Journal of Research of the National Bureau of Standards Vol. 57, No. 3, September 1956 Research Paper 2704

# A High-Voltage Pulse Generator and Tests on an Improved Deflecting System of a Cold-Cathode Oscillograph

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An improved deflecting system for a cold-cathode oscillograph is described. This deflecting system reduces transit-time errors and eliminates errors due to impedance mismatch between the signal coaxial cable and the deflector.

A high-voltage pulse generator for producing single pulses in the millimicrosecond range was devised, and its use in testing the improved deflecting system is explained.

## 1. Introduction

The cathode-ray oscillograph (CRO) is an indispensable tool in high-voltage surge testing and research. However, the requirements of an oscillograph for his work are somewhat different from the requirements of the usual laboratory oscillograph. Sensitivity is not an important factor, as ample signal voltage is available. In fact, the available voltage is usually limited only by the flashover voltage of the fittings on the coaxial cable between the source of voltage to be measured and the oscillograph.

High writing speed is essential, as it is necessary to record on film single transients of a few millimicroseconds duration. This high writing speed is attained by the use of high accelerating voltages for the electron beam, and in many cases by allowing the beam to impinge directly on the photographic emulsion instead of photographing a trace on a fluorescent screen.

Cold-cathode oscillographs of the type described by Ackermann [1][1] are used in the Bureau's high-voltage laboratory and are entirely adequate for ordinary surge work.

However, with increasing interest in studies of steep-front voltage surges, the ability to faithfully record single transients of approximately 50-mμsec duration is essential. A resolution of time intervals of the order of 1 mμsec for the current and voltage records is highly desirable in making a detailed study of the mechanism of spark breakdown.

To record such rapid transient variations, a very high writing speed is necessary. Also, recording errors in the oscillograph, as well as those arising from its connections, which are not significant at lower recording speeds, become important and must be reduced or eliminated. Actually, a sufficiently high writing speed was insured by using Park's method of beam intensification [2]. Therefore, the recording errors introduced by the oscillograph itself and the means adopted for their elimination form the basis of the present study.

[1] Figures in brackets indicate the literature references at the end of this paper.

## 2. Cathode-Ray Oscillograph Recording Errors

Two sources of error of the CRO are of concern when very short transients are to be recorded. One is the error due to the transit time of the electrons in passing from one end to the other of the deflecting plates. The other error is caused by impedance mismatch in connecting the deflecting-plate circuit to a coaxial cable.

A typical cold-cathode oscillograph uses an accelerating voltage of 50 kv and has deflecting plates 3 cm long. The velocity of 50-kv electrons is $1.23\times10^{10}$ cm/sec, and the corresponding transit time is $2.44\times10^{-10}$ sec. The error arising from transit time will be 5 percent at 600 Mc for a pure sine wave.[2] For a steeply rising pulse the error will be 5 percent for a pulse with a rise time of 2.4 mμsec.[3] Although, in general, for steep-front surge work, errors due to transit time can be neglected, pulses with rise times of this order occur and are important in studies of spark breakdown.

The errors arising from impedance mismatch are more serious. The capacitance of the deflecting plates in the typical cold-cathode oscillograph is about $15\times10^{-12}$ f, and the associated lead inductance is about 2 or $3\times10^{-8}$ h, giving rise to a resonance frequency of 200 to 300 Mc. Hence, for all but somewhat lower frequencies, disturbing reflections will occur, giving rise to a voltage at the deflecting plates that may be quite different from the voltage at the input end of the cable. Furthermore, steeply rising pulses will force the deflecting-plate system into oscillation. The modification of an oscillograph to reduce these errors and the testing of the modified oscillograph are now described.

## 3. Redesign of Deflecting Plate System

One method of reducing the above errors is to drastically reduce the size of the deflecting plates. The transit time can in this way be reduced by the

[2] See the appendix, which is a discussion by I. H. Park of transit-time errors. This discussion is an excerpt from an unpublished informal communication.

[3] These values do not take into account the effect of fringing field at the ends of the plates. This increases the effective length of the plates and increases the error due to transit time.

same factor as the reduction in plate length. However, as the plates must be moved closer together to increase sensitivity and reduce the fringing field, errors due to mismatch will not be reduced by the same factor. The local oscillations that occur at a higher frequency may be unimportant.

Miniaturization, however, introduces several problems. It requires that the diameter of the writing spot be made extremely small, and this in turn requires a short-focus electron lens close to the screen. The net result is that only small deflections can be employed, and these must be examined microscopically or enlarged photographically. The extent of the modifications required by this approach appeared to make this solution impractical for our existing oscillograph.

Another method of reducing the recording errors is to replace the deflecting plates by a traveling-wave deflector. A traveling-wave deflector of one type can be arranged as a segment of a transmission line that periodically passes back and forth across the axis of the beam [3,4]. This segment of the line can be so designed that the phase velocity of a wave along the axis of the deflector is the same as the velocity of the electron beam. Also, the deflector can be designed to match the impedance of the connecting signal coaxial cable up to very high frequencies.

A distributed constant line of this type, shown in figure 1, was installed in an oscillograph. The deflector consists of a single slotted plate or flat ribbon mounted asymmetrically between two ground plates. The electron beam is deflected by the field between the ribbon and the more remote ground plate. If a a single ground plate had been used, the spacing necessary to obtain the required capacitance would not have been sufficient to admit the electron beam. The deflector was designed to have an impedance of 52 ohms. The 52-ohm input cable entered the oscillograph housing and was connected to the upper end of the deflector. To provide a suitable termination, a second 52-ohm cable was connected to the lower end of the deflector. The cable was brought out of the oscillograph and was terminated with a 52-ohm resistor corresponding to its characteristic impedance.

