DEFLECTION POTENSIOMETERS FOR CURRENT AND VOLTAGE MEASUREMENTS

By H. B. Brooks

1. INTRODUCTION

The deflection potentiometer differs from potentiometers hitherto used in one essential feature, namely, the use which it makes of the galvanometer. The latter has formerly been regarded as a mere indicator of the absence of a difference of potential, dials or slide wires being adjusted to give the condition of no current through the galvanometer. In the deflection potentiometer, as the name indicates, the normal condition of the galvanometer is that of deflection due to the passage of a current through it, and its reading gives several figures of the result.

In previous papers¹ the writer has given an outline of the principles on which have been based two forms of deflection potentiometer used by this Bureau. Both of these were constructed for voltage measurements only, one being in use in the photometric laboratory and one in electrical instrument testing.

It was early recognized² that current measurements were possible with this type of potentiometer, using a suitable shunt, and in view of the advantages already secured in voltage measurements by the use of the deflection potentiometer, efforts were made to secure the special high-grade pivoted galvanometer necessary for such a potentiometer for use with shunts. At first it was thought desirable to plan a potentiometer for current measurements only, but as the work progressed it became evident that one instrument

¹ This Bulletin, 2, p. 225; 1906 (Reprint No. 33). ⁴, p. 275; 1908 (Reprint No. 79).
² This Bulletin, 2, p. 237; 1906 (Reprint No. 33).
could be made to measure both current and voltage. The object of the present paper is to give a brief description of two recent forms of deflection potentiometer, which are quite similar in design, and are called for convenience Model 3 and Model 5, respectively. The theory of the use of current shunts with the deflection potentiometer will then be given, and the most suitable values for such shunts will be pointed out.

The need for electrical measuring instruments which combine accuracy and speed is continually becoming more apparent. It is not sufficient that one can get an accurate value, given steady sources of current and voltage, skilled help, and plenty of time. Instruments are wanted which will give the desired accuracy even on unsteady circuits, the result being quickly obtained; the manipulation must be simple and capable of being quickly comprehended by a workman of average intelligence. External disturbing influences (such as temperature changes and stray magnetic fields) must not cause errors in the result, as corrections for temperature are time-consuming, and stray magnetic fields can not always be avoided.3

2. OUTLINE OF POTENTIOMETER THEORY

In Fig. 1, (a) represents in outline a simple potentiometer for measuring voltages not exceeding that of the auxiliary battery. A battery $e_1$ sends a current through a resistance $r_1,r_2$ which may be varied to enable the current to be kept constant as the emf of $e_1$ changes. $E$ is a source (battery or dynamo) whose emf is to be measured. The positive poles of $E$ and $e_1$ are directly opposed, and the negative pole of $E$ is connected (through a galvanometer) to such a point on the resistance that the emf $E$ is balanced by the fall of potential or "drop" in the portion $r_1$ of the resistance. This condition is shown by the absence of a deflection, and by Ohm's law we have:

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3 The need for this class of standard instrument is well stated by William Bradshaw in the following words (Electric Journal, 3, p. 394; 1906): "The refinement obtainable in electrical measurements by the use of null potentiometers is well known, but they are not adapted for measuring alternating current or unsteady direct current. The ideal instrument for this service is one that approaches the accuracy of the null potentiometer and allows the ease and quickness of manipulation necessary in the ordinary test room."
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\[
E = \frac{r_1}{r_1 + r_2} \times e_1, \text{ or } E = r_1 \times \frac{e_1}{r_1 + r_2}
\]

Hence if the current \( e_1/(r_1 + r_2) \) is adjusted to a convenient integral value, \( E \) may be read off from the value of \( r_1 \) which gives a balance. The usual and preferable method of adjusting the current to the proper value consists in replacing \( E \) by a standard cell, setting \( r_1 \) to a value proportional to the known value of the cell, and varying \( r_1 + r_2 \) until no deflection occurs. In most forms of potentiometers

the substitution of the standard cell for \( E \) is made by a suitable switch. Values of \( r_1 \) are frequently not marked in ohms, but in volts, on the assumption of a particular working current, or method of applying the standard cell.

