Recommendations for Specifying Idealized Bust Surrogates for the Testing of Female Stab Resistant Armor

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Abstract

This document describes recommendations provided to the National Institute of Justice’s (NIJ) Special Technical Committee (STC) for the revision of the Stab Resistant Body Armor Standard related to the specification of idealized bust surrogates for armor testing. This document summarizes the literature reviewed in making recommendations, relevant data collected, and the rationale behind the recommendations. The STC and NIJ are under no obligation to use these recommendations, they are provided for informational and documentation purposes only.

1 Introduction

Female stab resistant armor recently became the subject of interest during the Special Technical Committee (STC) Meeting, convened by the National Institute of Justice (NIJ), to develop a revised standard for the performance and testing of stab resistant body armor. The current NIJ Standard-0115.00, published in September 2000, describes performance requirements and protection classes for stab-resistant body armor. Standardized stab implements are specified for the protection classes defined by the standard. Armor may be engineered to meet the requirements for the Spike protection class, the Edged Blade protection class, or both. The test methods described in the standard were intended to apply to all conventional armor designs, and options were included to accommodate shaped or structured armor designs, including female armor designs. The guidelines for supporting shaped armor for testing are described in the standard, part of which is excerpted below [1].

For all flat and flexible armor samples, a slab of composite backing material shall be used. For shaped female armors or where armors have a preformed curve, it may not be possible for the rear face of the armor sample to lie in close contact with the surface of the composite backing material. If this occurs, then the following should be attempted in the order described:

1. If the armor design permits, the armor shall be flattened so that the curved portions of the armor make good contact with the backing material.

2. If this cannot be done, then the backing material shall be laid over a curved wooden form to ensure that the armor lies in contact with the surface of the backing material.

3. If it is still not possible to achieve good contact between the armor and the backing material, then smaller pieces of the neoprene backing material shall be cut, and then stacked together to make good contact with the tight curvature of the armor.
Currently, 287 models of body armor are listed on the NIJ stab-resistant body armor Compliant Products List (CPL) [2]. Of that total, 229 are Spike protection class only, 18 are Edged Blade protection class only, and the remaining 40 are some combination of Spike and Edged Blade protection classes [2]. Only 13 female stab-resistant body armor models are listed on the CPL, and all 13 belong to the Spike-only protection class [2]. On a percentage basis, female-specific stab-resistant body armor models represent less than 5% of the stab-resistant models listed on the CPL. Since spike-resistant armor materials are typically soft and flexible, many female armor models rely entirely on a darted and seamed armor carrier with traditional flat armor panels inserted in the carrier. These panels conform to the shape of the carrier in an effort to provide comfortable armor to female users. During discussions with the STC, none of the laboratory representatives could recall seeing a female vest that included cups, darts, or stitching as part of the armor panel design. The implication of this observation is that none of the female armor models are believed to be non-planar or shaped, but instead would have been tested the same as any other flat and flexible armor.

Another source of information with ties to the market demand for body armor can be found in the public records of the Bulletproof Vest Partnership (BVP) Grant Program [3]. The BVP records were examined for the period FY2007 through FY2012, inclusive, to determine the quantities of stab-resistant armor requested on grant applications. Of the 13 currently listed female stab-resistant models, only three of them were covered by grant request applications, for a total of 540 units of body armor, over the six year period. During that same period, applications covered requests for 1,079,987 units of body armor (either ballistic-resistant, stab-resistant, or both). Ballistic-resistant body armor accounted for 962,476 units. Stab-resistant body armor accounted for 117,511 units. The 540 units of female body armor represents only 0.46% of the total number of units of stab-resistant body armor units on BVP applications during this six-year period. The BVP does not report on combination armors, which tend to be grouped with ballistic-resistant armor, so if female officers are wearing combination stab and ballistic armor, it is difficult to tell from the BVP statistics.

Whether by model count or units requested on BVP applications, female stab-resistant body armor appears to represent a small fraction of the market demand. The corrections community represents the largest user group of stab-resistant body armor, and interestingly, surveys of corrections facilities indicate that females represent a much larger proportion of the corrections officer workforce than suggested by the body armor market demand indicators. For example, an article published by Cheeseman in 2012, states [4] “According to the American Correctional Associations September 30, 2007 report on Adult Correctional Personnel by Gender and Race, women represented 37 percent of adult correctional personnel (144,274, excluding federal prison personnel) and 51 percent of juvenile corrections personnel. The ratios of male to female personnel varied from state to state. This is in sharp
contrast to 1969 when women made up 12 percent of the correctional work force.” These incongruous findings suggest that female corrections officers are either not wearing body armor, are wearing body armor obtained without BVP funding, or are wearing body armor typically worn by males, without structuring to conform to their bodies [5, 6]. The Committee’s renewed interest in female armor stems from several concerns and questions that arose during discussions.

1. Are female officers being adequately protected from stab and spike threats?
2. Do supported and unsupported regions of armor (such as the cleavage area on a female armor) behave differently during testing?
3. How does one design idealized female bust surrogates for non-planar areas of armor?

