U. S. Department of Commerce
National Bureau of Standards

Research Paper RP2108
Volume 45, July 1950

Part of the Journal of Research of the National Bureau of Standards

Calibration of Precision Airplane Mapping Cameras

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An instrument is described that permits the registration of all the information necessary to calibrate a precision airplane camera on a single negative. Twenty-five collimators arranged at 7.5° intervals along two meridians at right angles provide optically distant targets. These targets are photographed by means of the camera to be tested. Measurements made on the negative yield information on the equivalent focal length, distortion, resolving power, prism effect, orientation of the lines joining opposite pairs of collimation index markers, and location of the principal point. This instrument was designed and built to make the calibration of precision cameras required for all precision cameras used in Government mapping projects. A brief account of the calibration of a typical camera and a discussion of the physical significance of calibrated focal length are given.

I. Introduction

The number of precision airplane mapping cameras submitted to the National Bureau of Standards for calibration has increased steadily since the first formal specification covering this type of camera was promulgated by the United States Department of Agriculture in March 1940. [1].[1] The demand for calibration of precision mapping cameras has increased so rapidly since 1945, that new equipment [2] for this work has been developed and built to supplement the precision lens testing camera that has been in use since 1935 [3]. The precision lens testing camera was primarily developed for use in photographically determining the equivalent focal length, distortion, and resolving power of lenses mounted in barrels or shutters. The instrument was later adapted to measuring these optical quantities for lenses mounted in cameras and further to locate the principal point in airplane mapping cameras [4].

With increased volume of work, it was evident that the method of calibration involving use of the precision lens testing camera possesses several deficiencies. First, four negatives are required, the making of which is a time consuming operation. Second, different sizes and shapes of the various makes of cameras cause difficulties in mounting them for test. Third, the entire field of the camera cannot be covered with a single photograph. The new camera calibrator was therefore designed to remedy these difficulties.

Inasmuch as the optical characteristics of lenses intended for use in airplane mapping cameras can be measured on the old precision-testing camera prior to their installation in cameras, the new calibrator was designed primarily as a pointing instrument. It provides 25 beams of parallel light from known directions. The information derived from it is therefore mainly of a directional nature and includes data on the uniformity or lack of uniformity in the distortion characteristics of lens-cone combinations. It permits more rapid and accurate location of the principal point, together with a determination of the equivalent focal length of the lens as mounted in the camera. It provides quantitative information on the magnitude of the prism effect and tangential distortion. Moreover it provides all this information on a single negative, thus avoiding possible changes that result from small movements between successive exposures. It provides a more stable support for the camera during test and also greater versatility in the types of cameras that can be tested.

The scope of the present paper is limited to a description of the instrument and the instruments

[1] Figures in brackets indicate the literature references at the end of this paper.

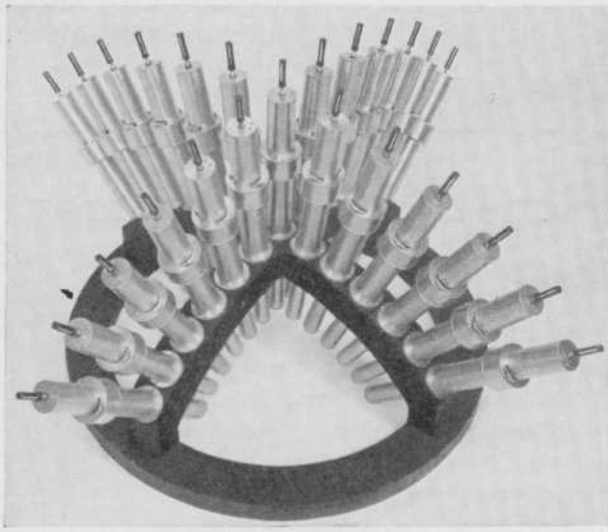

FIGURE 1. *Collimator bank of camera calibrator.*

Collimator bank before installation in the instrument. It consists of 25 collimators arranged in the form of a cross. When mounted in the camera calibrator, the central collimator points vertically upward with the remaining 24 collimators spaced at 7.5° intervals along the two meridians.

developed for use in its calibration. In addition, the calibration of a precision airplane mapping camera as conducted on the new calibrator is described. The manner of interpreting the negative is given for completeness.

II. Description of the Camera Calibrator

The camera calibrator and the details of its construction are shown in figures 1 and 2. Figure 1 shows the heart of the new instrument, which is a bank of 25 collimators arranged in the form of a cross. In use, it is mounted beneath a table as shown in figure 2. The central collimator points vertically upward, and the remaining 24 collimators are spaced at 7.5° intervals along the four arms of the cross from 0° to 45°. Departure from the 5° interval used in the Bureau's lens testing camera [3] is necessitated by space limitations. The collimators are of the fixed-focus type with the tubes cut to proper length so that each reticle lies in the focal plane of the corresponding objective. The collimators are mounted in a special casting, whose inner and outer envelopes are spherical. The collimator axes are normal to the spherical surfaces and correspondingly point toward the center of curvature. Accuracy of pointing of the collimators depends to a large extent upon the accuracy of counterbored holes and flanges by which the collimators are mounted, but the reticles may be moved about a little in the focal planes of the collimators so as to correct small deviations in the pointings. These adjustments are made with capstan screws, so arranged as to afford complete translational freedom of the reticle. After adjustments, clamping by appropriate screws guards against any subsequent movement. The clear apertures of the collimators are 18 mm in diameter, which is somewhat smaller than those of the lenses commonly used on precision mapping cameras. This, however, does not impair the instrument so far as its main purpose is concerned. To guard the collimators against disturbance the lamp housings, mounted independently on the arms of the cross, completely surround the target end of the collimators but do not touch them at any point. A 6-v. frosted flashlight bulb serves to illuminate the reticle. Provision is made for the insertion of a filter and glass diffusing screen between the lamp and the target.

The complete instrument is shown in figure 2. The table top is a flat steel plate having a large

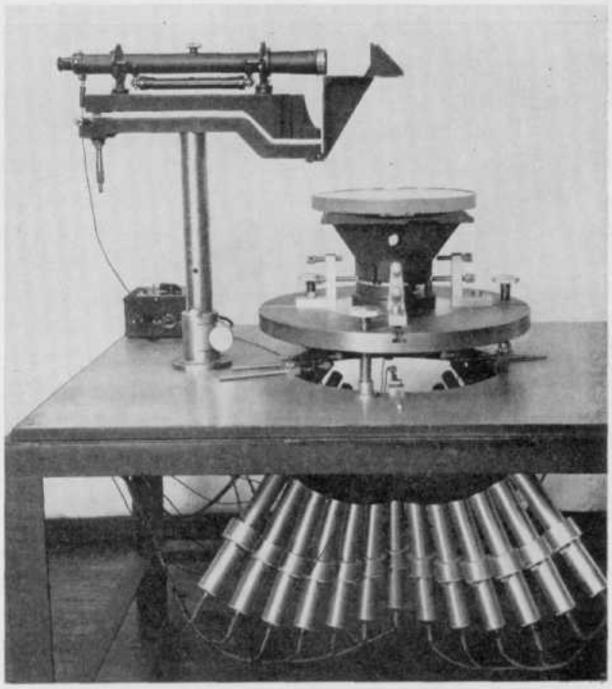

FIGURE 2. *Camera calibrator.*

Instrument as arranged for use. The camera under test points vertically downward. Its proper orientation is effected by the autocollimating telescope and the plane-parallel piece of glass placed on the focal plane of the camera. Images on a finished negative appear every 7.5° along each diagonal, with the collimation index markers registered on each side of the negative.

circular opening in its center. The collimator bank is mounted beneath the table and centered with respect to this opening in the table top. The camera holder consists of a tripod placed over the central opening in the table. A circular opening in the center of the tripod permits the light from the 25 collimators to reach the lens of any camera placed on the holder. The focal plane of the camera can be adjusted to normality with the axis of the central collimator with the aid of the adjusting screws on the tripod and the autocollimating telescope, whose axis is bent 90° by the prism mounted in front of its objective. The adjusting screws also provide a limited amount of vertical adjustment to compensate differences in distance between the front surface of the camera lens and the front end of the camera of the various types of camera. Four flashlight bulbs mounted on adjustable arms reaching out into the circular opening of the table top serve to illuminate the collimation index markers of the camera under test.

