

NISTIR 8307

Interoperability Assessment 2019: Contactless-to-Contact Fingerprint Capture

John Libert
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John Libert
Bruce Bandini
Kenneth Ko
Shahram Orandi
Craig Watson
*Information Access Division
Information Technology Laboratory*

John Grantham
*Systems Plus, Inc.
Rockville, MD*

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Walter Copan, NIST Director and Under Secretary of Commerce for Standards and Technology

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4.2.5 Index Finger Only

The single finger error results summarized in Section 4.2.4 could be misinterpreted in that it is unlikely that a single-finger capture would be applied with equal likelihood to any of the ten (or eight) fingers available. The most likely choice for a single finger capture would be the index finger. At least one contactless (smartphone) device being used in a study being conducted in the United Kingdom (UK) captures only a single index finger, as does a contact device currently used by police in the UK¹¹. Hence, we evaluate FNIR in Figure 13 for Finger 02 (index finger of the right hand) for both LE and COM matchers, and in Figure 14 for Finger 07 (index finger of the left hand).

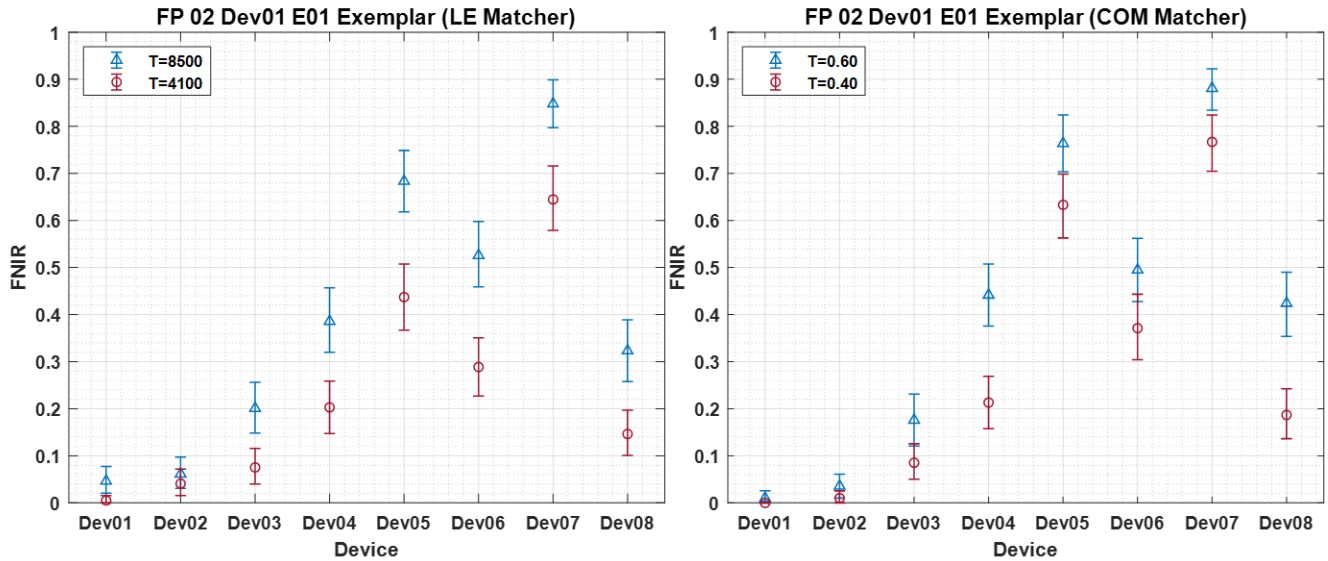


Figure 13 – Finger 02 matched to encounter #1 (E01) samples acquired on the control device, Dev01, using both matchers

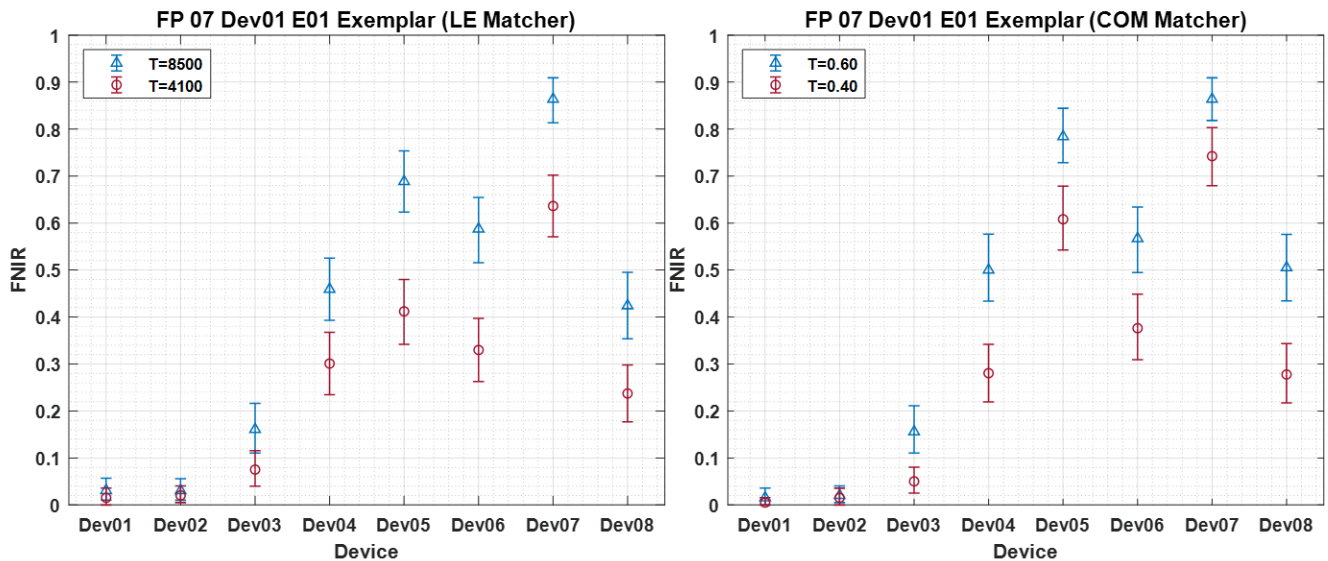


Figure 14 – Finger 07 matched to encounter #1 (E01) samples acquired on the control device, Dev01, using both matchers

¹¹ Pers. Comm., Richard Case, Forensics and Identity Team, Policing and Security Group, Counter Terrorism and Security Division Defense, Science and Technology Laboratory, Ministry of Defense, United Kingdom.

Interestingly, right- and left-hand performance results are not the same. Capture procedures are identical for the two hands, but there may be an order difference with typical capture of the right hand first. Unfortunately, most of the applications are fixed in such way that randomizing the order of capture is not possible without changes to the application software, which could only be performed by the manufacturer of a device.

4.2.6 Two Fingers

With the addition of the middle finger (03) to the index finger (02) of the right hand, the FNIR performance improves for all devices as see in Figure 15.

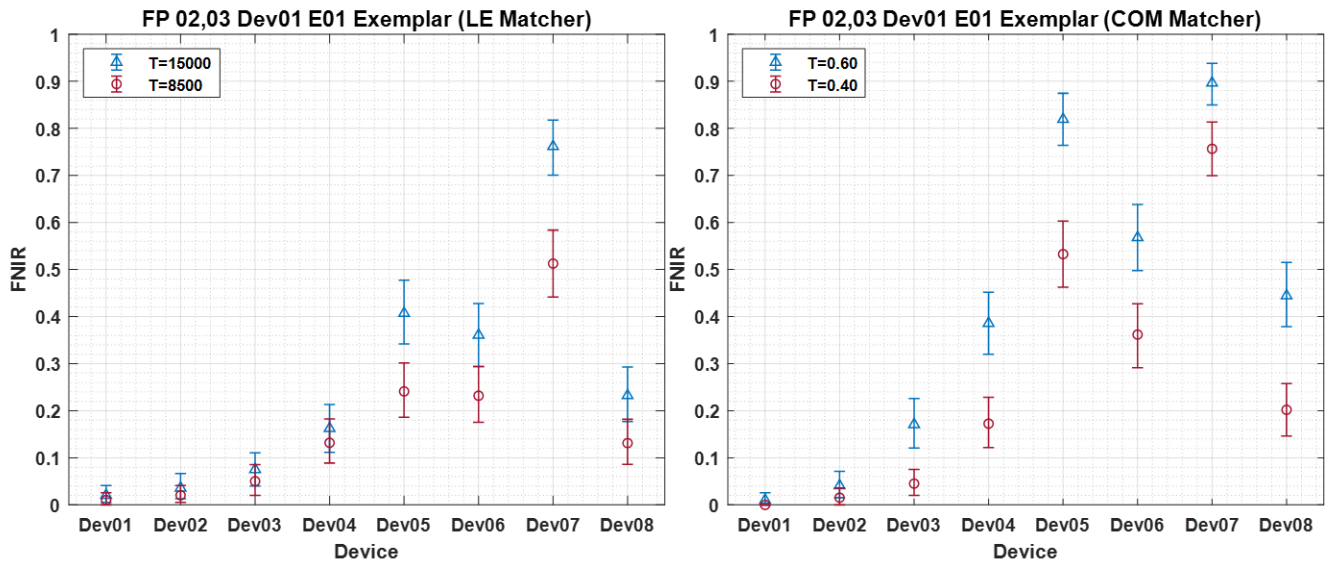


Figure 15 – Fingers 02 and 03 matched to encounter #1 (E01) samples acquired on the control device, Dev01, using both matchers

Similar results were obtained for the index (07) and middle (08) finger of the left hand in Figure 16.