When voltages much in excess of 2 volts are to be measured, it is convenient in most cases to use a volt box, as shown in Fig. 1 (b). \( R \) is a resistance high enough to be connected in parallel with the source \( E \) without appreciably changing the potential difference of the points to which it is connected. A suitable fraction of this resistance, \( R/p \), is connected to the circuit \( r_1 r_2 \) just as the
source $E$ was connected in Fig. 1 (a). When $r_1$ has been adjusted so that the galvanometer shows no deflection, we have:

$$\frac{1}{\rho} E = \frac{r_1}{r_1 + r_2} e_1, \text{ or } E = r_1 \times \frac{e_1}{r_1 + r_2} \times \rho$$

where $\rho$ is the ratio of the whole resistance $R$ to the portion $R/\rho$; or $\rho$ is the multiplying factor of the volt box.

The measurement of current is next to be considered. Fig. 1 (c) shows a source which passes the current $i$ through a standard resistance or "shunt" $R$. When balance is obtained as before, we have

$$R i = r_1 \times \frac{e_1}{r_1 + r_2}, \text{ or } i = \frac{1}{R} \times r_1 \times \frac{e_1}{r_1 + r_2}$$

Here the fall of potential over the shunt, as shown by the value of $r_1$, must be divided 4 by $R$, the resistance of the shunt.

In order to adjust $r_1$ to give no deflection of the galvanometer, it is necessary that the value $E$ to be measured be quite steady. In some forms of potentiometer the resistance $r_1$ consists of a series of four or five dials, each of which has a value per step one-tenth that of a step on the dial above. The dials are set in succession, beginning with the highest, until balance is obtained. In other forms one dial is used, a calibrated slide wire taking the place of the three or four lower dials. In the deflection potentiometer two dials may be used, but it has been found desirable thus far to use but one dial, thus reducing to a minimum the manipulation necessary.

If we assume in Fig. 1 (a) that balance is not obtained, then the current $i_g$ through the galvanometer is given by the expression

$$i_g = \frac{e_1 \frac{r_1}{r_1 + r_2} - E}{r_g + \frac{r_1 r_2}{r_1 + r_2}}$$

4 It is convenient instead to multiply by $i/R$, the "conductance" of the shunt, especially for shunts of values below 1 ohm, as is generally the case.
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This expression shows that the galvanometer current is equal to the difference between the voltage indicated by the dial or slide wire setting \( r_1 \) and the unknown emf \( E \), divided by the total resistance in the galvanometer circuit. This total resistance is to be figured as if the sources \( E \) and \( e_1 \) contained no emf. This formula shows the possibility of reading any desired part of the result on the galvanometer, provided its scale is properly calibrated and the total resistance of the galvanometer circuit is kept at a proper value.

The use of a volt box, as shown in Fig. 1 (b), gives an expression similar to the preceding. The value as read from the potentiometer dial and galvanometer scale is multiplied by the factor of the volt box used, to get the unknown voltage under measurement.

When a shunt is used, as in Fig. 1 (c), the current through the shunt is in general not equal to the line current which is to be measured, being greater or less than the line current by the amount of the galvanometer current. It has been found that a simple expedient will take this fact into account, and make the reading of the deflection potentiometer (when divided by \( R \), the resistance of the shunt) give accurately the value of the line current. It is only necessary to count in \( R \) as part of the galvanometer circuit, and provide means for keeping this circuit of constant resistance when different values of \( R \) are used. The proof of this is reserved for a later paragraph (see p. 410).

Reduced to its lowest terms, the principle of operation of the deflection potentiometer is very simple, as an illustration will show.

Fig. 2 is a repetition of Fig. 1 (a), except that the storage cell \( e_1 \) and the source \( E \) are not shown. The gap left in the lower circuit by the removal of the storage cell is closed by a wire of resistance equal to that of the cell, which is usually negligible. We thus have the equivalent of a potentiometer with its auxiliary current suppressed. It may be shown that if we regard what is left (galvanometer plus the resistances \( r_1 \) and \( r_2 \) in parallel) as a voltmeter, calibrating it to read the potential difference between

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* This Bulletin, 2, p. 230; 1906 (Reprint No. 33). 4, p. 281; 1908 (Reprint No. 79).
the points marked + and −, we may then replace the auxiliary cell $e_1$ and make measurements either with the null method or with a combination of null and deflection methods. In the latter case the two actions are simply superposed, neither interfering with the other. The galvanometer having been calibrated with a definite resistance in circuit, namely, $r_1$ and $r_2$ in parallel, we have to provide the means (not shown in Fig. 2) for keeping

![Fig. 2](image)

the galvanometer circuit of constant resistance as $r_1$ and $r_2$ vary with the changes of the null part of the result.

