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A TRANSFORMER METHOD FOR MEASURING HIGH ALTERNATING VOLTAGES AND ITS COMPARISON WITH AN ABSOLUTE ELECTROMETER

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ABSTRACT

Two entirely distinct methods available at the National Bureau of Standards for the measurement of the effective value of alternating voltages in the range from 10,000 to 100,000 volts are described. A comparison of these methods over a wide range of conditions involving 64 independent determinations has shown them to agree within 0.02 percent.

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I. INTRODUCTION

Electrical measurements are universally made in terms of the practical electromagnetic system of units, including even the rare, though sometimes important, cases in which electrostatic instruments are used. The values of electrical units are maintained with high accuracy at the several national standardizing laboratories by means of standard cells and wire coils of 1-ohm resistance. In any type of measurement the attainable accuracy tends to become less as the magnitude of the quantity measured becomes very much larger or smaller than that of the quantity embodied in the primary standard. This decrease in accuracy may be more marked in some types of quantity than in others, and may be expected to be large when the nature of the quantity requires the use of radically different types of measuring apparatus in different ranges.

An example of this situation is found in the measurement of high alternating voltages, for the growth of engineering enterprise has brought a demand for the accurate measurement of voltages at least 300,000 times that of a standard cell. The inherent limitations of available insulating media, particularly the atmosphere, and other factors, have led to the use of a number of different types of measuring apparatus to cover different parts of this total range. In view of this situation it seemed desirable that some national laboratory should develop an alternative method of measurement, which should be simple and direct and at the same time as different as possible from the conventional methods of the electrical engineer; and should compare the values obtained when the same voltage is measured by both methods. Any major discrepancy between the results would then be an indication of the probable existence of hitherto unrecognized errors in the conventional stepping-up method; and conversely a satisfactory agreement would add material assurance of freedom from such errors and cement a solid foundation for the future extension of the range of measurement along whichever line appeared to be the more feasible.

The alternative method selected as being applicable in the range from 10,000 to 275,000 volts and as giving promise of having adequate accuracy, was the use of the Brooks absolute attracted-disk electrometer. A brief discussion of this type of instrument is given in section IV (p. 329), and a more detailed account of its construction, operation, possibilities, and limitations will be found elsewhere.[1] Its advantage for the present purpose lies in the fact that it is completely different from the usual methods of measurement. The major differences are as follows: The full potential is applied directly to the instrument; its indications are primarily in absolute electrostatic units; and the transfer between alternating and direct quantities occurs in an electrostatic instead of in an electrodynamic system. The purpose of this paper is to describe the electromagnetic and electrostatic methods briefly and to state the results of the intercomparison of the two methods which has been carried up to 100,000 volts. Because some features of the electromagnetic process of measurement as regularly applied at this Bureau have not been published hitherto, this part of the procedure is described in somewhat more detail than is the electrostatic part, which has already been described.

In this work the two measuring equipments were supplied by the voltage developed at the secondary terminals of a 10-kva, 25-cycle, 400:1, oil-insulated, self-cooled, step-up transformer (designated as "10–B"). The primary of this transformer was supplied at about 250 volts, 60 cycles, from two line terminals of a 60-cycle, 3-phase, star-connected, 6-pole, 15-kva, revolving-field alternator, which, in turn, was driven by a direct-connected 30 hp, 230-volt d-c motor. The motor was supplied by a 48-ampere, 240-volt storage battery, and another similar battery supplied the alternator field circuits. By frequent attention to the brushes of both machines, the fluctuations in voltage could usually be kept within a few parts in 10,000, although abrupt changes of ten times this amount were occasionally noted. These fluctuations in voltage constitute the main limitation on the precision with which voltage measurements can be made by the electromagnetic method, and were of at least equal importance with the disturbances caused by air currents, which were a principal cause of uncertainty in the case of the electrometer measurements.

II. ELECTROMAGNETIC METHOD

Figure 1 shows the circuits used in the electromagnetic measuring process. The primary winding P of the voltage transformer T was connected directly across the terminals at which the voltage V_x to be measured was developed. The secondary voltage (about 120 volts)

[1] Herbert B. Brooks, Francis M. Defandorf, and Francis B. Silsbee. *An absolute electrometer for the measurement of high alternating voltages*, J. Research NBS **20**, 253 (1938) RP 1078.

of transformer T was applied to and measured by the special electrodynamic voltmeter Z of the suppressed-zero type, the series resistor of which is shown at R. This voltmeter in turn was calibrated at frequent intervals by comparing it by means of a specialized form of potentiometer D-S with a saturated standard cell C. This cell in turn was compared once a week with the reference group of standard cells maintained at this Bureau. The auxiliary current for the potentiometer circuit was supplied by the 200-volt battery B.

In most of the measurements reported herein the step-down transformer, indicated at T, figure 1, was a 25-cycle, 10-kva transformer designated as "10–A". Its primary winding was in 4 sections which

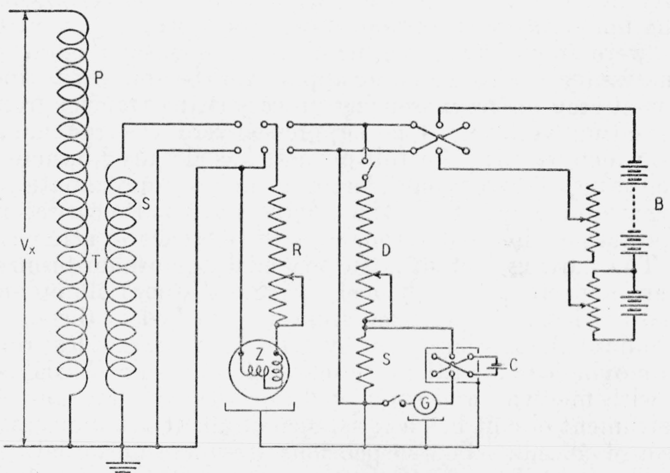

Figure 1.—*Circuits used in electromagnetic measurement.*

The primary P of the step-down transformer T is connected to the unknown voltage V_x. The secondary S supplies the reflecting electrodynamic voltmeter Z which has a series resistor R. This voltmeter is calibrated at frequent intervals using direct current from the battery B. The voltage then applied to the instrument is measured by the resistance voltage-divider D-S in terms of the electromotive force of the standard cell C.

by suitable series or parallel groupings gave voltage ratings of 100,000, 50,000, or 25,000 volts. Its secondary winding was in 2 sections and could be connected for ratings of 255 or 127.5 volts.

Studying now, in reverse order, the various steps in the measuring process, we may consider the probable accuracy of each step and the principle on which it is based. The comparison of the standard cell with the reference group can easily be made to 0.0001 percent. The cell was kept in a jacketed thermostatically controlled container and showed no changes exceeding 2 parts in 1,000,000 during the 6 months during which comparisons were in progress.