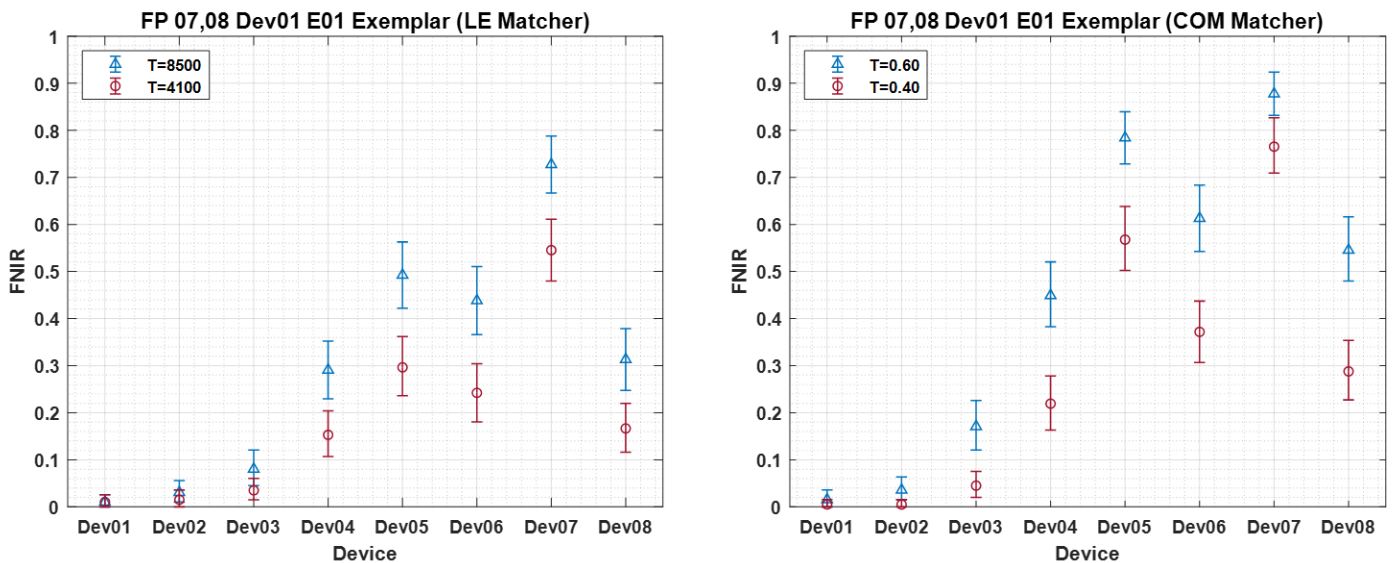


Figure 16 – Fingers 07 and 08 matched to encounter #1 (E01) samples acquired on the control device, Dev01, using both matchers

4.2.7 Three/Four Fingers

Here, the LE and COM matchers are treated somewhat differently. The LE matcher has its own internal fusion algorithm such that adding low scoring little fingers (05 or 10) does not degrade scores derived from the other three fingers. For the COM matcher, the scores of the three fingers are simply averaged, and the little fingers are dropped from the scoring.

In any case, we see that adding additional fingers reduces FNIR even more than that observed for two-finger combinations (02,03 or 07,08 as shown in section 4.2.6.) for some devices, as seen in Figure 17.

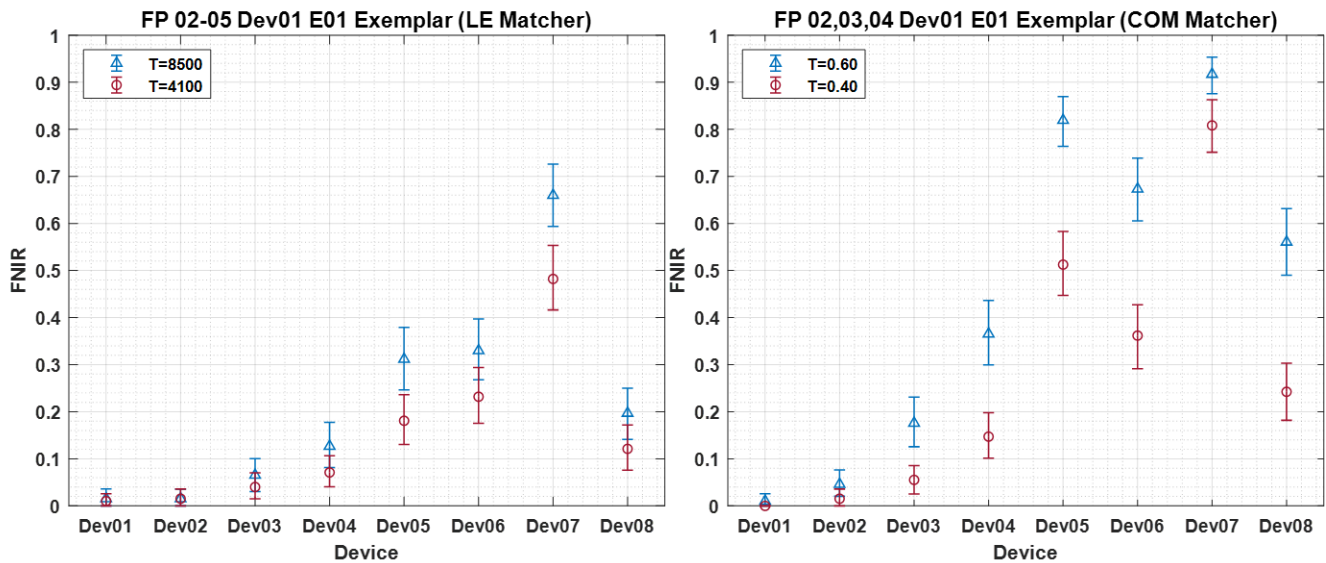


Figure 17 – Four fingers of right hand matched to encounter #1 (E01) samples acquired on the control device, Dev01, using LE matcher, shown beside the first three fingers of the right hand matched to encounter #1 (E01) samples acquired on the control device, Dev01, using COM matcher

As with the right hand, we see similar improvement when adding additional fingers to the index and middle fingers (see Figure 18).

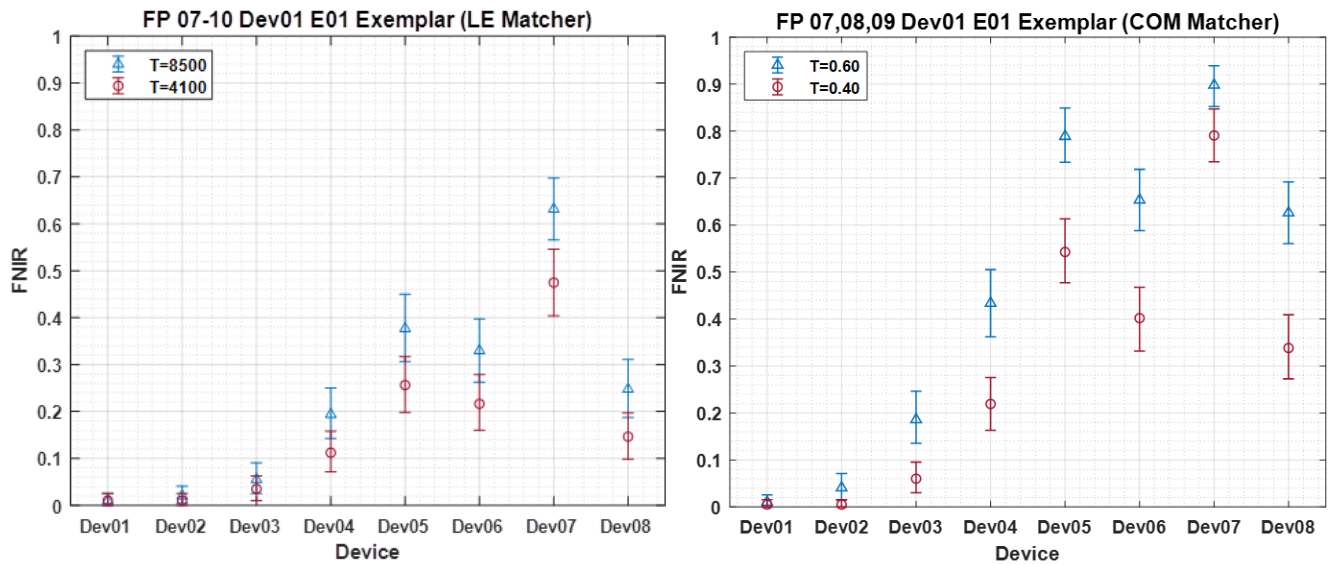


Figure 18 – Four fingers of left hand matched to encounter #1 (E01) samples acquired on the control device, Dev01, using LE matcher, shown beside the first three fingers of the left hand matched to encounter #1 (E01) samples acquired on the control device, Dev01, using COM matcher

4.2.8 One + One Index Finger Combination

A potential two-finger combination might include the two index fingers. The results for this test case are shown in Figure 19. We see that both contact devices exhibit near zero error with both matchers. The stationary contactless device, Dev03, shows an FNIR of around 0.05 for both matchers. With the LE matcher Dev08, a smartphone application, performs fourth best with an FNIR just under 0.1 for the LE matcher.

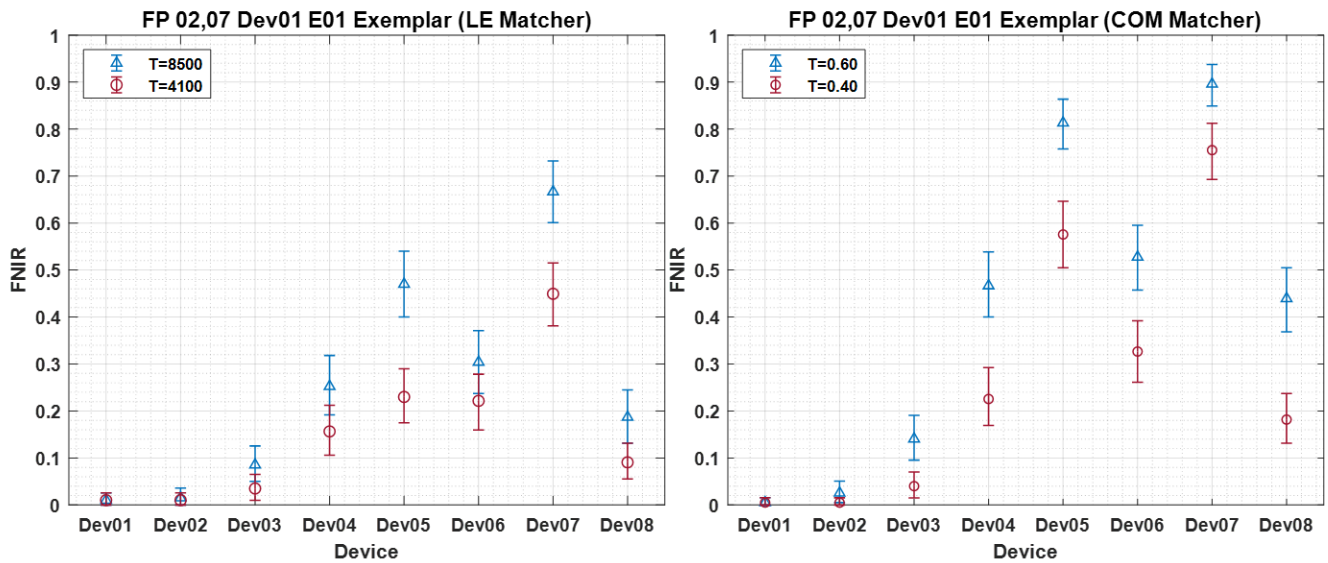


Figure 19 – Index fingers of each hand matched to encounter #1 (E01) samples acquired on the control device, Dev01, using both matchers

4.2.9 Two + Two Finger Combination

Improved performance is observed for most devices using the index and middle fingers of both the right and left hand (i.e., fingers 02, 03 and 07, 08). FNIR approaches zero for the two contact devices, Dev01 and Dev02, and is nearly matched by Dev03, a stationary contactless device, as seen in Figure 20. Remarkably, smartphone application, Dev08, improves considerably over its single finger performance, notably with the LE matcher. Figure 11 and Figure 12 verify that single finger match scores tend to be among the highest for fingers 02, 03 and 07, 08 for all devices.

