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NIST IR 8530

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NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
U.S. DEPARTMENT OF COMMERCE

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David Flater
Raghu Kacker
Information Technology Laboratory

Doug Foxvog
Ontologist Emeritus

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July 2024

U.S. Department of Commerce
Gina M. Raimondo, Secretary

National Institute of Standards and Technology
Laurie E. Locascio, NIST Director and Under Secretary of Commerce for Standards and Technology

Publication History
Approved by the NIST Editorial Review Board on 2023-08-16

How to cite this NIST Technical Series Publication:
Flater D, Foxvog D, Kacker R (2024) A unified model of core metrological concepts. (National Institute of Standards and Technology, Gaithersburg, MD), NIST IR 8530. https://doi.org/10.6028/NIST.IR.8530

Author ORCID iDs
David Flater: 0000-0002-2546-2530
Doug Foxvog: 0000-0003-4223-4051
Raghu Kacker: 0000-0002-7666-3391

Abstract

The definitions of core metrological terms, especially *quantity*, *quantity value*, and *unit*, have been the subject of years of wrangling in standards organizations. A chronic disagreement exists over whether quantities are conceptualized primarily as mathematical entities or as physical ones. This report provides a unified model into which all of the competing definitions can be mapped and productively integrated. This model should enable software that interacts with measurement systems to be more insulated from changing and conflicting metrological terminology.

Keywords

Digitalisation; International System of Units; quantity; quantity value; software; unit; vocabulary.

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Preface

All core term definitions that appear herein are incidental examples used only to demonstrate and validate the methods, or are necessary to make the text comprehensible but are not meant as recommendations.

Acknowledgments

Thanks to NIST colleagues David L. Duewer, Paul E. Black, John Henry J. Scott, and Barbara Guttman for constructive commentary.

1. Introduction

With the era of modern metrology being 150 years old, one might assume that core concepts and vocabulary would have been harmonized and settled by now. In fact, there have been different understandings of core concepts throughout this history [1], and those differences continue to drive evolution of standards such as the International Vocabulary of Metrology (VIM) [2–4].

Any designed system is constructed and operates according to a system of concepts. The terms associated with these concepts are of no consequence to the operation of the system but are chosen to improve human understanding of the system. Unfortunately, the connotations and denotations of terms used in metrology have shifted over time in such a way that any direct assignment of terms in the context of software implementations of quantity calculus and metrological reasoning becomes a problem. Physical scientists can prioritize operational needs over terminology, but those working on automation and digitalisation [5] are not in a position to redefine the core terms of the scientific fields that they are given to work with or to arbitrate an end to chronic disagreements therein.

To make progress, in this report we choose the path of understanding and integrating the competing viewpoints in a unified model. Where terms have had multiple conflicting definitions, we place each concept within a cohesive structure. The result is not another vocabulary standard or concept system [6] but a model that software documentation, ontologies, and vocabulary standards can reference to distinguish their own positions within the broader context. Identifying concepts by their place in this model rather than with metrological terms allows software that interacts with measurement systems to be insulated from changing and conflicting terminology: a shift in the meanings of terms will no longer result in an invalidation of the system's description.

The remainder of the report proceeds as follows. Section 2 reviews examples of vocabularies and ontologies where different interpretations are evident. Section 3 provides a "big picture" semantic model centered on quantities (broadly understood, covering all known definitions). Section 4 shows how the differences among vocabularies and ontologies can be sorted out using the unified model. Section 5 makes concluding remarks.

2. Related work and examples

2.1. Vocabularies

The origins of core metrological concepts in modern use are attributed mainly to Maxwell [7]. Despite the existence of this common ancestor, significant differences among vocabularies have evolved.

A division between "realist" and "synthetiker" or "concrete" and "abstract" interpretations was observed by König [8] and later described by Silsbee [9] and de Boer [10, §2.3.2]. De

Boer explored the division at some length, but here we will only quote the introduction and invite the reader to consult the source document for further explanation:

> König called scientists who are used to working and thinking on the "abstract" level of mathematical models and theories *Synthetiker*, systematicians or theoreticians. At the other extreme then stand the *Realists*, metrologists, engineers or the pragmatic experimentalists, speaking the language of the "concrete" level. This distinction was taken over by Silsbee.
>
> Of course in reality scientists will always hold a position somewhere between these extremes and may switch between the two extremes, called here the abstract and the concrete levels. However, taking into account the existence of these two points of view is essential to avoid confusion in scientific discussion.

Silsbee in fact introduced a vocabulary (see Appendix A) that was intended to distinguish the two sets of concepts so that they could coexist without confusion, but it was not followed and standard vocabulary has since evolved apart from it.

A synthetiker interpretation of core metrological concepts does not mean pure mathematics. It is mathematics applied to the modelling of real physical phenomena.

We now provide some example sets of definitions on the spectrum from realist to synthetiker.

Example 1 has a realist flavor and is based on proposals that appeared in the Consultative Committee for Units Working Group on Core Metrological Terms (CCU-WG-CMT) that in turn were inspired by various versions of the VIM (see Appendix B).

> **Quantity:** property of a phenomenon, body or substance that can be compared [by ratio or by order] to others of the same kind
>
> **Quantity value:** expression of a quantity in terms of a reference
>
> **Unit:** quantity adopted by convention as a reference

Example 2 has a more synthetiker flavor and is a revision of previous work of R. Kacker [11]:

> **Quantity:** magnitude of a property of an individual phenomenon, body, or substance
>
> **Quantity value:** definite magnitude that may be assigned to a quantity of the same kind
>
> **Unit of measurement:** definition of a fixed quantity that is established to be one unit of that kind of quantity, for use as a reference to assign a value to another quantity of the same kind by comparing it with a physical realization of that definition

Example 3 is a translation of relevant excerpts from the 1978 edition of Deutsches Institut für Normung (DIN) standard 1313 [12]. It emphasizes the algebraic interpretation and has an even stronger synthetiker flavor. Notably, it specifically deems vectors and tensors to be single quantities, whereas most of the other examples quoted herein have assumed quantities to be scalar.

