# CHARACTERISTICS OF RADIATION PYROMETERS

By George K. Burgess and Paul D. Foote

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I. INTRODUCTION

Some 10 years ago the Bureau of Standards published an account
of an investigation of the subject "Optical pyrometry," 1 in which
were stated the principles involved in the measurement of high
temperatures by optical methods and descriptions given of the
instruments then in use, together with certain applications.

In the meantime there has been a marked development in the
construction of instruments designed for the measurement of tem-
perature, based on the use of the total radiation from hot sub-
stances. It is the object of the present paper to give an account
of the principles of radiation pyrometry, considering mainly the
various types of radiation instruments, their calibration, sources
of error, including the various factors which may influence their
indications, and their adaptability to the measurement of tempera-
tures.

It will be shown that, when suitably designed, adequately cali-
brated, and correctly used there are available several trustworthy

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1 Waidner and Burgess, Bureau of Standards Scientific Paper No. 11.
Radiation Pyrometers

Instruments of this type having many advantages in practical use. It will also be made clear that the radiation pyrometer is subject to many influences which may seriously limit the accuracy of its readings unless proper precautions are taken. Although some of the facts here emphasized are not unknown, it was considered desirable to give a rather complete discussion of the subject of radiation pyrometry even at the risk of some repetition of certain facts, the consequences of which have not always been sufficiently considered in pyrometric practice.

The discussion has, however, been limited, in the main, to those types of instruments which are portable and which are in general use as pyrometers, although it is recognized that there are other special energy receiving and measuring devices which have been of great service in the laboratory and which could be or have been adapted to the measurement of temperature.

The experiments here described and many of the problems attacked are the outcome of the accumulated experience of the pyrometric laboratory of the Bureau of Standards, and, therefore, represent in part the development of suggestions coming from others, including, in particular, Messrs. Waidner, Buckingham, and Kanolt.

II. PRINCIPLES OF RADIATION PYROMETRY

1. FUNDAMENTAL FORMULAS

The radiation pyrometer in the forms constructed at present is a temperature-measuring instrument, primarily based upon the idea involved in the Stefan-Boltzmann equation of energy transfer by radiation from a black body:

\[ J = \sigma T^4 = \int_0^\infty J_\lambda d\lambda \]

Where \( J \) is the total amount of energy radiated per unit area and unit time from a black body \(^2\) at an absolute temperature \( T \), and

\(^2\) The expression "black body" is synonymous with "complete," "total," or "integral" radiator or absorber of radiant energy. The absorption coefficient of a black body is unity; i.e., it absorbs all the radiation falling upon it, whatever be the intensity or wave length of this radiation. The absorption of radiation of any wave length will change the temperature of the black body and alter the spectral distribution of the emitted radiation in a definite manner, the spectral distribution of the energy absorbed having only an indirect relation to the spectral distribution of emitted energy.
\( \sigma \) is an empirical constant the value of which depends only on the units of measurement.

If a black body of area \( A_1 \) and absolute temperature \( T_1 \) is radiating to a black body of area \( A_2 \) and absolute temperature \( T_2 \), the two surfaces being parallel, in alignment, and distant apart by \( R \), and if \( T_1 \) is considerably greater than \( T_2 \), and if \( A_1 \) and \( A_2 \) are small and \( R \) is large, the energy transferred by radiation from surface \( A_1 \) to surface \( A_2 \) in unit time is given by the following expression:

\[
J = \frac{\sigma}{\pi}(T_1^4 - T_2^4)A_1 A_2 \frac{1}{R^2}
\]

If \( \sigma \) is determined in watts cm\(^{-2}\) deg\(^{-4}\), the quantity \( J \) represents the energy transferred per unit time, in watts.

The total radiant energy of a material is represented by vibrations of all possible wave lengths from 0 to \( \infty \). The energy associated with waves of lengths lying between \( \lambda \) and \( \lambda + d\lambda \) may be expressed by \( J_1 \), and for a black body the distribution of the energy in the spectrum, \( J_1 = f(\lambda) \), is satisfactorily given by the Wien-Planck relation:

\[
J_1 = C_1 \lambda^{-5}(e^{\frac{\lambda}{kT}} - 1)^{-1}
\]

It would be possible to construct a radiation pyrometer which made use of either of the two displacement laws of Wien:

\[
\lambda_{\text{max}} T = \text{constant}
\]
\[
J_{\text{max}} T^{-5} = \text{constant}
\]

where \( \lambda_{\text{max}} \) is the wave length corresponding to the maximum intensity of radiation from a black body at an absolute temperature \( T \), and \( J_{\text{max}} \) is the numerical value of the intensity at this wave length. Both of these laws have a theoretical as well as an experimental basis.

It would also be possible to construct a radiation pyrometer which utilized the radiation as heat confined in a narrow spectral region (equation 2), but the sensitiveness of such a spectral radiation pyrometer, as well as that of an instrument designed for locating the position or wave length of the maximum energy emission, or for the measurement of the intensity of this emission,
at least in the forms practicable for the ordinary use required of pyrometers, is evidently very much less than that readily obtainable with the radiation pyrometers which use the radiant energy of the entire spectrum. These latter are often called total radiation pyrometers or sometimes total or integral receivers.

As distinguished from radiation pyrometers, instruments which are based upon the principle of estimation or comparison of radiation intensities in the visible spectrum, usually of a narrow spectral band, by means of the eye, are known as optical pyrometers. (See equation 2.) The characteristics and behavior of this type of instrument have been described elsewhere. In what follows we shall consider only the behavior of total radiation pyrometers.

The energy received by a total radiation pyrometer may be measured in a variety of ways: calorimetrically, e. g., certain pyrheliometers; thermoelectrically, e. g., the thermopile; electrically, e. g., the bolometer; mechanically, e. g., the angular deflection of a bimetallic spiral spring or the elongation of a metallic strip; and radiometrically, e. g., the pressure of radiation exerted on delicate vanes mounted in vacuo, etc.

The thermoelectric and the mechanical (bimetallic spring) methods are the only total radiation methods which have been quite generally applied strictly for the purpose of temperature measurement.

The quantity of energy a body receives by radiation from another body depends on certain conditions relative to each of the two bodies, viz, area of surface, distance apart, emissive and absorbing power, and temperature.

2. GEOMETRY OF THE RADIATION PYROMETER

Let us first consider the question of the areas of the source and the pyrometer receiver. In general it is desirable to have the area of the receiver very small, partly for the purpose of requiring a small area of the radiating source, a matter of great practical importance. The effective sizes of both the receiver and source may be decreased merely by the use of diaphragms suitably placed, but usually, for practical purposes in such a diaphragmed
system, the amount of energy received by the pyrometer would be too minute to be measured satisfactorily by either the thermoelectric or mechanical method. Some device is accordingly necessary for condensing the energy at the surface of the small receiver. This may be accomplished by locating the receiver at the focal point of a convex lens or concave mirror or at the apex of a hollow reflecting cone.

A fundamental condition, which should be satisfied by a radiation pyrometer, is invariability of the pyrometer reading when the distance from the source to the receiver is altered. This requires that the solid angle of the cone of rays entering the receiver remain constant. Such a condition may be secured by a proper arrangement of diaphragms.

![Diagram](image)

**Fig. 1.—Cone type of fixed focus radiation pyrometer**

The cone receiver is made independent of distance to the source, \( A \), by locating at a short distance in front of the cone a small diaphragm opening, \( B \), which acts as a secondary source, as shown in Fig. 1.

The amount of radiation falling on any element of area at \( D \) is proportional to the solid angle \( HDA' \), which is independent of the distance from the point \( D \) to the source. This is true of any point on the base of the cone \( DD' \). Hence, the total quantity of energy entering the cone is independent of the distance from the pyrometer to the source, provided the source is of sufficient size. The minimum-sized source for any distance is determined by the lines \( A''D' \) and \( A'D \), which intersect at \( C \), forming the angle \( \alpha \), called the aperture of the instrument. Thus, for the distance \( CA \), the diameter of the source must be at least \( A'A'' \), for the distance \( CP \), the diameter must be \( P'P'' \), or for any distance \( x \) measured from the point \( C \),

\[
\text{Least diameter of source} = \frac{x \times \text{(diameter of diaphragm } B)}{BC}
\]
When the device for condensing the energy upon the receiver is a concave mirror or a convex lens, two different designs of the instrument are possible.

1. The receiver and source are located at the conjugate foci of the mirror or lens.

2. The receiver and a front diaphragm of the pyrometer are located at the conjugate foci of the mirror or lens.

In case No. 1 the pyrometer must be provided with a focusing arrangement, whereby the distance from the lens or mirror to the receiver may be varied, depending upon the distance from the pyrometer to the source. It is possible to so construct the pyrometer that, as far as the geometrical optics is concerned, the readings of the instrument do not change with distance from the source to the pyrometer, Fig. 2.

Let \( ds' \) = the area of the receiver. (This is fixed by the mechanical construction of the pyrometer. For example, it may be either a small receiving disk fastened to the thermocouple, or a limiting diaphragm immediately in front of the receiver and practically coincident with it.)

\( ds = \) the section of the source of which \( ds' \) is the image.

\( u = \) distance from source to mirror or lens.

\( v = \) distance from image to mirror or lens.
The total energy in the cone $mdsn$ is given by the following equation:

$$J = 2\pi ids \int_0^\theta \sin \theta \cos \theta d\theta = \pi ids \sin^2 \theta,$$

where $i$ is the intensity of radiation of the source.

By the Helmholtz reciprocity theorem one may write a similar expression for the image space:

$$J' = \pi i'ds' \sin^2 \theta'.$$

If there is no loss of radiation by absorption and reflection, (1) and (2) are equivalent. Therefore, since by the condition of magnification of the optical system $ds/ds' = u^2/v^2$, the intensity $i$ is equal to the intensity $i'$. When radiation is lost at the lens or mirror by absorption (and reflection also in the case of the lens), $i'$ is less than $i$, so that $i' = Ki$, where $K$ is to all practical purposes a constant independent of $u$ or $v$.

Hence, $J' = K \pi i'ds' \sin^2 \theta' = i \cdot \text{constant} \cdot \sin^2 \theta'$, since $ds'$ is fixed in size. But $J'$ represents the amount of radiation absorbed by the receiver, and this can be independent of the focusing distance only when $\sin^2 \theta' = h^2/v^2 = \text{constant}$.

The angle $\theta'$, and hence $\sin^2 \theta'$, is made constant by locating a properly-sized exit pupil, $P$, at a certain distance from $ds'$. When this is done, the energy falling upon the receiver is proportional only to the intensity of the radiating source. The readings of the pyrometer will then be independent of the focusing distance, provided the source is of sufficient size that its image completely covers the receiver $ds'$. The size of the opening $P$ is determined such that for all focusing distances or variations in $v$ the cone $mds'n$ always intersects the mirror or lens surface.

The lens, or mirror (neglecting absorption and lost reflection), produces exactly the effect upon the receiver which would be obtained without the lens or mirror if the source having the size of the exit pupil $P$ were located at $P$, or a source represented by $mn$ located at the position $mn$, or a source of diameter $2x\cdot \tan \theta'$ located at $x$, where $x$ is the optical distance of the receiver from the source. It is obvious that a great advantage is gained by the use of a condensing device in diminishing the size of source required, especially at great focusing distances.
In case No. 2, the front diaphragm $B$, of the pyrometer and the receiver $b$, are placed at the conjugate foci of the lens or mirror $DD'$, as illustrated in Fig. 3.

These latter instruments are of fixed focus, the front diaphragm $B$ acting as a secondary source. The smallest-sized source which can be used without necessitating correction to the readings of the pyrometer must have a diameter $A'A''$ such that,

$$\text{Diameter of source} = \frac{x \text{(diameter of diaphragm } B)}{BC}$$

where $x$ is the distance from the source to the point $c$. The angular aperture of the instrument is $\alpha$.

![Diagram](image)

**Fig. 3.—Mirror and lens type of fixed focus radiation pyrometer**

3. THE RADIATION PYROMETER AS A "BLACK" RECEIVER

Energy receivers may be divided into three classes, as follows:

1. A "black" receiver is one which absorbs all the energy falling upon it and reflects none, whatever be the wave length of the incident radiation. Its absorption coefficient is accordingly unity.

2. A "gray" receiver is one having an absorption coefficient which is independent of the wave length of the incident radiation, the value of the coefficient being less than unity.

3. A "selective" receiver is one having an absorption coefficient which is a function of the wave length of the incident radiation.

If a radiation pyrometer is to be employed for the direct measurement of apparent temperatures of nonblack substances (see Sec. VII), it is essential that the receiver be either black or gray.
It is practically impossible to obtain reliable results with a selective receiver, as may be seen from the following ideal and extreme example: Suppose a material emitted selective radiation in a region confined exclusively between \( \lambda = 1 \mu \) and \( \lambda = 2 \mu \). A pyrometer having a selective receiver which absorbed radiation of wave lengths 3 to 4\( \mu \) only would show no reading whatever when sighted upon this material, while one for which the region of selective absorption lay between 1 and 2\( \mu \) would present a maximum indication. It is apparent that selective receivers can be adapted to the measurement of apparent temperatures on non-black bodies only when the value of the reflection coefficient for a single wave length, \( R_1 \), never becomes 1 (a condition which actually is always fulfilled), and when one possesses a complete knowledge of the equations \( R_1 = f(\lambda) \) and \( E_1 = f(\lambda, T) \), where \( E_1 \) is the emission coefficient of the nonblack body. Even assuming that these two functions are known for a given material, which is far from being true, corrections of an exceedingly complicated nature would then have to be applied to every reading unless some method of empirical calibration were devised, this empirical calibration of course being made for each and every material sighted upon.

Consequently, it is desirable to know under which type of receiver the ordinary radiation pyrometer may be classed. As an example, let us consider the Fény mirror instrument, one which apparently may show some deviation from the ideal black receiver. The receiving disk of this pyrometer is thoroughly blackened, and the observations of Coblentz \(^4\) would indicate that a properly blackened surface is practically nonselectively absorbing, the absorption coefficient for all wave lengths being approximately 0.99, when the radiation is normally incident. The gold mirror, however, is selectively reflecting for wave lengths shorter than 1\( \mu \), as may be seen from the S-shaped curve of Fig. 4. The values of \( R_1 \), which refer to normal incidence upon a gold surface, were taken from the Smithsonian tables and from recent papers of Hagen and Rubens, Tool, and Tate. The reflection coefficient for \( \lambda = 0.4 \mu \), is less than 0.30, rising to 0.97 at 1\( \mu \). From 1\( \mu \) to the extreme infra red (25\( \mu \)) the coefficient is more nearly constant, slightly increasing from the value 0.97 to 0.98\( \mu \).

\(^4\) Coblentz, Bureau of Standards Scientific Paper No. 196.
It is desired to consider what effect the selective reflection of gold in the region $\lambda < 1 \mu$ has upon the reading of the pyrometer, when the instrument is sighted upon a black body at various temperatures. If $R$ denotes the fractional part of the total energy reflected by the gold surface when radiation from a black body falls normally upon the surface, and $R_i$ the ordinary monochromatic reflection coefficient of a gold surface at room temperature, for radiation at normal incidence, the value of $R$ is given by the ratio of the product of $R_i$ and the Planck spectral distribution function $J_\lambda$, integrated from $\lambda = 0$ to $\lambda = \infty$ to the integral of $J_\lambda$ between the same limits.

$$R = \frac{\int_0^\infty C_i R_i \lambda^{-5} (e^{\alpha \lambda T} - 1)^{-1} d\lambda}{\int_0^\infty C_1 \lambda^{-5} (e^{\alpha \lambda T} - 1)^{-1} d\lambda}$$

**Fig. 4.—Reflection coefficient of gold as a function of the wave length**
The above expression was evaluated mathematically, but since the function \( R \) is difficult to express in a form which permits a simple computation of \( R \), the integration is better performed graphically. Accordingly, curves were drawn of \( R \) vs. \( \lambda \) and of \( J_1 \) vs. \( \lambda \) and integrated by means of a planimeter. The ratio of the two areas thus obtained is the value of \( R \) for any given temperature of the black body. The values of \( R \) obtained in this manner are represented by Fig. 5. It is apparent from this curve that for a variation in temperature of the black body up to about 2500° C, the selective reflection of gold in the visible spectrum does not alter the amount of total energy emitted by the black body and reflected from the gold surface by more than 2 per cent (equivalent to a possible maximum error in absolute temperature measurement of five-tenths per cent, if the pyrometer were calibrated at low temperatures only). That a larger error is not incurred is of course due to the fact that the greater part of the energy incident upon the mirror is comprised of wave lengths outside the region
of selective reflection of gold. Accordingly, when the pyrometer is sighted upon a black body at temperatures less than 2500° C, the instrument may be considered a gray receiver, approximately 3 per cent of the energy incident upon the mirror being lost by absorption at this surface (dissipated by conduction, convection, and radiation to the outside case), and an additional 1 per cent being lost by reflection at the blackened thermocouple disk. A gray receiver is obviously just as satisfactory for radiometric purposes as an ideal black receiver, the only effect of the nonblackness of the gray receiver being a constant percentage diminution in the effective energy absorbed.

A Féry mirror pyrometer, however, calibrated at temperatures in the region 500° to 1500° C could not be used to determine the apparent temperature of the sun, for example, without a careful consideration of the corrections for the selective reflection of the gold, because the maximum portion of the solar energy is confined to the region of the selectivity of gold. The apparent temperatures measured would be far smaller than the true apparent temperatures. Even at a temperature of the black body of 4000° C the gold mirror reflects but 87 per cent of the total energy falling upon it, and from the slope of the curve of Fig. 5 it is seen that at the sun's temperature (say 6000° C) the reflection coefficient will be somewhere in the neighborhood of 60 to 70 per cent. A reflection coefficient of 68 per cent means an error in temperature measurement of some 500° at 6000° C. Without doubt a silver or platinum mirror would be found much more satisfactory than the gold mirror for these high temperatures, but even these materials could not be used without applying large corrections.

