HIGH-FREQUENCY AMMETERS

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CONTENTS

I. INTRODUCTION

1. SCOPE OF INVESTIGATION

Instruments for the measurement of current of radiotelegraphic frequencies, about 100 000 to 2 000 000 cycles per second, generally depend upon the thermal effect of the current. The electrodynamic effect of the current has not been very successfully utilized; because when the wire is coiled up to form an electrodynamometer, conditions are favorable (impedance large and capacity large) for part of the current to flow through the dielectric instead of the wire, in amount varying with the frequency. An electrodynamic instrument which is useful for some purposes utilizes the eddy currents in a conducting ring suspended within a coil [1] which carries the current to be measured; however, the deflection varies with frequency, so that the instrument can not be considered in the usual sense an ammeter. The thermal effect, on the other hand, can readily be measured in a simple straight wire as well as in any other form of circuit, so that a suitable form of circuit is most readily attained in the thermal ammeter. The heat production may be measured by any thermometric method, and the following are in use: Expansion, calorimetry, change of electric resistance, and thermoelectric effect.

Observation of the heat production in a simple single wire is by no means a complete solution of the high-frequency ammeter problem. Consider the equation:

$$H = RI^2, \tag{1}$$

in which H is the rate of heat production, R is the resistance, and I is the effective current. The indicated current in any thermal ammeter depends upon H. It is well known that the resistance of any conductor increases as the frequency of the current increases, hence in general the indicated current will vary with frequency. For this change to be negligible, the conductor must be of smaller and smaller cross section as the frequency is increased. For the frequencies under consideration, this error will be greater than 1 per cent unless the diameter of the wire is less than a few tenths of a millimeter. Still there is nothing serious about this if only

[1] J. A. Fleming: "The principles of electric wave telegraphy," p. 201; 1910. G. W. Pierce: Phys. Rev. 20, p. 226; 1905.

small currents are to be measured. But when larger currents than about 3 amperes have to be measured, the single wire will not suffice; the wire must be of so small a diameter that its resistance is not changed by frequency, but then the large currents will overheat it, i. e., alter its properties. To measure large high-frequency currents, therefore, the current must have more than one path, and it is common to use (*a*), wires in parallel, (*b*), different portions of the same wire in parallel, or, (*c*), broad, thin strips of metal.

It is a common dictum that ammeters used in measuring currents of even moderately low frequency must not be shunted. This is obvious from consideration of the expression relating the current I_1 in the instrument and I_2 in the shunt, neglecting mutual inductance,

$$\frac{I_1^2}{I_2^2}=\frac{R_2^2+p^2L_2^2}{R_1^2+p^2L_1^2},$$

since usually the inductances of the instrument and the shunt are in a different ratio from that of the resistances, and hence the distribution of the current varies with frequency. In fact, even if $\frac{L_2}{L_1}=\frac{R_2}{R_1}$, the current distribution in an actual case is likely to vary with frequency because of the mutual inductance. Now, any high-frequency ammeter in which the circuit within the ammeter itself consists of more than a single elementary filament—or its closest approximation, a fine wire—in reality involves shunting, and needs most careful consideration before it can be pronounced free from error. When the indicated current depends on the heat production in just one branch of such an instrument the error due to change of current distribution on high frequencies may be very great. When the indicated current depends on the heat production of the whole current, the error will be of a smaller order of magnitude, but may be appreciable when the change of current distribution is great, inasmuch as the total heat production in any system increases as the distribution on direct current is departed from. The increase of total heat production is equivalent to an increase of R, the resistance of the system as a whole, in equation (1).

It has been known for some time that most of the ammeters in common use for measuring large high-frequency currents are subject to large errors. Users of the ammeters have put a number of them in series in high-frequency circuits and have been puzzled at the divergence of their readings. Various sources of error have been suspected, namely, change of current distribution, resistance change, eddy currents, capacity, and inductive action from other portions of the circuit. It was the purpose of this investigation to sift out these possible errors and to determine the magnitudes of the effects that need to be considered in high-frequency current measurement. This has involved a critical study, experimental and theoretical, of typical ammeters at various frequencies. The investigation includes more than what is usually connoted by the term "hot-wire" ammeter, viz, the commercial expansion instrument. It includes all the instruments whose indications depend on the thermal effect of the current, some of which are capable of high precision.

2. NEED FOR THIS WORK

There is great practical necessity for reliable ammeters in high-frequency work. A hot-wire ammeter is found in every radio station, and in some stations high-frequency currents of 300 amperes are used. The Government inspectors charged with enforcing the laws regulating power radiation must have portable ammeters upon which they can rely. The measurement of current is far more of a cardinal operation at high frequency than on direct current, for there is no potentiometer for precise measurement of high-frequency electromotive force and no Wheatstone bridge for precise measurement of high-frequency resistance. The design of resistances to carry large high-freqeuncy currents involves the same principles as the design of the circuits in ammeters. In the present state of high-frequency measurements great accuracy is not required, but it is at least desired to have current measurement correct to 1 per cent.

3. PLAN OF TREATMENT

The apparatus used and the experimental procedure will first be described, and then the three main types of ammeter will be treated in order without reference to experimental details. The means of measuring the heat production in the ammeter is a thermometric problem, affecting the precision and not necessarily the accuracy of the instrument; this investigation deals primarily with the electrical problem, or the form of the circuit within the ammeter. The experimental results obtained will be presented along with the theoretical discussion of each type. This is done because the main interest of the work lies not in the particular results here obtained, but in their illustration of the principles which govern the performance of high-frequency ammeters.

II. EXPERIMENTAL PROCEDURE

The general method of experiment was to observe simultaneously the thermal effects in the ammeter under test and in an ammeter which could be considered standard, the two being in series, using alternately high-frequency and low-frequency current. Any low frequency was equivalent to direct current, because the effects under investigation became appreciable only at very high frequencies, as will be shown.

1. CIRCUITS

The high-frequency current was produced by the oscillatory discharge of condensers; and the low-frequency current was

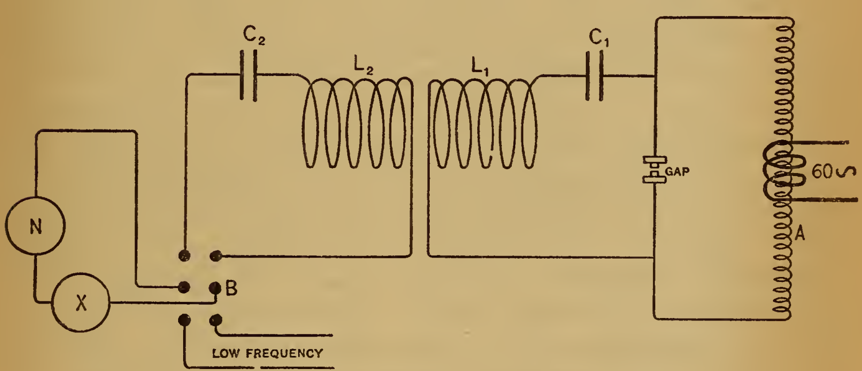

Fig. 1.—*Diagram of apparatus*

obtained from an ordinary generator. The condensers were of the Leyden jar type, and the inductances were the open-coil type used in radiotelegraphic work. A diagram of the apparatus is given in Fig. 1. A is an induction coil, supplied with 60-cycle alternating current; it is, in other words, a transformer for high voltages. The spark gap was either a rotating gap giving about

500 sparks per second, or a stationary gap with an air blast, using as electrodes small tungsten disks secured in copper and aluminum disks which helped to dissipate the heat. Both forms of spark gap gave sufficiently constant current. In Fig. 1, B is a double-throw switch, enabling either the high-frequency or low-frequency source of current to be connected to the ammeters X and N, one of which was under test and the other was the standard.

Experimenters[2] upon high-frequency resistances have used what purported to be a null method by balancing the thermal effect of X against the thermal effect in a similar auxiliary specimen, X′, carrying direct current. This additional experimental complication did not seem worth while in these experiments, since the observations in any case depend on a deflection, that of N. (To be sure, it could be made more nearly a null method by balancing the thermal effect of N also against an N′ carrying direct current, but this would involve still further experimental complication.) In addition, the thermal effect of X in some cases can not be readily balanced, e. g., in an expansion type commercial hot-wire ammeter.

A secondary circuit (L_2, C_2) was used because it is in similar secondary circuits that high-frequency current is usually measured. The frequency was measured by means of a wavemeter brought near the secondary circuit. The frequencies used were from 100 000 to 1 500 000, because this is the range of frequencies in radiotelegraphy; and furthermore, because it was found that the large and interesting changes of current distribution occur in this range, owing to the inductance becoming a determining factor in the impedance. It was found impracticable to go to higher frequencies, because at 1 500 000 capacity effects began to be appreciable; part of the current went through the dielectric instead of through the wire, so that two instruments in series did not carry the same current. The experiments on this point are given below.

2. STANDARD INSTRUMENTS

In considering the ammeters herein investigated, it must be borne in mind that the requirements are very different from those in the measurement of small currents, in which sensibility is the

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[2] J. A. Fleming: Electrician, 64, p. 381: 1909. H. W. Edwards: Phys. Rev., 33, p. 184; 1911.

desideratum. In an instrument for measuring very large currents the problem is, on the other hand, to dissipate the heat produced. The measurement of the heat production presents little difficulty, and in the instruments investigated the thermometric methods employed were expansion, electric resistance, and thermoelectric effect. The expansion of the hot wire or strip was utilized in the commercial ammeters. The electric resistance of the hot wire or strip can be conveniently utilized only in a particular form of instrument, as will be shown. The thermoelectric effect was advantageously employed to measure the current distribution among the parts of experimental ammeters.

For currents up to about 2 amperes a single fine wire served as the circuit of a standard instrument. The wire had to be fine enough so that its resistance was the same at the high frequencies as at very low frequencies. The work proceeds on the

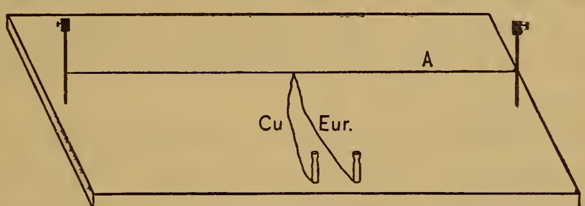

Fig. 2.—*Standard instrument of one wire*

assumption that such an instrument is correct at all frequencies. Two such instruments, one having a copper wire and the other a Eureka [3] wire, were compared and found to agree at all frequencies; this, together with the consistency of the results obtained throughout the work, and the quantitative agreement of the effects experimentally obtained with the theoretical calculations, furnish the justification of the assumption. A one-wire instrument, using a thermocouple to measure the heat production, is shown in Fig. 2. A is the fine wire which constitutes the essential part of the instrument. It is soldered to vertical copper rods, mounted on a wooden or hard-rubber base. The current enters the "hot wire" A through these vertical rods at right angles to it. A copper Eureka thermocouple is hard soldered to the middle point of A and connected to the binding posts and

[3] Eureka, "Advance," "Ia Ia," and constantan, all have practically the same properties.

thence to a galvanometer. A cover over the whole instrument protected the hot wire and the thermocouple wires from air currents.

For the measurement of relatively small currents such an instrument was used in which A was a Eureka wire 0.05 mm in diameter. It was suitable for the measurement of current up to about 0.3 ampere, at which the temperature of A was about 75° C and the emf in the thermocouple circuit was about 3 millivolts. In a Eureka wire of that size the increase of resistance for a frequency of 1 000 000 over the direct-current resistance is readily shown by Kelvin's formula [4] to be exceedingly small, less than 0.001 per cent.

For the measurement of currents up to 1.2 amperes a similar instruments was used in which the wire A was copper, 0.08 mm in diameter. Because of its lower resistivity it could carry more current than the Eureka wire for a given amount of heating, but its change of resistance with frequency is greater. However, the increment [4] of resistance at a frequency of 1 000 000 is found to be less than 0.3 per cent.

To measure currents greater than 1.2 amperes, up to 10 amperes, a two-wire instrument was used as standard. As will be shown theoretically in Section III and experimentally in the fourth paragraph below, two wires in parallel constitute as valid an instrument as one wire, provided the two wires are exactly similar in length, cross section, resistance, and configuration, and are fine enough not to change in resistance with the frequency. A system of more than two wires in parallel, however, is likely to be in error at high frequencies. Even in the case of two wires there is difficulty because of the requirement of exact similarity of the two wires. Wires of very small diameter vary considerably in cross section and hardness, and hence two wires which have apparently the same dimensions and consequently the same self-inductances may have resistances several per cent different. This is a most insidious source of error and is found to affect a number of the instruments investigated in this paper. Because of this possible difference of resistance the current distribution in the two-wire instrument is likely to vary with the frequency,

[4] Kelvin: Math. and Phys. Papers, vol. 3, p. 491; 1889. Bull. Bur. Standards, 8, p. 174; 1911.

and hence the indications of a thermocouple on one of the wires would not give true current values. If, however, the heat production in the whole instrument be measured, instead of the heat production in one wire only, the instrument is practically free from error; since, as will be shown in Section IV, the change of current distribution is a first-order change, while the change of the total heat production is a second-order change.

The total heat production in the two-wire instrument is readily measured by the rise of its electrical resistance when heated by the passage of current. An instrument on this plan was made as follows: Four uprights, bearing copper lead wires, were set in a wooden base at the corners of a rhombus about 10 cm on a side. At points about 3 cm above the base a fine copper wire was soldered as shown. The heating current was introduced

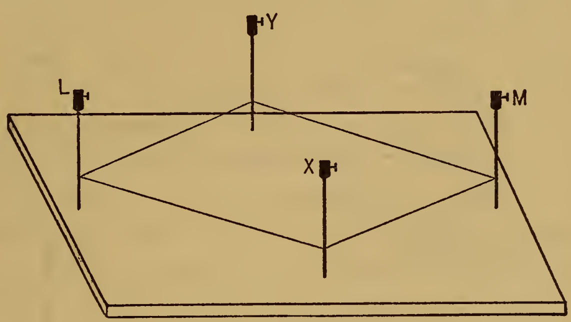

Fig. 3.—*Standard instrument of two wires*

by one pair of leads, say L and M, and the resistance was measured between X and Y, these points being connected to a Wheatstone bridge. The fine copper wire was 0.1 mm in diameter. With this it was found possible to measure currents up to 10 amperes by placing the instrument in oil. This method of measuring alternating currents is due to Paalzow and Rubens,[5] and is sometimes called a "bolometer-bridge" method. It has been used[6] as a detector of very small currents of high frequency, making its sensibility very high by putting it in a vacuum. Here current-carrying ability rather than sensibility was desired. This method as a means of measuring large currents of high frequency is believed to be novel.

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[5] Wied. Ann., 37, p. 529; 1889.
[6] Tissot (see Fleming: "The Principles of Electric Wave Telegraphy," p. 200; 1910.)

The arrangement for measuring the resistance of the device is shown diagrammatically in Fig. 4. K is a tapping key in the battery circuit. A closed galvanometer circuit was used, thus eliminating errors of false zero. G is a sensitive moving-coil galvanometer. Its sensitivity was such as to permit the 400-ohm bridge arm to be adjusted closer than the limit of precision set by the fluctuations of the high-frequency current. The current through the standard, X Y L M, from the bridge battery was of the order of 0.01 of the heating current, and therefore quite negligible. It was not found convenient to calibrate this standard on direct current, although it is theoretically possible to do so. For

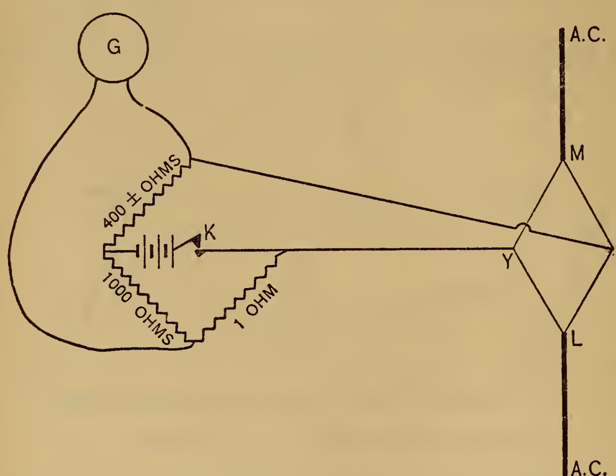

Fig. 4.—*Apparatus auxiliary to the two-wire standard*

a heating current entering at L and M, X and Y need to be so adjusted as to be equipotential points; then no portion of the heating current flows in the bridge used to measure the resistance. However, it is difficult to make this adjustment exactly, and it is moreover unnecessary, as a calibration by low-frequency alternating current is just as good as a direct-current calibration for the purposes of this work. Consequently, the points X and Y were simply made approximately equipotential points, but not adjustable. As seen in Fig. 3, the four leads were perpendicular to the plane of the hot wire, so that there could be no inductive effects between the leads and the hot wire. Some light on the possibility of inductive dissimilarities at high frequency could be obtained by interchanging the heating current from the terminals L and M to the terminals X and Y. This was done at the highest frequency for which the instrument was used, 750 000, and no differences could be detected in the results.

Measurement of electric resistance of the heated system is a thermometric method peculiarly applicable to the two-wire instrument. It is readily seen that it can not be conveniently used for systems of other numbers of wires, because of the necessity of keeping the high-frequency current out of the bridge used to measure the resistance.