III. Test Chart

The test charts of the new instrument include resolving power patterns, although the limited size of the collimator objectives precludes the evaluation of the resolving power of the camera lens at its maximum aperture. However, the inclusion of the chart does permit study of the resolving power at reduced aperture. Since a new test chart for use in the precision lens testing camera had been under consideration for some time, it was decided to make a new chart at this time and to use a reduced version of it as the target reticle of the camera calibrator. This new test chart is shown in figure 3. The chart was first made on a large scale and then photographed to the desired size, using Eastman high resolution plates, type 649GH. The size of the target in the 0° collimator is 8 mm square. Large scale charts are also made and subsequently reduced for each of the various angular separations from the axis, so that the cosine correction in the vertical dimensions and cosine-squared correction in the horizontal dimension of each target pattern can be made. These corrections are necessary in order that the corresponding patterns will all be imaged at the same size regardless of angular separation from the axis for a given lens-camera combination. The three-line patterns reduce in size in a geometrical progression, the ratio being $\sqrt[4]{2}$, or approximately 1.189. Closer steps in the series are admittedly desirable, for example $\sqrt[6]{2}$, but this smaller ratio could only have been achieved at the expense of compressing the range of permissible values because of the small size of the reticle. The spacings present in the patterns in the reticle range from 2.3 to 161 lines per millimeter in 26 steps. These patterns, when imaged by the lens under test, are changed by the ratio of the focal length of collimator lens and lens under test. Table 1 shows the range of resolution in the image plane for the three focal lengths most frequently encountered. The nominal maximum *f*-number of the lenses are given. Since the diameter of the collimator lens reduces the effective aperture of the lens under test, the effective operating *f*-number is given for each lens. The line showing limit of resolution shows the maximum theoretical resolving power for each lens at its effective *f*-number. With this chart therefore, the upper limit of resolution is determined by the performance of the lens and limitations of the emulsion of the photographic plate and not by insufficient range of the resolving power patterns. A considerable amount of work has been done in other laboratories, wherein a circular target is used instead of a line target.

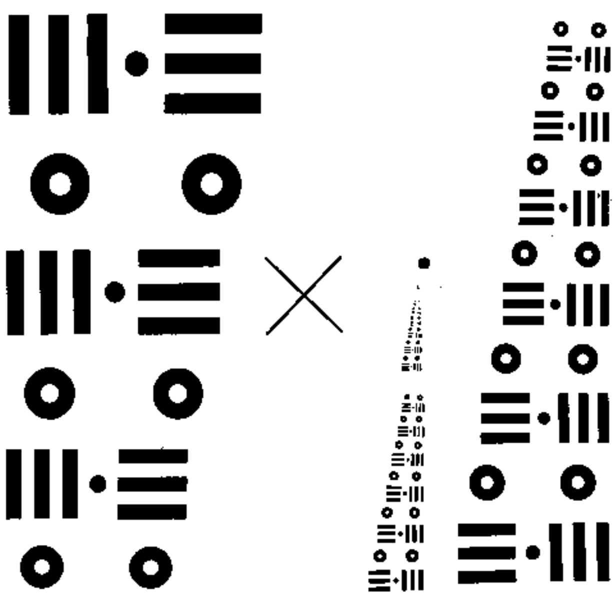

FIGURE 3. *Test chart.*

The test chart that is used in preparing the reticle for use in each collimator consists of patterns of parallel lines in two orientations with spaces varying in geometric progression by steps equal to $\sqrt[4]{2}$. The circle targets are included for comparative study of line and circle targets.

TABLE 1. *Limits placed on effective f-number by the diameter of the collimator objectives of the camera calibrator for three lenses of different focal lengths; the maximum theoretical resolution of these lenses operating at these f-numbers, and the range of the resolving powers obtainable for these different focal lengths provided by the test charts of the camera calibrator*

Equivalent focal length, mm	130	150	210
Usual nominal *f*-number	6.3	6.3	6.8
Effective *f*-number	7.2	8.3	11.7
Maximum theoretical resolution in lines per mm, as determined by effective *f*-number	198	172	122
Minimum resolution in lines per mm provided by test chart	4.7	4.1	2.9
Maximum resolution in lines per mm provided by test chart	332	289	205

A series of circular patterns is therefore included in the test chart for comparison of results obtained with it with the results obtained with the 3-line patterns.

The cross in the center of the chart is the fiducial mark with reference to which the angles separating the collimators are measured. When imaged on a camera test negative, the measured separations of the crosses for various collimators serve for the determination of equivalent focal length and distortion.

The National Bureau of Standards is at present using a high-contrast target and high-contrast fine-grained plates in the testing of photographic objectives and airplane mapping cameras. This is being done because existing Government specifications require that lenses for use in aerial mapping projects be tested under these conditions. It is recognized that there may be merit in the procedures followed in other laboratories that require the use of low-contrast targets and fast emulsions comparable to those used in actual aerial photography. Investigations are now in progress at this Bureau on the effect of contrast of target on performance of lenses. When this investigation is concluded, it is probable that a different type of target will be recommended for use in the testing of photographic objectives and cameras with corresponding changes in the required values of the resolving power for certification of lenses for Government use.

IV. Calibration of the Instrument

Adjustment of the angular relationships and measurement of the angles separating the collimators in each meridian is necessary before the instrument can be used in the calibration of precision cameras.

1. Adjustment of Angular Relationships

It is required that the angles between corresponding collimators on opposite sides of the center be as nearly equal as possible, and moreover that the axes of the beams of parallel light emerging from the collimator in a given meridian be parallel to and in near coincidence with a common plane. This adjustment is made before the collimator bank is mounted under the table. The collimator bank is placed face downward over the opening in the table top, as shown in figure 4. The lamp housings are removed to expose the target reticles. A front surface mirror is placed at the center of curvature of the spherical casting. The mirror, shown at A, is mounted in a special device that permits easy leveling and rotation about a horizontal axis.

A microscope equipped with a vertical illuminator is mounted over the reticle of the central collimator, and the mirror surface is brought into a position normal to the axis of the central collimator by means of the leveling screws, with the collimator, mirror, and microscope serving as an autocollimator. If now the $-7.5°$ reticle on one side is illuminated with the desk lamp, the