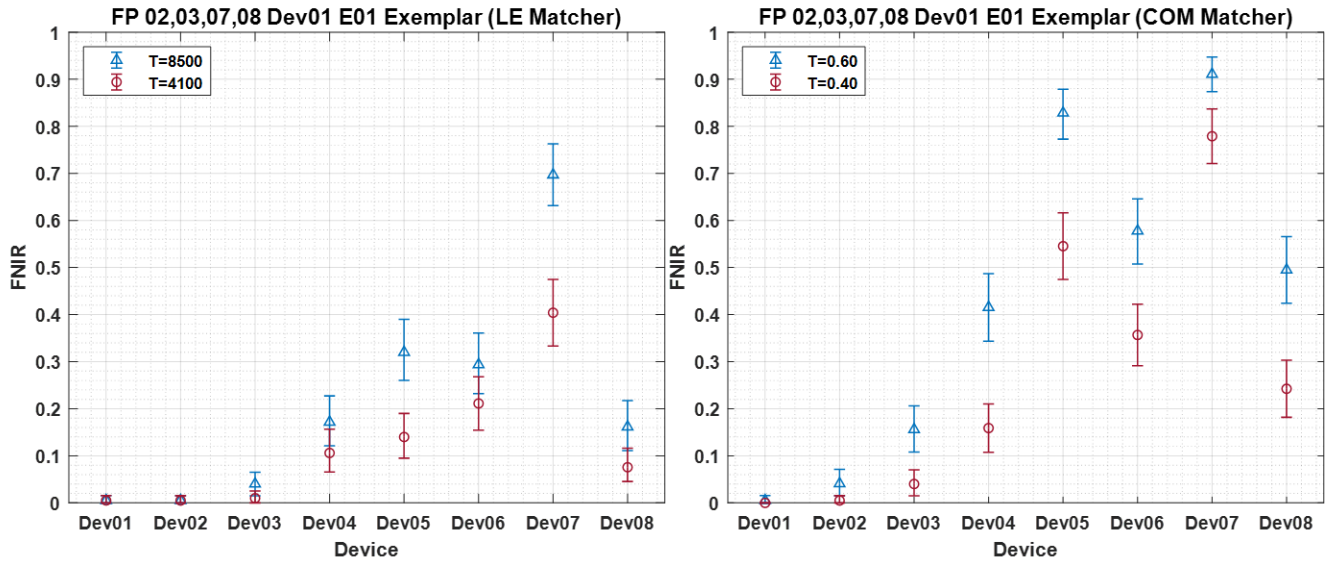


Figure 20 – Two + Two finger combination appears to produce the best performance for several devices, though with some disparity between the two matchers.

4.2.10 Six/Eight Fingers

Using all eight fingers (or six with the COM matcher, as explained in section 4.2.7) provides advantage for some DUTs, although the two + two combination is better for others, such as Dev08. As observed in [1], four-finger captures with contactless devices are challenged by geometric misalignment, lighting, and limited depth of field of the devices, especially smartphones.

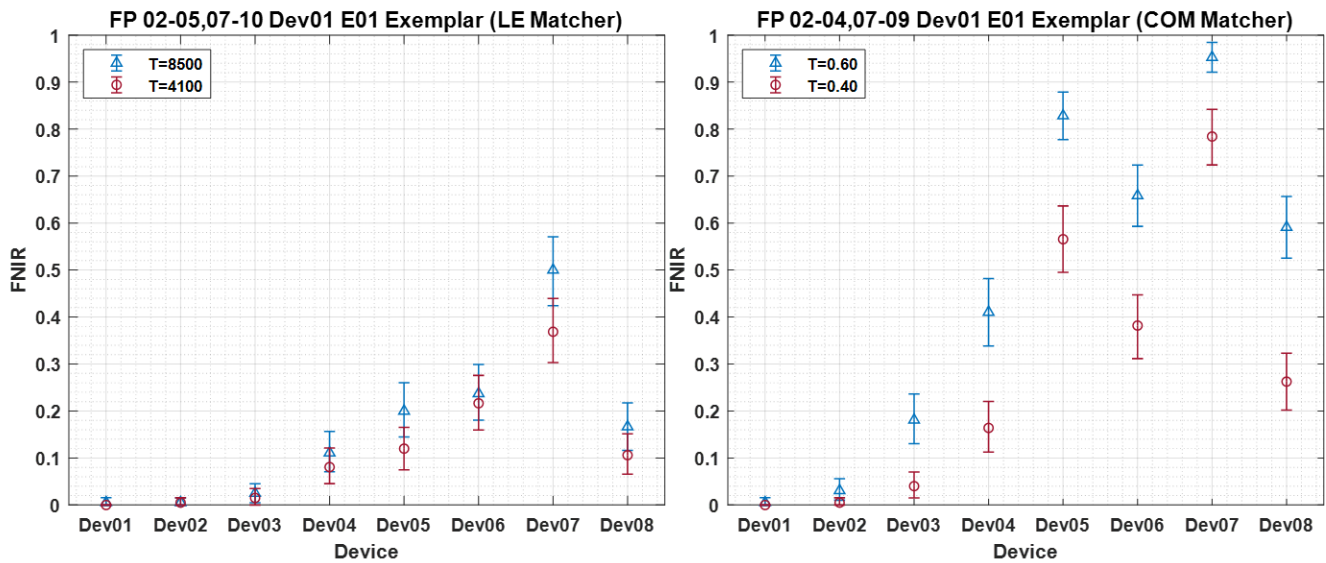


Figure 21 – Eight finger combination generates lower error for most devices using the LE matcher, shown beside the six-finger combination, which generates lower error for most devices using COM matcher

4.2.11 Eight Fingers with Dev 03 As Exemplar (LE Matcher Only)

Dev03 emerged as the best performing of the contactless devices, so match error was examined with both contact and contactless probes matched to the Dev03 contactless fingerprints as exemplars using only the LE matcher for this experiment.

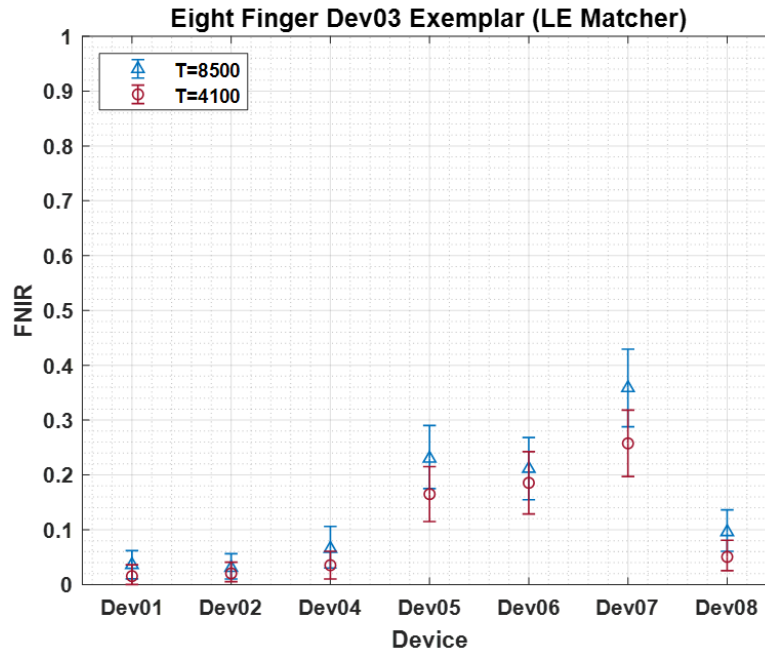


Figure 22 – FNIR with Dev03 prints as exemplars probed by prints from other devices, using LE matcher only.

Figure 22 shows some reduction in FNIR for Dev04 and Dev08. One possible cause for this may be that such devices perform better when matched against rolled exemplars. We have noted, while monitoring display of images during the registration process, that out-of-plane rotation of some fingers reduces the overlap such fingers might have with plain contact exemplars. Dev03 offers a somewhat larger sample of friction ridge that might increase match scores for these rotated contactless print samples. Another possibility is that contactless fingerprint impressions more closely resemble other contactless fingerprint impressions than they do exemplars from contact devices. The best test to determine the cause of the observed performance in Figure 22 would be to examine contactless print matching against rolled contact impressions – a notable goal for future data collection efforts.

4.3 Match Error Performance Summaries

The performance data presented in graphical form above is provided in tabular form in this section. Each table displays the error at one of two of the selected operating thresholds (LE matcher only) for selected finger combinations. First, False Negative Identification Rate (FNIR) is converted to True Positive Identification Rate (TPIR) in percent via the formula

$$TPIR(\%) = (1 - FNIR) * 100 \quad (1)$$

Note that FPIR values are adjusted such that they are no lower than the upper limit of 95 % confidence interval for a binomial experiment consisting of N trials. By the Rule of Three, this interval is 0 to $3/N$ [6][7].

Table 2 – TPIR/FPIR at threshold 8 500 (%)

	Single Finger		Fingers 02, 03		Fingers 02, 07		Fingers 02,03,07,08		Fingers 02,03,04,05		Fingers 07,08,09,10		Eight Fingers	
	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²
Dev01	91.94	0.19	97.94	1.55	98.97	1.55	99.48	1.55	98.45	1.55	98.97	1.55	99.48	1.55
Dev02	82.60	0.19	96.41	1.54	98.46	1.54	99.49	1.53	98.47	1.53	97.96	1.53	99.49	1.53
Dev03	71.11	0.19	92.46	1.51	91.46	1.51	95.98	1.51	93.47	1.51	94.47	1.51	97.49	1.51
Dev04	52.16	0.19	83.76	1.52	74.75	1.52	82.83	1.52	87.31	1.52	80.61	1.53	88.89	1.52
Dev05	26.52	0.19	59.30	1.51	53.00	1.50	68.00	1.50	68.84	1.51	62.31	1.51	80.00	1.50
Dev06	24.76	0.19	63.92	1.55	69.59	1.55	70.62	1.55	67.01	1.55	67.01	1.55	76.29	1.55
Dev07	7.98	0.19	23.86	1.52	33.33	1.52	30.30	1.52	34.01	1.52	36.87	1.52	50.00	1.52
Dev08	35.23	0.19	76.77	1.52	81.31	1.52	83.84	1.52	80.30	1.52	75.25	1.52	83.33	1.52

Table 3 – TPIR/FPIR at threshold 4 100 (%)

	Single Finger		Fingers 02, 03		Fingers 02, 07		Fingers 02,03,07,08		Fingers 02,03,04,05		Fingers 07,08,09,10		Eight Fingers	
	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²	TPIR	FPIR ¹²
Dev01	96.00	0.26	98.97	1.55	98.97	1.55	99.48	1.55	98.97	1.55	98.97	1.55	99.81	1.55
Dev02	91.49	0.19	97.95	1.54	98.97	1.54	99.49	1.53	98.47	1.53	98.98	1.53	99.49	1.53
Dev03	83.73	0.19	94.97	1.51	96.48	1.51	98.99	1.51	95.98	1.51	96.48	1.51	98.49	1.51
Dev04	68.83	0.25	86.80	1.52	84.34	1.52	89.39	1.52	92.89	1.52	88.78	1.53	91.92	1.52
Dev05	45.51	0.19	75.88	1.51	77.00	1.50	86.00	1.50	81.91	1.51	74.37	1.51	88.00	1.50
Dev06	45.84	0.19	76.80	1.55	77.84	1.55	78.87	1.55	76.80	1.55	78.35	1.55	78.35	1.55
Dev07	20.42	0.19	48.73	1.52	55.05	1.52	59.60	1.52	51.78	1.52	52.53	1.52	63.13	1.52
Dev08	53.28	0.19	86.87	1.52	90.91	1.52	92.42	1.52	87.88	1.52	85.35	1.52	89.39	1.52

¹² Note that grouping samples into finger combinations results in reducing the value of N , which increases the value of the $3/N$ upper limit. As a result, this leads to an increase in the computed FPIR values shown in the Tables Table 2 and Table 3.