Figure 2 shows an oscillogram of an 18-Mc wave that was obtained with this deflector. The asymmetry of the positive and negative half-cycles arises from the magnetic field caused by the current along the deflector ribbon to the terminating cable. The component of the magnetic field normal to the plane of the ribbon gives the electrons a velocity component in the sweep direction. This error increases as the beam is deflected toward the ribbon (that is, for positive deflections) because the magnetic field is stronger near the ribbon. In designing or using any traveling-wave deflector, the effect of the magnetic field must be carefully evaluated. This deflector was removed from the oscillograph and replaced by the deflector shown in figure 3.

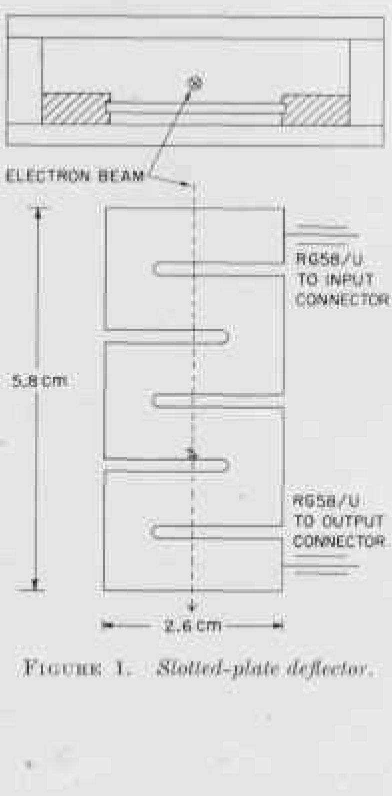


FIGURE 1. *Slotted-plate deflector.*

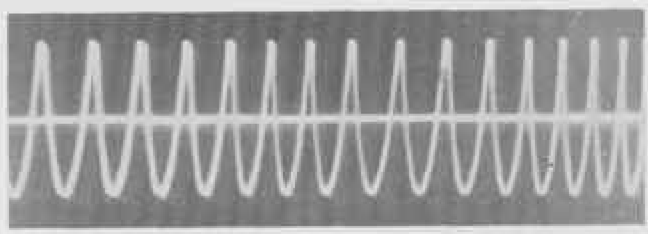

FIGURE 2. *Oscillogram of an 18-Mc sine wave obtained with the deflector shown in figure 1.*

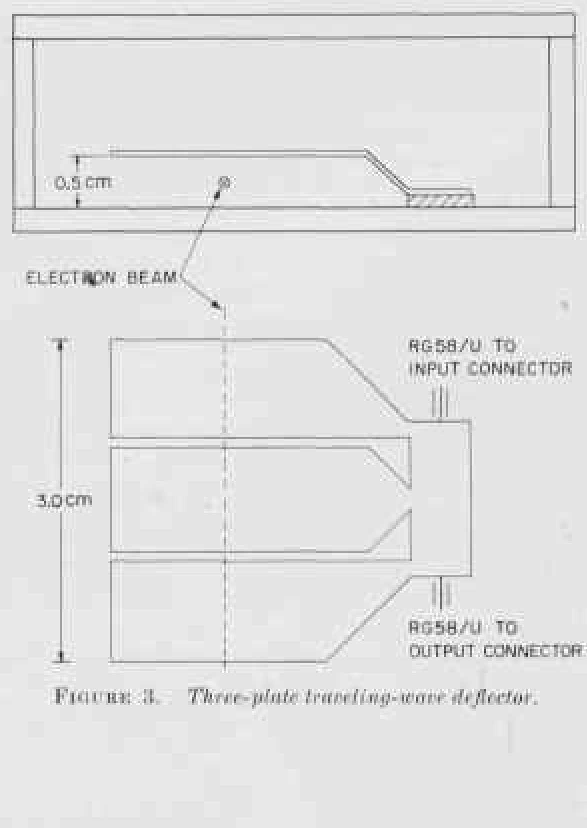


FIGURE 3. *Three-plate traveling-wave deflector.*

A deflector of this type is difficult to analyze quantitatively. However, to a first approximation, it may be considered as a three-section low-pass filter. The capacitive elements of the filter arise from the three plates, each 1 cm long, supported parallel to and 0.5 cm distant from a grounded plate. The electron beam travels between the grounded plate and the three plates in succession. The inductive elements are the segments of the "line" between the plates that protrude to one side. In this way the current-carrying elements are kept away from the electron beam.[4]

To the extent that the deflector can be considered as a low-pass filter, values of inductance and capacitance per section were computed to be $2.35\times10^{-9}$ h and $0.88\times10^{-12}$ f, respectively. From low-pass filter-design equations, the characteristic impedance was computed to be 52 ohms. Subsequent tests showing no appreciable reflections proved this to be close to the true value. The cutoff frequency was computed to be 7,000 Mc (cutoff frequency $=1/\pi\sqrt{LC}$). At frequencies substantially below cutoff, the time of travel of a wave through the filter is given by the equation $T=\sqrt{LC}$, where T is the travel time per section.

Substituting the values of $L$ and $C$ computed above, $T=0.455\times10^{-10}$ sec/section, or $1.36\times10^{10}$ sec for the entire deflector. The electron transit time is $2.44\times10^{-10}$ sec (see section 2).

This design of course is somewhat of a compromise. The wave velocity could be decreased by increasing $L$ or $C$ or both, but this would result in a decrease in the cutoff frequency. In addition to the error arising from mismatch of the wave and beam velocities, there also exists the error due to the time of transit of electrons past the individual deflecting plates.

Input and output cables were connected to the three-plate deflector in the same manner as for the first ribbon deflector. Oscillograms of sinusoidal voltages made by using the three-plate deflector showed no asymmetry, as the line current for this case is at a relatively large distance from the electron beam as compared with the ribbon plate deflector.

Further tests involved comparison of this oscillograph with an unmodified oscillograph.

Two cold-cathode oscillographs were available. One was unmodified, except for the beam intensification and faster sweeps that had been added. The other, also provided with beam intensification and faster sweeps, had been modified by substituting the traveling-wave deflector (and matched terminating cable) described above and shown in figure 3.