There is plenty of evidence among the results of this investigation that the thermometric methods used for measuring the heat production do not affect the behavior of the instruments with change of frequency. If certain electrical conditions are satisfied, two instruments agree as well at one frequency as another. For example, the two-wire resistance instrument just described was compared with a one-wire instrument in which the indications were produced by the expansion of the hot wire, at a very low frequency (equivalent to direct current), at 300 000 and at 750 000. As shown by the plotted results, Fig. 5, the two instruments agree throughout. Thus, an answer is obtained to any possible objection to the resistance instrument arising from questionable constancy of the resistance-temperature coefficient of a wire carrying oscillating currents of high frequency (the electrons reversing their direction of drift with great rapidity).

3. TAKING THE OBSERVATIONS

In comparing any two ammeters the experimental procedure was to pass high-frequency current through the two in series for a certain length of time, usually one minute, recording the deflections, then quickly to throw the switch (Fig. 1) from high frequency to low frequency, and allow an approximately equal low-frequency current to flow one minute, recording the deflections; then high frequency again, then low frequency again, and finally high frequency again. Thus, three high-frequency observations were obtained, with two low-frequency observations sandwiched between them. This method of alternating the observations, with the currents run successively for equal intervals of time, eliminated any errors due to thermal or other drifts. A sample set of observations is given in Table 1; the deflections for the

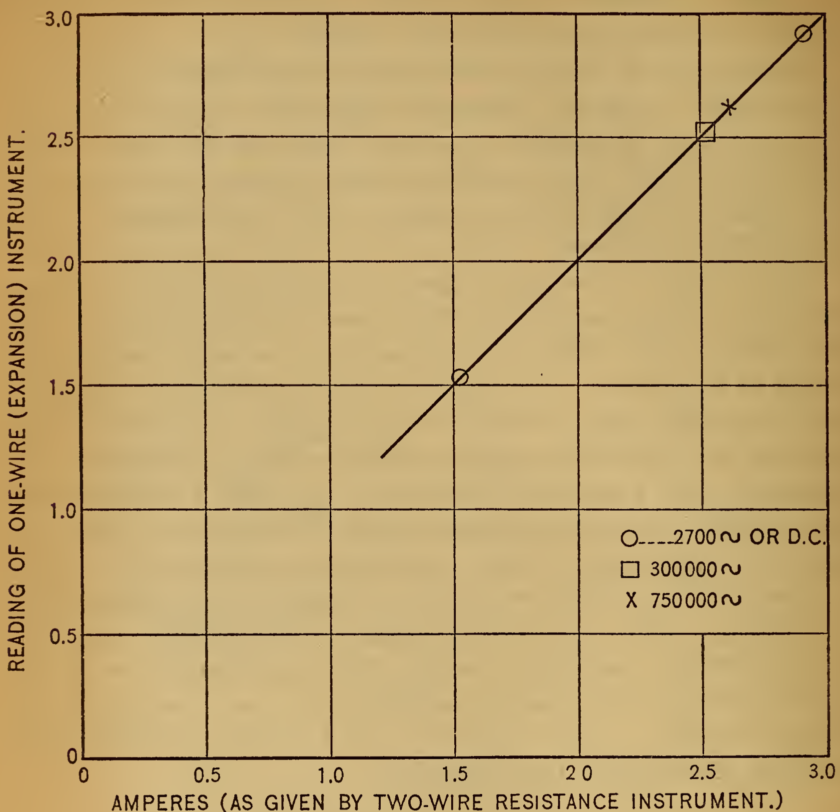

Fig. 5.—*Comparison of two standard instruments*

"three-wire instrument" being those of a suspended-coil galvanometer, read with lamp and scale, and the deflections of "Standard B" being those of a sensitive pointer-type galvanometer; the galvanometer in both cases measuring the thermocouple current.

TABLE 1

Frequency	Three-wire instrument	Standard B
1 500 000	38.0	30.4
60	39.1	28.0
1 500 000	38.2	30.4
60	39.1	27.8
1 500 000	38.9	31.2
Mean 1 500 000	38.37	30.67
Mean 60	39.10	27.90

In some cases, with more than one observer, three instruments in series were simultaneously observed. Instead of estimating the average deflection while the current was on, each observer took the instantaneous deflection and recorded it as many times as he could during the time. The mean deflection is then found arithmetically instead of mentally. A set of such observations is given in Table 2; the observations given under "two-wire standard" are the resistances in the variable arm of the measuring Wheatstone bridge; and those under the other two instruments are the currents indicated on their scales. The data give a good idea of the degree of constancy of the current.

The calibrations of each instrument with respect to a standard instrument, and of the experimental standards with respect to a laboratory standard ammeter, at a low frequency, were plotted. By interpolation from these curves, and from the means obtained from any set of observations, as at the end of Table 2, the high-frequency deflections of the instruments under test corresponding to the low-frequency deflections, for equal current in the standard instrument, are obtained. The results may be plotted as performance curves, or they give at once $\frac{I_a}{(I_a)_0}$, the ratio of the current indicated at high frequency to the current indicated at low frequency or direct current, with the same total current in the circuit. Suppose the instrument under test is one composed of several wires in parallel with a thermocouple on the wire *a*. Suppose

TABLE 2

Frequency	Two-wire standard	"Unshunted" ammeter	Seven-wire ammeter
	398.6	+0.03	−0.03
2700	461.2	6.48	6.15
	460.5	6.49	6.14
	460.0	6.43	6.10
		6.45	6.15
	Mean 460.6	6.46	6.14
750 000	459.4	6.58	8.85
	457.7	6.60	8.80
	457.2	6.50	8.80
		6.58	8.75
		6.50	
	Mean 458.4	6.55	8.80
2700	460.9	6.48	6.15
	460.3	6.49	6.14
	460.3	6.46	6.15
	460.0	6.49	6.15
	460.2	6.48	6.18
	Mean 460.3	6.48	6.15
750 000	458.0	6.41	8.80
	457.0	6.55	8.75
	457.0	6.52	8.75
	457.3	6.53	8.70
		6.55	8.80
	Mean 457.3	6.51	8.76
2700	460.3	6.48	6.18
	459.8	6.52	6.18
	459.8	6.48	6.18
		6.50	6.20
		6.50	6.19
		6.57	
	Mean 459.9	6.50	6.19
	393.0	+0.05	−0.01
Mean 2700	460.27	6.48	6.16
Mean 750 000	457.85	6.53	8.78

that the per cent change, k, of current in the wire a can be calculated for any desired frequency from the inductances, etc., as in Section III below. Then, using I_a for the current in the wire a, and I for the whole current in the circuit, and the subscript $_o$ to denote direct-current or low-frequency values, the theoretically predicted relation is

$$\frac{I_a}{I}=\frac{(I_a)_o}{(I)_o}(1+k)$$

For the same total current on high frequency as on low frequency, i. e., for $I=(I)_o$,

$$\frac{I_a}{(I_a)_o}=1+k \tag{2}$$

Now, coming to the experimental performance of the instrument, the indicating device is such that

$$\text{Indicated } I=\text{constant}\times I_a \tag{3}$$

or,

$$\frac{\text{Indicated } I}{(\text{Indicated } I)_o}=\frac{I_a}{(I_a)_o},$$

which, by equation (2), $=1+k$. That is,

$$\frac{\text{Indicated } I-(\text{Indicated } I)_o}{(\text{Indicated } I)_o}=k \tag{4}$$

Computation from the experimental results is made according to equation (4), thus obtaining by experiment a quantity also theoretically calculable. This reasoning is subject to the limitation that equation (3) may not be exactly fulfilled when the current distribution changes and the wires are not all at the same temperature. Thus the wire in which the current is measured may be influenced by convection and radiation from its neighbors; this will always tend to equalize the temperatures and reduce the apparent change of current distribution. The effect of convection is more variable when the measurement is made thermoelectrically, being determined by the temperature at a single point, than in other methods, such as the expansion of the wire, in which the temperature throughout the length of the wire is integrated. Calculation of the change of indicated current with frequency, as made according to equation (4), is such as to insure correct relationships of the curves at different frequencies, independent of temporary changes of calibration such as might arise, e. g., from temperature change. Thus, the direct-current calibration points before and after high-frequency observations may not lie exactly on the same curve; but the change of calibration does not affect interpretation of the curves, for the high-frequency points are plotted at the correct distances from the direct-current curve as drawn. The relation of indicated high-frequency current to the actual current flowing is not the important thing, but rather the relation of indicated high-frequency current to indicated direct-current or low-frequency current for equal total current in the circuit.

4. SOURCES OF ERROR

(a) ACCIDENTAL ERRORS

The precision of an observation in general was 0.5 per cent of the current measured, and the precision of a mean of several observations was about 0.2 per cent or 0.3 per cent. The chief source of accidental error was the lack of constancy of the high-frequency current, together with the different lag constants of the two galvanometers. In the strip instruments described later, the thermal lag of the strips themselves was appreciable, and in one case was greater than the mechanical lag of the galvanometer used on the standard instrument. A necessary precaution in the observations was to shield each galvanometer by connecting one of its binding posts to its case. Before this was done, irregular leakage of direct currents from lighting circuits was the cause of peculiar and puzzling results. A few possible sources of constant error in the observations will now be treated.

(b) EDDY CURRENTS

It has been suspected that eddy currents in neighboring conductors might affect the readings of hot-wire ammeters. This was investigated by greatly exaggerating the conditions that might occur in practice. A copper sheet 2 mm thick and about 6 cm square was placed very close to one of the wires of the two-wire standard instrument, which was in series with two other instruments. At a frequency of 750 000, the presence of the sheet made no perceptible difference in the currents indicated. The same thing was tried with an iron sheet with the same result. The absence of eddy current effect upon high-frequency ammeters is also confirmed by the experience of a commercial manufacturer of instruments;[7] the confidence of the company in respect to this point being so great that they inclose their best high-frequency ammeter in an iron case.

(c) INDUCTIVE EFFECT OF LEADS, ETC.

It is very reasonable to suppose that outside parts of a circuit, particularly the leads, might inductively affect an ammeter at high frequency. The effect of distant portions of the circuit was tested by changing the orientation of instruments by 180°, and also by moving the instruments into different positions at distances 15 to 80 cm from the secondary circuit inductance coil. No changes in reading were noticeable. It was otherwise with the leads, however. In the cases tried, the readings were not affected by the presence of the leads when extending straight

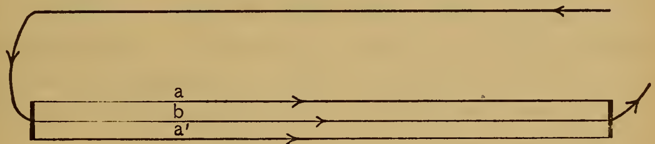

Fig. 6.—*Experiment upon proximity of lead wire*

out from the binding posts to some distance, as they would usually be connected. But when a lead was turned and run along the side of an instrument which consisted of more than a single wire, there were appreciable changes. These were very evidently due to the greater mutual inductance of the lead upon the parts of the ammeter circuit nearest to it than upon the more remote parts. The changes were of the algebraic sign and of a magnitude to be expected from experience with the effects described in later sections of this paper. For example, with a frequency of 1 000 000, upon placing the lead wire in the plane of and parallel to the three-wire instrument of Fig. 9 below, about 5 cm distant from the three wires, as shown in Fig. 6, the current in the wire *a'* decreased 1.5 per cent. Upon reversing the direction of the lead, the current in the same wire increased 1.5 per cent. This effect

[7] R. Hartmann-Kempf: Elektrotech. Zs., 32, p. 1134; 1911. G. Eichhorn: Jahrbuch d. Drahtl. Tel., 5, p. 517; 1912.

could be strikingly illustrated[8] by replacing the three-wire system of Fig. 6 by a vacuum tube; the high-frequency current would *visibly* crowd to the side of tube nearest the lead when the lead was directed as in Fig. 6, and vice versa. Similar results were obtained with the instrument of Fig. 19 below. With the lead adjacent and parallel to the long, thick copper wire ST of this instrument, 4 cm distant from it, with the current in the two opposed, the current in the right section of the instrument increased 3 per cent at a frequency of 500 000. These effects are large enough to suggest the need for caution in any radiotelegraphic circuit, particularly when the ammeter is very close to other parts of the circuit.

(d) LOW FREQUENCY

The results of experiments throughout this investigation were found not to depend on the low frequency with which the high-frequency effects were compared. Frequencies of 60, of 550, and of 2700 cycles were used, and all gave consistent results. Any frequency of this order is practically equivalent to zero frequency or direct current, in these experiments, because the changes of current distribution in the instruments here used only begin at frequencies of the order of 100 000.

(e) CAPACITY

Electrostatic capacity was found to affect the experiments at the highest frequency used, 1 500 000, but at no lower frequency. The effect was serious, causing two ammeters in series to carry different amounts of current; but, as will be shown, a way was found to eliminate it. There was no indication that the capacity of the high-frequency circuit itself affected matters. The observed result was found to be due rather to the relatively large capacity of the measuring galvanometers and leads, used in the thermocouple circuits of the instruments. This effect was at first very puzzling. Two instruments whose readings agreed at all lower frequencies showed an unaccountable difference of 5 per cent in the current indicated at 1 500 000. This difference was consistently maintained until one day the difference was found to have changed very suddenly from + 5 per cent to − 5 per cent.

[8] J. J. Thomson: "Recent Researches," p. 511; 1893.

This was surprising, and the source of trouble was sought by altering various connections of different parts of the circuit. It was found that the effect changed sign when the leads a and b to the two instruments X and N, in Fig. 7, were interchanged. That the difference was due to an actual difference in the current flowing in the two instruments was suggested by the fact that if the observer touched the case of one of the galvanometers, a continuous spark passed, and the deflections of the galvanometers changed, even though the person stood on insulating blocks. The existence of a capacity current to the ground was thus manifested. The effects were further studied, exaggerating them by connecting a wire from a metal pipe lying on the floor to various parts of the instru-

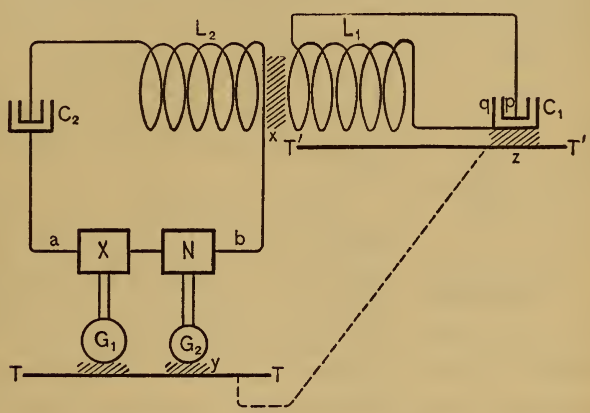

Fig. 7.—*Diagram showing path of dielectric current*

ments and their circuit. Current left the circuit and flowed along this wire, changing the deflections of the instruments by amounts depending on where the connection was made. This indicated strongly that the observed errors were due to part of the current leaving the circuit from some point between the two instruments, and as the instruments and their attached galvanometers were well insulated by hard rubber blocks it appeared to be a capacity current.

The effect is explained by electrostatic induction of current across the adjacent ends of the primary and secondary inductance coils. The electrostatic circuit is completed through the instruments and galvanometers, thence as a dielectric current to the wooden table T T, thence partly as dielectric and partly conduction current to the slate table T′ T′, and thence as a dielectric current to the outer coating of the Leyden jar C_1. The capacity in the circuit thus completed simply acts as a condenser in parallel with the Leyden jar. This is made clearer by Fig. 8, which is equivalent to those portions of Fig. 7 concerned in this phenomenon. The instantaneous current electrostatically induced seemed to be opposite in direction to that electromagnetically induced in the secondary circuit, so as to reduce the whole current flowing in the instruments. The current in the instrument to which the lead b (Fig. 7) was connected was less than the current in the other instrument, because part of this dielectric current left the circuit through the galvanometer of the former and so never reached the latter. The theory was corroborated by interchanging the connections of the coil L_2 in Fig. 7. Then the current was less in the instrument to which the lead a was connected, as expected.

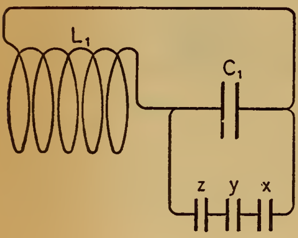

Fig. 8.—*Theoretical equivalent of Fig. 7*

If this explanation were valid, it was thought that it might be possible to eliminate the effects, for the purposes of ammeter comparison, by shunting the dielectric current around the ammeters in some way. It is only required that the currents in the two instruments be the same, regardless of the current in other parts of the circuit. It was successfully accomplished by connecting a wire from the lead b (Fig. 7) to a metal plate under the primary Leyden jar C_1, the jar standing upon hard-rubber blocks on the plate. This provided a path of much lower impedance than through the ammeters and tables. When this device was used, the current through the two ammeters was the same, within the limits of observational error, as shown by interchanging the leads a and b. These results were obtained repeatedly with a number of different pairs of instruments. The success of this scheme to shunt the dielectric current out of the ammeters is strong evidence for the explanation postulated for the observed effects. If the trouble had been caused by the distributed capacity of the ammeter wires themselves, then neither this scheme nor any other could make the current in the two ammeters equal.

