THE CORRECTION FOR EMERGENT STEM OF THE MERCURIAL THERMOMETER

By Edgar Buckingham

CONTENTS

Two important methods for determining the "correction for emergent stem" to be applied to the reading of a mercurial thermometer when the thermometer is not used with "total immersion" consist, first, in the use of an auxiliary stem as described by Guillaume [1]; and, second, in the use of the "fadenthermometer" or thread thermometer as described by Mahlke. [2] The original accounts of these two methods are not available in English and neither of them has been discussed very fully. In view of this and of the further fact that the standard of precision of mercurial as of other thermometry has continued to advance since the original publications, it seems worth while to treat the subject in some detail. For various suggestions as to both form and substance the writer is much indebted to other members of the heat division, and especially to Dr. H. C. Dickinson.

1. THE STANDARD SCALE

It is universally considered desirable that statements of temperature should be made in terms of the scale of the ideal-gas thermometer as an ultimate standard. This scale, also called the "thermodynamic scale," is at present represented by a series of

---

[1] Zeitschrift für Instrumentenkunde, 12, p. 69; 1892. 13, p. 155; 1893.   [2] Ibid., 13, p. 58; 1893.

numerical values assigned to certain reproducible "fixed points" at which primary standard determinations of temperature have been made by using gas thermometers filled with helium, hydrogen, nitrogen, or air. Such a gas thermometer gives *directly* only readings in terms of its own individual scale, but if the properties of the gas have been sufficiently investigated, corrections may be applied which convert these readings into values of the temperature on the ideal-gas scale. Usually, though not always, these corrections are less than the uncertainties of even the best determinations of temperature by the gas thermometer.

On account of the difficulties of accurate work with the gas thermometer, some discrepancies still exist in the values obtained for definite fixed points, and the corrections for reducing to the ideal-gas scale are also somewhat in doubt, though this latter uncertainty is relatively unimportant. The series of numbers to be adopted as the most probable values, on the ideal-gas scale, of the temperatures of any set of fixed points, is therefore to a small extent a matter of opinion.

The series of fixed points and the numerical values for their temperatures adopted by the authorities of a given standardizing laboratory constitute the foundation of the standard scale of that laboratory and provide, so to speak, a certain number of base points. The definition of the scale is completed by stating the means used for interpolation between the base points in determining intermediate temperatures.

When a thermometric instrument for such interpolation has been standardized at two or more base points, it becomes a secondary standard for the given range. For determining temperatures between any two adjacent base points, various secondary standards, dependent for their action on entirely different physical principles, may often be used, and each form of instrument may have its particular advantages and disadvantages of precision, convenience, etc., for a given piece of experimental work. It is therefore usual to have each interval between the base points covered by two or more secondary standards of different nature. The greater the number of different methods of interpolation, and the simpler the means adequate to attaining a satisfactory agreement of the instruments in defining a single

scale over the interval in question, the greater is the confidence usually felt that the scale so defined would agree with an interpolation by a primary standard gas thermometer. It is by no means certain that this confidence is always justified.

The standard scale of another laboratory may be slightly different because of the adoption of other fixed points or of other numerical values as most probable, and also by the adoption of other methods of interpolation. But if the definitions of the scales are clear in these respects, each standard scale is definite and the relation between two such scales, if not immediately evident, may be found by simple experiments capable of a precision far surpassing the accuracy with which the standard scales represent the ideal-gas scale.

Up to about 1,650° C, there are, with one or two exceptions, no very serious divergences of opinion as to the base points, and the various methods used for interpolation also agree closely; so that the expression "standard gas scale" may be regarded as sufficiently precise in meaning for our present purposes, even though we feel sure that a slightly closer approximation to the ultimate standard scale of the ideal gas will be attained in the future.[3]

2. THE MERCURY-IN-GLASS SCALE

Let us consider a mercurial thermometer which has a capillary stem of exactly uniform cross section. Let the glass be such that its changes of volume with temperature are reversible and show neither frictional nor viscous hysteresis. Let a mark be placed on the stem exactly at the point where the end of the mercury column stands when the mercury and its surrounding glass are at the temperature of the ice point, and let another mark be similarly placed for the steam point. The external as well as the internal pressures on the bulb must be the same when the two marks are made. Let the stem be provided with a scale of equal parts [4] upon which these marks are at 0° and 100°, respectively.

---

[3] See Note I at the end of this paper.
[4] The scale divisions are to be of equal length when the glass on which the graduations are made is all at the same temperature.

Then wherever the end of the mercury column may stand, if the whole of the mercury and the surrounding glass are at the same temperature, and if the internal and external pressures on the walls of the bulb are the same as when the 0° and 100° marks were made, the reading on the scale is *by definition* the numerical value of the temperature, on the centigrade scale of this thermometer.

The ideal thermometer just described is, like the ideal gas, nonexistent. The actual mercurial thermometer is a more or less approximate representation of it, and by the application of suitable corrections, the readings of the actual thermometer may be reduced to those of the ideal thermometer at the same uniform temperature. These corrections are as follows: (*a*) The scale correction to allow for errors of graduation; (*b*) the calibration correction to allow for nonuniformity of bore; (*c*) the external pressure correction to allow for the effect on the volume of the bulb, of variations in the outside pressure; (*d*) the internal pressure correction to allow for similar effects due to variations of pressure within; (*e*) the zero correction to allow for the fact that at the temperature of the ice point the reading is not exactly at the zero mark of the scale; (*f*) the fundamental-interval correction to allow for the fact that the difference of the readings at the temperatures of the ice and steam points is not exactly 100° of the scale.

