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MEASUREMENT OF RATE OF FLOW OF WATER THROUGH FILTER PAPER

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ABSTRACT

A method is described for measuring the rate of filtration of water through filter papers, in which the paper is used as a cone in the usual manner and special apparatus is not required. An equation is derived, with which a water-filtration coefficient is determined from the filtration data. Data are presented also to show the effect of continued filtration on this coefficient. A correlation is shown between the air permeability of filter paper and the rate of filtration, enabling one to make a good estimate of the speed of a filter paper without wetting it. A definite procedure is recommended for determining the rate of flow of water through filter paper.

CONTENTS

I. INTRODUCTION

Suitable filter paper is an item of considerable importance to the analytical chemist. Until recently, most of the filter paper intended for analytical use has come from Germany, England, and Sweden. The war has made the supply from these sources uncertain. As a consequence, American manufacturers have undertaken the making of grades of analytical filter papers that hitherto have not been produced in this country.

Heretofore, the chemist has relied mainly on established brands and grades, with which he has become familiar, as guides in the choice of suitable filter papers. It seems desirable to provide manufacturers and consumers with better means of determining the suitability of filter paper intended for analytical use, especially since new products are now included in the available supply.

An investigation was undertaken at the National Bureau of Standards to supply the needed information. A number of the important physical and chemical properties of available filter papers are being studied, and will in due time be reported upon. One phase of this study is the evaluation of the rate of filtration, or speed, as this property is frequently called. This article is confined to reporting the development of a method of measuring the rate of flow of water through filter paper.

II. PREVIOUS WORK

Herzberg [1] used a constant-head apparatus, based on the principle of the Mariotte flask, which forced water through a horizontal disk of filter paper 10 cm^2 in area and discharged it into a measuring flask. With a head of 5 cm of water at 20°C, the instrument was used to measure either the time of filtration of 100 ml of water or the volume of water filtered in 1 minute. He reported a range of filtering rates of 23 to 760 ml/min per 100 cm^2 of area for 30 filter papers.

Griffin and Parish [2] reported difficulty in maintaining a constant head with the Herzberg apparatus, and were unable to obtain consistent results with it. They devised a filtering cell, 2 inches in diameter, in which a horizontal filter circle supported by a wire screen was subjected to a head of 9 inches of water, held constant by overflowing an excess of water at the top of a standpipe. The filtrate was caught in a measuring flask, and the time required to filter 100 ml of water at 20°C was reported. They gave values of 8 to 120 seconds for typical papers. They also gave data on the effect of temperature.

III. CONE METHOD

If one could make suitable measurements of filtration through filter paper folded in the usual manner to form a cone, some obvious advantages would result. Such a procedure would adhere more nearly to the conditions of normal filtering, and there would be no need for special apparatus required when disks are used.

Griffin and Parish observe, however, that, although the most common method of appraising filter papers has been to judge the relative performance when they are folded and supported as cones, several uncertainties characterize such a procedure, making it unreliable and ill-adapted for supplying data suitable for record and reference. They point out that the results depend upon the design of the funnel, its angle, the dimensions of the stem, the manner of folding the filter paper, the amount of water used, and the manner of pouring it in.

These difficulties can be surmounted with little effect on the innate simplicity of the testing procedure. The effect of the funnel can be eliminated by supporting the filter cone freely in a wire loop, or in a conical wire helix if the cone is large. This method of support was used in the tests to be described. The paper can be folded in a standard manner. All the water used was poured in at once.

Of the several factors that ordinarily contribute to the uncertainty in measuring rate of flow through a conical filter, the one which presents the greatest difficulty is the change in head and filtering area as the water level falls in the cone during the testing period. Preliminary experiments with disks of filter paper indicated that Darcy's law should be applicable, that is, for a given filter paper the rate of filtering of water is proportional to the area of the filter and to the head. With this premise a conical filter lends itself readily to the derivation of a water-filtration coefficient.