In general, nonblack radiating materials at the temperatures ordinarily available in technical or laboratory work (less than 2000° C), even when selectively radiating, emit the greater portion of their energy in wave lengths outside the region of the selective reflection of gold. The radiation pyrometer accordingly acts as a gray receiver and may be employed for the determination of the total emissivity of these substances. In the case of gold and copper at high temperatures, however, the selective emission is high in the visible spectrum and low in the infra red. Since the selective reflection of the gold mirror of the Féry pyrometer is
low in the visible spectrum and high in the infra red, the total 
emissivity of gold and copper determined by the Fény instrument 
would be too small, and the pyrometer can not in this case be 
considered a gray receiver. The pyrometer may, of course, be 
empirically calibrated to read the true temperatures of glowing 
copper or gold, or a table may be prepared for correcting the 
apparent temperatures measured by the Fény to true tempera-
tures, but these apparent temperatures are not those which one 
would obtain with a black or gray receiver.

4. THERMOELECTRIC RECEIVER AND GALVANOMETER

The proper design of the thermoelectric receiver and indicating 
galvanometer of a radiation pyrometer for technical use is quite 
different from that of a thermopile as usually employed for radio-
metric investigation in the laboratory. In the case of the latter 
instrument experience appears to show that the most satisfactory 
working conditions are obtained when the resistance of the gal-
vanometer is approximately equal to that of the thermopile. In 
general the thermopile has a low resistance—less than 10 ohms—
so that in order to obtain a galvanometer resistance of this mag-
nitude the entire galvanometer electrical circuit is usually con-
structed of copper. Since copper possesses a large temperature 
coefficient of resistance, the sensibility of this type of galva-
mometer will vary with change of room temperature. Also changes 
in the resistance of the thermopile and lead wires due to fluctua-
tions of room temperature will seriously alter the galvanometer 
readings. Accordingly, while such a system may be suitable for 
laboratory use, where a calibration may be made frequently, it is 
not to be preferred for technical work. The galvanometer read-
ings, therefore, should not be influenced by changes of room tem-
perature. Furthermore, the galvanometer should have a short 
period, be dead beat or suitably damped, and have a readily con-
trolled zero, as well as be sufficiently robust and sensitive.

Of late the design of moving coil or d’Arsonval galvanometers 
has progressed so rapidly that even with the portable pivot type 
suspension coils the sensibility is so high that a swamping resis-
tance of zero temperature coefficient may be placed in series with

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6 This condition need not be satisfied exactly, variations of 50 to 200 per cent being permissible, cf. Cob-
the coil, and still the instrument will be of sufficient sensitivity for pyrometric practice. The swamping resistance is located inside the galvanometer, so that in what follows the resistance of the galvanometer is understood to include this resistance, as well as that of the coil; in other words, it is the resistance measured across the galvanometer binding posts. The effect of the swamping resistance is to reduce to practically zero the temperature coefficient of the galvanometer and the dependence of the deflection upon variations in resistance of the thermocouple circuit. The elimination of the effect of change of resistance in the thermocouple and lead wires is of extreme practical importance. It is also desirable that the galvanometer read true emfs of the pyrometer and that it will still read correct emf values when used with pyrometers of different resistances. All of these conditions are satisfied when the resistance of the galvanometer is high in comparison with that of the couple and leads, as is evident from the following discussion:

If \( R_1 \) is the resistance of the thermocouple and the lead wires, \( R_2 \) that of the galvanometer, and \( e_0 \) the true emf of the thermocouple, then the emf, \( e \), indicated by the galvanometer, becomes

\[ e = e_0 \frac{R_2}{R_1 + R_2}. \]

If the galvanometer has been properly calibrated to read the emf at its terminals, the value \( e_0 \), actually indicated by the galvanometer when connected to the pyrometer, will be too low by the factor \( R_2/(R_1 + R_2) \). It would be possible, of course, to allow for this factor in the calibration of the instrument if \( R_1 \) remained fixed. But even though \( R_1 \) varies considerably in a single pyrometer, and especially for different pyrometers, the effect of this variation may be swamped by making \( R_2 \) at least 100 times \( R_1 \). The factor then becomes constant for all practical purposes and nearly equal to 1. A number of galvanometers are available which are sufficiently sensitive for use with a radiation pyrometer and which have a high resistance, 100 to 500 ohms, and a negligible temperature coefficient, are robust and portable, and highly satisfactory for technical purposes.

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6 Other devices are also employed for this purpose, such as varying automatically the strength of the magnetic field, cf Thwing, J. Frank. Inst., 165, pp. 363-370; 1908. Various shunting arrangements in the electrical circuit are also employed.
The thermoelectric power of the couple should be as high as possible, but since the couple must be robust and able to resist mechanical shocks which it is likely to receive, certain couples showing extremely high thermoelectric power can not be used. Iron-constantan, copper-constantan, and several other metals and alloys, some of secret composition, have been employed satisfactorily.

The couple should be of very small wire and rather long in order that heat conduction from the hot to cold junction be a minimum. The cold junction should be carefully screened from radiation from the furnace and the entire couple thoroughly protected from air currents. In general the hot junction has a small metal receiving disk soldered to it, although it is sometimes possible to simply flatten the couple at the junction in order to form a satisfactory receiver. The heat capacity of the receiving system should be small in order to have a quick-acting pyrometer. The receiver should be carefully blackened, so that very little of the energy falling upon it is reflected. The resistance of the couple should be small in comparison with that of the galvanometer. The size of the receiver should be small in order to obtain by the mirror or lens condensers a sufficiently large image without requiring too large a source. And, finally, the hot junction and receiver must not be overheated. It would be possible to actually melt the receiver by the use of a too highly condensing system of mirrors.

The following statement summarizes the essential conditions to be satisfied by the well-constructed thermoelectric radiation pyrometer. Some of the conditions are somewhat contradictory, in which case an adjustment must be made. For example, it might be necessary in a given case to sacrifice low resistance of the couple in favor of high emf.

**Galvanometer.**—(1) High resistance (this refers to the resistance across the terminals of the instrument); (2) negligible temperature coefficient; (3) satisfactory sensibility; (4) robust and portable; and (5) short period and critically damped.

**Receiving system.**—(1) High thermoelectric power; (2) low resistance; (3) robust mechanically; (4) the couple wires and the receiving disk small in size; (5) cold junction screened from radiation and air currents; (6) heat capacity small (quick acting, i. e.,
no lag); (7) black receiver; and (8) maximum temperature attained by receiver should not be too great.

Pyrometer as a whole.—(1) Indications independent of distance from the source to the pyrometer; (2) indications independent of the size of the source above the minimum size required by the instrument; (3) size of source required by the instrument, small; and (4) pyrometer strong mechanically and convenient to handle.

The use of the thermoelectric radiation pyrometer and the characteristics of the mechanical type receiver will be discussed later in the paper.

5. STATEMENT OF THE SOURCES OF ERROR

The errors which may occur in total radiation pyrometry may be classified as follows: (1) Limitations or approximations of the fundamental formulas; (2) imperfections of the radiating source or uncertainties in its radiometric properties; (3) effects of the intervening medium, i.e., air more or less charged with water vapor and gases such as CO and CO₂; (4) construction of the pyrometric receiver; and (5) errors of the measuring or recording instruments. Of these, No. 1 can be exactly determined and allowed for; No. 2 can usually be eliminated when suitable precautions are taken; No. 3 is uncertain, but, in general, is small; No. 4 is of the greatest practical interest and will be considered in detail; and, finally, No. 5 may readily be made negligible by a suitable choice of instruments.

In the ideal radiation pyrometer, the energy \( J \), received from the radiating source at an absolute temperature \( T \), by the receiver at an absolute \( T_0 \), is proportional to the factor \( (T^4 - T_0^4) \), viz,

\[
J = \text{const.} \ (T^4 - T_0^4),
\]

as follows directly from the Stefan-Boltzmann radiation law (equation 1, p. 93). Various factors enter, however, into the actual construction of the radiation pyrometer which slightly alter this ideal relation. For example, consider the thermoelectric type of radiation pyrometer, in which the energy of the radiator is indirectly measured by the emf developed in a thermoelectric circuit. Here, the emf developed is not exactly proportional to the temperature of the receiver, and the temperature of the receiver is not
exactly proportional to the energy received. While the proportionality relation may exist to all practical purposes over a very small temperature range of the radiating source, e. g., \( T = 1000 \) to 1001, etc., it does not hold for the wide variation in emf or in \( T \) which is met with in practice.

Again, mechanical defects in construction, some of which appear impossible to remedy, may cause deviations from the ideal condition. Stray reflection, selective reflection, and convection currents in the pyrometer must necessarily vary in magnitude, depending upon the temperature of the radiating source. The temperature of the hot junction of the thermocouple, \( T_o \), increases necessarily with \( T^4 \), the relation being but approximately linear. The loss of energy expressed as a fraction of the energy incident at the receiver is entirely different for different values of \( T_o \), not merely because of changes in radiation from the receiver, but mainly because of the different rates of energy loss by conduction and by convection currents, i. e., departure from Newton's law of cooling.

For these reasons the radiation pyrometer does not follow exactly the Stefan-Boltzmann radiation law. The deviation from this relation, however, is so small that a quite similar expression can be used. Thus, for the thermoelectric radiation pyrometer the relation between the emf developed, \( e \), and the absolute temperatures of the radiating source, \( T \), and of the receiver, \( T_o \), is expressed by the equation,

\[
e = a \ (T^b - T_o^b)
\]

where \( a \) and \( b \) are empirical constants. The basis for this equation is mainly experimental. Usually it is possible to neglect the \( T_o^b \) term at higher values of \( T \), so that \( e = a T^b \). This expression may be written \( e = (aT^b) \ T_o^{b-4} \), where the term \( T_o^{b-4} \) is a small correction factor necessary to take care of the departure of the instrument from the ideal conditions. This form of equation gives to the performance of the pyrometer a somewhat theoretical significance.

If, in the above equation, \( b \) is arbitrarily chosen of value 4, the calibration computed from the theoretical Stefan-Boltzmann relation, on the basis of the determination of the emf at one standardization temperature, will in general be greatly in error.
Among the sources of error to which the various forms of total radiation pyrometer are subject are:

Lag, or the time required for the pyrometer to come to equilibrium and so give a constant reading when sighted on a hot body. For certain instruments this effect may vary with distance and size of source as well as with temperature.

For the thermoelectric receivers, the resistance of the thermocouple wires and its variation with the temperature of the radiator.

Dirt or dust on and oxidation of the optical system.

The effect of distance of pyrometer to radiator, as influenced by absorption of the intervening medium and by the angular size of the radiator.

Optical imperfections of the pyrometer and its geometry as related to that of the radiator or source.

Lack of "blackness" of the source and of the pyrometer.

These various sources of error will be considered in detail in Section VI, after describing the usual types of instrument and their calibration.

III. TYPES OF RADIATION PYROMETER

The important characteristics in the construction of the several radiation pyrometers available will be briefly considered.

1. MIRROR AND THERMOCOUPLE PYROMETER

(a) Adjustable Focus.—This type of instrument is represented by the Féry thermoelectric pyrometer, Fig. 6.

The image of the source sighted upon must be sharply focused, by adjusting the focusing pinion, upon a limiting diaphragm immediately in front of the thermocouple receiver. This adjustment is greatly facilitated by means of an ingenious device due to Féry. Two semicircular mirrors, inclined to one another at an angle of 5° to 10°, Fig. 7 (a), are mounted in the thermocouple box, an opening of about 1.5 mm at the center of the mirrors forming the limiting diaphragm immediately in front of the couple. The observer views the image of the furnace formed by the large concave mirror $MM'$ and reflected by the focusing mirrors $xx'$ and $yy'$ through the small telescope at $D$. If the image is not correctly focused at
$O$, the line of intersection of the two small mirrors, the image appears broken in half, as shown by (b). Correct focus is obtained when the two halves of the image are in alignment (c). Thus, until correct focus is obtained certain straight lines of the source appear broken in the image. This breaking of a line is quite evident from a consideration of (d). Suppose that the pyrometer were incorrectly focused upon a line source, the image falling at position $AB$ instead of at $O$. The image of the line reflected from the mirror $yy'$ lies at $A''B''$ and that reflected from the mirror $xx'$ at $A'B'$. To the observer at $D$, the projections of these images appear as two distinct arrows, thus, $\rightarrow \rightarrow$. As the pyrometer is brought nearer into the correct focus, by turning

---

*This drawing has been distorted intentionally, for the purpose of more clearly illustrating the principle rather than the actual course of the rays in a given instrument.*
Fig. 7.—Optical system of the Firy mirror type radiation pyrometer
the pinion screw and thus moving the large concave mirror in the direction $OD$, the points $P'$ and $P''$ of the reflected images move along the lines $P'O$ and $P''O$, coinciding at $O$ when the correct focus is obtained. The two arrows are then superposed, forming a single image.

In the earlier type of the Féry pyrometer the large concave mirror was constructed of glass silvered on the outside surface. On account of the oxidizing of a silver surface in air the silver was later deposited on the back of the mirror. Glass itself is a very good reflector for the infra-red rays, hence, with a mirror silvered on the back, the heat rays are reflected in part from the front air-glass surface and in part from the back glass-silver surface. These two groups of rays are brought to the same focal point by making the radii of curvature of the two surfaces slightly different. For all practical purposes, however, the same radius of curvature may be used for each surface if the distance between the front and back surfaces of the glass be small, i. e., 1 to 2 mm.

In some of the later types of the instrument gold mirrors have been employed. Until recently the gold has been deposited upon the front surface of glass by a special "burning-in" process similar to glazing of porcelain but such surfaces show large local aberrations, so that at present an electrolytic deposit of gold upon copper is preferred. Even these surfaces show pitting and local aberration to some extent. The small focusing mirrors are either of glass silvered or gilded or of copper gold plated. The concave mirror is usually about 6.8 cm diameter, 7.5 cm focal length, and 15 cm radius of curvature. The thermoelement is usually iron vs. constantan or Heil's alloy vs. constantan.

This type of pyrometer is generally used at a minimum temperature of 500° or 600° C., although it would be possible to empirically calibrate the instrument for a much lower range, provided the indicating device were made sufficiently sensitive to measure the small emf's developed at low temperatures. The upper temperature range is limited, by the excessive heating of the receiving system, to about 1500° C. However, the cover to the front of the pyrometer is provided with a sectored diaphragm

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8 Féry, Revue Scientifique, (5), 8, pp. 264-272; 1907.
which may be adjusted to reduce the radiation falling upon the receiver by any desired amount, and in this manner the upper temperature range of the instrument is practically unlimited.

The pyrometer, without the front diaphragm, develops an emf of usually several millivolts at 1000° C., the magnitude of course depending upon the construction of the individual instrument.

It has been objected that this type of pyrometer requires a large source upon which to sight, and as generally used such is the case. But it is quite simple to reduce the size of the limiting diaphragm of this pyrometer, just as Gillette reduced the aperture of the Thwing, and thus permit much smaller sources to be employed, at a sacrifice of course in both cases, of the magnitude of the emf developed. Such a procedure can not be called an "improvement of the pyrometer," but it is rather an adaptation of the instrument for some special work. The diameter of the opening in the receiving diaphragm of the Féry pyrometer is usually about 0.15 cm. The relation between the diameter of source $D_s$, diameter of image $D_i$, and focal length of the concave mirror, $f$, is given by the expression:

$$D_s = D_i \left( \frac{u}{f} - 1 \right)$$

where $u$ is the distance from the source to the mirror. Thus, for a mirror of focal length 7.6 cm, the following table shows the diameter of source required for several focusing distances in order that the image cover the 0.15 cm receiver. If the receiver had an opening of half this magnitude sources of half the diameter indicated would be required.

<table>
<thead>
<tr>
<th>$u$ (cm)</th>
<th>Diameter of source (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>1.2</td>
</tr>
<tr>
<td>80</td>
<td>1.4</td>
</tr>
<tr>
<td>100</td>
<td>1.8</td>
</tr>
<tr>
<td>150</td>
<td>3.1</td>
</tr>
<tr>
<td>200</td>
<td>4.2</td>
</tr>
<tr>
<td>300</td>
<td>6.3</td>
</tr>
<tr>
<td>500</td>
<td>10.7</td>
</tr>
</tbody>
</table>

The selection of a satisfactory sectored diaphragm for use with the Fény pyrometer at higher temperatures is rather difficult. Originally a blackened diaphragm was used, but this absorbed a considerable amount of energy and reradiated from the back face to the receiver. To overcome this effect a gold-plated diaphragm is employed. However, reflection from this gold surface back to the source may alter the properties of the radiation of the source as noted on page 155. This effect is in general of small, if any, consequence, but for certain special work it must be considered. Certainly the gold-plated diaphragm is to be preferred to the blackened diaphragm unless the latter were water-cooled.

(b) Fixed Focus Pyrometers.—The front diaphragm of the Foster pyrometer (diameter 2.5 cm), which acts as a secondary source, is placed about 51 cm in front of a mirror of 2.5 cm diameter and about 2.4 cm focal length. The diameter of the source sighted upon must be at least one-tenth the distance from the source to the wing nut at the middle of the pyrometer tube. The mirror is usually of glass gilded on the front surface. The deposit of gold is very thin and may be removed by touching with the finger. Gold-plated copper mirrors would be more satisfactory. The method of mounting the thermoelement is shown in Fig. 8.

