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Abstract

The pixel is often used as a unit of measurement in information technology. Unfortunately, there is not a simple, consistently-applicable answer to what it is a measure *of*. This report analyzes the pixel unit through the lens of metrology and shows how one entity, the pixel, actually gives rise to a group of different units.

Keywords

Measurement; metrology; pixel; unit.

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1. Introduction

As a broad generalization, pixels are the elementary constituents of raster graphics images, imaging sensors, and displays. The term is etymologically derived from *picture element*. This general sense of the term pixel has remained stable since it was introduced to the field of computing. But down in the details, the term means different things in different contexts. Following are some representative quotes, with links to later sections that apply to them:

1. "A pixel is a *point* sample. It exists only at a point. For a color picture, a pixel might actually contain three samples, one for each primary color contributing to the picture at the sampling point." [1] (Sec. 3.1)
2. "A pixel is a little square... a point sample of a continuous function... a fuzzy blob... three tiny colored rectangles... a tradeoff between blurring and aliasing... a quantization of intensity... a projection of the color spectrum... a transparency... a piece of colored glass... a bunch of other stuff... an information-destroying technique" [2] (Sec. 3.1)
3. "Smallest element that is capable of generating the full functionality of a display" [3, 845-32-060]. (Sec. 3.1)
4. "The area of the smallest element of a picture that can be distinguished from its neighboring elements" [3, 723-05-31] (Sec. 3.3)
5. "A pixel is an N-bit value, where N is the number of bit planes used in a particular window or pixmap (that is, the depth of the window or pixmap). A pixel in a window indexes a colormap to derive an actual color to be displayed." [4, Glossary] (Sec. 3.1)
6. "1 px = 1/96th of 1 in" [5, §5.2] ($= 264\frac{7}{12}$ µm $= 264.58\bar{3}$ µm) (Sec. 3.5)
7. "The ***reference pixel*** is the visual angle of one pixel on a device with a device pixel density of 96 dpi and a distance from the reader of an arm's length. For a nominal arm's length of 28 inches, the visual angle is therefore about 0.0213 degrees." [5, §5.2] ($\approx 3.72 \times 10^{-4}$ rad) (Sec. 3.6)
8. "When a coordinate or length value is a number without a unit identifier (e.g., '25'), then the given coordinate or length is assumed to be in user units (i.e., a value in the current user coordinate system). ... One *px* unit is defined to be equal to one user unit. Thus, a length of '5 px' is the same as a length of '5'." [6, §7.10] (Sec. 3.4)

In abstract senses, the pixel often serves as a unit of measurement in information technology (IT). Unfortunately, there is not a simple, consistently-applicable answer to what it is a measure *of*. To be fit for use, the definition of a unit must answer that question above all others. But the situation is far from hopeless. We simply need to analyze the pixel through the lens of metrology and identify the different units that can properly be associated with it.

The remainder of this report proceeds as follows. Section 2 reviews the concept of units in physical metrology. Section 3 reviews the uses of pixel units in information technology. Section 4 brings the previous two topics together to rationalize the interpretation of pixel units. Section 5 gives a summary and concluding remarks.

2. Units in metrology

The following quote is from the Preliminary of *A Treatise on Electricity and Magnetism* by James Clerk Maxwell [7], the earliest reference belonging to the canon of modern metrology:

> Every expression of a Quantity consists of two factors or components. One of these is the name of a certain known quantity of the same kind as the quantity to be expressed, which is taken as a standard of reference. The other component is the number of times the standard is to be taken in order to make up the required quantity. The standard quantity is technically called the Unit, and the number is called the Numerical Value of the quantity.

The model thus introduced acquired a more algebraic description through the evolution of quantity calculus, the method of doing calculations with physical quantities [8]. The product of a number and a unit is a single abstract value that can be expressed in an infinite number of ways, in terms of the same unit or others of the same kind.

Continuity is usually assumed as a defining feature of measured quantities [9, §A][10, §21][11–13]. For example, if x is a positive quantity, it is assumed that $x/2$ exists and is a positive quantity that is smaller than x. That assumption is false for counts (e.g., of bits, radioactive decays, or hen's eggs) and other "quantal" quantities that are constrained to exist in integral multiples of a quantum (e.g., electric charge, amount of substance, or photon intensity) [14]. The error introduced by treating these quantities as if they were continuous can be insignificant in context, but this has never been the case for those quantities whose values are expressed in pixels: being "off by one" in software can be very bad [15].