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[1] W. Herzberg, Papierprüfung [7th ed.] p. 109 (1932). 5th ed. (1921) J. Springer, Berlin.
[2] R. C. Griffin and H. C. Parish, Ind. Eng. Chem. **14** 199 (1922).

Figure 1 represents a 60-degree conical filter made by folding a filter circle in the usual manner. Let k be the water-filtration coefficient, defined as the volume of water filtered per second through unit area under unit head.[3] A quantity of water is poured into the cone, and at any time, t, thereafter, the volume remaining is v, having a depth h.

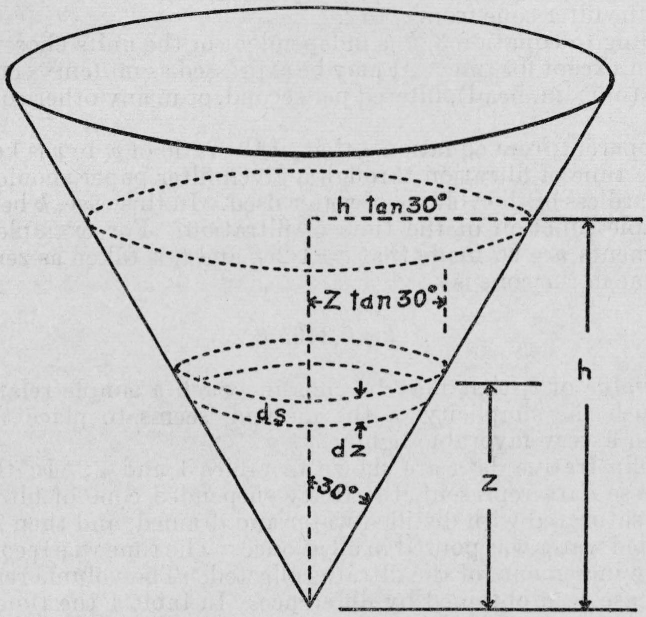

FIGURE 1.—*Sixty-degree conical filter containing water.*

Consider an elementary surface, ds, of the cone at a depth, $h-z$, having a height, dz, a slant height of dz sec 30° ($=2\sqrt{3}/3\ dz$), and a circumference at midheight of $2\pi z$ tan 30° ($=2\sqrt{3}/3\pi z$). Therefore, the area, $ds=4/3\pi z dz$. The rate of flow through any small area is the water-filtration coefficient multiplied by the area and the head operating on that area (Darcy's law). The rate of change of the volume in the cone with time is negative. Hence,

$$dv/dt=-k\int_0^h(h-z)ds=-4/3\pi k(h\int_0^h zdz-\int_0^h z^2dz)=-2/9\pi kh^3. \quad (1)$$

At the time t, the volume of water remaining in the cone is

$$v=1/3\pi h(h \tan 30°)^2=1/9\pi h^3.$$

Therefore,

$$dv/dt=-2kv. \quad (2)$$

[3] Although the conical filter is composed of three layers over half its area, and only one over the other half, this density distribution is constant for all values of z (fig. 1) and therefore will not affect the setting up or the solution of the differential equations. This fact does mean, however, that k is associated with this particular arrangement of the filter paper and would not have the same value for some other arrangement, such as a disk, for example. The coefficients for disk and cone would, however, be proportional.

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The solution of equation 2 gives

$$k=\frac{ln\ v_1/v}{2(t-t_1)}=\frac{1.151\ \log v_1/v}{t-t_1}, \tag{3}$$

in which $t-t_1$ is the time interval required to decrease the volume of water in the filter cone from v_1 to v.

According to equation 3, k is independent of the units chosen for its expression, except for time. It may be expressed as ml/(cm$^2\times$cm head), or as in.^{3}/(in.$^2\times$in. head), filtered per second, or in any other consistent units.