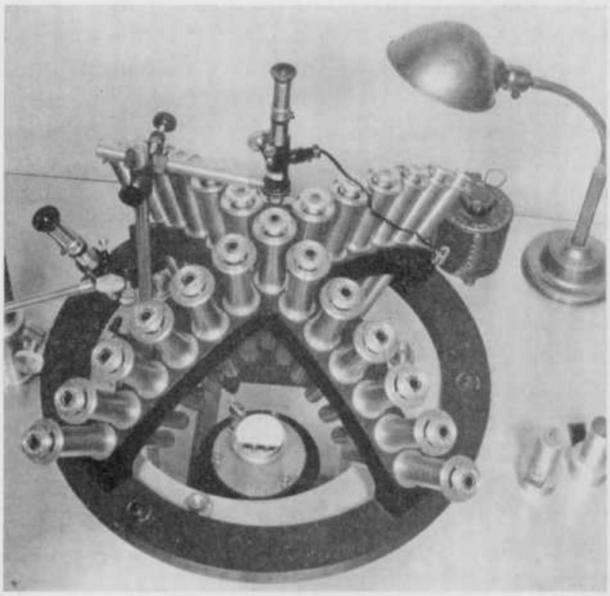

FIGURE 4. *Arrangement of apparatus for adjusting the angular relationships.*

second microscope may be used to view the reticle of the $+7.5°$ collimator on the opposite side while adjusting the reticle to bring it into coincidence with the reflected image of the $-7.5°$ reticle. The two $7.5°$ angles are then equal, and the axes of the $+7.5°$, $0°$, and $-7.5°$ collimators fall in one plane. The microscope with the vertical illuminator is then brought over the $+7.5°$ reticle and the mirror rotated and made normal to the axis of this collimator. The $+15°$ and $+22.5°$ collimators are then adjusted. The process is repeated by using the $+22.5°$ collimator as the autocollimator to adjust the $+30°$, $+37.5°$, and $+45°$ collimators. On returning and using the $0°$ collimator as autocollimator, all of the remaining collimators on the same meridian can be adjusted. Following this adjustment it can be stated that the axes of all collimators in a given meridian lie in the same plane, opposite angles are equal, and moreover angles between adjacent collimators are equal. The entire process is repeated to adjust the collimators on the meridian at right angles to the first meridian. It must be mentioned that, although the angles separating collimators along a given meridian are equal, the corresponding angles between collimators in different meridians are not exactly equal. The accuracy of the initial boring of the casting is attested by the fact that no difficulty was encountered in equalizing the angles and making the collimators coplanar, although the amount of lateral adjustment available was less than 1 mm.

2. Measurement of the Angles

The angles between adjacent collimators are determined by a comparison method using a reflecting biprism and telescope, as shown in figure 5. The standard of comparison is a biprism having a known angle between the two surfaces forming the obtuse angle. The two surfaces are aluminized and form two mirrors that maintain a constant angle with respect to one another. The angle formed between one surface and the extension of the other is $3.7600°$. Ideally this angle should be one-half of the $7.5°$ angle between the collimators, but its present value is sufficiently close, and moreover the amount of departure from $3.75°$ is known. The biprism is mounted in a special device that permits rotation of the biprism about a horizontal axis coincident with the edge separating the two aluminized surfaces. As shown in figure 5, this device is mounted on the camera holder in such a manner that the edge of the biprism coincides with the center of curvature and is normal to the plane in which the collimator axes for a given meridian lie. The error that arises from unsymmetrical use of the collimator objectives is made negligibly small by the use of collimator objectives that have very small longitudinal spherical aberration and making certain that the reticles lie in the focal plane of the objectives.

To measure an angle, the telescope, equipped with a micrometer for varying its pointing, is sighted at the center line of the biprism. The biprism is rotated about its edge until the image

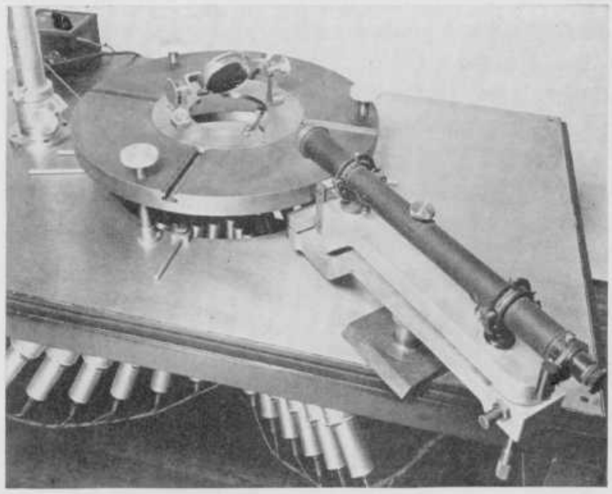

FIGURE 5. *Reflecting biprism for measurement of angles.*

reflected by one face of the biprism of the reticle in one collimator coincides with the cross-hairs of the viewing telescope. The lower telescope micrometer reading is recorded. The reticle of the first collimator is darkened and the second one illuminated. The image reflected by the second face of the biprism then appears in the field of view of the telescope. If the angle between collimators is exactly twice the biprism angle, no shift is evident. If the second image is shifted, the lower telescope micrometer is used to correct the pointing and the difference in micrometer reading noted. The difference in micrometer reading for the two pointings when translated into angular shift gives the difference in angle between the telescope and twice the biprism angle. By rotating the biprism, the angles separating

TABLE 2. *Values of the individual angles separating adjacent collimators and their deviation from 7.5° for each collimator bank*

Collimator bank	I		II		III		IV	
Nominal angle	Angle	Deviation	Angle	Deviation	Angle	Deviation	Angle	Deviation
0 to 7.5	7.4963	−0.0037	7.4946	−0.0054	7.5032	+0.0032	7.5041	+0.0041
7.5 to 15	7.4972	−.0028	7.4978	−.0022	7.5038	+.0038	7.5038	+.0038
15 to 22.5	7.4985	−.0015	7.4960	−.0040	7.5061	+.0061	7.5096	+.0096
22.5 to 30	7.4937	−.0063	7.4955	−.0045	7.5021	+.0021	7.5138	+.0138
30 to 37.5	7.4940	−.0060	7.4930	−.0070	7.5069	+.0069	7.5080	+.0080
37.5 to 45	7.4981	−.0019	7.4992	−.0008	7.5070	+.0070	7.5082	+.0082

each successive collimator can be determined for all the collimators lying in the meridian. The biprism holder and viewing telescope are then rotated 90° about a vertical axis, and the above process is repeated to obtain the angular separations of the collimators in the second meridian. The measured values of the angles are given in table 2 to show the agreement between the various angles, together with their departure from 7.5°. The tolerance on angle for boring the casting was ±1 min. (±0.0167°); the table shows that the results obtained indicate the accuracy of boring was amply close.

V. Operation of the Camera Calibrator

To calibrate a camera on the instrument, one must be sure that the focal plane of the camera is normal to the axis of the central collimator. To do this the operator adjusts the micrometers on the viewing telescope until the image of the center cross on the 0° collimator coincides with the cross-hairs of the viewing telescope. The camera cone is then placed on the camera holder, adjusted laterally until the lens of the camera is coaxial with the 0° collimator. The camera is then adjusted vertically until the images formed by the outer collimators show a minimum of vignetting, the camera cone is rotated until the two diameters along which the two perpendicular rows of images fall coincide with the diagonals of the focal plane. The camera is then clamped to prevent its further motion. An optically flat plane-parallel glass plate aluminized on its upper surface, is placed on the focal plane of the camera. Then using the viewing telescope as an autocollimating telescope, the focal plane of the camera is adjusted to normality with the axis of the autocollimating telescope by small adjustments of the screws supporting the camera holder. Flashlight bulbs attached to the ends of small rods supported by the table top are then moved about until the light from the bulbs illuminate each of the four collimation index markers mounted in the camera cone. The rods are then clamped. The plane parallel is removed; the room is darkened, and a photographic plate is placed on the focal plane of the camera, and an exposure is made. It is customary to place a heavy piece of flat glass over the photographic plate during exposure to hold the plate in place and help to flatten it.

The finished negative shows the images of the targets every 7.5° from the center to the corner along each diagonal. A schematic drawing of a finished negative is shown in figure 6. The collimation markers are registered on the sides of the negative, so that there is little likelihood of the target images being fogged by the light that registers the markers.