The International System of Units (SI) is the current standard of what was previously and more generally called the metric system. It defines units for time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity based on fundamental constants of nature. Units for other kinds of quantities are derived algebraically from these seven.

Counted quantities, such as a number of pixels, are misfits in the system. The current (9th) SI brochure [16] accepts the inclusion of counted quantities within the scope of the system but regards them as all being of the same kind, with the associated unit one (§2.3.3), or alternately "just numbers" (§5.4.7).

Unit one is an abstract algebraic construct. Referring to it does nothing to distinguish different kinds of counts from one another. In practice, we cannot afford the mistake of equating bits to bytes or neutrons to pixels, so we often indicate units of counting. Reconciling this practice with SI guidance on the use of unit symbols and names remains a topic of ongoing discussion [17–21].

The international measurement system is designed to ensure that measurements of the same kind of quantity taken from different contexts are meaningfully comparable. A precondition for this is that units are constant. They do not change size based on context. As shown by the examples in Sec. 1, some uses of the pixel as an apparent unit violate this criterion.

3. Pixel units in information technology

3.1. Countable entity type

Often, a count of entities is a surrogate for some other kind of quantity, like counting traffic lights to characterize distance along a street. But in its simplest and most honest form, a count is just a count. Sometimes, 5 pixels just means 5 pixels, and the only question is to which kind of pixel it refers.

Taking at face value the Xlib glossary's definition "A pixel is an N-bit value" [4, Glossary], we can interpret the pixel as a data type of size N b (b being the somewhat standard symbol for the unit bit). N, "the depth of the window or pixmap," is context-dependent and variable. A number of pixels, therefore, specifies a number of logical data values but not a particular amount of data.

There has been chronic confusion about the relationship between these stored data values and their analogs in the material world [1, 22]. In the simplest possible scenario, a pixel in a digital image, pixmap, or canvas would correspond 1:1 with a picture element of an image input device (scanner or sensor) or image output device (display or printer). Often, however, this is not the case. For example, in a camera sensor using a Bayer filter [23], different color components are neither sampled from the same point nor grouped in a 3:1 correspondence with full-color pixels. A direct mapping from sensor elements to image pixels is impossible, irrespective of scaling. Similarly, for various types of displays and printers, a direct mapping from image pixels to output elements is impossible at any scale. So, although a data value sometimes literally controls "a little square" on a display, the pixel entity is first and foremost an abstraction existing in the software viewpoint.

3.2. Context-dependent length

In software interfaces to displays, many parameters are passed as integers that signify a number of pixels. In these uses, the pixel serves as a context-dependent unit of length. It is equal to the [vertical or horizontal] quantum of length, which is also context-dependent.

These uses of number of pixels as a surrogate for length can be identified with the subtypes of length enumerated in item 3-1 of the International System of Quantities [24]. For example, looking at parameter naming conventions in Xlib:

- 3-1.2, width, breadth: unsigned int width
- 3-1.3, height, depth, altitude: unsigned int height[1]
- 3-1.10, position vector (specified via Cartesian coordinates): int x, int y and variants (e.g., xoffset, yoffset)
- 3-1.11, displacement: int dx, int dy

When quantal length and continuous length meet, there is error, truncation, or rounding that has to be dealt with in a context-appropriate manner. An example of this appears in the current snapshot of the Hypertext Markup Language (HTML) living standard: "User agents working with integer values for frame widths (as opposed to user agents that can lay frames out with subpixel accuracy) are expected to distribute the remainder first to the last entry whose unit is *relative*, then equally (not proportionally) to each entry whose unit is *percentage*, then equally (not proportionally) to each entry whose unit is *absolute*, and finally, failing all else, to the last entry" [25, §15.6].

