy by the cylinder or sphere, or patch on the wall, is assumed to be the sum of the rates for the individual plane waves. This is equivalent to assuming that, on the average, no power is developed at the absorbing surface by the sound pressure of one incident plane wave acting on the particle velocity of another incident plane wave.

The first assumption fixes the statistical distribution of the sound waves incident upon the absorber. The third assumption is basically the rule of superposition for rates of absorption. It is a considerable advantage to have such a rule, as the calculation of rates of absorption for plane waves is in general much simpler than for any other kind of incident wave.

A corollary of the first two assumptions is that the time-averaged intensity of the plane wave coming from any direction is $Wcd\Omega/4\pi$.

2. FLAT ABSORBER

Suppose a flat patch of absorbent material is placed on the wall of the reverberation room (see fig. 2). Let the effective area of the patch be A_f for a plane wave at normal incidence. A_f is the product of the geometrical area of the patch and its absorption coefficient. The product of A_f and the intensity gives the rate of absorption of sound energy from the plane wave. A_f will include the effects of diffraction near the edges of the patch, and might be either greater or less than the geometrical area of the patch. For a plane wave at angle of incidence ϑ, the effective area is assumed to be $A_f \cos \vartheta$.

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[1] Figures in brackets indicate the literature references at the end of this paper.

Then the rate of absorption of sound energy by the patch is

$$P_f = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} A_f \cos\vartheta \frac{WcR^2 \sin\vartheta \, d\phi d\vartheta}{4\pi R^2}$$

$$= \frac{A_f W_c}{4} \tag{15}$$

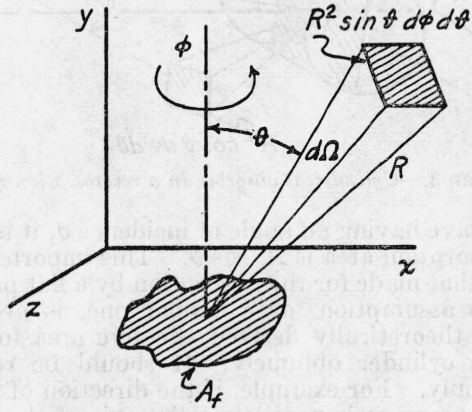

FIGURE 2.—*Flat absorber on the wall of a reverberation room.*

Hence the differential equation for the decay of sound in the room and its solution are

$$V\frac{dW}{dt} + \frac{A_f c}{4} W = 0, \quad W = W_0 e^{-\left(\frac{A_f c}{4V}\right)t} \tag{16}$$

Finally, if T is the time required for W to decay by a factor of e, then

$$A_f = \frac{4V}{cT}. \tag{17}$$

This is the Sabine expression generally used for calculation of absorption coefficients from reverberation-room measurements of decay time.

3. CYLINDRICAL ABSORBER

Suppose an absorbent cylinder, of length l great enough so that end effects can be neglected, is placed at a distance from the walls of the reverberation room. Let the effective area of the cylinder be A_c (see fig. 3) for a plane wave whose direction of propagation is perpendicular to the axis of the cylinder. The product of A_c and the intensity gives the rate of absorption of sound energy from the plane wave. It is clear that $A_c = 2\pi a l \alpha_p$, where α_p is as defined in section III. As A_c includes the effects of diffraction (computed in section III), it might be either greater or less than the geometrical area $2al$ of the actual cylinder projected on the plane wave front. The cylinder shown in figure 3 is formed by revolving A_c about the axis of the actual absorbent cylinder.

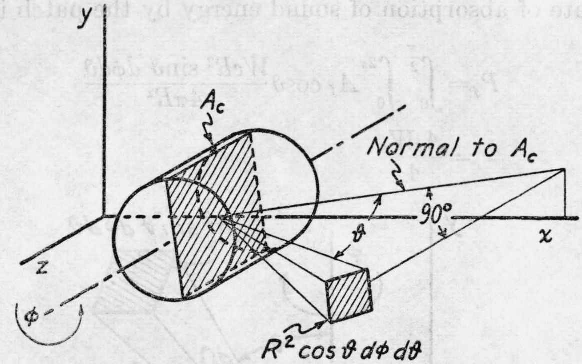

FIGURE 3.—*Cylindrical absorber in a reverberation room.*

For a plane wave having an angle of incidence ϑ, it is assumed that the effective absorption area is $A_c \cos\vartheta$. This important assumption is analogous to that made for the absorption by a flat patch at oblique incidence. The assumption, or a similar one, is necessary in the absence of any theoretically derived effective area for plane waves that strike the cylinder obliquely. It should be regarded as an approximation only. For example, if the direction of propagation of the incident plane wave is parallel to the axis of the cylinder, then $A_c \cos\vartheta = 0$, which is evidently not true.