4.4 Matcher Tests: Area Under the ROC Curve

The current datasets are small in comparison to the hundreds of thousands of scores usually encountered in our biometrics testing [11]. As with data analyses in medical research often restricted to small samples, ROC analysis may be summarized effectively by the Area Under the (ROC) Curve (AUC) [12]-[16].

The AUC of a classifier is equivalent to the probability that the classifier will rank a randomly chosen positive instance higher than a randomly chosen negative instance [17]. If the trapezoidal rule is employed, the AUC is equivalent to the Mann-Whitney statistic formed by independent and identically distributed (i.i.d.) genuine scores and impostor scores [10]-[18].

Let $f_G(s)$ and $f_I(s)$ denote the continuous probability density functions (pdf) of the genuine scores and the impostor scores at a score $s \in \{s\}$, respectively. Then, the AUC can be expressed as

$$\text{AUC} = \int_{-\infty}^{+\infty} \left[\int_s^{+\infty} f_G(t) dt \right] \times f_I(s) ds . \quad (1)$$

Its derivation can be found in [18].

Let $P_i(s)$, where $s \in \{s\}$ and $i \in \{G, I\}$, denote the discrete empirical probabilities of genuine scores and impostor scores occurring at a score s , respectively. Then, by employing the trapezoidal rule, the AUC expressed in Eq. (1) can be estimated as follows:

$$\begin{aligned} \hat{A} &= \sum_{s=s_{\max}}^{s_{\min}} \text{trapezoid}(s) \\ &= \sum_{s=s_{\max}}^{s_{\min}} \text{triangle}(s) + \sum_{s=s_{\max}}^{s_{\min}} \text{rectangle}(s) \\ &= \sum_{s=s_{\max}}^{s_{\min}} P_I(s) \times \left[\frac{1}{2} \times P_G(s) + \sum_{\tau=s+1}^{s_{\max}} P_G(\tau) \right] . \end{aligned} \quad (2)$$

In the figures to follow, we display the AUC computed considering the genuine and impostor score distributions displayed as a point. The distributions are resampled with replacement to generate 2 000 replicate distributions and the computation repeated to form a distribution of AUC values from which we determine the upper and lower 95 % confidence limits on the AUC. The confidence interval thus determined by the bootstrap procedure is displayed around the initially computed value.

4.4.1 Single Finger

Examining Figure 23, we see that some devices exhibit slightly better performance using the COM matcher. As was pointed out previously the single finger scores include all fingers, including the little fingers.

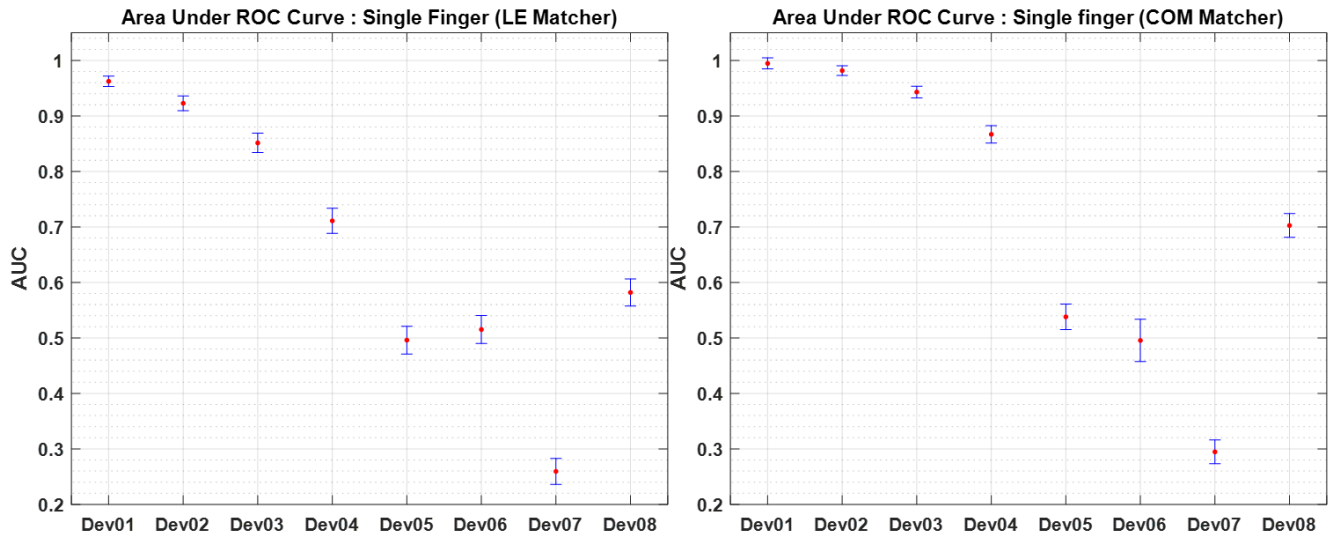


Figure 23 – AUC for single finger from both matchers

4.4.2 Index Finger Only

Figure 24 and Figure 25 display AUC for single index fingers matched with LE and COM matchers, respectively. It is notable that with only a single index finger, the contact devices, Dev01 and Dev02, score very well. Dev08 also scores well at an AUC of 0.9 with only the single index finger.

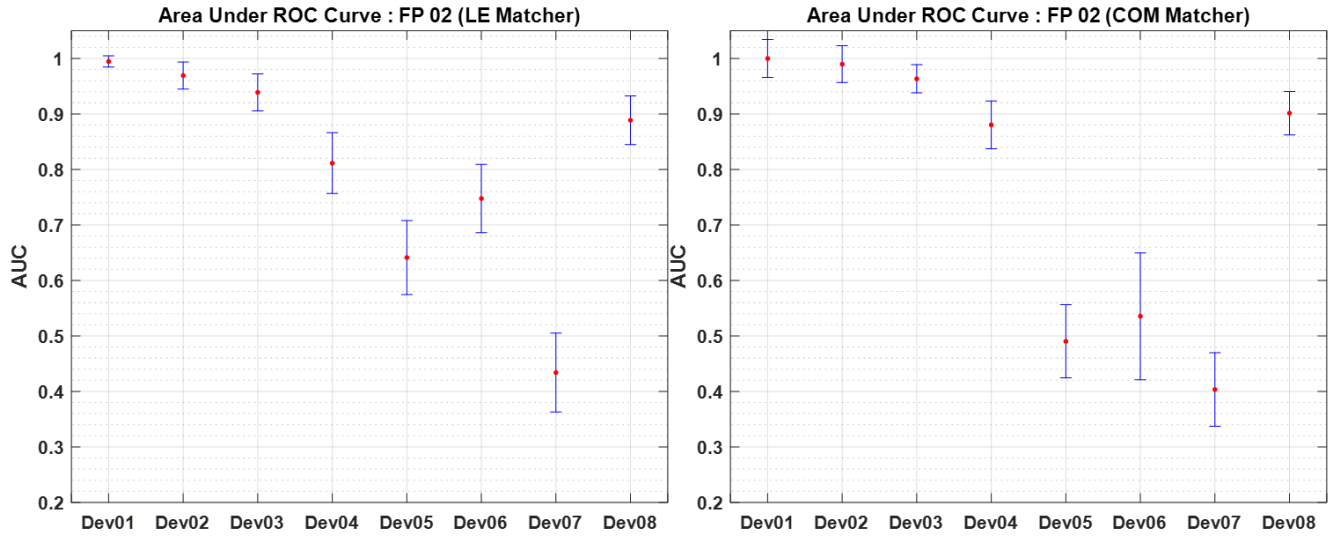


Figure 24 – AUC for index finger from right hand using both matchers

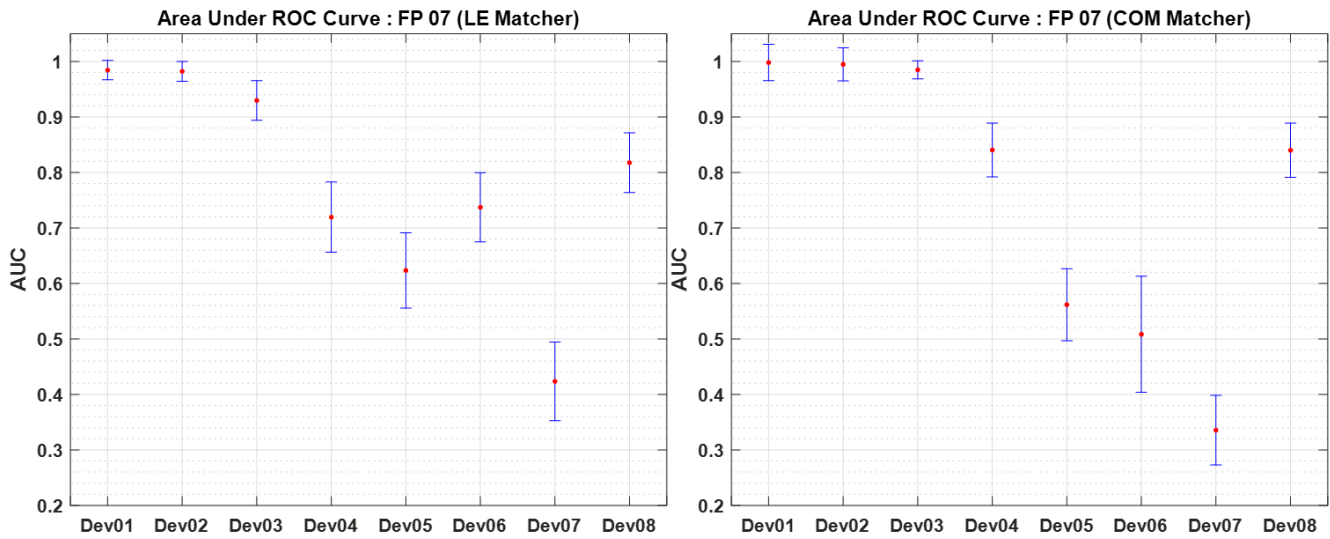


Figure 25 – AUC for index finger from left hand using both matchers

4.4.3 Two Fingers

In Figure 26, with the addition of the middle finger to the index, the second stationary contactless device, Dev04, and smartphone, Dev08, are shown to approach or exceed the 90 % mark with Dev03 and the two contact devices.

