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## A CRITICAL STUDY OF SOME FACTORS AFFECTING THE BREAKING STRENGTH AND ELONGATION OF COTTON YARNS

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### ABSTRACT

The results of single-strand tests to determine the breaking strength and elongation of cotton yarns varying in yarn number and spun with four twist multipliers are reported. Pendulum and inclined-plane types of testing machines and two rates of loading were used in the tests. The results give information regarding the corrections of these machines, variability of the yarns, and the number of tests required for a given precision and probability. The effect of rate of loading on the breaking strength and elongation is discussed. The tests on the inclined-plane machine yielded a significantly higher breaking strength and lower elongation than the tests on the pendulum machine. These unexpected differences are not attributable to a difference in the rates of loading between the two machines. They are explainable on the basis of the mechanics of the two types of machines.

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### I. INTRODUCTION

The results obtained in testing textiles for breaking strength and elongation vary with the type of testing machine used and the rate of loading. Comprehensive and systematic studies of these variations have been rather limited in scope, and the variations continue to present a serious problem in the preparation of textile specifications and in the testing of textiles. The need for further information on the effect of rate of loading on the breaking strength of cotton yarns by Subcommittee B-1, Section II on Machines, of Committee D-13, American Society for Testing Materials, and by others led to the present work.

The tests were planned to yield data which could be treated readily by statistical methods and to cover not only rate of loading and type

<sup>1</sup> Richard S. Cleveland collaborated in this study before he left the National Bureau of Standards in April 1940.

of testing machine but also uniformity of the yarns in order to show the number of tests required for a given precision and probability in the results. Further, the effects of yarn number and twist on the breaking strength and elongation of cotton yarns received attention.

## II. TESTING MACHINES AND CALIBRATIONS

Two types of testing machines were used. They are known as type *A* and type *B* in ASTM Designation D 76-39. Type *A* is often referred to as the pendulum-type machine, while type *B* is known as the constant specimen-rate-of-load machine,<sup>2</sup> and is commonly referred to as the "Incline Plane Serigraph."

The tests on the type *A* machine were made with 500- and 2,000-g capacities. The machine was operated at speeds of 5 and 12 inches per minute for the movement of the lower jaws. The speed of 5 inches per minute was obtained by the use of an auxiliary pulley. Under these conditions each yarn was loaded at two rates which differed by a factor of 2 or more.

The tests on the type *B* machine were made with 250-, 500-, 1,000-, and 2,000-g capacities. The 2,000-g capacity was attained by adding an auxiliary weight to the cart. Under these conditions each yarn was loaded at two rates which differed by a factor of 2.

Before the tests were made, the two machines were calibrated. Since published information regarding methods of calibration and of the results obtained are meager, and since those obtained on machine *B* are of special interest, the results are presented in detail.

The type *A* machine was calibrated according to the methods given in ASTM Designations D 76-39 and D 179-38, as well as by the swinging-pendulum and by the calibrated-spring methods.

In the method given in D 76-39, a weight is suspended freely from the upper jaws and the equilibrium position of the pendulum is determined by disengaging the pawls, bringing the pendulum slightly below its equilibrium position, engaging the pawls, and then releasing the pendulum and allowing it to swing up to the equilibrium position with the pawls engaged. The applied weight minus the indicated load is equal to the correction. This correction is plotted against the indicated load in *A* of figures 1 and 2 for the 500- and 2,000-g capacities, respectively.

In the method given in D 179-38, the weight is lowered by means of the lower jaws until it is suspended freely from the upper jaws and with the pawls of the pendulum engaged. The correction is equal to the applied weight minus the indicated load when the weight is completely suspended and the pendulum has come to rest. This correction is plotted against the indicated load in *B* of figures 1 and 2 for the 500- and 2,000-g capacities, respectively. The calibration for the 500-g capacity was then repeated with the pawls disengaged. The correction is plotted against the indicated load in *B'* of figure 1. The difference between *B* and *B'* is approximately 2 percent of the indicated load and represents the friction due to the dragging of the pawls.

In the swinging-pendulum method the pawls were disengaged, and with a weight suspended from the upper jaws, the pendulum was displaced from its equilibrium position and released to swing freely.

<sup>2</sup> G. B. Haven, *A constant load rate testing machine for textiles*, Proc. Am. Soc. Testing Mtls. 23, pt. 2, 640-654 (1923).

The correction is equal to the applied weight minus the load indicated after the pendulum has come to rest. This correction is plotted against the indicated load in *C* of figure 1 for the 500-g capacity. This correction remained practically constant and equal to  $-15$  g throughout the 500-g load range. The larger numerical value of this correction compared with the corrections obtained by the preceding

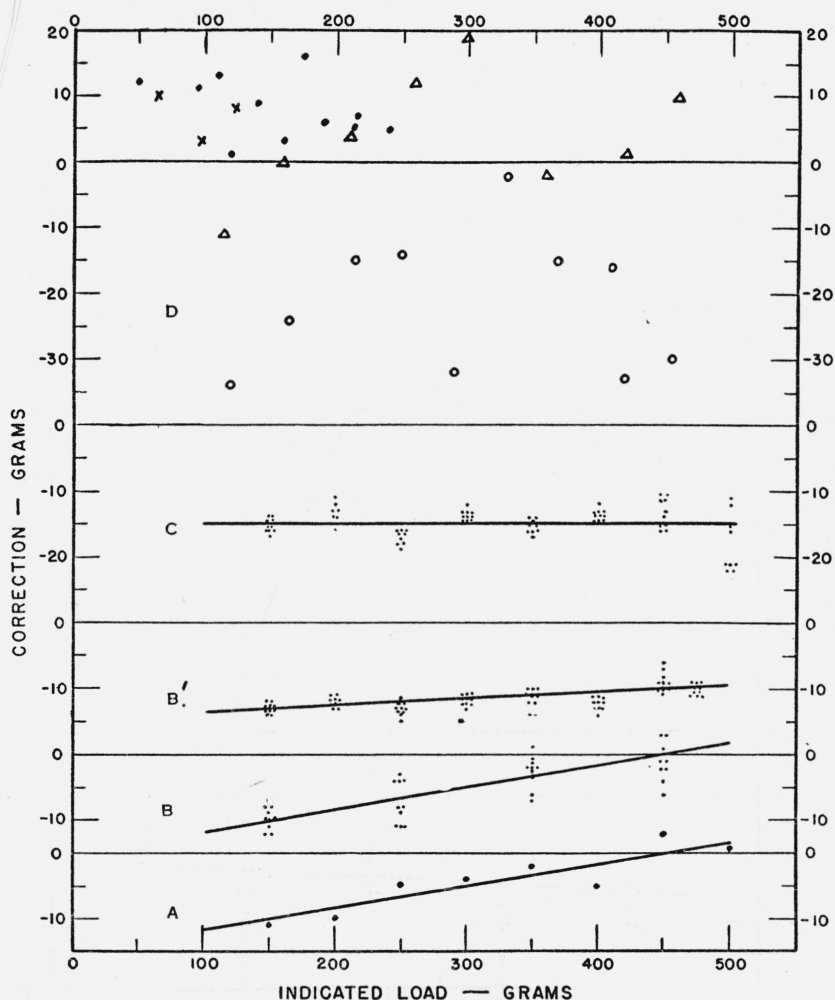


FIGURE 1.—Calibration correction of the type A machine, 500-g capacity, using different methods of calibration.

methods is chiefly attributable to the different ways in which the bearing friction of the pendulum and the friction due to the dragging of the pawls enter into the calibration procedure.