3. ARRANGEMENTS OF CIRCUITS

A number of arrangements are possible in planning the circuits of a deflection potentiometer. It is not even necessary that the galvanometer circuit be of constant resistance; in fact, the first instrument constructed has a different value of galvanometer circuit resistance for each setting of the main dial. It is much more convenient, however, as regards facility of construction and subsequent checking, to use the plan of constant resistance. For detailed discussion of various possible plans of circuits the reader is referred to preceding articles.⁶

⁶ This Bulletin, 2, pp. 230–234; 1906 (Reprint No. 33). 4, pp. 276–286; 1908 (Reprint No. 79).
The plan of circuits used in the instruments about to be described is shown diagrammatically in Fig. 3.

In this figure, $E$ is the source whose voltage is to be measured, using a volt box of resistance $R$. The "potentiometer wire" AB is supplied with current from the auxiliary storage cell $e_1$, this current being regulated by the rheostat $r_3$ in series with the cell and the rheostat $r_6$ in shunt with the potentiometer wire. It will be noted that as $r_3$ is increased, $r_6$ is decreased; both of these changes tend to reduce the current flowing through the potentiometer wire AB. The values of $r_3$ and $r_6$ are so chosen that their resultant resistance in parallel is a constant; thus the resistance from the point B through the regulating rheostats and storage battery to the point A is constant for all settings of the rheostats. This simple expedient removed a very considerable difficulty in the design of convenient circuits for deflection potentiometers.

The resultant resistance in the potentiometer circuit proper thus consists of the resistance $r_1$ shunted by the sum of $r_2$ and the constant resistance of the regulating rheostats. This resultant resistance is zero when the sliding contact is at the point $A$, and increases as the slider moves to the right up to a point somewhat past the middle of AB, then decreases to a minimum value (greater than zero) at the point B. The resistance of the compensating rheostat $r_4$ is such that (for any setting) the sum of $r_4$ and the resultant resistance through the potentiometer is constant. The remainder of the resistance in the galvanometer circuit consists of the portion $R/p$ of the volt box shunted by the rest of the volt box, the resultant resistance being figured as if the source $E$ had zero resistance and no emf. If a volt box is not used, the voltage to be measured is applied to the points which (in Fig. 3) are shown as joined by the coil $R/p$. If the galvanometer circuit was of correct resistance when the volt box

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7 It has been asked whether the deflection potentiometer would not thus be in error if the source $E$ contains appreciable resistance. It has been shown (this Bulletin, 4, pp. 294–297 (Reprint No. 79)) that no error is introduced, whatever the internal resistance of the source. The quantity to be measured is (in such a case) not the internal or total emf of the source, but the difference of potential at the terminals of the volt box $R$, as in any potentiometer measurement with a volt box.
was used, it is necessary (when working without the volt box) to insert in series with the galvanometer a ballast resistance equal to the resultant of $R/p$ shunted by the remainder of the volt box. The potentiometer will now measure the voltage at its terminals.\(^8\)

When current measurements are to be made, the terminals of the potentiometer (volt box having been removed) are connected to the potential points of a suitable shunt, whose resistance is to be counted in as part of the required ballast resistance. If another shunt of different value is used, a suitable change must be made in the ballast resistance. By proper arrangement of the dials and switches, the values of resistance for the different circuits are

\(^8\) If it were desired to run "pressure wires" of appreciable resistance from the potentiometer to the points where the voltage is to be measured, the resistance of these pressure wires should be counted as part of this ballast resistance.
obtained automatically; no attention on the part of the operator being required, except when changing shunts.

By providing a suitable switch, it is possible to throw the potentiometer terminals quickly from a current shunt to a volt box, or from one shunt to another, or one volt box to another. In such work as checking indicating wattmeters, or in watthour meter testing, it is convenient to go quickly from voltage to current, and vice versa.

A double-throw switch for this purpose is provided in addition to a similar switch used for making the change from unknown quantity to standard cell.

4. DESCRIPTION OF INSTRUMENTS

Figure 4 shows a plan of circuits of the Model 3 potentiometer, which has been designed for general measurements of current and voltage in laboratories whose requirements include a reasonable degree of accuracy combined with speed of working.