The first question touches on several aspects of protection that are beyond the scope of research that could be performed in the limited time the committee has been engaged in a standard review and revision process. Key to addressing the first question are answers to still more questions:

1. Is the female anatomy more susceptible to injury compared to the male anatomy?
2. Are current armor designs worn by females exposing them, in some manner, to more severe threats?
3. Are the current body armor test and evaluation methods appropriate for both the female anatomy and contemporary body armor design strategies?

While not definitive, the committee was not aware of data indicating that current levels of protection were inadequate. Future research directed toward these questions posed above would help to resolve the uncertainty, although such research is not presently planned at NIST. Given the apparent limited usage of female armor, addressing some of the questions will be difficult.

The second question concerning supported and unsupported regions of armor will be the focus of continuing research at NIST. Recent research led to the development of an instrumented drop mass for characterizing the stab impact event and foam materials used in the composite backing material for standardized testing. Research that utilizes the same instrumented drop mass to study impacts on supported, unsupported, and non-planar regions of armor is currently underway and will be the subject of a future report.

The third question concerning the design of an idealized bust surrogate is the subject of this document, which describes NIST’s efforts to assist in developing a suitable bust surrogate.
The Committee’s sense is that improvements in test methods, including the specification of an idealized bust simulant, will stimulate greater interest in female body armor and lead to greater market demand.

2 Definition of idealized surrogates for non-planar armor testing

At the June 2012 meeting of the stab resistant armor STC, the Committee established requirements to base the bust surrogates on common bra sizes. The committee wanted two bra cup sizes defined, one smaller and one larger to cover a large percentage of the female population. The committee also established a requirement for the bust shapes to be simple, primarily to minimize variability and cost in forming the surrogates. A request was made during the meeting for NIST to provide recommended dimensions for idealized surrogates for testing non-planar female stab resistant body armor. NIST agreed to a limited amount of research to advise the committee. The committee’s approach toward supporting shaped or non-planar armor is different in a significant manner from the approach in NIJ Standard–0115.00, the salient portion of which was previously discussed [1]. The difference lies in how non-planar armor should be filled, with the current standard including provisions for shaping the contour of the backing material to ensure that the armor model lies against the surface of the backing material. The committee’s approach was to define a shape, which presumably means that any non-planar armor models submitted for testing would have to be made with curvatures that conformed to the pre-defined surrogate shapes, assuming the manufacturer believes that close contact between armor and backing material is important. Alternatively, the manufacturer may not hold such a view, and instead opt to submit armor with non-planar shapes that do not conform to the surrogate. Therein lies a key point: during compliance testing under the envisioned new test protocol, there is no assurance that body armor will be in firm contact with the backing material. Similarly, in field use, there is no assurance that body armor will be in contact with the body. This observation suggests that the development of a bust surrogate may not be as important as understanding the impact of non-supported areas on armor performance.

In an effort to inform the committee about some of the questions raised, a limited research effort began with an examination of available data to understand common bust dimensions for females in the United States. The next part was to identify the best sizes and shapes for the surrogates. The final part was to specify practical methods of making these surrogates for use in testing.
3 Determination of common bust sizes of females in the United States

While many media articles frequently discuss topics such as the most common bust size in the United States, finding real data on this topic is difficult. Several reports [7, 8, 9, 10], in addition to internet searches, were used to gather information about this topic. The most helpful of these reports was the United States Air Force Research Laboratory’s (AFRL) “CAESAR: Summary Statistics for the Adult Population (Ages 18-65) of the United States of America” [7]. This report was a survey of the civilian populations of three countries, the US, the Netherlands, and Italy. The data gathered was weighted using data from the 1990 US Census. The data related to bust size is reported as a bust/chest measurement and an under-band measurement, which is the torso circumference measured under the breasts. Using common bra sizing charts [11], these measurements were converted into bra sizes\(^1\). The under-band circumference is determined and is denoted here as \(C_{ub}\), (see Figure 1)\(^2\). Traditionally, 5 inches (12.7 cm) are added to this number, and the result in inches is simply rounded (e.g., 31.5 is rounded to 32). If the measurement is even, then it is used as the band size, \(L_b\). If the measurement results in an odd number, 1 inch (2.54 cm) is added to the measurement to generate an even number for \(L_b\). (US band sizes are all even numbers.) Next, the circumference of the fullest part of the bust area is measured (see Figure 2), in inches. The cup size is determined from Equation 1, where \(C_b\) is the circumference of the fullest part of the bust, \(L_b\) is the band size as determined previously, and \(L_c\) is the cup size [11].

\[
L_c = C_b - L_b
\]  

2 The quantity \(L_c\) can be converted to common “letter” cup sizes using Table 1.

<table>
<thead>
<tr>
<th>If (L_c) is</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Then cup size is</td>
<td>AA</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>DD</td>
</tr>
</tbody>
</table>

Table 1: Cup size conversion chart.

\(^1\)Certain commercial equipment, instruments, or materials are identified in this paper in order to specify the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for this purpose.

\(^2\)Units: The units in this document are primarily given in SI units. As the US industry standard for body armor and other clothing measurements is inch-pound units, in many cases the numbers reported here are conversions from inch-pound units to SI units. To avoid confusion with the discussion of the determination of band sizes in this paragraph, inch-pound and SI units are both reported.
Using Equation 1 and Table 1, the measurements reported in the AFRL study were converted into cup size measurements. The results of this effort are shown in Table 2 [7].