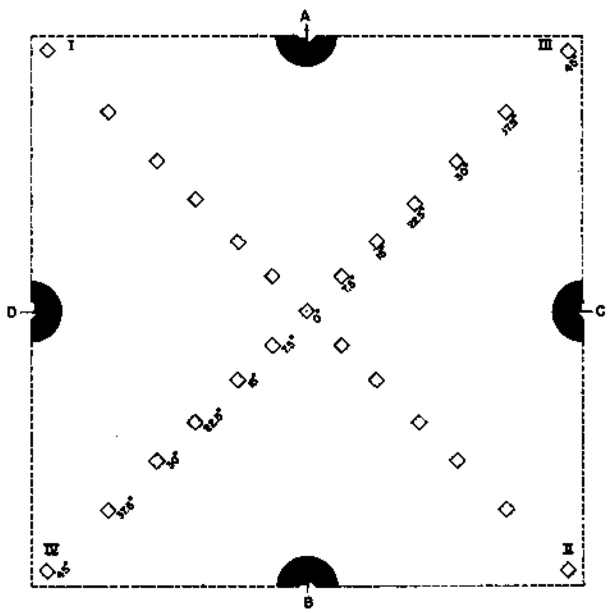

FIGURE 6. *Schematic drawing of test negative obtained with camera calibrator.*

Eastman V–G spectroscopic plates are generally used. In those instances, where a precision camera is to be used for infrared photography, Eastman IV–R spectroscopic plates are used.

VI. Calibration of a Precision Camera

Calibration of a precision camera consists of the location of the principal point with respect to the collimation index markers, determination of the angle formed by the intersection of lines joining opposite pairs of collimation index markers, determination of the equivalent focal length of the lens as mounted in the camera, determination of the distortion, and resolving power from the center to the corner of the image plane. Detail requirements relating to all these quantities are contained in specifications set up by various governmental mapping agencies concerning cameras that are to be used in Government mapping projects [5]. It is further stipulated that precision type cameras intended for use in these projects be tested for compliance with specifications by this Bureau.

It frequently happens that a camera on final examination does not comply with the requirements, but that it can be brought into compliance with the requirements by small lateral movements of the lens with respect to the collimation index markers or by small movements of the markers themselves. Following these adjustments, a recheck of the camera shows that it complies. To ensure that nothing can then happen to change this adjustment, holes are drilled and dowels placed so that the movable members remain fixed with respect to one another.

VII. Interpretation of the Negative

With the new camera calibrator, all of the information necessary to evaluate performance of the lens-cone combination is contained in a single negative. However, it is customary to make two negatives and combine the results of measurements from these to minimize any possible error. A schematic drawing of a typical negative is shown in figure 6. The markers are designated *A*, *B*, *C*, and *D* for convenience. The letter *A* is reserved for the markers bearing the arrow indicating direction of flight. The Roman numerals *I*, *II*, *III*, and *IV* serve to designate the rows of images formed by the four collimator banks radiating out from the center. The central image formed by the 0° collimator is designated the center cross (*C. C.*). The intersection of lines joining opposite pairs of collimation index markers is designated the center of collimation, (*C* of *C*).

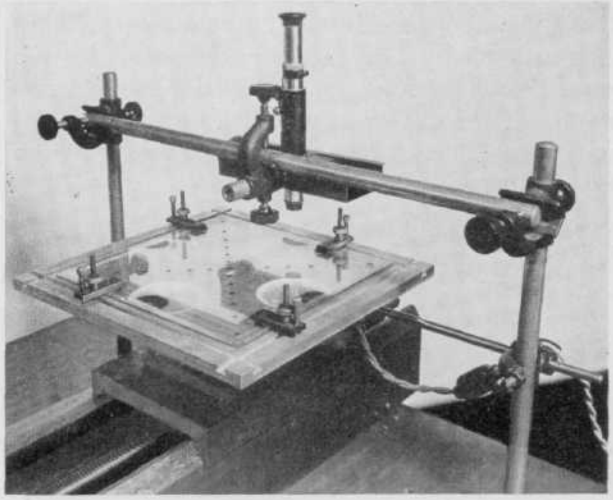

FIGURE 7. *Equipment used in the determination of the 90° condition.*

1. Determination of the 90° Condition

The first measurements on a given negative are made to determine whether or not the lines *AB* and *CD* are perpendicular within ±1 min. This determination is made on a special device shown in figure 7.[2] The central feature of this device consists of a piece of plate glass bearing four short radial diamond lines cut in the surface near the end of a 4.5-in. radius. The error in the positioning of the lines does not exceed ±5 sec. This plate is embedded with the lines uppermost in the metal frame that bears the plate clamps, so that the surface of the metal frame and the upper surface of the glass lie in the same plane. The negative is placed, emulsion side down, over the glass plate in such a manner that the index lines of the collimation markers and the four diamond lines are in near coincidence. The negative is then clamped down, so that no relative movement can occur during measurement. The metal frame is mounted on a spindle, so that a given marker can be brought under the viewing microscope. The separation of

[2] The authors wish to express appreciation to William P. Tayman, who developed this device for holding the 90° standard plate and the negative in contact during measurement, which represents a definite advance over the earlier equipment used in making this determination.

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diamond line and index line on the negative are then measured. This process is repeated for the remaining three markers. From these four measurements, the departure and the direction of departure of the collimation index markers from the 90° condition can be determined.

2. Location of the Center Cross with Respect to the Center of Collimation

The center cross, as used herein, is the point where an infinitely distant object point lying on a line perpendicular to the focal plane is imaged by the camera lens. The coordinates of the center cross are determined with respect to the rectangular coordinate system formed by the lines joining opposite pairs of collimation index markers having the center of collimation as origin. These measurements are made with the aid of the device shown in figure 8. The negative is placed, emulsion side uppermost, upon the flat metal plate and so oriented that the line AB is approximately parallel to the bench ways upon which the slide rests. The negative is then clamped to prevent any movement with respect to the metal plate. By moving the slide along the bench ways, the measuring microscope can be set in turn on marker A, center cross, and marker B and readings taken. From this data the lateral displacement of the center cross from the line AB can be obtained. The negative is then rotated

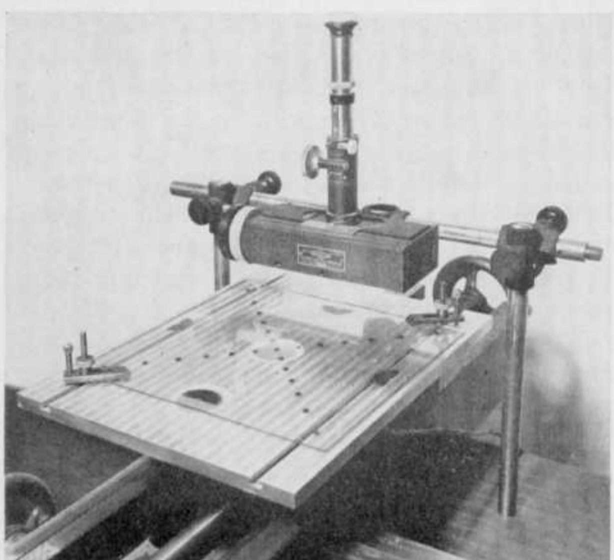

FIGURE 8. *Equipment used in locating the center cross with respect to the center of collimation.*

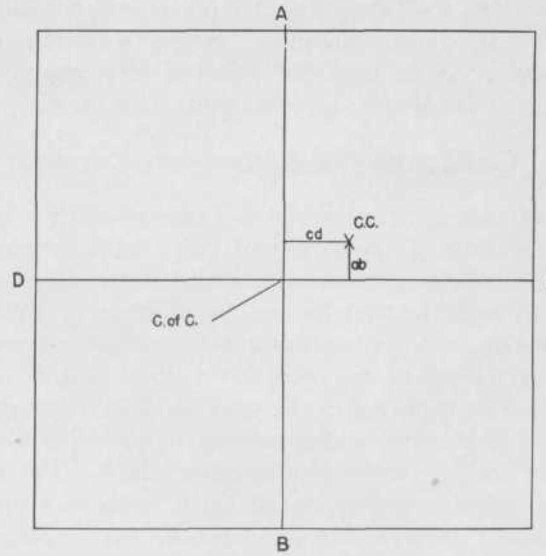

FIGURE 9. *Schematic drawing showing the center cross with respect to the center of the collimation.*

90° and similar settings made to determine the lateral displacement from the line CD. The results of measurement on a typical negative are shown schematically in figure 9. The magnitudes of the displacement are exaggerated for purposes of clarity in the figure.

