THE TESTING OF POTENTIOMETERS

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I. INTRODUCTION

Potentiometers are being used extensively in electrical measurements in which the precision required is higher than can be obtained by the use of deflection instruments. With a potentiometer and a standard cell, electromotive forces are measured directly. By the use of a standard resistance in connection with the potentiometer and standard cell, measurements may be made from which the current or power may be calculated. If in addition the time is measured we have data from which the energy may be calculated.

In general, potentiometers are constructed very much like other resistance apparatus. They are made up of a number of coils of wire of various resistances connected in various ways. Dial switches, plugs, or sliding contacts are provided to make the necessary changes in the connections to the various coils. The coils are usually made of manganin wire and are adjusted so that the resistances are nearly proportional to the readings of the dials.

In measuring electromotive forces (or potential differences) with a potentiometer, the ratio of the unknown electromotive force to the known electromotive force is determined from the ratio of two resistances, which during the measurement carry a current. (See Fig. 1.) The drop in potential in one resistance, R_s, is made equal to the known electromotive force, S, usually by adjusting the current, I, the equality or balance being indicated by a zero deflection of the galvanometer. This gives

$$S = R_s I$$

The other resistance R_e is adjusted so that its drop of potential balances the unknown electromotive force E. This gives

$$E = R_e I$$

Therefore

$$E = S R_e / R_s \tag{1}$$

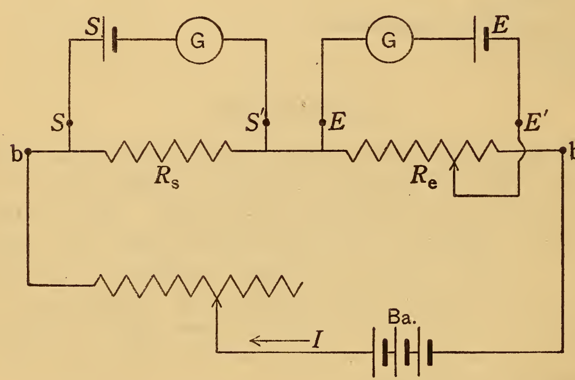

FIG. 1.—*Connections for comparing two electromotive forces by potentiometer method*

The test therefore consists in making such measurements as are necessary for the determination of the ratio of R_e to R_s for all settings of both R_e and R_s.

When a potentiometer is used in conjunction with a standard resistance and standard cell for determining the current or power, the measurement with the potentiometer is in reality that of a potential difference. The test just referred to is therefore all that is required.

The resistance of the coils is not always accurately adjusted, nor does it remain constant but changes somewhat with time. The greatest change occurs within one year after construction and may amount to as much as several hundredths per cent. Later the changes seldom amount to as much as one hundredth per cent during a year, in well-constructed potentiometers. Therefore potentiometers which are to be used in measurements of high precision [1] should be calibrated occasionally, especially during the first few years. If this is done and the corrections applied, errors on account of lack of adjustment of the coils or on account of changes in their resistance can be eliminated.

A number of potentiometers are tested each year at the Bureau of Standards. These include potentiometers tested for manufacturers, public-service companies, educational institutions, the State governments and various departments of the National Government. Potentiometers of the Feussner [2] type (see Fig. 2) and Crompton [3] (or slide wire) type (see Fig. 5) are the more usual, but occasionally potentiometers of the Raps [4] type or potentiometers of the split circuit [5] type, usually designed for use with thermocouples, are tested. The testing of these required a considerable amount of work on account of its having been necessary to make several kinds of measurements on each potentiometer and to use different methods for different types of potentiometers. For this reason the testing of potentiometers has received special consideration.

The purpose of this paper is to bring to the attention of students and users of potentiometers work done at the Bureau of Standards in connection with the testing of such apparatus. To do this we shall (*a*) discuss the theory of the potentiometer in such a way as to show the corrections which must be applied if results of a high accuracy are to be obtained, (*b*) describe briefly methods and arrangements used in testing potentiometers, and (*c*) describe apparatus designed for and used in testing potentiometers.

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[1] That is, where the accuracy is 1 in 10 000 or better.

[2] Zs. f. Instrk., **10**, p. 113; 1890.

[3] Electrician, London, **31**, p. 32; 1893.

[4] Eektrotech. Zs. **15**, p. 215; 1895.

[5] Diesselhorst: Zs. f. Instrk., **26**, p. 173–297, 1906; **28**, p. 1; 1908. White: Phys. Rev., **25**, p. 334; 1907.

2. THEORY AND CORRECTIONS OF POTENTIOMETERS

Any potentiometer can be considered to be a system or network of conductors having three[6] pairs of terminals. The battery or other means of supplying a current, I, to the potentiometer is connected to one pair, the I-terminals; the known or standard electromotive force is connected to the second pair, the S-terminals; and the unknown electromotive force is connected to the third pair, the E-terminals. The current I flowing through the resistance of the potentiometer causes a difference in potential, E, between the E-terminals and, S, between the S-terminals. The ratio of E to I is equal to the resistance R_e and the ratio of S to I is equal to the resistance R_s.

The resistance between the battery terminals, the total resistance of the potentiometer, is designated by the letter T. By manipulation of the various switches, R_e and R_s can be changed but the total resistance T remains approximately constant and would remain constant if the potentiometer were mechanically perfect and correctly adjusted.

In comparing two electromotive forces, one, whose value is usually known and which we shall call S, is connected to the S-terminals and either the resistance R_s or the current I adjusted so that

$$R_sI=S,$$

then the other electromotive force, usually of an unknown value and which we shall call E, is connected to the E-terminals and the resistance R_e adjusted so that

$$R_eI'=E.$$

Therefore
$$E=SR_eI'/R_sI, \qquad (2)$$

which differs from the simple case considered above in that the total current through the potentiometer is not necessarily the same when the two balances are made. In order to express the value of E in volts it is therefore necessary to know the value of the ratio R_eI'/R_sI and the value of S in volts.

[6] For convenience a switch and a fourth pair of terminals are provided for connecting a galvanometer either into one of the leads to the E-terminals or into one of the leads to the S-terminals. In some cases more than one pair of E-terminals are provided and in other cases in effect the E-terminals and S-terminals are identical. (See p. 10.)

The value of S is determined by comparison with the electromotive force of Weston Normal Cells set up according to definite specifications; but as the standard cell constitutes no part of the potentiometer it need not be considered further here. What we shall be concerned with, then, is the determination of the ratio of R_eI' to R_sI. A complete calibration consists in the determination of this ratio for all possible values of both R_e and R_s.

In most potentiometers the total resistance is nearly independent of the value of both R_e and R_s, so that if the conditions in the external battery circuit are constant the current will be nearly constant. In any case the ratio I'/I is nearly unity and depends only slightly on the values of R_e and R_s, so we can say

$$I'/I = 1 + h \tag{3}$$

where h is a small correction, which is different for different values of R_e and R_s. Therefore

$$E = SR_e(1 + h)/R_s \tag{4}$$

If the electromotive force of the source supplying the current is constant, we have

$$\frac{I'}{I} = \frac{T_s + R}{T_e + R} \tag{5}$$

or

$$h = \frac{T_s - T_e}{T_e + R} \tag{6}$$

where T_s is the total resistance of the potentiometer at the first balance and T_e is the total resistance of the potentiometer at the second balance and R is the resistance external to the potentiometer.

In many cases, for example potentiometers of the Crompton type, it will be evident from the construction that the total resistance is constant. In other cases, where when a dial switch is changed by one step a resistance is removed from one part of the circuit and another resistance inserted in another part of the circuit for the purpose of keeping the total resistance constant, a few measurements of the total resistance for certain values of R_e

and R_s will be required. If these show that h is negligibly small[7] for all values of both R_e and R_s, the potentiometer is said to be compensated.

In such a case the ratio R_eI' to R_sI is equal to the ratio of R_e to R_s. Compensation, therefore, makes the apparatus much easier to calibrate, as well as much more convenient to use. From this point on, unless the matter is specifically mentioned, we shall assume that the potentiometer under consideration is compensated.

The resistances R_e and R_s are usually varied by the manipulation of three sets of switches, which may be of various forms, including plugs and contacts on a slide wire. These three sets may be called the e-switches, the s-switches, and the f-switches (range or factor switches), and we shall call the reading corresponding to their settings e, s, and f.

The e-switches change the value of R_e over a large range, the s-switches change the value of R_s usually over a small range, and the f-switches change the value of R_e or R_s, or both, by a factor, usually .1 or 10.

If the resistance R_e is independent of the reading s, if the resistance R_s is independent of the reading e, and if a change in the reading f changes the ratio of R_e to R_s corresponding to any reading of e and of s by the same proportional amount as that corresponding to any other reading of e and of s, as is usually the case (even in potentiometers which are not completely compensated), then all possible values of the ratio of R_e to R_s can be determined from a number of measurements equal to the number of possible readings of the e-switches plus the number of possible readings of the s-switches plus the number of possible readings of the f-switches. It will therefore be convenient to introduce into our equation expressing the relation between the unknown and known electromotive force three corrections, one for each of the readings e, s, and f.

The e-switches are regularly so marked that the reading e is approximately proportional to the resistance R_e so we may write

$$R_e = K(e + a) \tag{7}$$

[7] In precision measurements, an error, to be negligibly small, must usually be less than 1 in 10 000 and in some cases less than 1 in 100 000.

where K is a constant and a is a small correction depending upon the reading e.