The Brown pyrometer is similar in principle to the Foster pyrometer, the distinguishing feature being the construction of the tube or case, which is made collapsible in sections for convenience of carrying. The mirror is of glass silvered on the front surface, having a focal length of 2.2 cm. The thermocouple is rather large, and could be improved by reducing the size of the wires and possibly style of mounting. A camera “finder” for convenience of sighting is mounted on the tube casing, but this, however, is not always properly adjusted. The couple is mounted about 76 cm from the front opening of the pyrometer. The arrangement of diaphragms is poor; each end of each tube section of the case acting as a diaphragm. As a result points on the outer edge of the mirror are screened almost entirely from
the radiation entering the front opening of the pyrometer. However, considering the extreme cone of rays which can fall upon the receiver, it may be seen that the diameter of the source must be at least one-twelfth the distance from the source to a point 29 to 30 cm back from the front opening of the pyrometer.

2. MIRROR AND SPIRAL SPRING PYROMETER (FÉRY "SPIRAL" PYROMETER)

The construction of this instrument is quite similar to that of the thermoelectric type except that the couple is replaced by a bimetallic spring spiral, Fig. 9, carrying an aluminum pointer $P$,

![Fig. 9.—Féry spiral pyrometer](image-url)

which turns over a dial $D$, graduated in degrees centigrade, in response to the differential expansion and uncoiling of the spring when radiation is concentrated upon it. In some of the instruments the spring is trimetallic, the thermal expansion coefficient of the intermediate metal being itself intermediate to that of the two outer metals. Thus, gold, platinum, and invar have been employed. The spiral is similar to that used in the metallic thermometer of Breguet. A strip 0.02 mm thick and 2 mm wide is coiled by several turns into a spiral 2 mm in diameter. The center of the spiral is connected by a shank to a small disk and on this disk is usually mounted the pointer. (In Fig. 9 a slightly
different mounting is shown. The shank is fixed and the pointer is mounted at the other end of the spiral.) Usually a mirror is placed behind the spiral so that the radiation which passes through and between the turns of the spiral is reflected back upon it.

It is of interest to consider the method of spacing of the temperature scale engraved on the instrument. Suppose that the scale is first spaced linearly or in terms of angular deflection of the pointer:

Let \( d = \) angular deflection.

\[ T_o = \text{absolute temperature of spiral.} \]

\[ T = \text{absolute temperature of furnace.} \]

\[ J = \text{energy falling upon spiral.} \]

The angular deflection of the pointer is approximately proportional to the temperature of the spiral; the temperature of the spiral is approximately proportional to the energy absorbed by it; this energy is approximately proportional to the fourth power of the absolute temperature of the furnace; or,

\[ d \propto T_o \propto J \propto T^4 \]

\[ \therefore d = \text{const} \cdot T^4 \]

Hence, determining the deflection corresponding to any one furnace temperature fixes the constant in the above relation and permits the computation of the temperatures corresponding to all other deflections.

Actually, the pyrometer does not exactly follow the fourth-power law but rather the relation:

\[ d = c \cdot T^b \]

where \( b \) is an empirical constant slightly different from 4. If a calibration is made at a number of different temperatures, the exponent \( b \) may be determined from the slope of the best straight line drawn through the observations, plotting \( \log d \) vs. \( \log T \).

The spiral pyrometer has an especial advantage in being self-contained, requiring no accessories such as lead wires, galvanometer, etc., but its accuracy is not equal to that of the thermoelectric instruments. (See p. 146.) The readings depend somewhat upon the position in which the pyrometer is held and upon the previous condition of the instrument. For example, tilting the case to the
right or left alters the reading, and slightly different readings may be expected when (1) the pyrometer has been sighted upon a source at a higher temperature immediately before taking a certain reading, and (2) when the initial source sighted upon was at a lower temperature.

3. LENS AND THERMOCOUPLE PYROMETER (FÉRY LENS THERMOELECTRIC PYROMETER)

Fig. 10 illustrates the construction of the laboratory type of Fény lens pyrometer. The image of the furnace is focused upon the junction of an iron-constantan thermocouple located behind the shield C. In the center of the shield there is a small opening of less than 1 mm which admits the radiation to the thermocouple junction. The image is viewed at the ocular O through the sectional openings of the shield. The ocular is first carefully focused upon the thermocouple junction, and the objective is then adjusted by the pinion P, either for distinctness of image or, preferably, until the effect of parallax is eliminated.

The difficulty in construction of this pyrometer is in realizing a material for the lens which is transparent for all radiations, so that the pyrometer may be calibrated directly in terms of the modified Stefan-Boltzmann law. This is effected by the use of a fluorite lens, which for temperatures above 900° C satisfies the conditions of not altering seriously the radiations transmitted through it; that is to say, the ratio of the radiation absorbed or reflected to the radiation transmitted is approximately constant.
At low temperatures a large proportion of the energy exists in the form of long wave lengths, and as fluorite has an absorption band in the infra-red (near 6 µ) it will absorb a considerable portion of the radiation (at still longer wave lengths fluorite exhibits metallic reflection), and therefore the Stefan-Boltzmann law can no longer be assumed. With a sufficiently sensitive galvanometer or potentiometer this pyrometer could, however, still be empirically calibrated for low temperatures.

The laboratory form of apparatus described above is not well suited for use in technical practice, and fluorite is difficult to procure of sufficient size, and, moreover, is extremely costly. An industrial pyrometer is made by substituting for the fluorite lens a much larger one of glass, and for the delicate galvanometer one of the same type and sensibility as is used in thermoelectric work; the resulting instrument is robust and sufficiently sensitive for all practical uses, and as made has a range from 800° to 1600° C, although the upper limit could readily be extended by the use of sectored diaphragms or rotating sectored disks.

The indications of the industrial form of this pyrometer will not obey Stefan's law, but the instrument may be readily calibrated empirically and the temperatures engraved on the scale of the pyrometer galvanometer.
The emf developed by the fluorite lens instrument is of the order 50 microvolts at 1000° C. Emf's of this magnitude may be conveniently and accurately measured in the laboratory by use of a specially designed potentiometer.

4. CONE THERMOELECTRIC PYROMETER

The construction of the Thwing pyrometer is essentially that discussed on page 96. Fig. 11 represents a section of the receiving system. Thwing has also designed a ventilated type receiving tube or box in which the diaphragms are open to the air, Fig. 12. Air circulation prevents the heating of the diaphragms, so that this instrument may be employed for permanent installation. Also, as will appear later, stray reflection is reduced to a minimum in this type of construction.

The aperture of the Thwing tube pyrometer is usually about 3.5°, requiring a source of diameter at least one-fifteenth of the distance measured from the source to a point 19 cm back of the front diaphragm, i. e., to the point midway between the diaphragms A and B, of Fig. 11, but, of course, any desired aperture may be obtained by altering the size or distance apart of these diaphragms. The emf developed by the usual Thwing pyrometers is much smaller than that of the Féry-mirror, Foster or Brown instruments, having the order of about 0.2 to 0.5 millivolts at 1000° C and about 1 millivolt at 1500° C.

5. NOTES ON CONSTRUCTION AND BEHAVIOR

All of the above-described pyrometers are provided by the makers with indicating galvanometers. Their proper design and construction are also of importance. The question of galvanometers and other points arising in the use and construction of these pyrometers are discussed on pages 104 and 134.

(a) In several Féry pyrometers examined the mirror support or carriage has proven unsatisfactory. The carriage should be rigid, so that the mirror is always perpendicular to the axis of the pyrometer. In one instrument the play was sufficient to cause a large deviation in the reading when the pyrometer was used in different positions. This is due to the difficulty of centering the
image upon the receiver when the mirror is tilted or out of alignment.

(b) The blackening used inside the tube case of the Thwing pyrometer is not as satisfactory as that used in the Brown or Foster instruments. On the other hand, the Brown and Foster pyrometers could be improved by cutting a screw thread inside the tubes, such as that used in the Thwing.

(c) What is the effect of locating the thermocouple receiver of the Foster or Brown pyrometer at a position slightly different from the focus conjugate to the front opening? The result of this shift in position of the receiver may be a decrease in emf, but geometrically the reading of the instrument is still independent of the distance from a sufficiently large source. As a particular example, consider the following case:

\[ u = 50 \text{ cm} = \text{distance of front diaphragm to mirror.} \]
\[ v = 5.6 \text{ cm} = \text{distance of receiver to mirror.} \]
\[ f = 5.0 \text{ cm} = \text{focal length of mirror.} \]
\[ d_o = 0.2 \text{ cm} = \text{diameter of receiver.} \]
\[ d_s = 1.79 \text{ cm} = \text{diameter of opening in front of diaphragm.} \]
\[ d_m = 2.5 \text{ cm} = \text{diameter of mirror.} \]

\( d_s \) is the image of \( d_o \), and the size of \( d_o \) determines the limiting size of the front diaphragm of the pyrometer.

Suppose the same size receiver were incorrectly located 6 cm instead of 5.6 cm from the mirror, all diaphragms and other construction remaining the same, then:

\[ v = 6 \text{ cm} = \text{distance from receiver to mirror.} \]
\[ u = 30 \text{ cm} = \text{distance from mirror to image of receiver.} \]
\[ d_s = 1 \text{ cm} = \text{diameter of image of receiver.} \]

The source may now be considered as lying at the point \( u = 30 \), and having a diameter of 1 cm. The front diaphragm, however, will be seen to act as a stop, preventing radiation from all parts of this new secondary source from reaching all parts of the concave mirror. This results in a decrease in the amount of energy received by the couple.

The solid angle subtended by the mirror at the receiver decreases when the receiver is moved from \( v = 5.6 \) to \( v = 6 \) cm. This also results in a decrease in amount of energy falling on the couple.
If the couple receives radiation directly on its front surface, the amount of energy thus received increases when the couple is moved from \( v = 5.6 \) to \( v = 6 \).

The net result of these three effects is a decrease in the reading of the pyrometer when the couple is located too far from the mirror.

Actually, stray reflection and the aberrations of the mirror seriously modify the effects described in the above statements, so that frequently the couple may show the highest emf when located at a position slightly different from the focal point conjugate to the front diaphragm. The best adjustment is probably obtained by experiment, but whatever be the location of the receiver, if approximately near the ideal position, the readings of the pyrometer will be no more dependent upon the distance to the source than when the receiver and front diaphragm are exactly at the conjugate foci of the concave mirror.

(d) It is frequently stated that great care must be taken to prevent heating of the front diaphragm of the fixed focus pyrometers on account of the expansion of the opening, and as a consequence an increase in the amount of energy falling upon the receiver. The front diaphragm should not become heated for reasons which will be discussed on pages 161–167, but any error arising from expansion is negligible. Thus, suppose the front diaphragm is constructed of brass and is uniformly heated to 125° C, or to 100° above room temperature. The increase in area is proportional to twice the linear coefficient of expansion and to the rise in temperature.

Hence, increase in area = \( 2 \times 0.000019 \times 100 \)

= 0.0038 or 0.38 per cent.

If all this excess energy transmitted by the increased opening fall upon the receiver, it would result in an error of less than 0.1 per cent in absolute temperature—i.e., about 2° at 2000° C. Actually, the front openings of the Foster and Brown are larger than the size required by the image of the receiver, so that a slight increase in the diameter of the opening merely increases the diameter of the image of the opening formed at the receiver, and thus affects very little the amount of energy reflected from the mirror and falling upon the receiver itself. The radiation coming
directly to the side of the receiver opposite the mirror, forming a very small proportion of the total energy received, would theoretically be increased by an increase in the size of the diaphragm. For these two instruments, therefore, barring stray reflection, the possible increase in emf of the receiving thermocouple due to expansion of the front diaphragm would probably not be greater than 0.001 per cent in the case cited, or, expressed in temperature, would be an inappreciable amount.

IV. METHODS OF CALIBRATION

1. PRIMARY CALIBRATION

The primary calibration of a radiation pyrometer is made by sighting upon a black body. This is realized experimentally by uniformly heating the walls of a hollow opaque inclosure and utilizing the radiation coming from the inside through a very small opening. Since, in general, the radiation pyrometer requires a rather large source at a convenient sighting distance, the problem of the construction of a suitable black-body furnace is much more difficult than is the case with one designed for calibrating an optical pyrometer of small aperture. Two slightly different types of black-body radiators have been employed in this work, one constructed of graphite and the other of Marquardt porcelain. (See also Sec. V, 3.)

(a) GRAPHITE BLACK BODY.—A solid Acheson graphite cylinder 16 by 8 cm is turned on a lathe into the form of a crucible with three diaphragms, as illustrated by D, Fig. 13.

The walls of the three inclosures thus formed are about 1 cm thick, each inclosure having the dimension of about 5 by 6 cm. This radiator is mounted in an electrically-heated resistance furnace of the Heraeus type, 60 by 8 cm inside dimensions. The back wall of the graphite radiator is placed 40 cm from the front opening of the furnace, this position giving the most satisfactory temperature distribution as determined experimentally. The temperature of the black body is measured by two thermocouples E and F, protected by glazed Marquardt porcelain tubes inserted into the back wall of the graphite cell and projecting 1 or 2 cm into its interior. The back of the furnace is closed with a kaolin
diaphragm, 2 or 3 cm of powdered Marquardt \((G)\) and a plug of asbestos wool. In front of the cell are located two or more kaolin or Marquardt porcelain diaphragms, which aid in mitigating air circulation. As a further aid in preventing air circulation it is sometimes advantageous to incline the entire furnace at an angle of about 30°. The furnace tube is of Marquardt porcelain wound with platinum tape 5 cm wide. An auxiliary heating coil 15 cm long wound with 1 cm platinum tape has been placed in the front part of the furnace, considerably improving the temperature distribution. If the pyrometer to be calibrated is of the focusing type, it is placed at any convenient distance \(A\), say 100 cm from \(D\) and usually focused on the opening of the inmost diaphragm. This opening is made of sufficient diameter so that its image just covers the receiver. The sizes of all other diaphragms are then determined geometrically by drawing the cone of rays to the pyrometer as shown in the figure.

Graphite has a high thermal conductivity, and hence the temperature distribution over the black body is easily made quite satisfactory; with practically no special adjustment of the auxiliary heating coils, variations of temperature greater than 5° C at 1300° C will not occur over a 10-cm length of the graphite cell.
Graphite in the open has an emissivity very nearly that of a black body. When constructed in the present form of a series of three diaphragmed hollow inclosures, one would expect the radiation from the inner inclosures to be about as "black" as could be realized experimentally. The ends of the porcelain thermocouple tubes, having a very low emissivity in the visible spectrum, are indistinguishable from the surrounding graphite walls at 1300° C. Optical pyrometers, which have been calibrated by sighting into a Lummer-Kurlbaum black-body furnace 11 or with a graphite black body totally immersed in pots of molten metal, 12 the latter method furnishing almost ideal black-body conditions, will give temperatures agreeing with the indications of the thermocouples

![Diagram](image)

**Fig. 14.—Porcelain black body**

$E$ and $F$ to within ±5°, when sighted on the inner diaphragm $D$. And finally the two inner diaphragms of the graphite black body are easily made indistinguishable to the eye from each other or from the openings of the diaphragms. All these conditions would indicate that black-body conditions have been satisfactorily met.

Graphite, however, usually contains impurities in small quantities, but sufficient to corrode the furnace and even contaminate the thermocouples, protected in glazed tubes, after prolonged use. The graphite oxidizes rather quickly even in an atmosphere quite free from drafts, and the black body crumbles away and frequently collapses after several days' continuous heating. There is also some question relative to the effect of the combustion products CO and CO$_2$ upon the indication of the pyrometer, although it is

not likely that serious error can occur from this source, since one would expect the hot CO and CO$_2$ gases to radiate in the region of their absorption.

(b) Porcelain Black Body (Fig. 14).—With the idea of eliminating any possible effect of furnace gases, and in order to do away with corroding effects of the impurities in the graphite, a black body has been constructed of Marquardt porcelain. The diaphragms are of thinner walls but similarly arranged, as in the case of graphite black body. The inner radiating inclosure was made hemispherical in order to secure reflection from the side walls, it being thought possible that such construction might be of some advantage. The thermocouples are used without the protecting tubes and are threaded through small holes in the back of the radiator. This type of radiator also furnishes satisfactory black-body conditions and is somewhat more convenient to use than the graphite form, although the temperature distribution is not quite as uniform, apparently, on account of the much lower thermal conductivity of porcelain. It is possible for a black body of this type to emit radiation of even greater intensity than that of a perfect black body at the temperature of the thermocouples on account of reflection from the somewhat hotter side walls. Whatever undesirable reflection takes place may be minimized by blackening the interior with nickel oxide. However, Coblentz\textsuperscript{13} from spectrobolometric measurements has observed practically no difference between a white porcelain diaphragmed black body and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig15.png}
\caption{Curve showing temperature distribution in porcelain black body}
\end{figure}

\textsuperscript{13}Coblentz, Bureau of Standards Scientific Paper No. 204.

6844°—15—9
the same radiator blackened. If any difference exists, it appears to be too small to be detected by the radiation pyrometers so far examined.

In Fig. 15 is given a curve illustrating the temperature distribution through a portion of the porcelain radiator. The temperatures were obtained by exploration with a thermocouple. For a distance of 12 cm the variation from 1160° is in no place greater than 3°. No special care was taken to secure high temperature uniformity, this distribution representing a case picked at random during an ordinary commercial test. With special attention given to the adjustment of the auxiliary heating coil, the variations can be somewhat reduced.