In the calibrated-spring method several helical springs which previously had been calibrated with known weights were used. A spring was mounted between the upper and lower jaws of the machine. The lower jaws were lowered by means of the motor, thus elongating

the spring and exerting a pull on the upper jaws and the pendulum. The applied load was ascertained from the elongation of the spring and its calibration plus the weight of the spring. The correction is equal to this applied load minus the indicated load. This correction is plotted in *D* of figures 1 and 2 for the 500- and 2,000-g capacities,

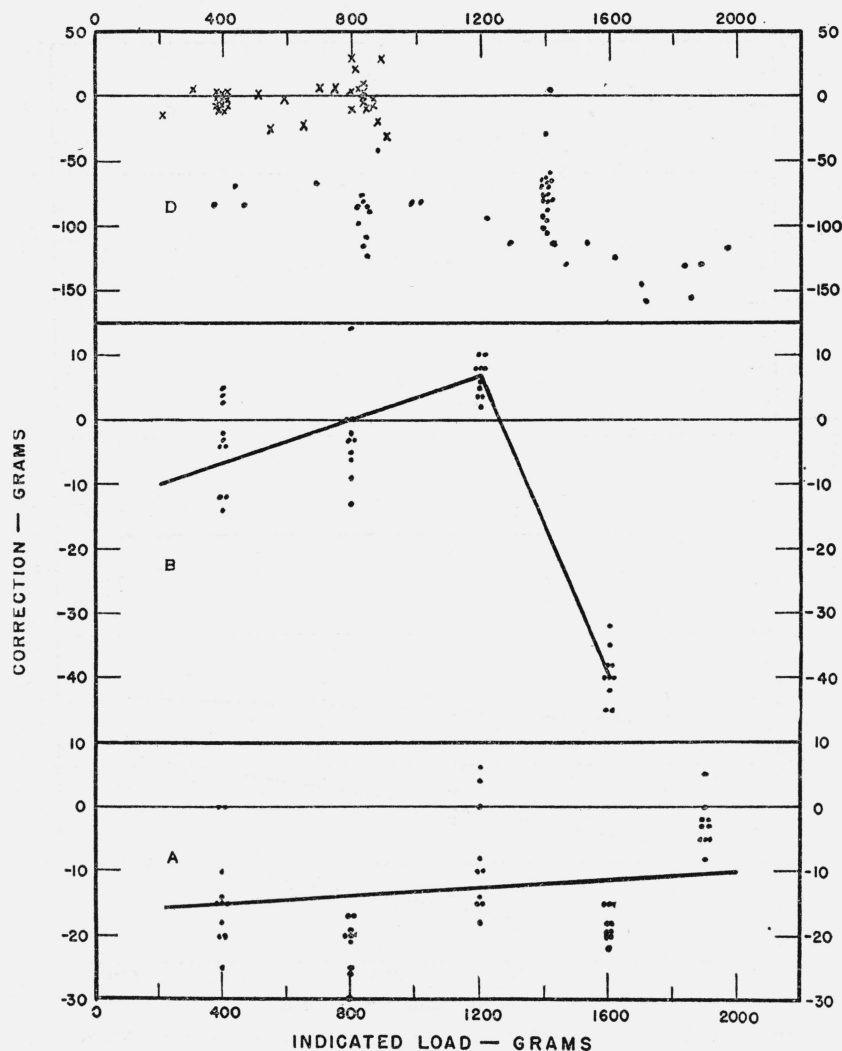


FIGURE 2.—Calibration correction of the Type A machine, 2,000-g capacity, using different methods of calibration.

respectively. The different symbols represent the results obtained with different springs. This method gave variable results, the variability being largely due to the inability of ascertaining the correct elongation of the springs. It is not considered a satisfactory method of calibration.

The best corrections to be applied to the test results obtained with this type A machine are believed to be those obtained by the two



ASTM calibration procedures. These two methods gave essentially the same values for the 500-g capacity. However, for the 2,000-g capacity the results by both methods were quite variable. The calibration corrections given in table 1, which were applied to the test results in this paper, are based upon the corrections derived from the two ASTM calibration methods.

TABLE 1.—*Calibration corrections for the type A machine*

500-g capacity		2,000-g capacity	
Indicated load	Correction	Indicated load	Correction
<i>g</i>	<i>g</i>	<i>g</i>	<i>g</i>
100	-12	400	-11
150	-10	500	-10
200	-8	600	-9
250	-7	700	-8
300	-5	800	-7
350	-4	900	-6
400	-2	1,000	-5
450	0	1,100	-4
500	+2	1,200	-3

The type *B* machine was calibrated by the weight and pulley method.<sup>3</sup> In this method a weight is fastened to one end of a string which passes over a pulley and has its other end fastened to the cart of the machine. The plane is inclined to a known angle and the weight is adjusted to such an amount that, when started, the cart would proceed down the plane with approximately uniform velocity. The weight corrected for pulley friction is taken as the applied load. The indicated load is the value which the machine records on the chart, namely, the chart ordinate of a uniform scale in inches. The applied load minus the indicated load is equal to the correction.

The correction for each individual calibration is plotted in figure 3 against the indicated load and chart ordinate. The straight line drawn for each capacity indicates zero correction at an indicated load equal to one-half of each capacity and a correction of minus and plus 2 percent of the full capacity at indicated loads equal to zero and full load, respectively. These straight lines represent the calibration data quite well. The deviations of the individual points from these lines probably are due largely to variations in the individual calibration determinations and to variations in the friction of the cart bearings.

The fact that straight lines, having the same percentage correction for each capacity, constitute a good fit of the data suggests that the source of the correction is common to the four capacities. A careful search was made and it was found that the source of this correction resided in the mechanism which moves the recording chart. A schematic drawing of this mechanism is shown in figure 4, where *P* represents the inclined plane. To it are fastened two pulleys, *A* and *B*, which may rotate. A cord, *C*, of length *l*, is fastened at one end to the table at *D*, passes over pulley *B*, and is fastened at the other end to pulley *A* at *E*. The movement of the chart is directly proportional to the angular motion of pulley *A*. The weight of the chart exerts a

<sup>3</sup> R. S. Cleveland, *Calibration of a "constant rate of loading" machine for testing the tensile properties of textiles*, Rayon Textile Monthly 18, 179-180, 230-232 (March and April 1937).

torque on pulley *A* and thereby places a tension on cord *C*. As the inclination of the plane is increased, by lowering the rack, *R*, at a constant speed, using a motor and reduction gear, the distance from *D* to *B* is decreased, and pulley *A* will rotate enough to take up the slack produced in the cord by the decrease in the distance from *D* to *B*. Although the change in the angle of inclination,  $\theta$ , is such that the

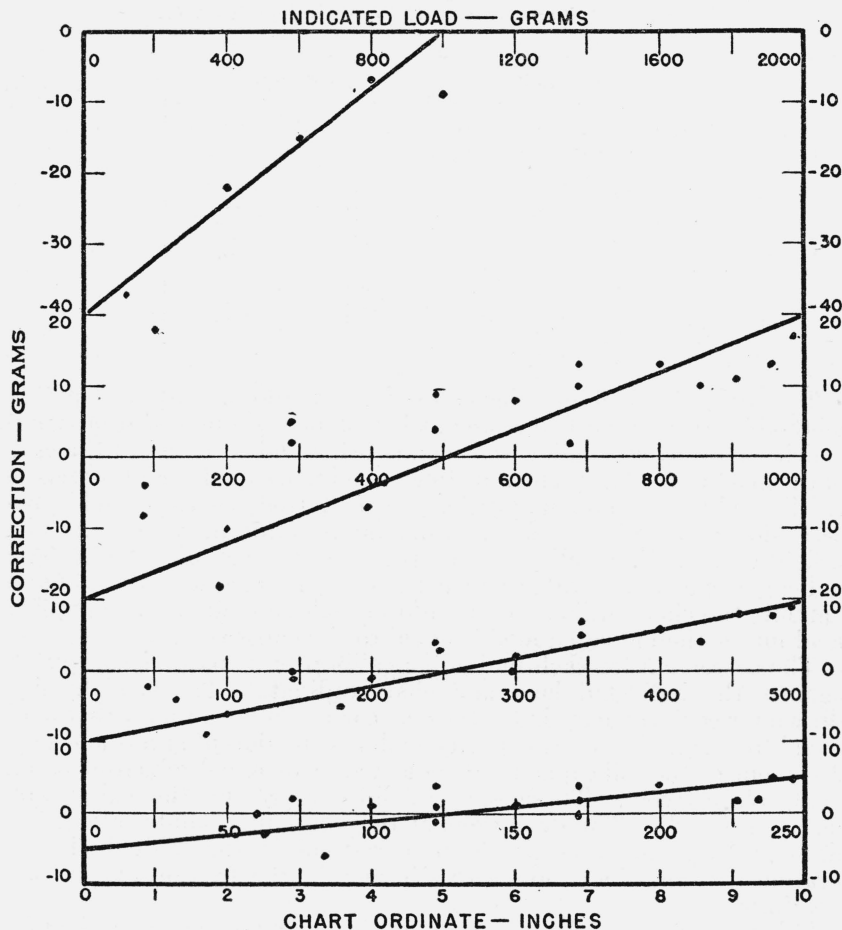


FIGURE 3.—Calibration correction of the type B machine for 250-, 500-, 1,000-, and 2,000-g capacity.