Also the s-switch is so marked that the reading s is approximately proportional to the resistance R_s so we may write

$$R_s = K's(1-b) \tag{8}$$

where K' is a constant which may or may not be the same as K and b is a small correction. We therefore have

$$E = \frac{S\,K}{s\,K'}(1+b)(e+a) \tag{9}$$

Here and in the equations which follow we neglect the second and higher powers and products of all small quantities.

Now if we let

$$K/K' = f(1+d) \tag{10}$$

where f, the reading of the range or factor switches is a simple number, usually an integral power of ten, and d is a correction which is small in potentiometers of good design and construction, we have

$$E = f(1+b+d)(e+a)S/s \tag{11}$$

In potentiometers having only one range there is no range or factor switch to be read, consequently a value for f must be supplied. This value is always to be taken as the nominal number of volts per unit of the reading e. Thus, for a potentiometer in which the reading e gives the resistance R_e in ohms the value of f to be supplied is the nominal number of volts per ohm. A better arrangement in this case would be to choose the unit of the reading e so as to make f unity, in which case the reading e is in volts.

In some cases it may not be possible to make the reading s equal to the value S of the known electromotive force. It is therefore necessary to introduce a correction c which may be defined by the equation

$$S/s = 1+c \tag{12}$$

so we have

$$E = f(1+b+c+d)(e+a) \tag{13}$$

Some potentiometers are not provided with s-switches and the difference between the value of the known (or standard) electromotive force and the standard cell value for which the potentiometer is adjusted may be fairly large. In such cases the correction c is of importance and must regularly be applied. To illustrate the application of the correction c, suppose that the potentiometer has no standard cell dial, but is adjusted for use with a standard cell whose voltage is 1.0185 and we wish to use it with a standard cell whose voltage is 1.0190. This makes

$$S = 1.0190 \text{ volt and } s = 1.0185 \text{ volt}$$

so

$$c = \frac{1.0190 - 1.0185}{1.0185} = +.0005$$

Since the three corrections a, b, and d corresponding to any readings e, s, and f are in effect but a single correction, which when applied to fe/s gives the ratio of the resistances R_e to R_s, two of these corrections may be chosen more or less arbitrarily, providing a proper value is assigned to the third. In the use of the potentiometer the corrections are more conveniently applied if d is made zero for that reading of f which is most used and if b is made zero for that reading s which is most used. Ordinarily[8] b is taken as zero for $s = 1.0185$ and in a well-constructed instrument this usually makes b negligibly small for values of s near 1.0185. In this way the entire correction for the readings most used is included in a and is therefore easily applied, since it is added directly to the reading e.

The reading e is that corresponding to the settings of the e-switches (dial switches or dial switch and slide wire). Since each of the e-switches changes R_e by amounts approximately proportional to changes in their readings we can say that

$$e = e_1 + e_2 + e_3 + \text{etc.} \tag{14}$$

where e_1 is the reading of the first switch, i. e., the one which changes R_e in the largest steps, e_2 is the reading of the second switch, i. e., the one which changes R_e in the next largest steps, etc. If the change in resistance R_e corresponding to a change in reading of

[8] The reason for choosing 1.0185 is that the electromotive force of the Weston Normal Cell at 20°C. is 1.0183 and that of the Weston unsaturated or portable cell is, on the average, about 1.0187 volt.

any one of the switches is independent of the readings of all of the other switches[9], as it usually is, we can say that

$$a = a_1 + a_2 + a_3 + \text{etc.} \qquad (15)$$

where a_1 depends upon the reading e_1,
where a_2 depends upon the reading e_2,
where a_3 depends upon the reading e_3, etc.

We may therefore write the complete formula, expressing the value of the unknown electromotive force in terms of the readings of the potentiometer and corrections, as follows:

$$E = f\,[(e_1 + a_1) + (e_2 + a_2) + (e_3 + a_3) + \text{etc.}]\,(1 + b + c + d). \qquad (16)$$

Here f is the reading of the range switch and d is its correction; e_1 is the reading of the first dial and a_1 is the correction to this reading; e_2 is the reading of the second dial and a_2 is its correction; e_3 is the reading of the third dial and a_3 is its correction, etc.; b is a correction to be applied on account of errors in the adjustment of the resistance R_s; and c is the amount in proportional parts by which the known electromotive force exceeds the reading s (either the reading of a standard cell dial switch, or the standard electromotive force for which the potentiometer is adjusted).

In the case of potentiometers of the Crompton type the reading e_2 is that corresponding to a setting of the contact on the slide wire. In such instruments usually 100 or 1000 divisions of the slide wire are equivalent to a step of the dial switch. If a reading of 1000 on the slide wire is equivalent to a reading of 0.1 on the dial (as is the case with the Leeds and Northrup potentiometer), we must consider that

$$e = e_1 + .0001 e_2. \qquad (17)$$

Since a_1, a_2, and a_3, etc., for any particular reading, are in effect a single correction, it is possible to choose all but one arbitrarily if the proper value is given to the one. It has been found convenient to choose a_2, a_3, etc., zero for the reading $e = 0$.

In some potentiometers it is necessary to change the f reading after balancing the known electromotive force and before balancing the unknown electromotive force. In this case the second reading

[9] This is not necessarily the case in potentiometers in which the Kelvin-Varley shunt scheme is used, nor in split circuit potentiometers unless the adjustment is very good.

of f divided by the first reading of f usually gives the value of f to be used.

In general equation (16) applies only in case the potentiometer is compensated, but if the known and unknown electromotive forces can be balanced without changing any of the e, s, or f-switches between balances, or if the balances can be made simultaneously, the corrections just considered are all that are necessary even though the potentiometer is not compensated.

However, as a matter of convenience, it is desirable that potentiometers be compensated so that changes in the settings of the dials do not change the total resistance by an appreciable amount. Should there be a decided lack of compensation, the simultaneous balance of both electromotive forces could be made only by successive approximations since an adjustment of R_e to balance against E would change the current and thus disturb the balance against the known electromotive force and then a change in current to reestablish this balance would disturb the balance against the unknown electromotive force.

In some potentiometers the known and unknown electromotive forces are in effect alternately connected to the same terminals so that the s and e-switches are identical. In this case

$$R_s = K\ (e_s + a_s) \text{ instead of } K's(1-b)$$

so that $b = -a_s/e_s$ and formula (13) becomes

$$E = f\ (1 - a_s/e_s + c + d)\ (e + a) \tag{18}$$

This formula [10] applies only if the potentiometer is compensated.

In general then the test of a potentiometer consists in determining the corrections a_1, a_2, a_3, etc., b and d for all possible reading of s, f, e_1, e_2, e_3, etc. In the case of a 5-dial potentiometer (5 e-dial switches each having 10 steps or 11 points) if the s-dial switch has 20 points (19 steps) and if there are two ranges or two readings for f, the total number of corrections is 77. In this case the possible number of settings of the ratio R_e to R_s or readings of the potentiometer is 6 442 040 to each of which the correction to be applied is obtained from the 77 corrections, one for each reading of each of the switches.

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[10] If there is no factor switch, or if its reading is the same for both balances, then $f=1$ and $d=0$.

3. METHODS OF TESTING

Since the test of a potentiometer must give the corrections to be applied to the readings to give the values of the electromotive forces measured, a simple and direct method is to use another potentiometer to furnish known electromotive forces. If these are measured by means of the potentiometer to be tested the corrections can be readily calculated. This method requires a potentiometer which has been previously calibrated and requires that two currents (one in each potentiometer) be maintained constant. For these reasons this method has been little used in the Bureau of Standards, though it has the advantage of giving the corrections directly and under operating conditions.

The method commonly used is to measure the relative values of the component resistances of the apparatus and from these to calculate the corrections. For the measurement of the relative values of the resistances a bridge is almost always used. Some of the bridge arrangements which have been used by the authors are described below.

The ratio of the resistances R_e and R_s can be determined either by direct measurement or by calculation from the measurement of the component parts; that is, the individual resistance coils or sections. To measure the resistances R_e and R_s directly requires apparatus whose corrections are known to an accuracy at least as high as that sought in the calibration of the potentiometer. If R_e and R_s are to be calculated from the values of their component resistances, the more important measurements should be comparisons of nearly equal resistances. We are therefore mainly concerned with the small differences between the different steps or coils so in general it is not necessary that we know accurately the corrections to the readings of the apparatus used in making the measurements. Good apparatus for which the corrections were sufficiently well known to permit of measuring R_e and R_s directly has not always been available. In general it has been our aim, therefore, to use such arrangements as will give the corrections to the readings of the potentiometer to an accuracy higher than that to which the corrections for the apparatus used in making the test are known. Some of the arrangements may therefore be suitable for use in those laboratories in which there is but a small amount of accurately adjusted or calibrated resistance apparatus.

Feussner Type.—The connections of a potentiometer of the Feussner type as improved by Brooks and made by O. Wolff are shown diagrammatically in Fig. 2. The relative values of the 1000 and 100-ohm coils in the first and second decades are measured in a Wheatstone bridge using the substitution method. If 1000 and 100-ohm resistance standards whose corrections are accurately known are available, they should be included in the measurements

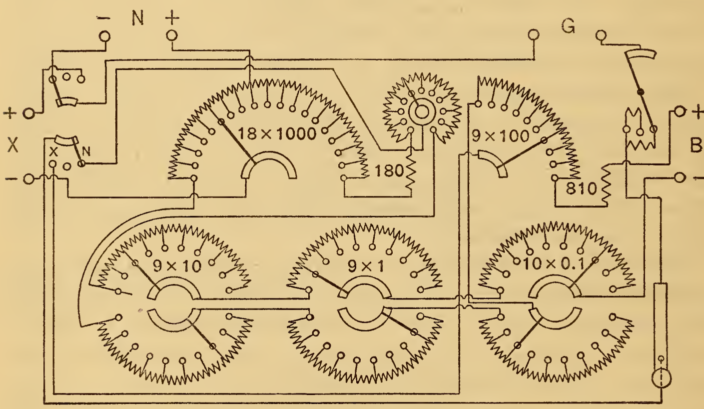

Fig. 2.—*Diagram of Wolff potentiometer*

of the 1000 and 100-ohm sections of the potentiometer. In this method the coils are connected successively in the same arm of the bridge and the changes in the setting of the bridge necessary to reestablish the balance are noted. The connections to the coils are made through flexible leads attached to the contact blocks by means of screws. If care is taken in making the screw connections the total connection resistance will be sufficiently constant, so that the changes in setting of the bridge will represent differences in the resistances of the coils. Since these differences are small the relative values of the resistances may be determined to a high accuracy with but a limited knowledge concerning the other resistances in the bridge.

