MAXIMUM USABLE FREQUENCIES FOR RADIO SKY-WAVE TRANSMISSION, 1933 TO 1937

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ABSTRACT

Graphs are presented to indicate the maximum usable frequencies for sky-wave transmission from June 1933 to December 1937. The graphs are given for March, June, and December to show equinoctial, summer, and winter conditions for each year. The factors which must be considered in deriving these graphs from vertical-incidence ionosphere measurements are outlined briefly. The principal factors are Snell's law, variation of virtual height of the ionosphere with frequency, the curvature of the ionosphere, and the effect of the earth's magnetic field.

The method of applying these graphs to simple and complex transmission paths is indicated.

The effects of sporadic E reflections, absorption, and scattered reflections are discussed briefly.

It is pointed out that the graphs and the ionosphere data from which they were derived may be used to estimate future diurnal, seasonal, and long-time variations of maximum usable frequencies.

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I. INTRODUCTION

The graphs of maximum usable frequencies for radio sky-wave transmission presented here were prepared from the vertical-incidence ionosphere measurements made by the National Bureau of Standards at Washington. These graphs, for any particular month, are derived from the average condition of the ionosphere for that month. The graphs give results representative of the winter, summer, and equinoctial periods from June 1933 to December 1937, inclusive.

1 A preliminary version of this paper was presented as paper C6 of the meeting in London in November 1937 of the special committee on radio-wave propagation.
The maximum usable frequency for any distance is defined as the highest frequency which can be used for radio sky-wave transmission over the given distance. Waves of frequencies higher than this penetrate through the ionosphere and are not returned to the earth; waves of lower frequencies are reflected and may be used for transmission over this distance. Graphs of maximum usable frequencies in terms of distance also give skip distances as a function of frequency. The maximum usable frequencies give more directly than skip distances information usually desired in connection with radio transmission; i.e., determination of the maximum frequency which can be used over a given distance is of more general value to an engineer than the minimum distance over which communication can be had on a given frequency.

II. RELATION OF VERTICAL TO OBLIQUE-INCIDENCE TRANSMISSION

Radio waves incident obliquely upon the layers of the ionosphere are more easily reflected than those incident vertically, that is, they can be reflected from regions of smaller ionization density. In general, for a given frequency of the transmitted wave, the larger the angle of incidence, the less is the ionization density required for reflection. Thus, if a given layer will reflect waves up to a certain frequency at vertical incidence, it will reflect waves of considerably higher frequencies at oblique incidence.

The angle of incidence is the angle between the ray and the normal to the lower boundary of the ionosphere at the point where the ray enters the ionosphere. The angle of incidence should not be confused with the angle of departure, which is the angle above the horizontal at which the ray is projected from the transmitting station. Although, in general, a transmitting station may radiate energy at all angles above the horizontal, only certain ray-paths, corresponding to a few definite angles of departure, are useful for transmission over a given distance at a given time. At vertical incidence the angle of incidence is zero, and increases for oblique incidence.

The angle of incidence depends on (1) the distance of transmission for one reflection from the ionosphere, and (2) the height of the layer. Given these two quantities, therefore, the angle of incidence may be calculated. The maximum usable frequency is given roughly, from this angle and the vertical-incidence critical frequency, by the following relation, known as the “secant law”:

\[ f' = f_c \sec \theta_1 \]

where

\[ f' = \text{maximum usable frequency}, \]
\[ f_c = \text{critical frequency or highest frequency reflected at vertical incidence from a given layer}, \] and
\[ \theta_1 = \text{angle of incidence}. \]

The given distance will be within the “skip-zone” for all higher frequencies.

The secant law does not take several important factors into account. The first and most important of these is the fact that the virtual
height of a given layer of the ionosphere increases with frequency, especially near the critical frequency. As a result, it is difficult to choose the proper height to be used in calculating the angle of incidence for a given distance. A method whereby, for a flat ionosphere, this difficulty may be overcome has been described by Smith.3

The second factor which must be considered is the curvature of the ionosphere. This introduces another modification in the calculation of the angle of incidence for a given distance and a given height of reflection. This results in a higher maximum usable frequency over a given distance than would be calculated on the basis of a flat ionosphere.

Still another factor that must be taken into account is the effect of the earth's magnetic field. In the presence of the earth's field a radio wave in the ionosphere is split into two components known as the ordinary ray and the extraordinary ray. These components are reflected from different levels in the ionosphere, and consequently have different critical frequencies. At Washington, the vertical-incidence critical frequency for the extraordinary ray is about 800 kc/s higher than for the ordinary ray, for frequencies in the neighborhood of 5,000 kc/s, and the separation is greater for lower frequencies. At oblique incidence the effect of the earth's magnetic field complicates the calculation of maximum usable frequencies. The maximum usable frequency for the extraordinary ray is always greater than for the ordinary ray. Except for short distances, the difference is not great, and decreases with increasing frequency and distance. For practical communication problems it is sufficiently accurate to make the calculations for the ordinary ray.