The magnitude of the observed effects is found to be consistent with calculation. The amounts of current flowing in the Leyden jar and in the complex dielectric path are proportional to the capacities of the paths. Since the current in the complex dielectric path was found by observation to be 5 per cent of the total secondary current, assuming the primary current roughly to be the same as the secondary, the capacities must be in the ratio 20 to 1. Since the capacity of the Leyden jar C_1 was 1500 micromicrofarads, the capacity of the complex dielectric path was therefore about 75 micromicrofarads. This is an exceedingly small capacity; and is a not unreasonable value for the circuit considered, even though the three capacities x, y, and z are in series. It may be recalled, for comparison, that the capacity of an isolated sphere equals its radius in centimeters, and that 1 micromicrofarad = 0.9 cm. The dimensions of the parts here concerned are smaller than 75 cm, but on the other hand they are by no means isolated. The reasons why the capacity effects were not appreciable at the lower frequencies were: (a) That the primary and secondary coils were separated a greater distance, and hence the capacity was smaller; and (b) that the current in the primary circuit was relatively smaller, for there was visible brush discharge from the primary Leyden jar at 1 500 000 and in no other case.

The capacity effect which has been investigated appears to have a decided bearing upon many experiments at radiotelegraphic frequencies. Similar leakage of current from primary [9] to secondary circuits is familiar in practical work. It can be eliminated for the purposes of certain kinds of measurements, as shown above.

III. THE PARALLEL WIRE AMMETER

The first type of ammeter circuit which suggests itself for carrying a large high-frequency current is a group of several fine wires in parallel. This has been used by a number of experimenters and instrument makers, but it does not appear that its limitations have been distinctly pointed out. As shown below, it is in some cases subject to most serious errors. The most important source of error is mutual inductance between the wires, which has usually been thought negligible. The readings of the instrument

[9] Electrician, 68, pp. 1017, 1063; 1912.

depend on the current in just one of the wires. It will be shown that changes in the distribution of current among the wires affect such an instrument to a much greater percentage amount than the percentage change of resistance of the system as a whole.

1. ILLUSTRATIVE THREE-WIRE INSTRUMENT

Let us consider a simple case, which illustrates the essential features of this type of instrument. Three copper wires, each 10 cm long and 0.08 mm in diameter, were placed parallel, 4.0 mm apart, all in one plane. They were soldered at the ends to conductors whose resistance or impedance could be neglected. The leads were connected to the ends of this system at right angles to its plane, and other parts of the circuit were some dis-

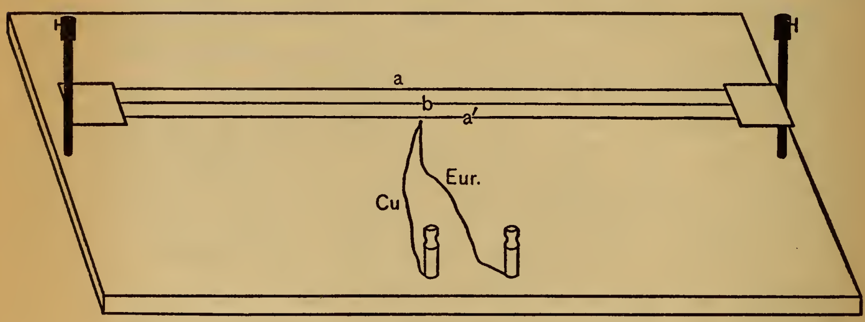

Fig. 9.—*Instrument of three parallel wires*

tance away. The current distribution in such a system can be calculated for any frequency and can also be experimentally measured by the methods described in Section II. A hard-soldered copper Eureka thermocouple was used, which could be soft-soldered to any wire.

Let R_a, R_b, $R_{a'}$ = resistance of a, b, a', respectively,

L = self-inductance of any one wire,

M_{ab}, $M_{aa'}$ = mutual inductances,

l = length of wire = 10.0 cm,

δ = diameter of wire = 0.008 cm,

d = distance between wires = 0.4 cm.

If a direct current be passed through the system, each wire carries one-third of the whole current, provided the wires all have exactly the same resistance. If, however, a current of high frequency be passed through the system, the two outer wires each carry more than one-third of the current and the middle wire less than one-third. This may be understood in a qualitative way from the well-known tendency of a high-frequency current to crowd toward the outer portions of a conductor as the frequency is raised. It is obtained quantitatively by considering the impedances of the system.

In the first place, the wires are so fine that the current distribution within the individual wire is not altered. The self and mutual inductances of the wires, neglecting bending of the current at the ends, may be calculated by well-known expressions,[10] and by the mode of connection assumed, no inductive action of other parts of the circuit need be considered. The frequencies dealt with are not so great but that for these small circuits the magnetic field may be considered as established instantaneously. The self-inductance of a single straight wire is given by the expression:

$$L = 2l\left(\log\frac{4l}{\delta} - 0.75\right) \tag{5}$$

For wires of such small diameter the high-frequency and low-frequency inductance are not appreciably different. The mutual inductance of two parallel wires when δ is small and l is large in comparison with d,

$$M = 2l\left(\log\frac{2l}{d} - 1 + \frac{d}{l}\right) \tag{6}$$

In the present case we find:

$$L = 20\ .\ (8.51 - 0.75) = 155.\ \text{cm}$$
$$M_{ab} = M_{a'b} = 20\ .\ (3.91 - 0.96) = 59.\ \text{cm}$$
$$M_{aa'} = 20\ .\ (3.22 - 0.92) = 46.\ \text{cm}$$

The calculations may be most easily understood by first considering frequencies so great that the resistance is small in comparison with the reactance. Expressions for the current distribution will be obtained by neglecting the resistance. Calling e the electromotive force between the two ends of the system,

---

[10] Rosa and Grover: This Bulletin, 8, pp. 150, 174; 1911.

and i_a, i_b, respectively, the instantaneous currents in a and b, and I_a, I_b the effective values of current,

$$e = L\frac{di_a}{dt} + M_{ab}\frac{di_b}{dt} + M_{aa'}\frac{di_a}{dt}$$

$$= L\frac{di_b}{dt} + 2M_{ab}\frac{di_a}{dt} \tag{7}$$

It is obvious that $i_a = i_{a'}$ since $M_{ab} = M_{a'b}$.
Integrating (7), there results at once:

$$(L + M_{aa'})I_a + M_{ab}I_b = LI_b + 2M_{ab}I_a \tag{8}$$

or,

$$(L + M_{aa'} - 2M_{ab})I_a = (L - M_{ab})I_b \tag{9}$$

$$\frac{I_a}{I_b} = \frac{L - M_{ab}}{L + M_{aa'} - 2M_{ab}} = \frac{155. - 59.}{155. + 46. - 118.} = 1.155$$

Denoting by I the whole current flowing,

$$I = 2I_a + I_b, \text{ or } 2 + \frac{I_b}{I_a} = \frac{I}{I_a}$$

Now, for very low frequencies or direct current, which will be denoted by the subscript $_0$, $\frac{(I_a)_0}{(I_b)_0} = \frac{R_b}{R_a}$, and $2(I_a)_0 + (I_b)_0 = (I)_0$, so that

$$2 + \frac{R_a}{R_b} = \frac{(I)_0}{(I_a)_0} \tag{10}$$

For $I = (I)_0$, i. e., the same total current in the circuit, $\frac{I_a}{(I_a)_0} = \frac{2 + \frac{R_a}{R_b}}{2 + \frac{I_b}{I_a}}$. If $R_a = R_b$, this ratio becomes $\frac{I_a}{(I_a)_0} = \frac{3}{2 + \frac{I_b}{I_a}}$, the numerical magnitude of which here $= 1.047$.
Similarly,

$$\frac{I_b}{(I_b)_0} = \frac{1 + 2\frac{R_b}{R_a}}{1 + 2\frac{I_a}{I_b}}$$

the numerical magnitude of which here $= 0.906$.

These are the limiting values of the currents for very high frequencies, i. e., for any frequency so high that the resistances are negligible in comparison with the reactances. It is shown below, both experimentally and theoretically, that this condition is pretty closely approached at a frequency of 1 500 000. If the temperature of one of the wires be measured by a thermocouple or expansion indicator or other thermometric device, the current indicated on low frequency would be exactly one-third the whole current flowing if the resistances of all three wires were exactly equal; and at any frequency higher than 1 500 000 or so the current indicated would be 9.4 per cent lower if the middle wire be used and 4.7 per cent higher if either outside wire be used.

For a system consisting of two parallel wires instead of three, a consideration of equations similar to (7) above and (11) below shows at once that each carries the same current at all frequencies, provided the two wires are exactly similar. The advantages and limitations of a two-wire system have been treated in Section II.

The current distribution in the three-wire system will now be calculated for frequencies such that the resistance can not be neglected. The potential difference between the ends is:

$$\begin{aligned} e &= R_a i_a + L\frac{di_a}{dt} + M_{ab}\frac{di_b}{dt} + M_{aa'}\frac{di_{a'}}{dt} \\ &= R_b i_b + L\frac{di_b}{dt} + M_{ab}\frac{di_a}{dt} + M_{ab}\frac{di_{a'}}{dt} \end{aligned} \tag{11}$$

By symmetry, $i_{a'} = i_a$, if $R_a = R_{a'}$; in any case understand by i_a the mean instantaneous current in a and a', and by R_a the mean resistance of a and a'. Since at every instant, $2i_a + i_b =$ the total instantaneous current, i,

$$i_b = i - 2i_a \tag{12}$$

Substitute in (11),

$$R_a i_a + (L + M_{aa'} - 2M_{ab})\frac{di_a}{dt} = R_b i - 2R_b i_a + (L - M_{ab})\frac{di}{dt} - 2(L - M_{ab})\frac{di_a}{dt}$$

Assuming that e is harmonic, and solving,[11]

$$[(R_a+2R_b)^2+p^2(3L+M_{aa'}-4M_{ab})^2]I_a^2=[R_b^2+p^2(L-M_{ab})^2]I^2 \quad (13)$$

in which $p=2\pi \times$ frequency. Similarly, solving for I_b^2,

$$\begin{aligned}&[(R_a+2R_b)^2+p^2(3L+M_{aa'}-4M_{ab})^2]I_b^2\\ &=[R_a^2+p^2(L+M_{aa'}-2M_{ab})^2]I^2\end{aligned} \quad (14)$$

By equation (10),

$$\frac{I_a}{(I_a)_o}=\sqrt{\frac{I_a^2}{I^2}\left(2+\frac{R_a}{R_b}\right)} \quad (15)$$

Similarly,

$$\frac{I_b}{(I_b)_o}=\sqrt{\frac{I_b^2}{I^2}\left(1+\frac{2R_b}{R_a}\right)} \quad (16)$$

The quantities under the radical sign in (15) and (16) are given in (13) and (14), respectively.

The significance of these solutions is made clearer by considering special cases. If the frequency is very low, equation (13) reduces to (10). If the resistances were equal, the current in one wire would be precisely one-third the whole current flowing, just as was shown above. In the instrument constructed to test these calculations, however, the resistances were not exactly equal. Copper wires of such small diameter vary greatly in cross section and hardness. By actual measurement the resistance of the wire b was found to be 0.352 ohm, and the mean resistance of a and a' was 0.347 ohm. The difference of cross section corresponding to

---

[11] Rewrite the differential equation:

$$(R_a+2R_b)i_a+(3L+M_{aa'}-4M_{ab})\frac{di_a}{dt}=(R_b)i+(L-M_{ab})\frac{di}{dt}.$$

Assuming e harmonic, it can be expressed in the ordinary way as proportional to $\varepsilon^{\sqrt{-1}\,pt}$, where ε is the Napierian base, p is 2π times the frequency, and t is time. Letting I_a and I represent effective values of current, and α and β angles of phase difference,

$$i_a=I_a\varepsilon^{\sqrt{-1}\,pt}\varepsilon^{\sqrt{-1}\,\alpha}$$

$$i=I\varepsilon^{\sqrt{-1}\,pt}\varepsilon^{\sqrt{-1}\,\beta}$$

Substituting in the differential equation,

$$\begin{aligned}&(R_a+2R_b)I_a\varepsilon^{\sqrt{-1}\,pt}\varepsilon^{\sqrt{-1}\,\alpha}+\sqrt{-1}\,p(3L+M_{aa'}-4M_{ab})I_a\varepsilon^{\sqrt{-1}\,pt}\varepsilon^{\sqrt{-1}\,\alpha}\\ &=R_bI\varepsilon^{\sqrt{-1}\,pt}\varepsilon^{\sqrt{-1}\,\beta}+\sqrt{-1}\,p(L-M_{ab})I\varepsilon^{\sqrt{-1}\,pt}\varepsilon^{\sqrt{-1}\,\beta}\end{aligned}$$

Simplifying,

$$[(R_a+2R_b)+\sqrt{-1}\,p(3L+M_{aa'}-4M_{ab})]I_a\varepsilon^{\sqrt{-1}\,\alpha}=[R_b+\sqrt{-1}\,p(L-M_{ab})]I\varepsilon^{\sqrt{-1}\,\beta}$$

Taking absolute values of these complex quantities, since the absoulte value of $\varepsilon^{\sqrt{-1}\,\alpha}$ and of $\varepsilon^{\sqrt{-1}\,\beta}$ equals 1,

$$[(R_a+2R_b)^2+p^2(3L+M_{aa'}-4M_{ab})^2]I_a^2=[R_b^2+p^2(L-M_{ab})^2]I^2$$

Thus equation (13) has been derived.

this difference of reisistance causes no appreciable difference in the self-inductances.

$$\frac{(I_a)_o}{(I)_o}=\frac{0.352}{0.347+0.704}=0.334_9,\text{ instead of exactly }0.3333$$

$$\frac{(I_b)_o}{(I)_o}=\frac{0.347}{0.347+0.704}=0.330_2,\text{ instead of exactly }0.3333$$

If the frequency is extremely high the resistance terms can be neglected in (13) and (14), giving

$$\frac{I_a}{I_b}=\frac{L-M_{ab}}{L+M_{aa'}-2M_{ab}},$$

the same as equation (9) above, which was independently deduced.

For any frequency whatever, substituting numerical values in (13) and (15), and letting f = frequency,

$$\frac{I_a}{(I_a)_o}=\sqrt{\frac{0.124+0.364(10)^{-12}f^2}{1.105+2.98\ (10)^{-12}f^2}}[2.986] \tag{17}$$

Similarly,

$$\frac{I_b}{(I_b)_o}=\sqrt{\frac{0.120+0.272(10)^{-12}f^2}{1.105+2.98\ (10)^{-12}f^2}}[3.029] \tag{18}$$

Values have been calculated from these expressions for four frequencies, and compared with the experimental observations. The results follow:

TABLE 3

Current Distribution in Three-Wire System

Frequency	Per cent increase of current in *a*		Per cent decrease of current in *b*	
	Calculated	Observed	Calculeted	Observed
	Per cent	Per cent	Per cent	Per cent
150 000	0.3	0.4	0.6	0.2
500 000	1.8	1.3	3.4	3.0
1 000 000	3.1	2.8	6.1	5.3
1 500 000	3.9	3.6	7.4	6.0
∞	4.3	....	8.5	....

The agreement of the observed values with the calculations is nearly as good as the experimental accuracy warrants. It should have been expected, however, that the observed values would be numerically smaller than the calculated values, because of the heat interchange among the three wires due to convection and radiation. The thermocouple method of observation measures primarily the *temperature* of one of the wires, from which the current in the wire is *inferred*. When the three wires are carrying different amounts of current their temperatures are to a certain extent equalized by the interchange of heat, and hence the wires will appear to be carrying currents more nearly equal than they actually are. This conclusion was corroborated by observations on an instrument similar to the one considered except that the wires were 2 mm instead of 4 mm apart. On account of the wires being closer together, the effect of heat interchange should be more marked and the per cent changes of current should be still smaller than the calculated values. Such was found to be the case, and, furthermore, the changes were not constant, but different on different days; apparently the amount of convection varied with different temperature or atmospheric conditions. Contrary to expectation, the results were not improved by placing this instrument in a partial vacuum under a pressure of 1 cm of mercury.

Table 3 shows that at a frequency of 1 500 000 the current distribution has become nearly that for infinite frequency. Of course, "infinite" must be understood simply as meaning such a frequency that the resistance is a negligible part of the impedance. For mathematically infinite frequency the current distribution is indeterminate. For any frequency higher than 1 500 000 there are practically no further changes, except that the resistance of the individual wires becomes appreciably greater; the ratio of currents, however, remains practically unchanged. Thus the remarkable fact becomes evident that the range of frequencies investigated, in which all the changes in current distribution take place, is just the range of frequencies utilized in radiotelegraphy.