In order to get the relative values of all of the coils, Σ1000 ohms (i. e., 1000 ohms resistance made up from the 100, 10, 1, and .1-ohm coils) is measured along with the 1000-ohm coils, also Σ100 ohms, made up from the 10, 1, and .1-ohm coils, is measured along with the 100-ohm coils. The 10, 1, and .1-ohm coils are measured in a Wheatstone bridge in the ordinary manner; that is, the three decades are connected into a Wheatstone bridge and the total resistance corresponding to the various settings of the dials measured. From the resistance corresponding to the different readings it is necessary to subtract the resistance found with the three dials set at zero. The odd part of the resistance across which the standard cell is balanced is also measured in a Wheatstone bridge in the ordinary manner.

In the 10, 1, and .1-ohm decades, double dials with compensating coils are used to maintain a constant total resistance. In checking the constancy of the total resistance all the coils except those in the three double decades are short circuited and the total resistance corresponding to the various settings of the double dials is then measured in a Wheatstone bridge. It will be noted that in all cases where measurements depend directly upon the accuracy of the bridge the quantity measured is less than 2 per cent of R_s or less than 2 per cent of R_e when $e_1 = 5000$ or more.

Besides the resistance in the various decades there may be an appreciable resistance in the connections between them, which would require a correction to be applied when the potentiometer was set to read zero. In this case a_1 for $e_1 = 0$ is the resistance R_e when e_1, e_2, e_3, etc., each equals zero. To determine this correction, which usually is small, the potentiometer is connected as it is in use, except that the E-terminals are short-circuited and a switch is included in the connection to the battery. When the e-switches all read zero, on closing the switch to the battery there will be a deflection of the galvanometer depending upon the magnitude of R_e and the magnitude of the test current. The resistance R_e is then obtained by comparing this deflection with the change in the deflection produced by changing the setting by a small amount (say one step on the lowest dial). Since the connections between the various decades include several switches, the

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constancy of this correction is a measure of the reliability of the dial switches.

A modification of the Carey-Foster method has also been used in measuring the 1000 and 100-ohm coils. With the connections as shown in Fig. 3*a* and with the galvanometer connected to g_1 a balance is made by changing either of the nearly equal ratio arms *A* or *B*. Then with the standard *S* and the link *L* interchanged and the galvanometer connected to g_2 a second balance is made by

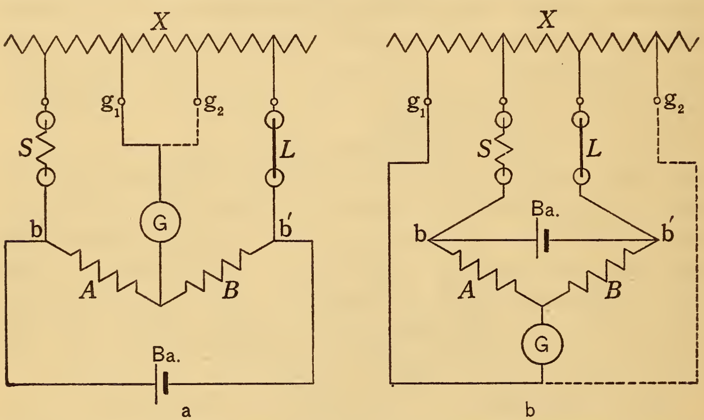

FIG. 3.—*Connections for comparing resistances of conductors connected in series by modified Carey-Foster method*

an adjustment of *A* or *B*. If *r* is the increase in the value of the ratio of *A* to *B* then[11] we have

$$X=(S-L)\ (1+r) \tag{19}$$

With the connections as shown in Fig. 3*b*

$$X=(S-L)\left(1+\frac{r}{2}\right) \tag{20}$$

With this latter connection the adjacent 1000 or 100-ohm coil is in the galvanometer circuit. By substituting one coil after another

[11] Equations (19) and (20), although not exact, give very accurate results if the coils in the potentiometer are nearly equal to $S-L$ and the connecting resistances are small compared with the resistance of one coil. As the method has been used the errors have seldom amounted to more than 0.001 per cent.

at X various values of r are obtained from which the relative values are directly obtained without an accurate knowledge of the value of $S-L$.

Another method of measuring a number of nearly equal coils connected in series has been used. The connections are as shown in Fig. 4. The bridge is balanced by adjusting A or B, (1) with the battery connected at b_1 and b'_1, (2) with the battery connected at b_2 and b'_2, (3) with ratio coils A and B interchanged and the battery connected at b_1 and b'_1, and (4) with ratio coils as in (3) and battery connected at b_2 and b'_2. From the observed changes in the ratio of

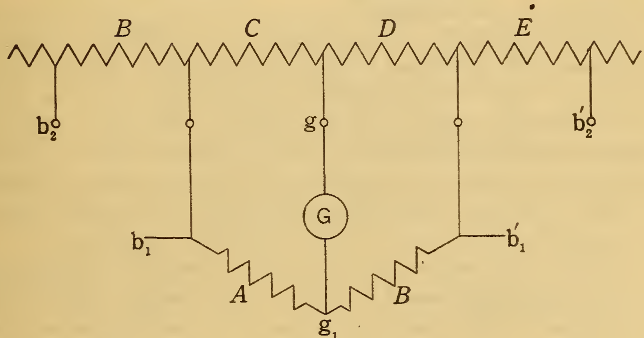

Fig. 4.—*Connections for comparing resistances of conductors connected in series by successive ratio method*

A to B the ratio of C to D is calculated. In a like manner we obtain the ratio of D to E, E to F, etc., from which the relative values are readily calculated. From such measurements the relative values of all the resistances of the first and second dial may be obtained.

From the relative values or values in ohms of all the resistance sections the corrections a_1, a_2, a_3, a_4, a_5, and b may be calculated. In case no standard resistances are used (or the values of the standard resistances are not accurately known) it is generally convenient to proceed with the calculation in the following way:

1. Subtract from the values found for the different sections of the first (or main) dial such a value as will make the mean remain-

der for all approximately zero, or better, the mean remainder for the 10 sections included in R_s equal to zero. This gives a series of corrections to the different sections which are smaller the better the sections are adjusted. The value of R_e when e_1, e_2, e_3, e_4, and e_5 are each zero is the value of a_1 for $e_1=0$. If to this we add the correction to the first section we have the value a_1 for $e_1=1000$. If to the value of a_1 for $e_1=1000$ we add the corrections to the second section we have the value of a_1 for $e_1=2000$. If to this we add the correction to the third section we have the value of a_1 for $e_1=3000$. In a like manner the values of a_1 for $e_1=$ 4000, 5000, 6000, etc., are obtained.

2. Subtract from the values found for the different sections of the second dial (including $\Sigma 100$) a value such as will make the sum of the values for those sections used to make up $\Sigma 1000$ equal to the correction found in (1) for $\Sigma 1000$. This gives a series of corrections the first of which is a_2 for $e_2=100$; the sum of the first and second is a_2 for $e_2=200$; the sum of the first, second, and third is a_2 for $e_2=300$, etc. For $e_2=0$, a_2 is zero, since the correction for e_2, e_3, e_4, and e_5 each equal to zero is included in a_1 for $e_1=0$.

3. If the value found by measuring $\Sigma 100$ directly is in good agreement with the value found in (2), we may consider that the units of resistance of the bridge and of the potentiometer are substantially equal. In this case the values of a_3, a_4, and a_5 may be obtained directly by subtracting the corresponding readings of e_3, e_4, and e_5 from the values of the resistances as measured with the bridge. Care should be taken to see that the value of R_e with e_3, e_4, and e_5 each equal to zero is first subtracted from the measured values as has been pointed out above. (See p. 13.) In case the value of $\Sigma 100$ as obtained directly with the bridge is not in good agreement with the value as obtained in (2), all values obtained with the bridge should be multiplied by the ratio of the latter to the former before subtracting to obtain a_3, a_4, and a_5.

4. Subtract from the values found for the resistance between the highest point on the first dial and the various points on the standard cell dial 183 for $s=1.0183$, 184 for $s=1.0184$, 185 for $s=1.0185$, etc. This gives a series the successive terms of which, when added to the sum of the corrections to the 10 sections of

the first dial which are included in R_s, are corrections pertaining to the corresponding readings of s. Dividing each term by 10 000 $\times s$ and changing the sign gives a series the different terms of which are the values of b corresponding to the readings s.