The main dial has 30 steps of 5 ohms each. In series with it is a coil of 1.80 ohms and a standard-cell dial of 10 steps of 0.01 ohm each. The Weston portable unsaturated standard cell only is used; it is balanced around the last two-thirds (100 ohms) of the main dial, plus the 1.80-ohm coil and the standard-cell dial. Thus it is possible to use standard cells whose values are from 1.0180 to 1.0190 volts inclusive. This covers the range of variation of these unsaturated cells sufficiently well, as cells may be bought with the specification that they shall fall within this range, and within several units of the lower end of the dial, to

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9 The writer is indebted for this and other valuable suggestions to Dr. C. H. Sharp. The use of a multiple-point switch for enabling measurements on any one of six circuits is a feature of the pioneer Crompton potentiometer. It was not thought desirable to carry this idea further than two circuits in the potentiometers here described. If occasion arises for quickly connecting a potentiometer to any one of three or more circuits, this can readily be done by combinations of double-throw switches, or by receptacles and plugs.

10 The figures given in this paragraph refer to the value of the 1.80-ohm coil as it has been adjusted to meet the change which occurred January 1, 1911, in the value of the international volt. As at first made, the coil had 1.862 ohms resistance, and cells having values of 1.0188 to 1.0198 volts (on the old basis) were provided for.
Fig. 4.—Plan of Circuit, Model 3 Potentiometer.
provide for the slight decrease of emf to be expected in a period of years. The standard current through the main dial coils is thus 0.01 ampere; it is furnished by a storage cell. The series rheostat \((r_3\) of the preceding discussion) has a minimum value of 20.85 ohms, and increases by 15 steps of 0.1 ohm each. The shunt rheostat \((r_6\) has a minimum value of 88.9 ohms, and increases by 15 steps to a maximum of 123.4 ohms. A fine rheostat of 0.5 ohm in the battery circuit covers any step of the coarse rheostat, and has a compensating resistance of 0.3 ohm in the galvanometer circuit. The circuits are so designed that the compensating resistance \((r_j\) of the main dial repeats at 90, 95, \ldots 150 the values for 80, 75, \ldots 20; hence a number of coils are saved by using cross connections.

The galvanometer key has a protective resistance of 2,400 ohms, which is in circuit on the first contact and is cut out on full depression. The total resistance in the galvanometer circuit under working conditions is 60 ohms between the binding posts marked "Volt Box," which with 40 ohms resultant resistance in the volt box makes up the normal total of 100 ohms. The total resistance measured between the binding posts marked "Shunt" is 100 ohms when the circular plug rheostat near these posts is plugged at the extreme right, and is less than this by amounts of 0.1, 0.2, 0.5 \ldots \ldots 40 ohms when the plug is placed in succession toward the left. This allows the total resistance to be kept 100 ohms when using shunts of the values just given, the resistance of the shunt being counted in the total.

A view of this instrument is given in Fig. 5. The galvanometer is mounted below the hard-rubber top, giving an improvement in appearance over the preceding model. The scale of the galvanometer is made direct-reading by the method of marking used.

The main dial is placed at the right so that the observer can use his right hand for setting it, and for recording results, while

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11 For the explanation of the large value which the fine rheostat must have in comparison with a step on the series rheostat, see this Bulletin, 4, pp. 284–285; 1908 (Reprint No. 79).
12 This Bulletin, 4, pp. 288–289 (Reprint No. 79).
the left hand is used to operate the galvanometer key. The coarse and fine rheostats are at the left; these require attention only at very infrequent intervals as compared with the main dial. The standard-cell dial is above the main dial; its setting remains unchanged so long as a cell of one particular value is used. The two double-throw switches are at the left of the galvanometer. While slight changes of level produce no appreciable error in the reading of the galvanometer, a small circular level is provided to enable the instrument to be kept in a definite plane when desired.

The aim in producing this particular model has been to supply a deflection potentiometer for general use in electrical engineering laboratories and instrument testing rooms, and in view of the fact that 0 to 1.5 volts is a range very frequently used for potentiometers, and since 150-division scales are convenient and much used in portable instruments, the range of this potentiometer is made 0 to 1.5 volts, \(^{13}\) the dial being marked from 0 to 150. The decimal point is not put in, since in most cases a potentiometer is used with a volt box or a shunt, introducing a multiplying factor. Ten divisions of the galvanometer scale equal one "dial unit" of 0.01 volt; hence one division corresponds to one millivolt. By estimation one can read tenths of a division, or parts in 15,000 of the total range.

