PRISM REFRACTOMETRY AND CERTAIN GONIOMETRICAL REQUIREMENTS FOR PRECISION

By L. W. Tilton

ABSTRACT

A surprisingly small gain in the accuracy of absolute measurements of index of refraction has been made since the experiments of Fraunhofer. The classical method of minimum deviation is fundamentally important and for solid or liquid media the most sensitive of absolute methods.

To obtain an accuracy of ± 1 unit in the sixth decimal place of index by this method, angles must be measured to fractions of a second and it would seem imperative to use a fully calibrated circle. Frequently, however, correction curves are based on comparatively few of the numerous graduations on a circle and relate only to the periodic errors of low frequency. In such cases it is still necessary, therefore, to resort to repetition on various parts of the circle in order to eliminate accidental errors and those of high frequency. On the other hand, with four or more equidistant microscopes and the proper advancing of the circle between observations, both accidental and periodic errors may be eliminated simultaneously and it becomes possible, even on uncalibrated circles of fairly high grade, to attain an accuracy approaching the order required.

Important steps in using any circle are (1) the slight displacements of the microscopes from their equidistant locations in such manner that the average readings are free from periodic errors of the degree subdivisions, and (2) frequent determinations of the error of the "runs" of the micrometers under the exact conditions of use. By these means the special error-free use of an uncalibrated circle is satisfactorily extended, so that it covers not merely a few cases of little more than theoretical interest but also many of the most important angle measurements required in minimum-deviation refractometry.

To minimize the effects of torsional strains in the male and female members of a conical bearing it is advisable to adhere strictly to uniform manipulative procedures in carefully planned observational programs.

CONTENTS

I. INTRODUCTION

Recent progress relating to the effects of heat history on the optical properties of glass [1] has resulted in the application of heat treatment to effect the removal of certain heterogeneities in glass.[2] Consequently, it now seems that optical-glass manufacturers can, by the elimination of temperature gradients in annealing furnaces, secure a greater degree of homogeneity within each melt than was formerly thought possible. Furthermore, an adequate control of annealing temperatures, together with the use of "annealing equilibrium coefficients of index of refraction," may even reduce appreciably the range of variation in the optical quality of annealed glass from melt to melt of the same type. Every advance in the uniformity of the properties of optical glass is of great importance because it furthers standardization in the manufacture of optical systems, and complete standardization is one of the chief requirements for American leadership in any industry.

Increased uniformity of refractive media results at once in a demand for more numerous and more accurate determinations of the index of refraction and of the partial dispersions than have been warranted in the past. Even at the present time optical-glass catalogues are customarily issued with indices to five decimals, and at least one firm makes some use of the sixth for some of the partial dispersions. Consequently, it is evident that a careful consideration of the conditions necessary for accuracy in the measurement of indices to six places of decimals is important.

This survey of requirements was in fact occasioned by the necessity of developing improved equipment for measurements on transparent media submitted to this bureau by commercial organizations which have already experienced the need of increased accuracy in index measurements. Such measurements were also required in providing adequate standards of index for the testing of commercial refractometers, some of which are now made to permit readings to the fifth decimal place and are used [3] in chemical analysis and in other work, both scientific and technical.

Before proceeding with the design of new or improved auxiliary apparatus, it seems desirable to investigate all the requirements for sixth decimal place accuracy in prism refractometry. Obviously, it is useless to proceed far toward satisfying any one requirement without giving consideration to all, and such a review must be made with

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[1] A. Q. Tool, L. W. Tilton, and E. E. Hill, Meeting Opt. Soc. Am., Ithaca, N. Y., 1925; Abstract in J. Opt. Soc. Am. and Rev. Sci. Inst., 12, pp. 490-491; 1926.

[2] L. W. Tilton, A. N. Finn, and A. Q. Tool, B. S. Sci. Papers (No. 572), 22, pp. 719-736; 1927-28. See also J. Amer. Ceramic Soc., 2, pp. 292-295; 1928.

[3] See, for example, D. A. Coleman and H. C. Fellows, United States Department of Agriculture Bulletin No. 1471, pp. 25-26; March, 1927; C. F. Hottes and W. A. Huelsen, Science, 65, p. 576; 1927; Frank Urban and V. W. Meloche, J. Am. Chem. Soc., 50, pp. 3003-3009; 1928.

sufficient detail and comprehensiveness to serve as a basis for the efficient direction of the effort to be expended in seeking higher accuracy.

An examination of the literature discloses comparatively few measurements of indices of solids for which any precision in the sixth decimal has been claimed, and, indeed, doubt has been expressed [4] concerning the extent to which all of the difficulties have been overcome in any one set of measurements. In view of this uncertainty, and also on account of the paucity of recent attempts at accurate refractometry with the spectrometer, it has been thought desirable to present, somewhat in detail, the course followed in this investigation.

The requirements which need discussion may be classified as pertaining to either (1) goniometry or (2) the knowledge and control of the conditions which affect the velocity of light in various media. The present paper, being introductory in character, deals only with a portion of the first of these categories, and it refers particularly to minimum-deviation spectrometry of the visible spectrum because the experimental work in progress in this refractometric laboratory is limited chiefly to that field.

The notation used is explained in the text, but for further convenience of reference the definitions of the various characters are summarized here as follows:

$A \equiv$ the refracting angle of a prism;

$A_a \equiv$ the refracting angle of a prism for use in the autocollimating method;

$a \equiv$ the temperature coefficient of relative index per 1° C.;

$\alpha \equiv$ the angular displacement of one microscope from its customary position diametrically opposite a second miscroscope;

$D \equiv$ the angle of minimum deviation produced by a prism;

$D_a \equiv$ the (essentially positive) angle of deviation of incident or emergent light at the refracting face of a prism when using the autocollimating method;

$d \equiv$ the number of subdivisions per degree of arc on a spectrometer circle;

$E \equiv$ the angle of refraction at the emergent face of a prism;

$e \equiv$ the angular eccentricity of a spectrometer circle;

$F \equiv$ the angle between emergent rays in the Féry-Martens autocollimating method and in the grazing incidence method;

$m \equiv$ the number of angularly equidistant microscopes for use in making readings on a spectrometer circle;

$N \equiv$ the number of degree subdivision scale graduations in one complete high-frequency error period;

[4] L. C. Martin, Opt. Measuring Instruments, p. 129 (London); 1924.

$n \equiv$ the relative index of refraction of a medium;
$n_0 \equiv$ the relative index of refraction of a medium at 0° C.;
$n_D \equiv$ the relative index of refraction of a medium for sodium light, wave length, 5,893 A;
$r \equiv$ the number of repetitions of a given angular measurement in an observational program;
$T_A \equiv$ the tolerance, in measuring refractive angle, which corresponds to an error of $\pm 1 \times 10^{-6}$ in the measurement of refractive index;
$T_{2D} \equiv$ the tolerance, in measuring (double) deviation, which corresponds to an error of $\pm 1 \times 10^{-6}$ in the measurement of refractive index;
$t \equiv$ the temperature in degrees C.;
$x \equiv$ the minimum number of complete traverses on a circle or on its equivalent "perfect arc" which permit the elimination of scale-graduation errors in the measurement of a given angle by a method of repetitions.