If 1000 and 100-ohm resistance standards, whose corrections are accurately known, are used we have data which so far we have not considered. This data may be used (a) as a check on the ratio of the mean value of the 1000-ohm sections to the mean value of the 100-ohm sections as determined by use of $\Sigma 1000$ measurement, or (b) to determine the value of resistance of the

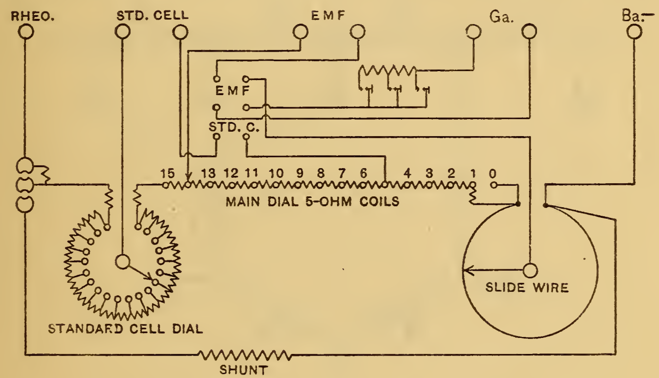

FIG. 5.—*Diagram of Leeds and Northrup potentiometer*

1000 and 100-ohm sections in ohms. If the values in ohms of the different sections are known, the corrections a_1, a_2, a_3, a_4, and a_5 are obtained by subtracting the readings e_1, e_2, e_3, e_4, and e_5 from the values of the corresponding resistances and the correction b is obtained by subtracting from the reading s the value of the resistance R_s and dividing the remainder by 10 000 $\times s$.

As has been pointed out above (p. 8) it is desirable to make the correction b zero for that setting of the standard cell dial most used. The first method outlined above for calculating the corrections usually makes b small while the second may not. It may therefore be desirable to replace this set of corrections by a new set in which b is zero for $s = 1.0185$ (or some other chosen value). If the new set is a'_1, a'_2, a'_3, a'_4, a'_5 and b'

$$b' = b - b_{1\cdot0185} \tag{21}$$

$$\left.\begin{aligned} a'_1 &= a_1 + e_1 b_{1\cdot0185} \\ a'_2 &= a_2 + e_2 b_{1\cdot0185} \\ a'_3 &= a_3 + e_3 b_{1\cdot0185} \\ \text{etc.}^{12} \end{aligned}\right\} \tag{22}$$

Crompton Type.—A diagram of the connections of a potentiometer of the Crompton type as made by the Leeds & Northrup

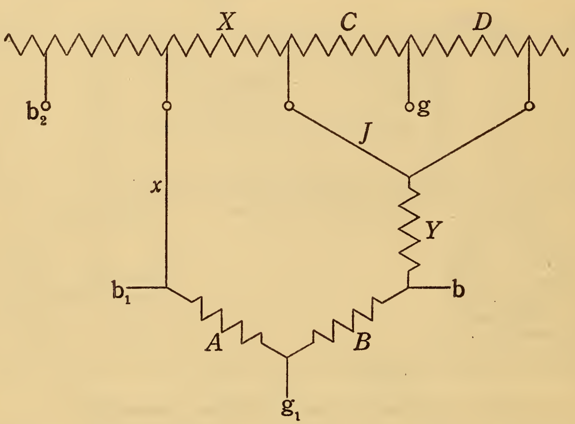

Fig. 6.—*Connections for comparing resistances of conductors connected in series by Thomson bridge method*

Co. is shown in Fig. 5. Several methods of testing this potentiometer have been used. On account of the low resistance of the coils (5 ohms) the resistances of the posts and connections introduce uncertainties in the measurements when the simple bridge method is used. In order to eliminate errors on account of these resistances a Thomson bridge method has been used. (See Fig. 6.) In the figure A and B represent variable ratio coils and X, C, and D are the approximately equal coils in the potentiometer. Y is an auxiliary resistance about equal to X.

[12] Here b is the old correction for any reading s for which the new correction b' is desired and $b_{1.0185}$ is the old correction for $s = 1.0185$.

Since $A/B = X/Y = C/D$ very nearly and J is small we have

$$X + x = AY/B, \text{ very closely.} \tag{23}$$

By shifting the battery connection from b_1 to b_2 and rebalancing, the connecting resistance x can be determined. By substituting one coil after another in the bridge, relative values of the coils are obtained. In this method it is seen that the two adjacent coils are used as the auxiliary ratio coils. The slide wire of this potentiometer can not be measured by this method on account of the high and uncertain resistance of the sliding contact.

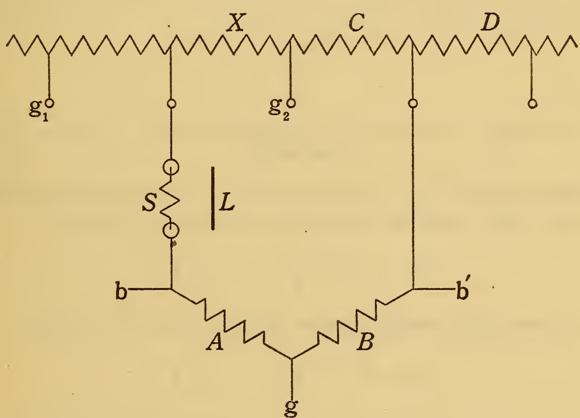

FIG. 7.—*Connections for comparing resistances of conductors connected in series by modified Callendar and Griffiths method*

The 5-ohm coils have also been measured by means of the modified Carey-Foster method as described on page 14 for the measurement of the 1000 and 100-ohm coils of the Wolff potentiometer.

The successive ratio method, described on page 15, and a modification of the Callendar and Griffiths bridge method have been used to measure the relative values of the 5-ohm coils. The connections for measurement by the latter method are shown in Fig. 7. With a 10-ohm coil at S and the galvanometer connected to g and g_1 a balance is made. The 10-ohm coil is then short-circuited by a conductor of very low resistance L, the galvanometer connection changed from g_1 to g_2, and the bridge rebalanced by a change of the ratio of A to B. If $(1+r_1)$ is the value of this ratio

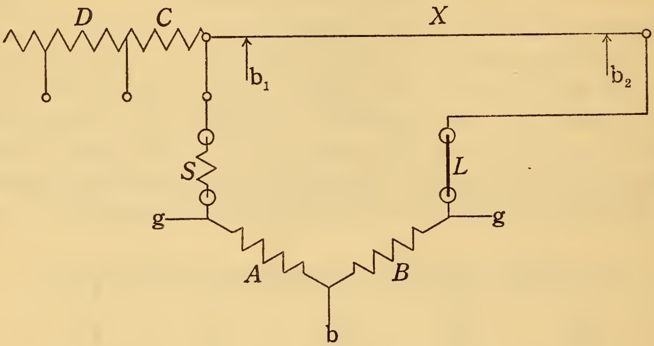

Fig. 8.—*Connections for measuring mean resistance per division of slide wire by Carey-Foster method*

in the first case and $(1+r'_1)$ is the value in the second case, where r_1 and r'_1 are both small in comparison with unity, then [13]

$$X_1 = \frac{(S-L)}{2}\left(1 - r_1 + \frac{r'_1}{2}\right) \tag{24}$$

Likewise from the measurements of the other coils we get

$$\left.\begin{aligned} X_2 &= \frac{(S-L)}{2}\left(1 - r_2 + \frac{r'_2}{2}\right) \\ X_3 &= \frac{(S-L)}{2}\left(1 - r_3 + \frac{r'_3}{2}\right) \end{aligned}\right. \tag{25}$$

Since $\frac{(S-L)}{2}$ is constant these measurements give the relative values of the coils.

The mean resistance per division of the slide wire has been measured by the Carey-Foster method with connections as shown in Fig. 8. This measurement gives two settings on the slide wire between which the resistance is $S-L$. If this difference is the same as that used in the measurement of the coils in the main dial, by

[13] This equation (24) is not exact, but gives very accurate results if the connections are of low resistance and the coils in the potentiometer are nearly equal in resistance to 1/2 $(S-L)$. For example, if none of the coils differ by more than 1/10 per cent from 1/2 $(S-L)$ and the resistance of connections is not more than 0.02 ohm, the error introduced by using the approximate formula is less than 1 in 100 000.

the modified Carey-Foster method, the mean resistance per division is obtained on the same basis as are the resistances of the coils in the main dial. In any case, relative values can be obtained by using standard resistances of known values.

Another arrangement is to make two settings 1000 divisions apart on the slide wire and measure the resistance between them by the modified Carey-Foster method described above. This measurement has been regularly made along with the measurement of the 5-ohm coils by the modified Carey-Foster method.

The Callendar and Griffiths method has been used to measure the mean resistance per division of the slide wire. The connections are as shown in Fig. 9. By balancing with the ratio connected first at m and then at n two settings on the slide wire are obtained. Then,

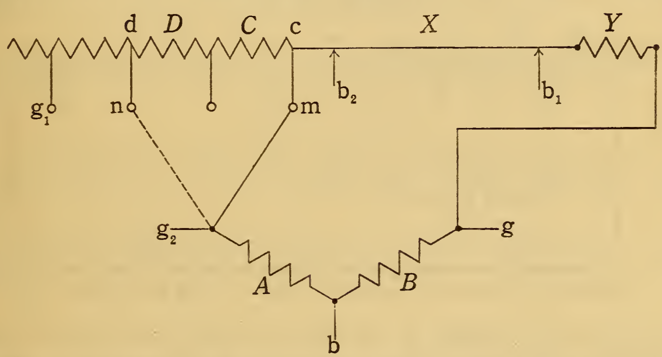

FIG. 9.—*Connections for comparing mean resistance per division of slide wire with the resistance of the first two 5-ohm coils of main dial Callendar and Griffiths method*

knowing the ratio A/B, the resistance between these two settings is obtained in terms of the first two 5-ohm coils. Therefore this method gives a value of this resistance on the basis on which the values of all the 5-ohm coils are expressed. In this method the use of auxiliary known standards is unnecessary but a resistance of about 5 ohms must be inserted at Y. The resistance A must be considered as including all the resistance from b to c (or d).