<table>
<thead>
<tr>
<th>Percentile</th>
<th>( C_b ) (cm)</th>
<th>( C_{ub} ) (cm)</th>
<th>( L_b ) (cm)</th>
<th>( L_c ) (cm)</th>
<th>( C_b ) (in)</th>
<th>( C_{ub} ) (in)</th>
<th>( L_b ) (in)</th>
<th>( L_c ) (in)</th>
<th>Cup size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>78.3</td>
<td>66.4</td>
<td>81.3</td>
<td>0.0</td>
<td>30.82</td>
<td>26.13</td>
<td>32</td>
<td>0</td>
<td>AA</td>
</tr>
<tr>
<td>2</td>
<td>79.3</td>
<td>67.5</td>
<td>81.3</td>
<td>0.0</td>
<td>31.23</td>
<td>26.58</td>
<td>32</td>
<td>0</td>
<td>AA</td>
</tr>
<tr>
<td>3</td>
<td>80.3</td>
<td>68.2</td>
<td>81.3</td>
<td>0.0</td>
<td>31.60</td>
<td>26.86</td>
<td>32</td>
<td>0</td>
<td>AA</td>
</tr>
<tr>
<td>5</td>
<td>81.4</td>
<td>69.0</td>
<td>81.3</td>
<td>0.0</td>
<td>32.05</td>
<td>27.17</td>
<td>32</td>
<td>0</td>
<td>AA</td>
</tr>
<tr>
<td>10</td>
<td>83.5</td>
<td>70.7</td>
<td>86.4</td>
<td>0.0</td>
<td>32.86</td>
<td>27.84</td>
<td>34</td>
<td>0</td>
<td>AA</td>
</tr>
<tr>
<td>20</td>
<td>86.4</td>
<td>72.8</td>
<td>86.4</td>
<td>0.0</td>
<td>34.02</td>
<td>28.66</td>
<td>34</td>
<td>0</td>
<td>AA</td>
</tr>
<tr>
<td>25</td>
<td>87.4</td>
<td>73.6</td>
<td>86.4</td>
<td>0.0</td>
<td>34.42</td>
<td>28.98</td>
<td>34</td>
<td>0</td>
<td>AA</td>
</tr>
<tr>
<td>50</td>
<td>93.2</td>
<td>77.7</td>
<td>91.4</td>
<td>2.5</td>
<td>36.68</td>
<td>30.61</td>
<td>36</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>75</td>
<td>102.2</td>
<td>84.0</td>
<td>96.5</td>
<td>5.1</td>
<td>40.25</td>
<td>33.09</td>
<td>38</td>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>80</td>
<td>105.8</td>
<td>86.1</td>
<td>101.6</td>
<td>5.1</td>
<td>41.64</td>
<td>33.91</td>
<td>40</td>
<td>2</td>
<td>B</td>
</tr>
<tr>
<td>90</td>
<td>114.1</td>
<td>92.9</td>
<td>106.7</td>
<td>7.6</td>
<td>44.91</td>
<td>36.56</td>
<td>42</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>95</td>
<td>120.8</td>
<td>100.2</td>
<td>111.8</td>
<td>10.2</td>
<td>47.54</td>
<td>39.43</td>
<td>44</td>
<td>4</td>
<td>D</td>
</tr>
<tr>
<td>97</td>
<td>127.5</td>
<td>104.6</td>
<td>116.8</td>
<td>10.2</td>
<td>50.21</td>
<td>41.19</td>
<td>46</td>
<td>4</td>
<td>D</td>
</tr>
<tr>
<td>98</td>
<td>131.5</td>
<td>109.6</td>
<td>121.9</td>
<td>10.2</td>
<td>51.78</td>
<td>43.13</td>
<td>48</td>
<td>4</td>
<td>D</td>
</tr>
<tr>
<td>99</td>
<td>137.4</td>
<td>114.2</td>
<td>127.0</td>
<td>10.2</td>
<td>54.11</td>
<td>44.95</td>
<td>50</td>
<td>4</td>
<td>D</td>
</tr>
</tbody>
</table>

Table 2: AFRL study bust size calculations [7].

In addition to the bust measurements presented in Table 2, the Appendix to the AFRL study also included data on self-reported bra sizes. These are summarized in Table 3 and simplified to include only cup sizes that were represented in more than 1% of the population surveyed.

The data from the AFRL study indicate the population’s mean bra size is a 36B, and the self-reported sizes show 34B, 36B, and 36C as representing the greatest percent of the population surveyed, however this study was performed over 10 years ago in three countries, and was weighted for the 1990 Census [7]. So it is possible that it is not representative of today’s female population. The popular media has recently reported larger average bust sizes for US women. For example, a 2012 USA Today article [12] reported that ten years ago, the most common bra size in the US was a 36C, and today it is a 36DD. Unfortunately, very little good data exist to objectively determine the current average bra size of US women. The bulk of the scientific literature is focused on breast cancer research and not on defining average bust size. Further complicating the matter is that the same bra size may sometimes be numbered differently by different manufacturers [13]. Even less is available for law enforcement and corrections officers, the target audience for this work. Law enforcement practitioners serving on the committee provided some data, the most useful being size information from 103 female
Figure 1: Demonstration of measurement of under-band circumference [11].