The thermocouples used in the measurement of the temperature of either type black body are of Pt — 10 Rh 90 Pt, and are carefully calibrated from time to time in terms of the melting points of zinc, antimony, and copper. The parabolic equation \( e = a + bt + ct^2 \) is used for interpolation between the points 300° to 1200° C. Above 1200° C corrections must be added to the temperatures determined by extrapolation of this thermocouple scale, in order that this empirical scale conform with the established gas thermometer scale, as follows:

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Correction to add</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>± 0</td>
</tr>
<tr>
<td>1300</td>
<td>+ 2</td>
</tr>
<tr>
<td>1400</td>
<td>+ 6</td>
</tr>
<tr>
<td>1500</td>
<td>+14</td>
</tr>
</tbody>
</table>

In the calibration of a radiation pyrometer the thermocouple temperatures are always checked by the readings of an optical pyrometer of the Holborn-Kurlbaum type, sighted from the position of the radiation pyrometer, into the black body.

By use of the above-described apparatus, the radiation pyrometer may be directly calibrated from 500° to 1500° C. Satisfactory observations above this temperature range have not as yet been obtained, though it is hoped eventually to develop a method for use at much higher temperatures.
The following table presents the calibration of one of the Bureau’s Féry mirror-pyrometers No. 111 by (1) the graphite radiator and (2) the porcelain radiator. The differences in the calibrations by the two methods are less than the experimental errors involved; and while the deviations are all in the same direction this may readily be due to the pyrometer receiver itself, as will be shown in the discussion of the source of errors of pyrometers in Section VI. Although this individual instrument happens to be the least subject to the various errors affecting pyrometers of any of its type so far examined, one is not justified in using it to compare minutely the merits of the two radiators.

**Féry Mirror-Pyrometer No. 111 (Emf vs. Temperature)**

<table>
<thead>
<tr>
<th>International millivolts</th>
<th>Degrees centigrade</th>
<th>Graphite-porcelain $\Delta t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Graphite radiator</td>
<td>Porcelain radiator</td>
</tr>
<tr>
<td>0.5</td>
<td>638</td>
<td>633</td>
</tr>
<tr>
<td>1.0</td>
<td>820</td>
<td>814</td>
</tr>
<tr>
<td>1.5</td>
<td>943</td>
<td>939</td>
</tr>
<tr>
<td>2.0</td>
<td>1040</td>
<td>1036</td>
</tr>
<tr>
<td>2.5</td>
<td>1119</td>
<td>1116</td>
</tr>
<tr>
<td>3.0</td>
<td>1187</td>
<td>1185</td>
</tr>
<tr>
<td>3.5</td>
<td>1248</td>
<td>1248</td>
</tr>
<tr>
<td>4.0</td>
<td>1304</td>
<td>1304</td>
</tr>
<tr>
<td>4.5</td>
<td>1354</td>
<td>1354</td>
</tr>
<tr>
<td>5.0</td>
<td>1400</td>
<td>1400</td>
</tr>
<tr>
<td>6.0</td>
<td>1484</td>
<td>1484</td>
</tr>
</tbody>
</table>

2. **SECONDARY CALIBRATION** (Illustrated in operation by Fig. 24)

The use of an electric furnace and black body of the type described above has two disadvantages: First, it requires several hours, at least 20 hours for $1500^\circ$ C, in order to reach temperature uniformity and satisfactory black-body conditions, and second, the size of the radiating source is rather small for certain types of pyrometers.

Once having obtained a primary calibration upon a standard radiation pyrometer, the pyrometer to be tested may be compared with the standard by sighting upon a uniform source which is not emitting exactly black-body radiation, provided this radiation is not of a too selective nature.
A source which satisfactorily meets this requirement is obtained by electrically heating sheet nickel in air. A firm and uniform coat of black nickel oxide (NiO) forms on the surface of the nickel thus heated. Such a strip can be used almost indefinitely up to 1300° C, and rapid changes of temperature can be made from 500° to 1300° C. The cooling from 500° to room temperature must be done slowly, or flaking of the oxide will occur. In the present work a strip of nickel usually 17 cm long (exposed section), 13 cm wide, and 0.015 cm thick was mounted vertically between water-cooled brass-clamp terminals. The size of the strips was limited by the current necessary to operate them, 1500 amperes being the maximum current available with the 10-kilowatt transformer employed. A strip of this size furnishes a source of circular area and diameter of 12 cm, which is uniform to within 2° at 1200° C over its entire surface. A careful survey of an area 8 cm in diameter on a 12 by 14 cm strip by means of a Holborn-Kurlbaum optical pyrometer showed the following temperature distribution:

<table>
<thead>
<tr>
<th>Zone radius</th>
<th>Temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm</td>
<td></td>
</tr>
<tr>
<td>0-1</td>
<td>1046.5</td>
</tr>
<tr>
<td>1-2</td>
<td>1046.0</td>
</tr>
<tr>
<td>2-3</td>
<td>1046.0</td>
</tr>
<tr>
<td>3-4</td>
<td>1045.5</td>
</tr>
</tbody>
</table>

The advantage gained in using a strip several cm longer than its width is marked. The temperature variation across the width of the strip is practically nil, the main variation occurring along the lower edge. Thus, the temperature gradient along a vertical section of the strip is not symmetrical, the bottom of the strip being cooler for several cm than the top. Using a strip 17 by 13 cm, the center of the 12-cm uniform temperature area is located 6.5 cm from either side, about 7.5 cm from the top, and 9.5 cm from the bottom.

Whereas heretofore it has been practically impossible to calibrate certain radiation pyrometers requiring a large source, with this type of radiator an intercomparison with a standard pyrometer can be satisfactorily made in an hour. This is illustrated by the
following calibration (an ordinary commercial test) of a Foster radiation pyrometer. Comparison was made with the Féry pyrometer No. 111 at 18 different temperatures. All of the observations are here presented, and the precision of the method may be judged from the deviations of the readings from the mean curve computed and drawn through them. Any systematic errors, or errors in the calibration of the standard pyrometer, can not be determined from this data.

**Foster Fixed-Focus Pyrometer No. 116**

<table>
<thead>
<tr>
<th>Observed emf (\text{millivolts})</th>
<th>Observed (\degree C)</th>
<th>Computed (\degree C)</th>
<th>(\Delta t) computed — observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.584</td>
<td>554</td>
<td>558</td>
<td>+4</td>
</tr>
<tr>
<td>0.760</td>
<td>612</td>
<td>612</td>
<td>0</td>
</tr>
<tr>
<td>1.035</td>
<td>686</td>
<td>685</td>
<td>-1</td>
</tr>
<tr>
<td>1.300</td>
<td>743</td>
<td>742</td>
<td>-1</td>
</tr>
<tr>
<td>1.505</td>
<td>789</td>
<td>785</td>
<td>-4</td>
</tr>
<tr>
<td>1.740</td>
<td>828</td>
<td>826</td>
<td>-2</td>
</tr>
<tr>
<td>1.913</td>
<td>852</td>
<td>853</td>
<td>+1</td>
</tr>
<tr>
<td>2.069</td>
<td>855</td>
<td>863</td>
<td>-2</td>
</tr>
<tr>
<td>2.227</td>
<td>898</td>
<td>898</td>
<td>0</td>
</tr>
<tr>
<td>2.277</td>
<td>908</td>
<td>905</td>
<td>-3</td>
</tr>
<tr>
<td>2.533</td>
<td>944</td>
<td>942</td>
<td>-2</td>
</tr>
<tr>
<td>2.850</td>
<td>974</td>
<td>978</td>
<td>+4</td>
</tr>
<tr>
<td>3.137</td>
<td>1008</td>
<td>1010</td>
<td>+2</td>
</tr>
<tr>
<td>4.102</td>
<td>1107</td>
<td>1103</td>
<td>-4</td>
</tr>
<tr>
<td>4.712</td>
<td>1151</td>
<td>1153</td>
<td>+2</td>
</tr>
<tr>
<td>5.248</td>
<td>1197</td>
<td>1194</td>
<td>-3</td>
</tr>
<tr>
<td>5.655</td>
<td>1225</td>
<td>1223</td>
<td>-2</td>
</tr>
<tr>
<td>6.282</td>
<td>1257</td>
<td>1264</td>
<td>+7</td>
</tr>
</tbody>
</table>

**Average deviation** ............................................... 2.4

In the use of this nickel-oxide source for the calibration of pyrometers it is essential that the instruments compared be of similar type, so that the departure from blackness of the strip will affect each pyrometer in the same manner. Large errors would be involved in the comparison of an optical and radiation pyrometer by this method unless the observations were corrected for the monochromatic and for the total emissivity of nickel oxide.\(^\text{14}\)

\(^{14}\) Burgess and Foote, Bureau of Standards Scientific Paper No. 224. This paper gives a complete discussion of these corrections.
3. COMPUTATION OF CALIBRATION DATA

The thermoelectric type of radiation pyrometer obeys the relation \( e = a(T - T_b) \), where \( e \) is the emf developed when the pyrometer is sighted on a black body at an absolute temperature \( T \), the temperature of the receiver being \( T_r \), and \( a \) and \( b \) are empirical constants. (See p. 107.) In general, \( T_b \) is negligible in comparison with \( T_r \); so that one may write \( e = aT_r \). The constants \( a \) and \( b \) must be determined empirically for each instrument.

Although two calibration points serve to determine \( a \) and \( b \), observations are usually made at five or more different temperatures, and the best curve is drawn through all the points. Since an exponential curve of the correct form is difficult to adjust graphically, the curve is rectified into a straight line by plotting \( \log e \) vs. \( \log T \). Thus, expressed in log form, the equation for the pyrometer becomes,

\[
\log e = \log a + b \log T
\]

which is a linear relation between \( \log e \) and \( \log T \), the slope of the straight line determining the constant \( b \). It is at once seen that in the process of rectification points which should possess equal weight on the \( e \) vs. \( T \) plot can not have equal weight on the \( \log e \) vs. \( \log T \) plot. It is possible to determine the reweight factors of the points on the log plot, but in general sufficient accuracy is obtained by slightly favoring the points at the higher temperatures, in drawing the best straight line through the observations. It would also be possible to adjust the straight line on the log plot by least squares for a minimum of \( \Sigma \delta \log e \), weighting each point differently to correct for the rectification, but the present accuracy of a radiation pyrometer does not warrant the procedure.

Having drawn the best straight line on the log plot, a plot or table of \( e \) vs. \( T \) may be obtained by computation of the exponential equation, the values of \( a \) and \( b \) following directly from the log plot; or, more conveniently, by reading from the log plot the values of \( \log e \) corresponding to various values of \( \log T \) with \( t(=T - 273) \), varying by, say, 50° intervals over the temperature range desired.

With the present equipment of the bureau a calibration may be experimentally determined to 1500° C. For the calibration of a
pyrometer at higher temperatures the straight line of the log plot is linearly extrapolated. Since the equation of the radiation pyrometer, although somewhat modified empirically, is still fundamentally based upon the Stefan-Boltzmann relation, which admits of extrapolation over any range, inasmuch as it defines a temperature scale, it is not likely that serious errors can occur in the extrapolation of the equation of the radiation pyrometer over a few hundred degrees interval above 1500° C. This statement, however, may be questioned, and can only be verified experimentally.

(a) **Effect of the $T_o$ Term.**—In the use of the above method of plotting the observations in terms of log $e$ and log $T$, it is frequently noticed that the curve through the observed points departs somewhat from the linear relation at the smaller values of $T$, the curve tending to curl up at the lower end when log $e$ represents the abscissa and log $T$ the ordinate. This is due to the effect of the $T_o$ term in the expression $e = a(T^b - T_o^b)$. On taking the logarithm of this expression and expanding log $(T^b - T_o^b)$, one obtains:

$$\log e = \log a + b \log T - \frac{0.4343}{b} \left[ \left( \frac{T_o}{T} \right)^b + \cdots \right]$$

Let $\phi = -\frac{0.4343}{b} \left( \frac{T_o}{T} \right)^b$.

Then $log e = \log a + b (\log T + \phi)$ which is a linear relation between log $e$ and $(\log T + \phi)$.

Hence, if instead of plotting log $T$ vs. log $e$, one plots (log $T + \phi$), the best straight line may then be drawn, as before, through the points thus obtained. The slope of this line is $b$ and the value of $a$ follows from computation. Since the quantity $\phi$ is only a small correction factor, one may assume the value of $b$ in the correction term to be equal to 4, so that

$$\phi = -\frac{0.4343}{4} \left( \frac{T_o}{T} \right)^4$$

If the pyrometer is so constructed that $T_o$ does not vary much from room temperature, the value 300° may be used for $T_o$ so that

$$\phi = -0.1086 \left( \frac{300}{T} \right)^4$$
In such a case the correction $p$ is most conveniently determined from a plot of $p$ vs. log $T$, as illustrated in Fig. 16, curve A.

![Correction to apply to log T in plotting vs. log e](image)

The corrections are negative, so that log $T$ is decreased by the value $p$ in plotting each observation. After the best straight line has been drawn on the log $T$ vs. log $e$ plot, each value of log $T$ having been decreased by the correction $p$, a new curve may be
drawn on the same plot by adding the numerical values of \( p \), for several values of \( \log e \), to the values of \( \log T \), read from the straight line. The new curve represents the actual calibration of the instrument and is in general slightly concave to the \( \log e \) axis at lower values of \( \log T \). This method of plotting the observations is illustrated in Fig. 17. The actual observations are represented by circles. The points are first replotted by correcting each value of \( \log T \) by \( p \), and the best straight line is drawn. Then the numerical values of \( p \) are added to the ordinates of the straight line and a new curve is drawn representing the final calibration.

(b) **Determination of the Exponent \( b \).**—The value of this exponent depends upon the construction of the individual instrument. Any defects or peculiarities in the pyrometer may seriously alter this constant, as will be shown later. It has sometimes been assumed\(^{15}\) that the exponent \( b \) for radiation pyrometers has a value of 4 within at least 1 per cent. That this assumption is not generally justified may be seen from the following table, which presents the values of \( b \) determined in the calibration of several radiation pyrometers submitted to the Bureau for test. The method of calibration employed was one of the two primary methods discussed above.

<table>
<thead>
<tr>
<th>Exponent</th>
<th>Type and number (arbitrary reference numbers)</th>
<th>Exponent</th>
<th>Type and number (arbitrary reference numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.85</td>
<td>Fény 130 without diaphragm</td>
<td>3.82</td>
<td>Fény 113 without diaphragm</td>
</tr>
<tr>
<td>3.91</td>
<td>Fény 100 with diaphragm</td>
<td>3.85</td>
<td>Fény 113 with diaphragm</td>
</tr>
<tr>
<td>4.14</td>
<td>Fény 101</td>
<td>3.50</td>
<td>Fény 114</td>
</tr>
<tr>
<td>4.12</td>
<td>Fény 102</td>
<td>4.50</td>
<td>Fény 105a without diaphragm</td>
</tr>
<tr>
<td>4.10</td>
<td>Fény 103</td>
<td>3.55</td>
<td>Fény 105a with diaphragm</td>
</tr>
<tr>
<td>3.96</td>
<td>Fény 104</td>
<td>3.69</td>
<td>Fény 109 fixed focus 107</td>
</tr>
<tr>
<td>3.75</td>
<td>Fény 105</td>
<td>3.28</td>
<td>Fény 109 fixed focus 116</td>
</tr>
<tr>
<td>3.80</td>
<td>Fény 106</td>
<td>3.63</td>
<td>Fény 117 without diaphragm</td>
</tr>
<tr>
<td>3.90</td>
<td>Foster fixed focus 107</td>
<td>3.82</td>
<td>Thwing 116 box receiver</td>
</tr>
<tr>
<td>4.13</td>
<td>Fény 108</td>
<td>4.22</td>
<td>Thwing 119 tube receiver</td>
</tr>
<tr>
<td>3.84</td>
<td>Fény 109 without diaphragm</td>
<td>4.26</td>
<td>Thwing 120 tube receiver</td>
</tr>
<tr>
<td>3.99</td>
<td>Fény 109 with diaphragm</td>
<td>4.32</td>
<td></td>
</tr>
<tr>
<td>4.17</td>
<td>Fluorite lens Fény 110</td>
<td>3.56</td>
<td></td>
</tr>
<tr>
<td>3.78</td>
<td>Fény 111 without diaphragm</td>
<td>3.68</td>
<td></td>
</tr>
<tr>
<td>3.90</td>
<td>Fény 111 with diaphragm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.87</td>
<td>Fény 112</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{15}\) Fény and Drecq, J. de Phys., 1 (5), pp. 551-9; 1911. Numerous other references might be quoted in this regard.
(c) Calibration of Féry Pyrometer With and Without Diaphragm.—The effect of the sector diaphragm used with the Féry pyrometer for extending its temperature range should reduce by a constant factor the amount of energy falling upon the receiver. Thus, if the calibration were represented by \( e = aT^b \) without the diaphragm and by \( Ke = aT^b \) with the diaphragm, the two curves on the \( \log e \) vs. \( \log T \) plot would be parallel, since the slope \( (b) \) of the line remains unaltered. In general, however, one finds that the two best curves are not exactly parallel, which means that the constant \( b \) does not have the same value with and without the diaphragm. The range of \( b \) with and without diaphragm for the same instrument is noted in the table of exponents above. That the constant \( b \) may be slightly different in the two cases is evidently quite possible when one considers that \( b \) differs from the value 4 mainly because of convection and conduction losses inside the receiver. These losses may be quite different for a given temperature \( T \), when the energy falls directly on the receiver and when it is reduced possibly 10 times by the sector diaphragm. It is quite possible also that the heating, however slight, of the sectored diaphragm and reradiation from it is in part responsible for variations in the value of \( b \). Thus, in general, for the highest accuracy, the calibration of the pyrometer with diaphragm should be made just as though one were using an entirely different instrument. (Cf, Fig. 17.) This is especially important if one desires to extrapolate the temperature scale above the points obtained in the calibration.

(d) Effects of Errors.—Errors of calibration, such as lag, or time required to reach the reading of equilibrium condition, the effect of distance from source, and the size of source sighted upon, etc., will be treated quantitatively in Section VI.