The volt box used with this potentiometer is a special one, having 5 ranges \(^{14}\) of 4.5, 15, 45, 150, and 300 volts. The only special point in the construction of this volt box is a plug switch, by means of which the resultant resistance introduced by the volt

\(^{13}\) While the nominal range is 0 to 1.5 volts, the actual range is from \(-0.03\) to \(+1.53\) volts; this extension of the usual range is often very convenient. For example, 150-volt voltmeters often require a little more than 150 volts for full-scale deflection; with a null potentiometer this requires changing the range of the volt box for the full-scale point. With the Model 3 potentiometer, using the 150-volt range of the volt box, the upper limit is 153 volts, which enables one to check an instrument 2 per cent in error.

\(^{14}\) It is believed that a more convenient set of ranges for the volt box is 3, 15, 30, 150, and 300. With these, the reading of the potentiometer is multiplied by 2 (except for the 15 and 150 ranges) and the decimal point located.
box into the galvanometer circuit is kept constant. The plan of circuits of this volt box is shown in Fig. 6.

The rubber-covered binding posts are located at the back of the potentiometer, and are mounted on a hard rubber apron extending down from the top, so that a cover may be placed over the potentiometer, when it is not in use, without disturbing connections.

The galvanometer requires a current of 10 microamperes per scale division. It has a coil resistance of about 10 ohms, and the total resistance in the galvanometer circuit for the condition of aperiodic motion (critical damping) is 100 ohms. On closing the circuit, the pointer comes to its deflected position (without passing it) in 1.2 seconds. The galvanometer was supplied by the Weston Electrical Instrument Co., of Newark, N. J., and the potentiometer was constructed and the galvanometer mounted by The Leeds & Northrup Co., of Philadelphia, Pa.

A view of the potentiometer with its accessories is shown in Fig. 7, which shows a wattmeter connected for test. The voltage
supply line is at the left, a slide resistance being used to set at the desired value. The current supply line (from a storage battery of 2 to 8 volts) is at the right. The current is controlled by a carbon rheostat, and flows through an oil-immersed\textsuperscript{15} shunt whose potential terminals are connected to the "shunt" terminals of the potentiometer. The volt box, storage cell, and standard cell are back of the potentiometer. This outfit is convenient for such work, as one observer at the potentiometer can hold the voltage constant and also measure the successive values of current required to produce the desired deflections of the wattmeter. It is desirable to have reversing switches at the wattmeter, to enable two readings to be made at each point tested, reversing the current through the wattmeter for the second reading, to get a mean value in which no error due to (constant) local field enters. These switches have not been shown in the figure.

The special requirements of photometric work make it desirable to have a volt box of high resistance. To meet this condition a deflection potentiometer has been devised which is termed Model 5. It differs from Model 3 in the values of its coil resistances and in the constants of its galvanometer. In general appearance it is a duplicate of Model 3, Fig. 5, and the plan of circuits, Fig. 4, applies to Model 5, except as to values of the coils, and the point on the main dial at which the standard cell tap is taken off. The main dial has 30 steps of 20 ohms each, the last 100 ohms of the dial being in series with a 1.80-ohm coil\textsuperscript{16} and a standard cell dial of 10 steps of 0.01 ohm each, providing for the same range of standard cell values (1.0180 to 1.0190 volts) as in Model 3. The coarse and fine rheostats have coils of 4 times the resistance of those of Model 3. The protective resistance in the galvanometer key is 96 000 ohms, being 24 times\textsuperscript{17} the normal total resistance

\textsuperscript{15} These are usually preferred in the work of the Bureau, as giving greater accuracy; but properly designed air-cooled shunts may be used in many cases.

\textsuperscript{16} See note 10, p. 403.

\textsuperscript{17} The normal maximum deflection of the galvanometer is 25 divisions, and with a protective resistance of 24 times the normal total, the deflection is to be brought to 1 division or less, by manipulating the main dial. If the key be now fully depressed, the reading will not be over 25 divisions.
Fig. 7.—Potentiometer and Accessories, with Wattmeter Connected for Test
(4000 ohms) in the galvanometer circuit. The design is such that a large part (3000 ohms) of this total resistance is available for the resultant resistance of the volt box. The volt box is similar to the one already described (see Figs. 6 and 7), but has the ranges 18 15, 45, 150, and 300 volts, the resistance of the 150-volt range being 75,000 ohms, and the other ranges in proportion. This high value reduces the power used by the volt box to 0.16 watt when 110 volts is applied to the 150-volt range. To secure this high volt-box resistance requires (in addition to a high-resistance galvanometer coil) the use of more voltage on the main dial, which has a range (when no volt box is used) of 0 to 6 volts instead of the usual 0 to 1.5 volts. The auxiliary current is supplied by a battery of four small storage cells in an oak case.