Figure 2: Demonstration of measurement of circumference of fullest portion of bust [11].
<table>
<thead>
<tr>
<th>Size</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 B</td>
<td>2.20</td>
</tr>
<tr>
<td>34 A</td>
<td>7.39</td>
</tr>
<tr>
<td><strong>34 B</strong></td>
<td><strong>15.70</strong></td>
</tr>
<tr>
<td>34 C</td>
<td>7.39</td>
</tr>
<tr>
<td>34 D</td>
<td>2.94</td>
</tr>
<tr>
<td>36 A</td>
<td>4.62</td>
</tr>
<tr>
<td><strong>36 B</strong></td>
<td><strong>12.99</strong></td>
</tr>
<tr>
<td><strong>36 C</strong></td>
<td><strong>12.07</strong></td>
</tr>
<tr>
<td>36 D</td>
<td>2.29</td>
</tr>
<tr>
<td>36 DD</td>
<td>1.12</td>
</tr>
<tr>
<td>38 B</td>
<td>3.11</td>
</tr>
<tr>
<td>38 C</td>
<td>5.72</td>
</tr>
<tr>
<td>38 D</td>
<td>2.11</td>
</tr>
<tr>
<td>38 DD</td>
<td>1.35</td>
</tr>
<tr>
<td>40 B</td>
<td>1.40</td>
</tr>
<tr>
<td>40 C</td>
<td>1.40</td>
</tr>
<tr>
<td>40 D</td>
<td>1.30</td>
</tr>
<tr>
<td>42 B</td>
<td>1.10</td>
</tr>
<tr>
<td><strong>SUM</strong></td>
<td><strong>88.13</strong></td>
</tr>
</tbody>
</table>

Table 3: AFRL study self-reported bra sizes representing greater than 1% of the reporting population. Sizes representing greater than 10% appear in bold [7].
correctional officers, including self-reported cup size [14]. These data indicated that B, C, and D were the most popular cup sizes for this population. The data are summarized in Table 4.

<table>
<thead>
<tr>
<th>Cup Size</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.80</td>
</tr>
<tr>
<td>B</td>
<td>31.07</td>
</tr>
<tr>
<td>C</td>
<td>27.18</td>
</tr>
<tr>
<td>D</td>
<td>23.30</td>
</tr>
<tr>
<td>DD or greater</td>
<td>11.65</td>
</tr>
</tbody>
</table>

Table 4: Self-reported cup size for 103 female correctional officers.

Another correctional agency provided data on vest sizes, but determining the relationship between the body armor manufacturer’s size nomenclature and bra or other clothing size is difficult. While the letter-scale bra cup size convention is ubiquitous, it is not standardized. Individual bra cup sizes and shapes vary among manufacturers and bra styles. Further complicating the problem is the fact that cup size and band size are related, but not in a well defined way. For example, an individual who wears a 36C bra may also find other sizes comfortable to wear. There is a relationship between band size and cup size, for example, a slightly smaller band size with a larger cup, 34D, or a slightly larger band size and a smaller cup, 38B, may be preferred. The implication is that the same breast volume could be considered a B, C, or D cup size, depending on body build and size, and the design approach taken by the bra manufacturer. Therefore, based on the available data, the Committee was comfortable with using two cup sizes, a B and something larger, for testing purposes [15]. The suggestion of this work for a larger size is to select a D cup because it represents a more extreme case. The next section of this report will discuss possible sizes and shapes of the surrogates to use in testing.

4 Options for bust surrogate sizes and shapes

4.1 Surrogate shape determination

The previous section concluded with a suggestion for two cup sizes to be selected for standardized testing, assuming that the standard should be prescribed with regard to the bust surrogate, but the matter of determining a suitable shape and dimension of bust surrogates for these two bra cup sizes also had to be addressed. After reviewing available literature [16, 17] from researchers focused on post-mastectomy reconstruction, several shapes were logical choices for further consideration and consistent with the committee’s objective for the shapes to be of relatively simple geometry. The profile
shapes of the various options were an inverted ‘V’ (sloped lines), a circular arc, a parabola, and a catenary. When these curves are described as a function of \( x \), rotated around the \( y \)-axis, and then cut by a plane perpendicular to the \( y \)-axis, they form a right circular cone, a spherical cap, a paraboloid, and a solid of revolution of a catenary, respectively, all of which have a circular base. The size of the solids generated are determined by the radius, \( r \), of the circular base, and the height, \( h \), of the solid. These sizing parameters would, ideally, correspond to typical values from anthropomorphic studies or common bra cup sizes and produce volumes that were consistent with anatomical studies of breast volume.