The rate of absorption of sound energy by the cylinder is

$$P_c = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} A_c \cos\vartheta \frac{WcR^2 \cos\vartheta d\phi d\vartheta}{4\pi R^2}$$

$$= \pi \frac{A_c W c}{4} \tag{18}$$

If, as before, T is the time required for W to decay by a factor of e, then

$$A_c = \frac{1}{\pi} \frac{4V}{cT}. \tag{19}$$

The right-hand side of this equation differs from that of the Sabine expression, eq 17, by the factor $1/\pi$. It would be advantageous to express the absorption coefficient and effective absorption area for cylinders in units such that the Sabine formula could be used to compute decay time. Therefore, we define the random incidence effective absorption area A_r by

$$A_r = \pi A_c = \frac{4V}{cT}. \tag{20}$$

Finally, the reverberation-room absorption coefficient α_r (introduced in eq 13) is

$$\alpha_r = \frac{A_r}{2\pi a l} = \frac{1}{2\pi a l} \frac{4V}{cT}. \tag{21}$$

The foregoing explains the statement made in section III (compare eq 12 and 13) that the effective absorption coefficient of a cylinder in a reverberation room is π times as great as the absorption coefficient for a plane wave incident normally to the axis. Section V contains the results of experiments designed to furnish a comparison between values of α_r computed from the normal impedance (eq 14) and values measured in a reverberation room (eq 21).

4. SPHERICAL ABSORBER

The statistical calculations for spherical absorbers do not suffer from the defect that an arbitrary assumption must be made, as was made for flat and cylindrical absorbers, regarding the absorption at oblique incidence. For this reason, the comparison between absorption coefficients computed from the normal impedance and coefficients measured in a reverberation room is more direct for spheres.

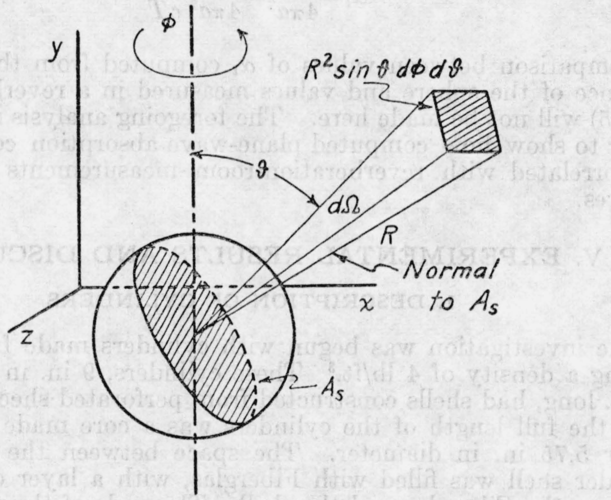

FIGURE 4.—*Spherical absorber in a reverberation room.*

Let the effective area of an absorbent sphere of radius a for a plane wave be A_s (see fig. 4). The product of A_s and the intensity gives the rate of absorption of sound energy from the plane wave. As for the flat patch and the cylinder, A_s includes the effects of diffraction and might be either greater or less than the geometrical area, πa^2, of the actual sphere projected on the plane wave front. The sphere shown in figure 4 is formed by revolving A_s about a diameter of the actual absorbent sphere.

The rate of absorption of sound energy by the sphere is

$$P_s = \int_0^\pi \int_0^{2\pi} A_s W c \frac{R^2 \sin \vartheta \, d\phi d\vartheta}{4\pi R^2}$$

$$= 4 \frac{A_s W c}{4} \tag{22}$$

If T is the time required for W to decay by a factor of e,

$$A_s = \frac{1}{4}\frac{4V}{cT}. \tag{23}$$

As for the absorbent cylinder, in order to express the absorption coefficient and effective area for spheres in units such that the Sabine formula can be used to compute decay time, we define the random incidence effective absorption area, A_t, by

$$A_t = 4A_s = \frac{4V}{cT}. \tag{24}$$

The reverberation-room absorption coefficient α_t of a sphere is

$$\alpha_t = \frac{A_t}{4\pi a^2} = \frac{1}{4\pi a^2}\frac{4V}{cT}. \tag{25}$$

Comparison between values of α_t computed from the normal impedance of the sphere and values measured in a reverberation room (eq 25) will not be made here. The foregoing analysis is presented in order to show how computed plane-wave absorption coefficients can be correlated with reverberation-room measurements on absorbent spheres.