The first stage in the stepping-up process occurs in the potentiometer circuit. The ratio (about 100:1) of the d-c voltage applied to the voltmeter to the electromotive force of the standard cell is dependent upon the ratio of the resistances across which the respective voltages are applied. These resistances were measured at intervals of about 5 months and the accuracy with which the ratio of their values was determined was at least 0.001 percent. No secular drift as great as 0.001 percent was observed. The principle involved in obtaining the ratio is that the resistance of several coils in series is equal to the sum of their separate resistances. This principle is valid except as imperfect insulation may shunt part of the current around one or more of the series coils. The resistance across which the galvanometer and standard cell were connected was 80 ohms. The total resistance of the potentiometer circuit ranged from 3,000 to 15,700 ohms, according to the voltage to be measured. For precision resistance boxes in this range, leakage errors should be negligible unless the insulation is seriously impaired as by exposure to an atmosphere of high humidity. To avoid this latter condition refrigerating equipment was employed to condense atmospheric moisture and thus to maintain the relative humidity in the laboratory below 60 percent. The errors resulting from the temperature rise produced by the working current of 0.012 ampere were found by experiment to be less than 0.001 percent. When necessary, corrections were applied for the ambient temperature.

The next step in the measuring process is the transfer from direct to alternating voltage. The suppressed-zero electrodynamic voltmeter Z which was used for this purpose has already [2] been described. This reflecting electrodynamometer is made approximately astatic by using two moving coils. The moving system also carries a mirror, by which its position is determined with reference to a telescope and scale. The moving system is suspended by phosphor-bronze strips, which are kept taut by a helical spring. A short aluminum boom which swings between two fixed stops is attached to the spindle and serves to limit the motion of the system to an angle of less than 3°.

Each moving coil is in the magnetic field of a pair of fixed coils also wound with fine wire and connected in series with the moving coils. The instrument circuit has a resistance of about 110 ohms and an inductance of 58 mh. The suspensions are under an initial torsion so that a current of about 0.040 ampere is required to cause the moving system to move away from the stop and deflect to the arbitrary zero point of the scale. A five-dial resistance box is in series with the instrument. The instrument is calibrated by applying a known direct voltage of about the same value as the alternating voltage to be measured and adjusting the series resistance until the instrument deflects to the desired point on the scale. To eliminate any residual effect of the terrestrial magnetic field, the applied voltage is reversed, and a second adjustment of the series resistance is made. The mean of the two values of series resistance is then used. The voltage is a linear function of the scale reading over a narrow range so that by calibration at two points 10 cm apart, an accuracy of 0.002 percent is obtained at intermediate points. The scale distance is 6 m. In the deflection position the axes of the moving coils are at right angles to those of the fixed coils so that the mutual inductance is nearly zero and the torque per unit current is near a maximum.

When an alternating voltage is being measured the series resistance is adjusted to bring the reading between the calibration points.

The effect of the self-inductance of the windings depends upon the value of the series resistance R. It usually amounts to about 0.004 percent and a suitable correction is applied. It is assumed that the construction of the instrument is such that errors from eddy currents, capacitance between the turns and layers of the winding, and residual capacitances shunting the coils of the series resistor are all negligible.

[2] F. K. Harris. *A suppressed-zero electrodynamic voltmeter.* BS J. Research **3**, 445 (1929) RP105.

As a check on the correctness of these assumptions the instrument was compared at frequencies of 600 and 1,200 cycles per second with two portable electrothermic indicating instruments. One of these was of the expanding hot-wire type and the other of the thermocouple type. When used with resistors to give a range from 90 to 150 volts, the net average discrepancy between the suppressed-zero voltmeter and the mean of the electrothermic instruments, after correcting for the self-inductance of the former, was only 0.12 percent at 1,200 cycles per second, and was one-fourth of this amount at 600 cycles per second, confirming the theoretical prediction that the residual frequency errors in the suppressed-zero voltmeter varied directly as the square of the frequency. On this basis these errors at 60 cycles per second would be less than 0.001 percent.

The suppressed-zero instrument was calibrated with direct current before and after each measurement with alternating current. The average difference in these two calibrations for a group of 50 pairs of calibrations is 0.002 percent and may be taken as a measure of the inherent precision of the instrument. In order to obtain this precision in the readings made with alternating voltage, in spite of the fluctuations then present, the mean of 10 readings was used at each determination. The frequent calibration served to eliminate temperature errors and rendered the nickel-wire compensating coils, described in RP105, superfluous. They were therefore removed to obviate any spurious magnetic effects.

The second stage of the stepping-up process occurs in the voltage transformer T which has a maximum nominal ratio of about 784 : 1. This is best separated for purposes of analysis, however, into two component factors of 196 : 1 and of 4 :1, respectively. The accuracy of the first factor rests directly on an experimental calibration of the transformer made while its high-voltage windings were connected in parallel, while that of the second depends primarily on the principle that when a number of windings are connected in series their voltages are additive.

The method used for the experimental calibration of the transformer was that originally suggested by Sharp and Crawford,[3] using the high-voltage shielded resistor and auxiliary circuits as described by Silsbee,[4] and is that regularly employed in testing voltage transformers at the Bureau.

The circuit used in this method is shown in figure 2. When switch T is closed to the left the voltage applied to the primary winding P of the transformer under test is also applied both to the "working" circuit which includes resistors W and R_2 and to the guard circuit connected in parallel with it, which includes resistors W' and R_2'. The voltage at the terminals of the secondary winding S of the transformer under test is opposed to the drop of potential in the adjustable section R_2 of the working circuit. When the vibration galvanometer G_1 indicates a balance, the ratio of the primary voltage to the secondary voltage is equal to the ratio of the total effective a-c resistance of W (together with that of the primary of M) to that of R_2. The mutual inductor M serves to balance any quadrature component of voltage resulting from the phase angle of the transformer.

[3] C. H. Sharp and W. W. Crawford. *Some recent developments in exact alternating-current measurements.* Trans. Am. Inst. Elec. Engrs. **29**, 1517 (1910).

[4] Francis B. Silsbee. *A shielded resistor for voltage transformer testing.* BS Sci. Pap. **20**, 489 (1924–1926) S516.

The shielded resistor is so constructed as to minimize errors from the capacitance currents which might flow from portions of the working resistor to ground. The resistor W (of manganin wound on micanite cards) is divided into 27 sections of about 20,000 ohms (16 cards) each, connected in series. Each section is enclosed in, but insulated from, an oil-filled brass box which serves as a shield which is connected to a tap from the guard circuit W'. Each tap is at a point such that the potential of the shield is the same as that at the midpoint of the resistor section enclosed by it. The capacitance currents flowing from the outer surfaces of the shields to ground flow in the

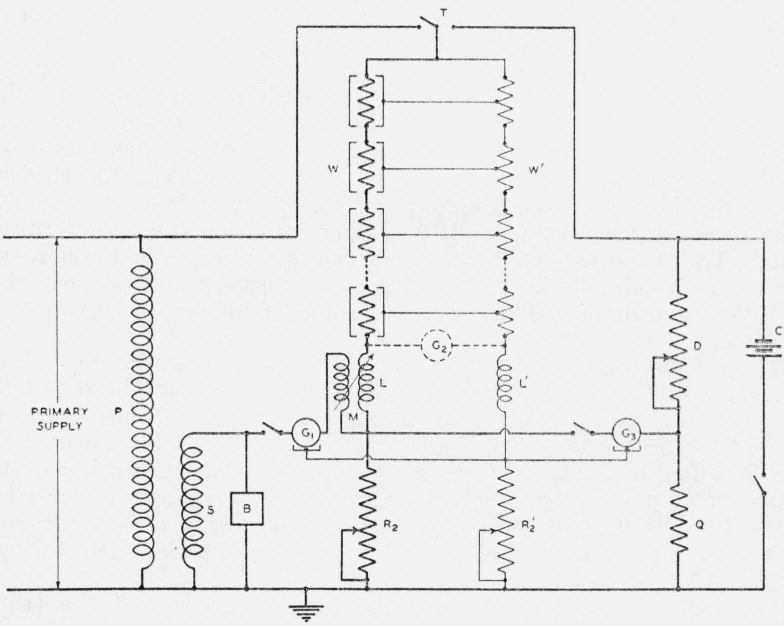

FIGURE 2.—*Circuit used in measuring the ratio and phase angle of voltage transformers.*