Comparison of the two oscillographs was made by applying a short high-voltage test pulse to each, observing the reflections resulting from discontinuities, and looking for shock-excited resonance effects. It should be noted that reflections from any discontinuity between the input end of the cable and the cable termination will have an adverse effect on the recorded waveshape. Possible sources of reflections are: cable fittings between the input end of the cable and the oscillograph, the input connector to the oscillograph, the deflecting system, the output connector of the oscillograph, and the termination itself if it is electrically close to the oscillograph.

If the test pulse is steeper, that is, has Fourier components of higher frequency, than any pulse for which the oscillograph is commonly used and if the magnitude of the reflections is only a small percentage of the incident pulse, then it can be assumed that the recorded waveshape of slower transients will be very nearly the true shape of those transients.

Because the sensitivity of the oscillograph is about 300 v/cm, the amplitude of the test pulse should be 500 to 1,000 v. The pulse should preferably be very short to facilitate identification of reflections. The rise (or decay) time should be less than 1 mμsec

## 4. Method of Generating Short Pulses

The first attempt to generate a suitable test pulse was to charge a short length of transmission line to a potential of 2,000 to 3,000 v, and then discharge this line by means of a suitable switch into the transmission line connected to the oscillograph. The switch used was a spark gap ("pulse gap") triggered by the ultraviolet light from an auxiliary spark gap [9] (or "trigger gap"). It was found that the average rise time that could be obtained in this manner was 2 to $4\times10^{-9}$ sec. Because of the time required for the current to build up in the pulse gap, a small capacitor is as effective as a short length of transmission line, and one was substituted.

The comparatively slow rate of rise can be partly attributed to the fact that there is little if any overvoltage on the pulse gap at the time of sparking.

If a high overvoltage is suddenly applied to the gap during the time that a copious supply of photons is available from the trigger gap, the pulse gap should fire much faster, giving a faster rate of rise. A special circuit shown in figure 4 was devised to provide the needed overvoltage.

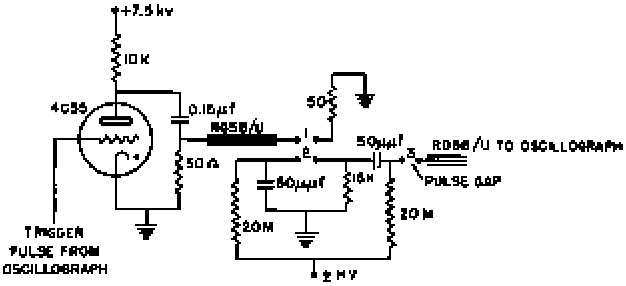


Figure 4. *Schematic diagram of voltage-doubling type of pulse generator.*

[4] This arrangement was suggested by J. H. Park.

Pulse gap 3 is adjusted to be slightly longer than gap 2. The sparkover voltage of gap 2 is determined by raising the charging voltage gradually until gap 2 fires (gap 2 is adjusted so that this breakdown voltage is usually about 2,500 v). For the actual tests the charging voltage is adjusted to be 50 to 100 v below this breakdown voltage. When gap 1 (the trigger gap) fires, gap 2 fires immediately, doubling the voltage on gap 3. An example of the pulse obtained is shown in figure 5. This pulse has a rise time of $1.5\times10^{-9}$ sec and a pulse width of $10\times10^{-9}$ sec. The recorded "hash" before the pulse is a transient from the firing of the trigger gap and can be largely eliminated by electrically shielding the gaps. It indicates how rapidly gaps 2 and 3 fire after trigger gap 1 fires. This oscillogram was obtained by using the modified oscillograph.

It was decided that, in order to obtain a faster rise time, a much more elaborate device would be needed [10]. It seemed expedient, therefore, to accept a rise time of 2 to $4\times10^{-9}$ sec, but to obtain a steeply falling pulse by chopping off the pulse obtained by discharging a capacitor through a single gap. This was accomplished by assembling a pulse generator in which another gap is arranged to fire on the rising front of the pulse, so that it will short-circuit the transmission line to the oscillograph. This offers the advantage that the chopping gap may be subjected to exceedingly high overvoltage so that it will break down very rapidly. It also has the advantage that the current drawn by the chopping gap during its early stages of breakdown merely decreases the apparent rise time of the pulse.

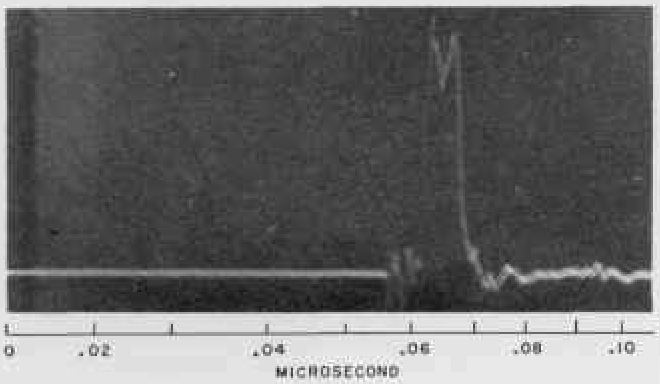


FIGURE 5. *Oscillogram of pulse obtained by using pulse generator shown in figure 4.*

Furthermore, as rise time is relatively less important if a relatively more steeply falling pulse can be obtained, a larger capacitor can be used to insure retention of a higher voltage across the gap during the breakdown process.

Pulses obtained in this way are shown in figure 6. Pulse width and also pulse amplitude can be controlled by adjusting the chopping gap so that it fires on the front or the tail of the wave. For the pulse oscillograms in figures 6, a, and 6, b, the spacing was about 0.001 in. For the oscillograms in figures 6, c, and 6, d, the spacing was increased. Once the chopping gap is set, the pulses repeat well, both in amplitude and duration.

Typical values obtained are: amplitude 800 v, rise time 2 to $4\times10^{-9}$ sec, and pulse width 4 to $10\times10^{-9}$ sec. The decay time is difficult to measure, but is probably of the order of 2 to $5\times10^{-10}$ sec. Figure 7 is a circuit diagram, and figure 8 is a drawing of the pulse generator.