V. METHODS OF USE

1. USE WITH GALVANOMETER OR WITH POTENTIOMETER

In the ordinary use of a thermoelectric radiation pyrometer a galvanometer is employed for the measurement of emf, but for the highest accuracy a potentiometer especially adapted to the measurement of small electromotive forces is desirable. Poten-
Radiation Pyrometers

Pyrometers are now available for the measurement of emf's as small as 0.0001 millivolts, and are so designed that internal thermal emf's are reduced to a minimum. The emf and battery circuit of the potentiometer should be provided with reversing switches for the elimination of stray thermal emf's in the lead wires and connections. Special attention should be given to the design of the reversing switches in order that they may not be a source for extraneous thermal emf's. In general, the same precautions apply for the use of a potentiometer with a radiation pyrometer as apply for its use in exact thermocouple work. It is often advisable to ground the pyrometer case, the potentiometer, galvanometer, and accessories.¹⁶

In the use of a potentiometer the resistance or length of the lead wires from the pyrometer, the resistance of the thermocouple, and the variation with temperature in the resistance of the pyrometer circuit produce no effect whatever upon the emf reading.

These sources of error are, however, inherent in the galvanometric method, and in addition there are the sources of error intrinsic to the galvanometer, such as change of sensibility with room temperature, etc. For practical purposes, however, galvanometers are available which are very satisfactory. Many pyrometer galvanometers have a resistance of over 100 ohms and a very low temperature coefficient. The sensibility is sufficient for ordinary pyrometric practice, and still the galvanometer is portable and mechanically robust.

The deflection potentiometer ¹⁷ appears not to have been used for radiopyrometric measurements, but without doubt this type of instrument would prove very desirable for laboratory work. The deflection potentiometer combines the potentiometric and galvanometric methods, the first figure of the value of the emf usually being given by a dial reading and the other figures by the galvanometer scale. The instrument is compact, portable, and accurate, and can be operated very rapidly.

(a) Extrapolation on Galvanometer Scale.—Frequently it is desired to calibrate a pyrometer and galvanometer indicator for a high temperature range. Thus, the Féry pyrometer and

¹⁷ Brooks, Bureau of Standards Scientific Papers Nos. 79 and 173.
galvanometer may be designed for a range 600° to 1300° C using the pyrometer without the sector diaphragm, and a range of 1300° to 2000° C when the diaphragm is employed. Three scales are, therefore, engraved by the makers upon the galvanometer, a scale of emf’s and two temperature scales, one, say, 600°–1300° C and the other 1300°–2000° C. Suppose that the highest temperature at which an observation may be taken during the calibration is 1400° C. The low range scale, using the pyrometer without the diaphragm, could thus be standardized, since the interpolation of the temperature scale of the galvanometer between observed points is permissible, especially if a large number of temperatures from 600° to 1300° C are used. Interpolation of the high range scale from 1300° to 1400° C may also be allowed, but direct extrapolation from 1400° to 2000° C of the temperature scale engraved on the instrument can not be justified. The extrapolation, however, may be effected in the following manner: The temperature-emf relation is obtained for the pyrometer, preferably by measurement on a potentiometer of the emf developed by the instrument, on open circuit, i. e., without the galvanometer, both with and without the diaphragm. These readings may be taken at the same temperatures at which the pyrometer and galvanometer are tested, the galvanometer being temporarily cut out of the circuit while the potentiometer is used.

As follows from the discussion, on page 130, the graph log e vs. log T may be linearly extrapolated. Thus, from observations made with the diaphragm opening, in the range 500° to 1400° C, it is possible to obtain values of e for the temperatures 1400° to 2000° C by extrapolation of the log plot. The following six steps may then be observed in the extrapolation of the galvanometer scale:

1. Plot of both temperature scales of the galvanometer vs. the galvanometer emf scale.

2. Plot of emf of pyrometer on open circuit vs. emf by galvanometer on closed circuit, using all observations, both with and without the sector diaphragm.

3. Extrapolation of log T vs. log e plot to the higher temperatures.
4. Reading on emf scale of the galvanometer corresponding to the true emf's at the higher temperatures. These values are obtained from (2).

5. Reading of the galvanometer high range temperature scale corresponding to the values on the galvanometer emf scale, obtained from (4).

6. Plot or table of galvanometer temperature scale from 1300° to 2000° C vs. correct temperatures.

Other methods of extrapolation might be employed, but the one described above has proven convenient and satisfactory, especially since, in general, corrections have to be applied to both the emf scale and the temperature scales of the galvanometer.

2. RECORDING RADIATION PYROMETERS

All the types of radiation pyrometer herein described may be made self-registering or automatically recording; and it is possible to obtain on a single sheet the distinctive, permanent records of several such instruments. For many technical processes, particularly those requiring a uniform temperature or regulation within narrow temperature limits, and those involving regular heating or cooling over definite temperatures or time intervals, the use of recording instruments is oftentimes highly desirable, as it is also in many cases in which a control over the operators is desired as well as of the operation. The recording apparatus may also be arranged with a signal or alarm adjusted to call attention when any desired temperature is reached or departed from.

The recording instrument is distinct from the pyrometer proper, and in general consists essentially of a galvanometer of suitable sensibility and range and of a recording mechanism which is preferably operated without interference with the galvanometer sensibility. The various types of recorders used with thermocouples are available for this purpose. In the case of certain radiation instruments the relatively small emf's developed necessitate slight

18 Descriptions of types of recording instruments, some of which are suitable for use with radiation pyrometers, are given in "The Measurement of High Temperatures," 3d ed., 1912, by Burgess and Le Chatelier; Recording Pyrometers (Trans. Faraday Soc., 10, p. 139, 1914), by C. R. Darling; and in the descriptive catalogues of several instrument makers.
modifications in the recorder. In many of the radiation instruments, however, the emf is nearly as great as that developed by the rare metal thermocouples, so that very little, if any, alterations need be made to the recorder. Unless photographic registration is used, the galvanometer and recording mechanism are usually inclosed in the same containing box, which should be dust proof and its interior readily accessible. Some of the more recently developed recording mechanisms are driven by an electric motor, and the stability, range, and sensibility in some cases have been greatly improved by using a recording potentiometer. In this latter system the pen, which gives a continuous record, is mechanically distinct from the galvanometer needle, which is practically always kept at zero deflection. Clockwork, combined with an electromagnetic or mechanical control acting directly on the deflecting needle, is a common method of control giving a discontinuous record.

The recorder may be located at a station distant from the pyrometer and, if the two are suitably designed and calibrated, may be operated in parallel with an indicating instrument, which may be placed near the pyrometer.

3. METHODS OF SIGHTING ON FURNACE

Two methods of sighting a variable focus pyrometer such as the Fény may be employed. The instrument may either be focused on the front opening of the furnace, Fig. 18, A, or upon some plane in the interior of the furnace, B. In general, method B is to be preferred for calibration purposes. The exact realization of black-body conditions for method A would require, in general, that the front diaphragm, a, be as hot as the furnace interior, which condition is practically impossible of being fulfilled even in the case of a furnace with compensating heating coils. Diaphragms c, d, e, and f may be located as shown in the figure, these being properly cut to fit the diverging cone of rays iag. The inner diaphragms will be more nearly the temperature of the radiator and, hence, will assist in maintaining black-body conditions. The radiator ihg is best constructed of some material of high emissivity, such as graphite. In the technical use of a pyrometer, such as the measurement of the temperature of a kiln,
the inside of the front wall, \( a \), is frequently as hot as the interior, and the opening is small in comparison with the rest of the heated inclosure, so that in such a case method \( A \) of focusing is quite satisfactory. When method \( B \) is used for calibration, the diaphragms \( c, d, e, \) and \( f \) are cut to fit the converging cone of rays from the pyrometer. The radiator \( bg\), located in the center of the furnace, is easily maintained at a uniform temperature over its entire surface, and since the opening, \( b \), is rather small in comparison with the area of surface \( bg\), black-body conditions are sufficiently well realized even when the radiator is constructed of white porcelain, as was seen from the discussion in Section IV, 1.

Method \( C \) should be employed for calibrating a fixed focus pyrometer such as the Thwing or Foster. The diaphragms \( c, d, e, \) and \( f \) are cut to fit the diverging cone of rays from the instrument. The radiator should be constructed of a material having a high emissivity.

Various modifications of a black-body furnace have been proposed. For example, the diaphragm \( a \) of method \( A \) might be silvered on the inside, in order that the reflection from its surface

---

**Fig. 18.—Methods of sighting pyrometer on a furnace**
may compensate for the small amount of radiation it contributes to the radiator.

In technical practice, diaphragmed furnaces are not generally available, and the methods described in Section VII, 2, may then be employed.

4. USE OF A RADIATION PYROMETER WITH A SOURCE OF INSUFFICIENT SIZE

It is sometimes necessary to use a radiation pyrometer at such a distance from a small source that the aperture of the instrument is not completely filled. Thus, with the Féry pyrometer, the image of the source formed at the receiver may be smaller than the limiting diaphragm immediately before the couple. The most satisfactory method of using the radiation pyrometer under such conditions is to construct a new limiting diaphragm of the proper size and recalibrate the pyrometer, sighting upon a black body.

Millochau 19 has suggested another method which may be employed if high accuracy is not desired. Assuming the 4th power law to hold one obtains

\[ K \delta_1 = T^4 \]  

where \( K \) is a constant, \( \delta_1 \) the galvanometer deflection, and \( T \) the absolute temperature of the furnace. The following empirical relation is assumed to hold when an absorbing screen of glass, gelatine, or mica is placed between the source and the pyrometer:

\[ \left( \frac{c}{T^4} + a \right) \delta_2 = T^4 \]

where \( c \) and \( a \) are constants, and \( \delta_2 \) the galvanometer deflection. Whence, from (1) and (2)

\[ T^4 = \frac{c \delta_2}{K \delta_1 - a \delta_2} \]

the constants \( c, a, \) and \( k \) being known.

Equation (3), although derived for the case in which the source is large enough to completely fill the pyrometer, is just as applicable when the source is of insufficient size. Thus, in order to

19 Millochau Compt. rend., 159, pp. 171-174; 1914.
obtain the temperature of a very small source, one obtains a reading \( \delta_1 \) of the galvanometer without the absorption glass before the pyrometer and a reading \( \delta_2 \) with the absorption glass, whence, from equation (3), the correct temperature may be computed, since \( K \) is a constant known from the calibration of the pyrometer, and \( a \) and \( c \) are constants depending upon the particular absorbing screen employed.

It is evident that equation (2) is purely empirical and can have little physical significance. It is, therefore, very doubtful if this method may be trusted with accuracy over a large temperature interval. The arbitrary form of equation (2) is apparent from the following discussion:

Let \( J_1 = \) energy emitted by black body at wave length \( \lambda \) to \( \lambda + d\lambda \).

\( P \lambda = \) transmission coefficient of the absorbing screen

\( J'_1 = \) energy transmitted by screen from \( \lambda \) to \( \lambda + d\lambda \)

\( J = \) total energy transmitted by screen of all wave lengths

\( J_1 = C_1 \lambda^{-5} \left( e^{\frac{c_2}{RT}} - 1 \right)^{-1} \), Planck's law.

\( J'_1 = P \lambda J_1 \)

\( J = \int_0^\infty P \lambda C_1 \lambda^{-5} \left( e^{\frac{c_2}{RT}} - 1 \right)^{-1} d\lambda \)

(4) galvanometer deflection \( \delta_2 = \alpha J'_1 = B \int_0^\infty P \lambda C_1 \lambda^{-5} \left( e^{\frac{c_2}{RT}} - 1 \right)^{-1} d\lambda \)

with screen \( (B = \text{const}) \)

According to equation (2):

\( \delta_2 = \frac{T^4}{c + aT^4} \times \text{const.} = \frac{T^8}{c + aT^4} \times \text{const.} \)

Thus, the integral expressed in equation (4) must have the form

\( \frac{T^8}{c + aT^4} \times \text{const.} \). This puts a very arbitrary limitation on the transmission coefficient \( P \lambda = f(\lambda) \). Since different materials have entirely different values of \( P \lambda = f(\lambda) \), one would not expect equation (2) to hold at all accurately. Accordingly this method must be used with extreme caution.

Probably a more satisfactory method to be employed with small sources is to compute the actual size of the image formed at the receiver (see p. 113) and correct the observed deflection,
making use of the assumption that the galvanometer deflection is proportional to the area of the image as long as the image is smaller than the limiting diaphragm. Thus, if the area of the opening to the receiver of the Féry pyrometer were $1 \text{ mm}^2$, and the area of the image of the source formed by the gold mirror were $0.5 \text{ mm}^2$, the correct temperature would be given by the value corresponding to a deflection twice that of the actually observed deflection. Errors due to aberrations of the gold mirror will affect the measurements to some extent. However, the assumption that the galvanometer deflection is proportional to the area of the image as long as the image is smaller than the limiting diaphragm before the receiver is fairly well justified by the section $AB$ of the curve in Fig. 25. Here the relation, diameter of the image vs. emf is parabolic, and hence, if (diameter of image)$^2$ were plotted against emf, the relation would be found linear.

As a check upon this method of using a Féry pyrometer the following rough measurements were made: The area of image required by the pyrometer was $1.77 \text{ mm}^2$. The source remained at approximately a constant temperature $1260^\circ \text{C}$, and its size was altered by means of water-cooled diaphragms.

<table>
<thead>
<tr>
<th>Area of image ($\text{mm}^2$)</th>
<th>Observed emf</th>
<th>Area of receiver ($\text{mm}^2$)</th>
<th>Observed emf/$\times$ ratio of areas</th>
<th>Temperature computed ($^\circ \text{C}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.302</td>
<td>1.04</td>
<td>5.86</td>
<td>6.09</td>
<td>1290</td>
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<tr>
<td>.695</td>
<td>2.33</td>
<td>2.55</td>
<td>5.94</td>
<td>1280</td>
</tr>
<tr>
<td>.807</td>
<td>2.78</td>
<td>2.19</td>
<td>6.09</td>
<td>1290</td>
</tr>
<tr>
<td>1.564</td>
<td>4.32</td>
<td>1.13</td>
<td>4.88</td>
<td>1200</td>
</tr>
<tr>
<td>1.77</td>
<td>5.64</td>
<td>1.00</td>
<td>5.64</td>
<td>1260</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td>1265</td>
</tr>
</tbody>
</table>

The above-computed temperatures have a wide range, but, without doubt, if sufficient care were taken, the accuracy could be increased, possibly to $\pm 20^\circ \text{C}$. This method certainly may be recommended in preference to that of Millochau, but neither method will give results as satisfactory as may be obtained by replacing the limiting diaphragm with one of smaller opening and then recalibrating the instrument by sighting upon a black body.
VI. DISCUSSION OF ERRORS

The sources of error mentioned in Section II, page 107, may now be considered in detail, including the determinations of their magnitudes and methods of elimination or correction.

The question of nonblackness of source and of receiver has already been treated in Section II, page 99. The effects of the $T_0$ term in the Stefan-Boltzmann equation and of the modifications in this equation on the calibration are given in Section IV, page 130. The errors of the electrical measuring devices, galvanometers, and potentiometers, do not need to be emphasized here. We shall limit ourselves to a study of the errors of the pyrometric receiver.

1. LAG

When a radiation pyrometer is exposed to the radiation from a source at a constant temperature, the pyrometer does not immediately indicate the temperature of the source, but exhibits a certain time lag during which the receiving system is heating up. Eventually, the receiver emits or loses by conduction, radiation, and convection as much heat as it receives, and a condition of equilibrium is maintained between the source and the receiver. Theoretically, this condition is reached only after an infinite time, but it is possible to so construct a pyrometer that for all practical purposes this state of equilibrium is attained within a few seconds, although few pyrometers show such good behavior. The following examples illustrate the lag effect in various amounts.

The behavior of Féry pyrometer No. 108, one of the earlier forms of the gold-mirror thermoelectric instruments, is illustrated in Fig. 19, curve A. The pyrometer was sighted into a black body at a temperature of $1335\degree$ C. At least 10 minutes were required for the instrument to reach this maximum temperature indication. Thus, if the reading had been taken after an exposure of 1 minute to the radiating source, the observed temperature would have been $1300\degree$ C, or in error by $35\degree$ C. A very much greater error would be incurred for a 30 seconds' exposure.

Féry pyrometer No. 109, sighted upon a furnace at a temperature of about $1313\degree$ C, showed a maximum reading of $1313\degree$ C in 20 seconds (curve B). The time required for this pyrometer to
reach its maximum indication is satisfactory, but the instrument shows a defect which is quite common in radiation pyrometers, namely, a rapid decrease in the emf immediately following the maximum emf. Thus, for an exposure of 20 minutes the temperature reading has dropped from $1313^\circ$ to $1305^\circ$ C. As seen

![Graph](image-url)

from the curve, the real point of equilibrium is reached after 20 minutes' exposure. It would be possible to calibrate this pyrometer so that the correct temperature would be indicated after 20 minutes' sighting upon the radiating source. With this particular instrument, however, it is better to make the calibration for gen-
eral use for a 20 seconds' exposure, and, if necessary, corrections may be applied whenever in actual use longer exposures are required.

In the case of pyrometer No. 108, the temperature reading even after 40 minutes' exposure is but 2° less than the maximum reading at 10 minutes. It is, therefore, not of serious importance whether the calibration is made for a 10, 20, or 30 minute exposure.

Curve C illustrates the behavior of Féry pyrometer No. 117. The maximum reading is obtained in about 30 seconds and is maintained thereafter. These three curves are typical of the action of all the pyrometers examined.