### 4.1.1 Right circular cone

The right circular cone is one of the simplest idealized shapes produced by rotating a profile described by Equation (2) around the \( y \)-axis. The slopes of the line segments, \( \pm \frac{h}{r} \), establish the constraints to ensure that the height and base radius conditions are met.

\[
f(x) = \begin{cases} 
\frac{-h}{r}x + h & 0 \leq x \leq r \\
\frac{h}{r}x + h & -r \leq x \leq 0 
\end{cases}
\]  

(2)

Expressions were derived for the volume and surface area of the right circular cone.

\[
V = \frac{\pi}{3} r^2 h \\
S = \pi r \sqrt{r^2 + h^2}
\]  

(3)

### 4.1.2 Spherical cap and hemisphere

The circular profile shape considered is described by Equation (4), where \( R \) represents the radius of the circle, as well as the radius of the sphere produced after rotation around the \( y \)-axis.

\[
x^2 + y^2 = R^2
\]  

(4)

Note the nomenclature: an uppercase \( R \) denotes the radius of the parent sphere, and if a spherical cap of height, \( h \), is taken from the sphere, the circular base of the spherical cap has a radius denoted by a lowercase \( r \). The spherical geometry is constrained by Equation (5), which requires that for any given specification of \( r \) and \( h \), the parent sphere from which the spherical cap is taken must have a radius \( R \).

\[
R = \frac{r^2 + h^2}{2h}
\]  

(5)
4 OPTIONS FOR BUST SURROGATE SIZES AND SHAPES

Expressions were derived for the volume and surface area of the spherical cap in terms of $r$ and $h$.

\[
V = \frac{\pi h}{6} (3r^2 + h^2) \quad (6)
\]

\[
S = \pi (r^2 + h^2)
\]

When $h=R$, then $r=R$, and these expressions reduce to the common formulas for a hemisphere:

\[
V = \frac{2\pi}{3} R^3 \quad (7)
\]

\[
S = 2\pi R^2
\]

For certain combinations of $r$ and $h$, the required value of $h$ may be greater than $R$, which implies that the spherical cap is more than half a sphere. Such a shape would have a smaller circular base than its largest circular cross-section. Due to this constraint, the spherical cap is not a preferred geometry.

4.1.3 Paraboloid

The parabola profile shape is described by Equation (8), where $a$ is a constant that adjusts the focal length, or flatness, of the parabola.

\[
f(x) = ax^2 \quad (8)
\]

To ensure that the paraboloid will satisfy requirements for height and base radius, $a$ is defined by Equation (9).

\[
a = \frac{h}{r^2} \quad (9)
\]

Expressions were derived for the volume and surface area of the paraboloid.

\[
V = \frac{\pi}{2} r^2 h \quad (10)
\]

\[
S = \frac{\pi r}{6h^2} \left[ (r^2 + 4h^2)^{3/2} - r^3 \right]
\]

The paraboloid geometry does not suffer from the same constraints as the spherical cap.
4.1.4 Catenary rotated around the y-axis

The catenary profile shape is described by Equation (11), where $a$ is a scaling parameter that adjusts the flatness of the catenary. The catenary, which is generally described as the shape that a chain or cable assumes under its own weight when supported on the ends [18, 16, 17], has been the focus of much research related to breast reconstruction because the family of catenary curves provides a great deal of flexibility to describe spans and positive and negative curvatures.

$$f(x) = \frac{a}{2} \left( e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$$  \hspace{1cm} (11)

For any combination of $r$ and $h$, the value for $a$ must first be determined from the implicit Equation (12).

$$a + h = \frac{a}{2} \left( e^{\frac{r}{a}} + e^{-\frac{r}{a}} \right)$$  \hspace{1cm} (12)

Expressions were derived for the volume and surface area of the catenary rotated around the y-axis:

$$V = \pi r^2 (a + h) - \pi a^2 \left( r - a \right) e^{\frac{r}{a}} - \left( r + a \right) e^{-\frac{r}{a}} + 2a$$  \hspace{1cm} (13)

$$S = 2\pi a \left[ \sqrt{h^2 + 2ah} \ln \left( \frac{a + h}{a} + \sqrt{\left( \frac{a + h}{a} \right)^2 - 1} \right) - h \right]$$

4.1.5 Other options

All of the shapes described thus far assume that two discrete breast surrogates would be used for body armor testing. Another option for consideration was also developed, and based on the idea that a single continuous ‘plank’ could be used as a surrogate for a bust that was already supported by a bra. Conceptually, the plank would be described as a wedge or a frustum of wedge. The sloped sides of these shapes would loosely approximate the desired bust form and the length would span the width of the non-planar bust region of the body armor. Similar to the approach that will be described for the discrete breast surrogates, the sizing parameters for the wedge or frustum of wedge would be guided by measured breast volumes and typical bra cup dimensions. This approach would not, of itself, produce a region where body armor was unsupported, such as that possibly produced by the inter-mammary cleft (cleavage). Variations on this approach could be considered to address this matter by either notching the wedge or frustum of wedge or using two shorter units separated by some distance; however the committee rejected this concept in favor of the discrete rounded shapes.

Elliptic versions of the cone and paraboloid, which are formed by a cutting plane that is not perpendicular to the y-axis, were also considered but not put forth to the committee because of their added complexity.
4.2 Determination of surrogate dimensions

Specifications for underwires, the reinforcing wires often used under a bra cup to provide additional support and shape provide an indication of the average diameter of bra cups for various combinations of cup and band size [19]. In order to estimate reasonable sizing parameters for the B cup and D cup surrogates, two approaches were taken to investigate the loose relationship between bra size and bra cup dimensions. One relied on measurements of actual bras, while the other considered breast volume measurements found in scientific literature.