2. INSTRUMENT OF SEVEN WIRES

A hot-wire ammeter employing a more complex type of parallel wire resistance was examined and tested with high-frequency currents. It was a commercial instrument with the ordinary arrangements for indicating expansion. The resistance consisted of seven bronze wires, each 8.6 cm long and 0.15 mm in diameter, spaced as shown in Fig. 10, six of them being 0.19 cm apart and the seventh, g, being at a distance of 0.95 cm from f. The flexible leads to g, each about 0.3 mm diameter, will be called m and m'. The indicating mechanism of the ammeter is a device for showing the expansion of the wire g. 9 amperes gave full-scale deflection. It will readily be seen that because of the mutual inductances the wire g will carry more current than any of the other wires when the frequency is high, while with direct current it will carry approximately the same as any other wire (actually a trifle less than one of the others because of the resistance of m and m'). Consequently the instrument will "read high" on high frequency. As shown below by calculation and by experiment, this effect is surprisingly large. The complete theoretical solution for all frequencies can not be obtained for such a com-

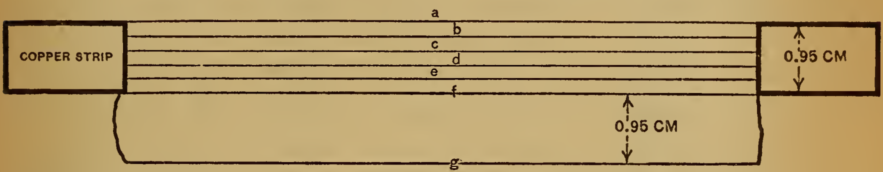

Fig. 10.—*Instrument of seven parallel wires*

plex case, but enough can be worked out to explain the instrument's behavior.

The inductances, calculated by (1) and (2) above and by a more accurate expression for mutual inductance in the cases of the wires farther apart, are as follows:

$$
\begin{aligned}
L_a &= \dots\dots\dots L_f = 120.2 \text{ cm} \\
L_g &= L_a + 2L_m \quad = 158.8 \\
M_{ab} &= M_{bc} = \dots = 60.7 \\
M_{ac} &= M_{bd} = \dots = 49.2 \\
M_{ad} &= M_{be} = \dots = 42.6 \\
M_{ae} &= M_{bf} = 38.0 \\
M_{af} &= M_{fg} = 34.6 \\
M_{eg} &= 31.7 \\
M_{dg} &= 29.5 \\
M_{cg} &= 27.4 \\
M_{bg} &= 25.7 \\
M_{ag} &= 24.3
\end{aligned}
$$

For frequencies so high that the resistance is negligible compared with the reactance, the condition that the potential difference be the same between the ends of each branch of the system, assuming harmonic electromotive force, is—

$$
\begin{aligned}
&L_aI_a+M_{ab}I_b+M_{ac}I_c+M_{ad}I_d+M_{ae}I_e+M_{af}I_f+M_{ag}I_g\\
=&M_{ab}I_a+L_bI_b+M_{ab}I_c+M_{ac}I_d+M_{ad}I_e+M_{ae}I_f+M_{bg}I_g\\
=&M_{ac}I_a+M_{ab}I_b+L_cI_c+M_{ab}I_d+M_{ac}I_e+M_{ad}I_f+M_{cg}I_g\\
=&M_{ad}I_a+M_{ac}I_b+M_{ab}I_c+L_dI_d+M_{ab}I_e+M_{ac}I_f+M_{dg}I_g\\
=&M_{ae}I_a+M_{ad}I_b+M_{ac}I_c+M_{ab}I_d+L_eI_e+M_{ab}I_f+M_{eg}I_g\\
=&M_{af}I_a+M_{ae}I_b+M_{ad}I_c+M_{ac}I_d+M_{ab}I_e+L_fI_f+M_{fg}I_g\\
=&M_{ag}I_a+M_{bg}I_b+M_{cg}I_c+M_{dg}I_d+M_{eg}I_e+M_{fg}I_f+L_gI_g
\end{aligned}
$$

Inserting numerical values:

$$
\begin{aligned}
&120.2I_a+60.7I_b+49.2I_c+42.6I_d+38.0I_e+34.6I_f+24.3I_g\\
=&60.7I_a+120.2I_b+60.7I_c+49.2I_d+42.6I_e+38.0I_f+25.7I_g\\
=&49.2I_a+60.7I_b+120.2I_c+60.7I_d+49.2I_e+42.6I_f+27.4I_g\\
=&42.6I_a+49.2I_b+60.7I_c+120.2I_d+60.7I_e+49.2I_f+29.5I_g\\
=&38.0I_a+42.6I_b+49.2I_c+60.7I_d+120.2I_e+60.7I_f+31.7I_g\\
=&34.6I_a+38.0I_b+42.6I_c+49.2I_d+60.7I_e+120.2I_f+34.6I_g\\
=&24.3I_a+25.7I_b+27.4I_c+29.5I_d+31.7I_e+34.6I_f+158.8I_g
\end{aligned}
$$

Combining the last line with each of the preceding, these equations become the first six of the following:

$$
\begin{aligned}
0&=95.9I_a+35.0I_b+21.8I_c+13.1I_d+6.3I_e+0-134.5I_g\\
0&=36.4I_a+94.5I_b+33.3I_c+19.7I_d+10.9I_e+3.4I_f-133.1I_g\\
0&=24.9I_a+35.0I_b+92.8I_c+31.2I_d+17.5I_e+8.0I_f-131.4I_g\\
0&=18.3I_a+23.5I_b+33.3I_c+90.7I_d+29.0I_e+14.6I_f-129.3I_g\\
0&=13.7I_a+16.9I_b+21.8I_c+31.2I_d+88.5I_e+26.1I_f-127.1I_g\\
0&=10.3I_a+12.3I_b+15.2I_c+19.7I_d+29.0I_e+85.6I_f-124.2I_g\\
I&=I_a+I_b+I_c+I_d+I_e+I_f+I_g
\end{aligned}
$$

The last equation expresses the obvious condition that the currents are all in the same phase when the resistance is negligible (I denotes the whole current in the circuit).

Solving these equations by the use of determinants, we find:

$$\frac{I_g}{I}=0.196$$

On low frequency, however, the ratio of the current in the wire g to the total current is determined solely by the resistances. The resistance of the instrument was found to be 0.027 ohm, and the resistance of the wire g, measured after severing its leads, was 0.213.

$$\therefore \frac{(I_g)_o}{(I)_o} = \frac{0.027}{0.213} = 0.127$$

For a given total current in the circuit, the ratio of the current in the wire g at an extremely high frequency to the current in g at low frequency is then

$$\frac{I_g}{(I_g)_o} = \frac{0.196}{0.127} = 1.55$$

That is, for a frequency so high that the resistance is a negligible part of the impedance, the instrument would read 55 per cent high.

The behavior of the instrument at any frequency whatever may be calculated approximately by the aid of assumptions. The wire g is so unsymmetrically situated that to a first approximation the variations among the currents in the other six wires may be neglected. Suppose each of the wires a to f replaced by a wire occupying the position s (Fig. 11). Let the mutual inductances of each of these six wires at s with respect to the other five be the same as the average mutual inductances when the wires occupy their positions a to f; i. e., the mutual inductances of one wire at s with respect to the others are taken to be approximately the same as the mutual inductances of the wire b with respect to a, c, d, e, and f. Let ΣM_s denote the sum of these five mutual inductances. The mutual inductances of the wires at s with respect to g are all the same, M_{gs}. The potential difference between the ends of any wire is:

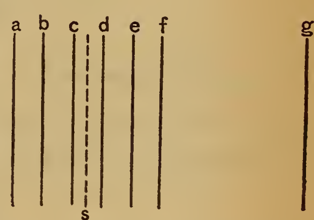

Fig. 11.—*Diagram for approximate calculation of seven-wire instrument*

$$e = R_s i_s + (L_s + \Sigma M_s)\frac{di_s}{dt} + M_{gs}\frac{di_g}{dt}$$

$$= R_g i_g + L_g\frac{di_g}{dt} + 6M_{gs}\frac{di_s}{dt}$$

Now, $i=6i_s+i_g$. $\therefore i_s=\frac{i-i_g}{6}$. Substitute this value for i_s.

$$\frac{R_s}{6}i+\left(\frac{L_s+\Sigma M_s}{6}-M_{gs}\right)\frac{di}{dt}=\left(R_g+\frac{R_s}{6}\right)i_g$$
$$+\left(L_g+\frac{L_s+\Sigma M_s}{6}-2M_{gs}\right)\frac{di_g}{dt}$$

Solving as in footnote 11, letting p denote 2π times the frequency,

$$\left(\frac{I_g}{I}\right)^2=\frac{\left(\frac{R_s}{6}\right)^2+p^2\left(\frac{L_s+\Sigma M_s}{6}-M_{gs}\right)^2}{\left(R_g+\frac{R_s}{6}\right)^2+p^2\left(L_g+\frac{L_s+\Sigma M_s}{6}-2M_{gs}\right)^2}$$

Inserting numerical values, $\Sigma M_s=251.2$ cm, and letting f denote the frequency:

$$\frac{I_g}{I}=\sqrt{\frac{0.00096+0.0439(10)^{-12}f^2}{0.0594+1.056(10)^{-12}f^2}}$$

This quantity divided by the value of $\frac{(I_g)_o}{(I)_o}$ gives the ratio of the indicated current at a frequency f to the indicated current at very low frequency. Thus for a frequency of 100 000,

$$\frac{I_g}{(I_g)_o}=\frac{0.141}{0.127}=1.11$$

and for a frequency of 750 000,

$$\frac{I_g}{(I_g)_o}=\frac{0.198}{0.127}=1.56$$

Therefore the ammeter should read 11 per cent high at a frequency of 100 000, and 56 per cent high at 750 000.

The actual performance of the instrument is shown by the curves of Fig. 12. The readings obtained at various frequencies are there plotted against the current. Throughout the scale, the readings at the different frequencies differ by approximately a constant per cent, as expected. At 100 000 the readings are 10 per cent high, and at 750 000 they are 46 per cent high. Measurements could not be made at higher frequencies than 750 000, because the instrument did not have a sufficiently open scale for observations below 6 amperes and currents as great as that could not be maintained steady enough at higher frequencies. Tne agreement between performance and theoretical calculation is quite satisfactory in view of the roughly approximate character of the assumptions made in the calculation of the preceding paragraph. Furthermore, the spacing of the wires *a* to *f* in Fig. 11 is not quite uniform, and the wire *a* is partly shunted by a wire three times its diameter (for purposes of adjustment). Heat inter-

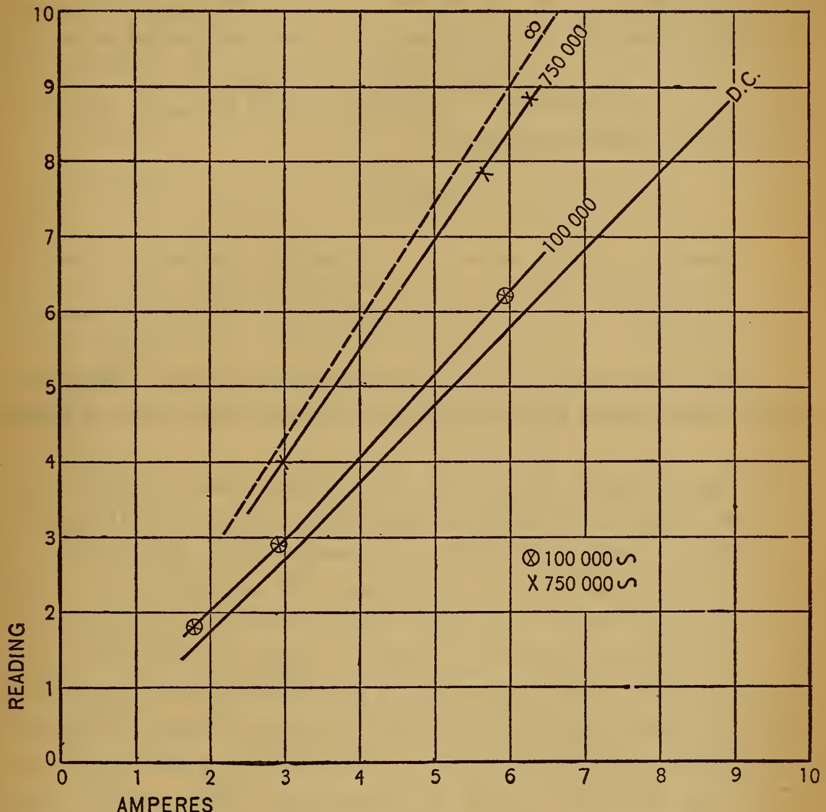

Fig. 12.—*Effect of frequency, seven-wire instrument*

change by convection and radiation, which would reduce the observed changes of reading, may be appreciable, although it is much less than it would be if the wire *g* were closer to the other wires. This thermal effect would make the changes appear less for larger currents, and the observations indicated this; but the relatively lower precision of the observations with smaller currents made it impossible to correct for the effect.

3. USE OF HIGH-RESISTANCE WIRES

The changes in current distribution at a given frequency can be much reduced by using wires of higher resistance. This was proved, experimentally and theoretically, in the case of the three-wire instrument of Fig. 9. An exactly similar instrument was constructed, the copper wires of 0.08 mm diameter being replaced by Eureka wires of 0.05 mm diameter. The resistivity of Eureka is about thirty times that of copper, and consideration of equations (17) and (18) of page 117 shows that the change of current in any wire, up to a frequency of 1 500 000, should be zero. Measurement gave the following results:

TABLE 4

Frequency	150 000	500 000	1 000 000	1 500 000
Decrease of current in *b*	−0.5 per cent	+0.4 per cent	−0.2 per cent	−0.1 per cent

The first two results were each a single observation; the departure of all the values from zero may be considered error of observation.

Thus the changes of current distribution can be moved up, as it were, to frequencies higher than those used in radiotelegraphy by employing wires of high resistance. The distribution of currents at a frequency so great that the resistance is a small part of the impedance is in nowise altered; that condition simply holds at a higher frequency. Even a wire of resistivity intermediate between that of copper and the high-resistance materials, such as platinum (resistivity 6.4 times copper), makes the changes of distribution negligible up to a frequency of 1 500 000, in the case of this particular type of instrument. The caution must be given, however, that the diameter of the wire should not be increased, as then the advantage of the higher resistivity of the wire disappears. This caution is necessary because instrumental requirements work against high resistance, for it increases the heat production, and the large heat production is the problem in constructing an ammeter for large currents; and again the designer will surely be tempted to use larger diameters when he substitutes higher resistivity wires for copper because he knows that the skin effect, or change of resistance with frequency of the wire itself, is less as the resistivity increases.

This advantage of using high-resistance metal for the working resistance holds as well for the flat-strip type of ammeter treated in Section V as for the parallel-wire type. The advantage is not quite so great, however, in the second class of instruments, the so-called unshunted type, as will be brought out in Section IV.

4. AMMETERS OF PREVIOUS EXPERIMENTERS

An ammeter of the parallel-wire type, used by H. W. Edwards [12] in investigating high-frequency resistance, has some points of interest. It consisted of 15 wires in parallel between two tri-

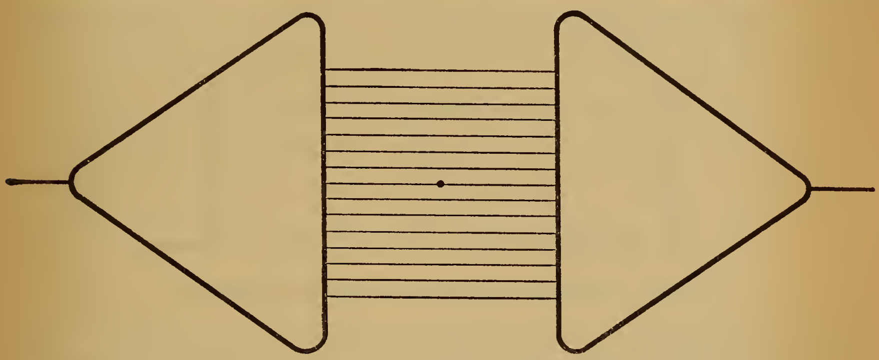

Fig. 13.—*Edwards' parallel-wire ammeter*

angular blocks of copper with a thermocouple soldered to the center wire. The wires were 3 cm long, 2 mm apart, and 0.08 mm in diameter. It is found by calculation that the self-inductance of one wire is 39. cm, and the 14 mutual inductances range from 11. to 3. cm. Comparison with the cases calculated above shows that for frequencies high enough to make the reactances comparable with the resistances, the instrument would read a good many per cent lower than on direct current. The change of distribution in the massive copper terminal blocks would increase the error. The latter effect is appreciable, as shown by the experiments on strip instruments in Section V; and, in fact, is

[12] Phys. Rev., 33, p. 184; 1911.

probably the only effect that would manifest itself at radiotelegraphic frequencies, as the wires were of high-resistance material. As Edwards' experiments were made at frequencies below 300 000, the instrument probably caused no error in his work.

A form of construction recommended by Fleming, and also described by A. H. Taylor,[13] is subject to still another error. It is shown in Fig. 14, and is simply a group of parallel wires between a pair of posts. Supposing that the wires could be spaced so far apart that the mutual inductances would have negligible effect, there would still be high-frequency errors if the self-inductances of all parts were not in the ratio of their resistances. The posts are of relatively large cross section, and their resistance is quite negligible in comparison with the resistance of the wires, but the self-inductance of the posts may be of the same

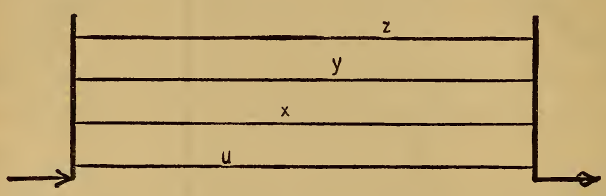

Fig. 14.—*Parallel-wire ammeter of various experimenters*

order of magnitude as the self-inductance of the wires. Then the high-frequency current in wire z will be much less than in wire u. This effect has been observed experimentally by Edwards (loc. cit.), the upper wire being found to carry 30 per cent less current than the lower at a frequency of 300 000. This effect is also illustrated experimentally in Section IV, following, in the present paper. In the ammeter constructed by Taylor the wires were of high-resistivity metal, but were of a relatively large diameter, 0.16 mm, so that the change of current distribution might be appreciable at radiotelegraphic frequencies.