change in  $\sin \theta$ , and therefore in the applied load,  $W \sin \theta$ , increases uniformly with time, it readily can be seen from figure 4 that the distance of the cord from *D* to *B* does not decrease uniformly with time. Therefore, the motion of the chart is not uniform, and the uniform load scale, ordinate on the chart, will be in error, depending upon the variation in the chart speed. The increase in the chart speed at zero load compared with that at full load was determined and found to be equal to 4.2 percent. The progressive change in the correction for each capacity, amounting to about 4 percent of each capac-

ity, is entirely attributable to the operation of the recording mechanism. If this condition were remedied the correction of this machine for each capacity would be substantially a constant and therefore could be held within a small tolerance by adjusting the weight of the

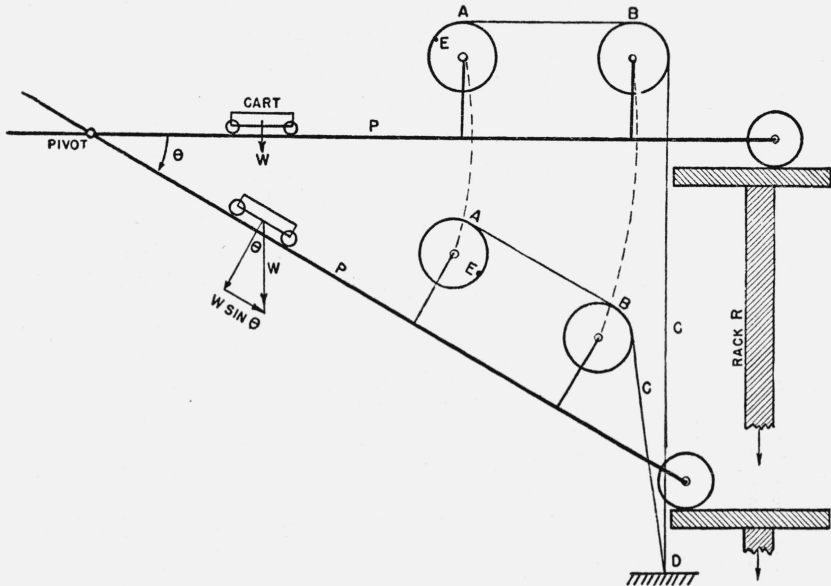


FIGURE 4.—Schematic drawing of type B machine.

cart. Table 2 contains the corrections applied in this paper for the type B machine.

TABLE 2.—Calibration corrections for the type B machine

250-g capacity		500-g capacity		1,000-g capacity		2,000-g capacity	
Indicated load	Correction	Indicated load	Correction	Indicated load	Correction	Indicated load	Correction
<i>g</i> 0	<i>g</i> -5	<i>g</i> 0	<i>g</i> -10	<i>g</i> 0	<i>g</i> -20	<i>g</i> 0	<i>g</i> -40
25	-4	50	-8	100	-16	100	-36
50	-3	100	-6	200	-12	200	-32
75	-2	150	-4	300	-8	300	-28
100	-1	200	-2	400	-4	400	-24
125	0	250	0	500	0	500	-20
150	1	300	2	600	4	600	-16
175	2	350	4	700	8	700	-12
200	3	400	6	800	12	800	-8
225	4	450	8	900	16	900	-4
250	5	500	10	1,000	20	1,000	0

III. MATERIALS, TESTING CONDITIONS, AND RESULTS

The cotton yarns for this investigation were spun on a small experimental spinning frame. Four bobbins of each kind of yarn were received. The nominal yarn numbers and twist multipliers of the yarns tested are listed in table 3.

TABLE 3.—Results of tests on cotton yarns

[The asterisk indicates that the difference is significant]

Typp number		Twist multiplier	Testing conditions				Breaking strength, ± standard error						Elongation at rupture, ± standard error	
Nominal	Actual		Type of testing machine	Speed of lower jaws	Capacity	Approximate rate of loading	Bobbin				Average	Combined average	Average	Combined average
							1	2	3	4				
50	51.6	3	A	$\left\{ \begin{array}{c} \text{in./min} \\ 5 \\ 12 \end{array} \right.$	$\frac{g}{500}$ 500	$\frac{g}{\text{den. min}}$ 7.2 17.2	$\frac{g}{162 \pm 3.9}$ 171 ± 4.3	$\frac{g}{164 \pm 4.1}$ 171 ± 3.4	$\frac{g}{148 \pm 4.5}$ 153 ± 3.9	$\frac{g}{174 \pm 3.8}$ 175 ± 4.6	$\frac{g}{162 \pm 2.1}$ 168 ± 2.1	$\left. \right\}$ 165 ± 1.5	$\left\{ \begin{array}{c} \% \\ 6.04 \pm 0.05 \\ 6.12 \pm 0.07 \end{array} \right.$	$\left. \right\}$ 6.08 ± 0.043
Significance ratio							1.6	1.3	0.8	0.2	2.0		1.0	
50	51.6	3	B		$\left\{ \begin{array}{c} 250 \\ 500 \end{array} \right.$	2.8 5.6	$\frac{160 \pm 3.8}{173 \pm 3.7}$	$\frac{163 \pm 3.6}{179 \pm 3.4}$	$\frac{158 \pm 4.1}{174 \pm 2.6}$	$\frac{174 \pm 3.4}{189 \pm 2.9}$	$\frac{164 \pm 1.9}{179 \pm 1.6}$	$\left. \right\}$ 171 ± 1.2	$\left\{ \begin{array}{c} 5.73 \pm 0.05 \\ 5.67 \pm 0.05 \end{array} \right.$	$\left. \right\}$ 5.70 ± 0.035
Significance ratio							2.4 $\frac{1}{2}$	3.2*	3.3*	3.4*	6.0*	3.2*	−0.8	−6.5*
50	54.2	4	A	$\left\{ \begin{array}{c} 5 \\ 12 \end{array} \right.$	$\frac{500}{500}$	7.2 17.2	$\frac{161 \pm 6.8}{169 \pm 3.4}$	$\frac{170 \pm 4.3}{182 \pm 4.3}$	$\frac{169 \pm 3.7}{185 \pm 3.6}$	$\frac{168 \pm 5.0}{177 \pm 5.0}$	$\frac{167 \pm 2.6}{178 \pm 2.1}$	$\left. \right\}$ 173 ± 1.6	$\left\{ \begin{array}{c} 6.38 \pm 0.06 \\ 6.71 \pm 0.07 \end{array} \right.$	$\left. \right\}$ 6.55 ± 0.05
Significance ratio							1.1	1.3	3.1*	1.3	3.3*		3.7*	
50	54.2	4	B		$\left\{ \begin{array}{c} 250 \\ 500 \end{array} \right.$	2.8 5.6	$\frac{175 \pm 5.2}{190 \pm 4.8}$	$\frac{183 \pm 2.8}{189 \pm 3.7}$	$\frac{170 \pm 3.1}{191 \pm 3.8}$	$\frac{179 \pm 5.0}{200 \pm 2.6}$	$\frac{177 \pm 2.1}{193 \pm 2.1}$	$\left. \right\}$ 185 ± 1.5	$\left\{ \begin{array}{c} 6.23 \pm 0.07 \\ 5.78 \pm 0.07 \end{array} \right.$	$\left. \right\}$ 6.01 ± 0.05
Significance ratio							1.9	1.3	4.2*	3.7*	8.0*	5.5*	−4.5*	−8.0*
50	49.6	5	A	$\left\{ \begin{array}{c} 5 \\ 12 \end{array} \right.$	$\frac{500}{500}$	6.4 15.4	$\frac{185 \pm 5.5}{184 \pm 6.1}$	$\frac{169 \pm 5.8}{173 \pm 5.7}$	$\frac{187 \pm 6.5}{184 \pm 6.3}$	$\frac{172 \pm 6.4}{175 \pm 6.1}$	$\frac{178 \pm 3.0}{179 \pm 3.0}$	$\left. \right\}$ 179 ± 2.1	$\left\{ \begin{array}{c} 8.15 \pm 0.09 \\ 8.18 \pm 0.09 \end{array} \right.$	$\left. \right\}$ 8.17 ± 0.06
Significance ratio							−0.1	0.5	−0.3	0.3	0.2		0.2	
50	49.6	5	B		$\left\{ \begin{array}{c} 250 \\ 500 \end{array} \right.$	2.8 5.6	$\frac{192 \pm 5.2}{198 \pm 5.9}$	$\frac{172 \pm 4.0}{191 \pm 5.6}$	$\frac{211 \pm 5.2}{201 \pm 5.0}$	$\frac{162 \pm 6.2}{176 \pm 4.2}$	$\frac{184 \pm 2.6}{192 \pm 2.6}$	$\left. \right\}$ 188 ± 1.8	$\left\{ \begin{array}{c} 7.74 \pm 0.11 \\ 6.93 \pm 0.10 \end{array} \right.$	$\left. \right\}$ 7.34 ± 0.07
Significance ratio							0.8	2.8	−1.4	1.9	2.2	3.2*	−5.4*	−3.5*