3.3. Context-dependent area

Two parts of International Electrotechnical Commission (IEC) standard 60050 [3] define the pixel as a context-dependent area:

- Telegraphy, facsimile, and data communication / Facsimile operational characteristics, 721-14-10 (picture element, pixel, scanned element): the area of the finest detail that can be reproduced effectively on the recording medium
- Broadcasting: Sound, television, data / Television: Picture analysis and display—Video signals, 723-05-31 (picture element, pixel): the area of the smallest element of a picture that can be distinguished from its neighboring elements

It is not evident from these definitions that the pixel so defined was used as a unit of measurement, but in principle it could be.

3.4. User unit of length

The pixel of Scalable Vector Graphics (SVG) 1.1 (Second Edition) [6, §7.10] is the unit of length in the current user coordinate system. This case differs from the context-dependent lengths described in Sec. 3.2 in that the pixels are not discrete elements of any kind, and

[1] *Depth* in Xlib refers to bit depth or "bits per pixel (bpp)." It does not correspond to the sense of the word used in item 3-1.3.

the lengths are certainly not quantized. This stands to reason, as pixels are the elementary constituents of *raster* graphics images, and SVG is for *vector* graphics.

This eccentric pixel definition was deleted from SVG Tiny 1.2 [26] and the current SVG 2 draft [27].

3.5. Physical length

In the current version of Cascading Style Sheets (CSS), units of length can be "anchored" in one of two ways [5, §5.2]:

> For a CSS device, these dimensions are ***anchored*** either
>
> i. by relating the physical units to their physical measurements, or
>
> ii. by relating the pixel unit to the reference pixel.

If the first option is chosen, the pixel is defined as a physical unit of length that is equal to $264.58\bar{3}$ µm (1/96 in).

Given this definition of the pixel, it follows that px^2 denotes an area and px^3 denotes a volume. But generally speaking, powers of the pixel unit other than unity are not used in practice.

3.6. Visual angle (angular size)

Now, for the second option (anchoring to the reference pixel). CSS defines the *reference pixel* as follows [5, §5.2]:

> The ***reference pixel*** is the visual angle of one pixel on a device with a device pixel density of 96 dpi and a distance from the reader of an arm's length. For a nominal arm's length of 28 inches, the visual angle is therefore about 0.0213 degrees. For reading at arm's length, 1 px thus corresponds to about 0.26 mm (1/96 inch).

The "recommended" CSS pixel is a quantized approximation to that angle that has the *device pixel* as its quantum:

> For screen media (including high-resolution devices), low-resolution devices, and devices with unusual viewing distances... it is recommended that the pixel unit refer to the whole number of device pixels that best approximates the reference pixel.

This version of the CSS pixel is variable both in its size and in the number of device pixels to which it corresponds. If the device pixel is smaller than the reference pixel, the CSS pixel is 2/3 to 4/3 times the reference pixel, or approximately $(3.72 \pm 1.24) \times 10^{-4}$ rad, with the largest deviation occurring when the device pixel is 2/3 times the reference pixel. If the device pixel is larger than the reference pixel, then the CSS pixel is the device pixel however large it may be, because a zero angle would mean a failure to produce output.

3.7. Resolution

Consider an image, sensor, or display that is described as 1000 px high. In the case of context-dependent length (Sec. 3.2), using the symbol H for the height of the example image, we could say $H = 10^3$ px, $H/\text{px} = 10^3$, and $\text{px} = 10^{-3}H$.

Resolution is subtly different. Without reference to any length, to say that an image is 1000 px high means that it is divided into 1000 separately-addressable rows of pixels. In lieu of the length H, we have only the image itself—an entity that is constituted of or reconstructed from pixels, "picture elements."

Although one can distinguish vertical, horizontal, and areal resolution as distinct quantities, one cannot apply the dimensional logic that applies to lengths and areas. Instead, one applies the logic of 2-dimensional arrays. Vertical resolution is the number of *rows* of pixels, horizontal resolution is the number of *columns* of pixels, and areal resolution is simply the number of pixels that comprise the image. Now, it is true that one can determine the number of rows or columns of pixels by counting the pixels in a single row or column, and this is in some sense the same as measuring the height or width of the image in pixels. But from a dimensional perspective, counting the pixels in a single row ($w \times 1$) or column ($1 \times h$) is just like counting the pixels in the whole image ($w \times h$). The horizontal pitch, vertical pitch, and areal footprint of the pixels do not enter into it.