The error would not be removed from this instrument by connecting the current leads at the middle points of the posts, for then the self-inductance in the paths of I_u and I_z would be greater than in the paths of I_x and I_y. This effect would decrease the

[13] Phys. Rev., 34, p. 365; 1912.

current in the outside wires, while the mutual inductances would increase it. However, if the leads be connected at opposite corners, at right angles to the plane of the instrument, as in Fig. 15, the self-inductance in each current path will be the same, and furthermore the leads will have no inductive action upon the instrument. If the leads were connected at opposite corners, as prolongations of the posts instead of at right angles to the posts, conditions would be almost as good; but each lead would act inductively on the adjacent section of the post somewhat more than on the other sections, and hence the middle sections of the post would carry somewhat more current than the end sections. This same principle is utilized in an instrument which employs thin, flat strips instead of wires, described in section V.

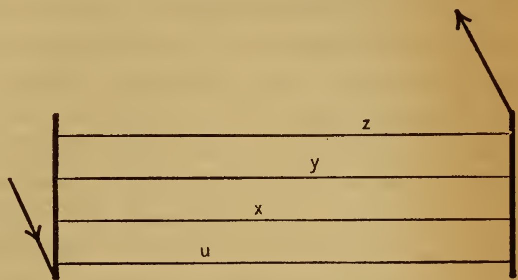

Fig. 15.—*Improved method of lead connection*

5. CYLINDRICAL ARRANGEMENT

The errors of the parallel-wire type of ammeter, due to mutual and self inductance, can be avoided by arranging the wires as equidistant elements of a cylinder and leading the current in to the centers of the ends of the cylinder. The mutual inductances of each wire with respect to the others are the same, and the self inductance is the same in each current path. The currents in the different wires must then be the same, at high frequencies. The current can be measured by a thermocouple or an expansion device on one of the wires. It turns out that this is so good a

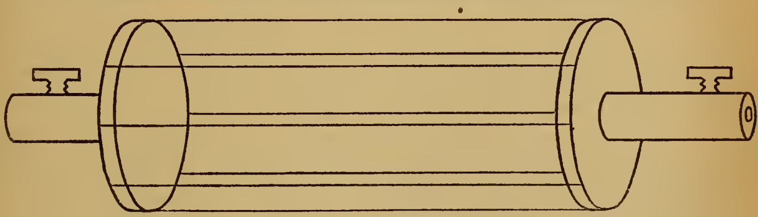

Fig. 16.—*Parallel-wire ammeter of cylindrical design*

high-frequency instrument that its only errors are due to its low-frequency current distribution. Great care is necessary to make sure that the wires all have the same resistance, otherwise they carry different amounts of current at low frequencies. In such an instrument, actually constructed of 8 manganin wires, 0.24 mm diameter and 6 cm long, the mechanicians found it difficult to solder on the wires without stretching and heating them unequally and consequently the resistances were not near enough to equality. Measurements showed that the increase of current in one of the wires from direct current was +4.8 per cent at 1 000 000, in another was 0.0 per cent, and in another −4.3 per cent. (These measurements were obtained by soldering the thermocouple successively on the three wires. Observations at other frequencies were consistent with those stated.) The greatest care in selection of wire and in construction is necessary to insure the equality of the resistances of the fine wires which must be used. This understood, the instrument may be a valuable one. The cylindrical arrangement of wires was first realized by Broca, and was described by him in Bulletin de la Société Internationale des Électriciens, **9**, p. 423; 1909. The symmetrical arrangement has also been utilized by R. Hartmann-Kempf [14], in the construction of instruments which employ thin metal strips instead of wires, described below in section V.

6. UTILIZATION OF WHOLE HEAT PRODUCTION

A way of escape from the current distribution errors of the parallel-wire type of ammeter is to make the deflection depend on the whole heat production in the system instead of on that in one branch. Now the whole heat production does change with change of frequency, because the resistance of the system considered as a whole increases as the current departs more and more from uniform distribution; but, as will be shown in the next section, the change of resistance of the whole system is of a smaller order of magnitude than the change of current distribution. In many cases, therefore, an instrument which is seriously in error if its reading depends on the current in one branch would have negligible error if it were so arranged that its readings depended on the

[14] See footnote 7, p. 107.

whole current. The calorimetric instrument, in which the reading depends for example on an air-thermometer device, avoids the direct errors of current distribution. The same thing holds for the expansion-indicating instrument of the so-called unshunted type, treated in the next section, but it will be there shown that in those particular instruments the dissymmetry of current distribution is likely to be so enormous that the change of resistance of the whole is not negligible. Another form of expansion-indicating instrument which avoids the direct errors of current distribution has been reported to the author. A number of wires parallel to one another were mechanically connected in such a way that the expansion of all of them contributed to the deflection. It so happened that the wires were less than 1 mm apart, so that the mutual inductances were large, and the change of current distribution was very great. Upon trial with high frequency currents the instrument was found to read high. The reading of any ammeter which depends on the whole heat production, or whole current through it, will either increase with frequency or will remain practically unchanged, but can not decrease.

This principle is readily applied to the instruments which employ a thermocouple as the means of indication. It was tried out in the three-wire case discussed first in the present

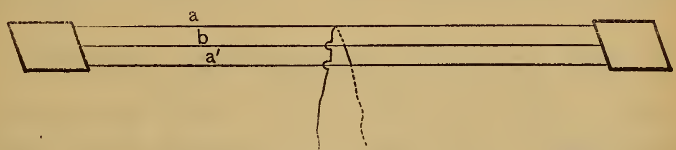

Fig. 17.—*Three-thermocouple instrument*

section. Three thermocouples in series were used, one on each wire, the direction of connection being alternately reversed. The resultant emf of such a series should be an average about equal to that of one couple. The construction was very simple, as shown in Fig. 17, a fine copper wire lead being soldered to the middle point of a, and at the same point a fine Eureka wire (shown dotted) which went to b, then a copper wire to a', and finally a Eureka lead; the two leads of course connect to a galvanometer. Theoretical calculation made as shown below, in the next section, shows that the change of reading should be negligible at all frequencies up to 1 500 000. Measurements at 500 000, 1 000 000, and 1 500 000 confirmed this; the increase of reading at 1 500 000 being 0.4 per cent, which was within the error of observation.

7. SUMMARY

Summarizing the investigation of the parallel-wire type of ammeter, it has been found that large errors with high-frequency currents occur in commonly used instruments, these errors being chiefly due to the mutual inductances and in some cases to self-inductances which had been thought negligible. These errors can be avoided for frequencies in the range employed in radiotelegraphy by using wires of high resistance. In some instruments in which the errors are due to the self-inductances of connecting parts they can be eliminated by suitable location of the current leads. Still another method of avoiding error is symmetrical location of the wires, such that each has the same mutual inductances with respect to the others. The errors are much reduced and in some cases eliminated entirely by making the deflection depend on the whole heat production instead of on the current in only part of the working resistance; this is possible in general for instruments whose deflections are produced either by calorimetric effect, expansion, or thermolectric effect.

IV. THE SO-CALLED UNSHUNTED AMMETER

Probably the type of ammeter most widely used in the past in radiotelegraphic work is that which employs but a single hot wire with different portions of its length joined in parallel. It will be shown that some of these have serious errors while others have not, and that the errors can be lessened. The construction is most readily understood from the diagram, Fig. 18. A and B are the current leads. They connect to thick copper bars, from which flexible metal strips take the current to several points of the hot wire, whose expansion is measured by the ordinary device (not shown in Fig. 18). Thus a single wire carries the whole current, and the instrument is therefore called unshunted. The resistances of the copper bar and metal strips are negligible in comparison with the resistance of the hot wire, and if the lengths of the sections in parallel are the same each carries the same current as any other on low frequency. But the inductances of these parts are by no means negligible, and consequently on high frequency the different portions of the wire carry different amounts of current. In fact, in practical cases the impedance of the hot wire itself is but a small part of the impedance of each current path for high frequencies. Even the mutual inductances of the different portions of the "hot wire" are not negligible and in themselves tend to cause more current to flow in the central

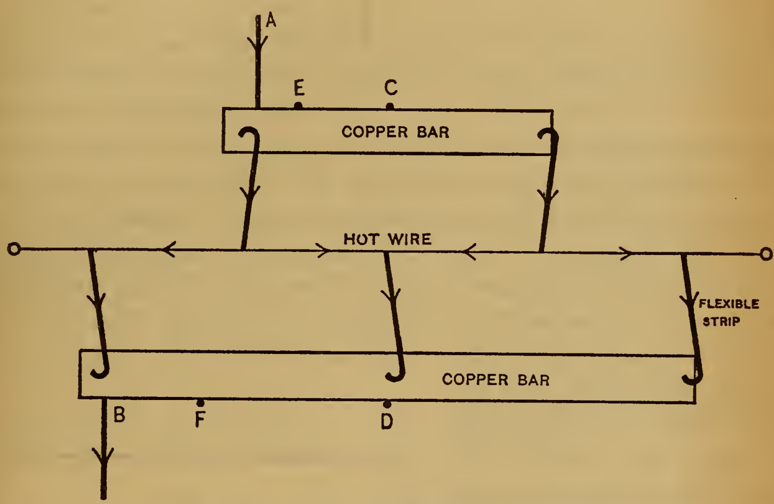

Fig. 18.—*Unshunted ammeter of four sections*

sections than in the outer sections. (This is rather unusual, as high-frequency currents tend to go to the outer portions of a conductor in general, but note that the direction of current is here reversed in adjacent sections). The changes in current distribution from the uniformity of direct-current distribution are in fact very large. They are equivalent to an increase in the resistance of the system as a whole, so that these instruments tend to read high on high frequency. It will be shown that the location of the leads A and B has an important influence in determining the error; and that some instruments of this type could be greatly improved by changing these points of connection.

1. ILLUSTRATIVE INSTRUMENT OF TWO SECTIONS

The actual distribution of current was studied, both experimentally and theoretically, in the case of a typical instrument of two sections. P O Q is a fine copper wire, 0.08 mm in diameter, soldered at the ends to the ends of P S T Q, which is a thick copper wire, 2.6 mm in diameter, bent at right angles in the two points S and T. This device exactly simulates the "unshunted" type of ammeter, P O Q being the "hot wire." Current leads were connected, perpendicular to the plane of this system, at the corner S and at O, the middle point of the fine wire. A thermocouple was soldered to some point on either P O or O Q, and its leads attached to a galvanometer; the deflection of the galvanometer was then a measure of the current flowing in P O or O Q. The length P O = O Q = 4.9 cm, and the length P S = T Q = 2.0 cm. For convenience in considering the current in the two branches of the system the diagram is redrawn, Fig. 20. The lengths P O and O Q, of Fig. 19, are here denoted by b and c, which have resistances R_1 and R_2, respectively. It could not be taken for certain that these two resistances were equal, although the lengths were equal, because such very thin copper wires are not uniform in diameter or in hardness. By measurement it was found that $R_1 = 0.168$ ohm and $R_2 = 0.179$ ohm.

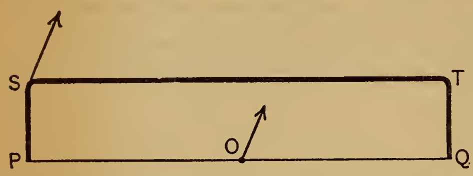

Fig. 19.—*Instrument with two sections of hot wire in parallel*

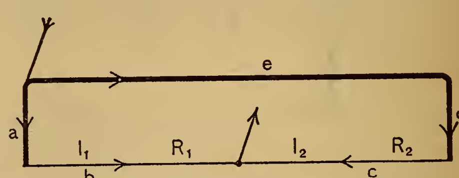

Fig. 20.—*Paths of current in two-section instrument*

For direct current or alternating current of low frequency R_1 and R_2 are the whole impedances of the two current paths, the resistance of the thick copper wire being quite negligible. But for high frequencies the inductances of these latter portions affect the impedance to an important extent. At very high frequencies, in fact, all resistances are negligible; and the self-inductance of e is actually greater than that of c; and, furthermore, the mutual inductances between e and b, between e and c, and between b and c appreciably affect the current distribution. No inductive action of outside portions of the circuit need be considered, as the leads are brought in at right angles to the wires of the system and other portions of the circuit are some distance away.

The self-inductances of b and c may be calculated by the simple expression:

$$L = 2l\left(\log\frac{4l}{\delta} - 0.75\right).$$

But the inductances of the thick wire portions, L_a, L_d, and L_e, are not the same on high as on low frequency because of change of current distribution in the thick wire itself. Thus for low frequencies the self-inductance of e is found by the simple calculation to be 75.0 cm, while for the high frequencies used in the experiments below it is found (see footnote 4) to be 71.0 cm. The mutual inductance of two parallel wires of length l and $\frac{1}{2}l$ and d cm apart, situated as e and b in Fig. 20, is readily shown to be:

$$M_{be} = l \log\frac{l+\sqrt{l^2+d^2}}{d} - \sqrt{l^2+d^2} + d. \qquad (19)$$

The mutual inductance of two adjacent wires in the same straight line, each of length l, such as b and c, is:

$$M_{bc} = 1.386l. \qquad (20)$$

The numerical values of the high-frequency inductances are found to be:

$$\begin{aligned} L_b &= L_c = 69.0 \text{ cm} \\ L_a &= L_d = 10.0 \\ L_e &= 71.0 \\ M_{bc} &= 6.8 \\ M_{be} &= M_{ce} = 12.7 \end{aligned}$$

In the actual instrument constructed the current lead was not attached exactly at the corner S, but 1 cm to the right of the corner. The slight alteration of L_e and departure from equality of M_{be} and M_{ce} produce negligible effects in the results.

The current distribution at any frequency may be calculated as follows. The potential difference between the ends of either wire is:

$$e = R_1 i_1 + (L_a + L_b)\frac{di_1}{dt} + (M_{be} - M_{bc})\frac{di_2}{dt}$$

$$= R_2 i_2 + (L_c + L_d + L_e)\frac{di_2}{dt} - 2M_{ce}\frac{di_2}{dt} + (M_{be} - M_{bc})\frac{di_1}{dt}$$

Since $L_c = L_b$, $L_d = L_a$, and $M_{ce} = M_{be}$,

$$R_1 i_1 + (L_a + L_b + M_{bc} - M_{be})\frac{di_1}{dt} = R_2 i_2$$

$$+ (L_a + L_b + L_e + M_{bc} - 3M_{be})\frac{di_2}{dt}$$

At every instant, $i_1 + i_2 = i$, the total instantaneous current, or, $i_2 = i - i_1$

$$\therefore\ (R_1 + R_2) i_1 + (2L_a + 2L_b + L_e + 2M_{bc} - 4M_{be})\frac{di_1}{dt}$$

$$= R_2 i + (L_a + L_b + L_e + M_{bc} - 3M_{be})\frac{di}{dt}$$

Assuming that e is harmonic and solving as in footnote 11,

$$\frac{I_1^2}{I^2} = \frac{R_2^2 + p^2(L_a + L_b + L_e + M_{bc} - 3M_{be})^2}{(R_1 + R_2)^2 + p^2(2L_a + 2L_b + L_e + 2M_{bc} - 4M_{be})^2}$$

For very low frequencies, which are denoted by the subscript $_o$,

$$\frac{(I)_o}{(I_1)_o} = 1 + \frac{(I_2)_o}{(I_1)_o} = 1 + \frac{R_1}{R_2}, \text{ or}$$

$$\therefore\ \frac{I_1}{(I_1)_o}\frac{(I)_o}{I} = \frac{I_1}{I}\left(1 + \frac{R_1}{R_2}\right)$$

For a given total current in the circuit, i. e., $(I)_o = I$,

$$\frac{I_1}{(I_1)_o} = \sqrt{\frac{R_2^2 + 4\pi^2 f^2 (L_a + L_b + L_e + M_{bc} - 3M_{be})^2}{(R_1 + R_2)^2 + 4\pi^2 f^2 (2L_a + 2L_b + L_e + 2M_{bc} - 4M_{be})^2}}\left[1 + \frac{R_1}{R_2}\right]$$

f denoting the frequency.

Inserting numerical values,

$$\frac{I_1}{(I_1)_o} = \sqrt{\frac{0.0320 + 0.556(10)^{-12} f^2}{0.120 \ \ + 1.45 \ \ (10)^{-12} f^2}}[1.939] \tag{21}$$

Similarly, for branch 2 of the instrument, it may be shown that the ratio of the indicated current at a frequency f to the indicated current at very low frequency is:

$$\frac{I_2}{(I_2)_0}=\sqrt{\frac{0.0282+0.211\,(10)^{-12}f^2}{0.120\ \ +1.45\ \ (10)^{-12}f^2}}[2.065] \qquad (22)$$

A correction was applied to these calculations to take account of the fact that the current heated the copper wires, so that their resistance while working was not the same as when measured cold.