The shielded resistor W (in series with L and R_2) is connected in parallel with the primary winding P of the transformer under test. The voltage induced in the secondary winding S of the transformer is opposed to the sum of the voltage drop in the adjustable resistor R_2 and the voltage induced in the secondary of the mutual inductor M. When a balance has been obtained, as indicated by the a-c galvanometer G_1, switch T is thrown to the right and the ratio of W to R_2 is determined by direct current in terms of the known resistances of D and Q, using d-c galvanometer G_3.

guard circuit only and hence can produce only a second-order disturbance in the working circuit.

The capacitance between the parts of each section of the working resistor and its enclosing shield acts as a shunt in parallel with the section. As has been shown elsewhere,[5] the effective resistance and inductance of each section can be measured individually at a convenient voltage and with the shield at the potential of the midpoint of the resistor. This value will remain unchanged when the section is used in series with others, provided only that the potential of each shield remain equal to that of the midpoint of the resistor within it.

Because of the considerable heat developed (3 kw in the combined circuits) and of the resulting changes in temperature and resistance,

[5] See footnote 4.

the ratio of the resistance of W (including that of L) to that of R_2 is measured just after each a-c balance while the circuits are still at their operating temperature. This is readily done with switch T thrown to the right so that W, R_2, and the accurately known resistors D and Q, form a Wheatstone bridge supplied at about 30 volts by the battery C.

From this description of connections and procedure it is evident that the accuracy of the calibration of the step-down transformer depends on (1) the precision of the a-c balance; (2) the accuracy of the d-c bridge measurement; and (3) the accuracy with which the effective resistance which the working circuit of the shielded resistor offers to the alternating current is exactly the same as that which it offers to a much more feeble direct current. By using as the a-c detector a sensitive vibration galvanometer, an easily detectable deflection is developed by a lack of balance of only 0.002 percent. The accuracy of the d-c bridge is at least 0.01 percent. The shielded construction greatly reduces the voltage across possible leakage paths, and measurements with a 1,000-volt d-c source showed that all such leakage was entirely negligible Failure of the working circuit of the shielded resistor to have the same resistance to a large alternating as to a small direct current may arise as a result either of self-heating or of capacitance. The relatively large thermal storage capacity of the oil in which the resistance material is immersed, and the short time which elapses between the cutting off of the alternating supply and the attaining of the d-c balance, insure that the oil temperature is very closely the same for the a-c and the d-c balances. The alternating current of 0.05 ampere heats the resistance wire 3.3° C above the oil with a resulting change in its resistance of about 0.013 percent. However, by auxiliary measurements on a single section at larger currents this heating effect was definitely measured and appropriate corrections are regularly applied, with the result that the uncertainty introduced by temperature into the corrected ratio is less than 0.01 percent.

The possibility of a difference in resistance between direct current and alternating current as a result of capacitance effects may be studied on the basis of the theory of the high-voltage shielded resistor. This theory enables the effective resistance (and inductance) of the complete working circuit to be calculated from measurements made on the sections separately, provided the potentials of the shields are also known. The calculated relationships have been directly checked by experiments using two sections in series, and indirectly by a variety of experiments in which the observed changes in impedance resulting from known changes in the shield potentials were found to agree with the computed changes. The unavoidable capacitance from the shields to ground and the resulting effects on the shield potentials may however introduce a second-order error in the resistance, the magnitude of which can be estimated only roughly.

Because of such second-order effects, the use of a shielded resistor having a resistive [6] guard circuit is not practicable for precise work at voltages above about 30,000. The second-order errors produced by the stray capacitances are proportional to $\omega^2 R_w R_g C_w C_g$ where R_w and C_w are, respectively, the total resistance of the working circuit and the total capacitance between it and the shields, while R_g and C_g are,

[6] By using a set of autotransformers having a rating of 75 kva to supply the shield potentials, Weller (Trans Am. Inst. Elec. Engrs. **48**, 790, (1929)) has successfully operated a shielded resistor up to 132,000 volts.

respectively, the total resistance of the guard circuit and the total effective capacitance between the shields and ground. If the voltage range of a shielded resistor is extended by adding more sections, each of the last four factors will increase in proportion to the voltage and the error will increase as the fourth power of the voltage. If the resistances are held constant and the voltage is increased, the resulting increase in heating will require the use of larger cooling surfaces in proportion to the square of the voltage so that the error, proportional to the product C_wC_g, is in this case also increased roughly as the fourth power of the voltage. Measurements of C_w and estimates of C_g indicate that in the equipment used, the second-order error may be 0.008 percent. Unfortunately, it is not feasible to measure C_g with sufficient definiteness to enable a correction to be applied. If this indeterminate error increases as the fourth power of the voltage it is evident that it will soon become excessive. The use of a third circuit to guard the guard circuit appears to be impracticably complex. It therefore appears that some different measuring process must be introduced to extend the measurement from the 30,000-volt limit of the shielded resistor to the 100,000-volt limit of transformer 10–A.

The additional measuring process which we have used is an application to the case of high-voltage step-down transformers of what may for brevity be called the "series-parallel principle." This well-known principle [7] is that an instrument transformer, the primary winding of which consists of a number of substantially equal sections, has the same ratio correction factor (i. e., quotient of true ratio divided by nominal ratio) and the same phase angle when used with any of the possible series or parallel groupings of these primary sections.

The measuring process therefore consists in using the shielded resistor to determine the ratio of transformer 10–A at voltages up to 30,000 with the four sections of its primary winding connected in parallel and then subsequently using the transformer with these sections connected in series. By the series-parallel principle, its ratio should then be exactly four times that under the former condition. Departures from this ideal factor of 4 may conceivably result from either of two causes: (1) Magnetic inequalities in the sections of the primary winding, and (2) currents flowing through the electrostatic capacitance which exists between various sections of the windings, and between the sections and the case.

For the first of these causes to be effective the mutual inductances between the secondary winding and the several primary sections must be different and also the exciting current must distribute unequally among the primary sections when they are connected in parallel. In any well designed transformer in which the same number of turns is used in each high-voltage winding, both of these disturbing conditions are necessarily extremely small; and the change in ratio correction factor with change in connections, depending as it does on the product of these two types of dissymmetry, will be of a still higher order of magnitude, vanishing if either of its two factors is zero. In the particular core-type transformer used (10–A) each of the two legs is encircled by four high-voltage coils placed end to end. Each of the four 25,000-volt sections of the primary winding consists of an end coil on one leg and an inner coil on the other leg. Each of the two

[7] This principle has also proved of great value in extending the range of current transformers. See BS J. Research **11**, 98 (1933) RP580.