> (§2.1.1) The qualitative and quantitative description of physical phenomena (bodies, processes, states) is carried out with *physical quantities*, hereinafter referred to as *quantities*. Quantities—which can be scalars, vectors, or tensors—describe measurable properties. Any specific value of a quantity (the *quantity value*, called measurement value in the field of measurement technology) can be expressed as a product:
>
> Quantity value = numerical value × unit
>
> (§2.1.2) A *quantity value* is invariant with respect to a change of unit. The numerical value and unit of a quantity value move in opposite directions, as the factors of a constant product. This also applies to the conversion to a decimal multiple or submultiple of a unit formed with a prefix.
>
> For example: The values 10 cm and 100 mm are the same quantity value.
>
> (§4.1.1) From any set of quantities that are comparable with each other by measurement, one quantity value can be selected as a reference quantity and used as a *unit*.

The differences among these examples are not merely substitutions of terms but different choices and factorings of concepts. For example, if one thinks in terms of Example 1, the algebraic *quantity value* described in Example 3 is inseparable from a *quantity*. The *quantity value* described in Example 1 is instead an expression.

2.2. Ontologies

Core metrological terms have been formalized to some extent in many ontologies, including (in alphabetical order, with no pretense of completeness):

- Cyc [13]
- Dybkaer's ontology on property for physical, chemical, and biological systems [14]
- EngMath [15]
- Extensible Observation Ontology (OBOE) [16]
- FIBO Quantities and Units Ontology [17]
- FOUnt Ontologies [18]
- OM [19, 20]
- QUDT [21]
- Semantic Web for Earth and Environmental Terminology (SWEET) [22]
- Suggested Upper Merged Ontology (SUMO) [23]

- SysML/QUDV [24]
- Units of measurement ontology (UO) in the Open Biological and Biomedical Ontology [25]
- Wolfram Data Framework (WDF) [26]

Ontologies vary in the degree to which they formalize and refine semantics. Some ontologies do little more than assign machine-readable identifiers to informal English definitions, so they simply “pass through” ambiguities and conflicts that exist among reference vocabularies. But for those that do formalize semantics, the interpretations of the metrological terms quantity and quantity value are inconsistent, just as they have been inconsistent in physical metrology literature and vocabulary standards.

In the following examples, please note that quoted descriptive text and identifiers formed from concatenated words are as incoming from the various sources.

Example 4 is the Ontology of Units of Measure (OM) [19, 20]. The following descriptions and notes are from the Web Ontology Language (OWL) encoding of OM version 2.0.50:

> (Quantity) A quantity is a representation of a quantifiable (standardised) aspect (such as length, mass, and time) of a phenomenon (e.g., a star, a molecule, or a food product). Quantities are classified according to similarity in their (implicit) metrological aspect, e.g. the length of my table and the length of my chair are both classified as length.
>
> (Unit) A unit of measure is a definite magnitude of a quantity, defined and adopted by convention or by law. It is used as a standard for measurement of the same quantity, where any other value of the quantity can be expressed as a simple multiple of the unit. For example, length is a quantity; the metre is a unit of length that represents a definite predetermined length.
>
> (Measure) A measure combines a number to a unit of measure. For example, “3 m” is a measure.
>
> (Point) A point is an element of an interval scale or a ratio scale, for example, 273.16 on the Kelvin scale indicates the triple point of water thermodynamic temperature.

Unit is a top-level class. Length is a subclass of Quantity. Metre is a Unit of type SingularUnit. SingularUnit is a subclass of Unit. A Quantity can be related to a Measure or a Point by the hasValue property.

Example 5 is Cyc [13]. Cyc includes formal definitions of QuantityType, Quantity, ScalarQuantity, VectorQuantity, PhysicalQuantity, MeasurableQuantity, and UnitOfMeasure, along with other quantity-related concepts. Quantity is a specialization of ScalarOrVectorInterval. Each instance of Quantity is an abstract individual that is at most partly numeric and that is used as measurement or a rating of a thing. Quantity is partitioned into two disjoint specializations: ScalarQuantity and VectorQuantity. Each PhysicalQuantity is a ScalarQuantity that can (at least in principle) be the measure of a physical, spatial, or temporal aspect of an individual. Instances of Quantity that are instances of both MeasurableQuantity and VectorQuantity include directionalized velocities and accelerations. Instances of TotallyOrderedQuantityType needn't be numeric (e.g., Dirtiness) nor measurable (e.g., Happiness). A UnitOfMeasure is not a quantity but a function that maps a numeric value to a MeasurableQuantity. Log-ratio quantities in Cyc do not fall under the concept of Quantity but are handled by predicates such as (#$pHOfSample SAMPLE NUMBER).

Example 6 is the EngMath ontology [15]:

> (§3.1) A physical quantity is a measure of some quantifiable aspect of the modeled world. Quantities "admit of degrees" in contrast to qualities, which are all-or-none (e.g., being pregnant). . . . For our purposes, what makes quantities "quantifiable" is the ability to combine them with algebraic operations. Physical quantities can be meaningfully added, multiplied, and raised to real-valued exponents. The types of quantities determine the conditions under which operations are allowed and the types of the results.
>
> (§3.6) Units of measure are quantities themselves (positive, scalar, constant quantities). A unit of measure is an absolute amount of something that can be used as a standard reference quantity.
>
> (§4.2) Physical quantities and physical dimensions are objects in the universe of discourse, and are not linguistic elements (e.g., "variables" of constraint expressions).
>
> (§4.3) The identity of quantities is independent of physical objects that might be modeled.
>
> (§4.4) Quantities exist as values that are related to, but independent of units and numbers. Quantities can be compared, multiplied, etc., without ever converting to reals or considering units of measure.