3. Location of the Principal Point with Respect to the Center Cross

The principal point of precision-type aerial mapping camera is defined as that point where a perpendicular dropped from the near nodal point of the lens meets the focal plane [6]. For an ideal lens, the principal point and the center cross coincide. In practice, however, the center cross is shifted away from the principal point either because of prism effect in the lens or because of nonparallelism of the surfaces of the filter on the front of the lens, which forms a part of the optical system of the camera. This is illustrated in figure 10, which is a schematic drawing showing the displacement of the axial ray and rays inclined at angle β with the axis before and after placing a prism in front of an ideal lens.

In figure 10, O is the principal point and is the point where light from an infinitely distant object point lying on the axis of an ideal lens would be imaged if no prism effect is present. On interposing a prism having a refractive index of 1.5 and angle α the axial ray is bent away from the normal by an amount $\epsilon_0=\alpha/2$ and now cuts the focal plane at O', which is now referred to as the center cross. To locate the position of the principal point with respect to the center cross the distance $O'O$ must be determined.

The determination of $O'O$ is made possible because the rays inclined at angle β are also displaced by amounts ϵ_1 and ϵ_2, and cut the focal plane at points X_1' and X_2' instead of X_1 and X_2. By assuming that the ray passing through the prism and meeting the focal plane at O' is near the region of minimum deviation, it follows that we may regard ϵ_1 as equal to ϵ_2 and can compute the relative magnitude of $\bar{\epsilon}$ in terms of ϵ_0. The magnitude of $\bar{\epsilon}$ is the average value of ϵ_1 and ϵ_2 and always greater than ϵ_0 and $X_2'\,X_2=X_1'\,X_1>O'O$. None of these quantities can be measured directly, but their magnitudes may be obtained as follows: From the figure,

$$O'X_2'=f[\tan(\beta+\epsilon_2)-\tan\epsilon_0]=OX_2'-O'O$$
$$O'X_1'=f[\tan(\beta-\epsilon_1)+\tan\epsilon_0]=OX_1'+O'O$$

or

$$O'X_2'=OX_2+X_2'\,X_2-O'O$$
$$O'X_1'=OX_1-X_1'\,X_1+O'O,$$

and

$$O'\,X_2'-O'X_1'=2(X_2'\,X_2-O'O)=\Delta D$$

since

$$X_2'\,X_2=X_1'\,X_1 \text{ and } OX_1=OX_2.$$

In the case of an actual lens having prism effect, X_1', O', and X_2' are the observed images and the separations $O'\,X_1'$ and $O'X_2'$ can be measured. Consequently, the value of ΔD can be determined for each pair of equal known angular separations from the axis from measurements on the negative. This information is used most conveniently by comparing the measured values of ΔD with values of ΔD computed for a definite set of values of prism angle, α, focal length f, and separation angle β. The computations can be readily performed and the observed values of ΔD can be interpreted in items of α sufficiently accurately by this process, because ΔD is linear in α for a fixed angle β for small value of prism angle α. Table 3 gives the computed values of $\bar{\epsilon}$, $X_2'X_2$, $O'O$, and ΔD for the case of a lens having a focal length of 150 mm combined with a prism of index 1.5 having an angle α of 0.05°. It may be noted from the table that ΔD increases rapidly with increasing β. For the case shown, ΔD is negligible at 7.5° but increases to 0.3 mm at 45°. It is evident from the table that the value of $O'O$ deduced at the wider angles is more sensitive than the value determined at the narrow angles. It is also clear that the presence of appreciably large prism effect may cause marked asymmetric distortion.

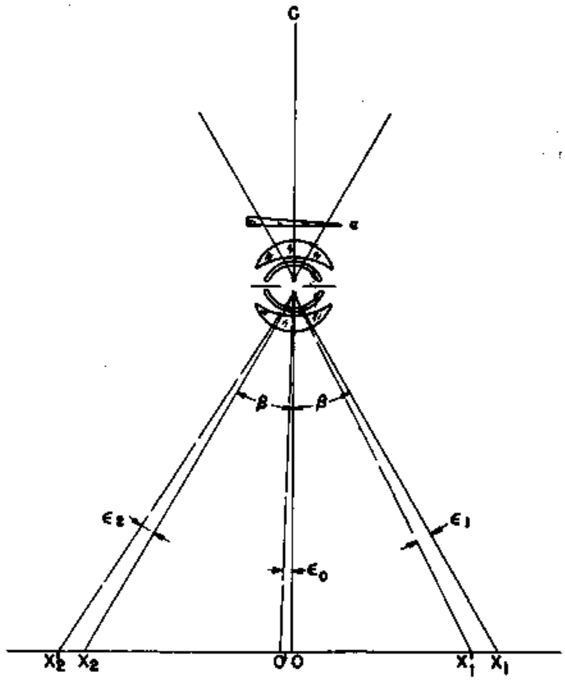

FIGURE 10. *Schematic drawing showing the image shift produced by interposing a thin prism in front of an ideal lens.*

TABLE 3. *Image displacements of various points in the focal plane for a prism of index 1.5 and angle $\alpha=0.05°$ combined with a lens whose focal length is 150 mm*

β	$\bar{\epsilon}$	$X_2'X_2=X_1'X_1$	$O'O$	$\Delta D=O'X_2'-O'X_1'$
Degrees	*Degrees*	*mm*	*mm*	*mm*
0	0.0250	0.065	0.065	0.000
7.5	.0254	.068	.065	.006
15	.0265	.074	.065	.018
22.5	.0283	.086	.065	.042
30	.0316	.110	.065	.090
37.5	.0364	.151	.065	.172
45	.0435	.228	.065	.326

Although it is feasible to determine the prism effect from a single negative from the camera calibrator by this process, it is customary to combine the results from two negatives with the camera being rotated 180° about its axis of symmetry between the marking of each negative. This procedure increases the accuracy and simplifies some of the computations. The coordinates of the principal point *P. P.*, with respect to the center cross, are measured along the diagonals of the focal plane, as indicated in figure 11, because the calibrator is so constructed that the images from the various collimations lie along the diagonals.

4. Location of Principal Point with Respect to Center of Collimation

To locate the principal point with respect to the center of collimation, it is only necessary to combine the results obtained in section VII, 2 and 3. Care must be taken to avoid errors in sign and to make allowance for the fact that the measurements are made in two coordinate systems rotated 45° with respect to one another. Figure 12 shows the resultant obtained by combining the measurements illustrated in figures 9 and 11.

The usual requirement in precision mapping cameras is that the principal point must not be distant from the center of collimation by more than 0.03 mm. In the event that this requirement is not met, it is customary to adjust the markers with respect to the lens until the separation of principal point and center of collimation is less than 0.03 mm. When this has been done, dowels are placed so that no relative movement of principal point and center of collimation can occur.