Average cup diameter, measured across the underwire, and depth measurements for a selection of different bra sizes are summarized in Table 5. Reported values are the mean of no less than 7 measurements of different bras, with size recorded as reported by the manufacturer. The diameter was measured across the widest point of the cup for width and the deepest point of the cup for depth.

<table>
<thead>
<tr>
<th>Bra Size</th>
<th>Mean Diameter (cm)</th>
<th>Diameter Deviation (cm)</th>
<th>Mean Cup Depth (cm)</th>
<th>Depth Deviation (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>36B</td>
<td>13.34</td>
<td>0.37</td>
<td>4.81</td>
<td>0.96</td>
</tr>
<tr>
<td>36D</td>
<td>15.32</td>
<td>0.22</td>
<td>9.45</td>
<td>0.22</td>
</tr>
<tr>
<td>38D</td>
<td>15.97</td>
<td>0.44</td>
<td>9.43</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 5: Critical dimensions measured from actual bras.

Inspection of Table 5 justifies the rationale for avoiding a hemispherical surrogate shape. Such a shape has a cup depth equal to the cup radius (one half the value of the cup diameter) regardless of the bra size. Inspection of actual bra cup dimensions reported in Table 5 reveals that the ratio of cup depth (the height of the cup) to cup radius ranges from 0.72 to 1.23, whereas evidence of a hemispherical geometry would yield a ratio of 1.0. The wide range reinforces the need for a surrogate shape whose sizing parameters can be adjusted independently for any given cup volume.

The mean bra cup dimensions reported in Table 5 were used to determine the volumes of various geometrical shapes possessing those dimensions, where the mean cup diameter was taken as $2r$ and the mean cup depth was taken as $h$ in the volume equations presented earlier. Table 6 and Table 7 present the results for the hemispherical and non-hemispherical geometries, respectively. Other parameters derived from sizing constraints mentioned earlier for the non-hemispherical shapes are also shown in Table 7.

Note that two sets of values are shown in Table 6 for the hemispherical geometry, depending on whether the radius of the hemisphere, $R$, is taken as the mean bra cup depth, $h$, or the radius of the bra cup base circle (half of the
mean cup diameter), \( r \). Due to the constraint imposed by the hemispherical geometry, \( h = r \), the height and diameter dimensions in Table 5 cannot be satisfied simultaneously.

<table>
<thead>
<tr>
<th>Bra Size</th>
<th>( r ) (cm)</th>
<th>( h ) (cm)</th>
<th>( V ) (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>36B</td>
<td>6.67</td>
<td>4.80</td>
<td>232</td>
</tr>
<tr>
<td>36D</td>
<td>7.66</td>
<td>9.45</td>
<td>1767</td>
</tr>
<tr>
<td>38D</td>
<td>7.99</td>
<td>9.42</td>
<td>1753</td>
</tr>
</tbody>
</table>

Table 6: Volumes of hemispherical shapes based on mean dimensions in Table 5.

<table>
<thead>
<tr>
<th>Bra Size</th>
<th>( r ) (cm)</th>
<th>( h ) (cm)</th>
<th>( V ) (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>36B</td>
<td>6.67</td>
<td>7.03</td>
<td>393</td>
</tr>
<tr>
<td>36D</td>
<td>7.66</td>
<td>7.83</td>
<td>1312</td>
</tr>
<tr>
<td>38D</td>
<td>7.99</td>
<td>8.10</td>
<td>1383</td>
</tr>
</tbody>
</table>

Table 7: Volumes of non-hemispherical shapes based on mean dimensions in Table 5.

Published breast volume measurements [20, 21] for various bra cup sizes were relied upon to assess the idealized surrogate geometries. The median values, as well as the minimum and maximum values of measured breast volumes are denoted by subscripts \( \text{med} \), \( \text{min} \), and \( \text{max} \), respectively, and shown in Table 8. The cited studies reveal considerable variability in measured breast volumes for each of the reported bra cup sizes.

<table>
<thead>
<tr>
<th>Cup Size</th>
<th>( V_{\text{med}} ) (cm(^3))</th>
<th>( V_{\text{min}} ) (cm(^3))</th>
<th>( V_{\text{max}} ) (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>225</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>B</td>
<td>425</td>
<td>150</td>
<td>1100</td>
</tr>
<tr>
<td>C</td>
<td>700</td>
<td>350</td>
<td>1800</td>
</tr>
<tr>
<td>D</td>
<td>950</td>
<td>250</td>
<td>2000</td>
</tr>
</tbody>
</table>

Table 8: Median, minimum, and maximum measured breast volumes reported by Ringberg [20].

Comparison of actual breast volumes (Table 8) with volumes based on idealized geometrical shapes whose dimensions were based on bra cup measurements (Table 6 and Table 7) reveals that the hemispherical geometry is
unreliable, either producing too little volume or too much volume depending on whether the desired bra cup height or bra cup base radius is given preference. The cone geometry appears to produce too little volume consistently. The spherical cap geometry produces too much volume for the larger cup size, while the rotated catenary and paraboloid geometries both yield comparable results and provide reasonably good matches to the median values of the measured breast volumes.