Example 7 is QUDT 2.1 [21]. The following descriptions are quoted from the main Quantities, Units, Dimensions, and Types (QUDT) schema, version 2.1.35:

> (qudt:Quantity) A quantity is the measurement of an observable property of a particular object, event, or physical system. A quantity is always associated with the context of measurement (i.e. the thing measured, the measured value, the accuracy of measurement, etc.) whereas the underlying quantity kind is independent of any particular measurement. Thus, length is a quantity kind while the height of a rocket is a specific quantity of length; its magnitude that may be expressed in meters, feet, inches, etc.
>
> (qudt:QuantityKind) A Quantity Kind is any observable property that can be measured and quantified numerically. Familiar examples include physical

> properties such as length, mass, time, force, energy, power, electric charge, etc.
>
> (qudt:QuantityValue) A Quantity Value expresses the magnitude and kind of a quantity and is given by the product of a numerical value *n* and a unit of measure *U*.
>
> (qudt:Unit) A unit of measure, or unit, is a particular quantity value that has been chosen as a scale for measuring other quantities [of] the same kind (more generally of equivalent dimension).

Again we find that the differences among examples are not merely substitutions of terms but different choices and factorings of concepts. For example, Example 7 says that a quantity "is always associated with the context of measurement" while Example 6 says "The identity of quantities is independent of physical objects that might be modeled."

In the next section, we introduce a unified model that integrates the different concepts.

3. Quantity model

3.1. Base vocabulary

We introduce a model that can be used to interpret vocabulary, but one needs a vocabulary with which to explain the model. The terms introduced in this subsection are the minimum needed to support the exposition. None of these is defined in the VIM [4].

Expression: term, symbol, or ensemble of terms and symbols that denotes something in the universe of discourse

Magnitude: size, extent, intensity, etc. (whether great or small) [27, defn. 2a][9, §10.3.b]

Property: attribute of a phenomenon, body, or substance that can be distinguished qualitatively

N.B., in the literature, the term *attribute* might have both qualitative and quantitative connotations and thus overlap with both *property* and *magnitude* as defined here (cf. VIM 2 [3]).

3.2. Simplified model

Table 1 characterizes the relevant semantic space with four "map" layers sandwiched between a linguistic level above and an objective level below.

Below the map, we all presumably inhabit a single material reality that is subject to empirical investigation, but anything that we say about it relies on abstractions and language. "A map *is not* the territory" [28].

In map layer 1, the universe can be factored into distinct phenomena, bodies, and substances in more than one way; for example, waves vs. particles.

In map layer 2, properties can be attributed to phenomena, bodies, and substances in more than one way; for example, one can attribute a gravitational force to bodies, or one can instead attribute curvature to spacetime.

In map layer 3, any layer 2 property can be quantified in more than one way; for example, one might have velocity as a base quantity, or one might have only the ratio of distance over time. Layer 3 is where the concepts corresponding to the physical anchors of the International System of Units (SI) reside; for example, the speed of light in vacuum (in SI 9 [29]) or the mass of the international prototype of the kilogram (in SI 8 [30]). It also includes realizations of units and normal measurands. These categories differ from one another primarily in how they interact with layer 4.

In map layer 4, any layer 3 quantity can be formally modelled in more than one way; for example, acid strength can be modelled as K_a, pK_a, or an ordinal scale of "strong" and "weak." In Table 1, "linear ratios," "log ratios," and "ordinal ranks or grades" correspond to what were called the ratio scale, logarithmic interval scale, and ordinal scale, respectively, in Stevens' hierarchy [31]. The alternatives for possible formalisms are limitless; for example, as suggested in Ref. [32], one could include cyclic ratio quantities like direction or compass bearing where full cycles can be dropped.

If two objects are said to have the same mass, length, or what have you, it means that those quantities (mass, length, ...) are separate entities at layer 3 to which the same algebraic value in layer 4 could be assigned.

Layer 4 quantities can be further subdivided in several different ways. This will be discussed in Sec. 3.5.

Above the map, anything in map layers 1–4 can be referred to in more than one way; for example, a quantity can be denoted by different letters or symbols and expressed in different units. In addition, linguistic expressions can denote other linguistic expressions or even themselves.

The mathematical entities of quantity calculus occupy the box 4A (linear ratios). When the SI brochure states "The value of a quantity is generally expressed as the product of a number and a unit" [29, §2.1] it is describing the relationship between a quantitative expression in the linguistic level and the entity in box 4A that it denotes. But the use of the term *quantity* in general metrology practice is broader in every direction:

- Vertically: in the "realist" interpretation of quantity calculus (described in Sec. 2.1), a strong identification is made between formal, mathematical entities and operations in layer 4 and "real" physical quantities, properties, and processes in the layers below [10, §2.3.2][8, 9].

Table 1. "Big picture" semantic model of quantities.

Above the map
Linguistic expressions may denote anything in layers 1–4 or in the linguistic level itself.

Map layers

Layer	Description	Contents			
4	Algebraic	A. Linear ratios	B. Log ratios	C. Ordinal ranks/grades	Etc.
3	Realist	"Real" (realized or reified) physical quantities			
2	Qualitative	Physical properties			
1	Conceptual	Phenomena, bodies, substances			

Below the map
The universe or material world is the "territory" to which layers 1–4 provide a "map."

- Horizontally: log ratios and ordinal ranks or grades are often included in the scope of the term *quantity* even though they cannot be manipulated under the same algebraic rules as linear ratios. The SI brochure refers to "logarithmic ratio quantities" [29, §4]. The VIM defines "ordinal quantity" [4, 1.26].

3.3. Elaborated model

A more elaborate rendering of the multi-layer model is shown in Fig. 1.