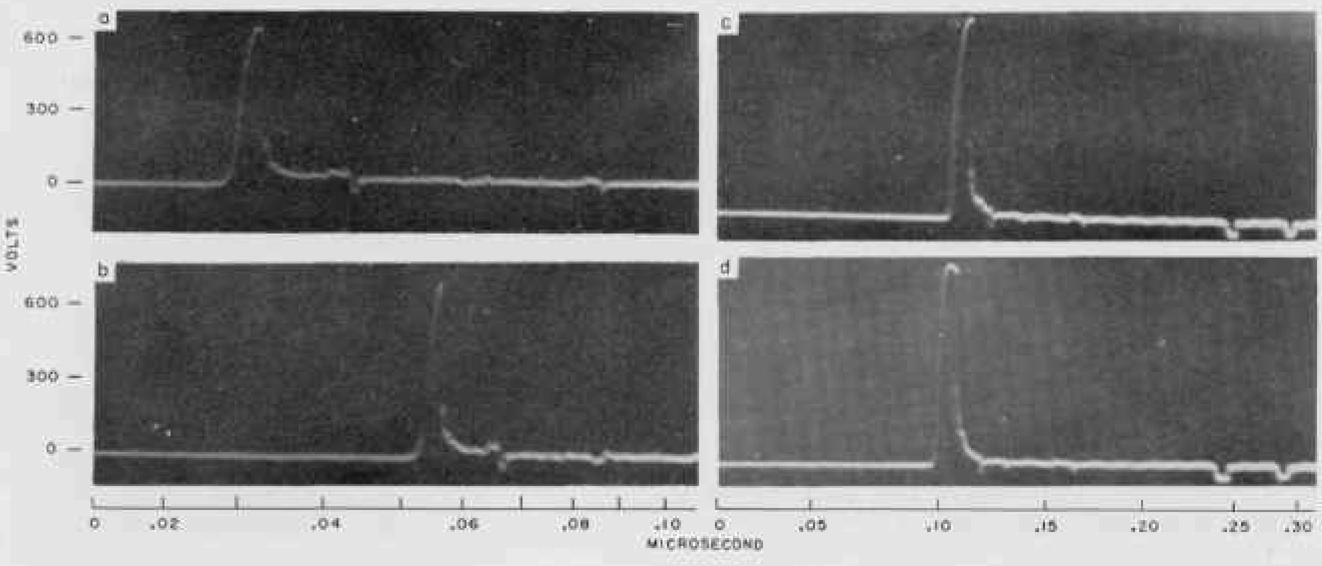


FIGURE 6. *Oscillograms of pulses obtained by using the pulse generator shown in figure 7.*

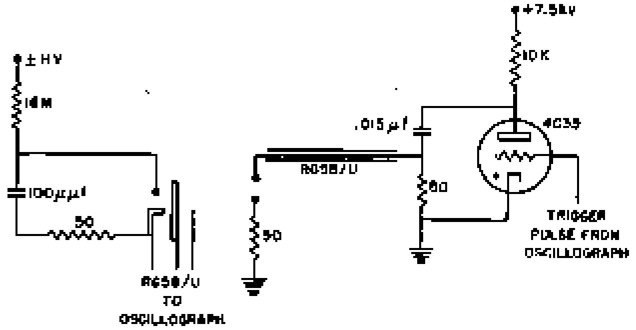


FIGURE 7. *Schematic diagram of a pulse generator designed to produce a steeply falling pulse.*

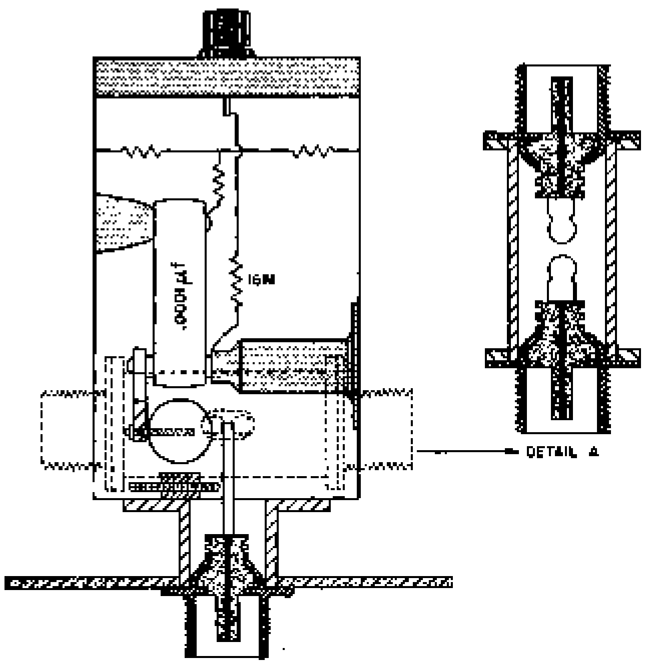


FIGURE 8. *Pulse generator (for schematic diagram see fig. 7).*

Detail A, trigger gap. This gap irradiates the sphere and chopping gaps.

## 5. Results of Tests

The modified and unmodified oscillographs were compared by using pulses of the type shown in figure 6. Representative diagrams for the test setups are shown in figure 9. The lengths of cable are indicated, and below each cable is shown the one-way time of travel of a pulse along that cable length. These travel times were determined by measuring the time interval required for a reflection to reach the oscillograph from a discontinuity at a known distance from the oscillograph. A value of 632 ft/μsec was obtained for the velocity of propagation.