50	49.2	6	A	{	5 12	500 500	5.8 13.9	147 ±7.6 171 ±6.0	140 ±5.1 145 ±4.2	137 ±5.8 160 ±4.0	145 ±6.6 157 ±5.0	142 ±3.2 158 ±2.4	} 150 ±2.0	{ 8.23 ±0.11 8.58 ±0.10	} 8.41 ±0.07
Significance ratio								2.5	0.8	3.2*	1.5	4.0*		1.7	
50	49.2	6	B	-----	{	250 500	2.8 5.6	155 ±3.0 169 ±4.1	160 ±2.1 171 ±4.6	158 ±5.1 175 ±4.4	160 ±6.0 172 ±5.2	158 ±2.2 172 ±2.4	} 165 ±1.6	{ 7.65 ±0.11 7.05 ±0.11	} 7.35 ±0.08
Significance ratio								2.8	2.2	2.5	1.5	4.2*	5.8*	-3.8*	-9.8*
25	25.2	3	A	{	5 12	500 500	3.9 9.3	194 ±6.0 205 ±6.1	212 ±6.8 208 ±4.9	232 ±9.1 245 ±6.4	183 ±6.3 193 ±8.0	205 ±3.6 213 ±3.2	} 209 ±2.4	{ 5.93 ±0.05 6.25 ±0.05	} 6.09 ±0.04
Significance ratio								1.3	-0.5	1.2	1.0	1.7		4.8*	
25	25.2	3	B	-----	{	500 1000	2.8 5.6	191 ±5.7 214 ±5.6	208 ±4.4 212 ±6.1	222 ±4.5 240 ±5.1	197 ±4.7 223 ±4.3	204 ±2.4 222 ±2.6	} 213 ±1.8	{ 5.76 ±0.06 5.73 ±0.04	} 5.75 ±0.04
Significance ratio								2.9	0.5	2.7	4.2*	5.1*	1.3	-0.4	-4.6*
25	25.5	4	A	{	5 12	500 500	3.9 9.4	229 ±4.5 232 ±6.9	259 ±7.1 266 ±5.7	246 ±7.2 258 ±7.3	206 ±6.1 245 ±5.7	235 ±3.2 250 ±3.2	} 242 ±2.3	{ 6.14 ±0.06 6.58 ±0.07	} 6.36 ±0.05
Significance ratio								0.4	0.8	1.2	4.6*	3.3*		4.9*	
25	25.5	4	B	-----	{	500 1000	2.8 5.6	285 ±6.1 268 ±7.7	260 ±6.4 264 ±5.8	271 ±6.9 262 ±5.6	267 ±7.5 261 ±7.8	271 ±3.4 264 ±3.4	} 267 ±2.4	{ 6.29 ±0.07 6.18 ±0.07	} 6.24 ±0.05
Significance ratio								-1.7	0.5	-1.0	-0.6	-1.5	7.6*	-1.1	-1.8
25	25.3	5	A	{	5 12	500 500	3.8 9.2	261 ±4.9 242 ±7.2	245 ±6.8 270 ±7.8	241 ±6.2 269 ±5.3	247 ±7.5 281 ±7.5	248 ±3.2 266 ±3.5	} 257 ±2.4	{ 6.95 ±0.08 7.40 ±0.10	} 7.18 ±0.06
Significance ratio								-2.2	2.4	3.4*	3.2*	3.8*		3.5*	
25	25.3	5	B	-----	{	500 1000	2.8 5.6	267 ±7.0 287 ±7.2	275 ±6.7 282 ±6.8	267 ±6.6 293 ±6.1	280 ±5.3 278 ±8.0	272 ±3.2 285 ±3.5	} 278 ±2.4	{ 7.32 ±0.09 7.49 ±0.09	} 7.41 ±0.6
Significance ratio								2.0	0.7	2.9	-0.2	2.8	6.2*	1.3	2.7
25	24.6	6	A	{	5 12	500 500	3.5 8.4	241 ±7.4 252 ±6.5	240 ±6.8 242 ±7.5	258 ±7.9 224 ±6.4	252 ±6.6 250 ±6.2	248 ±3.6 242 ±3.3	} 245 ±2.4	{ 8.48 ±0.10 8.24 ±0.11	} 8.36 ±0.07
Significance ratio								1.1	0.2	-3.4*	-0.2	-1.2		-1.6	
25	24.6	6	B	-----	{	500 1000	2.8 5.6	258 ±6.9 267 ±6.1	258 ±7.3 276 ±7.5	273 ±8.2 270 ±7.1	277 ±5.5 299 ±7.1	267 ±3.5 278 ±3.5	} 273 ±2.5	{ 8.10 ±0.10 8.29 ±0.11	} 8.20 ±0.07
Significance ratio								1.0	1.7	-0.3	2.4	1.8	8.0*	1.3	-1.5