The Weston galvanometer used requires a current of 1 microampere per scale division. It has a coil resistance of about 500 ohms, and the total resistance for aperiodic19 motion is 4000 ohms. The pointer comes to rest in 1 second after closing the circuit.

The high fundamental range of Model 5 potentiometer makes it unsuitable for the measurement of heavy currents, but as photometric work has little or no occasion to exceed 15 amperes, the power spent in the shunt is not important. For use in the laboratories of central stations, electrical engineering departments of colleges, instrument makers, public service commissions, municipal testing bureaus, and others who wish to make electrical measurements over usual commercial ranges with speed and accuracy, the Model 3 potentiometer is to be preferred.

18 It would probably be more convenient to have a 30-volt range instead of 45; see note 14, p. 406.
19 It should be more generally recognized that a moving-coil galvanometer ought to be so proportioned to the circuit in which it is to be used that the condition of critical damping exists. If the external resistance is below the value required for this condition, the galvanometer coil will "creep" slowly to its position of rest, resulting in loss of time and greater liability of error. If the external resistance is too high, the motion is periodic, and time is lost while the observer waits for the oscillations to subside. If the condition of critical damping can not be had, it is better to have slight underdamping.
The accuracy of measurement of the two models is the same, and a conservative figure is 0.4 of a galvanometer scale division (0.04 dial unit), which is equivalent to one twenty-fifth of 1 per cent for measurements at two-thirds of the main dial. In the two instruments in use at this Bureau, the accuracy is about twice as good as the above.

5. USE OF A CURRENT SHUNT WITH THE DEFLECTION POTENTIOMETER

In referring to the use of a current shunt with the deflection potentiometer, it was stated that no error is caused by the abstraction of some of the line current from the shunt to operate the galvanometer, provided the resistance of the shunt is counted in as a part of the normal total resistance in the galvanometer circuit. The proof of this statement will now be outlined.

In Fig. 8 a source $E$ provides the current $i$ whose value is to be measured. With the potentiometer balanced so that the galvanometer current is zero, all of the current $i$ flows through the standard resistance or shunt $R$. With a current $i_g$ flowing through the galvanometer in the direction indicated, the current through $R$ is less than $i$; with a current $i_g$ in the opposite direction, the current in $R$ is greater than $i$. For the sake of generality it is assumed that there is other resistance $R'$ in the source circuit. The potentiometer consists of the circuit $r_1 + r_2$ supplied with current from the auxiliary battery $e_1$. A sliding contact connected to one terminal of the galvanometer enables the value of $r_1$ to be varied while $r_1 + r_2$ remains constant. For simplicity the regulating rheostats $r_3$ and $r_6$ of the preceding figure are not shown, since it may be seen that their resultant resistance may be thought of as part of $r_2$. The compensating resistance $r_4$ also is omitted, as it is in series with the galvanometer only, and may be thought of as a part of the galvanometer resistance.

From Kirchhoff's laws we have:

$$i_1 r_1 + (i_1 - i_g) r_2 - e_1 = 0$$

$$i_1 r'_1 + i_g r'_g - (i - i_g) R = 0$$
and the solution for \( i \) gives

\[
e_i \frac{r_1}{r_1 + r_2} + i_g \left( \frac{r_g + \frac{r_2^2}{r_1 + r_2} + R}{r_1 + r_2} \right) = \frac{e_i r_1}{r_1 + r_2} + i_g \frac{r_2}{r_1 + r_2}
\]

(31)

The first term in the numerator is the emf indicated by the setting \( r_1 \) on the potentiometer wire \( r_1 + r_2 \). The second term is the ohmic drop due to the galvanometer current \( i_g \) assumed to be flowing around the closed path through the potentiometer and through the shunt \( R \); hence, if the galvanometer be calibrated so that it indicates correctly as a voltmeter when it has in series with it a resistance equal to \( r_1 r_2 / (r_1 + r_2) + R \), the sum of its reading and the null reading \( e_i r_1 / (r_1 + r_2) \) will (when divided by \( R \), the resistance of the shunt) give the value of the current \( i \) to be measured. The important thing about this expression for \( i \) is...
that it does not contain\textsuperscript{20} the terms $E$ and $R'$, the emf and the resistance in the source. Hence, all that is necessary is to provide means for keeping the galvanometer circuit of the proper constant resistance when using different shunts, to get a measurement of the line current $i$ which shall be free from error, regardless of the amount of current taken from the shunt to operate the galvanometer.