Figures 10, a, and 10, b, show the results obtained

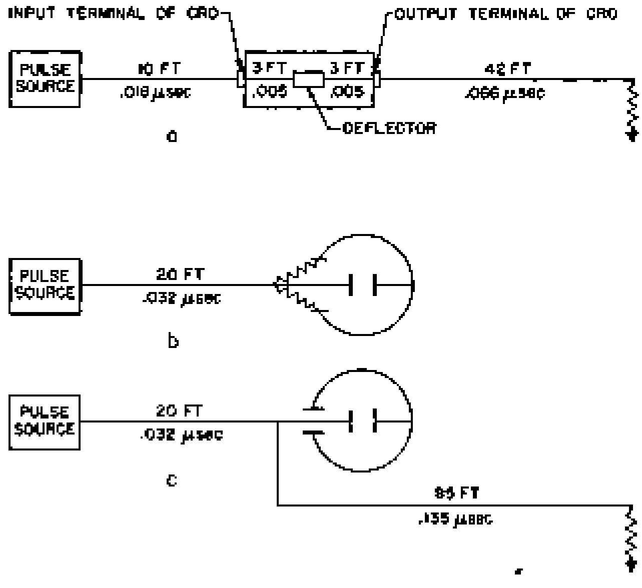


FIGURE 9. *Test setups for comparison of oscillographs.*

a, For modified oscillograph; b, for unmodified oscillograph.

when the test pulse is applied to the modified oscillograph. The most prominent reflections in figure 10, a, occur 0.010, 0.032, and 0.052 μsec after the steeply falling portion of the pulse. By comparing with the travel times in figure 9, a, it is seen that these reflections can be identified as follows:

1. The pip at 0.010 μsec is a reflection of the main pulse from the output terminal of the oscillograph back to the deflector. It is born 0.005 μsec after the main pulse reaches the deflector and arrives at the deflector 0.005 μsec later:

$$0.005+0.005=0.010 \ \mu\text{sec}.$$

2. The pip at 0.032 μsec arises from a reflection at the input terminal of the oscillograph. The pip is born 0.005 μsec before the main pulse reaches the deflector. It proceeds back to the pulse source where it sees a low impedance due to the residual ionization in the pulse gaps, is reversed in sign, and travels back to the deflector:

$$-0.005+0.016+0.016+0.005=0.032 \ \mu\text{sec}.$$

3. The pip described in 1 above travels back to the pulse source, where it encounters an open circuit and is reflected back to the deflector:

$$0.010+0.005+0.016+0.016+0.005=0.052 \ \mu\text{sec}.$$

It will be seen from figure 9, a, that the time of travel from the oscillograph to the termination and return is greater than the sweep length in figure 10, a. Hence this oscillogram does not show any reflections from the termination. Figure 10, b, shows the results when a longer sweep is used. Here the pip occurring at 0.142 μsec results from a reflection at the termination:

$$0.005+0.066+0.066+0.005=0.142 \text{ μsec.}$$

The pip at 0.184 μsec is the 0.142-μsec pip after it has traveled to the source and returned to the deflector:

$$0.142+0.005+0.016+0.016+0.005=0.184 \text{ μsec.}$$

Figures 10, c, and 10, d, show the results when the test pulse is applied to the unmodified oscillograph.

In Figure 10, c, the termination was directly at the deflecting plates. The deflections occurring at 0.064 μsec are first reflections from the deflecting plates and termination to the source and return. The waves at 0.128 μsec are second reflections (see fig. 9, b).

Figure 10, d, shows the results when the termination is at the end of an 85-ft cable. This cable is long enough that no reflections from the termination will occur during the time of the sweep. The waves at 0.064 μsec are first reflections from the deflecting plates only, to the source and return. Waves at 0.128 μsec are second reflections. The serious discontinuity that exists at the plates is quite apparent. The capacitance of the deflecting plates and their associated inductance are forced into oscillation, and some of this voltage is coupled into the sweep circuit, as evidenced by the fact that the trace is not always single-valued.

Comparison of figures 10, c, and 10, d, shows that the reflections occurring at 0.064 and 0.128 μsec are about the same amplitude in the two oscillograms. This is to be expected because the termination is nearly matched to the cable and its replacement by a length of cable does not greatly affect the discontinuity.

In figure 10, c, the amplitude of the local oscillations is lower than in figure 10, d, because the terminating resistor (in fig. 9, b) is in series with the local oscillating circuit, and its effective resistance at these high frequencies is apparently higher than the effective resistance of the 52-ohm cable used to replace it in figure 9, c, the arrangement for which the record in figure 10, d, was obtained.

The charging voltage and gap adjustment of the pulser were kept, as nearly as possible, the same for pulses in figures 10, b, and 10, d. Also the sweep lengths are not greatly different. Furthermore, in each case the termination is at a considerable distance from the oscillograph. Because of the small magnitude of the reflections in figure 10, b, we can assume that the shape of the pulse shown in figure 10, b, is not only a true representation of shape of the pulse generated by the pulser but is also of correct amplitude. The great improvement in recording obtained by the use of the three-plate deflector with suitable terminations as compared with the unmodified deflector with single plates is made evident on comparing the records in figures 10, b, and 10, d.