127.5-volt sections of the secondary winding consists of 38 turns on one leg and 37 turns on the other leg. The 38-turn coils are adjacent to and cover nearly the full length of the leg and the 37-turn coils are wound immediately over them.

The close magnetic equality of the primary sections is shown by the fact that the successive ratios observed when each section was used alone as a primary differed from their mean by less than 0.02 percent, and that the leakage impedances observed with the secondary short-circuited and each primary section in turn used alone as a primary, differed from their mean by less than 0.12 percent or 10 ohms in 8,900. These results show that this arrangement of coils gives an exceedingly close equalization of the magnetic effects of the several sections, but perhaps at the cost of somewhat increased capacitance effects.

The system of currents flowing in the various capacitances which exist both between the sections of the primary winding and between the primary winding and the secondary winding and ground is so complex that its effect on the ratio cannot be accurately evaluated, but rough approximations can be arrived at both by theoretical and by experimental methods. The total capacitance in the windings bushings, etc., of the particular transformer (10–A) used in this work, is so large that the leading current drawn by it more than offsets the lagging current required to magnetize the core, with the result that the no-load current of the transformer at 60 cycles per second leads the applied voltage. When the transformer is used, with its high-voltage winding as the secondary, the ratio is very materially affected by this leading current. At first sight, therefore, it might well be expected that such capacitance currents would have a very considerable effect on the step-down ratio of the transformer also. It appears, however, both from theory and from experiment that in this latter case such effects of capacitance on ratio are in reality surprisingly small. As a first step in considering the possible effects of capacitance on a step-down transformer it may be noted that: (1) A capacitance (as that of a bushing) which shunts the entire primary winding will have no effect on the ratio; (2) a capacitance which is connected between the midpoint and either terminal of the primary winding will have no effect on the ratio; (3) a capacitance which shunts more than half of the primary winding will have an effect which is greater than, and opposite in sign to, that of an equal capacitance which shunts less than half of the winding (hence the small capacitances spanning large sections tend to compensate for the effect of the larger capacitances between adjacent turns and layers); and (4) capacitance shunting any part of the primary winding will affect the ratio and phase angle only to the extent that the various parts of the primary winding are not sufficiently closely coupled to form an autotransformer of perfect (zero) regulation.

By making a number of simplifying assumptions the change ΔN in the ratio of a step-down transformer, produced by an assumed small capacitance C shunting a particular portion of the primary winding, can be estimated as $\Delta N = (1-\alpha)q\omega^2 CL_1$, where α is the coefficient of coupling between the section of the primary winding which is shunted and the remainder of it, and L_1 is the total (open-circuit) self-inductance of the primary. The numerical coefficient q depends on what fraction of the winding is assumed to be shunted and on the location of this particular fraction, and has as a maximum the value 0.09.

Measurements by the charge and discharge method indicated that the capacitance between the entire high-voltage winding and ground was about 1,700 $\mu\mu$f, and that the capacitance between the sides of adjacent sections was about 150 $\mu\mu$f. The coefficient of coupling ($\alpha = M/(L_1L_2)^{1/2}$) between the primary and the secondary windings departs from unity by only 0.0014 and the coupling between sections of the primary is presumably considerably closer than this.

Inserting these values into the expression for the change in ratio gives as an upper limit for the effect of shunting capacitance at 60 cycles per second the value 0.06 percent. Not only is this value very small compared to what might have been expected, but also it should be nearly independent of the flux density, and hence, for any one connection, independent of the voltage at which the transformer is operating. This follows because the factor $(1-\alpha)$, which is in effect the ratio of the leakage flux in the air space between the coils to the mutual flux in the iron core, varies inversely with the permeability, while the primary inductance L_1 is roughly proportional to the permeability.

Transformer 10–A is provided with two sections of low-voltage winding, which can be connected in series or in parallel to give rated secondary voltages of 255 or of 127.5 volts, respectively. By reason of the capacitance existing between the primary and the secondary windings there is, in normal operation, a certain capacitance current (of the order of 10 ma) flowing through portions, at least, of the secondary winding and returning to the primary circuit through the ground connections of the two circuits. This current affects the secondary terminal voltage by a few thousandths percent so that in general each of the six different combinations of connections of the primary and secondary windings yields a slightly different ratio correction factor.

The small magnitude of the capacitance effects predicted theoretically, led to the development of the following experimental procedure for extending the range of measurement by the step-down transformer beyond the limit set by the shielded resistor. The transformer is first tested with its four primary sections in parallel, using the shielded resistor up to its limit of 30,000 volts, which thus covers the full range of the transformer (up to 150 volts) with the secondary sections in parallel. These results may be plotted as in curve *AB* in the insert of figure 3 in which the ordinates are ratio correction factor (i. e., true ratio divided by nominal ratio) and the abscissas are secondary voltage (per coil). The primary sections are then connected in series-parallel and again tested up to the 30,000-volt limit of the resistor, which now corresponds to only 75 volts per coil on the secondary. These results will yield a curve such as *CD*. The primary connections are now changed to the series arrangement and again tested up to the 30,000-volt limit, which, in this case, corresponds to only 37.5 volts per coil on the secondary. These data will give a curve such as *EF*. For any given abscissa the flux density in the core is the same for all three curves and the difference in the ordinates is the result of capacitance currents. As mentioned above, it is to be expected that the differences in ratio correction factor will be nearly independent of voltage, and on this basis the upper curves may be extended at a constant distance above the lower as indicated by the dotted lines, and the ratio correction factor at 60,000 volts may be taken as that given by point *G* and that at 120,000 volts as that given by point *H*.

When the experimental process just outlined is performed on transformer 10–A, results such as those plotted along the curve *KL* in figure 3 are obtained. Here, to avoid confusion, points for only three of the six possible combinations of connections have been included and a curve has been drawn only for the parallel-parallel connection. The striking feature of these data is that the differences in ratio correction factor between the several connections are exceedingly small and only slightly exceed the precision of the measurements. In other words, the performance of the transformer is almost independent of capacitance effects. Similar measurements on a large number of other voltage transformers (usually of somewhat lower

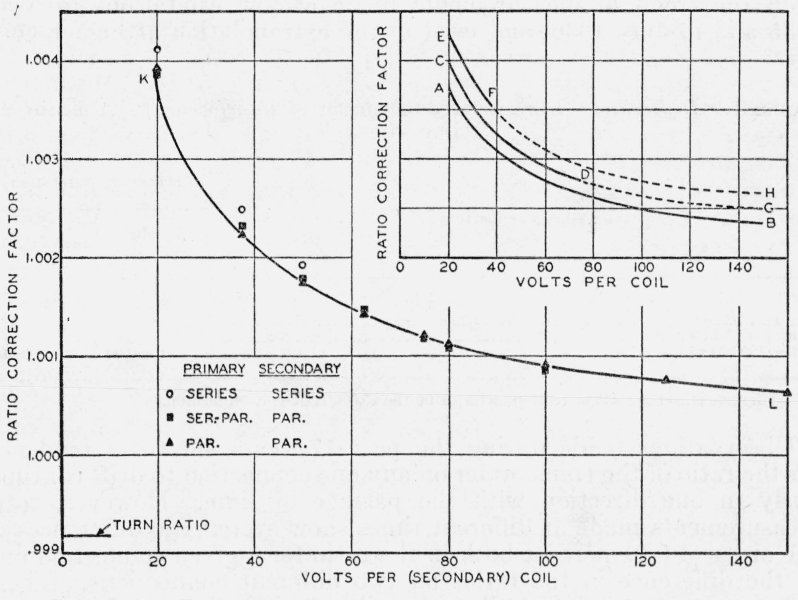

FIGURE 3.—*Ratio correction factor as a function of secondary voltage.*

Ordinates are the quotient obtained by dividing the true ratio of the transformer by the nominal ratio. Points shown by different symbols correspond to different connections of the primary or secondary windings as indicated. Insert shows the same variables plotted to an arbitrary scale in exaggerated form to illustrate the effects of capacitance and the process of "guided extrapolation."

range) have always shown only very small differences in ratio correction factor and phase angle when the primary windings were changed from parallel to series connection, and it appears that this process has very valuable possibilities as a means for extending the range of precision measurements at high voltages.