TABLE 3.—Results of tests on cotton yarns—Continued

Typn number		Twist multi- plier	Testing conditions				Breaking strength, ± standard error						Elongation at rupture, ± standard error	
Nom- inal	Actual		Type of test- ing ma- chine	Speed of lower jaws	Capac- ity	Approx- imate rate of loading	Bobbin				Average	Combined average	Average	Combined average
							1	2	3	4				
12.5	12.1	3	A	$\left\{ \begin{array}{c} in./min \\ 5 \\ 12 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 2000 \\ 2000 \end{array} \right.$	$\left\{ \begin{array}{c} g/den. min. \\ 4.6 \\ 11.0 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 493 \pm 9.0 \\ 489 \pm 6.9 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 420 \pm 5.9 \\ 410 \pm 6.5 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 454 \pm 12.0 \\ 440 \pm 6.9 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 431 \pm 8.7 \\ 450 \pm 9.8 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 450 \pm 4.6 \\ 447 \pm 3.8 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 448 \pm 3.0 \end{array} \right.$	$\left\{ \begin{array}{c} \% \\ 7.76 \pm 0.04 \\ 7.84 \pm 0.07 \end{array} \right.$	$\left\{ \begin{array}{c} \% \\ 7.80 \pm 0.04 \end{array} \right.$
Significance ratio							-0.4	-1.1	-1.0	1.5	-0.5		1.0	
12.5	12.1	3	B	-----	$\left\{ \begin{array}{c} 1000 \\ 2000 \end{array} \right.$	$\left\{ \begin{array}{c} 2.8 \\ 5.6 \end{array} \right.$	$\left\{ \begin{array}{c} 500 \pm 9.2 \\ 557 \pm 6.9 \end{array} \right.$	$\left\{ \begin{array}{c} 463 \pm 8.7 \\ 511 \pm 9.0 \end{array} \right.$	$\left\{ \begin{array}{c} 439 \pm 9.1 \\ 470 \pm 4.6 \end{array} \right.$	$\left\{ \begin{array}{c} 481 \pm 9.9 \\ 549 \pm 10.8 \end{array} \right.$	$\left\{ \begin{array}{c} 471 \pm 4.6 \\ 522 \pm 4.1 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 496 \pm 3.1 \end{array} \right.$	$\left\{ \begin{array}{c} 7.51 \pm 0.05 \\ 7.60 \pm 0.04 \end{array} \right.$	$\left\{ \begin{array}{c} 7.56 \pm 0.03 \end{array} \right.$
Significance ratio							5.0*	3.8*	3.0*	4.7*	8.2*	11.1*	1.7	-4.3*
12.5	11.8	4	A	$\left\{ \begin{array}{c} 5 \\ 12 \end{array} \right.$	$\left\{ \begin{array}{c} 2000 \\ 2000 \end{array} \right.$	$\left\{ \begin{array}{c} 4.8 \\ 11.5 \end{array} \right.$	$\left\{ \begin{array}{c} 602 \pm 11.1 \\ 592 \pm 11.8 \end{array} \right.$	$\left\{ \begin{array}{c} 585 \pm 11.2 \\ 572 \pm 11.1 \end{array} \right.$	$\left\{ \begin{array}{c} 632 \pm 11.0 \\ 577 \pm 11.9 \end{array} \right.$	$\left\{ \begin{array}{c} 566 \pm 9.4 \\ 580 \pm 6.5 \end{array} \right.$	$\left\{ \begin{array}{c} 596 \pm 5.4 \\ 580 \pm 5.3 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 588 \pm 3.8 \end{array} \right.$	$\left\{ \begin{array}{c} 8.99 \pm 0.06 \\ 9.55 \pm 0.07 \end{array} \right.$	$\left\{ \begin{array}{c} 9.27 \pm 0.05 \end{array} \right.$
Significance ratio							-0.6	-0.8	-3.4*	1.2	-2.1		6.2*	
12.5	11.8	4	B	-----	$\left\{ \begin{array}{c} 1000 \\ 2000 \end{array} \right.$	$\left\{ \begin{array}{c} 2.8 \\ 5.6 \end{array} \right.$	$\left\{ \begin{array}{c} 627 \pm 12.0 \\ 693 \pm 8.3 \end{array} \right.$	$\left\{ \begin{array}{c} 585 \pm 11.2 \\ 639 \pm 6.7 \end{array} \right.$	$\left\{ \begin{array}{c} 603 \pm 6.8 \\ 641 \pm 10.9 \end{array} \right.$	$\left\{ \begin{array}{c} 596 \pm 8.6 \\ 636 \pm 8.4 \end{array} \right.$	$\left\{ \begin{array}{c} 603 \pm 4.9 \\ 652 \pm 4.4 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 628 \pm 3.3 \end{array} \right.$	$\left\{ \begin{array}{c} 9.50 \pm 0.06 \\ 9.40 \pm 0.05 \end{array} \right.$	$\left\{ \begin{array}{c} 9.45 \pm 0.04 \end{array} \right.$
Significance ratio							4.5*	4.2*	3.0*	3.3*	7.4*	8.0*	-1.3	3.0*
12.5	12.2	5	A	$\left\{ \begin{array}{c} 5 \\ 12 \end{array} \right.$	$\left\{ \begin{array}{c} 2000 \\ 2000 \end{array} \right.$	$\left\{ \begin{array}{c} 4.7 \\ 11.4 \end{array} \right.$	$\left\{ \begin{array}{c} 547 \pm 9.3 \\ 592 \pm 12.2 \end{array} \right.$	$\left\{ \begin{array}{c} 608 \pm 9.2 \\ 590 \pm 10.4 \end{array} \right.$	$\left\{ \begin{array}{c} 587 \pm 10.2 \\ 596 \pm 12.5 \end{array} \right.$	$\left\{ \begin{array}{c} 595 \pm 11.9 \\ 596 \pm 12.1 \end{array} \right.$	$\left\{ \begin{array}{c} 584 \pm 5.1 \\ 594 \pm 5.9 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 589 \pm 3.9 \end{array} \right.$	$\left\{ \begin{array}{c} 9.38 \pm 0.07 \\ 9.90 \pm 0.07 \end{array} \right.$	$\left\{ \begin{array}{c} 9.64 \pm 0.05 \end{array} \right.$
Significance ratio							2.9	-1.3	0.5	0.1	1.3		5.2*	
12.5	12.2	5	B	-----	$\left\{ \begin{array}{c} 1000 \\ 2000 \end{array} \right.$	$\left\{ \begin{array}{c} 2.8 \\ 5.6 \end{array} \right.$	$\left\{ \begin{array}{c} 598 \pm 8.3 \\ 615 \pm 11.8 \end{array} \right.$	$\left\{ \begin{array}{c} 585 \pm 8.3 \\ 607 \pm 11.0 \end{array} \right.$	$\left\{ \begin{array}{c} 599 \pm 11.0 \\ 626 \pm 15.3 \end{array} \right.$	$\left\{ \begin{array}{c} 619 \pm 9.6 \\ 627 \pm 10.5 \end{array} \right.$	$\left\{ \begin{array}{c} 600 \pm 4.7 \\ 619 \pm 6.2 \end{array} \right.$	$\left\{ \begin{array}{c} g \\ 609 \pm 3.9 \end{array} \right.$	$\left\{ \begin{array}{c} 9.59 \pm 0.06 \\ 9.28 \pm 0.08 \end{array} \right.$	$\left\{ \begin{array}{c} 9.44 \pm 0.05 \end{array} \right.$
Significance ratio							1.2	1.6	1.4	0.6	2.4	3.6*	-3.1*	-2.9