Methods of obtaining maximum usable frequencies, taking into account the earth's magnetic field and the curvature of the ionosphere, have been presented by Smith.4 These methods have been used in preparing figures 1 to 14.

Figure 2.—Maximum usable frequencies, latitude of Washington, D. C., December 1933.

Figure 3.—Maximum usable frequencies, latitude of Washington, D. C., March 1934.

Figure 4.—Maximum usable frequencies, latitude of Washington, D. C., June 1934.
III. TRANSMISSION PATH

The geographical part of the ionosphere which controls long-distance high-frequency radio transmission is that part traversed by the wave in passing from the transmitter to the receiver. The nearest part of the ionosphere thus traversed may be as much as 1,000 to 2,000 km from the transmitter, and a similar distance from the receiver. Ionosphere conditions at this point may be considered to be the same as conditions at the transmitter or receiver, at the same local time, provided no great differences of latitude exist. Consequently, depending...
Figure 7.—Maximum usable frequencies, latitude of Washington, D. C., June 1935.

Figure 8.—Maximum usable frequencies, latitude of Washington, D. C., December 1935.
Figure 9.—Maximum usable frequencies, latitude of Washington, D. C., March 1936.

Figure 10.—Maximum usable frequencies, latitude of Washington, D. C., June 1936.
on the time of day, different frequencies might be necessary for transmission in different directions.

Take, for example, stations at Washington, D.C., North Platte, Nebr., and San Francisco, Calif., in the eastern, central, and western parts of the United States, respectively. The distances between Washington to North Platte and between North Platte and San Francisco are approximately 2,000 km. Also take the time as 0715 local time at North Platte in December 1937 (see fig. 14). Then the

![Graph showing maximum usable frequencies](image)

**Figure 11.**—Maximum usable frequencies, latitude of Washington, D.C., December 1936.

local time at the reflection point between North Platte and Washington would be about 0800 and the local time at the reflection point between North Platte and San Francisco would be about 0630. By interpolating linearly between the graphs for successive times in figure 14 it is evident that, at this time, the highest frequency (about 19,000 kc/s) which can be used between North Platte and Washington is much greater than the highest frequency (about 8,300 kc/s) which can be used over the different path between North Platte and San Francisco. This does not mean, however, that different frequencies should be used for two-way communication over the same path, such as North Platte to Washington and Washington to North Platte,
Figure 12.—Maximum usable frequencies, latitude of Washington, D. C., March 1937.

Figure 13.—Maximum usable frequencies, latitude of Washington, D. C., June 1937.
since the maximum usable frequency is the same for transmission in opposite directions over the same path.

For single-reflection transmission the controlling portion of the ionosphere is halfway between the terminal points. For multi-reflection transmission the ionosphere along the entire path, except for that portion within several hundred kilometers of the terminal points, must be considered. Because of large differences in local time and latitude encountered in long transmission paths involving more than

![Figure 14](image_url)

**Figure 14.—Maximum usable frequencies, latitude of Washington, D. C., December 1937.**

one reflection, widely different conditions sometimes prevail over different parts of these paths. In such cases the frequency will have to be lowered to satisfy conditions in the part of the path in which the critical frequency is lowest. Under such conditions absorption is likely to occur in the part of the path with the higher critical frequencies. Normally the ionosphere does not vary greatly over a latitude range somewhat exceeding that of the United States.

The maximum possible distance for single-reflection transmission, corresponding to zero angle of departure, is about 2,400 km for E-layer transmission, and about 3,500 to 4,400 km for $F_r$-layer trans-
mission, depending on the virtual height of the layer. Practically, it is usually impossible to accomplish high-frequency transmission at this zero angle of departure over land because of absorption at the earth's surface. If a practical lower limit of $3 \frac{1}{2}^\circ$ is assumed for the angle of departure over land, the maximum distance for single-reflection transmission is about 1,700 km for $E$-layer transmission and 2,800 to 3,600 km for $F_2$-layer transmission, depending on the height. Single-reflection transmission may often be possible at greater distances, while at the same time multireflection transmission over the same path may be more efficient.

IV. GRAPHS OF MAXIMUM USABLE FREQUENCIES

The maximum usable frequencies given in figures 1 to 14 represent monthly average conditions. The graphs for December represent winter conditions which hold for several months centered on the winter solstice, those for June represent summer conditions which hold for several months centered on the summer solstice, and those for March represent the spring and fall equinoctial conditions or transition conditions between winter and summer. The seasonal variations in characteristics of the ionosphere are given in detail in the papers referred to in footnote 1.

Day-to-day variations over short periods, such as a few weeks, are usually small (less than 10 percent from the average) and are given in detail in the papers referred to in footnote 1. Large variations, however, do occur during periods of ionosphere storms, which are usually associated with magnetic disturbances. At such times the virtual heights of the $F_2$ layer are increased and the critical frequencies decreased, resulting in lower maximum usable frequencies. The $F_2$ region becomes diffuse and unstable, resulting in high absorption and poor quality of transmission.