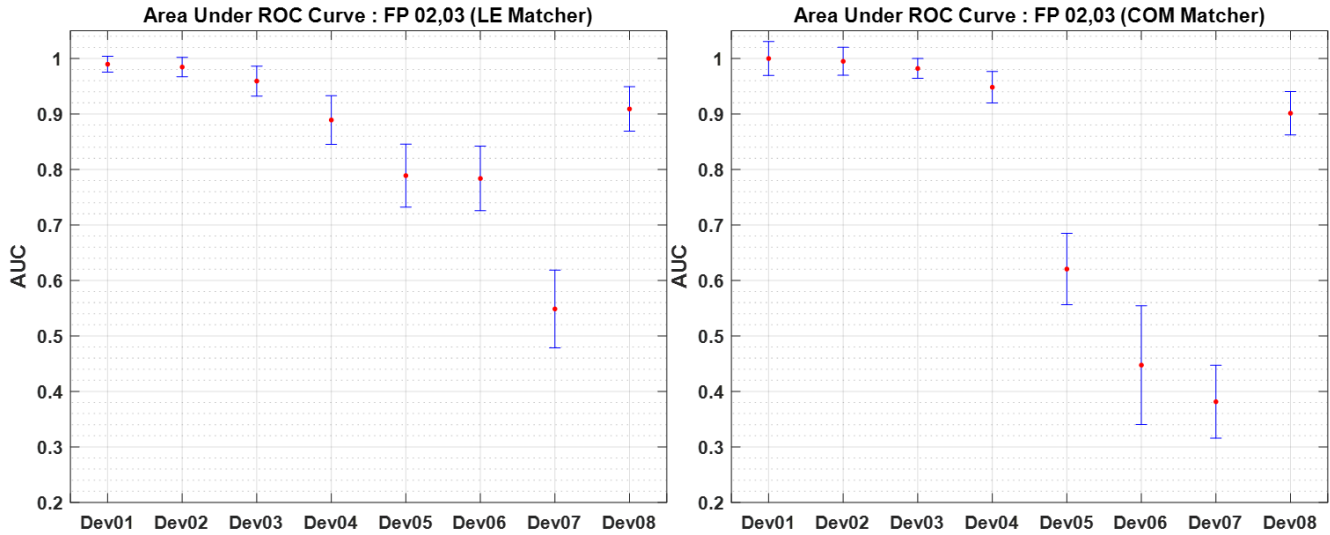


Figure 26 – AUC for index and middle fingers from right hand using both matchers

The first two fingers of the left hand exhibit performance similar to that for the right hand except that Dev03 achieves a very high AUC with a tighter confidence interval with the COM matcher.

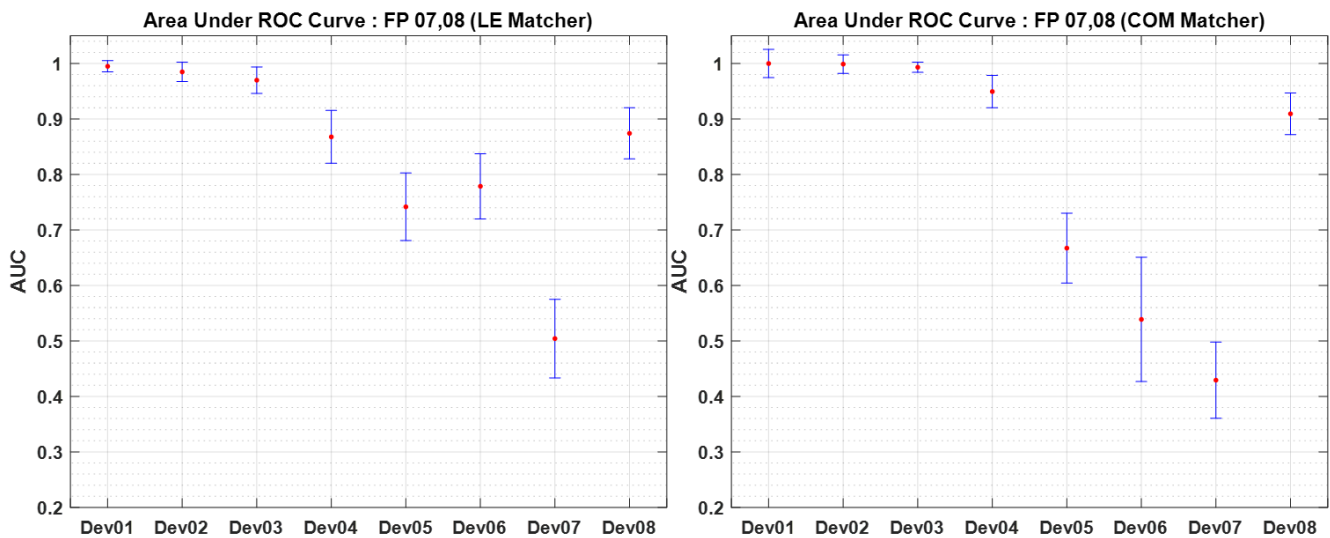


Figure 27 – AUC for index and middle fingers from left hand using both matchers

4.4.4 Three/Four Fingers

With additional sampled fingers of the right hand, further improvement can be seen with Dev04 breaking the 90 % mark with both matchers along with Dev08 for the LE matcher.

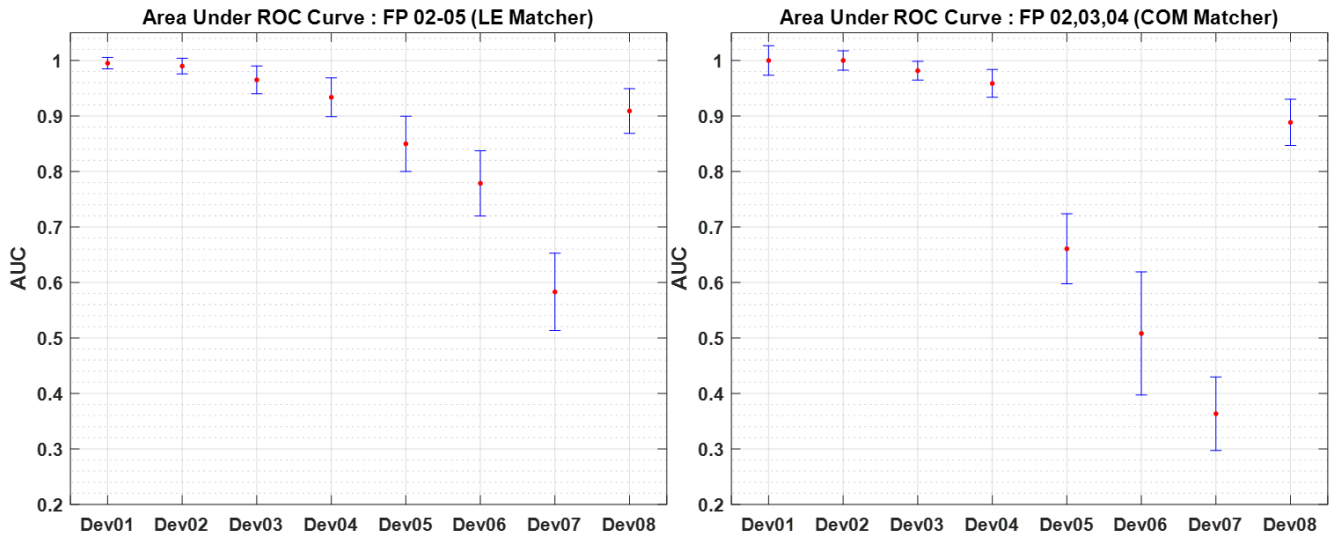


Figure 28 – AUC for four fingers from right hand using LE matcher shown beside AUC for first three fingers using COM matcher

A similar pattern is observed for the fingers of the left hand with near-perfect AUC scores for the contact devices and a score above 0.99 for the stationary contactless device, Dev03, with the COM matcher

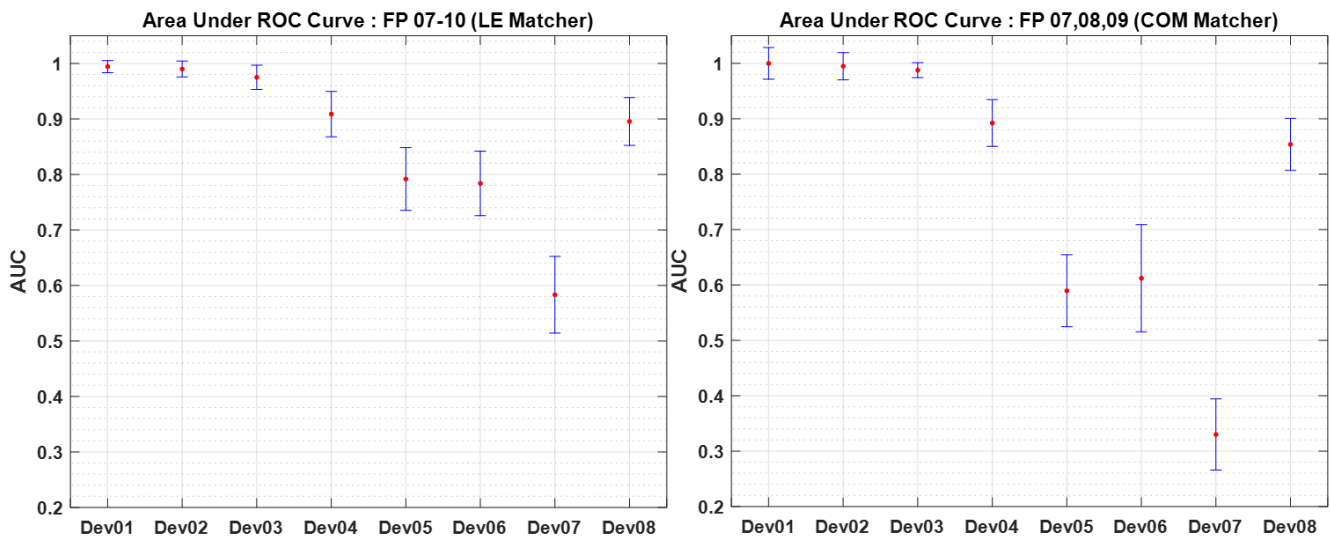


Figure 29 – AUC for four fingers from left hand using LE matcher shown beside AUC for first three fingers from left hand using COM matcher

4.4.5 One + One Index Finger Combination

As shown in Figure 30, with two index fingers, the two contact devices display AUC values at or exceeding 98 %, Dev03 almost reaching 98 %, and one of the mobile contactless devices, Dev08, almost at 94 %.