This use of the pixel is quite common, as the resolution of sensors and displays is quoted in megapixels or more specifically as the numbers of columns and rows of pixels; e.g., the same display could be described as 1920×1080 px or as 2 Mpx.

4. One pixel, multiple units

Let us revisit the quotation given in Sec. 2: "Every expression of a Quantity consists of two factors or components. One of these is the name of a certain known quantity *of the same kind as the quantity to be expressed*" (emphasis added).

There are different kinds of quantities associated with the pixel that are not equivalent to a simple count of 1 pixel nor to one another, but all of which have been expressed in "pixels."

Number of pixels has been used as a surrogate for both length and area, leading to dimensional confusion. The sleight-of-hand involved in this form of surrogacy was broken down by Cooper and Humphry: "An entity is not a magnitude of a specific property. … Hence, an entity cannot itself be a unit, but the *magnitude* of a quantitative attribute possessed by an entity may, in principle, be a measurement unit." "Counting has a primary ontological function in enumerating entities, phenomena, and events; whereas it has secondary ontological function in estimating the magnitude of a continuous attribute such as mass or spatial volume" [13].

Just as a photon can be said to have different properties (frequency, polarization, velocity), so can a pixel (horizontal pitch, vertical pitch, angular size, bit depth, etc.). Some properties are universal and constant for all photons and pixels, while others depend on the specific case or object of measurement.

When a single unit symbol may represent quantities of different kinds, there is an opportunity to think that two quantities are comparable when they are not. In ordinary numerical algebra, we would normally assume that each occurrence of a symbol within the bounds of a particular discussion denotes the same thing. To mitigate the risk of confusion, we must treat quantities as distinct that SI defines as algebraically the same, "with the associated unit one." We must furthermore take care to distinguish different properties of the pixel as different units when a simple count of pixels is acting as a surrogate for another quantity, such as length, area, or angular size.

For a concrete example, Table 1 suggests one approach to distinguishing the various pixel units, with the assumption that vertical and horizontal pitch are equal.

The counting unit pel is subtyped to distinguish stored data items from elements of a display when the mapping is nontrivial. When no confusion can occur, the general counting unit can be used.

The abstract unit px is inherently context-specific. Comparison, however approximate, of lengths expressed in px requires knowledge of or an assumption about the possible range of display densities. When the abstract lengths being compared are from the same context, you know that the display density was unchanged, so they can be compared, and their ratio is meaningful. When the variable-sized CSS pixel is used (Sec. 3.6), comparisons of abstract lengths associated with different displays are correspondingly vague.

5. Conclusion

The pixel is often used in IT practice as a unit of measurement. From a metrological point of view, this unit is an amalgamation of different units that are used to express different kinds of quantities, the least of which is a simple count of pixels. Furthermore, some of the ostensible pixel units change size depending on context and are, thus, not units in the conventional sense.

A software implementation of quantity calculus is unlikely to detect if different kinds and/or sizes of "pixels" are mistakenly combined in a calculation if they are all identified in the same way. To prevent these errors, the units that are not comparable with one another should be distinguished. In human-readable text, this would mean using different symbols or at least different subscripts to the px symbol. In software, it would mean

Table 1. Example disambiguation of the pixel unit.

Kind of quantity	Definition	Context
Number of entities	1 pel is one picture element	Resolution; storage
Number of entities	1 dpel is one picture element of a display	Resolution
Number of entities	1 ipel is one picture element of a digital image	Storage
Length (abstract)	1 px is the quantum of length (in the case of integer-typed parameters) or the "user unit" of length (in the case of float-typed parameters)	Source code
Area (abstract)	1 px^2 derived from ibid.	Source code
Length (physical)	$l_{px} = 264\frac{7}{12}$ µm	CSS option 1
Area (physical)	$A_{px} = \left(l_{px}\right)^2$	CSS option 1
Visual angle (physical)	$\alpha_{px} = 2 \arctan \frac{1}{5376}$	CSS option 2

assigning distinct identifiers in whatever schema is used to identify units.

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