If the thermometer has been suitably constructed, all these corrections may be determined by experiment. By the determination of the corrections, the thermometer is converted into a primary standard thermometer which gives, after the application to its readings of these several corrections, numerical values of temperature in terms of the mercury-in-glass scale of this particular thermometer.[5] It is not our purpose here to discuss these various corrections, but merely to call attention to the fact that by means of them, we may find the readings of an ideally perfect thermometer made of the same glass as the actual thermometer, so that we may dismiss the corrections from further consideration and refer at once to the ideal thermometer as if it did exist, and to its scale of temperature as something definite.

[5] These corrections have been discussed by Messrs. Waidner and Dickinson in this Bulletin, **3**, p. 663, where full references will be found.

The indications of a mercury-in-glass thermometer depend on the relative expansion of mercury and glass; and since glass is not a precisely reproducible substance and different kinds of glass expand and contract differently, no two mercurial thermometers define precisely the same scale. Different thermometers, well made of as nearly as possible the same kind of good thermometer glass, do, however, define almost exactly the same scale, which is then known as the mercury-in-glass scale of that particular glass. A determination of temperature even by a primary standard mercurial thermometer has no precise meaning except in connection with one particular glass, and to give it any general significance the result must be expressed in terms of some scale which is better defined, or more easily reproducible. The scale actually used for this purpose is the standard gas scale described in section 1, and to reduce a primary standard determination of temperature on any mercury-in-glass scale to the standard gas scale, one further correction, the "gas-scale correction," is needed.

At ordinary temperatures, the gas-scale correction is comparatively small for the common thermometer glasses, but at high temperatures it may be very large. According to Mahlke's observations,[6] at 500° C of the standard gas scale, a mercurial thermometer made of Jena 59$^{\mathrm{III}}$ borosilicate glass reads 527°.8, so that the gas-scale correction is −27°.8.

Ordinarily, mercurial thermometers are not primary standards but are standardized by comparison with other thermometers. Thus, in any case, the corrections given by the test reduce the readings to some other scale than the mercury-in-glass scale of the thermometer being tested, and it is most convenient to make the reduction directly to the standard gas scale, the temperature of the bath in which the test is made being given by some instrument which has been standardized as a secondary interpolation instrument for the gas scale, as described in section 1.

3. THE CORRECTION FOR EMERGENT STEM

It was assumed in the preceding section that whenever the mercurial thermometer was read, the whole of the mercury and the surrounding glass were at a uniform temperature, for if this

---

[6] Zeitschrift für Instrumentenkunde, **15**, p. 178; 1895.

is not the case the reading has by itself no definite meaning. In order, therefore, that the results of testing a mercurial thermometer should be capable of precise interpretation, the corrections are stated for "total immersion," i. e., they refer to readings taken when the whole of the thermometer, at least up to the end of the mercury column in the stem, is immersed or enclosed in a bath or space of uniform temperature.

In practice, mercurial thermometers are often used with only partial immersion, and if the temperature of the space into which the bulb of the thermometer is inserted is markedly different from that of the surroundings of the emergent stem, the temperature of a part of the stem and its enclosed mercury column will be different from that of the bulb and, furthermore, will not be uniform. If, for example, a high temperature is to be determined, the emergent stem will be colder, the mercury column shorter, and the reading lower than if the thermometer were totally immersed. To allow for this fact, a "correction for emergent stem," or more briefly a "stem correction," must be applied. This stem correction is of sensible importance more frequently than is sometimes recognized, and may in extreme cases amount to as much as 30° C or even more.

The temperature of the glass above the end of the mercury column has no direct influence on the reading. But mercurial thermometers for use above 200° or 250° C have to be filled above the mercury with nitrogen or other inert gas under pressure, to prevent the mercury from boiling, so that raising the whole of such a thermometer to a high temperature might, by increasing the internal pressure beyond that intended by the maker, cause an explosion and the destruction of the thermometer. In any high-temperature mercurial thermometer used with the gas space at a lower temperature than that to be measured, there is a troublesome tendency for the mercury to distill off and condense in the colder part of the gas space. It may be impossible to reunite the resulting drops of mercury with the column below, so that an error is produced equivalent to a permanent depression of the ice point. To reduce this distillation, the temperature of the meniscus should be kept as low as possible, so that high-temperature mercurial thermometers must, in general, be used with a portion of

the stem emergent and at a much lower temperature than that of the bulb.

In a large majority of cases it is needless to attempt to determine the stem correction closer than to about 5 per cent of itself. For frequently when the correction is large, which occurs usually only in work at high temperatures, such causes as unsteadiness of the temperature or failure of the thermometer to repeat its readings under identical conditions, make it impossible to be certain of the final result of the measurement of a temperature closer than to 5 per cent of the value of the stem correction, no matter how exactly this value itself may be known. We shall speak of such work as "ordinary work," in distinction from "work of high precision," in which a greater final certainly in the result is attainable and the highest possible accuracy is desired.

The magnitude of the stem correction can be found either directly or by calculation. Since it obviously depends on the length and temperature of the emergent stem, it can, in principle, be calculated from measurements of these quantities in connection with data on the relative expansion of mercury in the glass of the stem; but certain difficulties in both the theory and the practice of this method make it advisable, in most cases, to use the much simpler direct method described by Guillaume. This will be treated first.