V. EXPERIMENTAL RESULTS AND DISCUSSION

1. DESCRIPTION OF CYLINDERS

The investigation was begun with cylinders made from Fiberglas having a density of 4 lb/ft.^{3} These cylinders, 9 in. in diameter and 97 in. long, had shells constructed from perforated sheet steel. Running the full length of the cylinders was a core made of perforated paper 5.75 in. in diameter. The space between the core and the cylinder shell was filled with Fiberglas, with a layer of scrim cloth between the Fiberglas and the shell. The ends of the cylinders were covered with perforated sheet metal.

Cylinders having one and two layers of cattle-hair felt were also investigated. From measurements on superposed layers and from measurements on the finished cylinders, it was found that the actual thickness of the felt was ⅞ in. The weight of the felt was 0.60 lb/ft.^{2} The cores for the felt-covered cylinders were 28-gage sheet-iron tubes, 4 in. in diameter and in sections approximately 10 ft long. After the first layer of felt was wrapped and the edges sewed together, the average radius of the finished cylinders was 2.88 in. The average radius after the second layer was added was 3.77 in.

2. IMPEDANCE MEASUREMENTS

Measurements of the normal acoustic impedance at the periphery of the Fiberglas cylinders were not feasible. Estimation of the impedance is difficult because of the perforated paper core. However, the functional relationship

$$\frac{Z}{\rho c}=2-\frac{36}{13}\left(\frac{i}{ka}\right) \tag{26}$$

was assumed for the impedance on the basis of measurements reported by Beranek [3] for materials similar to the Fiberglas.

The results of normal impedance measurements for one and two layers of hair felt with rigid backing are shown in figure 5. Each plotted point represents the average of determinations on 15 samples selected at random from the original roll of felt. A modified Flanders [4] impedance tube was used to make the measurements. For both one and two layers of felt, the reactance curve at frequencies below 1,000 c/s is approximately that of a column of air having the same thickness as the felt.

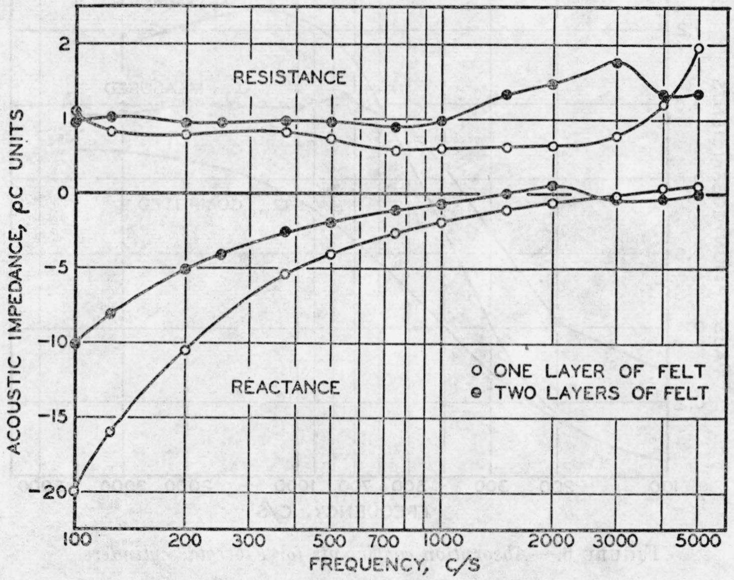

FIGURE 5.—*Impedance of hair felt for sound at normal incidence.*

3. ABSORPTION MEASUREMENTS

Reverberation-room measurements of the absorption coefficient α_r (defined by eq 21) were made on the Fiberglas cylinders and are plotted in figure 6. The absorption coefficients computed from the assumed impedance function (eq 26) by means of eq 14 are plotted in the same figure. We shall refer to these as the theoretical coefficients. The computed absorption coefficients α_n for normal incidence are also plotted. These are obtained by means of the usual equation

$$\alpha_n=1-\left|\frac{\frac{Z}{\rho c}-1}{\frac{Z}{\rho c}+1}\right|^2. \tag{27}$$