As a further confirmation experiments were carried out at 60 cycles with an additional capacitance of 1,900 $\mu\mu$f intentionally connected between the ¾ point and ground, and also, as a confirmation of the theory, between the ½ point and ground, on the primary of 10–A. These showed no change, when the capacitance was connected, exceeding the limits set by the sensitivity, which were 0.02 percent in ratio and 0.1 minute in phase angle.

From the foregoing discussion it would appear that the ratio of the transformer on the series connection can be relied upon to nearly the same accuracy as that to which it can be calibrated on the parallel connection.

The slight differences in ratio correction factor when the transformer is operating with different connections but at the same volts per coil and hence at the same flux density, are not as exactly constant as is suggested by the sketch inserted in figure 3 but show a tendency to decrease somewhat with increase in voltage for any one connection. For different connections table 1 lists the best mean values for the differences obtained by subtracting the ratio correction factors of transformer 10–A used in the parallel-parallel connection from those for the other five possible combinations of connections, as estimated from the trend of measurements made at low excitations in May 1935 and in July 1936, and used in the extrapolation at higher voltages.

TABLE 1.—*Differences* [a] *in ratio correction factor of transformer 10–A on various connections*

Primary connection	Secondary connection	
	Parallel (127.5 v)	Series (255 v)
	Percent	*Percent*
Parallel (25,000 v)	[a] 0	+0.003
Series-parallel (50,000 v)	−.005	−.005
Series (100,000 v)	±.000	±.000

[a] With respect to the ratio correction factor using the 25,000/127.5-volt connection.

Observations made during the past 17 years show no tendency for the ratio of the transformer on any one connection to drift continuously in one direction with the passage of time. However, ratio measurements made at different times show systematic differences of the order of 0.01 percent both in the ratio for a given connection and in the difference in the ratios on two different connections. These differences cannot be attributed to known effects of differences in temperature, for the effect of temperature on the resistance of the copper windings is easily computed and found to be negligible, and its effect on the magnetizing current was measured by observing this current at several temperatures and was also found to be negligible.

Tests have also been made before and immediately after the transformer core was magnetized by passing direct current through one winding. These also showed no permanent changes in performance. Nevertheless the differences in rato at different times though very small seem to be systematic and not attributable to the accidental errors of the measurement. Their origin is at present unknown.

In order to minimize the effect of these systematic differences on the comparison reported here, values of transformer ratio have been used which are interpolated between values observed in May 1935 and in July 1936, since most of the comparisons with the electrostatic method were made between December 1935 and June 1936.

The voltage assigned to the standard cell was of course in terms of the international volt as maintained at the National Bureau of

Standards since 1910. To convert the final result into absolute electromagnetic units it is necessary to multiply the observed value by the ratio of the international volt to the absolute volt. The factor [8] 1.00035 has been taken as giving this ratio with sufficient accuracy for the present purpose.

While the accuracy of each step in the measuring process just outlined is fairly high, the great number of steps involved gives rise to the possibility of a considerable error resulting from the cumulative effect of many small ones. To summarize the situation there is given in the first column of table 2 a list of the various sources of error and in the second column an estimate of the maximum error which each source might have introduced under unusually bad conditions or by reason of unrecognized systematic errors. In the third column is given a less pessimistic estimate of the uncertainty (estimated probable error) which very probably has been contributed from each source. The arithmetic sum of the second column then gives an indication of the greatest error which could have accumulated in the final result. The geometric sum (square root of the sum of the squares) of the items in the third column gives, on the other hand, a reasonable estimate of the uncertainty believed likely to be present in the electromagnetic measurement.

TABLE 2.—*Probable uncertainty and maximum error from various sources in the electromagnetic measurements*

	Maximum error	Probable uncertainty
	Percent	*Percent*
Comparison of standard cell with reference cells	0.002	<0.001
Changes in cell during use	.020	.002
Ratio of resistances in potentiometer circuit	.010	.001
Leakage shunting potentiometer	.010	.001
Self-heating of potentiometer [a]	.002	.001
Reading error, of d-c galvanometer	.002	.001
Inductance and capacitance of voltmeter [a]	.005	.001
Reading error of voltmeter	.010	.002
Temperature drift of voltmeter	.005	.001
Balancing error on alternating current	.002	.001
D-c measurement of shielded resistor	.010	.005
Temperature of shielded resistor [a]	.010	.002
Capacitance of shielded resistor [a]	.010	.002
Magnetic inequalities in transformer	.002	<.001
Capacitance of transformer [a]	.020	.010
Secular drift of transformer since test [a]	.020	.005
Arithmetic sum	0.140	
Square root of sum of squares		0.013

[a] Values given are estimated possible residuals not accounted for by the correction normally applied for this effect.

III. ELECTROSTATIC METHOD

The absolute attracted-disk electrometer used in these measurements is shown in figure 4 and a schematic cross section is given in figure 5. The hemispherical metal "dome" E at the top of the instrument encloses a sensitive balance D which serves to measure the force of attraction between the movable aluminum disk A, which hangs from one of its arms, and the lower plate B of the electrometer. The disk hangs with a small clearance (0.01 cm) in a central opening in the large upper plate C which carries the dome, and which serves as a

[8] This factor also includes the ratio, as determined in 1937, of the international volt as maintained at the National Bureau of Standards to the mean international volt.

guard ring to minimize the concentration of charge at the periphery of the disk. Suitable bushings adapt the opening in the guard ring to suit the diameter of the disk used. At the wide spacings (110 cm) needed for an applied potential difference of 275,000 volts for which the electrometer is designed, the diameters (100 cm) of the upper and of the lower plate would not be sufficient to prevent an appreciable bulging of the lines of force and a corresponding distortion of the electric field at the disk. To obviate this effect and to approximate the ideal uniform electric field throughout the region between the plates without having recourse to plates of excessive diameter (i. e., 6 m), a series of guard hoops H are placed as seen in figure 4 at equal intervals (2 cm center-to-center) between the planes of the two plates. These hoops are supported mechanically by pins inserted in three of the six fused-silica pillars which support the upper plate. The hoops are connected electrically to suitable equally spaced taps on a potential divider G. This divider consists of a stack of equal micanite capacitors which are connected in series with each other, the whole stack being in parallel with the electrometer. With this construction the electric field just inside the guard hoops is very nearly vertical and uniform, except for slight ripples caused by the finite thickness and spacing of the hoops. The ripples become inappreciable a short distance in from the hoops.