It is apparent from equation 3 that, if the ratio of v_1 to v is kept constant, the time of filtration through a given filter paper should be the same regardless of the volume of water used. In this case, k becomes a very simple function of the time of filtration. For example, if the measurements are so made that $v=1/2v_1$, and t_1 is taken as zero when the volume in the cone is v_1,

$$k=0.3465/t \tag{4}$$

for any value of v_1 that may be chosen. Such a simple relation, together with the simplicity of the method, seems to place the cone method in a very favorable light.

Some illustrative data are shown in tables 1 and 2. In the tests which these data represent, the freely suspended cone of filter paper was first saturated with distilled water and drained, and then 25 ml of the distilled water was poured in all at once. The time was recorded for successive increments of the filtrate collected. The volume remaining in each case was obtained by difference. In table 1 the time in seconds is tabulated against the volume remaining in the cone, at the intervals required to filter 3 ml of water.

In the first part of table 2 are shown values of the water-filtration coefficient, calculated by equation 3, for the intervals of volume change indicated in column 1 of table 1. In the second part of table 2 are shown the values of the coefficient, calculated by equation 4, when the volume in the cone has been reduced by one-half. The coefficient K is 10^4k.

TABLE 1.—*Data obtained by the cone method with six kinds of filter papers*

Volume in cone	Filtering time of sample—					
	1	2	3	4	5	6
ml	*sec*	*sec*	*sec*	*sec*	*sec*	*sec*
25	0	0	0	0	0	0
22	68	31	21	15	7	3
19	137	64	42	31	14	5
16	213	103	67	48	22	7
13	306	150	97	67	31	11
10	407	209	135	89	44	15
7	--------	281	186	120	60	21
4	--------	387	--------	163	88	29

TABLE 2.—*Water-filtration coefficients*[1] *calculated from the data in table 1*

Change in volume		Value of K for sample—					
From	To	1	2	3	4	5	6
ml	*ml*						
25	22	10	21	32	44	95	221
22	19	11	22	34	45	103	361
19	16	11	22	34	50	106	425
16	13	11	22	34	54	115	259
13	10	13	22	35	60	101	328
10	7	--------	22	35	58	112	297
7	4	--------	23	--------	65	100	350
24	12	11	22	35	50	112	347
22	11	11	22	35	52	108	347
20	10	12	22	34	54	105	315
18	9	--------	22	35	56	108	389
16	8	--------	22	35	57	105	315
14	7	--------	22	35	58	105	289
12	6	--------	22	33	59	108	315
10	5	--------	22	--------	61	112	315
8	4	--------	22	--------	64	105	315

[1] $K=10^4\,k$. It may conveniently be thought of as the number of milliliters of water filtered through a square meter of filter paper in 1 second under a head of 1 cm of water.

IV. EFFECT OF CONTINUED FILTRATION ON RATE OF FLOW

Previous work has indicated that the speed of filter paper decreases with time of filtering. Griffin and Parish (see footnote 2) published data showing this effect for a number of filter papers. In one case the speed at the end of 2 hours of continuous filtration had decreased to only 2.5 percent of the value at the beginning. Some further data are given in table 3, which were obtained by the cone method with water not previously filtered. In these tests also the speed decreased progressively as filtration was continued.

TABLE 3.—*Change in speed of filter paper with continued flow of water through specimen*

K, determined after filtering	Water-filtration coefficients[1] for specimen—		
	1	2	[2] 3
ml	*K*	*K*	*K*
0	64	58	53
100	57	47	47
200	50	40	40
500	46	--	--
FILTER REVERSED			
0	71	49	59
100	67	--	54
200	63	41	41

[1] see note under table 2.
[2] Specimen 3 had been soaked for 24 hours before being tested.

Griffin and Parish attribute this effect to "hydration" and swelling of the fibers. Swelling of course takes place, and must have an effect on the speed, at least during the initial stage of filtering, since it makes the paper thicker and affects the size of the pores. Swelling, however, does not seem to explain the change in speed with time of filtering. One of the filter circles reported on in table 3 was soaked for 24 hours before being tested. It behaved very much like the ones not soaked. Continued flow of water through the filter seems necessary to cause slowing of the speed. It is quite possible that swelling and thickening of the paper may make the interstices larger rather than smaller. The data discussed in the next section indicate that the paper probably becomes more permeable to water as a result of swelling.