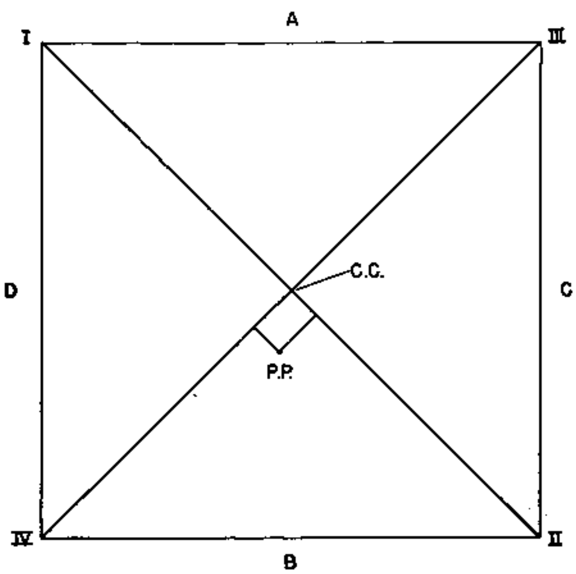

FIGURE 11. *Schematic drawing showing the principal point with respect to the center cross.*

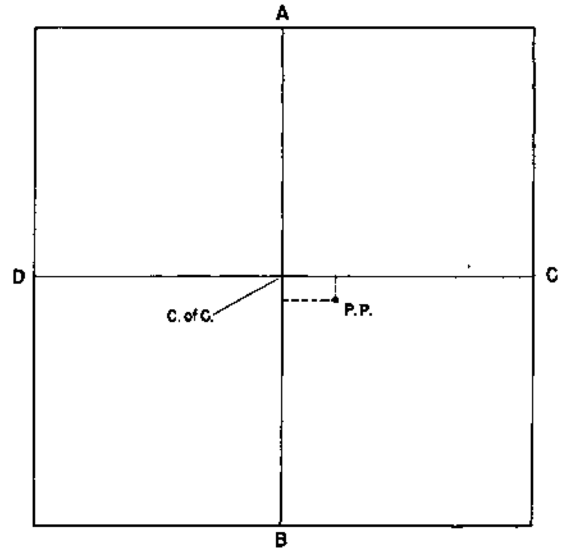

FIGURE 12. *Schematic drawing showing the principal point with respect to the center of collimation.*

5. Determination of Equivalent Focal Length and Distortion

The same negative from which the location of principal point is made is used to determine the equivalent focal length and distortion. In fact the same measurements, described in section VII, 3, for use in locating the principal point with respect to the center cross are used.

The equivalent focal length [6] is defined theoretically by the equation

$$f=\lim_{\beta\to 0}\frac{y^1}{\tan\beta} \qquad (1)$$

where y' is the distance from the principal focus to the center of the image in the image space focal plane of an infinitely distant object point that lies in a direction making an angle β with the axis of the objective. If a photographic objective were free from distortion, the quotient would be invariant with respect to β. For many photographic objectives, the distortion is negligible for points distant from the center of the useful field by not more than one-fifth of its radius. Consequently, it is often possible to obtain a satisfactorily accurate value of f by a single determination of β and y' for a point lying near the axis.

The negative made for an airplane camera cone on the camera calibrator has images at the center of the field and at six known angles proceeding in 7.5° steps from the center to the edge of the useful field. Consequently, on substituting the measured separation from the 0° to 7.5° image and the measured value of tan β into eq 1., the value of the equivalent focal length can be determined. The 0° to 7.5° separation occurs four times on the negative along radii separated by 90° of azimuth, so that four independent determinations of f are possible from a single negative. The accuracy of this determination is further increased by the fact that the measurements made on opposite sides along the same diameter automatically compensate for any error of plate tipping. It must be mentioned that practically the term equivalent focal length as used for a lens cone combination is actually the scale factor for use in interpreting distances measured on our aerial photograph in the central area. Inasmuch as photographic objectives are mounted in the aerial camera in such manner as to yield best over-all definition, it is clear that the equivalent focal length as above determined is the scale factor for the plane of best average definition.

Having established the value of the equivalent focal length for the lens camera combination, the distortion referred to the equivalent focal length can be readily found. To evaluate the distortion let y_1', y_2', y_3', . . . be the separation of the images on the negative from the central image corresponding to the angles in the object space β_1, β_2, β_3. Let products $f \tan \beta_1$, $f \tan \beta_2$, $f \tan \beta_3$, be computed. Then the values of the distortion D_1, D_2, D_3 . . ., are given by the relation

$$D_1 = y_1' - f \tan \beta_1$$

$$D_2 = y_2' - f \tan \beta_2$$

$$D_3 = y_3' - f \tan \beta_3$$

$$. \; . \; . \; . \; .$$

Positive values of the distortion indicates a displacement of the image away from the center of the negative.

6. Determination of Calibrated Focal Length

The lenses used in aerial mapping are not free from distortion. Consequently the value of the equivalent focal length determined for the vicinity of the axis is not necessarily the best scale factor for use in interpreting measurements made on the negative at points well removed from the axial area. If one holds fast to the value of equivalent focal length determined for the axial region, the measurements made on the negative in the axial region are free from distortion, whereas in the extra-axial regions the errors from distortion may become serious.

The customary procedure for alleviating this condition is to use a new scale factor that provides a better over-all accuracy in interpreting distances measured on the negative, although it does so at the expense of introducing negligibly small errors in the axial region. This new scale factor is called the calibrated focal length [6]. The term calibrated focal length, its significance, and the manner of its evaluation have been sources of considerable misunderstanding in photogrammetric circles since the inception of the term. It is proposed in the following paragraphs to present a study of the variation of equivalent focal length and distortion that occur for a lens affected with distortion when the equivalent focal length is based on the expression

$$f = \frac{y'}{\tan \beta},$$

and β is allowed to take any value at finite intervals between 7.5° and 45°. It is hoped that careful consideration of this analysis will lead to better understanding of the physical significance of the calibrated focal length.

In table 4, the row marked y' lists the measured separations from the axial image of the images occurring at 7.5° intervals out to 45°. These measurements are from an actual negative made on a lens-cone combination. In the second row, values of f, the equivalent focal length, are given where each value of f is obtained from the equation

$$f = \frac{y'}{\tan \beta}.$$

It is clear that a different value of f is obtained for each value of β. This variation in f appears because of the distortion in the lens. It is possible to determine the values of the distortion for each of the values of f at each angular separation from the axis. This has been done, and the results are shown in the table. One of the natural consequences of this procedure is that, for whatever value of the equivalent focal length is determined, the distortion is necessarily zero for that value of β when the distortion is evaluated with respect to that particular focal length. The location and the magnitude of the positions of maximum positive and negative values also change for each value of f. These effects are shown graphically in figure 13. It may be noted in figure 13 that the curves of distortion are very similar, and that the