The added detail brings out some additional features:

- In the algebraic layer, a distinction is made between quantities and values. Section 3.4 will explain.
- The concept of error belongs to the relationship between a single quantity value and the "real" physical quantity that it corresponds to. Its evaluation presupposes that a single "true" value exists, which can be an onerous ontological commitment to make [33, 34].
- The concepts of precision, bias, and accuracy belong to a measurement. In this model, the only place to discuss them is in the relationship between the set of quantity values produced by a measurement and the "real" physical quantity that they purport to quantify.
- Kinds of quantities are classes of quantities where the classification lattice is determined by metrological comparability. For example (from the VIM [4, 1.1]), radius and wavelength are subclasses of length—different in their most specific classifica-

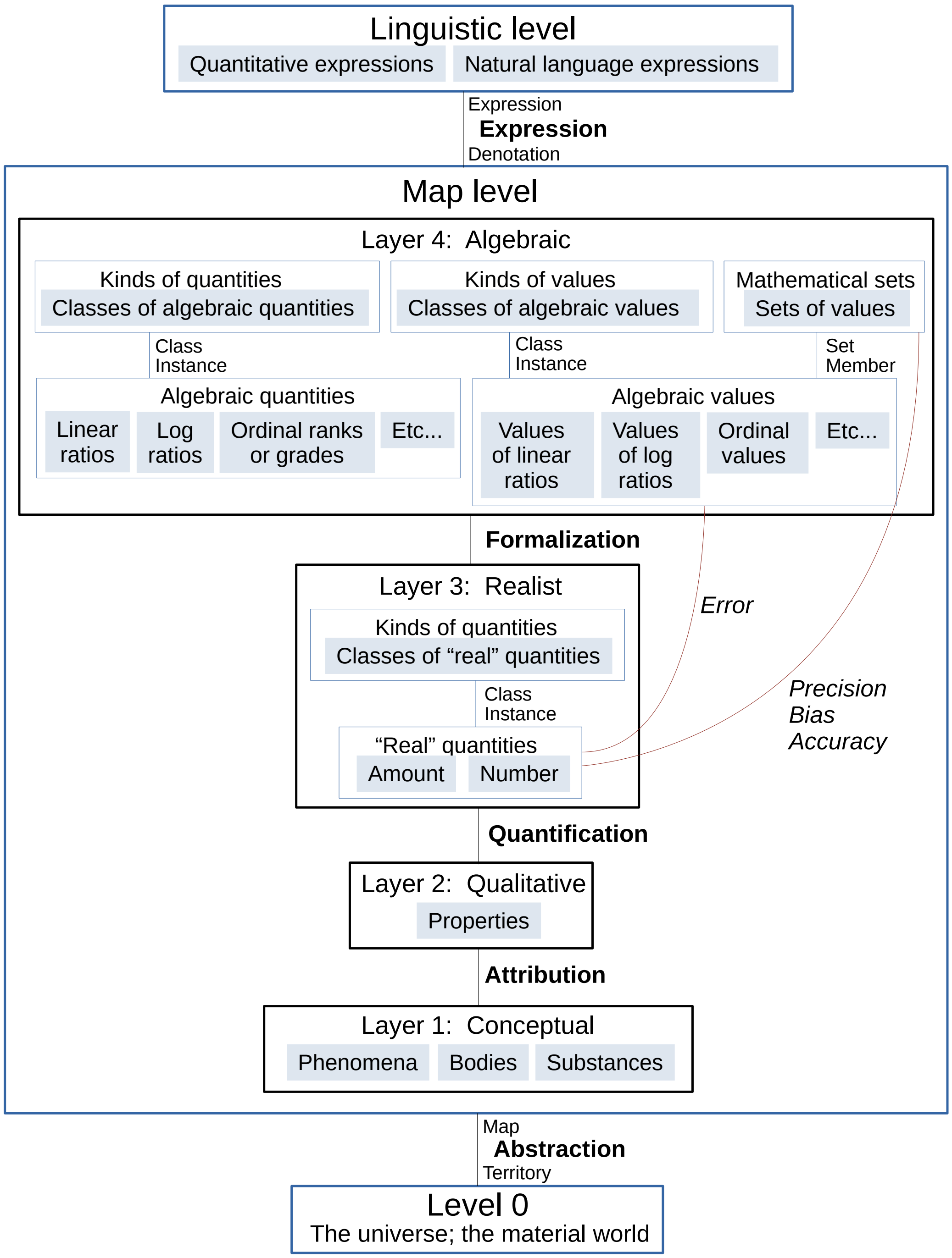

Figure 1. Elaborated semantic model of quantities.

tions but comparable in a more general sense as simple lengths if their differences are immaterial for the purpose at hand.

The values in layer 4 are not simply numbers: they are dimensional entities. Neither are they linguistic expressions: no specific choice of unit is entailed by them. They are the values of an abstract algebra that is dimensional rather than just numeric. VIM 3 has no more applicable term for them than *quantity*: *quantity value* in VIM 3 refers to the linguistic expression with specified unit, not to the abstract algebraic value [4].

3.4. Distinguishing quantities from values

The algebraic layer of Fig. 1 makes a distinction between quantities and values that comes from algebraic vocabulary. Quantities are normally identified with symbols such as x and y. Two quantities that have the same value are equal (e.g., $x = y = 5$), but two values that are equal are simply the same value (e.g., though it has many different expressions, there is only one 5). This means that quantities and values have different *identity criteria*, which are necessary and sufficient conditions for identity: mathematical equality is sufficient to identify a value but not to identify a quantity.[1]

The distinction just described is not always reflected in metrological vocabulary. Some would say that "a quantity value *is a* quantity." While that structure may suffice for some purposes, the difference in the identity criteria of quantities and values limits the ways in which they can be related in a valid conceptual model [35]. Formalizing metrological vocabulary thus requires disentangling these concepts, and interpreting sources where they remain entangled becomes a challenge.

The identity criterion for a layer 4 quantity is its definition (e.g., the speed of light in vacuum, the distance between two graduation lines on the international prototype metre bar at 0 °C, or an algebraic function of other quantities). The identity criterion of a quantity value is the value itself.

The three ways in which the value of a quantity might be determined are:

1. By definition,
2. By assignment of the result of a measurement, or
3. By algebraic derivation from other quantities.

When units were based on physical artifacts, the values of the associated quantities were set by definition, not measurement. The distance between two graduation lines on the international prototype metre bar at 0 °C was set to 1 m by definition. The corresponding distance on a copy was determined by comparison with the former (measurement). The distance on the international prototype metre bar and the corresponding distance on a

[1] In metrology, where measured quantity values come with uncertainty, we can say only that quantities are plausibly equal or that the results are metrologically compatible. Unlike equality, this relation is not transitive.

copy of it are two different quantities. They could have the same value in principle, but not in practice.