4. THE USE OF AN AUXILIARY STEM

Let SB (Fig. 1) be the thermometer of which the stem correction is to be determined, immersed to the level h in a bath or space of uniform or nearly uniform temperature. Let the bulb B and the lower part of the stem up at least to the level d, have the uniform temperature $t°$, expressed in the standard gas scale. The upper part of the stem and enclosed mercury column ending in the meniscus at c, have a different temperature, lower let us say.

Let A be an auxiliary stem similar in material and construction to a portion of the working stem S and of approximately the same inside and outside dimensions. It is to be sealed at both ends and the space above the mercury is to be exhausted or filled with gas under pressure to correspond with the conditions in S. A is provided, in the vicinity of the meniscus, with a scale of equal parts, e. g., a millimeter scale. The length l of the mercury

column in A must be sufficient that when A is placed parallel and close to the working stem S and with its meniscus at the same level c, its lower end at d reaches into the region of uniform temperature. The magnitude of l is of little importance so long as it is great enough. It is assumed that both A and S are of sensibly uniform bore.
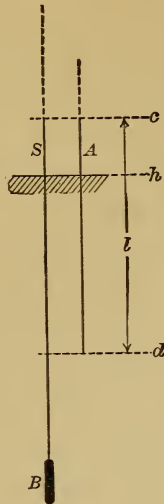

Fig. 1

When A and S are thus placed parallel and close together, the temperature at any given level may be assumed to be the same in both, this temperature being, on the average, lower than t^o between c and d, if t^o is above the temperature of the surroundings of the emergent stem. If A were now totally immersed and thus brought to the temperature t^o, its meniscus would rise by a distance $\varDelta$ in consequence of the rise in temperature of most or all of its elements. Since S is of the same glass as A, the same increase of length of the mercury column cd in S would occur if the temperature of each of its elements were also raised to t^o by total immersion, while no change would occur in the volume of the mercury below d since that is already at the temperature t^o. Hence the meniscus in S would rise by the same amount $\varDelta$, and $\varDelta$ is the linear magnitude of the stem correction to be applied to the reading on the working stem S to reduce the reading to the condition of total immersion. The correction *in degrees* is then

given by the equation

$$K = n\Delta \qquad (1)$$

in which n is the number of degrees of the scale of S, at the level c, included in one scale division of the auxiliary stem A. The problem is now to determine the value of Δ from observations on A, and for this purpose we need, in addition to the reading when in the position shown in the figure, the reading with total immersion at the temperature t°.

If the conditions are similar to those during the comparison of high-temperature thermometers in a well-stirred oil bath, the operation is very simple. After an observation of the position of its meniscus when at the level c, the auxiliary stem is lowered to a position of total immersion, left a short time (to be determined by trial) till it assumes the uniform temperature t°, then quickly raised just far enough to make another observation possible, and read immediately. The glass of the stem being thick and a poor conductor, the reading after raising changes but slowly, and if made at once may be taken as the reading for total immersion. The difference of the two readings is the value of Δ in terms of the scale of A.

Under the above conditions, when the space of uniform temperature is large enough to permit of total immersion of the auxiliary stem, the method just given is the simplest and most expeditious possible for finding the stem correction. If A satisfies the conditions of being made of the same glass as S, and of approximately the same dimensions, it requires no preliminary study. It is not necessary that the length of its divisions be known in millimeters. All that is needed for converting Δ into degrees of the working stem S is that the number n of degrees on S per scale division of A shall be known. This may be found, after removal from the bath, by using A as a scale with which to measure the degrees on S. Since the two scales are graduated on glass of the same kind, the fact that the comparison is made at room temperature instead of the higher temperature does not introduce any error. It is not necessary that the value of t° be known, even approximately, so that the stem correction may be applied to the reading of S before any other corrections and even if the other

corrections—zero correction, reduction to gas scale, etc.—are unknown.

If the space of uniform temperature is not long enough for total immersion of the auxiliary stem, the foregoing method can not be used; and if the space, though long enough, is changing in temperature rather rapidly, the time required may be sufficient to introduce a sensible error—the reading after total immersion corresponding to a different temperature from that of the bulb B when the original reading was made. In either case, the difficulty may be surmounted by standardizing the scale of the auxiliary stem, once for all, by total immersion in baths of known temperatures so that its reading with total immersion at any temperature $t°$ may be found from a table of the results of the standardization. If we know the values of the relative expansion of mercury in the given glass, and the value, in scale divisions, of the length of the column at the ice point, the scale reading for total immersion at any other temperature, $t°$, may also be computed from that at the ice point, and an experimental standardization is then unnecessary. Since the value of $t°$, which is usually the quantity sought, is involved in this process, successive approximations may be needed. For both these reasons the method loses, in such cases, something of its primitive simplicity and directness.

The stipulation was made that A and S should be of sensibly uniform bore, though it will be noted that we have, in reality, used only the condition that the cross-sectional areas of the bores should have the same ratio at all levels, not that each should be constant. The condition was put in the simpler form because uniformity of bore is always the ideal aimed at, and the stems of well-made thermometers do in practice have nearly cylindrical bores. A complete treatment of the general case in which the cross-section ratio varies with the level will not be attempted because it would certainly not be worth while from a practical standpoint. Accurate determinations of temperature can not be obtained with poorly made mercurial thermometers, and it may safely be assumed that thermometers which are satisfactory in other respects will have stems of sufficiently uniform caliber that, when used with auxiliary stems of a similar degree of uniformity, the error introduced into the stem correction by assuming both stems to be exactly uniform, will not be serious.