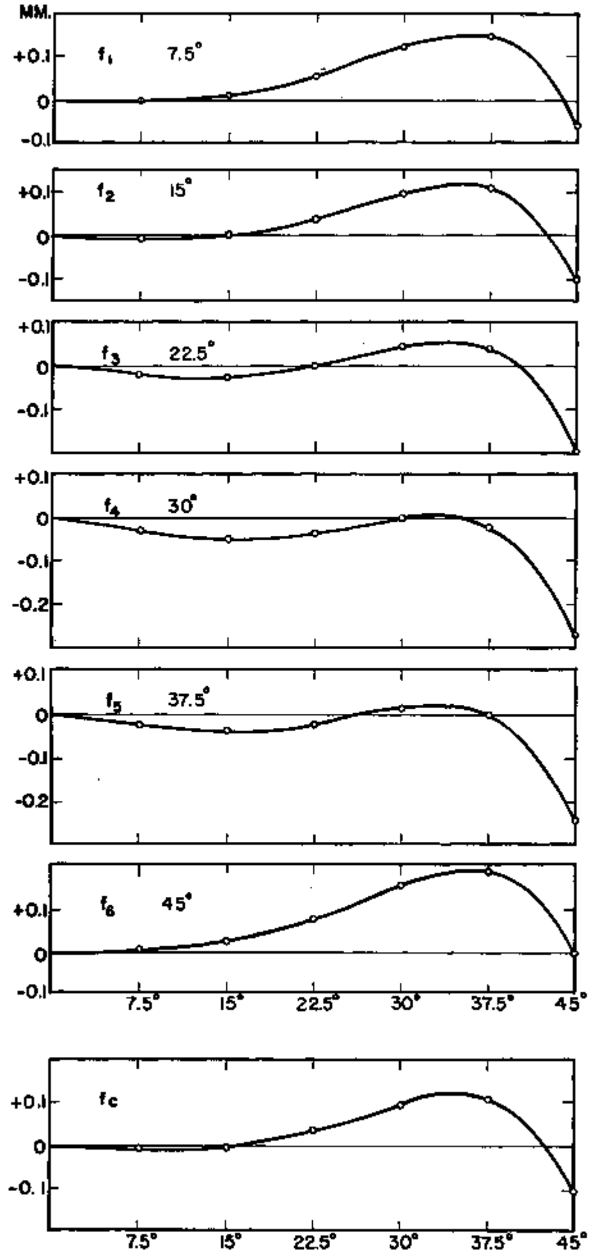

FIGURE 13. *Variation of distortion in the same image plane with equivalent focal length computed from the relation $f=y'/\tan\beta$ for varying angles of β.*

The lowest curve showing the distortion referred to the calibrated focal length indicates that the calibrated focal length is equal to the equivalent focal length for values of β of approximately 15° and 42.5°.

TABLE 4. *Variation of the values of equivalent focal length and distortion with angle upon which the computations are based for a typical lens*

	7.5°	15°	22.5°	30°	37.50°	45°
	y'_1	y'_2	y'_3	y'_4	y'_5	y'_6
	mm	*mm*	*mm*	*mm*	*mm*	*mm*
y'	20.064	40.487	62.182	88.112	117.086	152.345
	f_1	f_2	f_3	f_4	f_5	f_6
$f=\frac{y'}{\tan\beta}$	152.400	152.443	152.535	152.614	152.589	152.345
Distortion referred to the equivalent focal length $f=y'/\tan\beta$						
	7.5°	15°	22.5°	30°	37.50°	45°
f_1	0.000	0.012	0.056	0.124	0.145	−0.055
f_2	−.006	.000	.038	.099	.112	−.098
f_3	−.018	−.025	.000	.046	.042	−.190
f_4	−.028	−.046	−.033	.000	−.019	−.269
f_5	−.025	−.039	−.022	.015	.000	−.244
f_6	.007	.026	.079	.156	.188	.000
Distortion referred to the calibrated focal length f_c=152.451 mm						
f_c	0.006	−0.002	0.035	0.094	0.106	−0.106

variation of the curves is very like that which would be produced by rotating the entire curve above the 0° point.

The map maker is primarily interested in the scale factor that introduces the least error wherever used throughout the entire area of the photograph. Thus, while use of the scale factor equal to the equivalent focal length at 7.5° gives zero error within the 7.5° zone, it is at the expense of large error in the 37.5° zone. Inasmuch as there is usually no preferred area in a given photograph, it is obvious that it would be preferable to use a scale factor that would reduce error in the region of large error even if a small error were introduced in the region of zero error. Considering the curves in figure 13, it is clear that use of f_2 instead of f_1 as a scale factor will give reduced distortion in the 37.5° zone with the introduction of a small amount of distortion at 7.5°. Selection of one of the other values of f changes the error pattern either for the better or for the worse.

It is, at present, standard practice to consider the preferred error pattern as that one for which the absolute values of maximum positive and maximum negative distortion are equal. The preferred value of equivalent focal length for which this condition prevails is called the Calibrated Focal Length. It seldom happens that the calibrated focal length coincides with the equivalent focal length at one of the fixed angles (i. e. one of the 7.5° intervals) so that it is necessary to compute the increment Δf that must be added to the equivalent focal length determined at one of the fixed angles (usually the 7.5° angle) to yield the calibrated focal length. This may be done with the aid of the following formula:

$$\Delta f = \frac{D_m + D_n}{\tan \beta_m + \tan \beta_n},$$

where D_m and D_n are the values of distortion existing at the angles β_m and β_n referred to the equivalent focal length f. The calibrated focal length f_c is then obtained from the relation

$$f_c = f + \Delta f.$$

The above formula is a general one, and the new values of distortion referred to the calibrated focal length at angles β_m and β_n are equal in magnitude and opposite in sign. Usually β_m and β_n are the angles at which maximum positive and maximum negative distortion occur. These equations are essentially the same as those reported in a paper by Sewell [8].

The values of the distortion referred to the calibrated focal length are shown in table 4 and figure 13. The computation is made for $\beta_m = 37.5°$ and $\beta_n = 45°$. It may be noted that the same value of f_c will be obtained regardless of which value of f is selected as a base of computation, so long as the corresponding values of distortion are used.

It must be emphasized that the determination of the calibrated focal length involves no shift whatever of the position of the focal plane of the camera cone with respect to the lens. The calibrated focal length is simply that value of the equivalent focal length that serves as the preferred scale factor in interpreting distances measured on the photograph.

7. Determination of the Resolving Power

The camera calibrator provides an excellent means of checking whether or not the camera lens as mounted in the camera is capable of producing usable definition from the center to each corner of the field. If, as sometimes happens, the resolution along the four radii from the center to the corners of the field is not uniformly good this will be detected on the negative and thus minimize the possibility of a camera being certified as satisfactory when the resolution satisfies minimum definition requirements along one diameter of the field but does not do so at some points along the diameter at right angles to the first. Table 5 lists the observed values of the resolving power for a typical 6-in. lens in the focal plane of the camera in which it is mounted. These resolvings powers are measured at effective aperture $f/8.3$ and may be slightly higher than the values that would be found at maximum aperature.

It is clear from this table that some variation exists in the values of the resolving power for the same angular separation from the axis on different meridians. For the lens shown, the variations are not such as to bring the resolving power down at any point to values lower than the usually specified minimum of 15 lines per millimeter for this type of lens.

TABLE 5. *Observed values of resolving power for a typical 6-in. lens as determined from a negative made with the camera calibrator*

Collimator bank	Resolving power for tangential lines in lines per millimeter with angular separation from the axis of—						
	0°	7.5°	15°	22.5°	30°	37.5°	45°
I	53	53	32	27	32	27	23
II	53	27	27	23	32	23	23
III	53	32	27	27	32	27	19
IV	53	39	27	19	27	23	23
Average	53	38	28	24	31	25	22
	Resolving power for radial lines in lines per millimeter with angular separation from the axis of—						
	0°	7.5°	15°	22.5°	30°	37.5°	45°
I	53	53	39	46	39	32	27
II	53	27	32	27	39	27	27
III	53	32	32	39	39	32	23
IV	53	39	32	32	39	32	27
Average	53	38	34	36	39	31	26

8. Flatness of Platen

The specifications for precision airplane mapping cameras [5] usually contain the requirement that the surface of the platen shall not depart from a plane by more than ± 0.0005 in. The platen is the flat surface against which the film is pressed to ensure its planeness during the instant of exposure. This requirement is included in the specifications, because small departures from flatness in this locating surface would be imparted to the film and would in turn affect the location of images on the final negative.