12.5	12.2	6	A	{	5 12	2000 2000	4.2 10.1	505 ±12.0 548 ±13.7	533 ±11.7 560 ±11.1	511 ±10.1 535 ±12.4	547 ±11.2 553 ±9.8	524 ±5.6 549 ±5.9	}	536 ±4.0	10.33 ±0.09 10.04 ±0.10	}	10.49 ±0.07
Significance ratio								2.4	1.7	1.5	0.4	3.1*			2.3		
12.5	12.2	6	B	-----	{	1000 2000	2.8 5.6	528 ±12.2 577 ±10.2	544 ±8.0 550 ±15.3	524 ±8.0 539 ±11.0	550 ±9.7 553 ±17.1	536 ±4.8 555 ±6.9	}	545 ±4.2	10.04 ±0.11 9.99 ±0.11	}	10.02 ±0.08
Significance ratio								3.1*	0.4	1.1	0.2	2.3		1.6	-0.3		-4.6*
6.25	6.22	3	A	{	5 12	2000 2000	2.9 7.0	793 ±20.2 739 ±27.8	706 ±17.0 748 ±17.1	955 ±29.9 923 ±31.2	809 ±25.4 863 ±25.0	816 ±11.8 818 ±12.9	}	817 ±8.7	8.06 ±0.06 7.96 ±0.06	}	8.01 ±0.04
Significance ratio								-1.6	1.7	-0.7	1.5	0.1			-1.2		
6.25	6.50	4	A	{	5 12	2000 2000	3.0 7.2	1237 ±20.8 1167 ±17.2	1033 ±18.8 1042 ±18.2	1112 ±18.1 1260 ±17.5	1070 ±17.2 1201 ±16.5	1113 ± 9.4 1168 ± 8.7	}	1140 ±6.4	9.84 ±0.09 9.70 ±0.06	}	9.77 ±0.05
Significance ratio								-2.6	0.3	5.9*	5.4*	4.3*			-1.3		
6.25	6.15	5	A	{	5 12	2000 2000	3.0 7.0	1266 ±26.7 1265 ±26.9	1251 ±20.9 1272 ±19.4	1050 ±30.6 1262 ±23.7	1088 ±29.0 1201 ±25.7	1164 ±13.5 1250 ±12.1	}	1207 ±9.1	10.91 ±0.12 11.74 ±0.11	}	11.33 ±0.08
Significance ratio								0	0.7	5.5*	2.9	4.7*			5.1*		
6.25	5.92	6	A	{	5 12	2000 2000	2.5 6.0	1193 ±22.6 1315 ±26.6	989 ±27.6 984 ±23.7	1203 ±27.0 1201 ±22.5	922 ±25.2 1092 ±25.3	1077 ±12.8 1148 ±12.3	}	1112 ±8.9	14.46 ±0.23 15.10 ±0.22	}	14.78 ±0.16
Significance ratio								3.5*	-0.1	0	4.7*	4.0*			2.0		

The yarns were conditioned for several days in an atmosphere of 65-percent relative humidity and 70° F and were tested in this atmosphere. The test specimens were mounted between two pairs of flat jaws spaced 10 inches apart. Before clamping the yarn in the second pair of jaws, a tension of 5 g was applied by the use of a suitable weight. Twenty-five breaks were made for the yarn on each bobbin for each of the testing conditions used. The various testing conditions, including type of machine, capacity of machine, speed of pulling jaws for the type *A* machine, and the approximate rate of loading expressed in grams per denier per minute are listed in table 3.<sup>4</sup>

The averages of the 25 breaks determined for the yarn on each bobbin, hereafter called the breaking strengths of the bobbins, for each of the testing conditions used are given in table 3, together with the standard error of each average. These values have been corrected, using the applicable corrections of the testing machines. The results of tests in which the specimen ruptured near either pair of jaws were not discarded. The number of ruptures near either pair of jaws was less than 5 percent, and the magnitudes of these ruptures were consistently as high as ruptures further removed from the jaws. Table 3 also contains, for each testing condition, the average breaking strength and the average elongation at rupture of each yarn (average of the results for the four bobbins), together with their respective standard errors. Finally, significance ratios (difference of two values divided by the standard error of the difference) are given directly below the pair of values chosen for comparison. In the following discussion a significance ratio numerically equal to 3 or more is considered to show a significant difference between any two values which are compared. A significance ratio less than 3 indicates either that the two values do not differ significantly or that the number of tests were insufficient to detect a significant difference.

#### IV. DISCUSSION OF RESULTS

##### 1. VARIATION IN BREAKING STRENGTH OF COTTON YARN

The variation in breaking strength of a cotton yarn includes the variation in breaking strength of the yarn on each bobbin, hereafter called the variation within bobbins, and the variation of the yarn between bobbins of the same kind of yarn, hereafter called the variation between bobbins. These variations are expressed on a comparable basis for the different yarns by the coefficient of variation. The average coefficients of variation of breaking strength within bobbins on the type *A* machine are 12.9 and 11.9 percent for the speeds of the lower jaws equal to 5 and 12 inches per minute, respectively. The difference between these two values is not significant, the significance ratio being only 1.5. The average coefficients of

<sup>4</sup> The rate of loading for type *A* machine is not constant throughout the test. It depends upon the characteristics of the machine, that is, capacity of machine and movement of upper jaws per unit load, and upon the elongation of the specimen per unit load. The values reported in this paper for the type *A* machine were computed as follows: The movement of the upper jaws, which corresponds to an indicated load equal to the breaking strength of a specimen, plus the elongation at rupture of the specimen is equal to the movement of the lower jaws in inches. This value divided by the speed of the lower jaws equals the time to rupture the specimen. The breaking load in grams divided by this time in minutes gives the rate of loading of the specimen in grams per minute. Dividing this value by the denier of the specimen converts the rate of loading to the unit used in table 3.



variation of breaking strength within bobbins on the type *B* machine are 11.0 and 10.6 percent for the low and high rates of loading, respectively. These two values do not differ significantly, the significance ratio being 0.6. However, the difference between these coefficients of variation for machines *A* and *B*, amounting to 14 percent, is significant, the significance ratio being 3.5. This difference of 14 percent in the variability of the breaking strength obtained on the two machines will be reflected in the results discussed in section 2, below.

The variation in breaking strength between bobbins of a given yarn may constitute a considerable portion of the variation of the yarn. It is apparent from the results given in table 3 that for some yarns the breaking strengths of the bobbins may differ significantly. The significance ratios of breaking strength between the possible combinations of pairs of bobbins for each yarn were computed. It was found that approximately 30 percent of these pairs had a significance ratio greater than 3 and, therefore, differed significantly.

## 2. EFFECT OF RATE OF LOADING ON BREAKING STRENGTH AND ELONGATION

The effect of rate of loading on the breaking strength and elongation of cotton yarns is shown by the data given in table 3. The following facts are based upon a critical study of these data.

When the rate of loading of the type *A* machine was increased by increasing the speed of the lower jaws from 5 to 12 inches per minute, 15 percent of the bobbins tested showed a significant increase in the breaking strength and 55 percent showed an increase which was not significant. Of the remaining bobbins, 27 percent showed a decrease which was not significant, and 3 percent showed a significant decrease in breaking strength.

When the rate of loading of the type *B* machine was increased from about 2.8 to 5.6 g per denier per minute, 31 percent of the bobbins tested showed a significant increase in the breaking strength and 56 percent showed an increase which was not significant. The remaining bobbins, 13 percent, showed a decrease in breaking strength which was not significant.

When the rate of loading of the type *A* machine was increased by increasing the speed of the lower jaws from 5 to 12 inches per minute, 50 percent of the yarns tested showed a significant increase in breaking strength and 31 percent showed an increase which was not significant. The remaining yarns, 19 percent, showed a decrease in breaking strength which was not significant.

When the rate of loading of the type *B* machine was increased from about 2.8 to 5.6 g per denier per minute, 50 percent of the yarns tested showed a significant increase in the breaking strength and 42 percent showed an increase which was not significant. The remaining yarns, 8 percent, showed a decrease in breaking strength which was not significant.

The over-all effect of increasing the rate of loading of the type *A* machine by increasing the speed of the lower jaws from 5 to 12 inches per minute was an increase of 3.8 percent in the breaking strength. This increase is highly significant, the significance ratio being 7.8.

The over-all effect of increasing the rate of loading of the type *B* machine from about 2.8 to 5.6 g per denier per minute was an increase

of 5.8 percent in the breaking strength. This increase is highly significant, the significance ratio being 12.3.

The breaking strength of the cotton yarns tested on the type *B* machine at the two rates of loading was 6.1 percent higher than the breaking strength of the same yarns when tested on the type *A* machine at the two speeds of the lower jaws. This difference was highly significant, the significance ratio being 18.0. This result was not expected, and it cannot be explained as an effect due to a difference in the rate of loading, since the average rate of loading for the type *A* machine, namely, 8.5 g per denier per minute, is approximately twice the average rate of loading of the type *B* machine, which is only about 4.2 g per denier per minute.

The effect of rate of loading on the breaking strength of the bobbins and the yarns is brought out more decisively on the type *B* machine than on the type *A* machine. This difference in the effect of rate of loading may result in part because of the difference in variability of the breaking strength obtained on the two machines.