5. THE "FADENTHERMOMETER" AND ITS USE

As noted above, the use of an auxiliary stem may require a preliminary standardization by total immersion. If we define the "mean temperature" of a column of mercury enclosed in glass as the uniform temperature at which the mercury would fill the tube between the same two marks on the glass, the auxiliary stem, after the standardization, may be regarded as a thermometer which indicates its own mean temperature [7] and therefore that of the adjacent portion *cd* of the working stem *S*. This mean temperature $f°$ is given in terms of the scale used in standardizing the auxiliary stem, which we assume to have been the gas scale.

As a thermometer, the simple auxiliary stem is very insensitive, for if the mercury column is 20 cm long it requires a change of of about 30° in the mean temperature to change the reading by 1 mm. The sensitiveness and precision of reading may evidently be increased by adding a still finer capillary stem in which the changes of level of the meniscus may be observed on a magnified scale, and by this modification Mahlke converts the simple auxiliary stem into a thermometer which differs from the more usual forms only in having a very elongated, thick-walled, cylindrical bulb of small total volume, and an unusually fine stem.

Like the simple auxiliary stem of which it is a development, the bulb of this "fadenthermometer" must be of similar construction and dimensions to the working stem with which it is to be used. The method of making sure that the temperature of the auxiliary instrument is the same at any level as that of the neighboring portion of the working stem, by having the two as nearly as possible geometrically similar and similarly placed, is thus retained. But the determination of the stem correction directly as a *length* Δ is abandoned, and a computed value of Δ, involving the length and mean temperature of the emergent stem, and the relative expansion [8] of mercury in glass is substituted for it. The fadenthermometer need not, therefore, be made of the same glass as

---

[7] See Note II at the end of the paper.
[8] See Note III at the end of the paper, also sec. 6

the working stem S.[9]   As in the case of the auxiliary stem, the length of the bulb of the fadenthermometer is of minor importance if it is sufficient, though it should not be excessive, as will be shown in section 8.

Let SB (Fig. 2) be the thermometer for which the stem correction is desired, the meniscus being at the level c above the surface of the bath h, so that a length ch *at least* is at a different mean temperature from the bulb B and the lower part of the stem S.
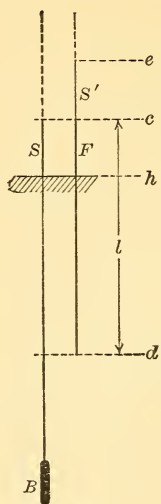

Fig. 2

As before, we may assume for the sake of concreteness that B is at a higher temperature than the emergent stem.

Let F be the bulb of the fadenthermometer set parallel and close to S with its upper end at the level c.   Let S' be the very fine thread of mercury, ending at e, in the stem of the fadenthermometer.   Its volume is small and we may, for the present, disregard the fact that its temperature is not quite the same as that of the bulb F.   The length of the bulb of the fadenthermometer must be sufficient that its lower end, at the level d, is within the region of uniform temperature containing the bulb of the main thermometer.   It may now be assumed that the mean temperature of F is the same as that of the length cd of the

---

[9] But see Note II at end of the paper.

working stem S, which includes all of S which is not at the same temperature as B.

Let the fadenthermometer have been standardized in terms of the gas scale by immersion in baths of uniform temperature so that its reading gives the mean temperature $f°$ of its bulb. Let $t°$ be the true temperature of B, also in terms of the gas scale. Let a_f be the total relative expansion of mercury in the glass of which the working stem is made, from $0°$ to $f°$, and let a_t be that from $0°$ to $t°$. Let l be the length in centimeters of the bulb of the fadenthermometer measured *at room temperature*. Let n be the number of degrees per centimeter on the scale of the working stem at the point where the reading is made, also measured when the stem is *at room temperature*. Then the value of the stem correction, K, is given in degrees by the equation [10]:

$$K = nl \, \frac{a_t - a_f}{1 + a_f} \tag{2}$$

If n, l, t, and f have been determined, it remains to find the values of a_t and a_f. For this purpose the relative expansion of mercury in the glass of the working stem must have been investigated and expressed in the form of a curve, a table, or an equation from which, knowing t and f, we find a_t and a_f.

This process may be modified by introducing the mean *coefficient* of relative expansion between $f°$ and $t°$—i. e., the average rate of increase of a, per degree, within this interval. This mean coefficient α is defined by the equation—

$$a_t - a_f = \alpha(t - f);$$

so that equation (2) may be put into the equivalent form—

$$K = \frac{nl\alpha(t - f)}{1 + f\alpha_f} \tag{3}$$

in which α_f is the mean coefficient of relative expansion between $0°$ and $f°$.

The value of $1 + a_f$ or $1 + f\alpha_f$ is always close to unity and may be computed with sufficient accuracy by setting $\alpha_f = 0.000\ 16$, for any glass and any value of f. When $f \lessgtr 500°$, the value of α_f lies

---

[10] See Note III at end of the paper, where the deduction of this equation is given.