It is therefore customary in the course of calibration of a precision camera to check the platen for flatness. It has not been considered expedient at this laboratory to make a detailed contour map of the entire platen surface. Instead the practice here has been to check the platen for flatness along selected lines or diameters of the platen and to conclude that the platen is satisfactorily plane if no departure from planes in excess of the specified tolerance occurs along these lines.

A simple device, shown in figure 14, has been constructed that permits a rapid measurement of the departure from flatness along a selected line with an accuracy of ± 0.0001 in. The device consists of a metal beam supported by two legs set 8 in. apart. Midway between these two legs, a sensitive dial indicator gage is mounted with the lower end of the plunger, which motivates the gage lying in the line common to the two main outer supports. The two outer supports and the

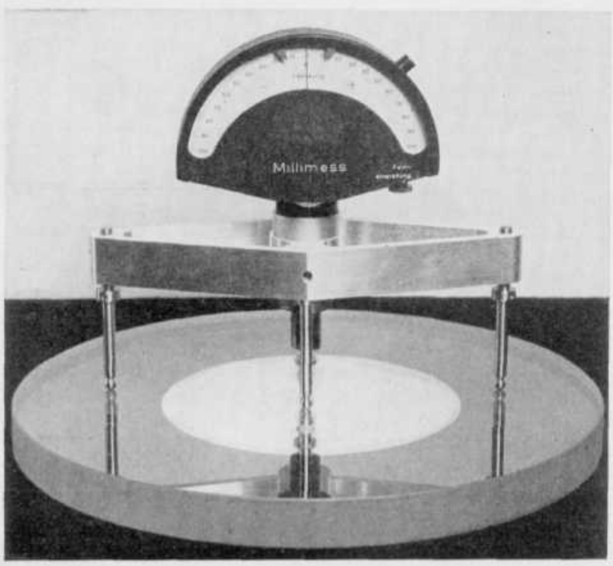

FIGURE 14. *Dial indicator device used for checking flatness of platen.*

The instrument is graduated in microns and permits the rapid measurement of departures from flatness with an accuracy of ± 0.0001 in.

central plunger have convex spherical surfaces at the lower portions, which touch the surface to be tested. The two smaller supports serve only to hold the instrument in a vertical position and to prevent the instrument from tipping over. The instrument may be rocked on the two outer legs. In use the small front leg is in contact with the surface being tested, and the movement of the plunger is in a direction closely perpendicular to the surface.

The instrument is graduated in microns. Before each use, it is placed on an optical flat to ensure that the gage reads zero for a truly flat surface. It is then placed on the platen under test and the reading noted. The instrument is moved across the platen, rotated 90°, and moved across again. It is also moved about the platen in other orientations. If the pointer on the gage does not depart from zero by more than $\pm 13\mu$ during the course of this operation, the platen is adjudged satisfactory.

VIII. Discussion

The method used at this Bureau for determining the location of the principal point of airplane cameras has been in use for some years. This type of work was first performed on the precision lens testing camera and it is now being performed on the new camera calibrator. In recent years, the question has been raised whether the principal point as herein defined and determined is the proper point to use in the photogrammetric sense in the interpretation of aerial photographs. In the absence of lens distortion, particularly of the asymmetric type induced by prism effect, there can be no question of the adequacy of the method and its interpretation.

However, in the presence of unbalanced distortion, there may be some merit in the criticisms that have been directed at this concept of the principal point. It is not the aim of the present authors either to refute or concur in the various other suggested concepts of what constitutes the best definition of principal point in cameras in the presence of asymmetric distortion. Rather, these matters are included in the present paper to show that cognizance of these alternative procedures is being taken; and it is hoped that ultimately some common basis may be found wherein these differences can be reconciled.

The most recent publication dealing with one of the alternative definitions of principal point is that of P. D. Carman of the National Research Council of Canada [9]. In that paper a proof is presented that indicates that a point having properties identical with the center cross as defined in section VII, 2 of the present paper, ought to be used as the principal point. This definition of principal point is accepted in Canada as evidenced by a paper by R. H. Field [10], who describes the method of locating the principal point used at the National Research Laboratories at Ottawa.

In the September 1948 issue of Photogrammetric Engineering, E. D. Sewell [8] presents the concept of a "Point of symmetry," which is a point about which all radial distortions are to be symmetrical. This point does not coincide with either the center cross or principal point as defined in the present paper, although all three points coincide in the absence of asymmetrical distortion.

To illustrate the differing locations of these points, consider the following case. A camera whose lens showed negligible prism effect was calibrated at this laboratory and the principal point and center of collimation brought into coincidence. Following this, a thin prism was placed in front of the lens with its base facing the direction A, and a test negative was made. The center cross was found to be displaced in the direction A by 0.239 mm. The principal point, as determined by the method described in this paper, was found to coincide still with the center of collimation within ± 0.002 mm. The point of symmetry was found to be displaced in the direction A by 0.496 mm (the computation being based on targets separated 37.5° from the axis and the 0° target following the method described by Sewell). The values herein given are far greater than one likely to be found in practice, as the angle of the prism used was approximately 12 min.

It is probable that in lens-cone combinations where little asymmetric distortion exists there is little to be said in favor of any one of these concepts over the others. When appreciable asymmetric distortion is present, the authors are not in a position to state which of these points ought to be used in photo-interpretation. This must be said in favor of the principal point as located by this Bureau: It is an invariant point for a given lens-cone combination; its location is not appreciably affected by placing filters of differing prismatic power in front of the lens; it is possible that our present procedure of reporting, however, should be amended to include magnitude of the prism deviation and its direction for a given lens-cone-filter combination so that the center cross can be located by those users who prefer it in their interpretation processes.

In the opinion of the authors, there are three courses that may be followed to eliminate these difficulties. The first course is to set an upper limit to the amount of prism deviation permissible on a lens-cone combination, possibly 0.015 mm for the axial ray. A lens-cone combination that has excessive prism deviation would then be unacceptable unless it were reworked to reduce the effect. That such a course is practicable is evidenced by the discussion contained in a paper [11] by J. V. Sharp and H. H. Hayes.

The second course of action is to make accurate measurements of the prism effect, either on the lens as mounted in its camera cone or on the lens alone. Having accurate knowledge of the magnitude and direction of the prism deviation, it should then be possible to neutralize the effect by using a filter on the front of the lens, which is itself a thin prism instead of the usual plane parallel. Other researches in progress at this laboratory indicate that such a course is feasible. It is possible that small residual effects may still remain, but it is believed that such residual effects will be negligible compared to the asymmetric distortions known to be produced by the prism effect.

The third course of action resembles the first, except that an upper limit on "tangential distortion" rather than prism deviation is set. This is preferred by J. V. Sharp, because it allows for the possibility [3] of the existence of a form of "tangential distortion" that deviates from the pattern predicted by the hypothesis that tangential distortion is produced by prism effect. Even in this case it is possible that a compromise involving partial neutralization of the prism effect may be advantageous.

The authors express their appreciation to other members of the staff of the National Bureau of Standards for assistance rendered during the vari-

[3] This possibility was brought to the senior author's attention in the course of an informal discussion.

ous phases of development of this equipment. Robert E. Ward of the Instrument Shop did a major portion of the machine work and in particular performed the highly accurate boring operation on the casting that holds the collimators in alinement. George L. Buzas constructed the target holders and the devices used in calibrating the instrument. Edgar C. Watts assisted in the design of testing devices and made the drawings used in this paper. William P. Tayman made the reticles used as targets in the collimators and is the principal operator of the completed instrument. Arthur A. Magill, Roland V. Shack, Irving Malitsky, and William P. Tayman assisted in the calibration of the instrument. Wilbur W. Brannon made the biprism used for calibration of the instrument.

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WASHINGTON, February 7, 1950.