II. CRITICAL REVIEW OF SIXTH DECIMAL PRISM REFRACTOMETRY

Fraunhofer [5] measured and published indices of refraction to the sixth decimal place, and a precision in index to this degree has figured prominently in several investigations [6] concerning objectives made from his glasses. The original index tables by Dutirou [7] also contained six decimals, and some indices of Merz [8] glasses have been published to the same order of precision. Aside from questions of instrumental and manipulative errors, however, the general inadequate reference to working conditions [9] makes it unlikely that even the fifth decimal place is accurate in any of this early work. An inspection of Fraunhofer's indices for the flint melt No. 23 (n_D = 1.633666), for which results were obtained with prisms of 60° and 45°, confirms this view. Comparing values for the solar lines B, C, D, E, F, G, and H, the differences in the indices of these prisms in units of the sixth decimal place are +32, +18, +1, −49, −24, −1, and +6, respectively. Although this series shows the presence of a large systematic error in the fifth decimal, it suggests, nevertheless, a

[5] Joseph von Fraunhofer, Denkschriften d. K. Akad. Wissenschaften zu München, 5, pp. 193–226; 1817; also Joseph von Fraunhofer's Gesammelte Schriften, p. 30 (München); 1888.

[6] For references, see S. Czapski u. O. Eppenstein, Grundzüge der Theorie der Optischen Instrumente, p. 565 (Leipzig); 1924.

[7] L'abbé Dutirou, Annales de Chimie et de Physique (3), 28, pp. 176–210; 1850, measured 18 glasses for each of 7 Fraunhofer lines, but the corrected values for the indices, pp. 501–502 of the same volume, are in every case reduced to five decimal places.

[8] Sigmund Merz, Zeitschrift f. Instrumentenkunde, 2, pp. 176–180; 1882.

[9] Dutirou records temperatures for all data, and Fraunhofer does also for some observations, but air pressures are lacking in all cases.

remarkably high degree of precision for measurements made over 100 years ago.

Two noteworthy series of measurements of this kind, published half a century ago, seem quite worthy of serious consideration as examples of accurate spectrometry. The first, by Hastings,[10] gives relative indices and temperature coefficients of five glasses for 10 or 11 wave lengths in the visible spectrum, the data being given for a temperature of 20° C. and an air pressure of 30 inches of mercury. The measurements were made by the method of minimum deviation, using a spectrometer with a circle of known periodic errors and provided with two microscopes reading directly to two seconds of arc. This work was very favorably mentioned by Müller [11] who a few years later was publishing his own more extensive results of the same character, the second of the two noteworthy series above mentioned. Müller gives full observational data on five glasses for from 7 to 12 wave lengths at room temperatures ranging from −6 to +28° C. Two spectrometers were employed, the microscopes reading directly to 5 seconds on one and to 2 seconds on the other, and correction formulas were used for the periodic errors of scale and screws. The results were corrected to a standard air pressure of 760 mm, and the indices, n_0 for 0° C., together with temperature coefficients, a, were determined for each wave length by least squares solutions of the equations, $n=n_0+at$. The probable errors of the results so adjusted were confined to the sixth decimal place, excepting for one line at the extreme violet end of the visible spectrum. The majority of the individual observations depart from the computed values by less than one unit of the fifth decimal place. Some confirmation of the precision of this work is also to be found in the highly satisfactory results which Hartmann [12] obtained when using these indices in a test of his dispersion formula.

The importance of air-pressure corrections is amply demonstrated by Hastings and by Müller in these precise measurements, but it seems to have escaped the notice of some later workers in this and related fields. Several years later the older measurements by Hopkinson [13] were mentioned by Taylor [14] as being perhaps the most accurate and reliable determinations of index of glass. Yet the results referred to, although given to six decimals, were obtained on a spectrometer with verniers reading to 10 seconds, and no mention was made of pressure or even of temperature. In the sixth decimal place data by Dufet [15] no pressures are given, and the last place is admittedly

[10] C. S. Hastings, Am. J. Sci., **15**, pp. 269–275; 1878.
[11] G. Müller, Publicationen des Astrophysikalischen Observatoriums zu Potsdam, **4**, pp. 149–216; 1885.
[12] J. Hartmann, Publicationen des Astrophysikalischen Observatoriums zu Potsdam, **12**, appendix, pp. 1–16; 1902.
[13] J. Hopkinson, Proc. Royal Soc., London, **26**, pp. 290–297; 1877.
[14] H. Dennis Taylor, Monthly Notices, Royal Astron. Soc., **54**, p. 69; 1893.
[15] H. Dufet, J. de Physique (3), **1**, pp. 163–177; 1892.

of little value. Conroy [16] worked on the index of water to six decimals but recorded no pressures, and since he worked at temperatures different from those of the room there is, furthermore, some question as to the temperature of the reference medium. Paschen [17] also, in connection with dispersion investigations on rock salt, sylvite, and fluorite, has published relative indices to the sixth decimal without an accompanying statement concerning air pressures.

Gifford's early work [18] on 27 glasses, for which sixth decimal indices were published for many wave lengths, including 13 in the visible region, is often referred to and used for purposes requiring exact values for the dispersion of glasses. But, aside from the lack of pressure observations, only one microscope was used on the circle. Furthermore, an unsymmetrical use of apertures may have resulted from the practice of translating the prism to vary the intensity of the slit image specularly reflected from the polished third side of the prism. Any one of these three factors could have caused errors affecting the fifth decimal place.

Pressure effects have received attention more recently in England. Gifford,[19] publishing indices for seven additional glasses, mentions an attempt to measure the effect; Guild [20] wrote a note on the subject; and Gifford [21] then published, for seven more glasses, indices corrected to standard conditions of 15° C. and 760 mm pressure. These last data by Gifford, and those previously given by Hastings and by Müller, and to which reference has already been made, constitute practically [22] all the published indices of optical glass for which both temperature and pressure conditions are precisely stated. The list may be extended to cover all goniometrically determined indices of solid media, it is thought, by the addition of references to Hastings'[23] indices of iceland spar to six decimal places, to the seventh place indices of quartz by Macé de Lépinay,[24] to Langley's [25] bolometrically determined indices of rock salt and of fluorite, to the sixth decimal place measurements on fluorite and on quartz made at the Reichsanstalt by Schönrock,[26] and also to the latter's seventh [27] decimal work

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[16] Sir John Conroy, Proc. Royal Soc., London, **58**, pp. 228–234; 1895.