A second analysis was performed by assuming that the breast surrogate volumes should be consistent with the median values for measured breast volumes shown in Table 8. With the exception of the hemispherical geometry, this analysis also relies on an assumed height, \( h \), of the breast surrogate shapes. To establish the surrogate heights for the non-hemispherical geometries, Table 1 and Table 5 were consulted. Table 5 suggests that the depth of the deepest point of the bra cup is consistent with the \( L_c \) values shown in Table 1; however, a better approximation of actual bra cup depths, which would correspond to the height of the idealized geometrical shape surrogate, can be obtained by multiplying the \( L_c \) values in Table 1 by a factor of 0.93. For the non-hemispherical geometries, the subsequent analyses used both sets of values, \( L_c \) and \( 0.93L_c \), for the heights of the surrogate shapes. By using the volumes in Table 8 and assuming various geometrical shapes, the base diameter of the surrogates, \( d \), and where appropriate, any other sizing constraints, were determined for each of the idealized geometrical shapes. The geometrical constraints mentioned previously for the hemisphere determine the radius (height) for a given volume, so it is not possible to assign a height of \( L_c \) and \( 0.93L_c \) while still satisfying the volume goals. Instead, for the hemispherical geometry, the base diameter, \( d \), (which is equal to \( 2h \)) that provides the desired volume was determined. As before, subscripts on the size parameters denote whether the parameters are associated with median, minimum, or maximum volumes. Results are shown in Table 9 for the hemisphere, Table 10 for the paraboloid, Table 11 for the cone, Table 12 for the spherical cap, and Table 13 for the rotated catenary.

The conclusion reached earlier about the unsuitability of the hemispherical geometry is further supported by the size parameters in Table 9, where hemisphere heights are too large for the smaller cup sizes and too small for the larger cup sizes when compared with bra cup dimensions in Table 5. Table 11 reveals that the right circular cone size parameters lead to base diameters that are too large relative to the bra cup dimensions in Table 5. Table 12 indicates that the spherical cap geometry suffers from a shape constraint issue mentioned earlier: for combinations of \( r \) and \( h \), typically those associated with the larger cup sizes, \( h \) tends to be greater than \( R \), which means that the spherical cap is more than half a sphere. Under this condition, the surrogate shape would have a smaller circular base than its largest circular cross-section. Of the geometries considered in depth, only the paraboloid and the rotated catenary offer the flexibility to adjust height and circular base radius independently. Importantly, when either of these two shapes are
sized to realistic heights and volumes, the resulting circular base diameters are similar to bra cup diameters. Of these two geometries, preference is given to the paraboloid because of its computational simplicity.

<table>
<thead>
<tr>
<th>Cup Size</th>
<th>( V_{med} )</th>
<th>( V_{min} )</th>
<th>( V_{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h_{med} )</td>
<td>( d_{med} )</td>
<td>( h_{min} )</td>
</tr>
<tr>
<td>A</td>
<td>4.75</td>
<td>9.51</td>
<td>3.63</td>
</tr>
<tr>
<td>B</td>
<td>5.88</td>
<td>11.75</td>
<td>4.15</td>
</tr>
<tr>
<td>C</td>
<td>6.94</td>
<td>13.88</td>
<td>5.51</td>
</tr>
</tbody>
</table>

Table 9: Hemisphere parameters for \( V_{med} \), \( V_{min} \), and \( V_{max} \) in Table 8.

<table>
<thead>
<tr>
<th>Cup Size</th>
<th>( h = L_c )</th>
<th>( h = 0.93L_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h )</td>
<td>( d_{med} )</td>
</tr>
<tr>
<td>A</td>
<td>2.54</td>
<td>15.02</td>
</tr>
<tr>
<td>B</td>
<td>5.08</td>
<td>14.60</td>
</tr>
<tr>
<td>C</td>
<td>7.62</td>
<td>15.29</td>
</tr>
<tr>
<td>D</td>
<td>10.16</td>
<td>15.43</td>
</tr>
</tbody>
</table>

Table 10: Paraboloid parameters for \( V_{med} \), \( V_{min} \), and \( V_{max} \) in Table 8.

<table>
<thead>
<tr>
<th>Cup Size</th>
<th>( h = L_c )</th>
<th>( h = 0.93L_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h )</td>
<td>( d_{med} )</td>
</tr>
<tr>
<td>A</td>
<td>2.54</td>
<td>18.39</td>
</tr>
<tr>
<td>B</td>
<td>5.08</td>
<td>17.88</td>
</tr>
<tr>
<td>C</td>
<td>7.62</td>
<td>18.73</td>
</tr>
<tr>
<td>D</td>
<td>10.16</td>
<td>18.90</td>
</tr>
</tbody>
</table>

Table 11: Right circular cone parameters for \( V_{med} \), \( V_{min} \), and \( V_{max} \) in Table 8.