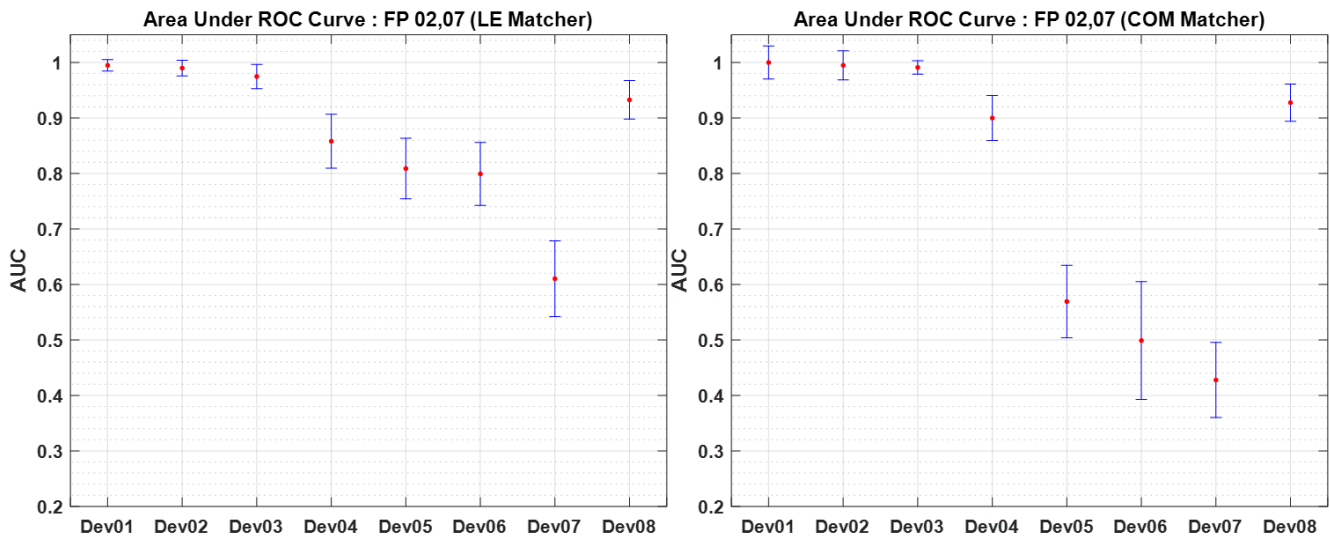


Figure 30 – AUC for index fingers from both hands using both matchers

4.4.6 Two + Two Finger Combination

Shown in Figure 31, with the index and middle fingers of both hands, Dev03 reaches 99 % for both matchers. Dev08 is at 94 % with Dev04 and mobile device Dev05 breaking the 90 % mark.

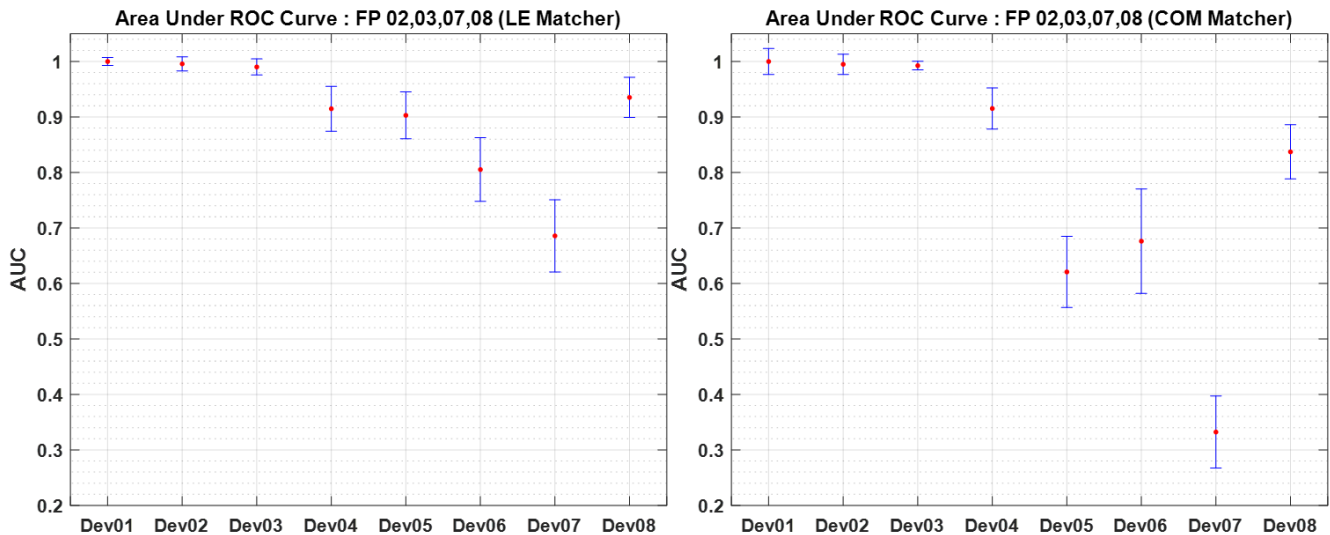


Figure 31 – AUC for index and middle fingers from both hands using both matchers

4.4.7 Six/Eight Fingers

It can be observed in Figure 32, when using all available fingers, the contact devices, Dev01 and Dev02 are over 99 % with tight confidence intervals.

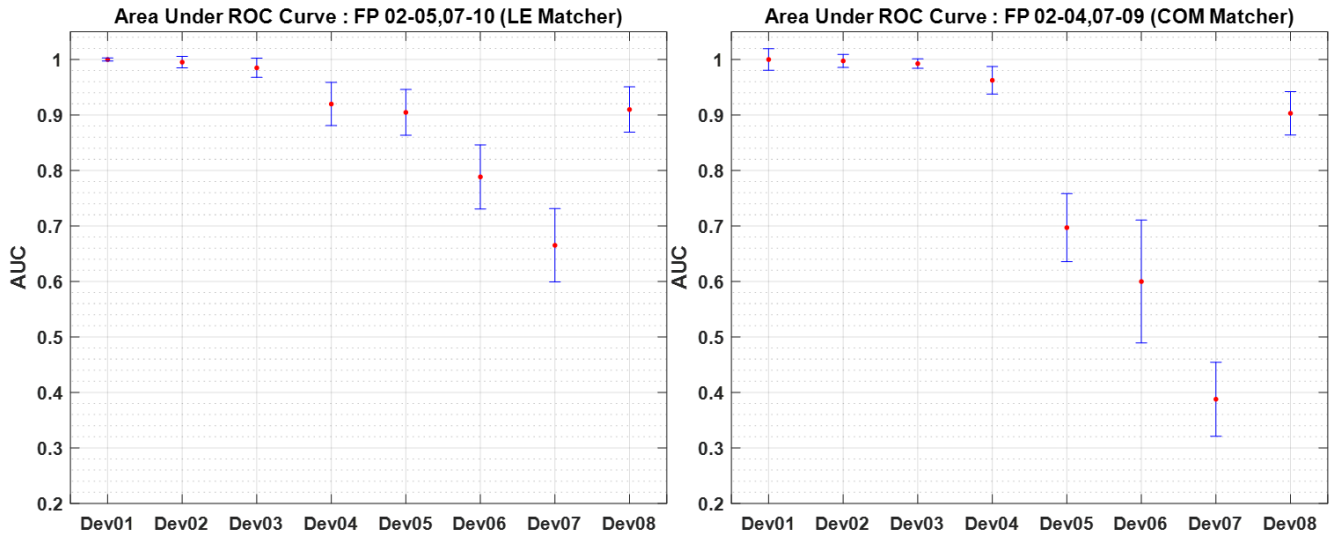


Figure 32 – AUC for all eight fingers from both hands using LE matcher shown beside AUC for the first three fingers from both hands using COM matcher

4.4.8 AUC Results by Device

In the following sections, we examine AUC for the various finger combinations for each device.

4.4.8.1 Dev01

The control contact device shows near perfect performance for all but the single finger (all finger) matching. It is worth noting that both exemplar and probe samples are acquired using the same Appendix F certified device for all Dev01 results. Figure 33 illustrates AUC values $\geq 99\%$ for most finger combinations, though the confidence intervals are broader for the COM matcher, even as the AUC is virtually 100% for all finger combinations.

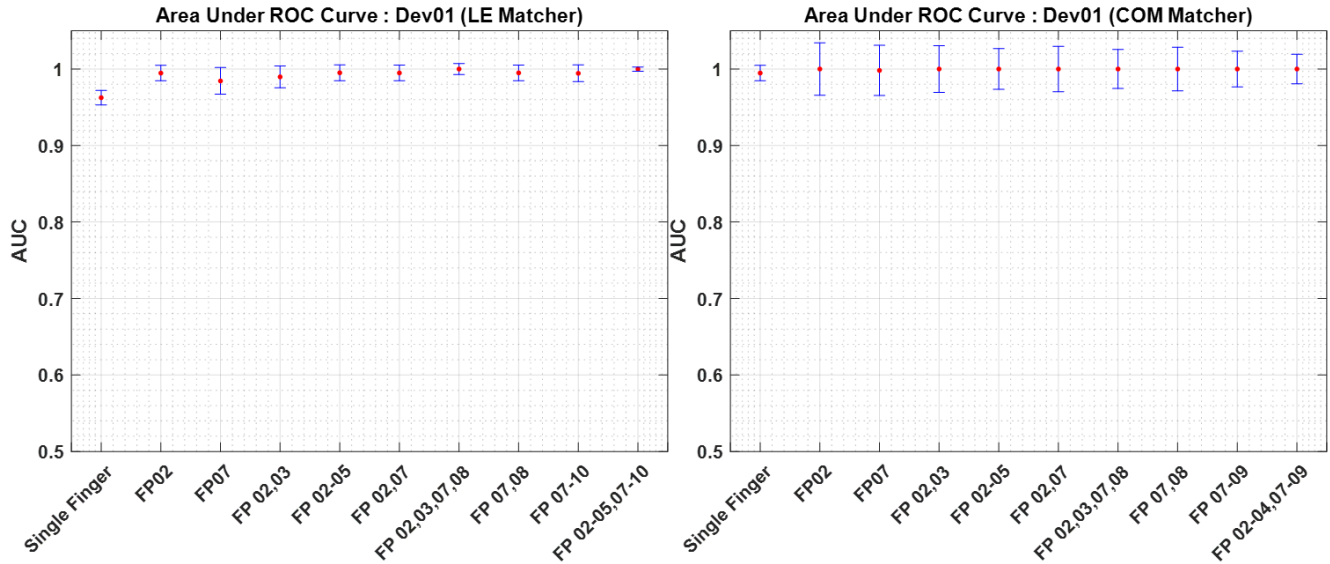


Figure 33 – AUC for all finger combinations with Dev01 using both matchers

4.4.8.2 Dev02

Here, probe samples acquired using an Appendix F device that uses a different imaging technology than that used to sample exemplar images are evaluated. As seen in Figure 34, performance remains high, but not quite reaching the level observed when both probe and exemplar are acquired using the same device (i.e. Dev01). However, in evaluating contactless fingerprint devices, it is important to acknowledge that some variation in performance is to be expected even among Appendix F certified devices.