That part of this resistance from g_2 to c (or d), although small, is generally unknown. It can, however, be readily measured by changing one of the galvanometer connections from g_1 to g_2 and noting the change in the setting of the slide wire contact necessary to reestablish the balance.

Another arrangement is to make two settings on the slide wire 1000 divisions apart and measure the resistance between them by the modified Callendar and Griffiths method described above. This measurement has been regularly made along with the measurement of the 5-ohm coils by the modified Callendar and Griffiths method.

A modification of the Matthiessen-Hockin method has also been used. A plan of the connections is shown in Fig. 10. Two balances are made, one with the battery connected to b and 2, and another at b′ and 3. The balances are made by adjusting the position of the slide wire contact. Under these conditions the following relation holds:

$$X = (A + a)C/B \tag{26}$$

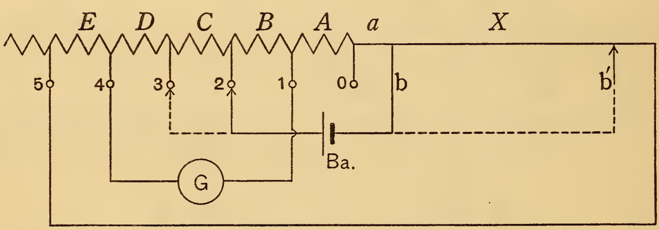

FIG. 10.—*Connections for comparing mean resistance per division of slide wire with the resistance of 5-ohm coils of main dial by Matthiessen and Hockin method*

From the two readings of the slide wire and the values of the first three 5-ohm coils the mean resistance per division of the slide wire is calculated.

The mean resistance per division of the slide wire can be measured in terms of the first three 5-ohm coils by forming the bridge shown in Fig. 11. Two balances are made by adjustment of the sliding contact.[14] In the first the points g_1, b, g, and b_1 are used as branch points and in the second the points g'_1, b′, g′, and b'_1 are used. If X is the resistance of the slide wire between the two points at which the sliding contact must be set to establish the two balances of the bridge, then

$$X = AC/B \tag{27}$$

very exactly, if A, B, and C are nearly equal.

[14] Wenner: Phys. Rev., **17**, p. 384; 1903.

Three 10-ohm resistance standards have also been used in conjunction with the slide wire and the first two 5-ohm coils of the potentiometer, as shown in Fig. 12, where B, C, and n represent the

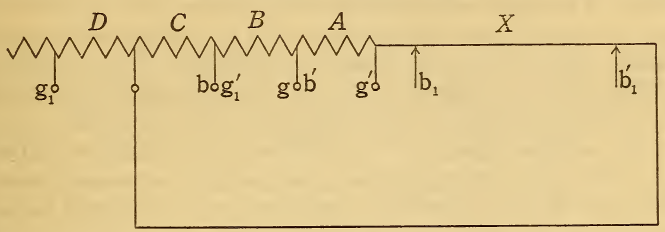

FIG. 11.—*Connection for comparing mean resistance per division of slide wire with the resistance of the first three 5-ohm coils of the main dial*

10-ohm resistance standards. This arrangement is a combination of the Matthiessen-Hockin and Thomson bridges and gives

$$X = (A + a)C/B \tag{28}$$

The resistance r need not be known but should be between 0.1 and 0.9 ohm so as to give suitable settings on the slide wire. The

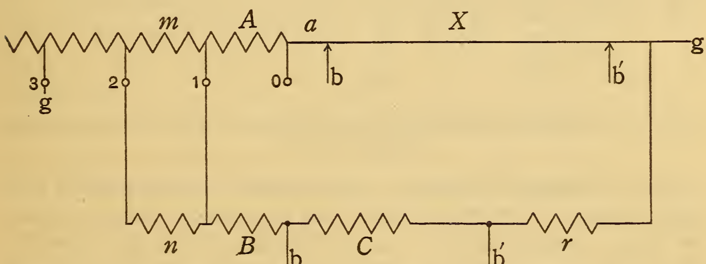

FIG. 12.—*Connection for comparing mean resistance per division of slide wire with that of first 5-ohm coil of the main dial using three nearly equal standard resistances*

error which would otherwise be introduced on account of the resistance of the connection between A and B is made negligibly small by using the coils m and n as shown.

The calibration of the slide wire of a potentiometer of the Crompton type may be made by several different methods. In the Carey-Foster the Strouhal and Barus,[15] and possibly some

[15] Bull., U. S. Geol. Sur., 14; 1885.

other methods the resistance or relative values of the resistance of short sections of the wire are measured. The calibration correction, therefore, at any point near the center depends upon a number of settings and readings in such a manner that its probable error is several times the probable error of a single setting and reading. Since we try to make the calibration to an accuracy as high as can be attained in the setting and reading of positions of the slide-wire contact such methods have not been used.

With suitable apparatus the Matthiessen and Hockin method is not only more convenient than the methods mentioned above but by its use the calibration corrections can be obtained to the

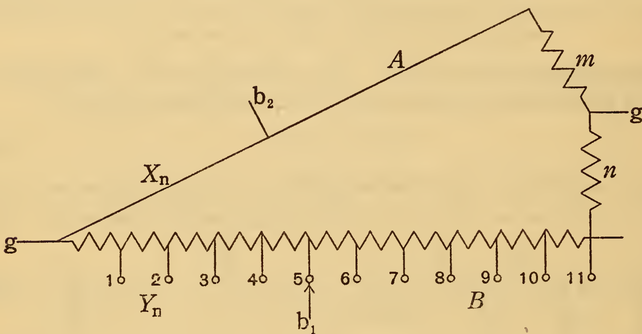

FIG. 13.—*Arrangement for calibrating slide wire by Matthiessen and Hockin method using coils of main dial*

accuracy attainable in setting and reading the positions of the sliding contact. The Matthiessen and Hockin method has therefore been regularly used in the calibration of the slide wire of potentiometers of the Crompton type. Connections as shown in Fig. 13 have been used in calibrating at 11 points the slide wires of Leeds and Northrup potentiometers. A simple Wheatstone bridge is made up of the first eleven 5-ohm coils, the slide wire and two resistances *m* and *n*. This bridge is balanced by the adjustment of the position of the slide-wire contact, to which one battery terminal is connected, with the other battery terminal connected successively to the different branch points between the 5-ohm coils.

Each of the balances gives

$$\frac{X_n}{Y_n}=\frac{A+m}{B+n}=\frac{A+m+X_n}{B+n+Y_n} \tag{29}$$

where X_n is the resistance of the slide wire between the zero end and the point at which the contact is set and Y_n is the resistance of the series of 5-ohm coils between the terminal g and the contact b_2. Since $A+m+X_n$ and $B+n+Y_n$ are constant

$$X_n=kY_n,\ X_1=kY_1,\ X_2=kY_2,\ X_3=kY_3,\ \text{etc.} \tag{30}$$

Since we know the relative values of the 5-ohm coils we can calculate the relative values of Y_1, Y_2, Y_3, etc. These are the relative values of the resistances of the slide wire corresponding to the various settings at which balances were made. By a preliminary adjustment of the resistances m and n these settings can be made to occur at points on the slide wire at which corrections are desired.

The calibration obtained by this method depends upon the accuracy of the setting and reading of the position of the slide wire contact and the accuracy to which the relative values of the 5-ohm coils are known. As the latter are regularly determined to an accuracy higher than the former can be made, the error in the calibration at any point at which a balance is made is approximately the error in the setting and reading at this point. In order to calibrate the slide wire at 22 points, it is only necessary to use 12 coils instead of 11 and arrange a 10-ohm resistance so that it can be made to shunt any two adjacent coils. By balancing the bridge, using the junction of the two coils as a branch point, the 5 ohms is halved, so that changes of 2½ ohms are obtained.

In the Leeds and Northrup potentiometer the standard cell is balanced across a resistance consisting of 10 of the 5-ohm coils in the main dial, an odd-valued coil and the coils in the standard cell dial. The measurement of the 5-ohm coils has been considered above. The odd-valued coil and the coils in the standard cell dial are measured by using the potentiometer connected, as shown in Fig. 14. The bridge so formed is balanced for all settings of the standard cell dial by adjusting the contact on the slide wire. A balance is also made with the battery connected to 15 instead of b_2. From the readings of the slide wire the corrections to the settings of the standard cell dial readings can be calculated,[16] since we also know the resistance between 14 and 15 and between 1 and the various points on the slide wire at which balances are made.

Since these measurements give either relative values of the resistances or their values in ohms the corrections can be calculated in much the same manner as described above for the Feussner type of potentiometer.