Now that units are based on defining constants, the values of the associated quantities are set by algebraic derivation from the defining constants. The values of the defining constants themselves are set by definition.

The value of the quantity *c*, the speed of light in vacuum, was once measured but is now defined. We still call the quantity *c* and think of it as effectively the same quantity. The definition of the quantity did not change; the change was only to how its value was determined.

On the other hand, as the SI units have been redefined from one edition of SI to the next, we are compelled to regard the associated quantities as different from one another even though they are called by the same names. The distance on the international prototype metre bar is not the same quantity as the length of the path travelled by light in vacuum during a time interval with a duration of 1/299 792 458 of a second. (The latter is a simplified explanation of the current official definition, which can be found in Ref. [29, §2.3.1].)

A quantity value could be attributed or assigned to any number of quantities, but its existence is not contingent on whether or not it is, in fact, attributed or assigned to any quantity. This is the same sense in which a real number exists regardless whether an expression for it has yet appeared anywhere.

3.5. Subdividing quantities

Layer 4 quantities can be further subdivided by full-range vs. non-negative, indefinite vs. definite, continuous vs. quantal, and exact vs. uncertain:

- Full-range vs. non-negative: Quantities corresponding to physical amounts or counts may be constrained to be non-negative or strictly positive when the excluded values have no physical interpretation. For example, there is negative charge, but there is no negative length. Negative quantities are produced in layer 4 by the ordinary rules of algebra (subtraction) even if all of the input quantities are constrained to be positive.
- Continuous vs. quantal (from Ref. [36]): A quantal quantity is a quantity that is constrained to exist in integral multiples of a quantum. The relevant dictionary definition of quantum is "a minimum amount of a physical quantity which can exist, and in multiples of which it can vary" [37, defn. 5].[2]

[2] The more general term *discrete quantity* says only that the values are countable, not necessarily that they are evenly spaced, but it is commonly used with the latter sense.

- Exact vs. uncertain: An exact quantity is equal to the algebraic product of a real number and a unit (if continuous) or an integer and a quantum (if quantal).[3] An uncertain quantity is characterized by a probability distribution over an interval of exact quantities, which is frequently (but not necessarily) determined by an exact estimate and a standard uncertainty that are produced through measurement. The probability distribution and interval are continuous for continuous quantities, discrete for quantal quantities.
- Indefinite vs. definite (based on Ref. [38, §3]): The word quantity has been used both for a quantity with a *nonspecific*, *unspecified*, or *undetermined* magnitude (e.g., the charge on steel ball BB163, an electrical charge $Q = \lambda \cdot (1\ \mathrm{As})$, where λ is a dimensionless number $-\infty < \lambda < \infty$), and for a quantity with a *specific* magnitude (e.g., 1.6×10^{-19} As or 3 As or 96 500 As). The quantity input for measurement is regarded as an indefinite quantity. The output of measurement is an assigned value, which is a definite quantity.

Any measured continuous quantity is an uncertain quantity due to the limited precision and accuracy of the measurement procedure. A measured quantal quantity is potentially but not necessarily exact [39]. Everyday examples of inexact quantal quantities include person counts in census and vote counts in elections.

Uncertainty can be expressed by the number of significant figures (e.g., 1.238 mm vs. 1.238000 mm),[4] by a symmetrical interval (e.g., (1.238 ± 0.001) mm or 1.238(1) mm [40, §7.2.2]), by an explicit probability distribution (e.g., `runif(n,min=1.237,max=1.129)*mm` [42]), and by other means.

3.6. The role of units

Units play the critical role of binding levels and layers together ("anchoring") so that quantitative expressions have a physical meaning.

For a defining constant (in SI 9) or prototype-defined unit (in earlier editions of SI), an exact value in layer 4 is bound to an invariant (or ideally invariant) physical quantity in layer 3 by definition. Layer 4 incorporates an assumption that these "invariant physical quantities" are truly invariant. The quantity values assigned to the defining constants are fixed and established (rather than measured) with zero uncertainty. Using these established values, additional values can be constructed through algebraic operations without introducing uncertainty. This is how the units of SI 9 are defined.

---

[3] For a quantity measured by counting, the quantum involved in this product is the algebraic representative of the entity or event type that was counted. Practices for expressing the result of a count vary and may involve a dimensionless counting unit, an implicit unit one as described in Ref. [29, §5.4.7], or no unit at all.
[4] Both the Guide to the expression of Uncertainty in Measurement (GUM) [40] and the Green Book [41] refer to significant digits, but neither one assigns an uncertainty interval as is suggested here.

For a linear ratio quantity, the association of layer 4 values with the layer 3 concept is determined through measurement using the previously defined units as a reference. The measurement process introduces uncertainty.

A unit is assigned multiple symbols at the linguistic level in different languages. These symbols are normally a written name, a spoken name, and a one or two character mathematical symbol that denotes the layer 4 value bound to the unit [29, §5.2]. While the mathematical symbols are standardized, the written and spoken names are ordinary words that vary among languages.

An expression in terms of an historical unit that lacks an exact definition in SI denotes the product of a real number and an uncertain quantity. Multiple references to a given historical unit in the same text presumably refer to the same quantity, so they can be added or subtracted as usual using the distributive property.

3.7. Kinds of quantities and their units

A complication to the functional view of units given in Sec. 3.6 is that different kinds of quantities may share a unit, in some sense, in SI [29, §2.3.4]. This practice has raised questions in physics [43, 44]. Nevertheless, the same practice has spread to information technology (IT), where the bit is ostensibly a unit both of data storage (equal to 1/8 of a byte, excluding historical uses) and of information-theoretic entropy (equal to the shannon).