The curves show that diffraction effects at frequencies corresponding to values of ka near unity (480 c/s) result in absorption coefficients greater than 1. At high frequencies, where the wave length is short in comparison with the circumference ($ka>>1$), the cylinder will cast an acoustic shadow, and only the material on the side of the cylinder that faces the oncoming plane wave will be effective in absorbing sound. It is interesting to note that the theoretical value of α_r shows this shadow effect, because as the frequency rises above 1,000 c/s, α_r decreases even though α_n continues to increase. There is general agreement between the theoretical and measured reverberation-room values of α_r, although a critical comparison of the two sets of values cannot be made as the normal impedance values on which the calculations are based are only estimated.

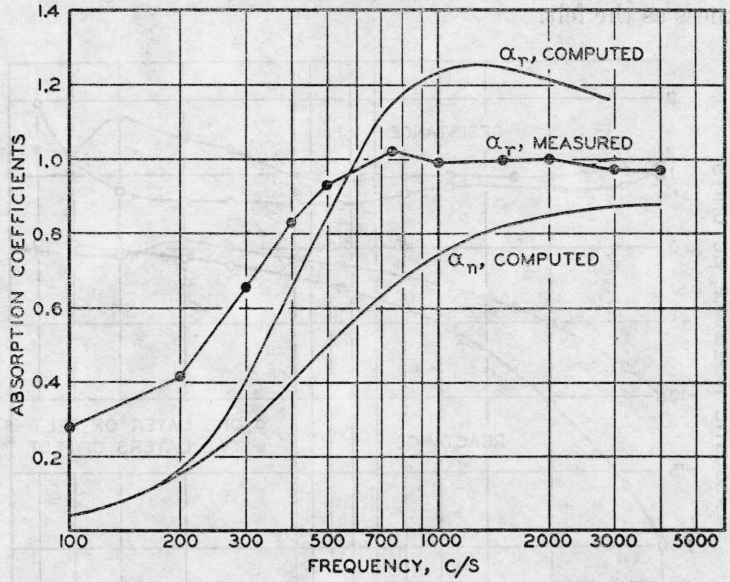

FIGURE 6.—*Absorption coefficients for Fiberglas cylinders.*

● = values of the absorption coefficient α_r measured in a reverberation room.
α_r, computed = theoretical absorption coefficients for random incidence computed from the impedance function (eq 26) by means of eq 14.
α_n, computed = absorption coefficients for normal incidence computed from the same impedance function by means of eq 27.

Figures 7 and 8 show curves for the cylinders having one and two layers of hair felt, respectively. The theoretical values of α_r were computed from the measured impedances given in figure 5 by means of eq 14. Also, the measured absorption coefficients for both one and two layers of hair felt laid flat on the floor of the reverberation room are plotted in figures 7 and 8, respectively. Comparison of the measured values for the flat layers with the corresponding measured cylinder coefficients shows that a given area of felt has considerably greater absorption, at higher frequencies, when wrapped on a cylinder than when applied to a flat surface.

Both the theoretical and measured reverberation-room coefficients

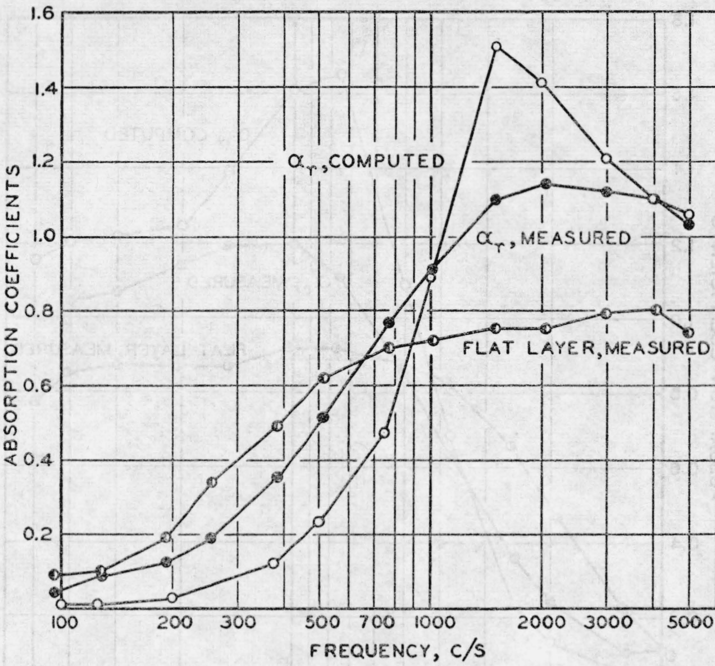

FIGURE 7.—*Absorption coefficients for cylinders with one layer of hair felt.*

○=theoretical values of the absorption coefficient α_r computed from inpedances given in figure 5 by means of eq 14.