A mirror fixed to the balance beam reflects the image of a vernier scale on to a ground-glass scale located at the control station about 2.5 m away from the electrometer, and serves to indicate the position of the beam and of the disk.

A measurement of voltage with this electrometer involves two operations. First, with no voltage applied but with a known mass M_F resting on the pan on the disk arm of the balance, the zero reading S_0 is observed. Second, by suitable manipulation of insulating control rods the disk stem is clamped, the mass M_F is removed, voltage is applied, and the stem is released. A second reading S_v is then taken, the voltage being adjusted to make the two readings nearly equal. The effective value of the applied voltage is then obtained in absolute electromagnetic volts by the following working formula:

$$V_m = \frac{4v\sqrt{2M_F g}(c+h_d)}{(r_b+r_d)}\left[1+\frac{A_v-A_o}{2M_F g}+\frac{\gamma(S_v-S_0)}{M_F}-\frac{f(S_v-S_m)}{2q}+\frac{fh_m}{2}\right.$$

$$\left.-\sum_{k=1}^{k=\infty} C_k G_k' - \frac{(\epsilon-1)}{2}+\phi_1(h_d/r_d)+\phi_2(s/r_h)\right] \qquad (1)$$

Taking up in succession the various quantities appearing in this working eq 1, v is the ratio of the cgs electrostatic to the absolute practical electromagnetic unit of voltage. The work of Rosa and Dorsey [9] on this ratio gave a value 299.71 in terms of the international ohm as realized at the National Bureau of Standards in 1907. We have, however, used for v the value 299.805, which embodies a correction of their data for the effect of moisture in the air on the same basis as that used in the present work, and a correction for an adjustment, made subsequent to their publication, in the value of the ohm

[9] E. B. Rosa and N. E. Dorsey. *A new determination of the ratio of the electromagnetic to the electrostatic unit of electricity.* Bul. BS **3**, 533 (1907) S65.

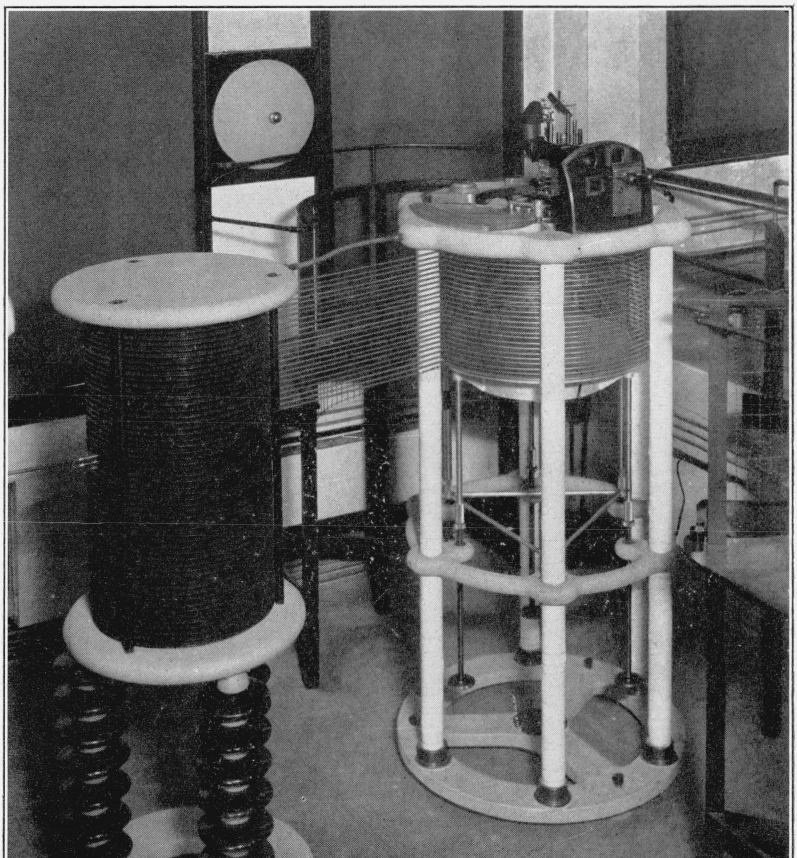

FIGURE 4.—*Brooks absolute electrometer.*

The electrometer proper is at the right; the balance mechanism visible above the upper plate is normally enclosed by a hemispherical metal cover. The capacitance potential divider is at the left.

as maintained at the Bureau, as well as for the ratio of the international ohm to the absolute ohm. For this latter ratio we have used 1.00048, which seems to be the best value from recent determinations.

The weight $M_F g$ of the mass removed from the disk arm when voltage was applied could be readily determined with ample accuracy (approaching 0.001 percent). The constant [10] of gravity g was taken as 980.08 cm/sec^2 and is probably accurate to 0.001 percent.

The separation c, as measured between the edge of the opening in the guard ring and the surface of the lower plate, is corrected by the

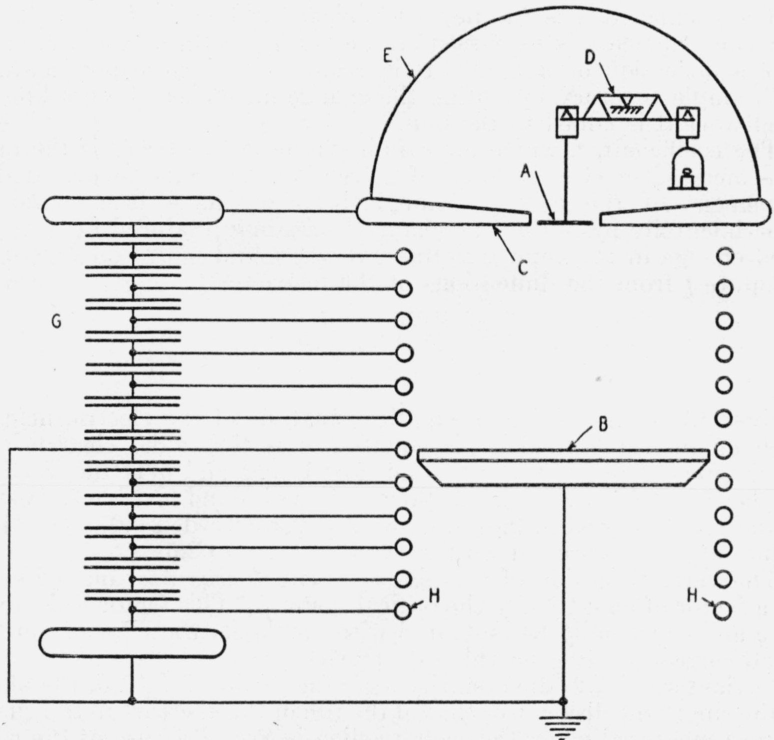

FIGURE 5.—*Schematic cross section of Brooks absolute electrometer.*

The attracted disk at A is suspended with its lower surface in the same plane as that of the guard ring C which is at the same potential. The electrostatic attraction is measured by the sensitive balance D. The horizontal hoops are each connected to a tap from the capacitance potential divider G. The two cross sections of each hoop appear as circles as at H–H. The capacitor units and hoops below the particular location at which the lower plate is used for a given voltage are short-circuited and grounded.

small difference h_d, which, as a first approximation, allows for the measured bulging upward of the average surface of the attracted disk above its rim.