Lane[4] observed a decrease in permeability of paper with continued flow of oil through it. In this case "hydration" and swelling would be absent or unimportant. He offered as a tentative explanation the orientation of fibers to partially block the flow, but did not consider this explanation altogether satisfactory because an equilibrium state was never reached. Likewise, the slowing of the speed of filter paper seems to go on indefinitely. In one instance during the present investigation, the speed of filtration with distilled water was still decreasing after 7 hours of continuous filtration.

When the flow through filter paper is reversed in direction by refolding the circle, the speed of filtration may be increased temporarily, as the data in table 3 show. It then begins to decrease as before. Lane observed a similar behavior when oil flowed through paper and thought it compatible with the fiber-orientation hypothesis.

Fungoid growths in distilled water have been reported,[5] and it has been suggested that the slowing of the speed of filter paper with time might be explained by the plugging of the pores with such material filtered. There has, however, been no evidence of the operation of this factor during the work reported here.

Muskat[6] observes that a very common cause of the plugging of a porous medium upon continuous flow of liquid through it is the evolution of air or gas dissolved in the liquid. He recommends as a remedy the use of distilled water, and the prefiltering of it before use. Such an explanation seems to be in keeping with many of the observations of the behavior of filter papers. Air bubbles collecting in the pores could explain the decrease of speed with time, even when long-continued; the failure of soaking to slow the speed noticeably; and the behavior of oil flowing through paper (see footnote 4); the temporary increase in the speed of filter paper when the flow of water is reversed in direction; and the fugitive nature of the slowing agent, as indicated by the fact that drying out a filter circle which has exhibited slowing usually restores it to substantially its original speed. This explanation, however, is not consistent with other observations. Boiling the water, or extracting the air by vacuum, does not serve the purpose of prefiltering it. Aerating water that has been prefiltered usually does not cause slowing of the speed of filter paper through which it is filtered.

Simon and Neth,[7] working with filters made of glass, porcelain,

[4] Wm. H. Lane, Paper Trade J. **117**, TS30 (1943).
[5] A. Griffiths and C. H. Knowles, Proc. Phys. Soc. **24**, 350 (1912).
[6] M. Muskat, The Flow of Homogeneous Fluids Through Porous Media, p. 90 (McGraw-Hill Book Company, Inc., New York, N. Y., (1937)).
[7] A. Simon and W. Neth, Z. anorg. allgem. Chem. **168**, 221 (1927).

and quartz, found that fresh water (distilled water not previously filtered) or fresh solutions of numerous chemical reagents progressively decreased in rate of filtration when successive portions were passed through the filters. Acetone and carbon tetrachloride behaved in the same way. When they were prefiltered, however, all passed through the filters repeatedly without decrease in rate of filtration. When the prefiltered water, solution, or reagent was boiled or allowed to stand for 1 or 2 days, it again behaved in each instance like fresh (unfiltered) liquid.

They found that a filter that had been clogged or slowed by filtering fresh water through it could be made to filter more rapidly again, either by filtering certain acids through it or by applying an electromotive force (110 or 220 volts) across the filter, provided the negative pole was placed on the discharge side of the filter. They also found that such a filter, clogged by filtering fresh water through it, retained a positively charged colloid while passing a negatively charged colloid of the same particle size, when the two sols were filtered through it.

From their observations they concluded that impurities in the water could not account for the clogging of the filters. They found that air already in the filter is sometimes a factor in its initial behavior, but concluded from experiments made in a vacuum with unfiltered water that air evolved from solution in the water could not be the chief factor involved in clogging the filters.

Simon and Neth concluded finally that a number of complicated factors are responsible for the change in the speed of filters as a result of the passage of liquids through them. Important among these are air already in the filter; order of filtration of various reagents; adsorption of ions; and the electrical condition of the filter and the solution, that is, polarization and potentials produced by flow of liquid through the filter.[8]

Most of the phenomena reported by Simon and Neth pertaining to the behavior of water when filtered through their rigid filters was observed during the present investigation when water was filtered through filter papers. No experiments were made with other liquids.