[17] F. Paschen, Annalen d. Physik, **26**, pp. 127, 134; 1908; **41**, pp. 670–672; 1913.

[18] J. W. Gifford, Proc. Royal Soc., London, **87**, pp. 189–193; 1912.

[19] J. W. Gifford, Proc. Royal Soc., London, **91**, pp. 319–321; 1915.

[20] J. Guild, Trans. Opt. Soc., London, **22**, pp. 153–155; 1920. See also Proc. Phys. Soc., London, **30**, p. 188; 1918.

[21] J. W. Gifford, Proc. Royal Soc., London, **100**, pp. 621–626; 1921–22.

[22] Some index data, measured by the writer, have been published, chiefly in a paper by L. W. Tilton, A. N. Finn, and A. Q. Tool, loc. cit.

[23] Charles S. Hastings, Am. J. Sci., **35**, pp. 60–73; 1888.

[24] J. Macé de Lépinay, Annales de Chimie et de Physique, (7), **5**, pp. 216–231; 1895; (7), **11**, pp. 112–116; 1897.

[25] S. P. Langley, Annals of the Astrophys. Observatory of the Smithsonian Institution, **1**, pp. 219–237; 1900.

[26] O. Schönrock, Zeitschrift f. Instrumentenkunde, **30**, pp. 185–186; 1910; **33**, p. 161; 1913; **34**, pp. 191–192; 1914.

[27] O. Schönrock, Zeitschrift f. Instrumentenkunde, **40**, pp. 93–96; 1920; **41**, pp. 103–105; 1921; **48**, pp. 275–276; 1928.

on the same materials at specified temperature, pressure, and humidity. It may be added that absolute indices of quartz for definite temperatures have been determined interferometrically by Macé de Lépinay and Buisson [28] to the seventh decimal place and also by Pérard [29] to the seventh and eighth places.

With reference to decimal places beyond the sixth, it should be mentioned that Macé de Lépinay's seventh place results, obtained by the use of a spectrometer, seemed precise to about one unit of the sixth but were later adjudged [30] too low in absolute value by five or more such units when compared with the interferometric measurements just referred to as made by Macé de Lépinay and Buisson. The opinion that the limits of accuracy in prism refractometry are reached at something scarcely to be called certainty in the fifth decimal place seems somewhat disturbing, especially if, as reported, it was held by one with Macé de Lépinay's experience. The explanation that this discrepancy between the results from the two methods is due to the extreme difficulty in obtaining accurate collimation with the spectrometer, appears untenable to the writer in view of the liberal tolerances which are permissible in collimation when making measurements of minimum deviation.[31] It is, in fact, considered as probable that the real cause was an unsuspected but avoidable systematic error in deviation measurements which, for example, could have occurred to approximately the extent in question through a variation in certain mechanical strains such as those mentioned in Section VII of this paper.

At and near the center of the visible spectrum, Schönrock's seventh decimal indices of flourite and quartz prisms, when compared with the computed values by the adjusted dispersion formulas, seem almost consistent to the nearest sixth decimal place. An intercomparison of Pérard's interferometrically obtained values for the 10 and 20 mm quartz end standards reveals inconsistencies in the sixth decimal, but the indices for the 50 and 100 mm standards seem consistent to within a few units of the seventh decimal. Only Schönrock and Pérard seem to have taken account of water vapor in the air, the variations in which, if neglected, completely vitiate the seventh decimal place of index and may affect the sixth place slightly. Pérard mentions the lack of carbon dioxide determinations which, in laboratory air, are quite important in the eighth place.

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[28] J. Macé de Lépinay and H. Buisson, Annales de Chimie et de Physique, (8), 2, pp. 78–108; 1904.

[29] Albert Pérard, J. de Physique (6) 3, pp. 252–260; 1922; (6), 8, pp. 344–352; 1927.

[30] J. Macé de Lépinay and H, Buisson, loc. cit., p. 108. See also Traveau et Memoirs, Bureau International des Poids et Mesures, 10 (Determination du Volume du Kilogramme d'Eau, Lépinay et al), pp. 9–10; 1910.

[31] Defective prism-angle measurement is practically eliminated as a contributory source of error in Macé de Lépinay's indices, because he averaged results involving adjusted values for all three refractive angles. The subject of collimation receives quantitative treatment in a forthcoming paper relating to prism quality and adjustment.

III. CONCERNING METHODS

The prism methods of measuring indices of refraction are the oldest and are fundamentally important. The interferometric methods are commonly used when dealing with gases, but they require in practice, for work on solids and liquids, previously determined index values of fairly high accuracy to permit a determination of the order of interference by the method of coincidences. Pérard, for his initial data, made use of the published indices of quartz to which he fitted a dispersion formula; and Peters,[32] using an interferometric method in measuring the index of glass at high temperatures, required preliminary spectrometric values of index correct within two units of the fourth decimal place, and of dispersion correct within two units of the fifth decimal place. Furthermore, these interferometric methods require two or more complete determinations on samples of progressively increasing thickness, to reach an accuracy of one unit of the sixth decimal place, and these successive samples must be constant in index to a degree not always found in the case of glass. Consequently, it is obviously desirable to increase the accuracy of spectrometer measurements of index and limit the use of the interferometer in such work to those cases where the use of the former instrument is unsuitable or inadequate. Improved quality in index determinations by the spectrometer is also of importance in lessening the labor involved in the application of interferometry to such special cases.

Guild [33] names, as the two methods of major importance in the precise refractometry of prisms, the classical minimum-deviation method of Newton-Fraunhofer and the grazing incidence (critical angle) method of Euler-Kohlrausch. He shows, in general, that the former possesses advantages in sensitivity. Relative ease of computation also favors the classical method, according to which the index, n, is given in terms of refracting angle, A, and angle of minimum deviation, D, by the formula

$$n=\frac{\sin\frac{A+D}{2}}{\sin A/2} \tag{1}$$

of Newton, as compared with the second method, for which there is the corresponding expression

$$n^2=1+\left(\frac{\cos A-\sin E}{\sin A}\right)^2 \tag{2}$$

of Kohlrausch, where E is the angle made by the emergent ray with reference to the face normal, positive if toward the refracting edge

[32] C. G. Peters, B. S. Sci. Papers (No. 521), 20, p. 640; 1926.

[33] J. Guild, Proc., Phys. Soc., London, 33, p. 136; 1921; or Nat. Phys. Laboratory, Collected Researches, 17, p. 9; 1922.

of the prism. Moreover, the character of the settings in minimum-deviation measurements compares favorably with the asymmetrical conditions necessary in critical-angle refractometry. Thus it is, perhaps, somewhat significant that all of the above-mentioned attempts at precise prism refractometry were carried out by the Newton-Fraunhofer method.