If we assume that the shunt $R$ is removed, and the object is to use the same potentiometer for voltage measurements, we find on solving for the emf of the source

$$E = e_1 \frac{r_1}{r_1 + r_2} + i_0 \left( \frac{r_1 r_2}{r_1 + r_2} + r_0 + R' \right)$$

(32)

Hence, to measure the total emf of the source, all that is necessary is to count in its internal resistance $R'$ in providing the proper total galvanometer resistance. The preceding equation may be written in the form

$$E - i_0 R' = e_1 \frac{r_1}{r_1 + r_2} + i_0 \left( \frac{r_1 r_2}{r_1 + r_2} + r_0 \right)$$

(33)

This shows that when the potentiometer is to measure the external difference of potential of a source whose resistance has any value $R'$, the resistance in the galvanometer circuit should be complete without using $R'$.

\textsuperscript{20} The terms $E$ and $R'$ are really in the expression in disguise, as the galvanometer current $i_g$ depends upon both. However, the value of $i_g$ is known from the reading of the galvanometer, so that no knowledge of conditions in the source is required. This is analogous to the case of the deflection potentiometer used with a volt box for voltage measurements. In the latter case the current through the galvanometer depends upon the value of the emf and resistance of the source, but the value of the voltage at the terminals of the volt box (which is the quantity to be measured) is obtained without the necessity of knowing anything about the source.

By taking the upper part of the network in Fig. 8, we get the equation

$$i R' + (i - i_g) R - E = 0.$$ 

Using this with the two Kirchhoff equations for the rest of the network will enable a more general solution for $i$ to be obtained, free from $i_g$ and containing only $e$ and $r$ terms. This is of no value in potentiometer design, but is of interest in some special cases.
It will thus be seen from the foregoing equations that the potentiometer ought not to be narrowly regarded as a null instrument only, since in the general case of any current flowing through the galvanometer which the latter can measure, the principles of operation of the resulting deflection potentiometer (or "generalized potentiometer") are simple, and may be conveniently carried out in the construction of accurate and quick-working standard instruments for current and voltage measurements.

With the older forms of potentiometer it has been the common practice to use current shunts whose values are decimal multiples or submultiples of an ohm, as 10, 1, 0.1, 0.01 ohm, and so on. When used with a 1.5-volt potentiometer, these shunts are convenient for certain ranges; for example, a 0.1-ohm shunt is just what is desired for use in checking a 15-ampere ammeter of 150 divisions, or a 10-ampere ammeter of 100 divisions. However, for a 3-ampere or 7.5-ampere instrument such a shunt is not the most convenient, as the reading of the potentiometer requires (in addition to the location of the decimal point) the use of the factor 2 (or 5). This requires some calculation during the test to determine for the various readings of the ammeter the corresponding settings of the potentiometer in order not to waste time hunting for the balance. It also requires computation in working up the results and in scrutinizing the work for possible errors. Inasmuch as the speed of working of the deflection potentiometer is much greater than that of null potentiometers, it is well to avoid loss of time in auxiliary matters, so as to bring the whole system of measurement up to a high state of efficiency. It is easy to choose convenient shunt values that will be free from the foregoing objections.

Assuming that the fundamental range of the potentiometer is, say, 150 "dial units", for rapid and convenient work one scale division on an instrument under test should correspond to 1 dial unit. To accomplish this, the current per scale division of the ammeter, multiplied by $R$, the resistance of the shunt, must equal 1 dial unit; or

$$R \text{ of shunt} = \frac{\text{emf corresponding to 1 dial unit}}{\text{amperes per division of ammeter}}.$$
Since in practice both numerator and denominator will have integral values, the shunt values called for are not abnormal, as examples will show. Take the case of Model 3 (or any 1.5-volt) potentiometer. One dial unit equals 0.01 volt; hence, for ammeters of 1, 2, 5, 10, and 20 amperes per scale division the current shunts required are 0.01, 0.005, 0.002, 0.001, and 0.0005 ohm, respectively. A single shunt will usually do for several ranges; thus the 0.01-ohm shunt is adapted for 100-ampere 100-division, 120-ampere 120-division, and 150-ampere 150-division instruments. The same shunts are equally convenient in testing wattmeters of corresponding current range; in this case 100 volts is applied to nominal 110-volt potential circuits.