When the rate of loading of the type *A* machine was increased by increasing the speed of the lower jaws from 5 to 12 inches per minute, 54 percent of the yarns tested showed a significant increase in the elongation, and 37 percent showed an increase which was not significant. The remaining yarns, 19 percent, showed a decrease in elongation which was not significant. The over-all effect of increasing the rate of loading on the type *A* machine was an increase of 3.3 percent in the elongation. This difference was highly significant, the significance ratio being 8.1. This effect was opposite to that expected from an increase in the rate of loading.

When the rate of loading of the type *B* machine was increased from about 2.8 to 5.6 g per denier per minute, 33 percent of the yarns tested showed a significant decrease in the elongation and 42 percent showed a decrease which was not significant. The remaining yarns, 25 percent, showed an increase which was not significant. The over-all effect of increasing the rate of loading of the type *B* machine was a decrease of 2.3 percent in the elongation. This difference was significant, the significance ratio being 5.2. This effect was in accordance with that expected from an increase in the rate of loading.

The elongation at rupture of the cotton yarns tested on the type *B* machine at the two rates of loading was 4.2 percent lower than the elongation of the same yarns when tested on the type *A* machine for the two speeds of the lower jaws. This difference was highly significant, the significance ratio being 14.0. This result and also the result that an increase in the rate of loading on the two types of machine produced opposite effects on the elongation were not expected. Further consideration is being given to these and the other unexpected results mentioned above.

Finally, an analysis of variance indicated that the effect of rate of loading on the breaking strength and the elongation, on either type of machine, did not vary significantly with either the yarn size or with the twist multiplier.

The over-all effects of the rate of loading and of the type of machine on the breaking strength and the elongation, based upon the average breaking strength and average elongation of the 50s, 25s, and 12.5s cotton yarns, are summarized in table 4. These values are plotted in

figure 5 against the logarithm of the rate of loading, the size of the dot being equal to the standard error. Castricum and Benson<sup>5</sup> found that the equation

$$S = S_1 + KS_1 \log R$$

gives a better representation of the relation between the rate of loading,  $R$ , and the breaking strength,  $S$ , than the linear function suggested by Bellinson.<sup>6</sup> In this equation,  $K$  is a constant and  $S_1$

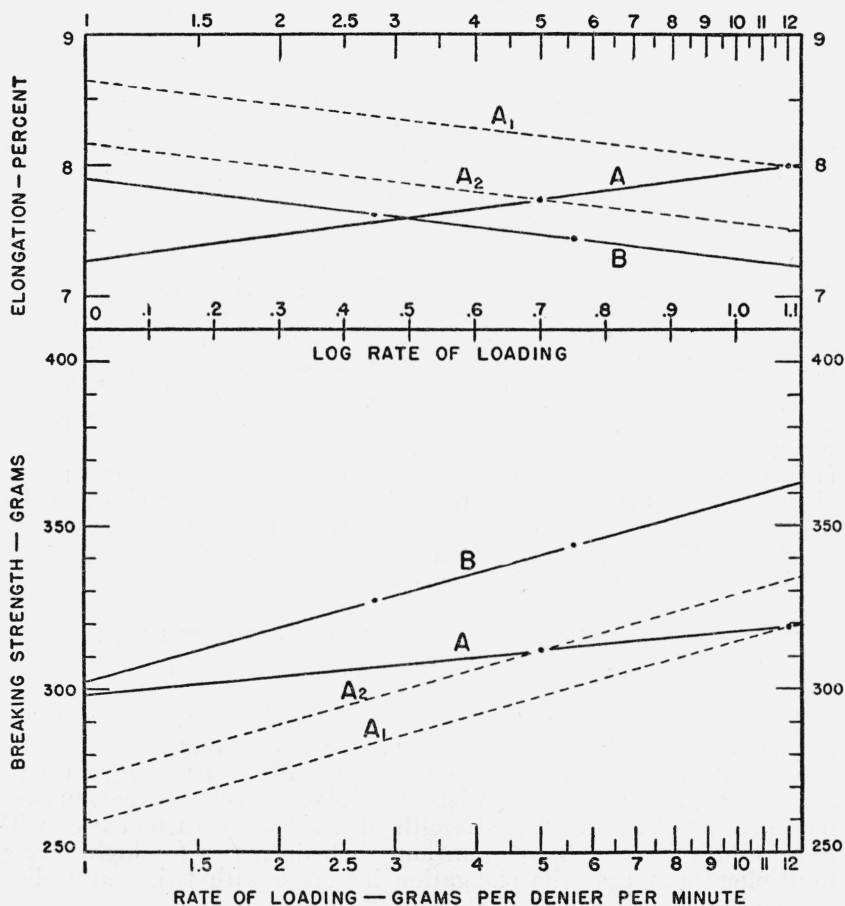


FIGURE 5.—Curves showing the effect of rate of loading and type of testing machine on the breaking strength and elongation of cotton yarns.

The broken lines represent the curves which should be obtained on the type A machine for the two conditions of operation, if it is assumed that the type B machine gives more nearly the correct effect of the rate of loading on the breaking strength and elongation of cotton yarns.

equals the breaking strength when  $R$  is unity. By using the solid curves, the effect of the same change in the rate of loading can be compared for the two types of machine. One would expect that a given change in the rate of loading for either machine would produce

<sup>5</sup> The effect of rate of loading on tensile strength of cord and yarn. ASTM Preprint (June 1941).

<sup>6</sup> Viscose rayon: Stress-strain properties. II. Effect of rate of load, Textile Research, 10, 316-322 (June 1940).

the same effect on the breaking strength and elongation. The same effect is obviously not obtained for the two machines.

TABLE 4.—Average breaking strength and average elongation based upon all of the tests on the 50s, 25s, and 12.5s cotton yarns

Type of machine	Average rate of loading	Number of tests	Breaking strength		Elongation	
			Average	Standard error	Average	Standard error
<i>A</i> -----	<i>g/den./min</i> <sup>a</sup> 5	1,200	<i>g</i> 312	<i>g</i> 1.13	<i>Percent</i> 7.73	<i>Percent</i> 0.022
<i>A</i> -----	<sup>b</sup> 12	1,200	319	1.11	8.00	.025
<i>B</i> -----	2.8	1,200	327	1.01	7.63	.024
<i>B</i> -----	5.6	1,200	344	1.13	7.45	.024

<sup>a</sup> The values which gave this average ranged from 3.5 to 7.2.

<sup>b</sup> The values which gave this average ranged from 8.4 to 17.2.

However, if it is assumed that the type *B* machine gives more nearly the correct effect of the rate of loading on the breaking strength and the elongation, then the broken lines represent the curves which should be obtained on the type *A* machine for the two conditions of operation, that is, with and without the use of the auxiliary pulley. On this assumption, the differences between curves *B* and *A*<sub>1</sub> and between *B* and *A*<sub>2</sub> are attributed to conditions which are believed to exist primarily in the type *A* machine. The differences between curves *A*<sub>1</sub> and *A*<sub>2</sub> for breaking strength and for elongation may be attributed to factors associated with the auxiliary pulley, which was mounted on a separate base and detached from the frame of the machine. The effects of a number of factors which may contribute to these differences are being considered.

### 3. EFFECT OF TWIST MULTIPLIER ON BREAKING STRENGTH AND ELONGATION

The effect of twist multiplier on the breaking strength and the elongation of the cotton yarns tested is shown in figure 6, where the average strength and average elongation of all the tests on each yarn are plotted against the twist multiplier. Previous conclusions, namely, that the breaking strength of a cotton yarn increases with twist multiplier to a maximum and then decreases for higher twist multipliers and that the elongation increases with twist multiplier, are confirmed.