For the range of frequencies in which changes in the current ratios occur, measurements were obtained and are compared with the values calculated from (21) and (22) in the following table:

TABLE 5

Current Distribution in "Unshunted" System

Frequency	Per cent increase of current in 1		Per cent decrease of current in 2	
	Calculated	Observed	Calculated	Observed
	Per cent	Per cent	Per cent	Per cent
150 000	4.5	4.2	4.2	4.4
500 000	15.2	14.3	15.4	15.0
1 000 000	18.6	17.7	19.4	18.7
1 500 000	19.2	19.1	20.3	19.5
∞	20.0	....	21.1	....

Of course, as before, "infinite" frequency simply means such a frequency that the resistance is a negligible part of the impedance. These results are also presented graphically in Fig. 21, in which the continuous curves give theoretical values and the dots inclosed by circles represent the observations.

The asymptotic approach of the current ratios to the values for infinite frequency is particularly well shown by these curves. Physically infinite frequency is practically attained. The agreement of the observations with the theory is all that could have been expected. There is no error here, however, due to heat interchange between the working parts, as in the case of the parallel wire instruments. The two parts of the hot wire were purposely made long and were soldered at their junction *O* (Fig. 19), to a heavy metal post to reduce their thermal effect upon each other. It was found, too, that the results obtained did not differ appreciably whether the thermocouple was soldered to the middle of one of the wires or close to its outer end.

The slight discrepancy between the theory and experiment is to be attributed to the insufficiency of the inductance calculations. In calculating the inductance of a finite portion of a circuit it is

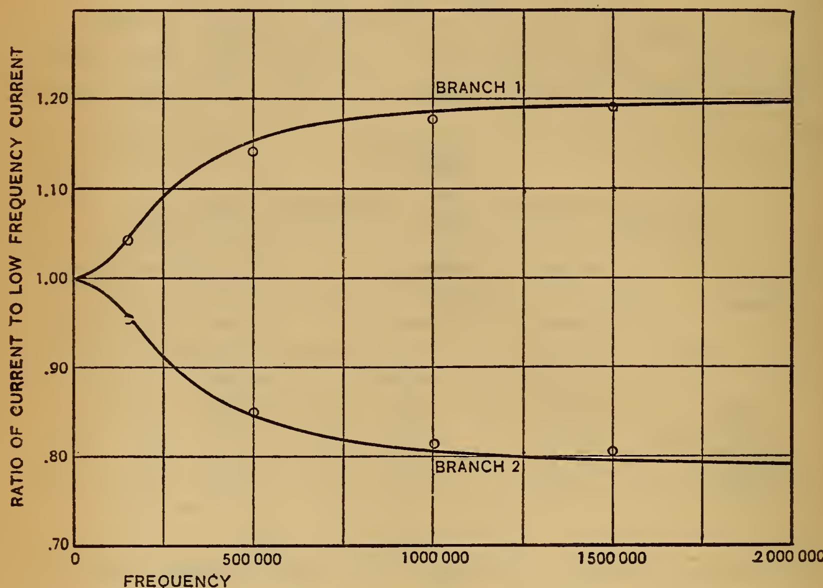

Fig. 21.—*Change of current distribution with frequency, two-section instrument*

assumed that the current is uniformly distributed about the axis of the conductor. This does not hold near a bend in the conductor; and furthermore the inductance of the current at the bend is neglected. Hence, one could not be sure how accurately the inductances of short lengths, such as those of the present case, are obtained by the theoretical formulas. The present experimental results are themselves a justification of the use of those formulas for approximate calculation even of such short lengths. Independent justification of the simple formulas for inductances of short linear conductors, in a somewhat more favorable case, is furnished by the direct inductance measurement [15] of Grover and Curtis.

The instrument under discussion not only illustrates the error in the "unshunted" type of instrument, but also the error treated in Section III in connection with Fig. 14. It is there brought out that while the resistance of the posts is negligible, the self-inductance is very important. The thick copper wire in the instrument here experimented upon plays the same part as the posts in Fig. 14, and hence the error there discussed is here experimentally realized.

2. USE OF HIGH-RESISTANCE WIRES

If the two-section instrument of Fig. 19 had a hot wire of some material of higher resistivity than copper, the changes of current distribution would be decreased. This is evident from a consideration of equations (21) and (22), pages 134, 135. It was proved experimentally by making a reproduction of the instrument, replacing the copper wire of 0.08 mm diameter by a Eureka wire of 0.05 mm diameter. The resistivity of Eureka wire is about thirty times that of copper, and calculation shows that up to a frequency of 1 500 000 the change of current distribution should be zero. The mean experimental results, for the ratio of high-frequency current to low-frequency current in the right section, were:

TABLE 6

Frequency	150 000	500 000	1 000 000	1 500 000
Per cent decrease of current	Per cent 0.0	Per cent 0.3	Per cent 0.2	Per cent 0.2

The difference between these results and zero is within the experimental error.

If, however, the instrument were made of a material of some intermediate resistivity, such as platinum or bronze, the changes of current distribution would be decidedly appreciable. In fact, if it were made of such material and the wires were of larger diameter than the copper wires (as they might be, since the skin effect is less as the resistivity increases), the errors might be fully as great

[15] This Bulletin, 8, p. 468, Reprint 175; 1911.

as when the hot wire is of copper. This will be illustrated in experiments to follow. There is not so great an advantage in using high-resistance metal in the hot wire of the "unshunted" type as in the parallel wire type of Section III because here the impedance of the hot wire itself plays a smaller part in determining the high-frequency current distribution.

3. INSTRUMENT OF FOUR SECTIONS

A commercial "unshunted" instrument of the four-section type sketched in Fig. 18 (p. 131) was considered and tested with high-

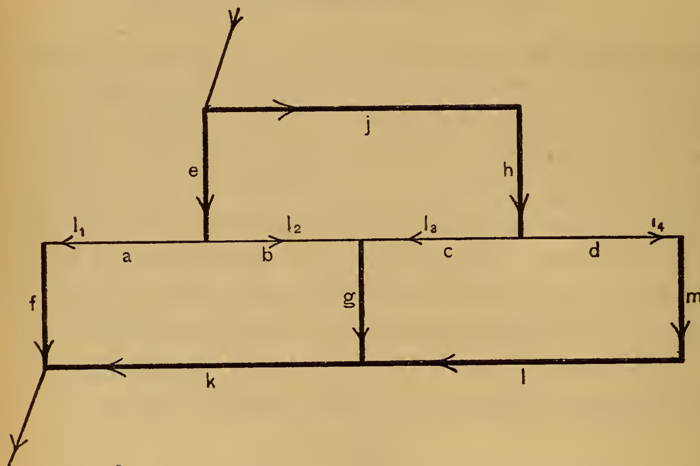

Fig. 22.—*Paths of current in four-section instrument*

frequency currents. The "hot wire" is of platinum 0.3 mm diameter and the working portion is 10. cm in length, so that the length of each section, *a*, *b*, *c*, *d* (Fig. 22) is 2.5 cm; 10 amperes give full scale deflection. The current has four paths in the instrument, which makes a system too complex for complete theoretical solution at all frequencies. However, the current distribution can be calculated for frequencies so great that the resistance is a negligible portion of the impedance, which will enable interpretation of the experimental results.

The two leads, shown at the extreme left corners, did not enter precisely at the corners; the effect of this will be considered later.

In the calculation it is necessary to take account of the self-inductance of each part, as well as the mutal inductances between parts, as follows, M_{ab}, M_{kl}, M_{jb}, M_{ka}, and the other equal mutual inductances of similarly situated parts. We may neglect the smaller mutual inductances M_{eh}, M_{fm}, M_{ac}, M_{ad}, M_{jk}, M_{kc}, and the similar mutual inductances equal to these. For frequencies so high that the resistance is negligible compared with the reactance, the condition that the potential difference be the same between the ends of each branch of the system is:

$$
\begin{aligned}
& L_e(I_1+I_2)+\underline{L_aI_1}+L_fI_1-M_{ab}I_2+M_{ka}(I_2+I_3+I_4) \\
= & L_e(I_1+I_2)+\underline{L_bI_2}+L_g(I_2+I_3)+L_k(I_2+I_3+I_4)+M_{jb}(I_3+I_4) \\
& \quad -M_{ab}I_1-M_{bc}I_3-M_{kb}(I_2+I_3+I_4)+M_{ka}I_1-M_{kb}I_2+M_{kl}I_4 \\
= & L_j(I_3+I_4)+L_h(I_3+I_4)+\underline{L_cI_3}+L_g(I_2+I_3)+L_k(I_2+I_3+I_4)+M_{jb}I_2 \\
& \quad -M_{jc}I_3-M_{jc}(I_3+I_4)-M_{bc}I_2-M_{cd}I_4+M_{lc}I_4+M_{ka}I. \\
& \quad -M_{kb}I_2+M_{kl}I_4 \\
= & L_j(I_3+I_4)+L_h(I_3+I_4)+\underline{L_dI_4}+L_mI_4+L_lI_4+L_k(I_2+I_3+I_4) \\
& \quad +M_{jb}I_2-M_{jc}I_3-M_{cd}I_3-2M_{ld}I_4+M_{lc}I_3+M_{kl}(I_2+I_3+I_4) \\
& \quad +M_{kl}I_4+M_{ka}I_1-M_{kb}I_2
\end{aligned}
$$

The four terms underlined in these equations are the only ones that might be taken into account, at first thought, in the consideration of the high-frequency behavior of this kind of instrument. As a matter of fact, these terms are quite small compared to others in the equations, as may be seen from the following numerical values of the inductances.

Many of the inductances in the equations are equal to one another, so that there are only six different ones. They may be calculated by (5), (19), (20), above, and the following formula for the self-inductance of a straight conductor of rectangular cross section, having width $=\alpha$ and thickness $=\beta$:

$$L=2l\left[\log\frac{2l}{\alpha+\beta}+0.5+\frac{0.2235(\alpha+\beta)}{l}\right] \qquad (22)$$

This gives the low frequency inductance, but comparison with the calculation (p. 133) for a round wire shows that the high-frequency value will be only slightly different. The calculation is only approximate anyway, for the bending of the current around corners is neglected. The six inductances were found to be:

$$L_e = 43.7 \text{ cm}$$
$$L_a = 25.3$$
$$L_k = 33.5$$
$$M_{ab} = 3.5$$
$$M_{kl} = 6.9$$
$$M_{ka} = 3.6$$

Inserting the numerical values, and adding an equation to express the summation of currents, there results:

$$\begin{aligned} 109.1I_1 + 3.4I_2 - 110.5I_3 - 216.2I_4 &= 0 \\ 40.2I_1 + 98.6I_2 - 40.4I_3 - 179.4I_4 &= 0 \\ 0 + 33.3I_2 + 58.4I_3 - 105.7I_4 &= 0 \\ I_1 + I_2 + I_3 + I_4 &= I \end{aligned}$$

Solving for the currents in the four sections:

$$I_1 = 0.495I$$
$$I_2 = 0.158I$$
$$I_3 = 0.190I$$
$$I_4 = 0.156I$$

(The sum of the four coefficients is 0.999, a sufficient check upon the computations). The dissymmetry of the current distribution is surprisingly great.

We come now to the essential characteristic of this type of instrument. The currents in all four sections affect the deflection, so that there will not be the enormous changes of deflection which would be expected if the current in just one section were measured. The error of the instrument is appreciable, nevertheless. It is well known that the direct current distribution of currents in any system is that of minimum heat production, so that the change of distribution with increase of frequency means an increase of total heat production. As the deflections of these instruments depend on the total heat production, they will read high on high frequency. The increase of total heat production for a given total current is equivalent to an increase in the resistance of the instrument; and it will be seen from the following that this change in resistance is of a smaller order of magnitude than the change in current distribution.

Consider a system of any n conductors in parallel between two points of a circuit, all of the same resistance and of such cross sections that the resistance of each individually is not appreciably different on high and low frequency.

Let R_k = resistance of any single branch of system,
R = high-frequency resistance of whole system,
$(R)_o$ = direct-current or low-frequency resistance of whole system,
n = number of branches of system,
I_k = high-frequency current in one branch,
$(I_k)_o$ = direct current or low-frequency current in one branch,
I = whole high-frequency current in circuit,
$(I)_o$ = whole direct current or low-frequency current in circuit,
H = rate of total heat production by high-frequency current,
$(H)_o$ = rate of total heat production by direct current or low-frequency current.

Since the resistances of all the branches are the same,

$$(R)_o = \frac{R_k}{n}$$

$$(I)_o = n(I_k)_o$$

$$(H)_o = \sum_{k=1}^{k=n} R_k (I_k)_o^2 \tag{23}$$

$$= n[R_k (I_k)_o^2] = n\left[n(R)_o \frac{(I)_o^2}{n^2}\right]$$

Or,

$$(H)_o = (R)_o (I)_o^2 \tag{24}$$

Similarly to (23), for high frequencies,

$$H = \sum_{k=1}^{k=n} R_k I_k^2$$

Or,

$$H = n(R)_o \sum_{k=1}^{k=n} I_k^2 \tag{25}$$

Similarly to (24), we can write:

$$H = RI^2, \tag{26}$$

this equation defining R, the high-frequency resistance of the system as a whole. For a given total current, by (24) and (26),

$$\frac{H}{(H)_0}=\frac{R}{(R)_0}, \tag{27}$$

or, obviously, the total rate of heat production is proportional to the resistance of the whole system, for a given current in the circuit.

We have, by (24) and (25),

$$\frac{H}{(H)_0}=\frac{n}{(I)_0^2}\sum_{k=1}^{k=n} I_k^2,$$

or, for the same total current on high frequency as on low, setting $I^2=(I)_0^2$, we have:

$$\frac{H}{(H)_0}=\frac{n}{I^2}\sum_{k=1}^{k=n} I_k^2 \tag{28}$$

In a hot-wire ammeter the scale and the mechanism for indicating the expansion are such as to satisfy the following relation:

$$\text{Indicated } I=\text{constant}\times\sqrt{H} \tag{29}$$

$$\therefore \frac{\text{Indicated } I \text{ for high frequency}}{\text{Indicated } I \text{ for low frequency}}=\sqrt{\frac{H}{(H)_0}} \tag{30}$$

Returning to the four-section ammeter under consideration, we have calculated above the numerical values of $\frac{I_k}{I}$ for each of the four sections. Inserting them in (28),

$$\frac{H}{(H)_0}=4.[(0.495)^2+(0.158)^2+(0.190)^2+(0.156)^2]=1.322$$

By (30), the ratio of the indicated currents $=\sqrt{1.322}=1.15$; i. e., the instrument should read 15 per cent high at a frequency so great that the resistance is a negligible portion of the impedance.

The foregoing calculations were repeated on the assumption that the leads of the instrument were connected at the middle of the two copper bars at the points C and D, Fig. 18, instead of at the points A and B as shown. The result of the calculation is that the change of reading would be negligible at all frequencies. The instrument would not read so much as 0.1 per cent high at "infinite" frequency. This result will appear reasonable upon inspection of Fig. 18 and consideration of the impedances in each current path. Hence, the location of the leads is of very great importance.

In the instrument tested the leads actually were attached at the points E and F, situated about two-thirds of the distance

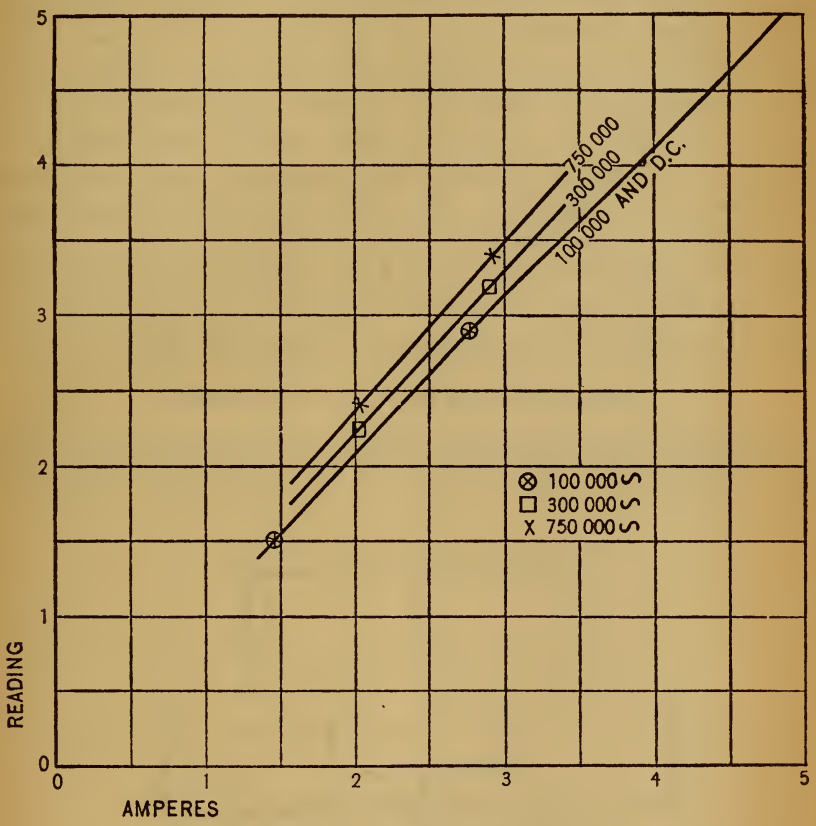

Fig. 23.—*Effect of frequency, four-section ammeter*

C A from C and two-thirds D B from D. The error of the instrument is certainly intermediate between the two cases already calculated, and we may roughly assume it to be two-thirds that calculated when the leads are connected at the corners. It should therefore read two-thirds of 15 per cent high at "infinite" frequency, i. e., 10 per cent high. The readings which the instrument gave experimentally are plotted in Fig. 23. The readings are found from the curves to be 11.5 per cent high at a frequency of 750 000, and 6 per cent high at 300 000, and at 100 000 to be the same as on direct current.