It should be mentioned, however, that for media in the form of prisms having low values for their refracting angles (less than approximately 65° at index 1.3 and 30° at 1.9), the tolerances in deviation measurements by the grazing incidence method are actually larger than those for the minimum deviation method. Furthermore, if one determines the (positive) angle, F, between the emergent rays from both faces of a refracting angle, the $-\sin E$ of equation (2) may be replaced by $+\cos\left(\frac{F-A}{2}\right)$ and then, by taking the partial derivative of n with respect to A for each form of the equation in turn, it is found that the tolerances in measuring refractive angle are not as unfavorable in this "double" variation of the grazing incidence method as in the above-mentioned case discussed by Guild.

In commercial refractometry, the methods of predominant importance are those of measuring a sample in contact with a medium of known higher index. A special case was used by Wollaston, but the names of Abbe and of Pulfrich are more often associated with some of the important variations. These methods offer interesting possibilities of obtaining great sensitiveness on special instruments of carefully determined constants, as is shown by discussions such as those of Guild,[34] Simeon,[35] Smith,[36] and Schultz,[37] but they do not lead to absolute or fundamental measurements and so have no place in the present discussion.

Variations of Abbe's autocollimating method of prism refractometry are finding some favor for precise work and his method should be compared with that of minimum deviation. The index is obtained by the equation

$$n=\frac{\sin E}{\sin A_a} \tag{3}$$

of Descartes, where E, although observed directly, may be expressed as A_a+D_a, the subscript, a, referring to the autocollimating method as distinguished from that of minimum deviation, and the refractional deviation, D_a, being considered in this case as an essentially positive angle equal in absolute value to the change in direction of the

[34] J. Guild, Proc., Phys. Soc., London, **30**, pp. 157–189; 1918; or Nat. Phys. Laboratory, Collected Researches, **14**, pp. 273–296; 1920.

[35] Frederick Simeon, Proc., Phys. Soc., London, **30**, pp. 190–203; 1918.

[36] T. Smith, Nat. Phys. Laboratory, Collected Researches, **14**, pp. 297–300; 1920.

[37] H. Schultz, Zeitschrift f. Technische Physik. **3**, pp. 90–93; 1922; Zeitschrift f. Instrumentenkunde, **48**, pp. 26–30; 1928.

incident or emergent light at the refracting face of the prism. For any prism of $A_a°$ which can be used in Abbe's method it is possible to use one of the same medium having an angle $A = 2A_a°$ in the method of Fraunhofer, and it is customary to use in the autocollimating method a prism of 30° with one face silvered. When the incident and emergent rays coincide in Abbe's method, the conditions which exist are then very closely related to those which obtain for the minimum-deviation setting of a 60° prism in the classical method. In fact, a comparison, assuming $A = 2A_a$, shows that $E = A/2 + D/2$.

Abbe's method automatically eliminates error due to prism table orientation. It offers those advantages in design relating to compactness of instrument and to a lightness of weight of all moving parts, the latter being particularly desirable since it reduces torsional and other mechanical errors. (See Sec. VII.) At the same time a telescope of large aperture may be used without resorting to an unfavorable aperture ratio. Furthermore, the use of a tangent screw for differential measurements renders such a spectrometer especially efficient for temperature coefficient and dispersion determinations.[38] This type of instrument also has certain very important merits in connection with a solution of the problems of temperature control; and, finally, the simplicity of the computations and the smallness of the required sample may be mentioned.

With respect to permissible errors in angular measurements, however, the Abbe method is decidedly inferior to that of minimum deviation, as is shown by taking the partial derivatives of n, as defined by equations (1) and (3), with respect to A, D, A_a, and E. This procedure with equation (1) yields equations (4) and (5) of the following group; and, similarly, the respectively corresponding expressions (6) and (7) are obtained from equation (3).

$$\frac{\partial n}{\partial A} = -\frac{\sin D/2}{2 \sin^2 A/2} \tag{4}$$

$$\frac{\partial n}{\partial D} = +\frac{n}{2 \tan \frac{(A+D)}{2}} \tag{5}$$

$$\frac{\partial n}{\partial A_a} = -\frac{\sin E \cos A_a}{\sin^2 A_a} \tag{6}$$

$$\frac{\partial n}{\partial E} = +\frac{n}{\tan E} \tag{7}$$

Thus the tolerances in the measurement of refracting angles are $2 \sin E \cos A_a \operatorname{cosec} D/2$ times as great in the minimum-deviation method as in that of autocollimation.

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[38] This appears to have been first pointed out by C. A. von Steinheil (Sitzungsberichte d. Münchener Akademie, **1**, pp. 47–51; 1863). For an example of its extensive use, see C. Pulfrich, Annalen d. Physik, **45**, pp. 613–619; 1892.

The value of this ratio varies widely for various indices and refracting angles but always equals or exceeds 2, a limit which corresponds to the condition of grazing incidence and emergence. When A is selected from the series of its most favorable values (see line $A = 2 \text{ arc tan } n^{-1}$ in fig. 1) this ratio in favor of the minimum-deviation method is $2n^2/(n^2-1)$, which for indices of 1.3, 1.5, 1.7, and 1.9 yields numerical values of 4.9, 3.6, 3.1, and 2.8, respectively.

Similarly, a comparison of equations (5) and (7) shows that a constant ratio of 2 exists between the tolerances for determinations of D and E. Moreover, since in making the observations for D it is customary to eliminate the direct or zero reading and measure $2D$ by observing the angle between right and left hand deviations, the sensitivity of the classical method as practiced is in this particular four times that of Abbe.[39] This serious handicap is in part offset by the increased sensitivity afforded by the autocollimating spectrometer in effecting coincidence of slit image and fiducial reference mark, the angular error of pointing as read on the circle corresponding in this case to only one-half the apparent error in the field of view.

Of the considerations which have been mentioned as relating to methods of prism refractometry, those of sensitivity and temperature control are the most important. The former has usually received more attention because the errors of circle division are frequently viewed as the limiting factors in index measurement; consequently, the minimum-deviation method has usually been chosen whenever the temperature-control problem can be satisfactorily handled.