Examination of the ammeter ranges given in a prominent maker's catalogue showed that over the whole range from 150 milli-amperes to 10,000 amperes, the following shunt values would be required (neglecting the decimal point): 1, 1.25*, 2, 2.5*, 3*, 4*, 5. The values marked * will seldom be needed, as the values 1, 2, and 5 (neglecting the decimal point) will provide for over 80 per cent of the ranges.

With such shunts, the observer at the ammeter sets successively on 10, 20, 30...... divisions of the scale, calling out these numbers; the observer at the potentiometer sets the main dial to the same numbers, and depresses the key. The small deflection of the galvanometer gives the correction to be applied to the instrument under test, one division of the galvanometer corresponding to 0.1 division of the ammeter; deflections to the right are + corrections, to the left, —. The work may be plotted on the form 21 shown in Fig. 9, putting a pencil mark on the proper vertical line; if the galvanometer reads 2 divisions to the right, the current is greater than the reading of the ammeter, and the pencil mark is put two divisions below the zero line of the chart; if the galvanometer reads 1 division to the left, the mark is put one division above the zero line on the chart. The scale points may be checked several times if desired, and a smooth curve drawn through the

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21 This form is recommended in the Meter Code of the National Electric Light Association; see report of committee on meters, N. E. L. A., for 1910, p. 141.
pencil marks. Thus a correction curve may be quickly drawn without recording a single figure, and without any computation; much time may be saved and errors avoided.

In this connection may be mentioned a neat arrangement of shunts which is used by a German manufacturer, and which would be convenient for use with the deflection potentiometer. As shown in Fig. 10, several shunts are soldered together, and their free ends are connected to a millivoltmeter. The line current enters at A, and may leave at B, C, or D. The section AB is of relatively low resistance and large carrying capacity; BC is of lower carrying capacity and higher resistance, and so on. The advantages of this method are as follows

1. In changing the range, but one connection has to be shifted.
2. The millivoltmeter circuit remaining closed, the connections may be well made, and errors due to dirty or loose contacts are avoided. Evidently, the millivoltmeter could have all connections soldered if the shunts were attached to it and did not need to be changed. The one movable contact is in the line, and variations of its resistance do not affect the accuracy of the result.

When applied to the deflection potentiometer, this set of shunts has the advantage that the plug rheostat ordinarily used for keeping constant galvanometer resistance for different shunts can be

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22 Hallo und Land; Elektrische und Magnetische Messungen und Messinstrumente, p. 254.
set to a value marked with the sum of the resistances of the set. Then the plug need not be changed, since no matter where the line current leaves the set, the resistance in the galvanometer circuit is constant.

6. CONCLUSION

The "Outline of Method of Design" given in a previous paper 23 has been revised and considerably extended, and will be published as a separate article. With it are included some notes on the design of moving-coil galvanometers, showing how to determine the important constants of an existing type of galvanometer, and what changes to make in spring strength, magnet strength, and size of wire in order to produce a galvanometer of certain desired constants.

Further progress in the development of the deflection potentiometer, such as the attainment of higher volt-box resistance, or lower fundamental ranges, will depend upon the improvements possible in the galvanometer. While the galvanometers used in the two potentiometers just described are of high grade, there is reason to look for still better performance due to certain changes of dimensions and material of coils. It is hoped that these improvements may be realized in the near future.

The deflection potentiometer combines, in suitable proportions, the accuracy and reliability of the (null) potentiometer with the quickness of working, ease of reading, and independence of fluctu-

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23 This Bulletin, 4, p. 298; 1908 (Reprint No. 79).
ations which characterize a well-damped deflection instrument. It makes the potentiometer no longer a time-consuming device, to be resorted to only at intervals when necessity demands, and avoided meanwhile by the use of "secondary standard" deflection instruments, which are but little better than the portable instruments they are used to check. With a deflection potentiometer in use, the manipulation and reading require but little more training, effort, or time than the reading of a portable instrument, and every day's work of instrument checking is done as it should be, namely, by reference to standard resistances and standard cells.

Washington, June 23, 1911.