46905°—12——2

between 0.000 15 and 0.000 18, and using these as limits, we have the following table of values of $1 + f\alpha_f$:

$f = 300°$	$400°$	$500°$
$1 + 0.000\ 15f = 1.045$	1.060	1.075
$1 + 0.000\ 16f = 1.048$	1.064	1.080
$1 + 0.000\ 17f = 1.051$	1.068	1.085
$1 + 0.000\ 18f = 1.054$	1.072	1.090

From this it appears that even at $f = 500°$, the greatest possible error due to using the fixed value $\alpha_f = 0.000\ 16$ is only 1 per cent, an amount which would invariably be negligible at such a high stem temperature. At lower values of f, the error is less, both because f is a factor of the error and because the true value of α_f is closer to 0.000 16. It is therefore always sufficiently exact to write equation (3) in the form

$$K = \frac{nl\alpha(t-f)}{1 + 0.000\ 16f} \tag{4}$$

For practical use this may be put into the more convenient form

$$K = nl\alpha(t-f) - A \tag{5}$$

where A is 1 per cent of K for each 60° in f. If, for example, $f = 300°$, $A = 5$ per cent of K so that in "ordinary work" A may be disregarded when f is less than 300°.

Since α depends on both t and f, it can not be represented exactly by a curve, a table with one argument, or an equation in one independent variable, so that it seems, at first sight, as if equations (3), (4), and (5) would be less convenient in practice than equation (2). In fact, however, to the degree of precision needed here, these equations are easier to use than equation (2), for the following reasons:

(*a*) The value of α does not vary very much, so that for many rough calculations it is sufficiently exact to use a constant value of α regardless of the values of f and t.

(*b*) The variation of α with f and t being slow, a comparatively small table with two arguments is sufficient to give the required value of α, by a single reading with no interpolation, or a very easy one, to an accuracy at least as high as that of the experimental data on the relative expansion.

(c) To the precision required in determining a stem correction this double-argument table may be replaced by one with the single argument $\dfrac{t+f}{2}$, which may also be made quite short without requiring any but the simplest interpolation.

We shall therefore use equations (3), (4), and (5) in preference to the apparently simpler but really less convenient equation (2). Numerical values of α will be considered in section 6, but we now proceed to the consideration of the quantities n and t, which appear in the second members of equations (3), (4), and (5).

The value of n may be readily found by measurement with a millimeter scale, and in a region of uniform graduation requires no further comment than that its percentage accuracy must be commensurate with that desired in K. High-temperature thermometers are often, however, graduated uniformly over intervals of 50° or 100° with abrupt changes in the length of the scale divisions at the even 50° or 100° marks, the change in n amounting sometimes to 6 or 7 per cent. If, as frequently happens in testing thermometers, the observed reading falls just below such a point while the correction carries the reading past it, the proper value of n will evidently be an appropriate weighted mean value. For ordinary work it is sufficiently exact to use the value of n for the interval in which the greater part of the correction falls, for this will never differ by more than 3 or 4 per cent from the correct mean value. When the highest accuracy is desired, the possibility of introducing an unnecessary error by neglecting this point should not be overlooked.

The temperature $t°$ in the equations is the temperature of the bulb B expressed, like f, in terms of the gas scale. Its value may be known, but more often it is the quantity sought and is to be found by applying the stem correction K to the observed reading. If t_1 is the reading after the application of all the known corrections, leaving only the stem correction outstanding, we have $t = t_1 + K$, and may easily derive an exact equation for K in terms of the observed temperature t_1. But it is more convenient to proceed by successive approximations, the first consisting in the use of the observed t_1 instead of t in equation (4) or (5). If K is small,

the first approximation may suffice, but if not, a second is easily made. For example, in equation (5) let

$$n = 8, \quad l = 19. \text{ cm}, \quad \alpha = 0.000 \; 17, \quad t_1 = 430°, \quad f = 200°.$$

We then have $nl\alpha = 0.0258$ and $A = 3.3$ per cent of K, so that the first approximation gives us

$$K = 0.0258 \; (430\text{–}200) - A = 5.93 - 0.20 = 5.73,$$

whence the value of t is $435°.73$. In the second approximation A will have sensibly the same value as in the first, and we have

$$K = 0.0258 \; (435.73\text{–}200) - A = 6.08 - 0.20 = 5.88,$$

whence the value of t is $435°.88$, or $0.15°$ higher than before, a difference which would usually but not always be negligible.

The error in t due to stopping at the first approximation K_1 is very nearly $\dfrac{K_1^2}{t_1 - f}$, so that it is easy to see whether, in any given case, a second approximation is needed. Frequently it is, but a third is probably always superfluous.

6. THE COEFFICIENT OF RELATIVE EXPANSION; NUMERICAL VALUES

If all temperatures could be expressed in terms of the mercury-in-glass scale of the glass of which the working stem is made, the coefficient of relative expansion would be constant by definition and could be determined by a dilatometer experiment between the ice and steam points. But since, for reasons already given, we express all our temperatures in terms of the standard gas scale, the value of α is not constant, and experimental data on expansion are needed over the whole range $f°$ to $t°$. Our experimental knowledge of the values of α is limited to a very few glasses. Such data as available are, with one exception, quoted in Hovestadt's "Jena Glass," [11] where references to the original papers will be found.