Whatever may be the cause of the slowing of the speed of filter paper, prefiltering the water consistently improves the behavior and substantially eliminates uncertainty in the test for speed. Using the same portion of water over and over is helpful, since each test prefilters it for the next test.

The effect of prefiltering is illustrated in figure 2. The usual behavior is shown by curve *A* when distilled water not prefiltered is used, fresh portions of the water being filtered through a given circle of filter paper for several successive filterings. Curve *A* is a composite of data for five circles from three kinds of filter papers. The water was from two sources. Sometimes, however, distilled water, not prefiltered, does not decrease the speed of filter paper in this manner. Curve *B* illustrates the behavior when prefiltered water is used. This curve is a composite of the data for 58 circles from 11 kinds of filter papers, obtained with water from three sources. A few of these tests were made with ordinary water from the city supply, after prefiltering it. Such water, not prefiltered, slows the paper much more rapidly than distilled water does. Of the 221 determinations used in arriving

[8] G. H. Bishop, F. Urban, and H. L. White, Colloid Symposium Annual 8, 137 (1930).

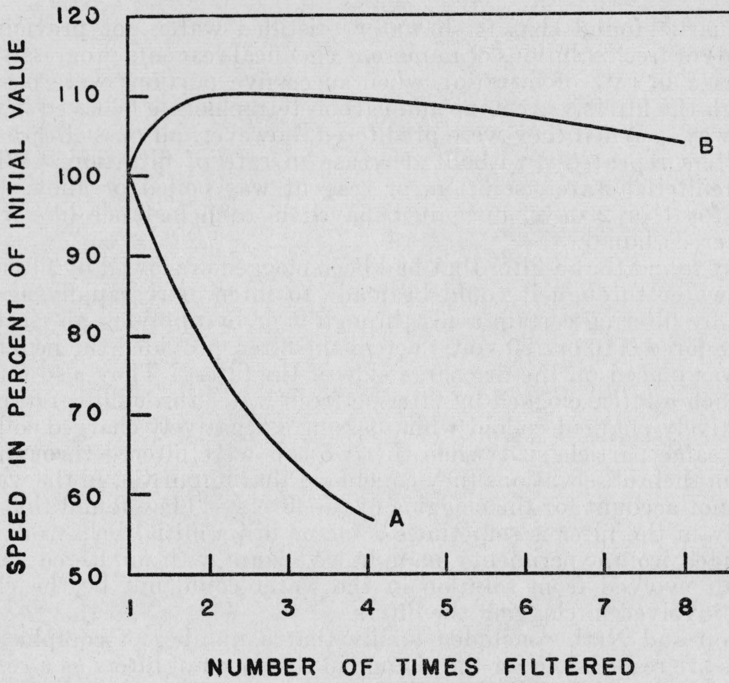

FIGURE 2.—*Change in rate of flow of water through filter paper.*

A, When distilled water is used which has not been prefiltered; and *B*, when prefiltered water is used.

at curve *B*, more than 90 percent were within ±15 percent of this composite curve.

V. RELATION OF AIR PERMEABILITY AND RATE OF FILTRATION

A knowledge of the air permeability of filter papers should be useful in throwing some light on what happens to the structure during filtration, and also in determining the approximate speed of a filter paper before using it.

According to familiar formulations that describe the flow of fluids through a capillary system, the ratio of the rates of flow (in terms of the volumes passing through per unit time, area, and driving pressure) of two fluids through a given capillary system should be equal to the reciprocal of the ratio of their respective viscosities. For air and water at room temperature, air flows through paper about 55 times as fast as water.