●=values of α_r measured in a reverberation room.

◐=absorption coefficients for a flat single layer of hair felt measured in a reverberation room.

α_r show again that absorbent cylinders can have absorption coefficients greater than unity at frequencies corresponding to values of ka near unity. However, the maximum theoretical coefficients are somewhat greater than the maximum measured values. The exact location of the frequency of maximum absorption depends, of course, on the way in which the normal impedance varies with frequency, as well as on the value of ka. Consequently, the measured frequencies of maximum absorption for the two types of cylinders correspond to different values of ka. For the cylinders with one layer of felt, figure 7 shows the value of ka to be 2.7. For the cylinders with two layers, figure 8 shows the value to be 2.1.

Although there is general agreement between the theoretical and measured curves in each case, the differences at some frequencies are too great to be ascribed to experimental uncertainty. The theoretical coefficients are too low at low frequencies, too high at frequencies near $ka=1$, and are in fairly good agreement with the measured coefficients at high frequencies.

The differences between the theoretical and measured values of α_r might be explained qualitatively as follows: (1) The original assumption that the normal impedance completely describes the state of vibration of the air at the surface of the cylinder might be too simple. Sound waves might be propagated through the felt tangentially to the surface. If this should occur, the boundary conditions to be used in

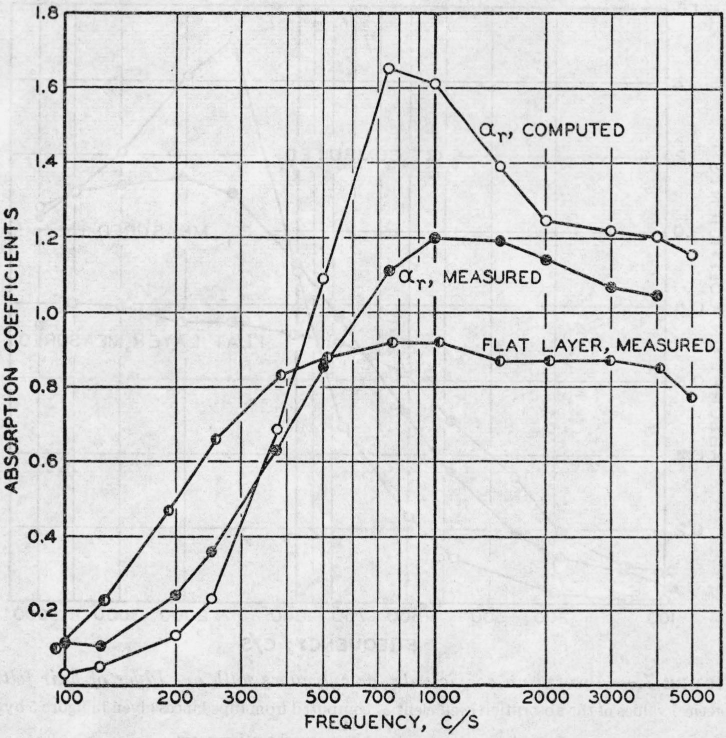

FIGURE 8.—*Absorption coefficients for cylinders with two layers of hair felt.*

○ = theoretical values of the absorption coefficient α_r computed from impedances given in figure 5 by means of eq 14.
● = values of α_r measured in a reverberation room.
◐ = absorption coefficients for a flat double layer of hair felt measured in a reverberation room.

solving the wave equation would be different. (2) The assumption that the absorption of a plane wave having an angle of incidence ϑ (see fig. 3) is directly proportional to cos ϑ is only an approximation, as has been mentioned before. (3) The statistical assumption that the wave motion is completely random in the particular reverberation room in which the measurements were made is a close approximation at higher frequencies, but might be incorrect at low frequencies.