Provision is made for reducing the range of the Leeds and Northrup potentiometer by a shunt and a series resistance, such

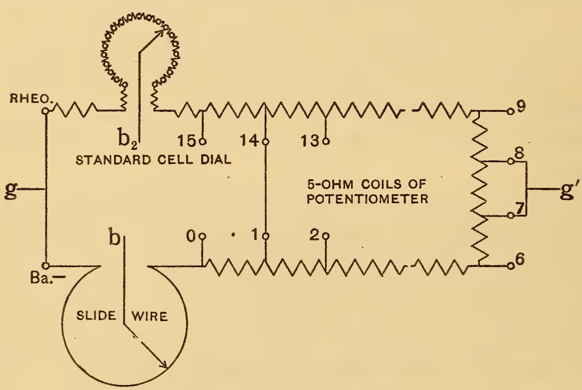

FIG. 14.—*Arrangement for calibrating standard cell dial by Matthiessen and Hockin method using some of the coils of the main dial*

that the current through the 5-ohm coils and slide wire is reduced to a tenth its normal value, while the total current drawn from the battery remains constant. Thus, if $\mathfrak{R}$ be the resistance of the potentiometer and $\mathfrak{S}$ that of the shunt, the reduction factor is $\mathfrak{S}/(\mathfrak{S}+\mathfrak{R})$. In order to determine the reduction factor, the

[16] In the calculation it is necessary to assume that the points 1 and 14 are at the same potential. These points are, therefore, connected together by a conductor of low resistance. Also one battery connection, *g*, is made at such a point on the connection (having a resistance of about 0.1 ohm) between the Rheo. and Ba—terminals as will bring the balance point on the slide wire very near the zero end when one terminal of the galvanometer is connected at 15 instead of b_2. In addition, the other battery connection ,g', is made to both points 7 and 8. With this arrangement, one step on the standard cell dial corresponds to one numbered division on the slide wire, but the corrections as determined depend upon several readings of the slide wire, among them some of those used in the calibration of the slide wire. The accuracy, therefore, is not as good as is sometimes desired.

ratio of $\mathfrak{R}$ to $\mathfrak{S}$ has been measured by a bridge method, and the factor calculated. In measuring this ratio extreme care must be taken, otherwise errors are introduced on account of connecting resistances. Where the reduction factor is nominally 0.1, $\mathfrak{R}/\mathfrak{S}$ should equal 1/9 and the compensation resistance 0.9 $\mathfrak{R}$. In this case the reading f may have a value of either 1 or 0.1. In stating the corrections the correction d is taken as zero for $f=1$, therefore for $f=0.1$,

$$d = 10\ \mathfrak{S}/(\mathfrak{R}+\mathfrak{S}) - 1 \tag{31}$$

In "split-circuit" or divided-circuit potentiometers, if provision is made for opening the parallel circuits, the relative values (or values in ohms) of the resistance sections may be determined by one or another of the methods described above. With the relative values known the corrections can be determined, though the calculations are often much more complicated than for potentiometers of the Feussner or Crompton type.

From what has been said it will be evident that the work involved in making the measurements and calculations necessary for determining the corrections to a potentiometer by any of the methods already considered is by no means small. As the Bureau of Standards is called upon to test a number of potentiometers each year it seemed desirable, in order to reduce the time required for making a test, to use a more direct method even though to do so might require the construction of special apparatus and some sacrifice as to accuracy. In any direct method the results obtained depend not only upon the measurements made but also directly upon the calibration of the apparatus used in making the test. Therefore, in general, the results can not be as accurate as those obtained by the methods discussed above.

4. RATIO-SET

A consideration of the matter led to the conclusion that the Matthiessen-Hockin method, with suitable apparatus, would give an accuracy sufficient for most of the tests, would result in a material saving of time in making the measurements, and could be made to give the data in such form that the calculation of the corrections would require but a small amount of time. It should

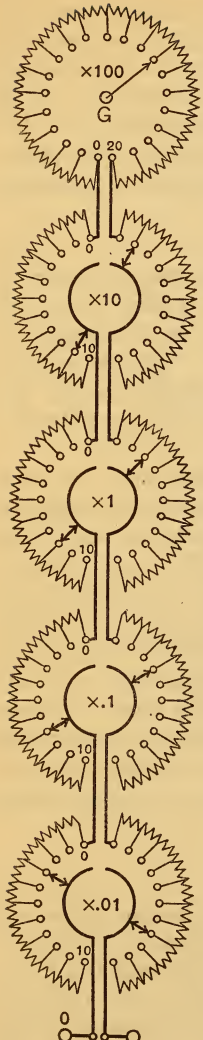

FIG. 15.—*Diagram of connections of ratio-set*

also be mentioned that according to this method a test can be made with a potentiometer under practically operating conditions. Accordingly, apparatus which we designate as a ratio-set was designed for use according to this method. The apparatus was constructed by O. Wolff, of Berlin, and delivered to the Bureau of Standards in the summer of 1912. It has been found very satisfactory not only in the testing of potentiometers but in general use in resistance measurements.

(A) DESCRIPTION

The ratio-set is made up of 100 resistance coils connected as shown in Fig. 15. It will be seen that it is equivalent to a long slide wire having a resistance of 2111.1 ohms and upon which contact can be made at intervals of 0.01 ohm. The resistance between the zero and the unmarked binding post is 2111.1 ohms and is independent of the settings of the dials while the resistance between the zero terminal and the galvanometer terminal (G) is variable in steps of 0.01 ohm from 0 to 2111.1 ohms. The resistance coils are made of manganin wire and excepting the 0.1 and 0.01-ohm sections are wound in a single layer on metal tubes. The 0.1 and 0.01-ohm sections, being made from short thick wires, require no special supports.

The four double dials are similar to the 10, 1, and 0.1-ohm dials of the Wolff potentiometer except that the usual screws for making independent connection to the terminal blocks are replaced by tapered holes. The 100-ohm dial is a simple one by means of which the galvanometer can be connected to either terminal of any 100-ohm coil. All of the dial switches are provided with clicking devices

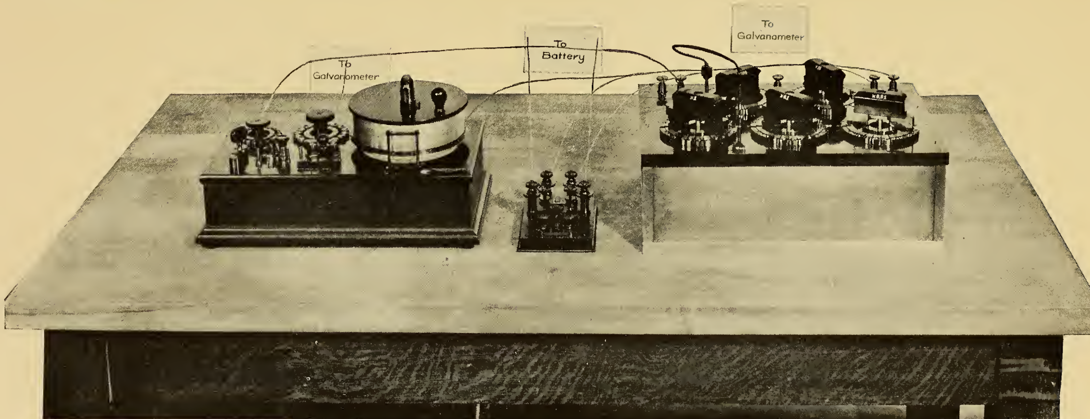

Fig. 16.—*Ratio-set connected to Leeds and Northrup potentiometer as used in testing the potentiometer*

to indicate the proper positions. In Fig. 16 the ratio-set is shown connected to a Leeds and Northrup potentiometer as it is used in calibrating the potentiometer.

(B) CALIBRATION

In order to obtain the complete calibration of the ratio-set the following measurements were necessary: (a) total resistance for all settings, (b) resistance between the zero and the galvanometer branch point for all settings, and (c) temperature coefficients of the coils.

The total resistance was measured by a simple Wheatstone bridge method and was found to be practically independent of the settings of the dial switches. The maximum variation was 0.0014 ohm or 7 parts in 10 000 000 of the total resistance. Variations due to changes in switch contact resistances amounted to 0.0001 ohm. The resistance between the zero and the galvanometer branch point for all settings was obtained by measuring it when the setting was zero and adding to this the measured changes produced on changing the switches. In the 100-ohm dial these changes are the actual resistances of the coils between branch points, but in the other dials the changes are the actual resistances between branch points plus any changes due to changes in the connections to the coils.

The 100-ohm coils were measured by the modified Carey-Foster method. The connections were made as shown in Fig. 3*a*. By this method the resistance between the post junctions is measured in terms of a standard resistance. This is the resistance required, since the posts are in the galvanometer circuit. The 10, 1, and 0.1-ohm coils were measured by a substitution method, using connections as shown in Fig. 17. With the switch as shown and the galvanometer connected to g_1, the bridge is balanced by adjusting the ratio A/B. The resistance in the left arm is $S+p+c$

where S is the resistance of the standard,

p is the resistance of the post,

and c is the resistance of the connections.

Now the switch is shifted to the dotted position, the standard resistance short-circuited by or interchanged with the link L, and the galvanometer changed to g_2. Suppose the ratio must be increased an amount r in proportional parts in order to restore the balance of the bridge. The resistance in the right arm is now

$$L+X+p'+c$$

where L is the resistance of the short-circuiting link,
p' is the resistance of the post,
X is the resistance of the coil,
and c is the connecting resistance assumed to be the same as before.

From the two balances of the bridge we have

$$(L+X+p'+c)=(S+p+c)\ (1+r) \tag{32}$$

and since L, p, and c are small in comparison with X and s, and r is small in comparison with unity,

$$X+p'-p=(S-L)\ (1+r) \tag{33}$$

approximately. Since $X+p'-p$ is the amount by which the resistance controlled by the switch is increased on changing the switch from g_2 to g_1, this method gives the resistance with which we are concerned in the use of the apparatus.

In the calibration an effort is made to get the corrections to 0.001 ohm or 0.1 step on the lowest dial. If care is taken, this accuracy can be obtained with a good quality bridge in measurements of 1 ohm and less. The 0.01-ohm coils (or sections) were and the 0.1-ohm coils might have been measured by connecting the center (or G) and 0-terminal to the X-terminals of the bridge, and measuring the resistances corresponding to all possible readings of the 0.01-ohm dial, with the other dials so set that none of the other resistances coils are included in the resistance measured. By this procedure also the measured resistances include any differences there may be in the resistances of the connections to the coils.