When a unit symbol may represent quantities of different kinds, there is an opportunity to think that two quantities are comparable when they are not. In ordinary numerical algebra, we would normally assume that each occurrence of a symbol within the bounds of a particular discussion denotes the same thing. Moreover, we would assume transitivity and from $\mathrm{Hz} = \mathrm{s}^{-1}$ and $\mathrm{Bq} = \mathrm{s}^{-1}$ conclude that Hz = Bq. The kind of confusion between hertz and becquerel is unlikely to happen in human dealings, but implementations of quantity calculus in software can and do reach such a conclusion if an exception to normal algebraic rules has not been implemented.

The invariant property referred to by the definition of a unit must be ill-defined if it is supposed to be measurable as disjoint kinds of quantities. Emerson would agree that the resulting units must be regarded as distinct, even if they are called by the same name [45]:

> A unit of a quantity is necessarily a quantity of the same kind. Every kind of dimensionless quantity, *were it to have a unit*, would have to have a unit of its own kind, even if it bore the same name as the putative units of all other kinds of dimensionless quantities, notably *one*. Dimensionless quantities of all kinds are evaluated as numbers and are at the same time quantities of different kinds. The magnitude of a relative area, for example, cannot be meaningfully compared with that of a Reynolds number.

But distinguishable kinds are not always disjoint. Comparability is not a binary, yes-or-no decision. Two counts that are initially of different kinds, such as a number of neutrons and a number of protons, can be comparable in a more general sense (a number of nucleons). To generalize them and add them together is to declare that the differences of kind are immaterial for the purpose at hand. Whether it is appropriate to do this is not an aspect of the quantities but, rather, depends on the context of use.

In the next section, we apply the unified model to the examples in Sec. 2 to show how the different interpretations of core metrological concepts are reconciled.

4. Application of quantity model

4.1. Vocabularies

Referring to the model introduced in Sec. 3, a realist viewpoint leads to definitions that span map layers 2–4 and the linguistic level while a synthetiker viewpoint leads to definitions that concentrate on layer 4.

Table 2 gives a proposed placement of the vocabulary concepts described in Sec. 2.1 into the unified model. Where the unified model has clarified and distinguished concepts that were not distinguished in cited references, the interpretations are correspondingly uncertain.

In Example 1, having a realist flavor, quantities and units are strongly identified with properties residing in layer 2 of our model. Units are a subclass of quantities. Quantity value refers to a linguistic expression rather than an algebraic entity. Thus, one is guided to think in concrete rather than abstract terms. In this example and others, we have made the interpretation that *unit* does not extend into logarithmic ratios because the “number times unit” narrative breaks down.

In Example 2, having a more synthetiker flavor, everything is a kind of “magnitude,” but not in the sense of the word that we used in Sec. 3. In this example, magnitude, consistent with Ref. [27, defn. 2b], is a count noun that refers to an entity in layer 3 or 4 in our model. A quantity is possibly indefinite. A quantity value is necessarily definite. A unit has an established value.

In Example 3, the focus is narrowed to layer 4 and the difference between quantities and quantity values is more distinct. One is less likely to say “a quantity value *is a* quantity” with this framework; however, the example text “one quantity value can be selected as a reference quantity” undermines the distinction.

In Sec. 2.1 we highlighted that the algebraic *quantity value* described in Example 3 is inseparable from a *quantity* in Example 1. That difference is reflected by Table 2 in that the Example 1 quantity concept covers algebraic values while Example 3 associates algebraic values with a separate term.

Table 2. Proposed placement of vocabulary concepts in the unified model.

L indicates the linguistic level; A through C denote linear ratios, log ratios, and ordinal ranks or grades, respectively.

Example	Concept name	Map	Location
Example 1	Quantity	2–4	Properties, "real" quantities, algebraic quantities, and algebraic values (ABC)
	Quantity value	L	Quantitative expressions
	Unit	3–4	"Real" quantities, algebraic quantities, and algebraic values (A)
Example 2	Quantity	3–4	"Real" quantities, algebraic quantities, and algebraic values (AB)
	Quantity value	4	Algebraic quantities (definite; see Sec. 3.5) and values (AB)
	Unit	4	Algebraic quantities (A) with established values
Example 3	Quantity	4	Algebraic quantities (AB)
	Quantity value	4	Algebraic values (AB)
	Unit	4	Algebraic values (A)

Metrological vocabulary has often disagreed about whether the referent of the *value* term is in layer 4 or in the linguistic level [1]. If the linguistic interpretation is used, "3 km" and "3000 m" are different quantity values. If the algebraic interpretation is used, they are two expressions that denote one and the same quantity value. The distinction is important: the expression depends on the choice of unit, and many different units could be used to express one algebraic value.

If one is concerned only with layer 4, one can switch entirely to the language of mathematics as was done in Raposo's model [46]. But this leaves us without a way to interpret metrological vocabulary as it has been defined and used in practice, particularly the realist senses.

4.2. Ontologies

Table 3 gives a proposed placement of the ontology concepts described in Sec. 2.2 into the unified model based on our reading of the definitions quoted, related context in the original sources, and supporting materials. The previous disclaimer about uncertainty applies.

In Example 5, Cyc [13] provides axiomatic theories at our map layers 1, 2, and 4 as well as the linguistic layer but does not distinguish the non-linguistic layers. At layer 1, it axiomatizes objects, substances, and events. At layer 2, it axiomatizes properties of the layer 1 concepts, including QuantityType and subclasses of that concept. At layer 4, it defines Quantity, PhysicalQuantity, ScalarQuantity, VectorQuantity, MeasurableQuantity, and UnitOfMeasure (and many specializations of the quantity terms and many instances

Table 3. Proposed placement of ontology concepts in the unified model.

L indicates the linguistic level; A through C denote linear ratios, log ratios, and ordinal ranks or grades, respectively.