Of the three conjectures, (1) and (2) will most likely explain the discrepancies observed. It is difficult to put these on a quantitative basis. The low theoretical values of α_r at low frequencies might be explained by (2), and the high theoretical values near $ka=1$ might be due to the tangential propagation of sound described in (1).

VI. EXPERIMENTAL PROCEDURE

1. IMPEDANCE MEASUREMENTS

The method used for measurement of the acoustic impedance of the felt was an adaptation of a method suggested by Flanders [4]. In the modified method, the unknown impedance was measured in terms of two known acoustic impedances and the complex ratio of one bridge balance reading to two others.

The impedance tube, 3.2 cm in diameter, was divided into two sections. One section was fitted with a loudspeaker at one end, and was tapped near the other end by a small hole communicating with a condenser microphone. The other section was fitted with a movable piston. The two sections of the tube either could be joined together or a ring carrying the test sample could be inserted between them. A closely fitting sleeve made the joints tight.

Two of the bridge-balance readings were for two known acoustic impedances, which were secured by setting the piston at the junction and at $\lambda/8$ from the junction. The third was obtained by inserting the felt sample in a ring between the two sections with the piston against the back of the sample. Sample rings ⅞ in. and 1¾ in. in length were used, and the felt samples were either compressed or fluffed out so that they just filled the sample ring. It was found that the felt sample disks fitted tightly enough in the tube so that cementing was unnecessary.

Simultaneous measurements of the acoustic impedance of a tube of arbitrary length served as checks on the accuracy of the felt impedance determinations.

2. ABSORPTION MEASUREMENTS

The reverberation chamber was a room 30 by 25 by 20 ft, with smooth plaster walls and a concrete floor. Double-brick wall construction prevented extraneous noise.

Rotating vanes, 8 by 16 ft over all, aided in the attainment of random wave conditions. Furthermore, the source of sound was modulated in frequency, or "warbled", through ± 10 percent of the nominal signal frequency. The loudspeakers were mounted on the vanes.

The sound was picked up by four stationary microphones. After preamplification, the outputs from the four microphones were fed into a commutating device that sampled the outputs successively 12 times a second. After further amplification, the output was filtered by means of a band pass filter.

The decay of the residual sound in the reverberation chamber was recorded on an automatic logarithmic power-level recorder working over a range of 50 decibels. Decay curves for the empty chamber were obtained before and after the absorbent materials were placed in the chamber in order to minimize any effects due to temperature and humidity changes. Care was taken to keep the amount of absorbent material in the chamber well below the quantity required to produce nonlogarithmic decays. In all, 10 decay curves for both the empty chamber and the chamber with material were recorded at each frequency. The final total absorption A was computed from the formula

$$A=\frac{4V}{c}\left(\log_e 10^5\right)\left(\frac{1}{T_M}-\frac{1}{T_E}\right), \tag{28}$$

where T_M and T_E are the 50-decibel decay times in the room containing the absorbent material and in the empty room, respectively.

The individual Fiberglas cylinders were joined together to form two long cylinders, each 32.3 ft long, and were supported on trestles with the centers of the cylinders 34 in. above the floor. The perpendicular distance between the two long cylinders was 12.1 ft.

Both the single-layer and double-layer felt-covered cylinders were arranged in two different ways. In one arrangement, three cylinders 30.4 ft long were assembled from the cylinder sections placed end to end. The perpendicular distance between the centers of the long cylinders was 15.7 ft. In the other arrangement, five cylinders 19.9 ft long were assembled from two sections each and were spaced 5 ft on centers. For the single-layer-covered cylinders, the centers were 50 in., and for the double layer, 54 in. above the floor of the reverberation chamber. All free ends of the cylinders were stuffed with hair felt to prevent any possible standing waves at the lower frequencies. All free ends were also capped with sheet iron.

The absorption coefficients of both one and two layers of felt laid flat on the floor of the reverberation room were determined on approximately square samples, each having an area of about 80 ft.^{2} The coefficients were extrapolated to what they would be for an infinite area by means of corrections empirically determined by Chrisler [5].