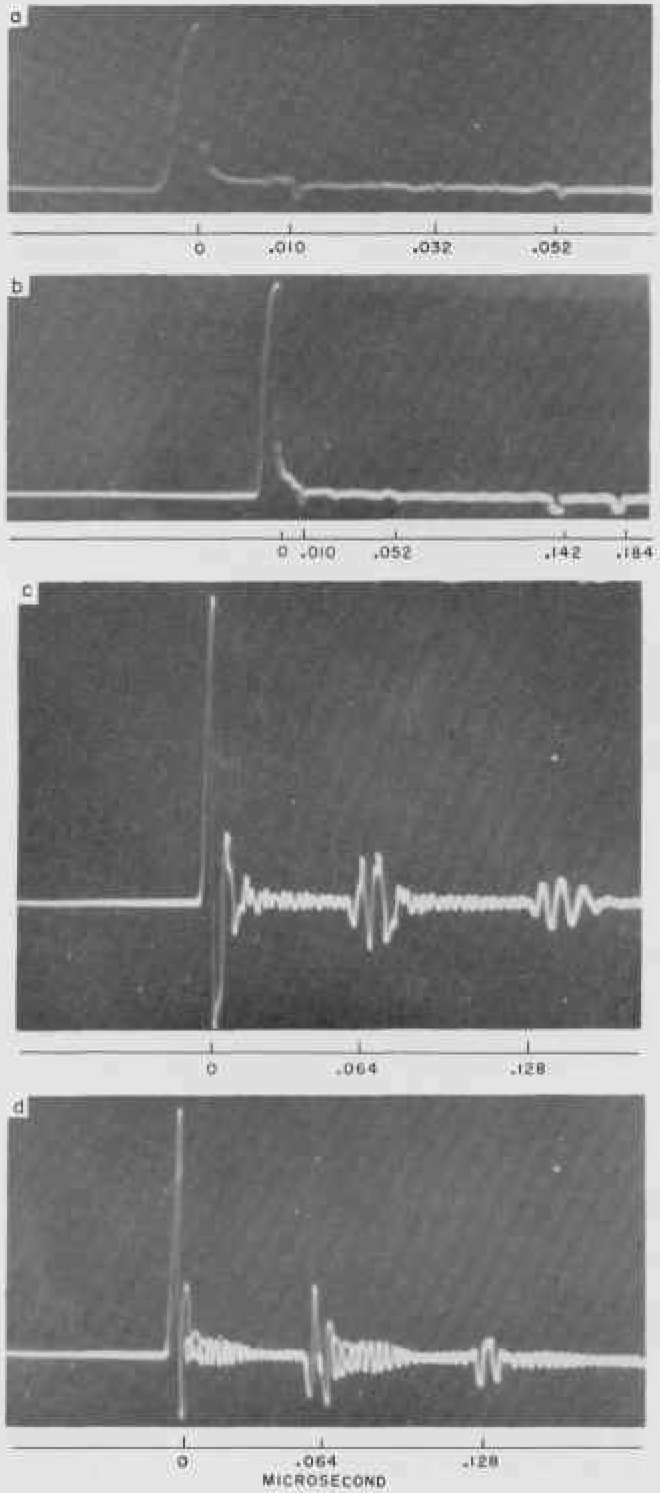


Figure 10. *Oscillograms of test pulse.*

a and b, Modified oscillograph; c and d, unmodified oscillograph.

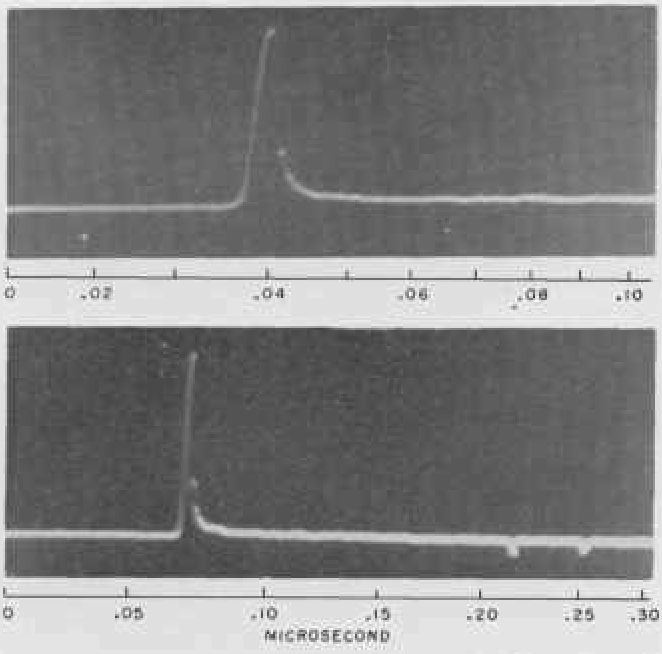


FIGURE 11. *Test pulse as recorded by modified oscillograph after installing improved input and output connectors.*

It is apparent from figures 9, a, and 10, a, that discontinuities at the input and output connectors of the modified oscillograph were present when these records were made. The connectors were "hybrid" connectors made in order to insure a vacuum-tight entry for the cable into the oscillograph. These hybrid connectors were subsequently replaced with constant-impedance fittings (pressurized bulkhead adapters), and the improvement obtained by this change is shown in the records of figure 11, where the corresponding discontinuities in the trace are greatly reduced.

## 6. Conclusions

1. Single transients of a few millimicroseconds duration can be faithfully recorded with a cold-cathode oscillograph if the deflecting system is appropriately designed and beam intensification is used.

2. For faithful recording at high writing speeds, the deflecting system of a CRO should be designed as a traveling-wave deflector properly matched to the signal-cable impedance. The design should insure that the magnetic field from the traveling wave of current is small in the region traversed by the electron beam.

3. A simple pulse generator has been developed that will produce very short high-voltage pulses of closely repeatable waveform. As is demonstrated in this paper, such a source of pulses, together with well-established transmission and reflection theory, provide a reliable basis both for comparing the high-speed performance of CRO's and for identifying and eliminating sources of reflection in the connected circuits. Distortions arising in the CRO records from circuit reflections of transients are readily discernible to a resolution within the millimicrosecond range.

4. Lacking a better and more faithful recorder of very short transient voltages for comparison with the cold-cathode CRO equipped with beam intensification and traveling-wave deflector, the use of pulses from the high-voltage pulse generator appears to provide the best method for verifying the recording reliability of this very high writing speed oscillograph.

## 7. References

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## 8. Appendix [3]

By J. H. Park

### 8.1. Deflection Error Due to Electron-Beam Transit Time

In slow-speed oscillography, the voltage applied to the deflection plates may be considered to be constant during the short time it takes for one electron of the beam to pass through the deflecting field (transit time), and the deflection on the screen is at all times directly proportional to the deflecting-plate voltage. However, as the rate of change of the deflecting-plate voltage is increased, a value will be reached such that the deflecting field will change an appreciable amount during transit time, and the deflection on the screen will not continue to be proportional to deflecting-plate voltage. The measurement error thus introduced depends upon (1) the time interval over which a change in voltage is to be measured, and (2) beam transit time. For certain functions of applied voltage, this transit-time error can be determined from theoretical considerations.