A constant-temperature room is the solution often adopted for temperature control, but it is very inadequate for work on liquids and seldom provides for work over a desirable range of temperature. Nevertheless, this plan provides uniformity of working conditions for the whole apparatus, it does not complicate the problems of instrument design, and it offers no interference with the usual procedure of operation. A satisfactory constant-temperature prism chamber or housing offers, on the other hand, a solution of the temperature-control problem which contrasts sharply with that of the constant-temperature

[39] Nonsilvering, or half silvering, of the 30° faces of the Abbe prism permits the use of the Féry-Martens autocollimating method (C. Féry, Compt. Rend., **119**, pp. 402–404; 1894; F. F. Martens, Verh. d. Deutschen Phys. Gesellschaft, **3**, p. 14; 1901) in which one measures A_a and the supplement, F, of $(2E-A_a)$, so that the equation for the index should be written as $n = \dfrac{\cos\left(\dfrac{F-A_a}{2}\right)}{\sin A_a}$. Under these conditions the partial derivatives are found to be $\dfrac{\partial n}{\partial A_a} = -\dfrac{\sin D_a + \sin E \cos A_a}{2 \sin^2 A_a}$, and $\dfrac{\partial n}{\partial F} = -\dfrac{n}{2 \tan E}$; and when compared with equations (4) and (5) the resulting tolerance ratios in favor of the minimum-deviation method are not so large as those given above. For refracting angles the value in this case is $(1+\sin E \cos A_a \operatorname{cosec} D/2)$. Although, like the previous similar expression, this always equals or exceeds 2, it reduces to $(2n^2-1)/(n^2-1)$ with optimum values of A, and thus gives for indices of 1.3, 1.5, 1.7, and 1.9 the ratio values 3.4, 2.8, 2.5, and 2.4, respectively. In deviation measurements the advantages of this variation of the autocollimating method are more marked and the ratio of tolerances in measuring $2D$ to those in determining F is only 2 nstead of the ratio of 4 which exists between the maximum permissible errors in $2D$ and the similar errors in E. But the loss of intensity is a serious matter for many of the spectral lines which it is desirable to use.

room when both of these proposals are compared along all the lines just mentioned. It is, therefore, evident that the best program should include both housing and room control.

A temperature-control housing for a minimum-deviation spectrometer must provide two windows, adjustable through wide limits in azimuth, with respect to the prism table and to each other. Furthermore, suitable jacketing means considerable weight to be sustained by the prism table, since the housing must rotate with the prism. Because of these and other difficulties, few minimum-deviation housings have been attempted. On the other hand, for Abbe's autocollimating method, satisfactory housings are relatively simple. They require but one window, and it is fixed with respect to the housing; their weight is sustained by supports which are entirely independent of the instrument; and their construction has been standardized for commercial production.[40] Furthermore, within a housing of this kind, a mirror suitably mounted with respect to the prism table [41] may take the place of a collimator and enable the autocollimating spectrometer to perform to a certain extent like an instrument of the more usual, or minimum-deviation, type.

For all index work with bright line sources, the mirror arrangement mentioned seems to combine in a favorable way the full sensitivity of the Fraunhofer method with many of the advantages of that of Abbe. This applies particularly to deviation measurements where, as compared with the classical method, the precision of pointings is doubled and the error corresponding to that of prism table orientation is halved. Accordingly, these considerations give a clear indication of one course to be followed whenever the construction of an entirely new spectrometer for precise work is contemplated. The instrument at hand in this laboratory was, however, of the minimum-deviation type, and certain exigencies required a more immediate solution which, if possible, should include provision for an adequate temperature-control housing sufficiently flexible in use to handle at minimum deviation the wide range of samples on which index measurements are required at this bureau.

IV. TOLERANCES IN ANGULAR MEASUREMENTS

Strictly speaking, accurate sixth decimal place indices are obtained only when the errors are confined to less than five units of the seventh place; but in this and in certain subsequent papers treating of the

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[40] Zeitschrift f. Instrumentenkunde, 9, p. 362; 1889.

[41] This is one of the valuable suggestions concerning autocollimation which were made by C. A. von Steinheil (loc. cit.) soon after J. Duboscq had built a half-prism spectroscope (using a 30° prism silvered on one side) and O. von Littrow (Sitzungsberichte d. Wiener Akademie, 47, pp. 26–32; 1863) had independently applied the principle of autocollimation to his multiple prism spectroscope. A mirror device of this kind is used on the Goerz autocollimating spectrometer as designed by F. Weidert. The feature receives detailed mention in a description of this instrument in a circular privately issued by the C. P. Goerz Co.

requirements for accuracy in measuring refractive indices the tolerance, T, to be considered in referring to any measurement will be that corresponding to an error of plus or minus one unit of the sixth decimal place of index.

From equations (4) and (5) the tolerances in the goniometric measurements for minimum-deviation prism refractometry may be written as

$$T_A = \pm 0.4125 \frac{\sin^2 A/2}{\sin D/2} \text{ seconds,} \tag{8}$$

and

$$T_{2D} = \pm 0.8251 \frac{\tan \frac{A+D}{2}}{n} \text{ seconds.} \tag{9}$$

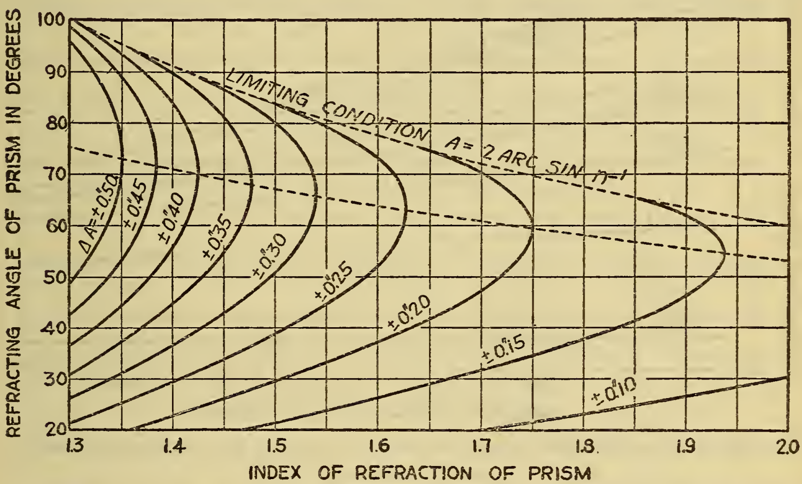

FIGURE 1.—*Iso-tolerance curves (in seconds of arc) for the measurement of prism angle*

For an accuracy of $\pm 1 \times 10^{-6}$ in a determination of refractive index by the method of minimum deviation, it is necessary that the refracting angle of a prism be measured within the limits of error, ΔA, which are expressed by these curves. Values of the most favorable prism angle, $A = 2$ arc tan n^{-1}, for work on media of a given index, are indicated by the undesignated dotted line. The latter shows not only the general merit of using 60° prisms, but also under what conditions it is advisable to depart from that usual value for the refracting angle.

From equations (8) and (9) curves have been computed which are very useful in choosing favorable working conditions for certain specific problems and, also, in making small corrections to previously computed data. These curves have been reproduced in Figures 1 and 2 for reference and for showing at a glance the wide variation in the values of the tolerances T_A and T_{2D}, respectively, for the various combinations of A and n which it may be necessary or desirable to use.