The radii r_b and r_d of the aperture in the guard ring and of the disk, respectively, were measured to ± 0.0002 cm and were corrected by 0.0017 percent per degree C when the electrometer was used at temperatures other than that at which the radii were measured.

[10] Paul R. Heyl and Guy S. Cook. *The value of gravity at Washington.* J. Research NBS **17,** 805 (1936) RP946.

Of the correction terms inside the brackets in the working eq 1, A_v and A_0 symbolize the force (in dynes) exerted on the disk by air currents at the time the two readings with voltage on and with voltage off were made. These forces probably constitute the principal limitation on the accuracy of the instrument. Since they cannot be measured and allowed for, the standard procedure is to take a second zero reading immediately after the voltage had been removed. If the zero readings show that the air conditions are substantially constant, it is assumed that the mean of the values of A_0 before and after the run is equal to A_v, so that the term involving air currents is zero.

The coefficient γ is the net mechanical "stiffness" of the balance. For convenience γ is expressed as the grams on the pan required to give a deflection of 2 cm.[11] It is readily obtained experimentally with ample accuracy by noting the change in reading when a known small weight is added to the pan.

The coefficient, f, which appears in the next two terms is the relative increase in electric force of attraction per unit downward displacement of the disk. Although it might have been obtained experimentally by noting the change in reading produced by a measured change in the applied voltage, it was found more convenient to compute f from the dimensions of the instrument using the formula

$$f=\frac{2}{c}+\frac{2}{\pi r_d}\left\{2\log_e\left(\frac{2c(1-e^{\frac{-\pi r_d}{c}})}{(a^2+h^2)^{1/2}}\right)-\frac{a}{h}\tan^{-1}\frac{h}{a}\right\} \tag{2}$$

derived from Snow's [12] mathematical analysis of the electric field in such an instrument. In this equation a is the radial clearance in centimeters between the disk and guard ring and h is the vertical displacement (if any) between them. It was found feasible to keep a in the neighborhood of 0.01 cm, and h never exceeded 0.07 cm. The values of f in the present work ranged from 1.0 to 2.5.

The vertical motion of the disk is in effect magnified on the scale by a factor of about 80 by the optical system. This factor is denoted by q and can readily be measured with ample accuracy for use in the small correction term in which it appears.

In the term $fh_m/2$, the quantity h_m is the vertical height of the plane of the rim of the disk above that of the rim of the aperture in the guard ring as measured when the scale reading is S_m. Because of the relatively large value of f it is necessary to know h_m with great accuracy. An error of 0.02 mm in h_m will produce 0.1-percent error in V_M and hence the determination of h_m by means of the coplanarity microscopes used for this purpose becomes a process of major importance in the measurement.

The correction term $\sum_{k=1}^{k=\infty} C_k G'_k$ is inserted in eq 1 to allow for the departures of the potentials of the individual guard hoops from their ideal values. The potential distribution as observed with a 1,000-cycle bridge is expressed as a Fourier series having coefficients G'_k. Each C_k is a factor, calculated theoretically, which connects the distortion of the potential at the outer boundary of the instrument with the attractive force at the disk. Except at very large spacings, the

[11] The factor 2 enters because of the quadratic relation between force and voltage.

[12] Chester Snow. *Effect of clearance and displacement of attracted disk, and also of a certain arrangement of conducting hoops, upon the constant of an electrometer.* BS J. Research **1**, 513 (1928) RP17.

values of C_k decrease rapidly with increase in k so that in practice it is never necessary to compute more than three terms of the series. The values of C_k are such that an accidental error of 1 percent in the experimental determination of the relative potential of a single hoop will change the values of G_k' enough to affect V_M by only 0.01 percent. A constant systematic error of this amount on all the hoops, however, would affect the values of G_k' cumulatively and make an error in V_M of ¼ percent. The errors actually present in the relative hoop-potential measurement probably did not exceed 0.02 percent and hence were quite negligible in their effect on the measurement of voltage.

The next correction term in eq 1 involves the dielectric constant ϵ of the air between the lower plate and the attracted disk and its guard ring. This quantity varies slightly with atmospheric pressure and temperature, and even more with humidity. Data were taken at the time of each weighing from which ϵ could be deduced and an appropriate correction applied. The uncertainty in this correction is probably not more than 0.001 percent, although experimental data on the dielectric constant of mixtures of air and water vapor at 60 cycles per second do not seem to be available. The corrections used were based on measurements [13] made at higher frequencies.

The term $\phi_1(h_d/r_d)$ is the result of early experiments which indicated that slight departures from flatness of the disk produced surprisingly large differences in the voltage required for a given attractive force, which were not sufficiently allowed for by merely adding h_d to c in the numerator. A concavity such that the area of a disk was on the average only 0.002 cm higher than its rim caused an increase of 0.1 percent in the voltage required to produce a given force. A similar though less serious effect is produced by conicality of the guard ring. With the technique finally developed the disks and guard rings were flat to within 0.0003 cm (i. e., the mean radius of curvature exceeded 1 km) and the correction represented by the term $\phi_1(h_d/r_d)$ and determined by experiments on disks of different curvature, was applied to allow for the slight residual curvature of the best disks.

At the widest separations there is a possibility that the electric field in the spaces between the guard hoops may be distorted enough to affect the force on the disk. Such an effect is represented in the working equation by the term $\phi_2(s/r_h)$. In the work here reported the greatest separation used was 48 cm. At this separation auxiliary tests were made with the spacing between hoops increased to 4 cm and to 6 cm. Under these conditions the term $\phi_2(s/r_h)$ is increased, respectively, to about 4 and 9 times its value at the normal spacing (2 cm). The observed changes in the indication of the electrometer in these auxiliary tests as compared with normal conditions, were +0.043 and +0.142 percent, respectively, from which it is to be inferred that the results obtained at normal spacing and 48-cm separation were 0.010 percent too high. A correction of this amount has been applied to the six points for which this separation was used.

In the last two columns of table 3 there are summarized, respectively, for each of the quantities concerned in the electrostatic method the maximum error, which could have occurred under unusually bad conditions, and the uncertainty (estimated probable error) which may reasonably be expected to be present.

[13] Delcelier, Quinchant, and Hirsch. *Inductive capacity of gases and humid air.* Onde Electrique **5**, 189 (1926).

A comparison of table 3 with table 2 shows that the accuracy of the electrostatic measurements here reported is limited primarily only by the air forces and the uncertainty in determining coplanarity, while the electromagnetic measurement is subject to a large number of sources of much smaller errors, and in addition involves the uncertainty which results because the series-parallel principle is here applied to a transformer having considerable capacitance.

TABLE 3.—*Probable uncertainty and maximum error from various sources in the electrostatic measurements*

	Maximum error	Probable uncertainty
	Percent	*Percent*
Ratio of units, v	0.010	0.003
Mass of weight[a], M_F	.008	.001
Gravity, g	<.001	<.001
Separation, c	.005	.003
Radius, $(r_b+r_d)/2$	.005	.002
Air forces	.050	.005
Reading error, S_o	.010	.002
Reading error, S_v	.010	.001
Stiffness coefficient, γ	<.001	<.001
Force coefficient, f	.001	<.001
Coplanarity[a], h_m	.020	.005
Potential distribution[a], G_k'	.010	.002
Dielectric constant[a], ϵ	.005	.001
Flatness of disk[a], $\phi_1(h_d/r_d)$	.010	.003
Distortion of field between hoops[a], $\phi_2(s/r_h)$	.015	.002
Arithmetic sum	0.160	
Square root of sum of squares		0.010

[a] Values given are estimated possible residuals not accounted for by the correction normally applied for this effect.