Example	Concept name	Map	Location
Example 4 (OM)	Quantity	3–4	Kinds of quantities (both layers)
	Unit	4	Algebraic quantities (definite) (A)
	Point	4	Algebraic values (AB)
	Measure	L	Quantitative expressions
Example 5 (Cyc)	ScalarQuantity	4	Algebraic values (AC)
	MeasurableQuantity	4	Algebraic values (AC)
	UnitOfMeasure	4	Algebraic functions (result is algebraic value (AB))
	TotallyOrderedQuantityType	4	Ordinal ranks or grades (C)
	pHOfSample	4	Log-ratio quantity (B)
Example 6	Quantity	4	Algebraic quantities and values (ABC)
(EngMath)	Unit	4	Algebraic quantities (definite) (A)
Example 7 (QUDT)	QuantityKind	2	Properties
	Quantity	3–4	Algebraic quantities (ABC) with associated “real” quantities
	QuantityValue	4–L	Algebraic values (ABC) or quantitative expressions (unclear)
	Unit	4	Algebraic values (A)

of UnitOfMeasure) along with other quantity-related concepts. MeasurableQuantity has the meaning of value at map layer 4. Cyc does not model the mass/length/duration of an individual entity as its own separate entity.

In Sec. 2.2 we highlighted that Example 7 says that a quantity “is always associated with the context of measurement” while Example 6 says “The identity of quantities is independent of physical objects that might be modeled.” That difference is reflected by Table 3 in that the Example 7 quantity concept protrudes into map layer 3 while the Example 6 does not.

The varying locations and coverage of concepts from different ontologies create work for industrial users of scientific data that refer to ostensibly standard models and vocabulary. The users must do their own analysis and construct yet another model that minimizes their own problems and confusion [47]. The unified model that we have provided reduces that burden.

5. Conclusion

In this report, we have

- Provided a unified model into which different definitions of metrological terms can be mapped and productively integrated,
- Demonstrated that realist and synthetiker interpretations of core metrological concepts can coexist, and
- Surveyed existing ontologies that can be interpreted using the unified model.

The volatility of and conflicts among standard definitions of metrological terms present challenges to automation. The connotations and denotations of terms used in metrology have shifted over time in such a way that any direct assignment of terms in the context of software implementations of quantity calculus and metrological reasoning becomes a problem. We have mitigated this problem by integrating competing conceptualizations into a single model. Identifying concepts by their place in this model rather than with metrological terms allows software that interacts with measurement systems to be insulated from changing and conflicting terminology.

Here are some specific examples:

1. For implementations of quantity calculus in high-level programming languages, conceptual integrity is improved by realizing the layer 4 linear ratio concepts with abstract data types and operations upon them *without specifying or exposing a unit*. It is difficult to explain and document this using metrological vocabulary that lacks a term for the abstract algebraic value as distinct from its expression in terms of a particular unit, is ambiguous about the physical and algebraic layers of reasoning, or shifts over time so that different readers arrive with conflicting assumptions. Our model insulates us from that problem by providing a compatibility layer with a specific place for the needed concept.
2. Whenever an attempt is made to formalize metrological vocabulary for use in IT, e.g., as either a class structure in a high-level programming language or the basis of an ontology, the constraints of logical integrity that are enforced by the IT system may reject the structure that a vocabulary standard entails. There are many ways that this can happen; e.g., invalid subsumption, class–metaclass confusion, class–individual confusion, or confusion among generic, partitive, and associative relationship types. The needs of a formal model for use in IT and the needs of a lexicon for natural language are different, and one cannot serve both at the same time. Our model enables a resolution by providing a stable foundation for a conceptual model, leaving the mapping to various vocabularies as a separate problem.
3. As was described in Sec. 4.2, industrial users of scientific data are forced to choose among reference models that take different interpretations of core concepts. The

burden is on them to integrate models and reconcile conflicts enough to meet the limited scope of their present needs. Our model reconciles the competing conceptualizations that exist so that industrial users need not be burdened with that analysis.

While our focus is IT, our model might nevertheless be useful to metrologists when the de facto vocabulary in use is imprecise or poorly factored for a particular purpose.

Our model leaves the question of what can or should be done to address the problematic variability of the kind of quantity that is associated with units (described in Sec. 3.7) open for further discussion.

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Appendix A. Silsbee's glossary

The following are selected definitions from Ref. [9, §10.3.b] reproduced with editorial changes.

Kind: That attribute of a physical quantity that distinguishes it qualitatively in regard to its physical nature, its relation to the phenomena, etc., from quantities of other kinds. Two physical quantities are of the same kind if operational methods are available for the meaningful comparison of their relative magnitudes.

Magnitude: That attribute of a quantity that distinguishes it quantitatively in regard to size, extent, intensity, etc., relative to other quantities of the same kind.

Measure: The number obtained by either (1) measuring a physical quantity by comparing it experimentally with a physical unit of the same kind, or (2) dividing a symbolic quantity by a symbolic unit of the same kind. Other writers have also used "magnitude," "value," "numerical value."

Quantity, physical: Any example of a "real" physical entity as conceived by the experimenter for the precise description of a phenomenon and operationally defined so as to be measurable. It is characterized by its kind and magnitude. Other writers have also used "entity," "physical entity," "magnitude," "quantity," "experimental quantity," "concrete quantity" for this concept.

Quantity, symbolic: Any example of an element that, in a mathematical model, corresponds to some physical quantity in nature. Other writers have also used "concrete quantity" (Maxwell), "abstract quantity," "mathematical variable," "magnitude," "idon" for this concept.

Realist: A fictitious character postulated to perform experimental measurement operations and to use mathematical manipulation on measure equations only. They deal only with physical quantities, physical units, and measure equations.

Synthetiker: A fictitious character postulated to use only quantity equations that express the relations among symbolic quantities. They derive symbolic units in terms of which they can formally write measures for symbolic quantities.

Unit: A particular sample of a quantity, either physical or symbolic, in terms of which the quantity can be measured or expressed quantitatively.

Unit, physical: A particular sample of a physical quantity of such magnitude that it is assigned the measure 1.

Unit, symbolic: A particular sample of a symbolic quantity of such magnitude that it is assigned the measure 1.

Appendix B. The VIM

The VIM is published by the Joint Committee for Guides in Metrology (JCGM) in English and French. It has been translated into other languages with the cooperation of National Metrology Institutes (NMIs).