The three most important thermometer glasses are the French "verre dur," Jena 16[III] "normal glass," and Jena 59[III] borosilicate glass. These and a few others have been very carefully studied between 0° and 100°, but at the higher temperatures we

---

[11] English translation by J. D. and A. Everett: Macmillan, 1902.

have data only on verre dur to 200°, 16[III] to 300°, and 59[III] to 500°. For these three glasses the values in the following table may be used in connection with equations (4) and (5); they are probably correct to somewhat better than 1 per cent:

Table of Mean Coefficients of Relative Expansion

Value of $\alpha \times 10^6$

$\frac{t+f}{2}$	Verre dur	16[III]	59[III]
50	158	158	164
100	158	158	164
150	158	158	165
200	159	159	167
225		160	169
250		161	171
275		162	173
300		164	175
325			177
350			179
375			181
400			183
425			186
450			189
475			193
500			198

The data used in computing the table were as follows: (a) The mean coefficients from 0° to 100° determined by Thiesen, Scheel, and Sell [12]; (b) the gas-scale corrections for 16[III] given by Wiebe and Böttcher [13]; (c) data on 59[III] obtained by Mahlke [14]; (d) the corrections of the verre dur scale to the hydrogen normal scale, given in the pamphlet of 1896, which is issued with the certificates for mercurial thermometers tested at the International Bureau of Weights and Measures.

---

[12] Zeitschrift für Instrumentenkunde, **16**, p. 55; 1896.
[13] Ibid., **10**, p. 245; 1890.
[14] Ibid., **15**, p. 171; 1895.

The values are given in each case up to the highest temperature for which experimental data are available, but it is evident that the use of the higher figures, when t and f are far apart and t therefore above the upper limit of the experiments, involves an extrapolation. Suppose, for example, that a thermometer with a stem of 16^{III} is being used at a temperature $t = 450°$ while the stem temperature is $f = 100°$. We then have $\frac{t+f}{2} = 275°$ for which the table gives $\alpha = 0.000\ 162$, but this value is quite uncertain, because we have no precise knowledge of how the glass behaves above $300°$. On the other hand, if $t = 300°$ and $f = 250°$, we again have $\frac{t+f}{2} = 275°$, but the value $\alpha = 0.000\ 162$ may now be relied upon.

Since both verre dur and 16^{III} thermometers are used up to $450°$ C, it is evident that our data for determining the correction for emergent stem are very deficient, but the situation is not quite so bad as it seems. As regards verre dur, it may be said that the behavior of this glass bears a close resemblance to that of Jena 16^{III} in so many respects that the values of α given for 16^{III} will probably not be in error by 5 per cent, even at $300°$, if applied to verre dur.

With respect to glasses which have not been investigated, we may make the following remarks: A comparison of the absolute expansion of mercury as determined by Callendar and Moss [15] with that of Jena 59^{III} glass, as determined by Holborn and Grüneisen,[16] in connection with Mahlke's [17] results on the relative expansion of mercury in 59^{III}, shows that for this glass the increase of α with temperature, denoting a departure of the total relative expansion from linearity, is accounted for mainly by the behavior of the mercury and only to a minor degree by that of the glass. And since the expansion of glass is only about one-tenth that of mercury, it seems probable that the change of α with temperature is not very different for different thermometer glasses, but is nearly parallel with the change for 59^{III}. The best we can do at present with stems of unknown composition or of glasses

[15] Phil. Trans. Roy. Soc., London, A 211, p. 1; 1911.
[16] Landolt and Börnstein, Tables, 3d ed., p. 201.
[17] Zeitschrift für Instrumentenkunde, 15, p. 171; 1895.

which have not been investigated is probably to use the values for
59^{III} rounded off to two significant figures. These values, thus
applied, will probably not be in error by over 10 per cent, except
at very high temperatures. This remark applies to the numerous
thermometers which have bulbs of 16^{III}, but stems of some softer
and less brittle glass. In all cases where the value of α is not
known by direct experiment, it is useless to attempt great per-
centage accuracy in determining the stem correction, and the
equation

$$K = n \, l \, \alpha \, (t - f),$$

with α from the table for 59^{III}, may be used with no further
refinements.

7. THE STEM ERROR OF THE FADENTHERMOMETER; MAHLKE'S METHOD OF SETTING

We have stipulated that the fadenthermometer shall have been
standardized so that its reading gives the mean temperature $f°$ of
its bulb in terms of the gas scale. If this has been done, equations
(2) to (5) determine the value of K; but the question arises whether
such a standardization is generally possible, and if not, how the
equations or the method of procedure are to be modified.

Let us first assume that the fadenthermometer has been stand-
ardized by total immersion. When it is in use in the position
shown in Fig. 2, its stem S' is not at the same mean temperature
as the bulb F, and its reading is subject to a secondary stem cor-
rection K' for the "emergent" stem of the fadenthermometer
itself, which must be applied to the observed reading f_1 so as to
give $f = f_1 + K'$ for use in the equations. The value of K' can be
found in the same manner as that of K if the mean temperature
of S' is known; and since it is evident that K' need be known only
roughly, this temperature may be found sufficiently well by a
thermometer placed with its bulb close [18] to the middle of S'.