Based on these studies, the recommended geometry for discrete surrogates is a paraboloid whose dimensions are similar to those shown in Table 10 for \( h = 0.93L_c \). A reasonable choice for the B cup surrogate has a base diameter of 15.14 cm (or 15 cm) and a height of 4.72 cm (or 4.7 cm), and the D cup surrogate has a base diameter of 16 cm and a height of 9.45 cm (or 9.5 cm).
\[
\begin{array}{cccccccc}
& h = L_c & & h = 0.93L_c & & \\
\hline \\
\text{Cup Size} & \text{h} & \text{R}_{\text{med}} & \text{d}_{\text{med}} & \text{R}_{\text{min}} & \text{d}_{\text{min}} & \text{R}_{\text{max}} & \text{d}_{\text{max}} \\
A & 2.54 & 11.95 & 14.73 & 5.78 & 9.57 & 25.52 & 22.20 \\
B & 5.08 & 6.94 & 13.37 & 3.54 & 6.39 & 15.26 & 22.74 \\
D & 10.16 & 6.32 & 10.02 & \text{null} & \text{null} & 9.55 & 19.07 \\
\hline
\end{array}
\]

Table 12: Spherical cap parameters for \(V_{\text{med}}\), \(V_{\text{min}}\), and \(V_{\text{max}}\) in Table 8.

\[
\begin{array}{cccccccc}
& h = L_c & & h = 0.93L_c & & \\
\hline \\
\text{Cup Size} & \text{h} & \text{a}_{\text{med}} & \text{d}_{\text{med}} & \text{a}_{\text{min}} & \text{d}_{\text{min}} & \text{a}_{\text{max}} & \text{d}_{\text{max}} \\
B & 5.08 & 5.7246 & 14.30 & 2.2499 & 8.32 & 14.0952 & 23.27 \\
D & 10.16 & 3.6838 & 14.73 & 1.2569 & 7.28 & 7.0562 & 21.73 \\
\hline
\end{array}
\]

Table 13: Rotated catenary parameters for \(V_{\text{med}}\), \(V_{\text{min}}\), and \(V_{\text{max}}\) in Table 8.
5 OPTIONS FOR CREATING BUST SURROGATES FOR TESTING

The methods described can be used to develop sizing recommendations for other cup sizes, some of which are shown in Table 10. Depending on surrogate materials and manufacturing methods, some adjustments to these dimensions may be necessary. For example, manufacturing a surrogate from multiple layers of materials having a uniform thickness may benefit from adjusting the total height of the surrogate to some multiple of the single-layer thickness. A surrogate whose size is adjusted may be reassessed using the equations and tables presented to confirm that the surrogate dimensions are a reasonable representation of actual bra cup sizes and breast volumes.

For comparison purposes, profiles of the various shapes are superimposed on each other in Figure 3 and Figure 4. The parameters selected for the profiles are as discussed above for the B and D cup. The catenary and the parabola appear nearly identical in this comparison, as would be expected given the similarities noted earlier.

5 Options for creating bust surrogates for testing

Many different ideas were discussed and debated for making the foam surrogates. This section will discuss several of these efforts.

5.1 Molding the foam

Efforts to mold the foam using heat were unsuccessful. An aluminum cylindrical mold (negative of cylindrical shape) was heated in an oven to 120°C. After reaching temperature, the mold was manually pressed into the polyethylene foam to create a foam cylinder. While the mold was able to shape the foam into a cylindrical shape, the process created a significant skin layer and it was difficult to carefully control the final shape of the sample. The presence of the skin layer indicates that the foam structure has been destroyed and there is a solid layer of polymer on the outside of the foam. There is concern that the inconsistent thickness of this layer and its presence on such a small foam part would unduly influence test results. In addition, the inability to create a consistent geometry would provide difficulties in mating armor to the foam for testing.

5.2 Cutting the foam

A hot wire foam cutting apparatus (Figure 5) was located and purchased for the purpose of cutting the foam into the desired shape. The blade could be curved to the appropriate shape and the wedge of foam could be rotated beneath the knife to cut the foam into the desired parabolic shape. However, once the device was obtained it was determined that the foam could not be
Figure 3: Comparison of possible geometric shapes for B cup surrogates. In this figure, the base was set to 15.1 cm and the height was set to 4.7 cm.
Figure 4: Comparison of possible geometric shapes for D cup surrogates. In this figure, the base was set to 16 cm and the height was set to 9.5 cm.
Figure 5: Hot wire foam cutter used to cut foam.
5 OPTIONS FOR CREATING BUST SURROGATES FOR TESTING

cut repeatably in this manner. The hot knife blade melted and caught in the foam, producing jagged, melted edges and a nonuniform shape.

Figure 6: Prototype foam stacks cut with scissors. Final cut stacks would use cutting dies for uniform shape and size.

5.3 Other efforts

Other ideas that were considered were using mold-able floral foam or craft foam to support the non-planar areas of the armor and create a flat surface for mounting to a typical foam pack. Another option was to fill the back of the armor in a similar manner with spray foam such as that used for insulation. However, when this option was attempted, large gaps and air pockets were created in the spray foam, meaning that the support structure was nonuniform and unsuitable for testing. Various methods of cutting stacks of stab foam into the desired shapes were also discussed [15]. The consensus was that using new materials added new and undesirable variability to the test, and would require a significant validation effort. Therefore, the STC determined that the best course of action was to prepare the foam surrogates from the existing foam backing materials used in the test method. Working within this constraint, NIST recommends using circles of foam cut and stacked to approximate the desired size and shape (Prototype of this concept shown in Figure 6). However, after this meeting additional work was done outside of NIST to look at materials other than the foams specified during the STC. Another option for more realistic and uniform bust surrogates would be to
have such surrogates machined from foam. Examining the feasibility and cost of this approach is an item for future work.

6 References