In general, the Thwing radiation pyrometers are very quick acting because of the simple type of receiver employed. The Féry pyrometers are complicated by the focusing device. These mirrors which are very essential for the convenience of focusing and operation may cause a great amount of trouble, as will be shown clearly on page 154.

(a) CAUSES OF LAG.—The lag of the reading of a radiation pyrometer may be decreased by reducing the heat capacity of the receiver to a minimum. The thermocouple wires should be of as small a diameter as is consistent with satisfactory mechanical strength, and the receiving disk, when used, should be constructed of very thin blackened tin or silver foil. The drop in emf after the maximum reading is attained should be practically eliminated. This drop is usually caused by heat conduction along the thermocouple wires from the hot to the cold junction, and by reradiation or convection from the sides of the receiving box, various diaphragms, focusing mirrors, etc. The effect of conduction may be reduced by locating the cold junction as far as possible from the hot junction, and by employing wires of small size. The couple should not be encumbered with a heavy supporting device. The cold junction in general must be very well screened from the direct radiation of the source. Silvered glass focusing mirrors used in some of the Féry pyrometers do not form a satisfactory screen. The heavy copper mirrors used in the latter type of the Féry instrument serve this purpose much better. The high heat

20 There are special instances where the type of construction may allow exposure of the cold junction. This construction will not be considered in the present paper.
conductivity of copper allows the heat absorbed by the diaphragm to be carried away to the outside of the receiving box where it is dissipated by radiation and convection. In the case of glass diaphragms, however, the heat absorbed is not carried to the outside so rapidly. The temperature of the diaphragm rises and the convection currents and radiation losses are set up inside the receiving box, which may affect both the hot and cold junctions of the thermocouple in a variety of ways. For the ideal pyrometer all diaphragms should be water-cooled, but the use of fairly heavy diaphragms of a good heat-conducting material such as copper, making tight contact with the outside case of the instrument or of the receiving box, will greatly reduce the effect of reradiation. All diaphragms should, of course, be cut with a bevel edge around the opening.

Besides the effect of lag due to the high heat capacity of the receiving system, the mechanical pyrometer such as the Féry spiral is subject to all the errors of hysteresis or elastic lag of the expansion device—i.e., the spiral spring. Some of these errors may be partially eliminated by setting the pointer at zero immediately before each temperature reading. If the zero reading is not frequently adjusted, very large errors may occur. Thus, Fig. 20 illustrates the shift of the position of the pointer for a period of 20 hours, when the front opening of the pyrometer was closed by the diaphragm at constant room temperature. While a 20-hour test is rather severe for this instrument, it will be noted that the shift even during a few moments is pronounced. This effect, combined with the elastic lag of the spring in attaining its maximum expansion when exposed to the radiating source and the thermal lag of the receiver, makes the instrument somewhat unsatisfactory for scientific work, although its compact and convenient form offsets in a measure these objections, especially for technical use.
No generalizations can be drawn as to the magnitude of the lag for the various types of pyrometers. Apparently no two pyrometers of the same type are constructed exactly alike. If a radiation pyrometer is to be used for highly accurate work, the effect of lag should be thoroughly examined, so that the calibration and all readings may be made with an exposure for which the change of lag with time is a minimum. Thus, pyrometer No. 108 may be best calibrated and used for exposures of 10 to 30 minutes, pyrometer No. 109 for exposures of either 20 seconds (maximum reading) or 20 to 30 minutes, and pyrometer No. 117 for exposures of 30 seconds to 30 minutes. Under certain conditions of use the pyrometer might be calibrated for a definite exposure such as one minute, but, as seen from Fig. 19, any wide variation in use from the one-minute time of exposure would give rise to a large error, especially in the case of No. 108.

It is usually more satisfactory to calibrate and use a pyrometer with an exposure sufficient to give a maximum reading. This would preclude No. 108 for the measurement of rapidly varying temperatures.

2. RESISTANCE OF THERMOCOUPLES USED IN RADIATION PYROMETERS AND VARIATION IN RESISTANCE WITH TEMPERATURE OF THE SOURCE

If the radiation pyrometer is to be used with a galvanometer, it is desirable that both the resistance of the thermocouple and its variation in resistance with the temperature of the source be small. These conditions are well satisfied by all the radiation pyrometers examined. Fig. 21 illustrates the method employed for determining the resistance for various temperatures of the source. The emf, \( e_o \), developed by the thermocouple on open circuit when the pyrometer was sighted upon the furnace at any definite temperature, and the potential drop, \( e \), across the resist-
The resistance of the couple, \( r_1 \), is given by the expression:

\[
r_1 = r_2 \left( \frac{e_0}{e} - 1 \right)
\]

The resistance of the couples of Fény mirror pyrometer No. 111, Thwing cone pyrometer No. 120, and a Brown pyrometer were found to be about 0.8, 0.6, and 0.2 ohms, respectively, with variations from these values of 2 or 3 per cent for a range in temperature of 500° to 1200° C. Since only a very short length of the couple is heated by the source, large variations in resistance would not be expected. The small changes in resistance of the couples observed will produce no effect upon the reading of a galvanometer indicator, even if the resistance of the galvanometer be as low as 10 ohms.

3. EFFECT OF DIRT AND OXIDATION UPON THE CONDENSING DEVICE

Whipple has observed that a slight film of oxide on the surface of the concave gold mirror of the Fény or Foster radiation pyrometers does not seriously alter the amount of radiation reflected (96 per cent),21 as the greater part of the energy exists in the form of long wave lengths, which the tarnished mirrors reflect without difficulty. This statement has frequently been interpreted to mean that in spite of dirt accumulation, stains, and scratches the gold surface remains unchanged in its reflecting power, but obviously such constancy is not the case. Pyrometers subjected to severe use in steel mills and other industries soon become coated with dust and dirt.

As an actual example of the errors to be expected from this source, the following table is given for a Fény mirror thermoelectric pyrometer No. 108. The instrument which was submitted to the Bureau for test was calibrated (1) in the condition received and (2) after the accumulation of dirt had been removed from the mirror. This case is not at all exceptional; many other instruments have been received which were just as dirty.

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21 Whipple, Engineering, 90, p. 142; 1910. (See also the present paper, p. 103.)
The differences in calibration with clean and with dirty mirrors, columns 4 and 7, vary from $100^\circ$ to $160^\circ$ C in the temperature range $600^\circ$ to $2000^\circ$ C. Since the ordinary calibration of the pyrometer is made for clean mirrors, the instrument as used with dirty mirrors may read too low by over $100^\circ$ C. It is interesting to note that the exponent $b$ in the emf relation $e = aT^b$ was approximately the same in the two calibrations, the main change being a shift of the straight line on the log $e$ vs. log $T$ plot, parallel to itself. The approximate constancy of $b$ would be expected, since the dirt merely alters the reflection coefficient of the mirror and diminishes the energy falling upon the couple, and hence the emf, by approximately a constant factor. Accordingly, only the constant $a$ can be seriously affected.

The importance of keeping the mirror free from dirt is therefore evident. When necessary, the mirror may be taken from the telescope and carefully washed with water, care being taken not to rub the surface of the earlier-type gilded mirrors on glass, as the gold is easily removed. Carefully polishing the mirror with soft rouge will frequently prove beneficial.

The statement of Whipple above, viz, that oxide films do not alter the reflection coefficient of the gold, is open to question. It would be interesting to measure the reflection coefficient of a freshly polished gold mirror for several wave lengths in the visible

<table>
<thead>
<tr>
<th>Indicator reading</th>
<th>Without diaphragm (temperature °C)</th>
<th>With diaphragm (temperature °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dirty mirror</td>
<td>Clean mirror</td>
</tr>
<tr>
<td>600</td>
<td>706</td>
<td>600</td>
</tr>
<tr>
<td>700</td>
<td>813</td>
<td>702</td>
</tr>
<tr>
<td>800</td>
<td>920</td>
<td>804</td>
</tr>
<tr>
<td>900</td>
<td>1026</td>
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<table>
<thead>
<tr>
<th>Dirty mirror</th>
<th>Clean mirror</th>
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<td>153</td>
</tr>
<tr>
<td>2051</td>
<td>1893</td>
<td>158</td>
</tr>
</tbody>
</table>
spectrum and infra-red, and to determine the coefficients of the same mirror after it had been exposed to the atmosphere for different lengths of time, one to five years.

The effect of dirt upon the surface of the aluminum cone of the Thwing pyrometer has not been tested, but without doubt the calibration would be altered in a manner quite similar to that observed in the case of the Féry pyrometer.

4. EFFECT OF DISTANCE AND SIZE OF SOURCE

As is seen on page 95, the simple geometrical optics of a radiation pyrometer indicates that the readings should be independent of the distance from the pyrometer to the radiating source, provided the uniform source were of sufficient size required by the constant aperture of the instrument. Enlarging the size of the source above this limiting value should produce no effect. Actually this ideal condition by no means obtains, and herein lies the most serious error to which radiation pyrometers are subject.

The causes for the dependence of the readings of a radiation pyrometer upon the distance from the instrument to the radiating source are two in number, (1) atmospheric absorption and (2) construction of the pyrometer. The errors due to atmospheric absorption are small in comparison with those occasioned by an unsatisfactory construction of the pyrometer. Quantitative measurements have not been made in the present work upon the absorption of the atmosphere. Dr. F. E. Fowle, of the Smithsonian Astrophysical Observatory, however, is at present making very extended measurements upon the absorption coefficient of air containing water vapor, from the ultra-violet to about 20 μ in the infra-red, and when these results are completed it will be possible to determine just what variation in the reading of the pyrometer will occur when the instrument is used close to the source and at several meters distance. It is possible that atmospheric absorption and absorption due to water vapor and gases such as CO and CO₂ which occur in the vicinity of most furnaces may account for discrepancies of 1 or more per cent in the emf or linear scale readings of a radiation pyrometer close to a source and at 3 meters distance. The effect of atmospheric absorption is to
diminish the reading of the pyrometer as the distance to the source is increased.

Errors arising from the size of the source, when this size is greater than that required by the aperture of the pyrometer, may be ascribed to the construction of the instrument. The various types of instrument examined will be discussed separately.

(a) Féry Mirror Thermoelectric Pyrometer—Mechanical Construction.—Many of the pyrometers of this type are not of constant aperture. Fig. 22 illustrates a typical example of this mechanical defect, observed for pyrometer No. 109. Referring to Section II, page 98, the energy J falling upon the receiver at A is proportional to $I \sin^2 \theta$. In order that $J$ may be independent of the focusing distance, $\sin^2 \theta$ must be constant. As constructed, the entire mirror is effective for all possible focal distances. The diameter of the mirror is 6.8 cm and its focal length 7.6 cm. Hence, for a focusing distance of 80 cm (mirror to source), $\sin^2 \theta = \sin^2 \theta_1 = 0.396$, and for 300 cm, $\sin^2 \theta = \sin^2 \theta_2 = 0.432$. Thus, the energy received by the thermocouple at A is 9 per cent greater for a focusing distance of 300 cm than for a focusing distance of 80 cm. The difference in emf of the thermocouple for these two distances would be approximately 9 per cent, and if the pyrometer were calibrated at 80 cm distance and used at 300 cm, the temperatures thus measured would be too high by approximately $\frac{1}{4} \times 9$ per cent = 2 per cent of the absolute temperature, i.e., 35° at 1500° C.

This particular pyrometer may be made of fixed aperture for any focusing distance greater than 80 cm, by locating at B a diaphragm having a 9 mm opening. The angle of aperture is then $\theta_1$. The size of the proper diaphragm opening is determined
so that the cone of rays diverging from $A$ and passing through the limiting diaphragm $B$ will always completely intersect the surface of the gold mirror at the shortest focusing distance $u$ employed (i. e., the longest distance $v$).

In the discussion of the independence of focusing distance of a constant aperture pyrometer (p. 98) it was assumed that the receiver is of very minute size. In actual practice the receiver, or entrance diaphragm immediately in front of the receiving surface, is about 1.5 mm diameter. An area of this size can not be considered theoretically as a point receiver. The discussion, therefore, rigorously applies not for the total energy falling upon the receiver, but for the intensity of the energy at a point in the center of the receiver. It would be possible similarly to compute the intensity at a number of points within the 1.5-mm image, but for practical purposes such a procedure is unnecessary, the simple discussion as given being sufficient.

Another factor altering the rigorousness of the simple geometrical relation is the aberration of the gold mirror. Besides showing the usual amount of spherical aberration for a mirror of this size and focal length, the mirror of every pyrometer examined shows all sorts of local aberrations, so that the image is fuzzy and distorted. This is especially true of the gilded mirrors on glass, but even the gold-plated copper mirrors in the later type instruments show this imperfection. It is impossible to say what effect these distortions have upon the reading of the pyrometer for various focusing distances. The Taylor Instrument Companies are experimenting with gold-plated speculum metal mirrors, and it is hoped that better mirrors may be obtained as a result of their investigations.

Still another factor modifying the simple geometrical relations, as discussed on page 98, is the shading effect of the small box containing the thermocouple receiver and of the supports ($K$, Fig. 6). It is easy to compute the shading effect of this system for a point of the source on the axis of the pyrometer and to determine the variation of the magnitude of this shading for various focusing distances, but to determine the integral effect of the shading upon the amount of energy falling on the receiver would require a consideration of the shading for every point of the source (image of
the receiver) for several focusing distances, the source of course being of a different size at each distance. A rough drawing will show qualitatively that the mean intensity of the total image decreases as the focusing distances decreases, in consequence of the shading of the thermocouple box and mounting. Hence, on account of this shading effect, the pyrometer reading will increase with increasing focusing distance.

**Convection Currents.**—The question of convection currents of hot air passing directly from the source to the receiver is not of serious importance. The effect, which is very small, usually tends to produce readings which increase with decreasing focusing distance. It has been suggested that convection currents of this nature might be reduced by placing a mica window on the front of the pyrometer. This would, however, reduce the readings of the instrument, due to reflection of a part of the radiation at the surfaces of the window, which, of course, is undesirable. Also, selective reflection of the mica window might seriously modify the relation discussed on page 107. Furthermore, the absorption of energy may heat the mica and give rise to new convection currents within the pyrometer.

As the pyrometer is brought nearer the source the temperature of the case and entire telescope increases somewhat, on account of convection from the source and absorption of the radiation by the metal case. This increase of temperature may either increase or decrease the pyrometer reading, depending upon whether it affects the hot or cold junction of the thermocouple. With ordinary care the Féry pyrometer will not become hot enough to have any serious error from this cause.

**Stray Reflection.**—Errors resulting from stray reflection may cause considerable trouble. The Féry pyrometer is blackened inside, but most of the stray reflection occurs at nearly grazing incidence, and under such a condition a surface blackened even with a thick deposit of soot from the acetylene flame becomes a fairly good reflector. The thermocouple box of all the instruments examined produced a rather intense caustic, caused by reflection from the side walls. Stray reflection can be reduced by cutting a sharp screw thread, preferably of small pitch, upon all
the surfaces likely to give any trouble. The thermocouple box should certainly have such a screw thread cut on the inside, and, undoubtedly, it would be worth while to cut the threads on the inside of the pyrometer case. Stray reflection for a source of given size generally causes the pyrometer readings to increase with decreasing focusing distance.

**Effect of Size of Image.**—It is obvious in the use of the Féry pyrometer that the source should be of a size sufficient for its image to cover the thermocouple receiver or the limiting diaphragm immediately in front of the receiver. (See p. 140 for a discussion of the use of the Féry with sources which are too small to meet this condition.) Usually this limiting diaphragm is the hole in the focusing mirrors, although in some of the instruments the limiting diaphragm is placed between the focusing mirrors and the thermocouple. As long as this opening is covered by the image of the source, it has usually been assumed that the reading of the pyrometer is independent of distance or size of source.

In general, the reading of the Féry pyrometer decreases with increasing focusing distance and with decreasing size of source, even though the image of the source always covers the receiver, although some instruments show this effect much more markedly than others. It is even possible to obtain a positive reading, as shown by Dr. Kanolt, when the pyrometer is sighted on a hole in a heated surface, even though the image of the hole covers the opening to the thermocouple receiver. The cause for this effect is best determined from a consideration of the size of the image of the source.

Errors which are surprising in magnitude, and which completely outclass those resulting from the preceding causes, may arise in the variation of the size of the image produced by the gold mirror of the Féry pyrometer. The size of the image may be altered by (1) varying the focusing distance, the size of the source remaining constant, and (2) by varying the size of the source, the focusing distance remaining constant.

In order to satisfactorily study the errors resulting from variations in the size of the image, it was necessary to devise a method whereby both the size of the source and the focusing distance could be conveniently and rapidly changed.
A large nickel strip (see p. 128) was employed as the radiator, in front of which, at a distance of 1 or 2 cm, were placed diaphragms of various openings. A number of polished brass diaphragms were first tried, but on account of the reflection of heat from the brass back to the strip the temperature of the strip was found to change seriously with openings of different size, as would be expected. Blackening the diaphragms reduces this effect of heat reflection, but the diaphragms then become so hot that they act as secondary sources of radiation. It was therefore necessary to employ water-cooled and thoroughly blackened diaphragms. For convenience of operating a series of diaphragms was constructed, as shown in Fig. 23. By a sliding arrangement and spring catch the various openings may be quickly shifted and centered in front of the strip. The water cooling maintains the temperature of the diaphragms at about that of the room, so that there is no effect of radiation from them, and the thoroughly blackened surface of the diaphragms absorbs practically all the heat falling upon it, so that the radiation loss from the surface of the strip behind the diaphragm is the same as that from the exposed surface of the strip. Consequently there is no variation in either the apparent or true temperature of the strip when the size of the diaphragm opening is changed. Except for very close distances from the pyrometer to the strip, the diaphragm openings act as the real source of the radiation. In this manner five sizes of the radiating source were obtained, circular areas having diameters of 1.95, 3.5, 5.6, 8.5, and 12 cm.
The pyrometer was mounted upon a carriage which rolled on parallel tracks similar to an ordinary photometer bench (Fig. 24). The strip and water-cooled diaphragms were properly adjusted at one end of the bench, and the apparatus aligned so that the image of the diaphragm opening employed was always centered upon the pyrometer receiver for all focusing distances (usually 80 to 300 cm).
The relation between the diameter of the source $D_s$, the diameter of the image $D_i$, and the focusing distance $u$ (measured from the source to the concave mirror) is given by the equation

$$D_s = D_i \left( \frac{u}{f} - 1 \right)$$

where $f$ is the focal length of the mirror, usually about 7.6 cm.