Concerning the choice of A in particular instances, it may be readily shown that the value of the refracting angle for the limiting condition of grazing incidence and emergence in minimum-deviation refractometry is $A=2$ arc sin n^{-1}, and from equation (1) it follows that the corresponding limit of deviation is $D=2$ arc cos n^{-1}. Moreover, by taking partial derivatives with respect to A of the tolerance T_A, expressed in terms of A and n, the condition for most favorable tolerance in prism-angle measurement is found to be $A=2$ arc tan n^{-1}. Corresponding to this condition, the deviation is expressible as $D=2$ arc cos $2n(n^2+1)^{-1}$. This special value of A reduces equation (9) to $T_{2D}=\pm 0.825$ seconds, showing that for all media a given error in deviation measurement corresponds to the same unit error in index provided the prism angle has this specified optimum value.

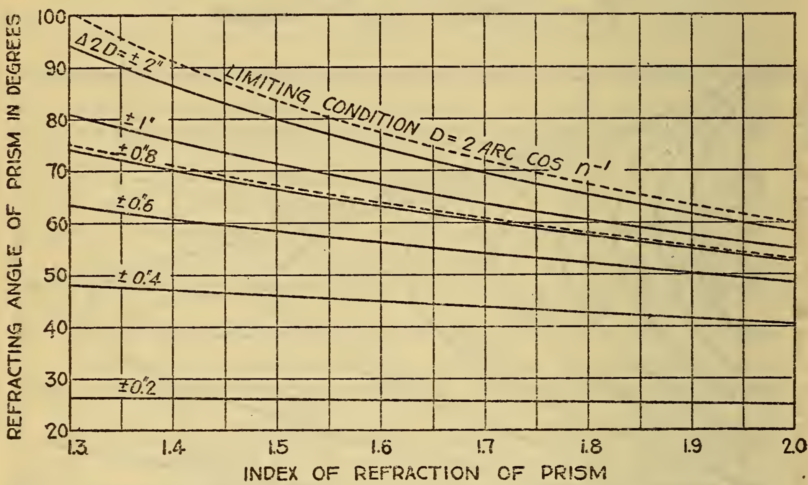

FIGURE 2.—*Iso-tolerance curves (in seconds of arc) for the measurement of double deviation*

For an accuracy of $\pm 1 \times 10^{-6}$ in a determination of refractive index by the method of minimum deviation, it is necessary that the angle of double deviation be measured within the limits of error, $\Delta 2D$, which are expressed by these curves. The line of limiting condition, $D=2$ arc cos n^{-1}, is identical with the limiting condition, $A=2$ arc sin n^{-1}, of Figure 1. Obviously, for this condition of grazing incidence and emergence, $(A+D)/2=90°$ and equation (9) shows that the limiting tolerance is $T_{2D}=\infty$. The undesignated dotted line, $D=2$ arc cos $2n(n^2+1)^{-1}$, is identical with the line of optimum prism angle which is similarly shown in Figure 1. This dotted line is also the iso-tolerance curve of ± 0.825 seconds in double deviation measurement which for all media is equivalent to unit error in the sixth decimal place of index, provided the prism angle has the optimum value, $A=2$ arc tan n^{-1}. When precise determinations of the partial dispersions are desired, even at some sacrifice of accuracy in the absolute values of the indices, it is evident that refracting angles should be chosen from the area above the undesignated dotted line. In selecting such angles it should be remembered, however, that there is a loss of intensity, and that there may be a serious increase in aberration, as the limiting condition of grazing emergence is approached.

From Figure 1 alone it appears that for the refractive angle of a prism of any given medium there exist equally favorable values above and below the optimum angular value for the prism, but Figure 2 shows that from this particular standpoint only the larger refracting angles should be chosen. It must be remembered, however, that there is a loss of intensity, and that there may be a serious increase in aberration, as the limiting condition of grazing emergence is approached.

It will be noticed that, except for media having low refractive indices, the tolerances in prism-angle measurement are less liberal than those for determinations of double deviation. This makes it apparent that, for the most accurate minimum-deviation refractometry of glass, it is important to choose only certain refractive angles readily measureable with the highest accuracy, or else to determine deviations for all three refracting angles of an equilateral prism polished on three sides and so almost eliminate error of prism-angle measurement. The latter course was followed, implicitly, by Hopkinson,[42] Macé de Lépiney,[43] and others, and explicitly, by Gifford,[44] who entirely avoided precise refractive angle determinations.

The exclusive use of equilateral prisms and the making of triple deviation measurements is, however, neither desirable nor practicable, even in precise refractometry. In determinations on liquids of low index, for example, the more favorable tolerances in angle measurement, together with the very considerable increase in sensitivity, both of which are gained by using large refracting angles, make it advantageous to use hollow prisms with an angle appreciably larger than 60°; and the resulting simplicity of prism construction and use is a factor worthy of mention. Certainly there is also, in the general practice of prism refractometry, a considerable demand for accurate single determinations, and it is therefore pertinent to carefully consider in detail these requisite tolerances for high precison. With reference to the possibility of working within the limits found to be necessary, it may be mentioned that in angle measurement by the use of a spectrometer a precision of the order of a few tenths of a second has been attained.[45] Brief references to particular methods of prism-angle measurement will be made in a future paper when the symmetrical use of apertures, the curvature of prism surfaces, and the accuracy of collimation are considered.

In this survey little attention need be given to errors made in the pointings of the telescope or to those made in settings with the micrometer microscopes. Whenever, in decreasing the magnitudes of these accidental errors, repetition becomes onerous, it is likely that such errors can be satisfactorily reduced by the substitution of larger apertures, by improving the quality of scale rulings, or by the use of optical systems which are more carefully designed and constructed. Frequently, however, the size of the prism itself determines the effective telescope aperture and, hence, the limit of precision attainable in a single telescope pointing. This is especially likely under the conditions of reduced width of beam which must exist at

[42] J. Hopkinson, loc. cit., p. 290.

[43] J. Macé de Lépinay, J. de Physique (2), **6**, p. 193; 1887.

[44] J. Gifford, Proc. Royal Soc., London, **70**, p. 329; 1902. (Note by R. T. Glazebrook.)

[45] O. Schönrock, Zeitschrift f. Instrumentenkunde, **40**, pp. 92–93; 1920; **46**, p. 174; 1926.

the oblique incidence and emergence of rays in minimum-deviation measurements, and prism size is given quantitative consideration in a forthcoming paper, particularly as an important contributing factor in the error of prism table orientation.

V. USE OF MULTIPLE MICROSCOPES IN MINIMIZING CIRCLE ERRORS

In considering the difficulties in reaching the requisite accuracy, either in refractive-angle or minimum-deviation measurements, the errors relating to the divided circle are of fundamental importance. No detailed reference to eccentricity is necessary, because most attempts at moderately high accuracy include the use of scale-reading devices at positions angularly equidistant on the circumference of the circle, and under these conditions errors due to eccentricity and to the ellipticity of the pivot are eliminated.[46] The marks themselves, however, usually have errors of position which are classed as periodic (systematic) and accidental (irregular) and are by no means negligible. Therefore, it is generally assumed necessary to calibrate a circle when the accuracy required is of the order of a second of arc or better; but, to avoid the labor of a complete calibration, a Fourier series approximately expressing the low frequency periodic errors as functions of the readings is often determined by the examination of a limited number of the circle divisions.[47] When this latter procedure is followed the resulting corrections, which are computed for application to the circle readings, include neither the accidental errors nor those of high frequency; and often, in order to eliminate such errors, it is still necessary, during the regular use of the instrument, to repeat all measurements on various parts of the circle.