A consideration of the resistances and inductances and a comparison with the calculations for the two-section instrument above indicate that the current distribution at a frequency of 750 000 is here practically the same as at "infinite" frequency. (The resistance of each of the four sections was 0.04 ohm.) The agreement between the theoretical value of 10 per cent for "infinite" frequency and the observed value of 11.5 per cent for 750 000 is quite satisfactory. There is some possible question as to whether the expansion of the hot wire for a given total heat production will be the same, when the distribution of heat along it is uniform, and when more heat is produced in some sections than others; this is the question of the validity of equation (29). Any error due to this cause will probably be small and will be reduced an unknown amount by heat conduction along the wire.

4. ADDITIONAL COMMERCIAL INSTRUMENTS

Although it has been shown that an instrument of the type under consideration can be improved by symmetrical location of

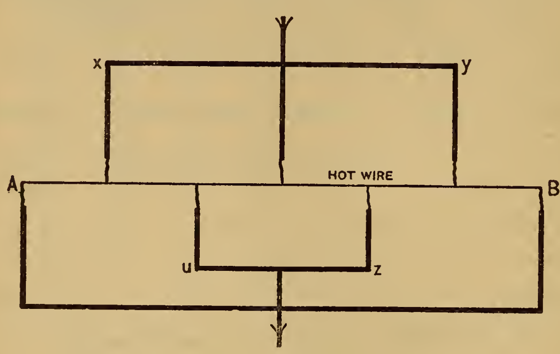

Fig. 24.—"*Unshunted*" *ammeter of six sections*

the current leads, it does not always follow that such instruments can thus be entirely freed from error. This is shown in the case of a six-section instrument represented in diagram in Fig. 24. The hot wire A B is about 10 cm long. The heavy lines indicate copper bars, which are connected to the hot wire by flexible wires. As indicated, the current enters and leaves at center points of the connecting bars. However, inspection of the diagram suggests that there are differences of current distribution because the inductances of the several current paths differ materially. It was found by a calculation similar to that made for the four-section instrument above that this instrument should read 8

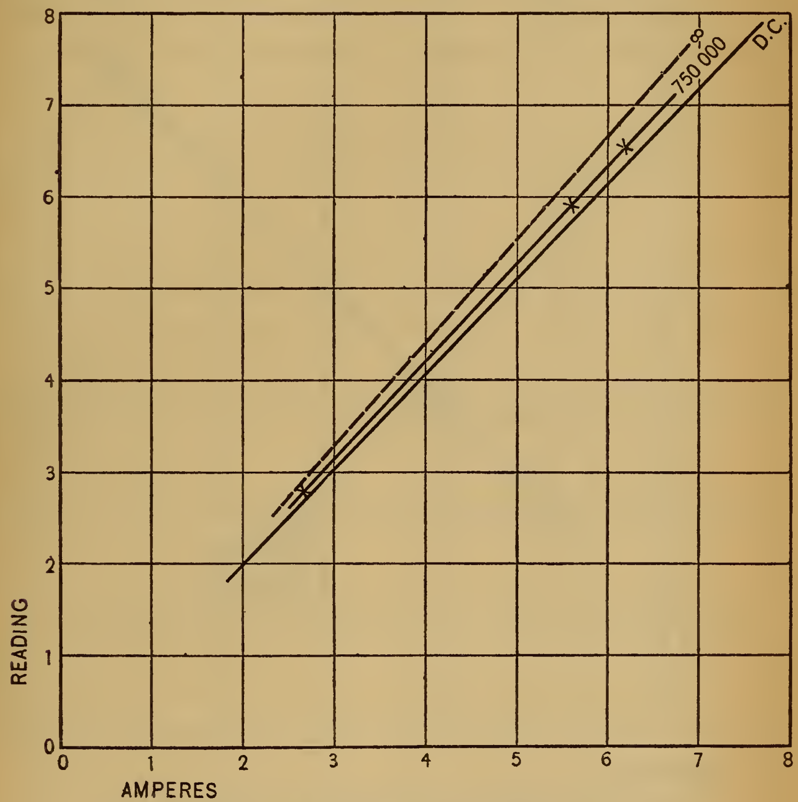

Fig. 25.—*Effect of frequency, six-section ammeter*

per cent high at a frequency so great that the resistances are negligible portions of the impedances. The results of actual measurement at 750 000 are plotted in Fig. 25, from which the increase of reading is found to be 3 per cent. It was not to be expected that the resistances would be negligible at 750 000, because the hot wire was of a high-resistance material. Nevertheless, the wire was relatively thick, having a diameter of 0.3 mm, so that the resistance was not very much greater than the resistance of the copper wires in the two-section experimental instrument described above. The observed change of 3 per cent at 750 000 is therefore of the proper theoretical order of magnitude.

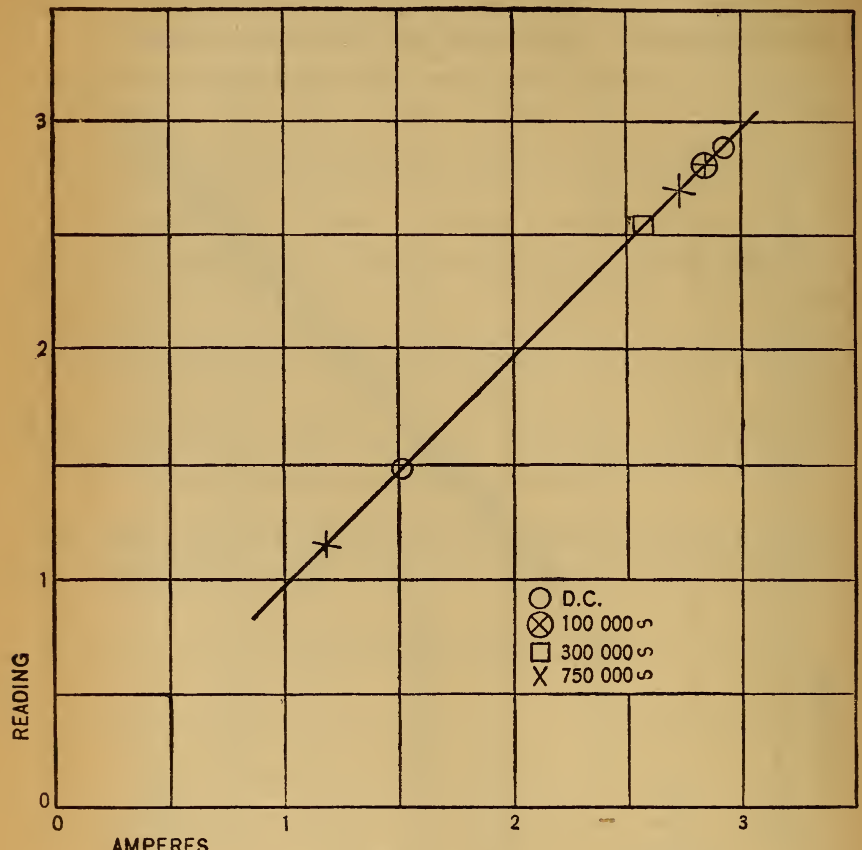

Fig. 26.—*Performance of instrument with two symmetrical sections*

The instrument was provided with a device for reducing its range, by opening the circuits at points *x*, *y*, *u*, *z*, putting the two halves of the hot wire in parallel. The range was then 3 amperes, while with six sections in parallel it should have been 9 amperes, with direct current or low-frequency current. The latter range, however, actually was 8.7 amperes, being reduced by the resistances in the connecting bars and wires (thus giving a hint of what was to be expected at high frequencies, when the inductances are all-important). Theoretical consideration of the instrument on low range, with merely the two halves of the hot wire in parallel, shows that there should be no changes of reading with frequency; and this was verified by observations at various frequencies, as shown in Fig. 26.

The results of measurements upon a commercial "unshunted"

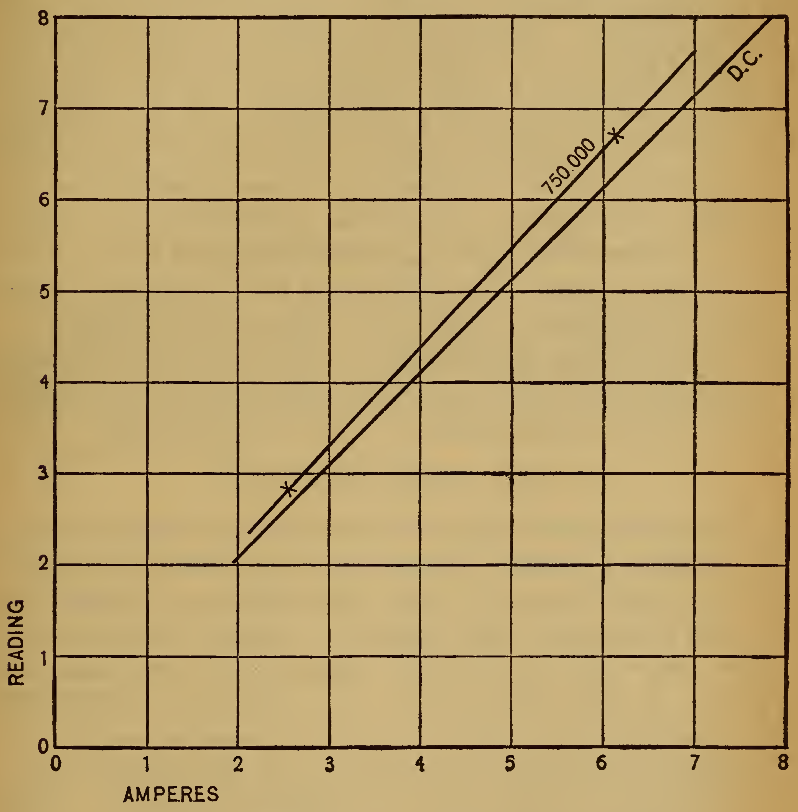

Fig. 27.—*Effect of frequency, commercial two-section ammeter*

ammeter of two sections are given in Fig. 27. The diagram of this instrument is about the same as Fig. 19. Its indicating device was of the ordinary expansion type. At a frequency of 750 000 the reading was 7 per cent high, which is the order of magnitude that theory would lead us to expect.

5. SUMMARY

To sum up the investigation of the so-called unshunted type of hot-wire ammeter, it has been seen that this type is subject to serious errors, due to the self and mutual inductances of parts which have ordinarily been thought negligible, and that the errors can be predicted from a theoretical consideration of the impedances of the parts. These errors can be avoided in some cases by symmetrical location of the current leads. The errors can be eliminated in the range of radiotelegraphic frequencies by using hot wires of very high resistance, but not by using wires of moderate resistivity, such as platinum or bronze, especially if not kept to very small diameters. Since most of each current path is made up of bars or strips of very low resistance but very considerable inductance, the current distribution at a given frequency is more affected by the inductances than in the parallel wire type, and therefore the distribution for theoretically infinite frequency is practically reached at a much lower frequency.

Having found these various errors in the two types of ammeter employing hot wires, let us turn to a consideration of instruments in which the wires are replaced by a thin strip of metal.

V. THE STRIP AMMETER

By the use of a thin strip or sheet of metal instead of wires as the working resistance, an ammeter can readily be made to carry very large currents. Such instruments are coming into use in the high-power radio stations. The first requirement is that the strip be so thin that its resistance does not change with frequency. In the instruments whose deflections are produced by the expansion of the sheet of metal it is found in practice that the mechanical inequalities of a thin metal sheet make it necessary to use only a narrow strip as the indicating portion. As the sheet must have considerable width in order to carry large currents, it must therefore be slit or otherwise subdivided. Consequently the deflections depend on the current in one part of the sheet and not on the whole current. As the frequency is increased, the deflection changes as the current distribution changes. The errors are much larger than they would be if the deflections depended on the resistance of the whole sheet.

The changes of current distribution and resistance with frequency have not been theoretically worked out for thin strips of finite width, but from the experiments made some conclusions can be drawn.

1. EXPERIMENTS ON CURRENT DISTRIBUTION IN STRIPS

The first requirement of these instruments, viz, that the strip must be thin enough not to change in resistance with frequency was emphasized by the following experiments. A strip of phosphor-bronze 0.07 mm thick, 6.2 mm wide, and 11.1 cm long was soldered at the ends to two wires at right angles to the strips. Two rectangular portions at the middle were cut away, as shown, leaving

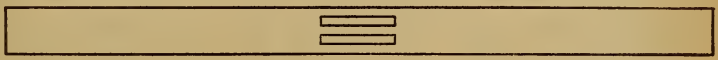

Fig. 28.—*Metal strip for investigation of current distribution*

three parallel strips 1 cm long and 1.2 mm wide separated by spaces of the same width. A thermocouple was soldered in turn on each of these three strips and the apparent change of current in each was observed for different frequencies. The results are summarized in Table 7.

TABLE 7

Per Cent Increase of Apparent Current in Phosphor-Bronze Strip

Frequency	150 000	500 000	1 000 000	1 500 000
	Per cent	Per cent	Per cent	Per cent
Middle strip............	+0.9	+4.6	+ 8.2	(+12.0)
Mean outside strip.....	+2.5	+8.6	+12.4	(+21.0)

The observations at 1 500 000 were made with insufficient sensibility and are uncertain, but all the results show a very marked increase of apparent current with increase of frequency, no matter at what point of the strip observations are made. This means a large increase of resistance with frequency. This effect would be much larger in a copper strip, and copper would have been used in this experiment, the purpose being to show the resistance change at its worst, but this could not be done because a copper strip with its low resistance did not get hot enough with the currents used to give measurable deflections. The resistivity of the phosphor-bronze was four times that of copper.

The foregoing experiment illustrates another effect, viz, the tendency of the current at high frequencies to crowd toward the outer edges of the conductor. In fact, the difference between the current in the middle and the outer strips was really greater than appears from Table 7, because the convection and conduction of heat tended to equalize the temperatures.

It would be expected that in strips of higher resistivity the changes of resistance and of current distribution would be less. This was proved by repeating the above experiment with a strip of high-resistance metal—resistivity thirty-six times that of copper. The dimensions were the same as in Fig. 28, except the thickness, which was 0.03 mm. The results of measurement were:

TABLE 8

Per Cent Increase of Current in High-Resistance Strip

Frequency	500 000	1 000 000	1 500 000
	Per cent	Per cent	Per cent
Middle strip	....	−0.5	−1.7
Mean outside strip	0.0	+0.3	+0.9

It appears that the strip is thin enough and the resistivity great enough so that there is no appreciable change of resistance up to 1 500 000. The change of current distribution, however, is perceptible, and is really greater than the results show, because of heat conduction and convection. Since the deflection of an ammeter depends on the temperature of the indicating strip, this equalization of temperature over the strip is actually an advantage, reducing the effects of change of current distribution. In the ammeters described below, the parts of the strip were not separated by spaces as wide as 1 mm, as in the present experiment, so the temperature differences would be still less. Finally, since ammeters for such large currents would rarely be used for frequencies as high as 1 000 000, it may be said that in an ammeter with the working strip made of this high-resistance metal no error is introduced by the strip itself.

2. EFFECT OF TERMINAL BLOCKS

Measurements were made on a simple ammeter of the strip type. A side view of the instrument is given in Fig. 29. B and B′ are massive brass blocks of square cross section. S is a vertical thin strip of resistance metal, soldered at the ends into the brass blocks. The strip is 0.07 mm thick, and the resistivity is about the same as that of the strip studied in the preceding paragraph. The mechanism for indicating the expansion is attached by a wire to the point I. The strip is slit as shown, so that only the expansion of the center portion is measured. The large brass blocks help to dissipate the heat, which is the main concern in an ammeter for large currents. The results of measurements are given in Fig. 30. As shown, its readings decreased 11 per cent at 750 000 and 4 per cent at 300 000, and were practically the same at 100 000

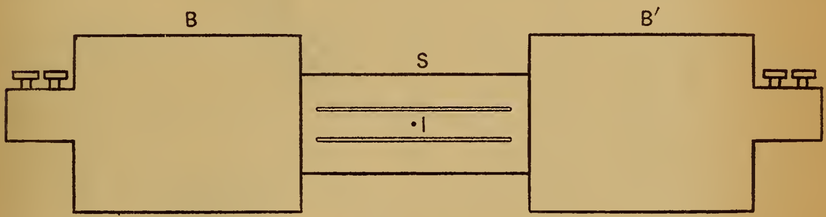

Fig. 29.—*Strip ammeter with rectangular terminal blocks*

as on direct current. This result is surprising, in view of the conclusion reached above, that the changes of current distribution in strips of this high resistivity should be negligible. All is explained when the massive terminal blocks are considered. The resistance of these is insignificant compared to the resistance of the strip, but their inductance is certainly comparable with that of the strip. For one thing, the path of the current in the blocks is longer than in the strip, and, moreover, is not of exceedingly greater cross section, because at these frequencies it is known that the current flows in a thin skin on the surface of such conductors. Thus more current will be flowing to the outer edges of the thin strip than to its middle portions.