Usually, and unless K is large, this secondary stem correction
may be neglected and f_1, the reading of the fadenthermometer,
may be identified with $f = f_1 + K'$, the true mean temperature of

---

[18] Guillaume suggests wrapping tin foil around the two.

the bulb F. This is not always permissible, however, as may be seen from the following example: Let $n = 10$, $l = 20^{cm}$, $\alpha = 0.000\ 17$, $t = 450°$, $f_1 = 300°$, as read without correction for the secondary stem error, and A therefore 5 per cent of K. Then we have, using the uncorrected value for f in equation (5)

$$K = 10 \times 20 \times 0.000\ 17 \times (450 - 300) - A = 4.85 \text{ degrees.}$$

Now, suppose that the stem of the fadenthermometer is at a mean temperature of only 100°, that the length of the thread in its stem is 10^{cm}, and that there are 30 degrees per centimeter on its scale. Then its stem correction will be approximately

$$K' = 30 \times 10 \times 0.000\ 16 \times (300 - 100) = 9°.6$$

where we have used α from 100° to 300° $= 0.000\ 16$, a sufficiently approximate value. Hence we have for the corrected value of f

$$f = f_1 + K' = 309°.6$$

and $A = 5.2$ per cent of K. A recomputation of K now gives us

$$K = 10 \times 20 \times 0.000\ 17 \times (450 - 309.6) - A = 4.53 \text{ degrees.}$$

In this case, therefore, neglecting the secondary stem correction would cause an error of over 0.3 degree in the primary stem correction K, an amount which may or may not be negligible under the given conditions.

Mahlke recommends a slightly modified method of setting the fadenthermometer, designed to eliminate this secondary stem error. The mercury in S' is always small in volume; its temperature also will seldom vary greatly between c and e; hence we shall make only a negligible error if we assume the mean temperature of S' to be the same as the temperature at its lower end c where it joins the bulb. If, then, we imagine the cylindrical bulb extended upward and the mercury now in S' run down without change of temperature into this extension; and if we next lower the whole fadenthermometer by a short distance m till this imaginary new position of the meniscus (instead of the end of the bulb F) is at the level c, the mean temperature of the mercury in the whole fadenthermometer will now be the same

as that of a length $(l+m)$ of the working stem S.   Hence we shall
have eliminated the secondary stem error if we use the observed
reading of the fadenthermometer but use $(l+m)$ in place of l.
Mahlke's actual procedure, therefore, is to estimate m from the
easily computed relative cross sections of S' and F, and set the
fadenthermometer so that the top of its bulb is below the level c
by this small amount m, adding m to the measured length l of the
bulb before making further computations for the primary correc-
tion K.

On account of the difficulty of construction, the transition from
stem to bulb in the fadenthermometer is usually somewhat irregu-
lar and there is frequently a slight enlargement at the junction.
The correct setting and the point which is to be considered as the
end of the bulb, from which l is measured, are thus often somewhat
uncertain, and it is doubtful whether Mahlke's method presents
any real advantage over the simpler method of setting always at
the same point and using a fixed value of l, applying the secondary
stem correction in the few cases where it is worth while.

8. ON THE SELECTION OF A FADENTHERMOMETER

It sometimes happens that several fadenthermometers are
available for determining a certain stem temperature.   The fol-
lowing principles should then govern the selection of the one to be
used.   Since it is a fundamental assumption that the temperature
distribution is the same along the fadenthermometer bulb as along
the working stem, the more alike these are the better.   If the
fadenthermometer is very 'much larger than the working stem,
especially of larger bore, it will read too high (when used for tem-
peratures above those of the surroundings) on account of longi-
tudinal conduction, so that very large diameters are undesirable.
A fadenthermometer of the ordinary type must not be used with
a working stem of the enclosed-scale or "einschluss" form nor vice
versa, as the fundamental condition regarding temperature dis-
tribution would not be at all closely fulfilled in either case.   These
remarks are equally applicable to the choice of an auxiliary stem
when several are available.

Of several fadenthermometers otherwise equally suitable, the
shortest which will reach into the region of uniform temperature

is to be preferred.   One reason for this is that the shorter the bulb *F* the lower will be its mean temperature and, therefore, the less the secondary correction for its emergent stem in which the mercury thread will be short.   A second reason appears from equation (4).   For the larger $(t-f)$, i. e., the smaller f is, the less is the error involved in using the observed temperature t_1 instead of the corrected temperature t, and the less, also, is the effect of any error in f.   Hence with a suitably short fadenthermometer several approximations may be permissible, which would not be sufficient for the desired accuracy with a longer one.

The emergent stem of the fadenthermometer should be short, both in degrees and in absolute value.   The stem is often made unnecessarily fine.   Beside making the emergent thread long at high temperatures and thus liable to have a very different mean temperature from that of the bulb, the use of a very fine capillary involves difficulties of construction which result in an irregular joining of the bulb and stem.   With a bulb 20 centimeters long, a stem of half the diameter of the bulb gives a scale of 7° or 8° per millimeter, which is quite open enough.   If accurate work is to be done conveniently, a series of fadenthermometers should be available and that one should be used which will give a reading as low down in the stem as possible, thereby reducing the secondary stem correction.

9. RELATIVE MERITS OF THE AUXILIARY STEM AND THE FADEN-THERMOMETER

It will have become evident, upon reading the preceding sections, that the theory of the use of the fadenthermometer is not altogether simple and that as an instrument of precision, it is open to certain practical objections, one difficulty arising from the fact that above 200° C, the coefficient of relative expansion has been investigated for only two glasses, one of which, Jena 16[III], is not entirely suitable for stems.