1. SPECIAL USE OF AN UNCALIBRATED CIRCLE

It should be emphasized, however, that in measuring certain commensurable angles all positional errors of scale marks, both periodic and accidental, are completely eliminated from the mean result of a few determinations taken with proper circle orientations.[48] As an obvious example, consider the important case of a 60° prism for which accurate angle measurement is desired. When using only one microscope, a correct value for the angle may be reached by averaging

[46] William Chauvenet, Spherical Astronomy, **2**, p. 51 (Philadelphia); 1891. There may exist, however, certain errors of the pivot which are not so eliminated.

[47] A method of this kind which has recently received considerable attention is that of H. Heuvelink, Zeitschrift f. Instrumentenkunde, **45**, pp. 70–84; 1925. See also L. Fritz und W. Uhink, ibid., **48**, pp. 53–68; 1928.

[48] J. Macé de Lépinay (J. de Physique, (2) **6**, p. 193; 1887), in measuring a double deviation of 84°, used two series of measurements involving a 90° difference in azimuth, and he pointed out that the process would entirely eliminate graduation errors for a double deviation of 90°. See also Annales de Chimie et de Physique (6), **10**, p. 73, note 4, 1887.

three measurements made on equidistant portions of the circle. Only three scale marks are used, and, if perfect functioning of the eyepiece micrometer is assured, the only requirement concerning the positions of these marks is that they be approximately 120° apart so that they will appear in turn in the field of view of the microscope as required. This, of course, is because the entire positional error of each line affects the results twice, but with unlike signs, and is thus eliminated from the average for the series.

The use of several equidistant scale-reading devices is of considerable importance in this connection, on account of the way in which their number, m, is related to the number of necessary repetitions, r. With four microscopes, every measurement of a 90° prism angle is entirely free from the errors of graduation of the circle. After each quadrant rotation the marks utilized are the same four used initially, and the circle has become in a sense a perfect quadrant. In general, the circle becomes equivalent to a perfect arc of $360°/m$, and a convenient equation

$$r = x . \frac{360}{mA} \tag{10}$$

gives the condition for perfect measurement of an angle of $A°$, provided that to the proportionality factor, x, there is assigned the lowest integral value which will yield an integral value for r. Obviously, the scale should be rotated between measurements through an angle of $360°/mr$, and x is the required number of complete traverses on the circle or on its equivalent perfect arc.

This error-free use of an uncalibrated circle is more or less well known; but, possibly because it is somewhat time consuming and requires considerable care for its successful application, it seems that it is avoided, whenever possible, by the use of a more accurate circle or one supposed to be accurately calibrated. In precise refractometry, however, where even the best available instrument often leaves much to be desired from the standpoint of the accuracy attainable, this special use of a circle should be of service. Its comparative neglect in this and related fields, especially in prism goniometry where the use of commensurable angles such as 30°, 45°, 60°, and 90° is quite prevalent, is probably due to the apparent limitation of the procedure to exactly commensurable angles. In reality the practical value of the method is very great because of certain possibilities in connection with errors of the subdivision of the degree intervals of circles. Whenever these errors are approximately eliminated (see pt. 2 of this section) this special use of a circle extends quite satisfactorily to all angles within 1° of the strictly commensurable values. Furthermore, the systematic errors of the degree scale marks are usually relatively unimportant over arcs of a few degrees; and, consequently, by a small number of repetitions, the mean results for many incommensurable angles may be rendered comparatively free from positional errors of the scale marks.[49]

Although the use of an uncalibrated circle in measuring double minimum deviations is, in general, less satisfactory than its use in measuring prism angles, a glance at Figures 1 and 2 shows that only one-half or one-quarter as much accuracy is required, particularly for work on 60° glass prisms. The possible use of dispersion equations also ameliorates the situation somewhat, especially if indices are determined for several different wave lengths. Furthermore, the means required to eliminate some of the various accidental errors, which in determinations of deviation are larger than in prism-angle measurement, may serve to minimize unknown errors of the circular scale. As an example of the latter possibility, reference may be made to the errors in telescope pointings. For all slit images, except those formed by using the brightest lines near the central portion of the visible spectrum, the precision of pointings is appreciably lower than for those made during prism-angle measurement; consequently, it is often desirable, especially if but few different wave lengths are used, to make a number of observations of each deviation, and these repetitions also reduce the deleterious effects of an imperfectly graduated scale provided a suitable observational program is followed. The accidental errors of graduation are, of course, reduced as the square root of the number of scale marks used, and, according to Woodward,[50] the periodic errors tend, in general, to be eliminated with repeated observations provided the circle is advanced each time through an arc of $360°/mr$, which is the same rule mentioned above when considering the strictly error-free measurement of commensurable angles.

Attention has already been called to the advantages of the optimum relationship between prism angle, A, and number of reading microscopes employed, namely, that condition expressed in equation (10) for making certain measurements entirely free from scale error. It should also be added that for any angular measurement the elimination of periodic error of given frequency becomes more nearly complete, and the process of elimination extends to periodic errors of higher frequency, as the number of scale-reading devices is increased.[51] Consequently, in the design of a spectrometer, it is pertinent to consider a device to permit the mounting of a variable number of equi-

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[49] B. Walter (Annalen d Physik, **46**, p. 424; 1892), by an "Umkreisungsverfahren," repeated measurements of refracting angle and of deviation until within ±30° of the starting point on the circle and reduced the errors from 50 seconds, for some single measurements, to 3 seconds or less for an average result of a number of repetitions not exceeding seven.

[50] R. S. Woodward, Report, Chief of Engineers, U. S. A., Appendix MM, p. 1947; 1879.

[51] Wm. Chauvenet, loc. cit., p. 53. If the most important periodic errors are those of long period, they are often almost eliminated from the mean of the readings of only four microscopes. (See L. C. Martin, loc. cit., p. 53.)

distant microscopes around the circle. Such an arrangement would also greatly facilitate complete initial calibration of the scale, and subsequent recalibrations which may be required after cleanings, in the event that it should seem desirable to provide for accurate measurement of incommensurable angles without repetitions.