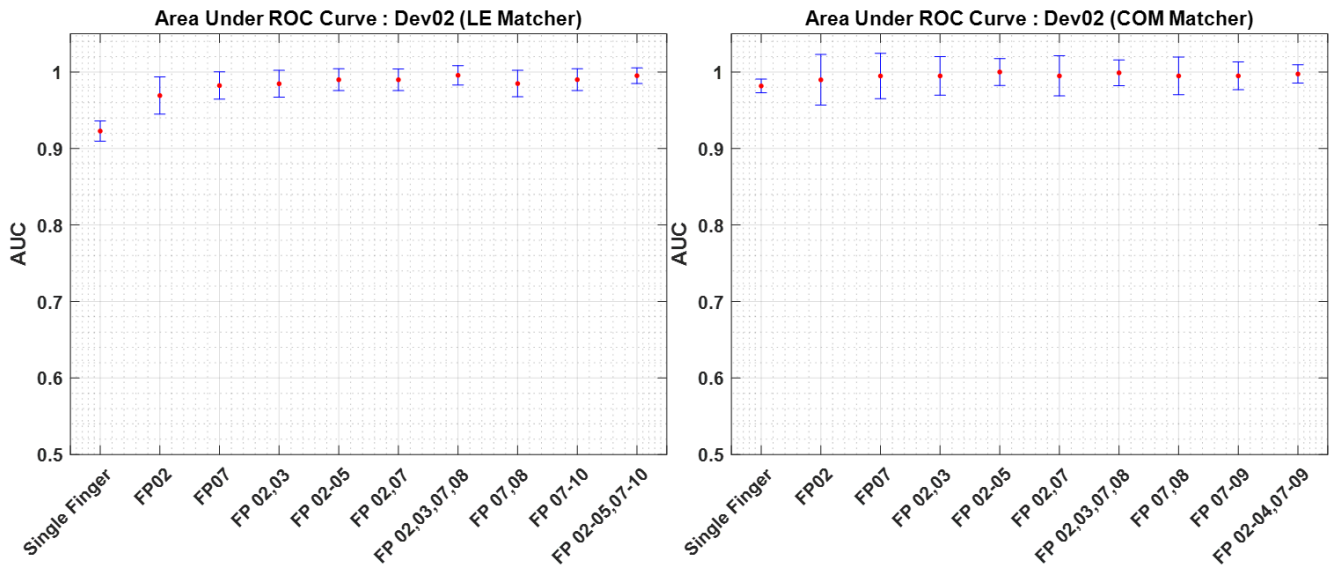


Figure 34 – AUC for all finger combinations with Dev02 using both matchers

4.4.8.3 Dev03

Figure 35 indicates that the Dev03 contactless device shows AUC at or near 99 % with some multiple finger combinations.

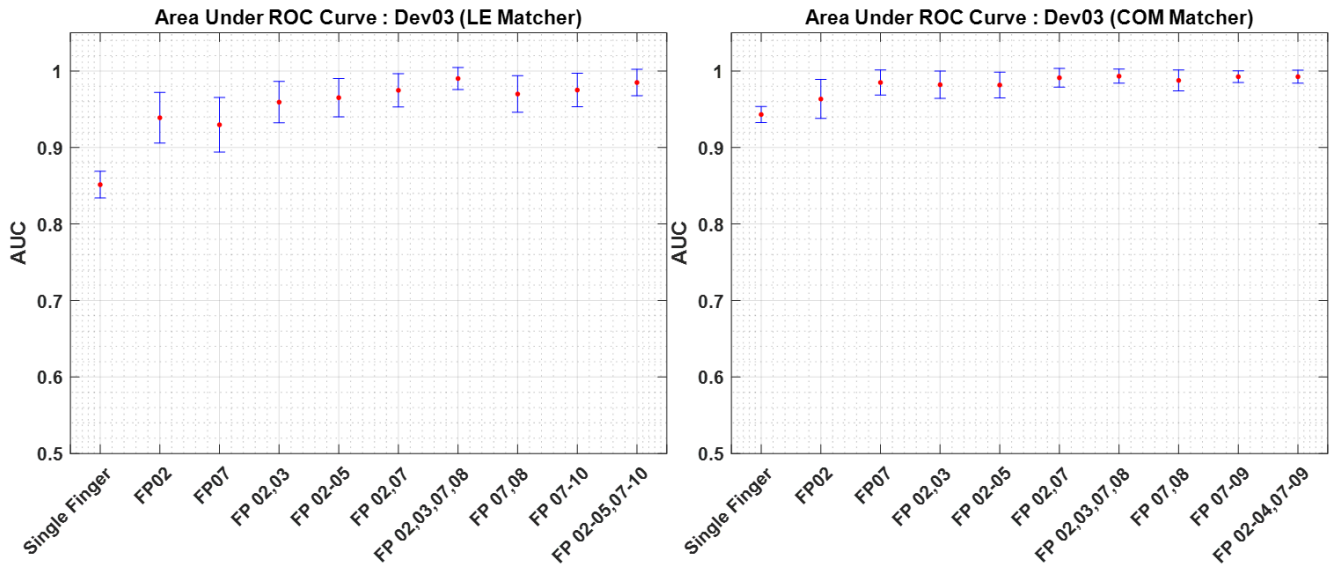


Figure 35 – AUC for all finger combinations with Dev03 using both matchers

4.4.8.4 Dev04

Figure 36 shows that Dev04 approaches or exceeds the 95 % mark with some multiple finger combinations.

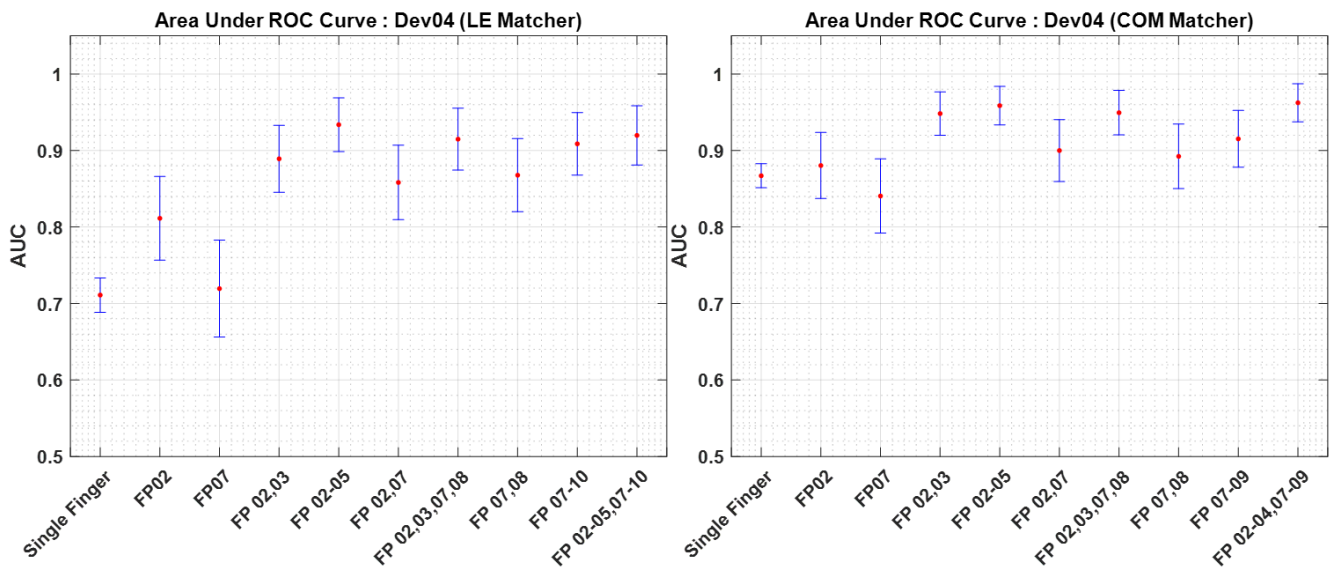


Figure 36 – AUC for all finger combinations with Dev04 using both matchers

4.4.8.5 Dev05

Figure 37 shows a few finger combinations that reach the 90 % AUC with the LE matcher. Interesting, but unexplained, is the much lower performance observed with the COM matcher. We suspect that this particular matcher may rely heavily on detail of the core region more so than the LE matcher.

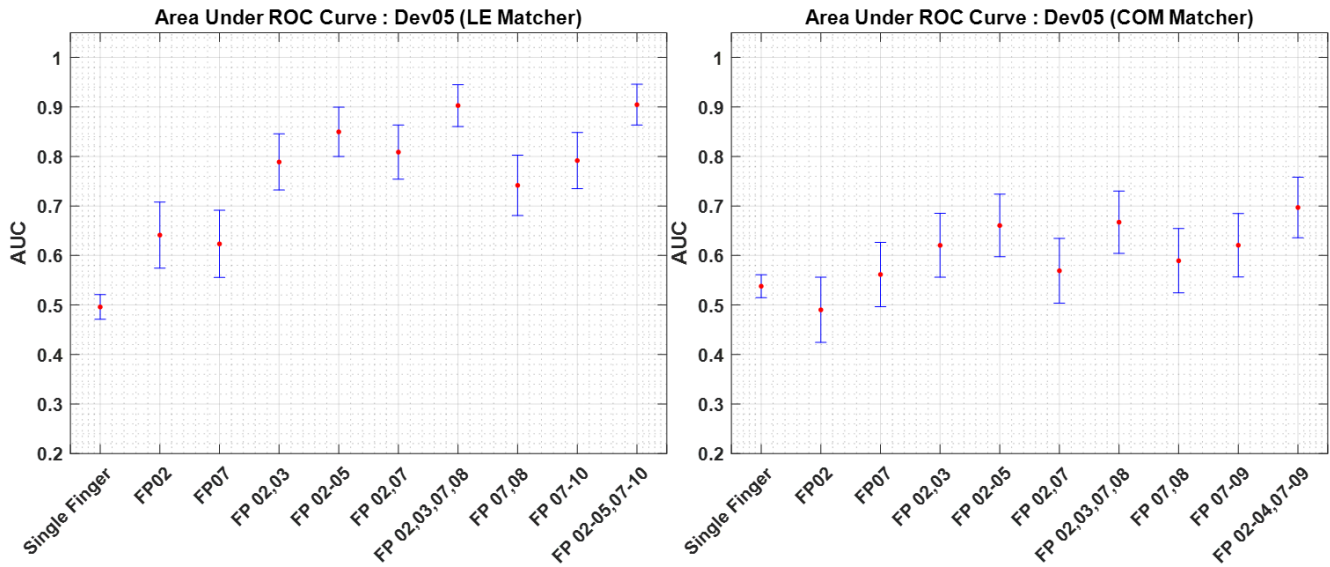


Figure 37 – AUC for all finger combinations with Dev05 using both matchers

4.4.8.6 Dev06

Figure 38 illustrates the disparity in performance with the two different matchers for Dev06.