[3] Part of an informal communication not readily available for reference.

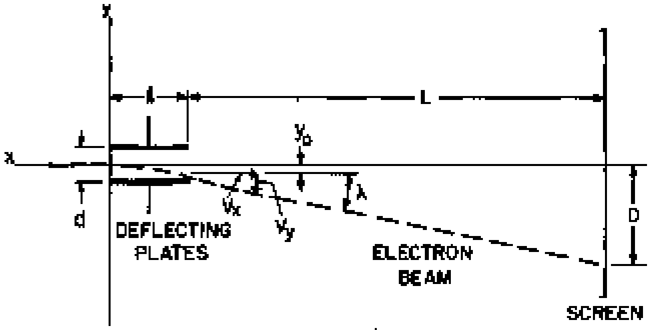


FIGURE 12. *Schematic diagram of beam deflection.*

A schematic diagram indicating how an electron beam is deflected during and after passage between a pair of deflecting plates is shown in figure 12. The voltage to be measured, $E=f(t)$, is impressed upon the deflecting plates. The electron beam has a constant velocity in the $x$ direction, $V_x$, which is proportional to the square root of the total voltage used in accelerating the electrons before they pass through the deflecting plates. Each electron of the beam, while passing through the electric field between the deflecting plates, will be given an acceleration in the $y$ direction, which at every instant is proportional to the voltage applied to the plates, $E$. While the beam is passing between the plates it will be given a small deflection, $y_0$, in the $y$ direction, but the deflection, $D$, at the screen, is the quantity being measured, and its magnitude is determined mainly by the velocity, $V_y$, imparted to the electrons while passing between the plates. Deflection $y_0$ will be small compared to $D$, provided $L \gg l$; thus to a close approximation

$$D=L\tan\lambda=L\frac{V_y}{V_x}. \tag{1}$$

A force in the $y$ direction is exerted on each electron while it passes between the plates, which at any instant is equal to the product of the charge on an electron, $e$, and the electric field, $E/d$, at that instant. Thus the instantaneous acceleration of each electron may be expressed as

$$\frac{d^2y}{dt^2}=\frac{eE}{md}, \tag{2}$$

where $m$ is the mass of an electron. The velocity, $V_y$, of an electron in the $y$ direction, just as it leaves the deflecting-plate field, may be expressed as the integral of acceleration over the time of travel between the plates, $\tau=l/V_x$. The deflection of the beam on the screen is determined by its two velocity components at the time it leaves the deflecting plate field. Calling this time $t'$,

$$V_y=\int_{t=t'-\tau}^{t=t'}\frac{eE}{md}\,dt \tag{3}$$

and

$$D=\frac{L}{V_x}\frac{e}{md}\int_{t=t'-\tau}^{t=t'}Edt. \tag{4}$$

For $E$ constant, or for rates of change of $E$ such that it can be considered constant over the short time, $\tau$, the deflection becomes

$$D=\frac{LeE}{V_xmd}\,\tau=\frac{eLl}{mdV_x^2}\,E=KE, \tag{5}$$

which is the familiar form used in low-speed oscillography.

For $E=E_0\sin\omega t$ the integral in eq (4) becomes

$$\int_{t=t'-\tau}^{t=t'}Edt=E_0\int_{t=t'-\tau}^{t=t'}\sin\omega t\,dt=\frac{E_0}{\omega}2\sin\frac{\omega\tau}{2}\sin\omega\left(t'-\frac{\tau}{2}\right).$$

The deflection, $D$, for $E=E_0\sin\omega t$ becomes

$$D=K'E_0\sin\omega\left(t'-\frac{\tau}{2}\right)\frac{2}{\omega\tau}\sin\frac{\omega\tau}{2}, \tag{6}$$

where $K'=eLl/mdl$.

Transit time, $\tau$, is always very small. For values of frequency such that $\omega\tau<0.05$, eq (6) becomes the same as eq (5). As the frequency of the signal applied to the deflecting plates increases a value will be reached where the transit-time error is appreciable. The error consists of a lag in phase and an attenuation, both of which are a function of the product $\omega\tau$. When a steady high frequency is being measured, the phase shift is of little consequence, but the attenuation imposes a limit on the maximum frequency that can be accurately measured. A graph of this attenuation plotted against the frequency being recorded is shown in figure 13 for various values of oscillograph transit time, $\tau$.

When a steeply rising voltage pulse is being measured, the voltage applied to the plates may be taken to be $E=\beta t$, with $E=0$ for negative values of $t$. The integral in eq (4) must then be taken in two steps:

$$\int_{t=t'-\tau}^{t=0}\beta tdt+\int_{t=0}^{t=t'}\beta tdt=\frac{\beta}{2}t^2\Big|_{t=t'-\tau}^{t=0}+\frac{\beta}{2}t^2\Big|_{t=0}^{t=t'}$$
$$=-\frac{\beta}{2}(t'^2-2t'\tau+\tau^2)+\frac{\beta t'^2}{2}.$$

For $t'<\tau$, the first integral is zero, so the deflection becomes

$$D=\frac{L}{V_x}\frac{e}{md}\tau\beta\frac{t'^2}{2\tau}=K\beta\frac{t'^2}{2\tau}. \tag{7}$$

For $t'>\tau$ both integrals must be taken, and their sum is $\beta\tau(t'-\tau/2)$, and the deflection for $t'>\tau$ becomes

$$D=\frac{L}{V_x}\frac{e}{md}\tau\beta\left(t'-\frac{\tau}{2}\right)=K\beta\left(t'-\frac{\tau}{2}\right). \tag{8}$$

The percentage difference between the deflection obtained on an oscillograph with transit time $\tau$ and that which would be obtained with zero transit