From the calibration of the ratio-set we get the resistance between the 0 and G posts (branch points to battery and galvanometer) for any reading of the ratio-set. However, as used it is only the relative values of these resistances that is required, so a unit is chosen which makes the corrections much smaller than the corrections necessary to give the values in ohms. For convenience in use a table has been constructed which gives the small correction which must be added to any reading to give the corresponding relative value of the resistance which we designate as A.

The temperature coefficients of the coils were obtained by repeating the calibration with the apparatus at another tempera-

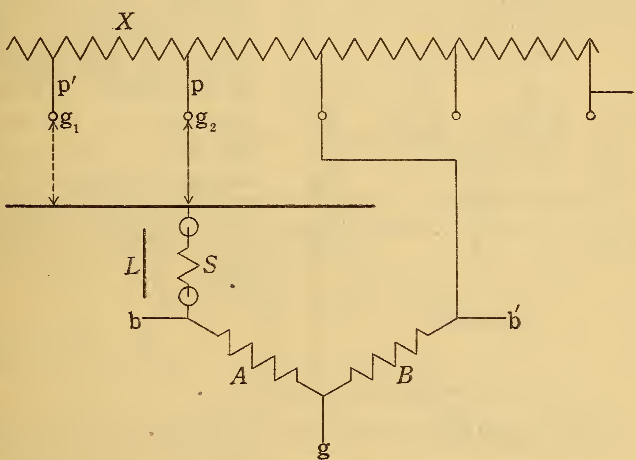

FIG. 17.—*Arrangement for measuring change in resistance corresponding to steps of a dial switch of the ratio-set*

ture. From the two measurements it was found that the temperature coefficients of the 100-ohm coils were very nearly equal, the greatest difference from the mean being 0.0002 per cent per degree centigrade. Since only relative resistances are required this would cause an error less than 0.000002 × 10 or 2 in 100 000 at a temperature differing 10° C. from that at which the corrections were determined.

(C) USE IN TESTING POTENTIOMETERS

To use the ratio-set in the calibration of a potentiometer connections are made as shown in Fig. 18. This gives us a Wheatstone bridge, two arms of which are formed by the ratio-set o G t and the other two by the potentiometer l m n o p q and its external resistance R. Suppose that the correction to the reading e is required, assuming that when the potentiometer is in use the known electromotive force is balanced when the reading was s. The potentiometer is set so that the reading is s and the bridge o o t G balanced by adjusting the dial switches of the ratio-set. Let the resistance A of the ratio-set be A_s. Now by balancing the bridge o p t G (i. e., with one galvanometer connection changed from o to p) we get A'_s. These give

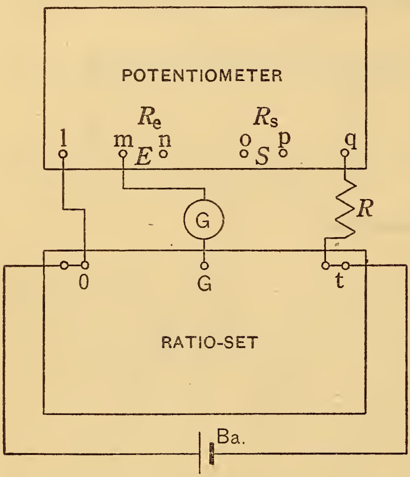

FIG. 18.—*Connections for calibrating any potentiometer by means of the ratio-set*

$$\frac{R_s}{A'_s - A_s} = \frac{1}{k} \tag{34}$$

where k is the ratio of the current through the potentiometer to that through the ratio-set. Let the potentiometer be set now so that its reading is e and two more balances made, first using o m t and G as branch points and then using o n t and G. Let the corresponding values of A be A_e and A'_e. Then

$$\frac{R_e}{A'_e - A_e} = \frac{1}{k'} \tag{35}$$

from which it follows that

$$\frac{R_e k'}{R_s k} = \frac{A'_e - A_e}{A'_s - A_s} \tag{36}$$

But $\frac{k'}{k}=\frac{I'}{I}$ of equation (2) since R is the external resistance as actually used. Therefore

$$\frac{E}{S}=\frac{A'_e-A_e}{A'_s-A_s} \tag{37}$$

from which the correction to the reading can be calculated if S is known. If the potentiometer is compensated $I'/I=1$ and is independent of the resistance R so that R may be chosen arbitrarily. It should be noticed that this is the Matthiessen-Hockin method of comparing R_e with R_s.

From equations (11) and (37) we get that

$$\frac{E}{S}=\frac{f}{s}(1+b+d)(e+a)=\frac{A'_e-A_e}{A'_s-A_s} \tag{38}$$

Since we wish to have b zero for $s=1.0185$ and d zero for the normal range ($f=f_0$) we get by substitution

$$\frac{f_0(e+a)}{1.0185}=\frac{A'_e-A_e}{A'_{1.0185}-A_{1.0185}}$$

from which

$$a=\frac{1}{f_0}\frac{(A'_e-A_e)1.0185}{(A'_{1.0185}-A_{1.0185})}-e \tag{39}$$

By substituting this in equation (38) we get when we put $f=f_0$ and $d=0$

$$b=1-\frac{1.0185}{(A'_{1.0185}-A_{1.0185})}\frac{A'_s-A_s}{s} \tag{40}$$

The correction d can be determined for any range by setting $e=e'$ any convenient large value, $s=1.0185$ and $f=$the reading of the range switch for which the correction is to be determined and making four more balances. These give the four resistances $\bar{A}_{e'}$, $\bar{A}'_{e'}$, $\bar{A}_{1.0185}$ and $\bar{A}'_{1.0185}$ and by substitution in (38) we obtain

$$\frac{f}{1.0185}(1+d)(e'+a')=\frac{\bar{A}'_{e'}-\bar{A}_{e'}}{\bar{A}'_{1.0185}-\bar{A}_{1.0185}} \tag{41}$$

Now from equation (39) we see that

$$e'+a'=\frac{1}{f_0}\frac{(A'_{e'}-A_{e'})1.0185}{(A'_{1.0185}-A_{1.0185})} \tag{42}$$

which when substituted in (41) gives

$$d=\frac{(\bar{A}'_{e'}-\bar{A}_{e'})}{(A'_{e'}-A_{e'})}\frac{(A'_{1.0185}-A_{1.0185})}{(\bar{A}'_{1.0185}-\bar{A}_{1.0185})}\frac{f_0}{f}-1 \tag{43}$$

Equations (39), (40), and (43) give all of the corrections to the potentiometer in terms of the resistances in the ratio-set which are known from its readings and its corrections.

The calibration of potentiometers of the Crompton type is comparatively easy. The total resistance is constant, the dials are independent, and, further, the resistance between one end of the potentiometer and any potential terminal is dependent on the setting of only one dial. Therefore the total number of balances necessary is of the same order as the total number of points on the dial and points on the slide wire for which corrections are to be determined. From the preceding discussion it will be seen that if

$$A'_{1.0185}-A_{1.0185}=u(1.0185) \tag{44}$$

where u is approximately a decimal multiple or submultiple of ten the calculations are simplified. In case of the Leeds and Northrup potentiometer it is convenient to make $u=1000$. This gives

$$a=\frac{A'_e-A_e}{1000}-e \tag{45}$$

$$b=s-\frac{A'_s-A_s}{1000}\text{ approximately} \tag{46}$$

and [17]

$$d=\frac{\bar{A}'_{e'}-\bar{A}_{e'}}{A'_{e'}-A_{e'}}\frac{1}{f}-1 \tag{47}$$

On the data sheet is given a complete record of the calibration of a Leeds and Northrup potentiometer for all readings of the standard

[17] Equation (47) differs from equation (43), since in the use of the potentiometer with $f=0.1$ (or 0.01) the standard cell is regularly balanced with $f=1$, so that $A'_{1.0185}-A_{1.0185}=\bar{A}'_{1.0185}-\bar{A}_{1.0185}$.