VII. ACCURACY OF MEASUREMENTS

The estimate of the accuracy of the impedance measurements of the hair felt was based on (1) the agreement between the computed and the experimental values of the impedance of a tube l cm long measured simultaneously with the felt samples, and (2) the precision as given by repeated measurements. It was found that the impedance of the tube computed from

$$\frac{Z}{\rho c} = -i \cot (kl) \tag{29}$$

agreed within 3 percent with the experimental value, except for those frequencies at which the impedance approached zero. In agreement with Flanders [4], it was also found that the resistive component was very small, provided the tube length did not approach $(n\lambda/2)$ $(n=1, 2, 3 \ldots)$. The resistive component, for a tube of length $l=6$ cm, fluctuated between negative and positive values, averaging about 2 percent of the reactive component. The results of repeated measurements on the 6-cm tube, and also on the felt, agreed within 4 percent through about 2,000 c/s. Above this frequency, the variation in repeated measurements on the tube averaged about 7 percent. The final uncertainty ascribed to the impedance measurements on the felt is 5 percent.

In evaluating the uncertainty in the measurement of absorption coefficients on the assumption that Sabine's formula holds for the reverberation chamber, all that needs to be considered is the repeatability of the decay times and the agreement by different observers in reading the decay curves. All other factors in the reverberation formula are known with relatively high accuracy. Due to the irregularities in the decay curves at the lowest frequencies, the uncertainty in the decay times may be as high as 6 percent. The uncertainty decreases rapidly with increasing frequency until, beginning at about

250 c/s, it is about 2 percent. As the absorption coefficients are directly proportional to the difference between the reciprocals of two decay times, the estimated final uncertainty is about 3 percent for the greater part of the frequency range.

Absorption measurements were made on cylinders of finite length, whereas the theoretical coefficients were deduced for infinitely long cylinders. As at almost all frequencies the absorption coefficients for the cylinders 19.9 ft long and 30.4 ft long mentioned above differed by less than the experimental uncertainty, it is believed that the end effects are negligible. The absorption coefficients given in figures 7 and 8 are the averages for the cylinders of different lengths.

VIII. TECHNIC OF CALCULATION

Bessel functions of high order and their derivatives were needed to compute α_r (eq 14). It was decided that the British Association Mathematical Tables [6] would yield accurate results more quickly than would any other available tables of Bessel functions. First, values of ka (which are the arguments of the Bessel functions) were computed to three decimal places from the measured circumferences of the cylinders and the frequencies used in making the normal impedance measurements. The values of ka used in the calculations ranged from about 0.13 to about 8.7. The Bessel functions of high order were computed from the British Association Tables for J_0, J_1, Y_0, and Y_1 by means of well-known recurrence formulas.

Enough terms in the infinite series of eq 14 were used so that the estimated sum of the remainder terms amounted to less than 0.1 percent of the whole sum. It is believed that the final computed values of the absorption coefficients α_r have errors (arising from the sequence of calculations) of less than 0.01.

IX. SUMMARY

A wave theory has been applied to the absorption of sound by an absorbent circular cylinder of infinite length. An exact expression, including the effects of diffraction, has been obtained for the absorption coefficient of a cylinder on which is incident a plane wave of sound traveling in a direction perpendicular to the axis of the cylinder. The derivation of the expression for the absorption coefficient was based on the assumption that the ratio of sound pressure at a point on the cylinder surface to the normal component of particle velocity was equal to the normal acoustic impedance of the surface. Theoretical coefficients were computed for Fiberglas cylinders by using assumed impedances and were computed for cylinders of hair felt by using impedances that were measured directly.

The theoretical coefficients were compared with experimental values obtained for cylinders placed in a reverberation room. In order to effect the comparison, it was necessary to develop the statistics for the absorption by a cylinder located in a random wave field. These statistics were shown to be different from the usual statistics that apply to the absorption by a flat patch on a wall of a reverberation room.

The theory predicts, and measurements confirm, that absorbent cylinders can have coefficients greater than unity. The agreement between the theoretical and experimental coefficients is fairly good, and is better than that usually found for flat patches of absorbent material placed in a reverberation room. The discrepancies between theoretical and experimental coefficients appear to be due either to wave propagation through the absorbent material in directions tangential to the surface, to absorption at oblique incidence that is not directly proportional to the cosine of the angle between the direction of propagation of the incident sound and a plane normal to the cylinder axis, or to a combination of these two effects.

The statistics for the absorption by a sphere located in a random wave field were developed. Comparison of theoretical and experimental absorption coefficients for spheres would be of great interest. There would be no uncertainty for spheres about the absorption at oblique incidence as there is for cylinders and flat patches of absorbent material.

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