Thus, for a given diameter of the source at any distance $u$, the diameter of the image may be computed. Fig. 25 represents the variation in emf with the diameter of the image of a uniform temperature source for Fény mirror pyrometer 105a. The limiting diaphragm immediately in front of the thermocouple receiver is the focusing mirror system, the opening in these mirrors being 0.15 cm. A 0.6-cm diaphragm is located immediately in front of the focusing mirrors, and the inside diameter of the thermocouple box is 1.1 cm. The focusing mirrors are of thin glass silvered on the back surface. In the present case the diameter of the image was varied from 0.05 to 0.9 cm by employing sources of diameters 2 to 11 cm at focal distances of 70 to 250 cm. In the section $AB$ of the curve the image was not large enough to cover the 0.15 opening in the focusing mirrors. For this range the emf is approximately proportional to the area of the image or to the $(\text{diameter})^2$, so that $AB$ is a parabola. If the focusing mirrors formed a perfect diaphragm completely shutting out all radiation except that passing through the opening, the point $B$ would represent a maximum reading and the curve would continue horizontally along the line $BC$. Actually the emf increases up to the point $E$, where the image has a diameter of about 1.1 cm, the inside diameter of the thermocouple box, although for all points from $B$ to $E$ the size of the image was sufficient to cover the receiver. The large increase along $BE$ is due to the heating of the silvered glass focusing mirrors, and the amount of this heating increases with the size of the image, until the image completely fills the inside of the thermocouple box. This heat is communicated to the thermocouple by radiation and by convection currents set up within the receiver. The errors in measurement resulting from variations in size of image are readily apparent. For example, suppose the instrument were calibrated by sighting.
at 150 cm distance upon a black body having an opening of 3 cm and were used for the measurement of the temperature of a source 11 cm in diameter at a distance of 100 cm. In the first case the image diameter is about 0.15 cm and in the latter 0.9 cm, corresponding on the curve to the points $B$ and $E$, respectively. The emf at $E$ is 227 per cent of that of $B$, so that as used the instrument would indicate emf's in error by 127 per cent. Suppose this instrument obeys the law $e = aT^4$, then by differentiation

$$\frac{\delta e}{e} = 4 \frac{\delta T}{T}$$

which states that the fractional error in the absolute temperature is one-fourth the fractional error in emf. Hence, an error of 127 per cent in emf is equivalent to an error of 32 per cent in the absolute temperature. Thus, for a source of 11 cm diameter, 100 cm distance, and at a temperature of 1500° C, this pyrometer would read about 2070° C, or a temperature too high by 570° C.

The example cited is an extreme case, and the variation of emf with diameter of the image was greater for this instrument than for any other examined. Actually, a pyrometer would not be calibrated with the minimum-sized image required by the optics of the instrument. It is rather difficult to center such an image, so that usually the image is made large enough to overlap the opening of the receiver. Probably an image diameter of approximately 0.4 cm is more often employed in calibration, and in general use, a variation in image diameter of from 0.2 to 1.1 cm. Thus calibrated, this pyrometer would indicate emf's too small by 32 per cent when the smaller image is used and too great by 17 per cent with the larger image.

Fig. 26 illustrates the variation of emf with image diameter for the Féry pyrometers No. 111 and No. 117. The size of image was varied by both altering the size of the source and of the focusing distance, as explained by the key inserted on the plot. The fact that the points obtained by either method coincide equally well with the curve precludes the possibility of any experimental errors, such as temperature gradient across the nickel strip used as a source or the change in temperature of the strip due to the use of various sized diaphragms. It also indicates that the error due to the atmospheric absorption is small, at least in comparison
with errors involved in the heating of the focusing mirrors. These curves represent the behavior of the two best acting pyrometers examined, but it is apparent that their behavior is far from being satisfactory. Thus, in the case of No. 111, the emf for a 1.1-cm image is 133 per cent of that obtained with an image of 0.15 cm, the diameter of the limiting diaphragm in front of the receiver, and for No. 117 this relation is 162 per cent. If these instruments were calibrated with an image diameter of 0.4 cm, the emf developed by No. 111 would be too great by 2 per cent for a 1.1-cm image and too small by 11 per cent for a 0.2-cm image, and by No. 117, too great by 2 per cent for the larger image and too small by 18 per cent for the smaller image, with errors of one-fourth these percentages when referred to absolute temperatures.

Pyrometer No. 111 has a metal diaphragm located between the silvered glass focusing mirrors and the receiver, while in No. 117 this diaphragm is absent, but the focusing mirrors are of gold-plated sheet copper. These two instruments show a smaller effect of variation in size of image than does pyrometer No. 105, for the
reason that the metal diaphragm in No. 111 and the copper mirrors in No. 117 are good heat conductors, in contrast with the glass-mirror diaphragm of No. 105, and allow part of the heat to be carried to the walls of the thermocouple box, where it is dissipated by radiation and convection on the outside, away from the thermocouple.

*Summary of Focusing Errors for a Féry Pyrometer.*—The principal error results from a variation in the size of the image of the source formed by the large condensing mirror, and is due to the heating of the limiting diaphragm or the focusing mirrors immediately in front of the thermocouple receiver. The amount of this heating increases with increasing size of image. On account of this fact, the pyrometer readings for a source of constant size decrease with increasing focusing distance; and for a constant focusing distance the readings increase with increasing size of source. With ordinary use of the pyrometer, errors of this type may amount to several hundred degrees in extreme cases, and, in general, to 50° or more, unless certain specified methods of procedure are employed, for example, the use of an image of a definite size for every measurement. Other errors to which the pyrometer is subject are of minor importance.

The following table presents a summary of the various errors the magnitudes of which depend upon the focusing distance. For the best construction of the Féry mirror pyrometer, it might be advisable to purposely make the instrument one of variable aperture in order that the error, arising thereby, may be used to annul the errors caused by convection currents and atmospheric absorption, etc.

**Effect Upon the Pyrometer Reading of Increasing the Focusing Distance**

<table>
<thead>
<tr>
<th>Reading increases on account of—</th>
<th>Reading decreases on account of—</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Variable aperture</td>
<td>1. Atmospheric absorption</td>
</tr>
<tr>
<td>2. Shading of concave mirror by thermocouple box</td>
<td>2. Convection currents from source to receiver</td>
</tr>
<tr>
<td></td>
<td>3. Stray reflection in receiver and telescope tube</td>
</tr>
<tr>
<td></td>
<td>4. Reradiation to couple from side walls of pyrometer</td>
</tr>
<tr>
<td></td>
<td>5. Image of source becoming smaller</td>
</tr>
</tbody>
</table>
(b) Féry Fluorite Lens Pyrometer.—Fig. 27 illustrates the effect of size of image upon the readings of the Féry lens type pyrometer. The readings increase with increasing diameter of the image just as is the case with the mirror instrument. The diameter of the receiver is less than 0.1 cm, but on account of the radiation and convection from the shield and limiting diaphragm to the receiver the readings continue to increase even with an image diameter of 0.9 cm.

It is interesting to note that Féry and Drecq employed an instrument of this type for the measurement of the temperature of the radiator used in the determination of the constant $\sigma$ of the Stefan-Boltzmann law. The value of $\sigma$ determined by Féry and Drecq is $6.51 \times 10^{-12}$ watt cm$^{-2}$ deg$^{-4}$, while the unweighted mean of all determinations by various investigators is about 5.75. The latest determination of this constant is 5.61. If the pyrometer used by Féry had been calibrated with a large image, as was quite likely the case, and used with a small image, the temperature measured would be too small, and hence the value of $\sigma$ too great.

(c) Foster Fixed-Focus Pyrometer.—This pyrometer is so constructed that, as far as the geometry of the instrument is concerned, the front diaphragm opening acts as the radiating source provided the actual source has a diameter at least one-tenth the distance from the actual source to the wing nut of the pyrometer. Thus if the source were enlarged to a diameter more than one-tenth this distance the readings of the instrument should be unaffected. Similarly, for any definite size of source, the readings should remain constant, as the distance from the wing nut to the source is increased up to the point where this distance becomes

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22 Féry and Drecq, J. de Phys., 1 (5) pp. 551-91; 1911.
ten times the diameter of the source. For still greater distances the front diaphragm is not sufficiently covered by the source, so that a rapid decrease in the readings will occur. Actually the readings of this pyrometer decrease when the distance from the source increases, even though the aperture of the instrument is always completely filled, on account of (1) atmospheric absorption, (2) radiation from the side walls and diaphragms of the front part of the pyrometer case to the thermocouple receiver, and (3) stray reflection from the side walls. The largest error is probably due to stray reflection and could be decreased by cutting a screw thread inside the entire length of the receiving tube.

However radiation from the heated side walls and diaphragms of the front part of the pyrometer may amount to a considerable fraction of the entire radiation received by the couple, even though the temperature of the case does not rise above 100° C. This radiation effect may be reduced by water-cooling the pyrometer case and by the use of heavy metal plate diaphragms (preferably copper), making good contact with the walls in order that the heat they receive may be readily conducted to the outside.
Both the effects of reflection and radiation from the side walls depend upon the size of the source, as well as the sighting distance. Fig. 28 illustrates the decrease in reading of a Foster pyrometer with increasing sighting distance (measured from the source to the front of the pyrometer) for three different sources having diameters 5.6, 8.5, and 12 cm. The arrows mark the point at which the given-sized source is insufficient to cover the front diaphragm of the pyrometer. It is only with the readings from zero distance up to this point that one is here concerned. The readings were taken rapidly, just as in actual practice, and hence the curves do not show the variation with distance as prominently as they would if several minutes were allowed for each determination. Also, at zero distance the water-cooled diaphragm in front of the nickel strip does not act as the real source, although this fact was not considered in drawing the curves. Thus, using the pyrometer under these favorable conditions, a 12 per cent variation in emf was observed over a range in distance at every point of which the aperture of the instrument was filled.

It is evident that the most satisfactory sighting distance from a source of definite size, both for calibration and use of this pyrometer, is that distance at which the slope of the curve, distance vs. emf, is a minimum, i. e., where the variation of emf with distance is small. The ratio of the best distance to the diameter of the source is roughly constant, as seen from the following table obtained from the curves of Fig. 28:

<table>
<thead>
<tr>
<th>Diameter source</th>
<th>Best distance—</th>
<th>Best distance, source diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>To front</td>
<td>To wing nut</td>
</tr>
<tr>
<td></td>
<td>diaphragm</td>
<td></td>
</tr>
<tr>
<td>cm</td>
<td>cm</td>
<td>cm</td>
</tr>
<tr>
<td>5.6</td>
<td>19-27</td>
<td>44-52</td>
</tr>
<tr>
<td>8.5</td>
<td>28-44</td>
<td>53-69</td>
</tr>
<tr>
<td>12.0</td>
<td>60-70</td>
<td>85-95</td>
</tr>
<tr>
<td></td>
<td>7.9-9.3</td>
<td>6.2-8.1</td>
</tr>
<tr>
<td></td>
<td>7.1-7.9</td>
<td></td>
</tr>
</tbody>
</table>

Thus, for the best results, this individual pyrometer should be calibrated and used at a distance from the source (measured to the wing nut) approximately seven to eight times the diameter of the source. Distances greater than ten times the diameter of the source are barred because of the aperture of the instrument, and
distances smaller than seven to eight times the diameter of the source should not be used on account of stray reflection and radiation from the walls of the pyrometer tube. Even after imposing this limitation upon the use of this pyrometer, the instrument for the highest accuracy should not be used with sources or sighting distances materially different from those used during the calibration, on account of atmospheric absorption and the fact that the amount of stray reflection from a source of constant angular dimensions (measuring from the wing nut) increases as the source is brought nearer the front opening of the pyrometer.

In general, therefore, it may be concluded that the readings of the Foster fixed-focus pyrometer decrease with increasing distance from the source, but that the errors to which this instrument is subject are very much smaller than those observed with the Féry pyrometer, as constructed at present.

(d) Thwing Pyrometers.—The Thwing pyrometers are, in general, subject to the same errors affecting the Foster pyrometer. The readings decrease with increasing sighting distance, as illustrated in Fig. 29. The readings here plotted were obtained with a source 12 cm in diameter, which is of sufficient size to fill the

![Fig. 29.—Emf as a function of the sighting distance. Thwing pyrometers](image-url)
aperture of the instruments at every distance represented. Curve A refers to a tube form pyrometer No. 119 in exactly the condition received. The instrument was then remodeled by completely water-cooling the receiving tube and the front diaphragm. In order to illustrate the importance of thoroughly blackening the inside of the pyrometer tube, the black paint was removed from the front part of the case, and in this condition curve B was obtained, representing the variation of emf reading with sighting distance. When the inside of the water-cooled tube was thoroughly blackened with smoke from an acetylene flame, curve C was obtained. The water-cooling prevented the pyrometer from becoming warm even when 1 or 2 cm from the radiating strip, so that curve C represents exposures for any length of time whatever, and hence can not be compared with curve A or with the curves of Fig. 28 obtained with the Foster pyrometer for short exposures. Since water-cooling maintains a uniform temperature of the pyrometer tube for all sighting distances, the variation in emf illustrated by curve C can not be due to stray radiation from the side walls. The decrease in emf must, therefore, be caused by atmospheric absorption, and more especially by stray reflection. The entire interior of the tube of the Thwing pyrometers is cut with a shallow screw thread; and even though this was thoroughly blackened with soot from the acetylene flame it is still probable, from a consideration of curve D, that a greater part of the variation with distance is due to stray reflection.

Curve D represents the variation of emf with distance for a box-type (Fig. 12) pyrometer. In this instrument there are no side walls to produce stray reflection, and hence the error from this source is practically nil. The decrease of the first 20 cm is due mainly to heating and reradiation from the front diaphragm, this blackened diaphragm becoming almost red-hot when the source is close. The variation in emf from 25 to 100 cm is less than 2 per cent. Apparently, then, if the front diaphragm of this pyrometer were thoroughly water-cooled, the readings would be practically independent of distance, except in so far as any effect of atmospheric absorption obtains.

e Brown Pyrometer.—Fig. 30 illustrates the variation of emf with sighting distance for a Brown pyrometer. The points
close to the source were taken as quickly as possible on account of the rapid heating of the thin wall of the pyrometer tube, the heating in a few seconds being sufficient to cause fuming of the inside-painted surface. Consequently the emf's observed at these positions are all too small. Even under this condition the variation in emf observed, from zero distance up to the maximum distance allowable by the aperture of the instrument, is about 65 per cent of the zero reading. Thus, if the pyrometer were calibrated on a 12-cm source at 115 cm distance and used at distances
close to the source, the instrument would indicate emf's too great by over 50 per cent, i.e., absolute temperatures too great by over 12 per cent.

On account of the construction of this pyrometer with no real front diaphragm the errors due to stray reflection are of considerable magnitude.

5. THE TEMPERATURE OF THE RECEIVER

In the discussion on page 131 it was assumed that the temperature of the hot junction of the thermocouple, $T_o$, did not vary much from that of the room. This assumption is approximately true in the case of some pyrometers; for example, the Thwing, but with certain Féry instruments, $T_o$, may rise very considerably.

The value of $T_o$ for any value of $T$ may be computed under certain assumptions. Thus, in the case of the Féry mirror pyrometer, let the concave mirror be considered a perfect reflector and suppose no heat is lost from the black receiver by either conduction or convection. Let the source sighted upon be a black body of temperature $T$. As far as the receiver is concerned the mirror itself acts as the source, subtending a solid angle at the receiver of about $0.2 \pi$. If the source subtended a solid angle of $4 \pi$, i.e., a spherical shell inclosing the receiver, $T_o$ would equal $T$. It follows then from the Stefan-Boltzmann law that

$$\frac{T_o^4}{T^4} = \frac{0.2\pi}{4\pi}$$

Hence,

$$T_o = 0.47 \ T$$

approximately.

Thus, for the given case cited, the absolute temperature of the receiver would be about half that of the radiator. Actually, $T_o$ never attains such high values on account of convection and conduction losses.

The temperature of the cold junction of the thermocouple receiver is, in general, closely that of the room, and should change very little for different values of $T$. Hence, if the temperature-emf relation of the thermocouple is known, the temperature of the hot junction or receiver, $T_o$, for any furnace temperature, $T$, may be determined from the emf developed.

As an example of the actual variation of $T_o$ with the temperature of the furnace, the following table is given for Féry pyrometer
No. 109. This instrument has an iron-constantan couple which develops an emf of about 0.054 millivolts per degree difference in temperature between its hot and cold junction. The temperature of the cold junction is considered that of the pyrometer case, 30° C.