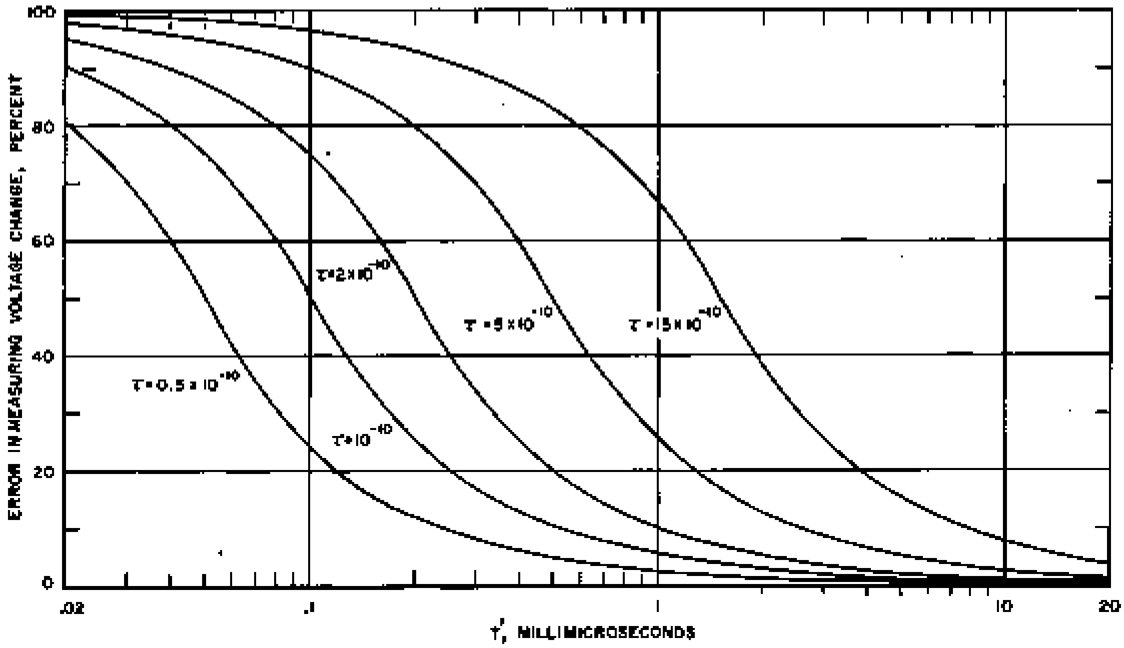


FIGURE 13. *Decrease in deflection at screen as frequency increases, for several values of electron beam transit time, τ, in seconds.*

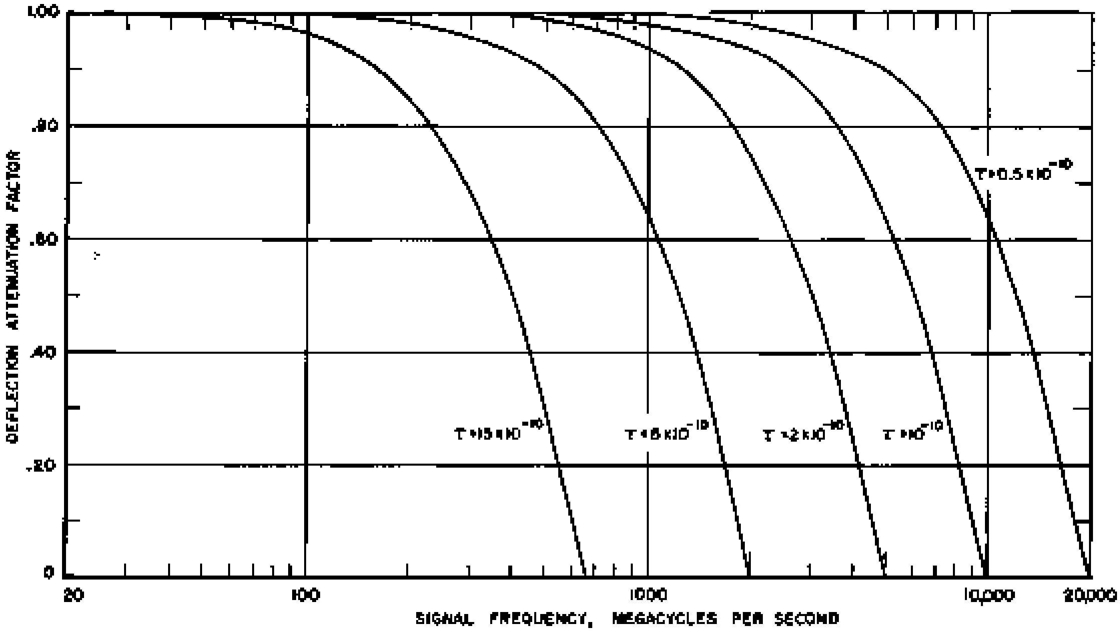


FIGURE 14. *Error in measuring a change in voltage that occurs during a given time, t'.*
The voltage is assumed to vary linearly with time. Results are shown for several beam transit times, τ, in seconds.

time, i. e., the percentage error due to transit time, becomes

$$\frac{K\beta t' - K\beta \frac{t'^2}{2\tau}}{K\beta t'} 100 = \left(1 - \frac{t'}{2\tau}\right) 100 \tag{9}$$

for $t' < \tau$, and

$$\frac{K\beta t' - K\beta \left(t' - \frac{\tau}{2}\right)}{K\beta t'} 100 = \frac{\tau}{2t'} 100 \tag{10}$$

for $t' > \tau$. In this derivation, $t'$ is the time interval over which it is desired to measure the change in voltage, and it has been assumed that the voltage varies linearly with time over this interval. From eq (9) it is seen that for measurements of change in voltage over time intervals, $t'$, equal to or less than the electron-beam transit time $\tau$, the errors will be very large (50 percent or greater). For $t' > \tau$, as seen from eq (10), the transit-time error decreases as time interval $t'$ increases. In order to keep errors to within 5 percent, the transit time, $\tau$, must be 1/10 or less of the time interval, $t'$, over which a change in voltage is to be measured. Curves of error in percentage, plotted against time interval $t'$, are shown in figure 14, for several values of transit time $\tau$.

WASHINGTON, April 11, 1956.