DATA SHEET FOR CALIBRATION OF LEEDS AND NORTHRUP POTENTIOMETER No. 6082

	Pot'meter reading	Ratio set readings at—		Relative resistances			Correction
	s	o	p	A_s	A'_s	A'_s-A_s	b
STANDARD CELL DIAL	1.0185	80.08	1098.58	80.80	1098.58	1018.50	±0.00000
	1.0175	80.98		80.98		1017.60	−.00010
	6	.89		.89		.69	−.00009
	7	.80		.80		.78	− 8
	8	.70		.70		.88	− 8
	9	.61		.61		.97	− 7
	1.0180	.52		.52		1018.06	− 6
	1	.43		.43		.15	− 5
	2	.34		.34		.24	− 4
	3	.25		.25		.33	− 3
	4	.17		.17		.41	− 1
	5	.08		.08		1018.50	± .00000
	6	79.98		79.98		.60	± 0
	7	.88		.88		.70	± 0
	8	.78		.78		.80	± 0
	9	.69		.69		.89	+ 1
	1.0190	.60		.60		.98	+ 2
	1	.50		.50		1019.08	+ 2
	2	.41		.41		.17	+ 3
	3	.32		.32		.26	+ 4
	4	.22	1098.58	.22	1098.58	.36	+ 4
	e	m	n	A_e	A'_e	A'_e-A_e	a
MAIN DIAL	0.00000	1598.56	1598.58	1598.57	1598.59	0.02	+0.00002
	.1	1498.58		1498.58	1598.59	100.01	+ 1
	.2	1398.59		1398.59		200.00	± 0
	.3	1298.59		1298.59		300.00	± 0
	.4	1198.58		1198.58		400.01	+ 1
	.5	1098.58		1098.58		500.01	+ 1
	.6	998.57		998.58		600.01	+ 1
	.7	898.57		898.57		700.02	+ 2
	.8	798.57		798.57		800.02	+ 2
	.9	698.56		698.56		900.03	+ 3
	1.0	598.55		598.55		1000.04	+ 4
	1.1	498.54		498.54		1100.05	+ 5
	1.2	398.53		398.53		1200.06	+ 6
	1.3	298.52		298.52		1300.07	+ 7
	1.4	198.50		198.50		1400.09	+ 9
	1.5	98.48	1598.58	98.49	1598.59	1500.10	+ 10
	s	o	p	A_s	A'_s	A'_s-A_s	b
SLIDE WIRE	1.0185	80.08	1098.58	80.08	1098.58	1018.50	+0.00000
	e	m	n	A_e	A'_e	A'_e-A_e	a
	0.00000	1598.56	1598.58	1598.57	1598.59	0.02	+0.00002
	.01	1598.56	1608.57	1598.57	1608.57	10.00	± 0
	.02		1618.57		1618.57	20.00	± 0
	.03		1628.59		1628.59	30.02	+ 2
	.04		1638.61		1638.61	40.04	+ 4
	05		1648.61		1648.61	50.04	+ 4
	.06		1658.59		1658.59	60.02	+ 2
	.07		1668.60		1668.60	70.03	+ 3
	.08		1678.61		1678.61	80.04	+ 4
	.09		1688.62		1688.63	90.06	+ 6
	.10	1598.56	1698.60	1598.57	1698.61	100.04	+ 4

Factor plug changed, f=.1

e'	m	n	A_e'	$A'_e{}'$	$A'_e{}'-A_e'$	d
1.50000	1538.55_6	1688.63_0	1538.55_9	1688.63_6	150.07_7	$+0.0004_5$

Date, June 2, 1914. Temperature, 26.4° C.

cell dial, all readings of the main dial, for 11 points on the slide wire and for both readings of the range plug. The connections used in making this test were as shown in Fig. 19. To make $u=1000$ four of the 100-ohm sections of the ratio-set were short-circuited and a resistance of about 0.555 ohm placed in series with the potentiometer. This series resistance was easily adjusted so as to make $u=1000.00$. The measurements then consisted in setting the switches of the ratio-set so as to balance the bridge with the potentiometer set at the readings for which corrections were desired. The calculations indicated by equations (45), (46), and (47) are of such a simple nature that the corrections were obtained

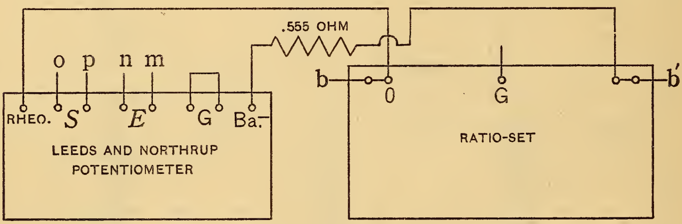

FIG. 19.—*Ratio-set connected to Leeds and Northrup potentiometer for calibrating the potentiometer. The arrangement is such that the corrections are obtained from the readings of the ratio-set with but very little calculation*

by inspection, comparing the readings with the corresponding relative resistances.

It remains to divide the reading e and the correction a in two parts as pointed out on page 8. When this is done we have the values of a_1, b, and d for all readings e_1, s, and f; and the values of a_2 for 11 readings e_2. The values of a_2 for other readings e_2 may be obtained by interpolation. We then have all the correction terms of equation (16) except c which is known from the reading s and the known electromotive force of the standard cell.

In the calibration of the Wolff potentiometer, the total resistance is first measured at the various settings of the dial switches to ascertain whether the potentiometer is compensated. This is done by short-circuiting the 100 and 1000-ohm coils and measuring the resistance of the three lower dials in a Wheatstone bridge.

As the resistance measured is less than 1 per cent of the total resistance the accuracy required in this measurement is not high. The calibration is then carried out in much the same way as for the Leeds and Northrup potentiometer. The connections used are shown in Fig. 20. In this case to obtain the corrections most directly from the readings we make $A'_{1.0185} - A_{1.0185} = 1018.5$, and since $R_{1.0185} = 10\ 185$ the ratio of resistances is 1 to 10, approximately. To make the currents have the desired ratio, auxiliary resistances of 290 and 821 ohms are used as shown. The observations are

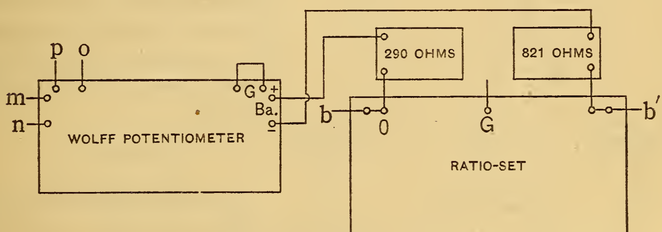

FIG. 20.—*Ratio-set connected to Wolff potentiometer. The arrangement is such that the corrections to the potentiometer may be obtained from the readings of the ratio-set by inspection*

made and the results tabulated in the same manner as was done for the Leeds and Northrup potentiometer.

To calibrate one of the older Wolff potentiometers which does not have a standard cell dial the procedure is approximately the same as that just given, fewer observations being necessary in this case. Here b, the correction to the standard cell dial reading, is calculated from the corrections to the reading e by use of the relation

$$b = -\frac{a_s}{e_s} \tag{48}$$

shown in the discussion of the theory of the potentiometer (equation 18). The arrangement for testing of other kinds of potentiometers readily suggest themselves, and it will be seen that any normal-range potentiometer can be completely calibrated by means of the ratio-set if the maximum resistance in the potentiometer between two potential terminals is less than twice the resistance across which the known electromotive force is balanced. By a normal-range potentiometer is meant one in which the steps on the first dial correspond to tenths of a volt. In these cases 100 ohms of the ratio-set can be made to correspond to one step of the first dial, and since there is a hundredth-ohm dial, the reading can be made directly to 0.00001 volt, which is sufficient for most purposes. However, with a good galvanometer, one can interpolate to 0.001 ohm, the ratio-set having been calibrated to this accuracy.

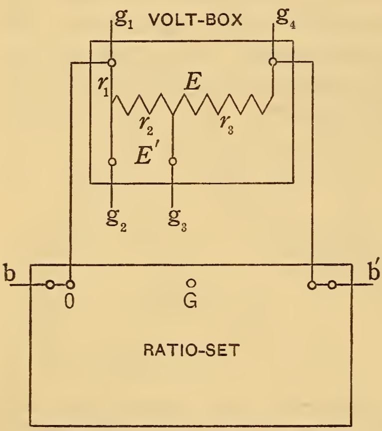

FIG. 21.—*Connections for determining factor (or ratio) of volt-box or potential-divider by means of ratio-set*

(D) OTHER USES

Volt boxes or potential dividers are used in connection with potentiometers for measuring electromotive forces or voltages above 1½ or 2 volts. In use the terminals (see Fig. 21) g_1 and g_4 are connected to the voltage E to be measured while the g_2 and g_3 are connected to the E-terminals of the potentiometer. If E' is the voltage across the E-terminals of the potentiometer and $f\ (1+d)$ is the factor of the volt box

$$E = E'f\ (1+d). \tag{49}$$

From the figure it will be evident that

$$f(1+d) = \frac{r_1+r_2+r_3}{r_2} \tag{50}$$

or

$$d = \frac{A_4 - A_1}{f(A_3 - A_2)} - 1 \tag{51}$$

where f is the nominal value or reading of the volt-box factor and

A_1, A_2, A_3, and A_4 are relative resistances of the ratio-set when the bridge is balanced successively with the galvanometer connected to g_1, g_2, g_3, and g_4.

The ratio-set is very useful in measuring resistances to which connections can not be made except through other resistances.

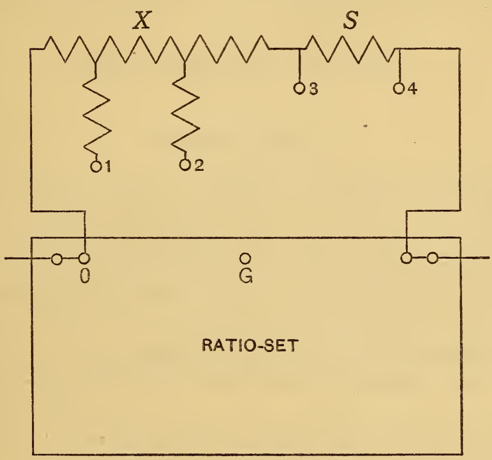

FIG. 22.—*Arrangement for measuring resistances to which connection can be made only through other resistances or resistance of four-terminal conductor by means of ratio-set*

The connections are made as shown in Fig. 22. By balancing at the four potential terminals we get the relation

$$X = \frac{A_2 - A_1}{A_4 - A_3} S, \tag{52}$$

which gives the unknown resistance in terms of the ratio of the differences in readings of the ratio-set and the value of the known resistance, S.

5. SUMMARY

1. An analysis is made of the relation between the readings of any potentiometer and the ratio of the known and unknown electromotive force.

2. A convenient way of stating the corrections to the readings of a potentiometer of good design and construction is given.

3. Methods of determining these corrections by resistance measurements are described.

4. A device for use in the calibration of potentiometers is described and the method of using it explained and illustrated.

WASHINGTON, June 10, 1914.