The air permeability of a number of filter papers was measured with the Carson [9] instrument. The same specimens were then used in determining the water-filtration coefficient by the cone method. The results are shown in table 4. The ratio varies from 24 to 40, being in all cases considerably less than the theoretical value. If, when filter paper is wet with water, the swelling of the fibers tends to reduce the size of the pores in the paper, as is commonly supposed, the ratio

[9] F. T. Carson, J. Research NBS **26**, 475 (1941) RP1390; see also BS J. Research **12**, 567 (1934) RP681.

in table 4 should be greater than 55 instead of less. It seems likely that the spaces between the fibers become larger as a result of swelling in the structure.

It is observed that in table 4 the papers are arranged in nearly the same order by the two measurements. This fact may be found useful as it affords a means of estimating the speed of filtration without wetting the paper.

TABLE 4.—*Relation of the air permeability of filter paper to the rate of filtration of water*

Sample	Water-filtration coefficient [1] K	Air permeability [1]	Ratio	Sample	Water-filtration coefficient [1] K	Air permeability [1]	Ratio
1	28	680	24	16	175	5,510	32
2	31	835	27	17	220	6,560	30
3	42	1,360	32	18	225	7,330	33
4	55	1,520	28	19	230	6,930	30
5	55	1,680	31	20	230	9,260	40
6	57	1,470	26	21	232	8,690	37
7	60	1,570	26	22	240	7,920	33
8	95	2,510	26	23	272	9,420	34
9	95	3,350	35	24	325	11,200	35
10	107	3,060	29	25	370	12,800	35
11	132	3,540	27	26	415	14,600	35
12	132	4,370	33	27	425	15,600	37
13	140	5,000	36	28	595	20,900	35
14	145	4,730	33	29	615	21,100	34
15	152	4,270	28				

[1] See note under table 2. The air permeability is expressed in the same unit as K.

VI. RECOMMENDED PROCEDURE

The following procedure is recommended for testing the speed of filtration of filter paper. Obtain distilled water (preferably freshly distilled) and prefilter it through filter paper that has a rate of flow at least as slow as that of the paper to be tested. Keep the temperature of the water constant during the tests. A temperature of 23° C is recommended.

Carefully fold a filter circle in the usual way to form a 60-degree cone. Place it in a 60-degree funnel, wet it thoroughly with water prepared in the manner indicated above, and press down the folds to expel all air pockets and to make the three plies smooth and in good contact. Pour out the excess water, remove the cone and suspend it freely over a burette by supporting it in a loop of wire. This loop should be of such a size that it will support the cone at about two-thirds of its height, reckoned from the apex. Immediately, while the cone is saturated with water, pour into the cone all at once a known volume of water which is approximately two-thirds the capacity of the cone. When one-fifth of the water has filtered into the burette, start a stopwatch. When half the remaining volume has filtered through, stop the stopwatch and record the time in seconds. Test 5 to 20 circles, depending on their variability, to obtain an average. Use the same portion of water for each test, adding to it from the prepared supply as found necessary. The average time can be used as an inverse measure of speed, but it is preferable to use the average time as t in the equation $K=10^4k=3465/t$ (eq 4) to obtain a water-filtration coefficient.

The data in table 5 illustrate the application of the method. Each result is the average of five tests.

TABLE 5.—*Data obtained by the recommended procedure*

Diameter of filter circle	Volume poured in	Filter paper *A* Water-filtration time,[1] *t*	Filter paper *A* Water-filtration coefficient,[2] *K*	Filter paper *B* Water-filtration time,[1] *t*	Filter paper *B* Water-filtration coefficient,[2] *K*
cm	*ml*	*sec*		*sec*	
5.5	3.5	95 ±4.9	36	14 ±2.9	250
7	5	89 ±3.6	39	15 ±1.8	230
9	10	90 ±5.0	39	15 ±1.9	230
11	25	87 ±8.9	40	17 ±3.9	200

[1] Time required to filter half the volume remaining when the stopwatch was started. The limits of uncertainty indicated were calculated by the method recommended in the ASTM Manual on Presentation of Data, Supplement A, for a probability of 9 in 10.
[2] See note under table 2.

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