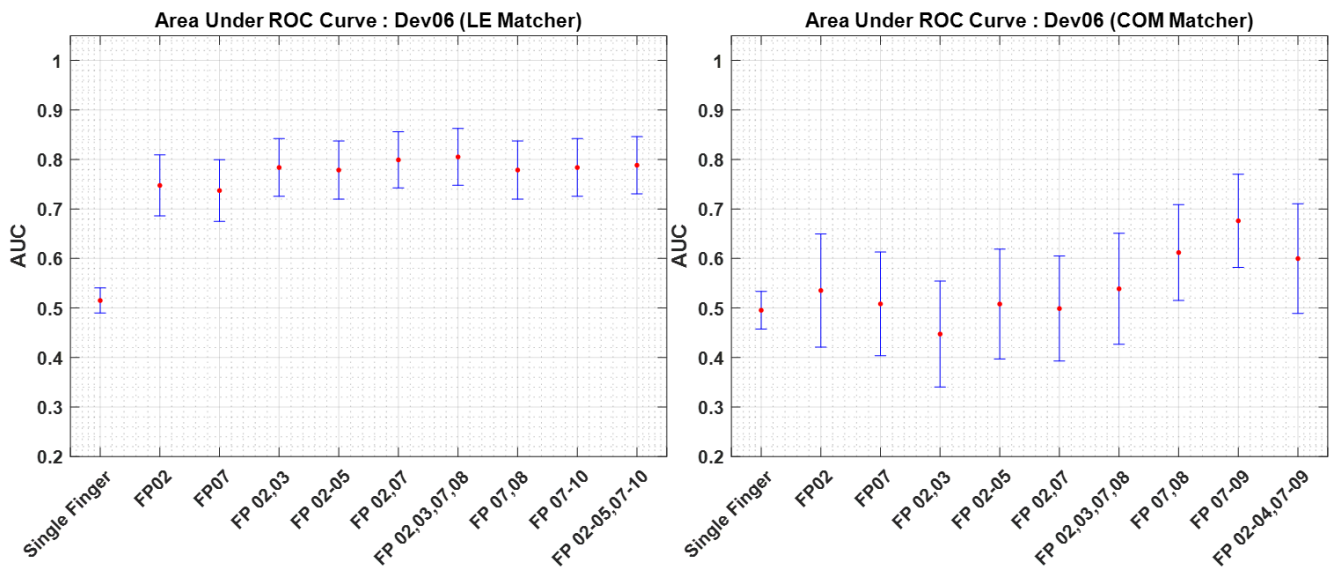


Figure 38 – AUC for all finger combinations with Dev06 using both matchers

4.4.8.7 Dev07

Dev07 exhibits anomalously poor performance across all finger combinations (see Figure 39) yet showing its best performance with the “two + two” finger combination found to perform well for all devices.

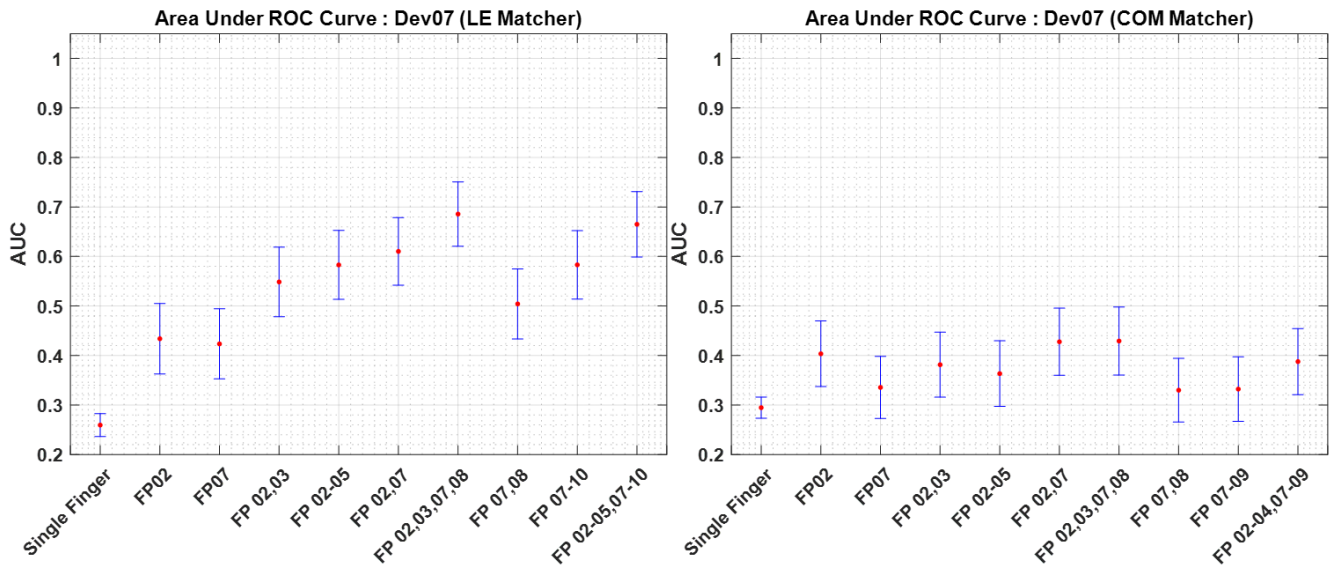


Figure 39 – AUC for all finger combinations with Dev07 using both matchers

4.4.8.8 Dev08

Figure 40 illustrates that Dev08 surpasses the 90 % mark with several finger combinations, with highest AUC with the “two + two” combination.

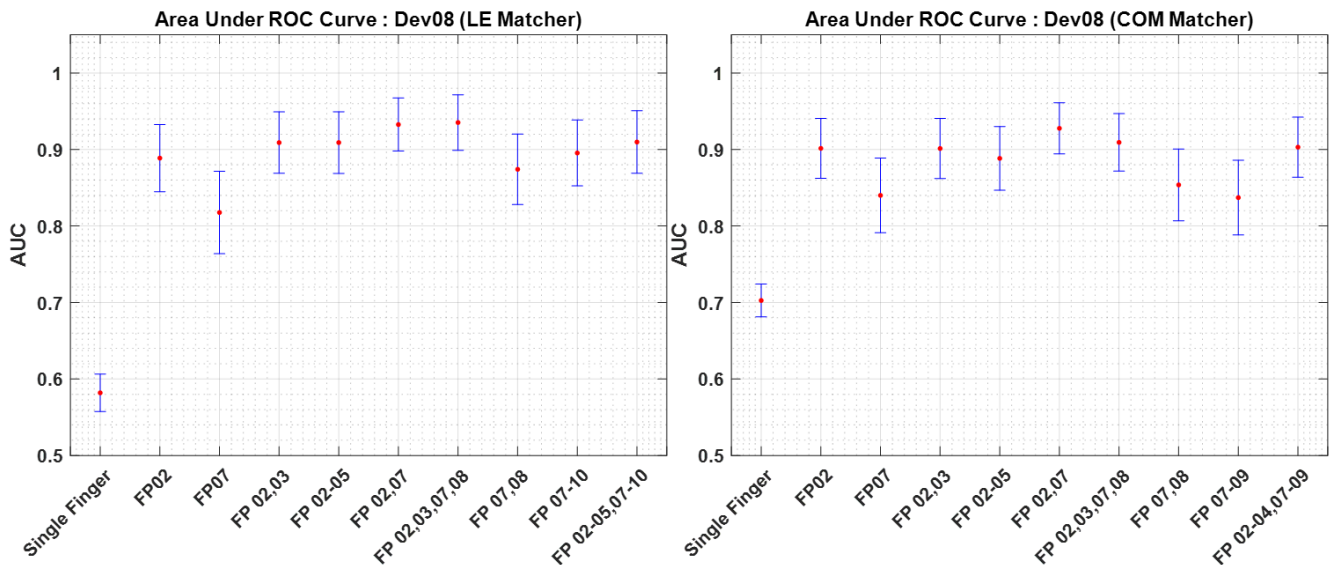


Figure 40 – AUC for all finger combinations with Dev08 using both matchers

5 Discussion and Conclusions

5.1 Metrics

A few of the comparison metrics detailed in [1] were applied to fingerprint images output from the devices under test (DUTs) and corresponding exemplars captured with the control FTIR device. We see that control of scale or sample rate continues to present a challenge for mobile contactless capture, though it appears to be much improved for stationary contactless capture, even to the extent that one stationary device has its entire distribution of measurements within the PIV range of 500 ppi \pm 10 ppi. Other contactless devices show a little over 50 % within the PIV range with medians at 500 ppi, however exhibiting larger dispersion. The matchers used in the current study expect images sampled at 500 ppi, hence the observed match performance correlates roughly with the order of the scale and several other metrics. Regarding variations in scale and minutiae displacement, it is worth noting that modern matchers demonstrate some, even if limited, invariance to rotation and robustness to limited scale variation or displacement of minutiae as observed with deformation of the skin during contact capture. The matchers used here demonstrate some of this robustness but do expect either explicit input of the sample rate of fingerprint images or that the input images are conformant to the default sample rate of 500 ppi.

5.2 Matching

One of the principal objectives of the current research effort is to verify that contactless matching is much improved when considering multiple fingers. This hypothesis is confirmed. Results show that all fingers are not created equal from the point of view of matching and that, while treating each finger of N subjects as independent might be attractive to increase sample size, the practice is not realistic from an operational point of view. Images captured from index and middle fingers of either hand appear to carry the most information to support matching for both contact and contactless devices. These two fingers of either hand, or preferably of both hands, show substantial improvement over most other combinations. For contactless devices, particularly smartphone applications, optimizing for focus and lighting of index and middle fingers should produce the best results. As we observed in [1] and confirmed via extensive interaction with the smartphone applications, maintaining focus over four fingers simultaneously is difficult given the small depth of field of smartphone cameras.

5.3 Other Conclusions and Observations

We have observed a trend across most metrics where the performance of the DUTs appear to fall into three tiers of performance. The first, highest performing tier unsurprisingly includes the two contact devices, followed by a second, slightly lower-performing tier consisting of the two stationary contactless devices, and finally a third tier consisting of the mobile smartphone contactless devices. This trend is not surprising in that it is logical to expect that measurements of interoperability will show lower performance when comparing samples collected from devices with increasing deviation from the capture methods and technology employed by the control device. This is borne out in the results seen in comparing the two contact devices, which generally show only small differences, as opposed to the stationary contactless devices which show greater difference followed by the mobile contactless devices, which show even greater difference.

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