2. ELIMINATION OF DEGREE-SUBDIVISION ERRORS

The periodic errors of the divided circle which have just been considered are those, principally of relatively low frequency, which relate only to the marks representing even degrees. On some circular dividing engines, however, the auxiliary gear governing the subdivision of degree intervals causes a periodic error whose period includes only a few of these subdivisions; [52] that is, a comparatively very high frequency periodicity is superimposed on the long periods which have been previously considered. These errors in subdivision of degree intervals sometimes approach or even exceed in magnitude [53] the errors of the "degree" graduations, and the importance of their elimination in the special use of an uncalibrated circle has already been pointed out. The avoidance of such errors is necessary also in any use of a scale which, as frequently happens, has been calibrated for the "degree" graduations only. Fortunately, the periodicity of these degree-subdivision errors can be easily determined; and such errors can be automatically eliminated from the mean of the readings by using N microscopes which are displaced from their equidistant positions [54] by fractions of a degree not exceeding $N/2d$, where N is the number of scale marks in each error period and d is the number of subdivisions per degree. Even with less than N microscopes these errors can be very satisfactorily reduced.

If the microscopes are not displaced by these fractions of a degree but are allowed to remain equidistant, then for a given telescope pointing it is evident that each microscope reading, and also the average reading, will contain degree-subdivision error to approximately the same extent. In general, this error will be different for two successive pointings and thus cause an erroneous angle measurement. Furthermore, in repeating, if the circle is advanced in azimuth through an arc measured in integral error periods (of N scale marks each), an error of this nature is not decreased with the number of observations.

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[52] Gustav Förster, Zeitschrift f. Instrumentenkunde, 33, pp. 44–51; 1913.

[53] J. Macé de Lépinay, Annales de Chimie et de Physique (7), 5, p. 218; 1895.

[54] In the elimination of the eccentricity, e, from the mean of the microscope readings, the maximum error due to the displacement of one of two microscopes over an arc of α degrees from its position diametrically opposite the second is $e \sin \alpha$. Thus, for example, a 10′ displacement can not cause such error in excess of $0.003e$, and the value of e seldom exceeds a few seconds on circles for precise work.

VI. MICROMETER ERRORS

Errors in the adjustment of the magnification of the microscopes, causing so-called error of the "runs" of the micrometers, are not sufficiently emphasized in prism goniometry. This may be due to the fact that the use of the necessary corrections is supposed to be well known, or possibly it is because such errors are minimized when both positive and negative "runs" are made; that is, when results are obtained from the means of readings on both scale marks which are found adjacent to the fiducial line in the microscope field. In using multiple microscopes and in repeating scale positions, however, such duplication of micrometer settings is not only unnecessary but may, in fact, lead to errors through vibrations communicated to the instrument during the time required for moving the cross hairs from mark to mark of the scale.

The matter can not always be handled with a simple fixed correction because, for example, differences in room temperature, possibly through effects produced on the oil film between the surfaces of a conical bearing, are quite sufficient to cause changes in the working distances of the microscopes and thus to result in appreciable variations in magnification. Similar variations may also be produced by mere reorientation of the circle or of the system carrying the microscopes, except under ideal adjustment of the bearings and mounting of the circle. Fortunately, frequent determinations of the average run correction may be made accurately and easily under the exact conditions of use by taking one half of all measurements with positive and the other half with negative runs, rather than always selecting the run of minimum length.[55] Indeed, in this way, by a proper balance of positive and negative runs for the repetitions included in an observational program, the error of the runs may be so minimized in the average results that the residual effects are negligible.

Progressive errors of the eyepiece micrometer screws and the distortion introduced by the optical systems of the microscopes will, to a large extent, be included in the error of the runs when the latter are determined under the conditions of actual use as above mentioned. When using multiple microscopes, the periodic error of the screws can be automatically eliminated from the mean readings by the proper adjustment of their individual zero-position drum readings, provided such periodic errors are somewhat similar for all the microscopes and are known roughly as functions of the drum readings. Another method which obviates the use of corrections for the periodic errors of the micrometer screws is based on the repetition of observations with the program of circle advance so chosen that at each

[55] References to negative as well as positive runs should not, of course, be construed as abrogating the use of the usual precautions to eliminate backlash.

micrometer the initial drum readings for each of the successive measurements are evenly distributed over one complete revolution of the screw.

VII. NONCONSTANT RESIDUAL MECHANICAL STRAINS IN THE INSTRUMENT

One additional topic will be mentioned in this résumé and discussion of certain goniometrical requirements in refractive index spectrometry. This is the need of good balance of all moving parts of the instrument and the necessity of freedom from residual horizontal stress components at the instant when a pointing is completed. These requirements must be fulfilled in order to avoid small creepings which may otherwise occur after pointings but before and during the various microscope settings.[56]

Such creepings are very likely to cause systematic rather than so-called accidental errors, and the writer has found that certain errors of the order of 0.5 second of arc in prism-angle measurement are explainable as the algebraic sum of torsional effects of the male and female members of the spectrometer cone bearing.

With a moving telescope and autocollimating eyepiece, the (stationary) prism table being supported from the upper part of the male cone and the (moving) microscope system from the lower part of the female cone, the prism does not remain exactly stationary, but turns through a small angle in the direction of the telescope rotation, whereas the microscope system "drags" [57] slightly with respect to the telescope. If the first effect predominates, the measured arcs are too large and the refracting angles as determined are, therefore, too small. In deviation measurements, only the second of these effects is present and again the values as observed are less than should be found, so that under such circumstances the computed indices of refraction may be either high or low, systematically, on account of torsional strain which varies during prism angle and deviation measurements.

Such results are viewed as due to the frictional contacts which cause strains of the cone members during their relative motion and also prevent their subsequent complete elastic restitution. If the residual strain varies in amount for successive telescope pointings, the result

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[56] The writer is averse to the use of clamps and slow-motion screws in making telescope pointings, feeling that small harmful residual stresses are more likely to exist through the use of such devices than is the case when the moving system is induced to take the desired orientation by a number of slight finger taps which produce swings alternating in direction and gradually decreasing in amplitude. It is obvious, however, that, even with optimum instrumental conditions, this method of making observations is not advantageous unless great care is taken to avoid vibrations during the time interval between the telescope pointings and the microscope settings.

[57] The effects discussed have, however, no reference to "drag" through mechanical contact other than that between the conical surfaces.

of a measurement of an angle is correspondingly incorrect. Errors of this kind are sensitive to defects in the conical surfaces, to mechanical interadjustment of the cone members, and also to variations in room temperature, possibly in the latter case through the effect of temperature on the oil film between the surfaces.

In designing a new instrument it is probably possible to avoid such defects to the extent required in sixth decimal place refractometry. In using an old instrument the effect of these errors may be greatly minimized in prism angle measurement, and also in deviation measurements if the collimator permits, by making observations not only on the angles themselves but also on their explements as is sometimes done for similar reasons when using the method of repetitions in geodetic surveying. Frequently, however, it will be necessary to make a statistical study of data taken under various conditions in order to show that these and similar errors are not present or to outline a working plan for their satisfactory elimination.

WASHINGTON, October, 1928.