IV. RESULTS OF COMPARISONS

The two methods of measurement outlined above were compared by using them simultaneously to measure the voltage developed at the secondary terminals of a step-up transformer. These comparisons were made under a variety of different conditions. The voltage measured ranged from 10,000 to 100,000 volts. The electrometer was used at separations (between disk and lower plate) of 6, 12, 24, and 48 cm. The system of guard hoops is unnecessary and was not used at the two smaller of these separations. Most of the measurements were made with the best 16-cm attracted disk, because this gives the greatest force for a given voltage and because the edge effects are less in proportion than with the disks of smaller diameter. However, three comparisons made while using the 10-cm disk are included.

The step-down transformer, 10–A, has two secondary windings, rated at 127.5 volts each, which can be used connected in series or in parallel. For each set of conditions comparisons were made using each of these two connections with a corresponding change in the ratio of the potentiometer circuit used to calibrate the electrodynamic voltmeter. The three possible connections of the sections of the primary, high-voltage winding, namely, in series for 100,000 volts, in series-parallel for 50,000 volts, and in parallel for 25,000 volts, were each used over the range corresponding to from 10,000 to 25,000 volts per section. Observations were made with both the series and the parallel connection of the secondary circuit over most of these ranges.

In all, 64 comparisons were made. The results of these are shown graphicly in figure 6, in which the relative differences between the values of voltage as determined by the two methods are plotted as ordinates. The points are arranged in the order in which the determinations were carried out, but the abscissas are not proportional to the time intervals between measurements. The separation between disk and lower plate in centimeters is indicated at the top of the figure and the connection of the transformer is indicated by the shape of the plotted symbol. An unweighted algebraic average of the differences between the two methods was found to be +0.006 percent, the plus sign indicating that the electrostatic method gave the higher result. The average taking the differences without regard to sign comes out ±0.010 percent. It may therefore be concluded that the two methods of high-voltage measurement are in agreement to within a few hundredths of one percent up to voltages of 100,000. The scattering is seen to be slightly less than the value 0.016 percent to be expected from combining the estimates in tables 2 and 3.

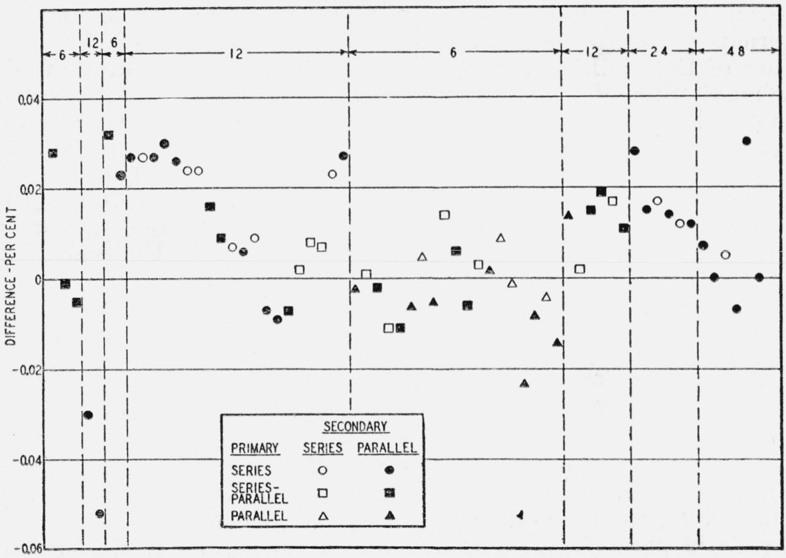

FIGURE 6.—*Graphic summary of results.*

Ordinates are the differences, expressed in percent, obtained by subtracting the value indicated for a given voltage by the electromagnetic method from the value indicated for the same voltage by the electrostatic method. The 64 independent comparisons are arranged in the order in which they were made. The first three points were obtained with a 10-cm disk in the electrometer, the remainder with a 16-cm disk. The separation between the electrometer plates is indicated at the top of the figure, and the manner of grouping the coils of the step-down transformer is indicated by the shape of the plotted symbol.

A more detailed study of the individual differences shows no systematic correlation between them and any one of the following variables: (a) Magnitude of voltage measured; (b) magnetic flux density in the core of the step-down transformer; (c) resistance of the d-c potentiometer; (d) separation of electrometer plates; or (e) diameter of electrometer disk. However, the differences depart somewhat from following a simple Gaussian distribution and give some indication of the occasional occurrence of abrupt changes of 0.01 or 0.02 percent in some factor entering the measurements. The origin of these changes has not yet been located, and they are so small as to be of negligible importance in the present state of the art of high-voltage measurement.

The agreement between the two methods is as good at the higher voltages, for which the values of transformer ratio were extrapolated, as at the lower voltages at which it could be measured directly. This fact is convincing evidence for the soundness of applying the series-parallel principle to transformers of the type here used.

It may be noted incidentally that if these measurements are regarded as a determination of the ratio of the electrostatic unit of electromotive force to the absolute volt, the mean of the results corresponds to the value 299.783 for this ratio. This value happens to be a trifle closer to the values which correspond to the more recently published results [14] for the velocity of light than is the value 299.805 derived from the work of Rosa and Dorsey.

V. CONCLUSION

From the foregoing it may be concluded that the measurement of alternating voltages of commercial frequency up to 100,000 volts by either of the methods here outlined is capable of an accuracy of a few hundredths of a percent. Inherent unsteadiness in the voltage supplied by rotating machinery at present sets a limit to the accuracy of any method of measurement, which limit is closely approached in the present work. If a need for still higher accuracy should arise in the future, there is some indication that it could be met by either method, provided the cause of the occasional slight but abrupt and systematic changes noted in the present work can be located and eliminated.

The agreement between the two methods in the range covered by the present work in spite of the presence of a large amount of capacitance in the transformer, gives good assurance of the soundness of the procedure here suggested of measuring the ratio correction factor of a transformer with its primary windings in parallel and using this value to extend the voltage range with the windings in series.

The use of a capacitance potential divider in place of the shielded resistor offers good promise of extending, much beyond the present 30,000 volts, the range over which a transformer can be tested with its coils in parallel. The application of the series-parallel principle to step-down transformers of still higher voltage rating therefore seems to offer almost unlimited possibilities for the precise measurement of the effective value of high voltage.

There is every expectation that the absolute electrometer, when installed in a laboratory affording sufficient clearances, will be capable of measurement with an accuracy of at least 0.1 percent up to the voltage, 275,000, for which it has been designed. Hence, if the series-parallel principle is found in future to be valid up to this voltage, it should be permissible to extend the application of this principle to still higher voltages without needing further checks by another electrometer of still higher range.

WASHINGTON, November 4, 1937.

[14] Michelson. Astrophys. J. **65**, 1 (1927) 299.796
Karolus and Mittelstaedt. Physik. Z. **29**, 698 (1929) 299.778
Michelson, Pease, and Pearson. Astrophys. J. **82**, 26 (1935) 299.774
Anderson. Rev. Sci. Instr. **8**, 239 (1937) 299.764