While the readings decreased when the expansion of the middle portion was measured, they should increase if the expansion of the upper or lower part of the strip were measured. It seemed worth while to verify this experimentally, and to do so the indicating mechanism was attached to a point on the upper part of the strip instead of at the point I. The readings then *increased* 8 per cent at 750 000 and 4 per cent at 300 000. The changes of current distribution are really larger than the results indicate, because the temperatures of different parts of the strip are equalized to a considerable extent by heat conduction and convection. The

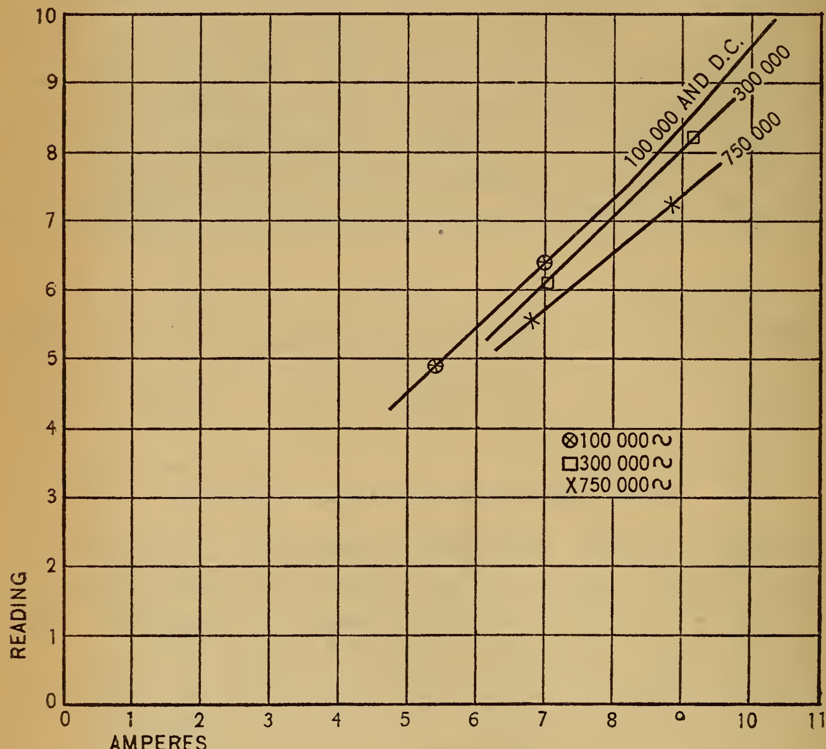

Fig. 30.—*Effect of frequency, strip ammeter*

existence of the heat flow between parts at different temperatures is verified by the fact that on high frequency the reading was observed to come up to its final value more slowly than on low frequency for which the heat production was the same in all parts. Furthermore, the effect of convection could be reduced by placing the strip in a horizontal plane, when the observed change from low to high frequency should increase. This was tried, and the increase was found to be 1 per cent, both at 750 000 and 300 000.

The terminal blocks can be so designed as to reduce their effect on the current distribution across the thin strip . For example, an improvement in this direction is to make the blocks wedge-shaped as in Fig. 31. The top sketch is a top view and the following one a side view of another ammeter tested. As the current passes along the block toward the strip, it tends more and more to become distributed uniformly with respect to the breadth of the strip. The results of measurements are plotted in Figs. 32 and 33 for two instruments of this type having strips 0.03 mm thick and of the same resistivity as before. There is no change of reading at

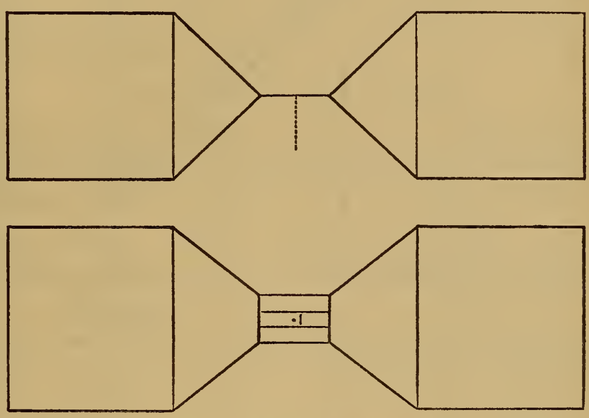

Fig. 31.—*Strip ammeter with wedge-shaped terminal blocks*

100 000 and 300 000, and at 750 000 the decrease is somewhat less than 3 per cent. The shaping of the terminal blocks decidedly improves the instrument.

A still further improvement is found in the instrument represented in diagram in Fig. 34. This design is due to Mr. F. W. Roller. The thin strip is soldered at its ends to rather long rods, at opposite ends of which the current is introduced. Each part of the strip has in series with it about the same amount of rod impedance, and hence no change of current distribution should take place. This was confirmed for frequencies up to 750 000.

The changes of current distribution with frequency in the terminal blocks of strip instruments are the counterparts of the

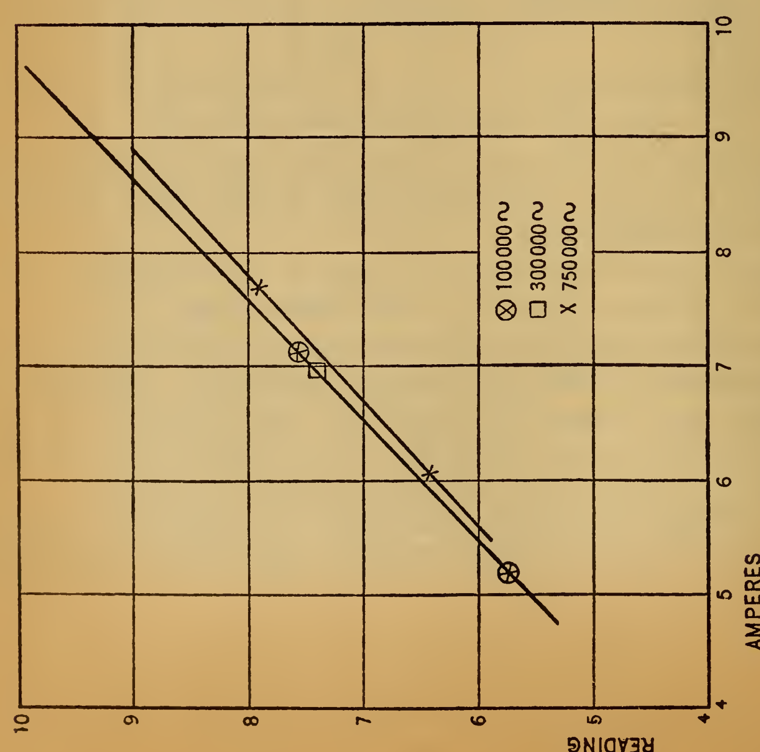

Fig. 32.—*Effect of frequency, wedge-block ammeter*

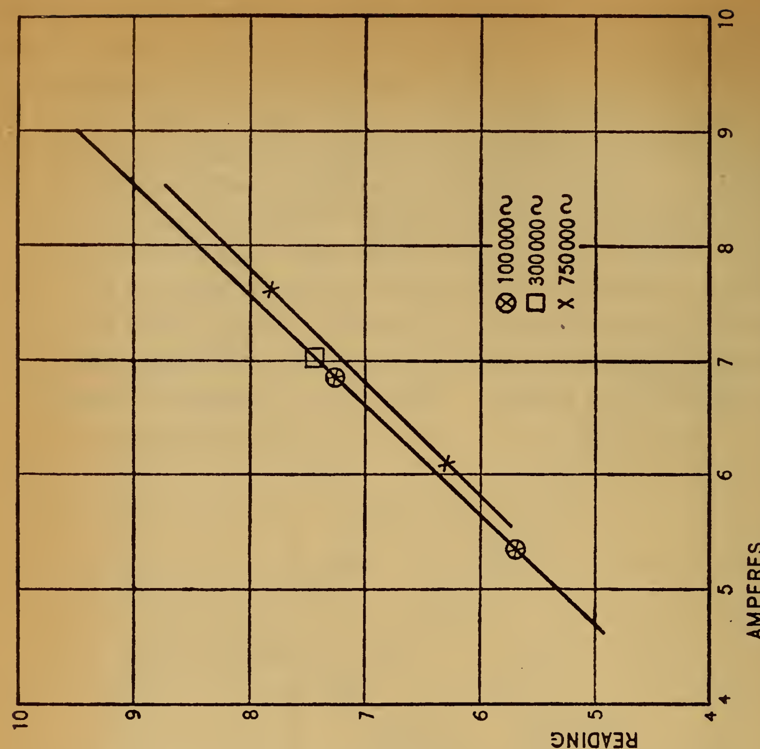

Fig. 33.—*Effect of frequency, another wedge-block ammeter*

effects in wire instruments due to the self-inductances of parts other than the hot wire, e. g., Figs. 14 and 19. The remedy, the

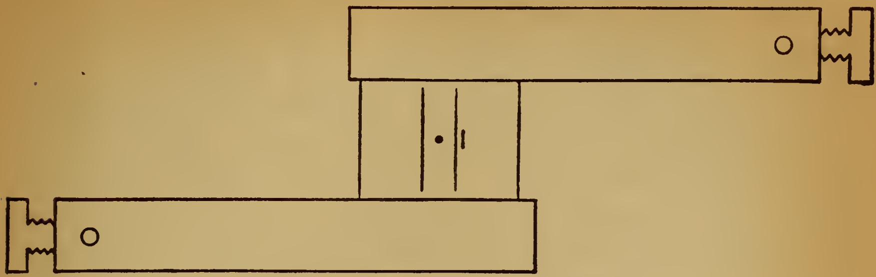

Fig. 34.—*Strip ammeter with rod terminals*

type of current lead shown in Fig. 34, is the counterpart of the disposition of leads suggested in Fig. 15.

3. CYLINDRICAL ARRANGEMENT

An instrument which avoids most of the difficulties is one in which strips are arranged in parallel equidistantly on a cylindrical surface, so that each has the same set of mutual inductances with respect to the others. This is the same principle as in the wire instrument shown in Fig. 16. If the strips are put closer and closer together this type approaches the limiting case of a thin tube, which theoretically has uniform current distribution on high and low frequencies. Such an instrument employing strips, the expansion of one of which is measured, has been designed by R. Hartmann-Kempf [16] and is now on the market. The strip must be thin enough and of such high resistivity that it does not change appreciably in resistance at the frequencies used. It is very difficult in practice to get such thin strips of uniform thickness, so that the resistances of the strips are likely to differ, although the inductances would be equal, which would cause considerable changes of reading with frequency. Strips of platinum or of platinum-rhodium are used in such instruments because of their excellent thermal qualities, but their rather low resistivity makes them subject to this source of error. This kind of strip instrument was not tried out because none was available to the writer, but its principle and performance are illustrated by the instrument of Fig. 16.

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[16] See footnote 7, p. 107.

4. SUMMARY

To summarize the consideration of the strip ammeter, the thin strip with its terminal blocks or other leads is a complex problem. By using thin enough strip of high enough resistivity the effects of resistance change and current distribution within the strip itself can be made negligible, but the current distribution in the terminals may greatly alter the current distribution in the strip at high frequencies. This can be avoided by suitable shaping and connection of the terminals. A good way to avoid the errors is to arrange strips in parallel on the surface of a cylinder. Great care is necessary to see that the different strips in this arrangement are sufficiently uniform in resistance.

In concluding the description of this investigation the author desires to express his thanks to Prof. E. B. Rosa, of the Bureau of Standards, and to Prof. E. P. Adams, of Princeton University, for their helpful encouragement, and to Mr. F. W. Roller, of New York City, for the loan of instruments.

VI. CONCLUSIONS

1. The circuit within a high-frequency ammeter must be of as simple form as possible. This requirement is best fulfilled by a single straight wire of very small diameter, and no other arrangement can be taken as à priori reliable at all frequencies. The heat production is readily measured in any form of circuit, and in consequence all successful ammeters for high frequency utilize the thermal effect. The thermometric means of measuring the heat production, whether expansion, calorimetric effect, resistance, or thermal emf, does not affect the accuracy. Two similar wires in parallel, the integrated heat production in the whole being measured, constitute a system nearly as reliable as a single wire. With such a system, in an oil bath, currents up to 10 amperes can be measured. For larger currents other combinations of current elements are necessary, and a great variety of ammeters have been developed and have been studied experimentally and theoretically in this investigation. Experimental instruments, and commercial instruments of three different companies, have been included. All the types in use are subject to errors when

used at the frequencies of radiotelegraphy, the errors being in some cases very large. In some the readings increase with increase of frequency and in others decrease. In most cases the design can be so changed as to eliminate the errors.

2. The current *in a conducting circuit* has no meaning for extremely high frequencies; for in any circuit, above a certain high frequency, the capacity between parts of the circuit and of auxiliary apparatus is so important that an appreciable fraction of the current is shunted through the dielectric and the current is of different amount in different parts of the wire circuit. This was demonstrated for the circuits used in these experiments at a frequency of 1 500 000 (wave length = 200 meters); for the current was found to be different in two ammeters in series, because of the large capacities to earth of lead wires and galvanometers. The apparent errors due to this effect were of the order of 5 per cent. A way was found to eliminate the effect, for the purposes of ammeter comparison.

3. The changes of current distribution, in the instruments whose working parts were of low-resistivity metal, all occurred in about the range of radiotelegraphic frequencies, 100 000 to 1 500 000. That is, the current distribution was constant for frequencies from 0 up to about 100 000, then underwent changes and became constant at different values for frequencies above about 1 500 000. The agreement of the range of these changes with the frequencies of radiotelegraphy is a remarkable coincidence. For these instruments and the circuits used, in view of this fact and conclusion (2), just above, it may be said that 1 500 000 is physically infinite frequency in two senses.

4. The order of agreement found between theoretical calculation and experiment shows that the ordinary formulas for self and mutual inductances of finite linear parts of a circuit hold for the short lengths used. In view of the frequencies used and the fact that the oscillations were somewhat damped, this is one of the more interesting results of the work from the standpoint of pure science.

5. An approximate experimental method for investigation of the current distribution in thin strips at high frequency has been devised, and used to obtain qualitative results. Further

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investigation of thin strips as to current distribution and resistance, both experimentally and theoretically, is recommended as a subject for research. The subject will increase in importance, for probably more current can be carried with a given skin effect error by a thin strip than by a round wire of the same cross section.

6. Most of the errors of commonly used high-frequency ammeters have been found to be due to the mutual inductances, or to self-inductances of parts, which had been supposed negligible

7. Some errors which have been suspected were found negligible. Eddy currents in adjacent masses of metal were found to produce no effect. The inductive actions of distant parts of the circuit and of the leads when brought straight in to the instrument were found negligible. In some cases the leads do change the readings appreciably, when close to and parallel to the working parts of the instrument, and some caution in regard to them is therefore necessary.

8. The use of high-resistance metals in the working parts, *keeping them of very small cross section*, eliminates errors in most cases. This expedient has the effect of moving the changes of current distribution up to frequencies higher than those with which it is desired to work. However, other considerations sometimes make the use of larger cross sections or of the lower resistivity materials desirable. In these cases, then, the effects of current distribution may be appreciable at ordinary working frequencies.

9. The location of the current leads is of great importance, particularly in the so-called unshunted ammeter, determining whether the error shall be large or inappreciable.

10. All errors due to inductive action of the leads can be avoided by bringing them in at right angles to the plane of the instrument. This is a very helpful arrangement in experimental instruments made to isolate and study particular effects.

11. In the hot-strip ammeters, if the strip be thin enough and of sufficiently high resistivity, the observed errors depend entirely on the current distribution in the terminal blocks, and can be eliminated by proper design.

12. An instrument free from theoretical objections consists of current elements arranged equidistantly on a cylindrical surface, the leads being brought in to the middle points of the ends of the cylinder. The current elements may be fine wires, or they may be replaced by thin strips of considerable width or by a continuous thin tube. The instrument has the limitation treated in the next paragraph, in common with the other types of ammeter.

13. The most insidious error of all is nonuniformity of resistance of the working parts, in any ammeter for large high-frequency currents. Two wires or strips of the same length and approximately the same cross section will have the same self-inductance, but the resistances may be quite different because of variations of hardness and small variations of cross section. Thus they may carry exactly the same currents at high frequency, but very different currents at low frequency. This error arises from the difficulty of obtaining and preserving wires and strips of such small cross section sufficiently uniform. This error was rather unexpected, but was surprisingly evident in a number of cases investigated. The practical result is that any high-frequency ammeter whatever, employing more than a single fine wire, is subject to change of current distribution.

14. These experiments furnish very good illustrations of the fact that the changes of current distribution within a particular system are changes of the first order of magnitude, compared to which the change of resistance of the whole is of the second order. By taking advantage of this principle, it has been shown that some of the types of ammeters can be greatly improved.

15. In conclusion, the various effects which determine the deflections of high-frequency ammeters have been isolated and critically studied, by experiment and by the aid of theoretical calculation. Some sources of error which had been suspected were found negligible, and some other effects were found to produce errors of surprisingly great magnitude. Ways of eliminating the various errors have been given.

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Fig. 7.—*Meters complete*