The graphs are given for single-reflection transmission at the latitude of Washington. Linear interpolations may be made between graphs for successive times. The conditions given in the curves are for local time at the geographical part of the ionosphere where the waves are reflected. For multireflection transmission, the distance scale on the curves must be multiplied by the number of hops. For example, for two-hop transmission, 3,000 km on the graphs should be read as 6,000 km. For a path which does not involve latitudes differing widely from that of Washington, the maximum usable frequency is then the lowest one of the several corresponding to the local times at the several points of reflection along the path.

As an example of the use of the graphs, let us consider two-hop transmission from Washington to San Francisco, 4,000 km, at 0900 EST in December 1937. The distance of each hop is 2,000 km, and the local times at the points of reflection are 0815 and 0645, at the eastern and western reflecting points, respectively. Interpolating between the graphs for successive times in figure 14 at the 2,000-km distance, we obtain the maximum usable frequency to be 21,000 kc/s for the first, or eastern hop, and 10,000 kc/s for the second, or western hop. Thus 10,000 kc/s is the highest frequency that can be used at this time for two-hop transmission over the given path.  

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The graphs for June show that both the $E$ and $F$ layers are effective in determining the maximum usable frequencies. For example, at noon the $F_2$ layer will be effective out to a short distance, say 500 km, because of its higher critical frequency; then the $E$ layer will be effective out to 1,600 km because of its lower height and resulting larger angles of incidence. Beyond about 1,600 km the intensity of single-hop $E$-layer transmission decreases because of the necessarily small angles of departure at the transmitting station. The maximum usable frequency is then that which will be transmitted by two-hop $E$-, or single-hop $F_2$-layer propagation. Either of these conditions gives a maximum usable frequency lower than does the single-hop $E$-layer transmission at about 1,600 km. Of the two conditions, the $F_2$ layer gives the higher maximum usable frequency for average conditions at Washington as indicated on the graphs. A similar decrease of maximum usable frequency occurs when the $F$-layer transmission must change from one-hop to two-hop propagation at about 3,500 km.

V. OTHER TRANSMISSION CHARACTERISTICS

1. SPORADIC E

Reflections from the $E$ region are often found at frequencies considerably in excess of the maximum daytime critical frequency for the $E$ layer. These differ in character from the regular refraction phenomena of the normal $E$ layer and occur at all seasons, but they are more prevalent during the summer months in Washington, sometimes persisting for several hours. Sporadic $E$ reflections seem to be of the nature of reflections from a sharp boundary, as suggested by Kirby and Judson,\(^1\) rather than of regular refraction from a region of intense ionization. Because of the sporadic nature of these reflections their effect has not been included in the graphs. Sporadic $E$, however, accounts for good transmission at higher frequencies than those indicated by the graphs for a small percentage of the time and at irregular intervals. Reports of long-distance transmission at frequencies as high as 56 megacycles, which occur mostly during the summer, appear to be by way of the sporadic $E$ layer.

2. ABSORPTION

In contrast to the sharply defined maximum usable frequencies shown by the present graphs, there are less well defined minimum usable frequencies for practical high-frequency sky-wave transmission for various distances. These lower frequency limits are determined by absorption. The absorption varies in general with time of day, season, frequency, and length of path. Absorption occurs mainly in the lower ionosphere, that is, in the $E$ layer and below. The absorption is greater during the summer day than during the winter day. It is greater for the lower high frequencies, that is, those frequencies which are close to or below the maximum usable frequency for the $E$ layer for a given distance. It is usually greater during the day than during the night, especially for these lower high frequencies. A high-frequency wave at large angles of incidence, corresponding to long distances, is absorbed like a much lower frequency wave at small angles of incidence, corresponding to short distances.

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3. SCATTERED REFLECTIONS

Usually, complex scattered reflections are observed at frequencies considerably higher than the $F_I$-critical frequencies. These reflections provide weak, poor-quality transmissions much above the maximum usable frequencies for $F_I$-layer transmissions over distances of several hundred kilometers.

VI. CONCLUSIONS

For practical applications the principal value of these graphs is to estimate transmission conditions in the future either diurnally, seasonally, or over longer periods. The ionosphere data of reference 1, from which these graphs were derived, indicate that the diurnal and seasonal characteristics are regular and may in general be predicted. In addition, there is a large variation of ionization densities with the 11-year sunspot cycle, as indicated by the increase of maximum usable frequencies from 1933 to 1937. At the end of the year 1937 the sunspot cycle was near a maximum and is expected to return to a minimum about 1944. In a rough way these graphs may be used for corresponding times on the descending part of the sunspot cycle. Current ionosphere data are given by the National Bureau of Standards in its weekly ionosphere bulletins broadcast from station WWV each Wednesday, and in reports on the characteristics of the ionosphere at Washington, D. C., published each month in the Proceedings of the Institute of Radio Engineers.

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