### 4. COEFFICIENT OF VARIATION OF COTTON YARNS OF VARIOUS TYPP NUMBERS AND TWIST MULTIPLIERS

Table 5 contains the coefficients of variation of breaking strength and of elongation of the various cotton yarns tested on the two types of machines. The general observation that the coefficient of variation of breaking strength is a minimum at the twist multiplier giving maximum strength confirms the results of our tests on cotton yarns of 8.4s and 67s typp number.<sup>7</sup> The coefficient of variation is also

<sup>7</sup> Herbert F. Schiefer and Daniel H. Taft, *Mechanical properties of cotton yarns*, J. Research NBS 15, 237-253 (1935) RP826.

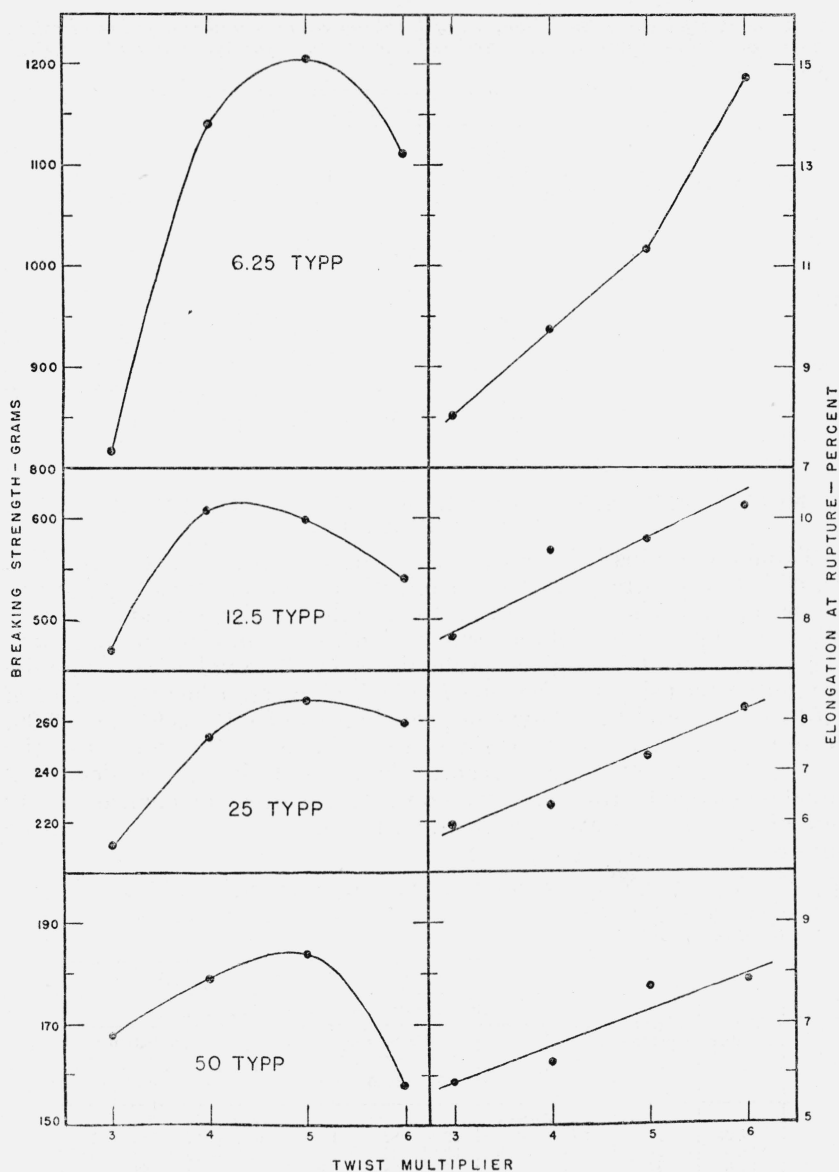


FIGURE 6.—Effect of twist multiplier on breaking strength and elongation of cotton yarns.

seen to vary with the typ number. The 12.5s yarn had a significantly lower coefficient of variation than the other yarns for both breaking strength and elongation. The average coefficient of variation of breaking strength on the type A machine was about 14 percent higher than the value for the same yarns when tested on the type B machine. This difference is significant. The coefficient of variation of elongation on the type A machine is about 7 percent lower than the value for the same yarns when tested on the type B machine.

TABLE 5.—Coefficient of variation,  $V$ , of breaking strength and elongation of cotton yarns of various typp numbers and twist multipliers for types A and B testing machines

Type of machine	Typp number	Coefficient of variation, $V$ (percent)									
		For breaking strength					For elongation				
		Twist multiplier				Average	Twist multiplier				Average
		3	4	5	6		3	4	5	6	
A.....	6.25	17.7	10.4	12.1	14.8	13.8	7.7	7.8	10.4	15.3	10.3
	12.5	11.1	9.3	9.5	11.3	10.3	7.3	7.1	7.5	8.6	7.6
	25	17.5	14.1	13.0	14.0	14.7	8.7	8.7	12.0	11.7	10.6
	50	12.3	12.3	16.9	17.7	14.8	9.5	9.5	11.2	12.1	10.7
Average.....		14.7	11.5	12.9	14.5	13.4	8.3	8.3	10.3	11.9	9.8
B.....	12.5	10.1	7.9	8.9	10.8	9.4	5.9	5.7	7.4	10.7	7.4
	25	12.1	12.4	11.9	13.0	12.4	9.1	11.0	11.7	12.9	11.2
	50	10.6	11.5	15.8	15.1	13.5	9.1	11.1	11.1	15.2	12.7
		10.9	10.6	12.2	13.0	11.7	8.0	9.3	11.2	12.9	10.4

## 5. NUMBER OF TESTS CORRESPONDING TO A GIVEN PRECISION AND PROBABILITY

Table 6 contains information regarding the coefficient of variation,  $V$ , of breaking strength and of elongation of cotton yarns which can be used as a basis for computing the number of single-strand tests,  $n$ , that are required to yield a result which will be in error by not more than  $\pm E$  percent for a given probability,  $P$ . The values of  $n$  may be computed by using the formula  $n = t^2 V^2 / E^2$ , where  $t$  has the values of 1.645, 1.960, and 2.576 for the probabilities of 0.90, 0.95, and 0.99, respectively. The values of  $n$  given in table 6 are for an error of the mean,  $E$ , equal to  $\pm 3$  percent. In general, if 100 single-strand breaking-strength tests are made on cotton yarns, the chances are 19 out of 20 that the average breaking strength so determined will be in error by less than  $\pm 3$  percent.

TABLE 6.—Coefficients of variation,  $V$ , and number of single-strand tests,  $n$ , required to yield a result for cotton yarns which will be in error by not more than  $\pm 3$  percent for the given probability

Typp number	Type of machine	N <sup>b</sup>	Computed values of <i>n</i> for cotton yarns							
			Breaking strength				Elongation			
			V	Probability			V	Probability		
				0.90	0.95	0.99		0.90	0.95	0.99
6.25	A	800	<i>Percent</i>	58	82	140	<i>Percent</i>	32	46	78
12.5	A	800	13.8	32	46	78	10.3	18	25	44
12.5	B	800	10.3	27	38	64	7.6	17	24	40
25	A	800	9.4	66	94	160	7.4	35	48	82
25	B	800	14.7	47	67	114	10.6	37	54	92
50	A	800	12.4	67	95	160	11.2	35	50	104
50	B	800	14.8	79	135	180	10.7	50	70	120
8.4 *	A	1200	13.5	56	79	135	12.7	22	32	52
67 *	A	2000	13.8	82	140	160	8.5	26	37	62
2/134 *	A	1200	14.7	66	94	160	9.2			
			15.7	76	107	180	-----	-----	-----	-----
Average			13.3	55	76	130	9.8	29	42	70

<sup>a</sup> Values of  $V$  taken from NBS Research Paper RP826, referred to in footnote 7. The 2/134 is a mercerized yarn of high commercial quality being spun from 1¾-inch staple Sea Island cotton.

<sup>b</sup>  $N$  represents the number of single-strand tests upon which the value of  $V$  is based.

WASHINGTON, July 9, 1941.



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