<table>
<thead>
<tr>
<th>Temperature of furnace</th>
<th>Emf</th>
<th>Temperature of receiver</th>
<th>T°</th>
<th>T° abs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>Millivolts</td>
<td>°C</td>
<td>° abs.</td>
<td>° abs.</td>
</tr>
<tr>
<td>700</td>
<td>1.004</td>
<td>49</td>
<td>322</td>
<td>973</td>
</tr>
<tr>
<td>800</td>
<td>1.462</td>
<td>57</td>
<td>330</td>
<td>1073</td>
</tr>
<tr>
<td>900</td>
<td>2.058</td>
<td>68</td>
<td>341</td>
<td>1173</td>
</tr>
<tr>
<td>1000</td>
<td>2.811</td>
<td>82</td>
<td>355</td>
<td>1273</td>
</tr>
<tr>
<td>1100</td>
<td>3.759</td>
<td>100</td>
<td>373</td>
<td>1373</td>
</tr>
<tr>
<td>1200</td>
<td>4.925</td>
<td>121</td>
<td>394</td>
<td>1473</td>
</tr>
<tr>
<td>1300</td>
<td>6.310</td>
<td>147</td>
<td>420</td>
<td>1573</td>
</tr>
<tr>
<td>1400</td>
<td>8.026</td>
<td>178</td>
<td>451</td>
<td>1673</td>
</tr>
<tr>
<td>1500</td>
<td>10.03</td>
<td>216</td>
<td>489</td>
<td>1773</td>
</tr>
</tbody>
</table>

Thus, when the pyrometer is sighted upon a source at a temperature 1500° C the receiver is at 216° C, the cold junction of the thermocouple being 30° C.

The temperature of the receiver of a Thwing pyrometer does not rise to this high value, as is seen from the following table for pyrometer No. 120. This pyrometer has a thermocouple of special composition, the temperature-emf relation of which was determined at the Bureau, and found to be very nearly linear, approximately 0.060 millivolts per degree difference in temperature of the hot and cold junctions. The cold junction temperature is taken as 30° C.

<table>
<thead>
<tr>
<th>Temperature of furnace</th>
<th>Emf</th>
<th>Temperature of receiver</th>
<th>T°</th>
<th>T° abs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>Millivolts</td>
<td>°C</td>
<td>° abs.</td>
<td>° abs.</td>
</tr>
<tr>
<td>700</td>
<td>0.17</td>
<td>33</td>
<td>306</td>
<td>973</td>
</tr>
<tr>
<td>800</td>
<td>0.24</td>
<td>34</td>
<td>307</td>
<td>1073</td>
</tr>
<tr>
<td>900</td>
<td>0.34</td>
<td>36</td>
<td>309</td>
<td>1173</td>
</tr>
<tr>
<td>1000</td>
<td>0.45</td>
<td>37</td>
<td>310</td>
<td>1273</td>
</tr>
<tr>
<td>1100</td>
<td>0.60</td>
<td>40</td>
<td>313</td>
<td>1373</td>
</tr>
<tr>
<td>1200</td>
<td>0.78</td>
<td>43</td>
<td>316</td>
<td>1473</td>
</tr>
<tr>
<td>1300</td>
<td>0.99</td>
<td>46</td>
<td>319</td>
<td>1573</td>
</tr>
<tr>
<td>1400</td>
<td>1.25</td>
<td>50</td>
<td>323</td>
<td>1673</td>
</tr>
<tr>
<td>1500</td>
<td>1.54</td>
<td>54</td>
<td>327</td>
<td>1773</td>
</tr>
</tbody>
</table>
The preceding two examples can not be taken to represent the receiver temperatures of all the Thwing and Féry pyrometers. Many instruments of apparently the same construction have been found to give quite different values. The statement, however, is generally made that the receiver of the Féry pyrometer does not rise above 80° C. A value of 200° C would be nearer the receiver temperature, when the pyrometer is sighted on a source at 1500° C, for the average run of the Féry instruments.

It is quite apparent if $T_0$ increases fast enough with increasing $T$ in the expression $e = a (T^b - T_0^b)$, that $T_0^b$ may have nearly as great weight at high values of $T$ (say 2000° C) as at low values (700° C). If $T_0$ remains nearly constant at room temperature, the curve drawn through the observations, plotted log $e$ as abscissas and log $T$ as ordinates, is a straight line at higher values of $T$ with a slight bending away from the line at lower temperatures, the values lying above the line in this region. For such an instrument the correction, $p$ (see p. 131), decreases with increasing values of log $T$. Frequently, however, $T_0$ increases so fast with increasing $T$, that the correction $p$ is approximately constant, and the best curve through the observations on the log $e$ vs. log $T$ plot is a straight line from 600° to 1500° C. Fig. 16, curve B, represents the correction $p$ for Féry pyrometer No. 109 discussed in this section. Values of $p$ are just beginning to increase at the higher temperatures. From log $T = 3.10$ to 3.24 (i.e., $t° C = 1000$ to 1500), $p$ is nearly constant.

The errors in a calibration of a pyrometer as a result of neglecting the $T_0$ term entirely are small, only a few degrees at the most in the range 500° to 1500° C. The question has been considered here mainly because of the general impression that the effect of the $T_0$ term must necessarily rapidly decrease with increasing $T$. Actually, in some cases the $T_0$ term is just about as important at $t = 1500° C$ as at $t = 700° C$.

VII. APPLICATIONS

1. DETERMINATION OF TOTAL EMISSIVITY OF NONBLACK MATERIALS

The total emissivity, $E$, of a nonblack material is defined as the ratio of the total energy ($\dot{J}'$), emitted by the material at a given temperature to that ($\dot{J}$) emitted by a black body at the same temperature. The apparent absolute temperature, $S$, of the non-
black radiator at a true absolute temperature, $T$, has a value such that the total energy emission is equivalent to that of a black body at an absolute temperature $S$. Hence, from the Stefan-Boltzmann relation, one obtains the following, $\sigma$ being an empirical constant:

$$J = \sigma (T^4 - T_o^4)$$
$$J' = \sigma (S^4 - T_o^4)$$

$$\frac{J'}{J} = E = \frac{S^4 - T_o^4}{T^4 - T_o^4} = \text{total emissivity}$$

(1)

If the energies $J$ and $J'$ are measured indirectly by the emf $(e)$ developed in a thermoelectric circuit, as in the case of the thermoelectric radiation pyrometer, the following equations are convenient to use:

$$e = a \ (T^b - T_o^b)$$
$$e' = a \ (S^b - T_o^b)$$

$$E = \frac{\left[\frac{e'}{a} + T_o^b\right]^{4/b} - T_o^4}{\left[\frac{e}{a} + T_o^b\right]^{4/b} - T_o^4} = \text{total emissivity}$$

(2)

In general, since the $T_o$ term is small, and since it occurs alike in numerator and denominator, the equations (1) and (2) may be written as follows:

$$(1)' \quad E = \left(\frac{S}{T}\right)^4$$

$$(2)' \quad E = \left(\frac{e'}{e}\right)^{4/b}$$

The small error in $E$, caused by neglecting the $T_o$ term (equation 1), is illustrated by the following table. The value of $T_o$ is assumed constant at 300° abs.

<table>
<thead>
<tr>
<th>Apparent temperature</th>
<th>$S^4/T^4$</th>
<th>Correction to subtract from $S^4/T^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.9 or 0.1</td>
<td>0.0020</td>
</tr>
<tr>
<td>600</td>
<td>.7 or .3</td>
<td>.0045</td>
</tr>
<tr>
<td>800</td>
<td>.9 or .1</td>
<td>.0056</td>
</tr>
<tr>
<td>&gt;1100</td>
<td>.7 or .3</td>
<td>.0082</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.0103</td>
</tr>
<tr>
<td></td>
<td>.9 or .1</td>
<td>.0103</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.0156</td>
</tr>
<tr>
<td></td>
<td>.9 or .1</td>
<td>.0156</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>&lt;.0085</td>
</tr>
</tbody>
</table>
The above derivation is on the assumption that the pyrometer employed may be properly considered as a black or gray receiver. (See p. 99.) This assumption is usually justifiable over a moderate range of temperature.

2. THE DETERMINATION OF TEMPERATURES

As follows from equation (1)' above, the relation between the observed absolute temperature $S$, measured by the radiation pyrometer, and the true absolute temperature $T$, of the material sighted upon, is:

$$S = \sqrt[4]{E} \cdot T,$$

or expressed in terms of the centigrade scale:

$$S^\circ C = \sqrt[4]{E} \cdot (t^\circ C + 273) - 273$$

where $E$ is the emissivity or characteristic constant of the material. For substances in the open $E$ is always less than 1, but frequently, in the case of the measurement of the temperature of furnaces, $E$ becomes sufficiently near unity that $s$ equals $t$ for all practical purposes—that is, the temperature measured by the pyrometer is the correct temperature of the furnace, since the furnace is emitting black-body radiation. Thus, if the pyrometer is sighted upon a small hole in the side of a kiln, the walls of which are at a uniform temperature, correct temperature values will be indicated by the pyrometer.

It is frequently desired to measure the temperature of a certain part of the furnace which is not uniformly heated or to obtain the temperature of the interior of a furnace when there are present, around the walls, dense clouds of cooler smoke, gases, or fumes. This may be done by sighting the pyrometer into a long porcelain or metal tube hemispherically closed at one end, the closed end being inserted in the furnace at the required position, as illustrated in Fig. 31.

If a considerably greater area of the tube than that radiating to the pyrometer receiver is uniformly heated, the temperatures measured will be approximately correct, since black-body conditions are then approximately obtained. In general, the tube must have a rather large diameter, larger than that of the pyrometer. This is quite evident in the use of a fixed focus pyrometer such as the Thwing, Foster, or Brown. The cone of rays falling
upon the receiver is diverging toward the source, measuring from a point within the pyrometer. Hence, if the tube were of the same diameter as the front opening of these pyrometers, the greater part of the area of the tube capable of radiating to the pyrometer receiver would be that included in the front part of the tube (i. e., the cold portion located in the wall of the kiln, Fig. 31). Hence, the temperature measured would be by no means that of the end of the tube A. The tube must, therefore, be large enough in diameter, so that the cone of rays drawn from the pyrometer will intersect its walls at a position where the temperature distribution from this point, at least, to the end A of the tube is uniform. If a long tube is required, it frequently happens that the diameter necessary to fulfill this condition is so great that the method becomes unsatisfactory. Moreover, in the use of large tubes, especially if the tube is of metal, the heat conduction away from the part of the furnace of which the temperature is desired becomes so high that correct temperature values can not be obtained. There appears to be no satisfactory method of readily meeting this difficulty, although reducing the aperture of the pyrometer will help, at a sacrifice, of course, of sensibility.

It is not always necessary to have the tube of at least the diameter of the pyrometer in the case of focusing instruments such
as the Féry, if the pyrometer is located some distance from the opening of the tube. If the tube is of smaller diameter than that of the mirror of the Féry, care must be taken that the cone of rays represented by lines drawn from the edges of the mirror to the limits of the required area of the source (of which the receiver is the image, i.e., the source of sufficient size to cover the hole in the focusing mirrors) lies wholly within the tube and does not cut the front end. Thus, if the pyrometer were sighted at such a distance that a source, say, 2 cm in diameter, were required, lines drawn from the top, bottom, and sides of this source to the corresponding points of the mirror must not cut the surface of the front part of the tube. The pyrometer, however, may be located at too great a distance from the front of the tube, thus requiring such a large source that lines drawn in the above manner will still intersect the tube. In this case the pyrometer must be focused at a closer distance, and if the cone still intersects the tube, the tube must be enlarged. Of course, it is not absolutely essential that these correct geometrical relations be realized, if a permanent installation is made and merely temperature control desired. In order to obtain true temperatures, however, with an incorrectly mounted system, the pyrometer would have to be calibrated in the exact position in which it is used, since in the above cases where the cone of rays intersects the tube, some part of the tube acts as a limiting diaphragm or entrance pupil and really becomes an essential unit of the system.

Whipple has slightly modified the Féry pyrometer by permanently mounting on the instrument a fire-clay, metallic, or graphite tube closed at one end, the pyrometer being focused on the bottom of the tube. This instrument has been employed for the purpose, among others, of measuring the temperature of molten metals, the tube being plunged directly into the metal.

When the radiation pyrometer is sighted directly upon objects in the open, it will read too low by amounts depending upon the emissivity of the material in question. This is especially true in the case of sighting upon molten metals which, when issuing from the furnace, generally present a clear surface, comparatively free from oxide. A slight film of oxide, in general, greatly increases the emissivity of the substance, and the pyrometer then reads more
nearly correct. Thus, the corrections necessary to apply to the observed readings to convert them to true temperatures are usually small in the case of solid iron or nickel at high temperatures in air, since the surface upon exposure to air almost immediately

Fig. 32.—True temperatures vs. temperatures observed with radiation pyrometer sighted upon materials in the open
becomes heavily oxidized. If a molten surface is exposed for a few seconds to the air, it also becomes coated with an oxide slag, having a higher emissivity than the pure metal. The great increase in emissivity of a surface after oxidation has taken place is a phenomenon well known to the workmen in a foundry, who find that they can approach quite close to the stream of metal when poured from the furnace, but that as soon as the metal strikes the molds and comes in contact with the air, although it is necessarily cooler at this stage, the heat radiated becomes so intense that one can not longer stand near the metal.  

The experimental investigation of the total radiation of non-black materials or metals in the open, at high temperatures, has been confined to an extremely small number of materials. The following table shows the true temperatures corresponding to apparent temperatures measured by a radiation pyrometer when sighted upon molten copper, copper oxide, iron oxide, nickel oxide, and platinum. Determinations for other metals and oxides are in progress. This table is represented graphically in Fig. 32.

<table>
<thead>
<tr>
<th>Observed temperature °C</th>
<th>True temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Molten iron 25</td>
</tr>
<tr>
<td>600</td>
<td>1130</td>
</tr>
<tr>
<td>700</td>
<td>1260</td>
</tr>
<tr>
<td>800</td>
<td>1200</td>
</tr>
<tr>
<td>900</td>
<td>1340</td>
</tr>
<tr>
<td>1000</td>
<td>1475</td>
</tr>
<tr>
<td>1100</td>
<td>1610</td>
</tr>
<tr>
<td>1200</td>
<td>1750</td>
</tr>
<tr>
<td>1300</td>
<td></td>
</tr>
</tbody>
</table>

24 Cases have been noted for some of the rarer metals where the oxide possessed the smaller emissivity. However, this condition is very exceptional and is never met in practice.
26 Burgess, Bureau of Standards Scientific Paper No. 121.
27 Burgess and Foote, Bureau of Standards Scientific Paper No. 249.
VIII. CONCLUSION

In the foregoing pages there have been given an account of the principles which form the basis for the operation of total radiation pyrometers, descriptions in some detail of representative types of this instrument, together with the results of an experimental study of their calibration and behavior under various conditions of use, and as modified by changing the several factors which may influence the readings of such pyrometers. A considerable portion of the text is devoted to the examination of the sources of error and their elimination or correction.

In all, there were carefully examined some 20 instruments, including all the ordinary types commonly met with in practice, such as the Féry, Foster, Thwing, and Brown pyrometers.

Although this investigation, which is largely concerned with instrumental details, does not readily lend itself to a brief summary, yet the following conclusions of a general nature may be mentioned:

The Stefan-Boltzmann law \( E=a (T^4-T_0^4) \) is not, in general, except by accident, obeyed in its exactness by any of the pyrometers examined. The similar equation \( E=a T^4 \cdot T_0^{b-4} \), in which \( b \) is slightly different from 4 (usually neglecting the \( T_0 \) term) is, however, obeyed with sufficient exactness by all total-radiation pyrometers.

The main factors influencing the value of the exponent \( b \) are the geometry and mechanical construction of the instrument; the value of \( b \) for 20 thermoelectric pyrometers ranged from about 3.5 to 4.5. The same instrument of the Féry type may have a different exponent, according to its use with or without the sectored diaphragm for increasing the temperature range.

In general, a radiation pyrometer behaves not as a "black" or total receiver of energy but as a "gray" receiver for the ordinary range of temperatures. For the Féry pyrometer with a gold mirror, for example, the effect of selective reflection of the gold mirror does not become practically appreciable until above 2500\(^\circ\) C but may cause an error of 500\(^\circ\) C at the temperature of the sun.
The auxiliary apparatus, galvanometer, or potentiometer, and recording devices, can be constructed so that their errors will be practically negligible. For the work of the highest accuracy the potentiometric method of measurement is to be preferred with thermoelectric radiation pyrometers.

The principal errors to which the several types of radiation pyrometers are subject are shown to lie in the design and mechanical construction of these instruments; and certain of these inherent errors, such as lag or slowness in reaching an equilibrium reading, require for satisfactory results that the pyrometer be calibrated and used under similar conditions of time of exposure, distance from, and aperture of source. Wide variations in the lag effect exist among apparently similar instruments, ranging from a few seconds to an hour or more. These and other errors of appreciable magnitude, such as stray reflection, convection currents, intervening atmosphere, size of source, tarnishing of receiving mirror, etc., are discussed at length in the text.

It is shown that errors greater than 100° C may readily be caused by dirt on or oxidation of mirror.

The magnitude of errors due to varying the focusing distance may amount to several hundred degrees, if suitable precautions are not taken.

A convenient method for the rapid comparison of different types of pyrometer, and for determining the effects of size of aperture and focusing distance, was devised, consisting of a wide nickel (oxide) strip heated electrically, a series of circular, water-cooled diaphragms, and an optical bench.

The methods in use at this Bureau for the calibration of radiation pyrometers by means of especially designed experimental "black bodies" are described in detail and methods of extrapolation outlined, as well as methods of use of radiation pyrometers, including methods of obtaining approximately correct temperatures for the case in which a source of insufficient size is sighted upon.

Finally, there is considered the application of the radiation pyrometer to the determination of the total emissivity of non-black substances and to the measurement of temperatures.
In conclusion, the writers desire to thank the Taylor Instrument companies, of Rochester, N. Y., and the Thwing and the Brown companies, of Philadelphia, for very kindly placing at their disposal several radiation pyrometers.

WASHINGTON, January 2, 1915.