Table 4, Table 5, and Table 6 quote the English and French definitions of *quantity*, *quantity value*, and *unit*, respectively, from the three editions of the VIM. Note the evolution of *value* in English from

1. An expression of a quantity, to
2. The magnitude of a quantity, and then
3. An expression of the magnitude of a quantity.

Table 4. Definitions of *quantity* in the three editions of the VIM.

Terms (EN, FR)	VIM 1, 1984–1987 [2]	VIM 2, 1993 [3]	VIM 3, 2008–2012 [4]
(measurable) quantity	An attribute of a phenomenon, body or substance, which may be distinguished qualitatively and determined quantitatively.	Attribute of a phenomenon, body or substance that may be distinguished qualitatively and determined quantitatively.	Property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference.
grandeur (mesurable)	Attribut d’un phénomène, d’un corps ou d’une substance, qui est susceptible d’être distingué qualitativement et déterminé quantitativement.	Attribut d’un phénomène, d’un corps ou d’une substance, qui est susceptible d’être distingué qualitativement et déterminé quantitativement.	Propriété d’un phénomène, d’un corps ou d’une substance, que l’on peut exprimer quantitativement sous forme d’un nombre et d’une référence.

Table 5. Definitions of *quantity value* in the three editions of the VIM.

Terms (EN, FR)	VIM 1, 1984–1987 [2]	VIM 2, 1993 [3]	VIM 3, 2008–2012 [4]
quantity value value of a quantity value	The expression of a quantity in terms of a number and an appropriate unit of measurement.	Magnitude of a particular quantity generally expressed as a unit of measurement multiplied by a number.	Number and reference together expressing magnitude of a quantity.
valeur d’une grandeur valeur	Expression d’une grandeur sous la forme d’un nombre et d’une unité de mesure appropriée.	Expression quantitative d’une grandeur particulière, généralement sous la forme d’une unité de mesure multipliée par un nombre.	Ensemble d’un nombre et d’une référence constituant l’expression quantitative d’une grandeur.

Table 6. Definitions of *unit* in the three editions of the VIM.

Terms (EN, FR)	VIM 1, 1984–1987 [2]	VIM 2, 1993 [3]	VIM 3, 2008–2012 [4]
measurement unit unit of measurement unit	A specific quantity, adopted by convention, used to express quantitatively quantities which have the same dimension.	Particular quantity, defined and adopted by convention, with which other quantities of the same kind are compared in order to express their magnitudes relative to that quantity.	Real scalar quantity, defined and adopted by convention, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number.
unité de mesure unité	Grandeur déterminée, adoptée par convention, utilisée pour exprimer quantitativement des grandeurs de même dimension.	Grandeur particulière, définie et adoptée par convention, à laquelle on compare les autres grandeurs de même nature pour les exprimer quantitativement par rapport à cette grandeur.	Grandeur scalaire réelle, définie et adoptée par convention, à laquelle on peut comparer toute autre grandeur de même nature pour exprimer le rapport des deux grandeurs sous la forme d'un nombre.

The French word *magnitude* is canonically used only in astronomy (star brightness) and geophysics (earthquake intensity) [48]. In the French VIM, the English word *magnitude* is replaced with *expression quantitative*, which makes the English and French versions of some definitions significantly different. The *expression quantitative* is simply the *valeur* as the French VIM defines it. Table 7 shows the impact on core terms for VIM 3.

Table 7. English, French, and English translation of French from VIM 3.

Terms	English	French	Translation
quantity grandeur	Property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference.	Propriété d'un phénomène, d'un corps ou d'une substance, que l'on peut exprimer quantitativement sous forme d'un nombre et d'une référence.	Property of a phenomenon, body, or substance, that one can express quantitatively in the form of a number and a reference.
value valeur	Number and reference together expressing magnitude of a quantity.	Ensemble d'un nombre et d'une référence constituant l'expression quantitative d'une grandeur.	Ensemble of a number and a reference constituting the quantitative expression of a quantity.
unit unité	Real scalar quantity, defined and adopted by convention, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number.	Grandeur scalaire réelle, définie et adoptée par convention, à laquelle on peut comparer toute autre grandeur de même nature pour exprimer le rapport des deux grandeurs sous la forme d'un nombre.	[Same as first column]

Table 8 shows the corresponding text from the published German translation of VIM 3 [49] along with the translator's notes and a translation back to English. Significantly, the translator's notes state that the definition of *unit* is wrong.

Table 8. Definitions from German translation of VIM 3 with translations back to English.

Terms (DE, EN)	Definition (DE)	Back-translation
Größe quantity	Eigenschaft eines Phänomens, eines Körpers oder einer Substanz, wobei die Eigenschaft einen Wert hat, der durch eine Zahl und eine Referenz ausgedrückt werden kann. *Anmerkung in der deutschsprachigen Fassung: Diese Definition (siehe auch Anmerkung 2) ist zu weit gefasst. Nach dem Verständnis im deutschen Sprachraum beschränkt sich der Begriff "Größe" auf verhältnisskalierte und intervallskalierte Merkmale, siehe auch DIN 1313. . . .*	Property of a phenomenon, body, or substance, where the property has a value that can be expressed by a number and a reference. *Note in the German language version: This definition (see also Note 2) is too broad. According to the understanding in the German-speaking world, the term "quantity" is limited to ratio-scaled and interval-scaled properties, see also DIN 1313. . . .*
Maßeinheit Einheit im Messwesen Einheit measurement unit unit of measurement unit	Reelle skalare Größe, durch Vereinbarung definiert und angenommen, mit der jede andere Größe gleicher Art verglichen werden kann, um das Verhältnis der beiden Größen als Zahl auszudrücken. *Anmerkung in der deutschsprachigen Fassung: Statt "Größe(n)" muss es hier korrekt "Größenwert(e)" heißen.*	Real scalar quantity, defined and accepted by agreement, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number. *Note in the German language version: Instead of "quantity(ies)" it should correctly read "quantity value(s)."*
Größenwert Wert einer Größe quantity value value of a quantity value	Zahlenwert und Referenz, die zusammen eine Größe quantitativ angeben.	Numerical value and reference, which together specify a quantity quantitatively.