The fadenthermometer was evolved from the simple auxiliary stem for the sake of giving a more open scale.   The openness of the scale has a certain practical advantage in that the reading requires less care and imposes less strain and fatigue on the observer; but when the stem of the fadenthermometer is as fine as it has often been made, the greater ease of reading the position

of the meniscus on the more open scale is far more than offset by the difficulty of finding the meniscus at all.

The ostensible object of introducing the fadenthermometer was the greater accuracy attainable by reducing the reading errors. It may be seen, however, upon considering the nature of the reading errors when the simple auxiliary stem is used, that the improvement in accuracy attained by the fadenthermometer is altogether illusory. With equally good graduation and illumination, the *linear magnitude* of the reading error is of the same order for the auxiliary stem as for the working stem. There is thus a possibility of adding together two similar reading errors. When the method of double reading, as described in section 4, is followed, we have, taking this second reading into account, the possibility of a still further increase in the sum of the reading errors, though this last possibility may be eliminated by standardizing the auxiliary stem once for all with sufficient care that the results may be regarded as exact relatively to the precision of a single reading, when the illumination in practical use is similar to that during the calibration. But in reality these reading errors are not important. If the temperature to be measured is steady, a number of readings may be taken and averaged, while if the temperature is unsteady or changing rapidly in one direction, it can not be determined with any great accuracy by a mercurial thermometer, no matter how exactly the individual readings may be made. Furthermore, at high temperatures even three times the probable accidental error of *reading* the position of the meniscus in either the working or the auxiliary stem usually falls well within the probable error of the final result on account of the failure of mercurial thermometers to repeat their readings exactly, at high temperatures.

For purposes of the highest attainable accuracy, where it is desirable to avoid all unnecessary errors, even small ones, the readings of the auxiliary stem, both during standardization and during the determination of a stem correction, may, if thought advisable, be made with a micrometer microscope. This presents no difficulty even when micrometric readings on the working stem would be very difficult,[19] because the meniscus in the auxiliary stem is so much more steady than that in the working stem.

---

[19] It is doubtful whether micrometric readings on the working stem are ever worth while.

It therefore appears that the use of the fadenthermometer presents no real advantage over the use of the auxiliary stem, while it is open to several objections.   Not the least of these is the difficulty of obtaining satisfactory fadenthermometers, whereas old thermometer stems, easily convertible for use as auxiliary stems, are often embarrassingly numerous.   The maker of any thermometer can easily supply a suitable auxiliary stem of the same glass and the glass need not have been studied as to its expansion.

There seems little doubt that the auxiliary stem method, which avoids several difficulties inherent in the use of the fadenthermometer is the best we have for work of high accuracy, as well as the most convenient in a large number of cases in which only ordinary accuracy is desired.

NOTES

NOTE I.—THE PRESENT STATUS OF THE STANDARD GAS SCALE

In adopting a set of values for the base points, the corrections for reducing to the ideal gas scale have, up to the present time, usually been ignored on account of their uncertainties and relative insignificance.   When this is done, the values adopted must, in strictness, be regarded as referred to or expressed in terms of the individual scales of the particular gas thermometers used in the primary standard determinations of the separate points.   And since no series of determinations has ever been made with a single instrument, or even with a single gas, over the whole range of temperatures accessible to the gas thermometer, it can not be said that there is any entirely consistent series of values for the base points except over limited ranges.

This appears at first sight to be a very unsatisfactory state of affairs, and in reality a good deal of haziness seems to exist as to the meaning of the term "standard gas scale." The *practical* indefiniteness of the term has, however, been of little or no importance in the past, because the differences between the different gas scales used or between any one of them and the ideal gas or thermodynamic scale have been less than the uncertainties of the determinations of temperature on the individual gas scales.   For this reason the various gas scales have in most, though not all

cases, been practically indistinguishable from one another and from the ideal gas scale, and the term "gas scale," with no further qualification, has been a sufficient specification of the *scale*, although the same numerical value for the temperature of a given fixed point referred to that scale may not have been accepted as most probable by all authorities.

The art of gas thermometry has, however, now advanced to the point where the accuracy of the determinations is comparable with that of our knowledge of the differences between the gas scales used in different pieces of work and of the probable corrections to the ideal gas scale. This makes a more precise definition of the scale used in any case desirable, and makes a greater consistency of statement possible by reducing the different gas scales to a common standard. The corrections are often appreciable and though they are not known with a high percentage accuracy, there is no doubt that applying them brings, in general, a slightly greater degree of consistency into the series of base-point values as determined by different gas thermometers.

The authority responsible for the series of values to be used for the base points of the scale of a standardizing laboratory is therefore faced by the question whether it is well to abandon a familiar set of values which have been in use, perhaps for a number of years, in favor of a new set which is only slightly different from the old one and is liable to further modifications of the same order of magnitude as those now proposed, whenever, in the course of a few years, the individual gas thermometer determinations and the determinations of the corrections to the ideal gas scale are more accurate. Up to the present time this question has almost universally been answered in the negative. It seems likely, however, that within a very few years the change will be made and that all standardizing laboratories will use, for reference, the nearest possible approach to the ideal gas scale.

The success of the movement which has resulted in international agreement on the practical values to be used for the electrical units points clearly to a similar international agreement with regard to the standard scale of temperature. Such an agreement has already existed for many years as regards the use of the hydrogen "normal scale" of the International Bureau of Weights and

Measures, for temperatures between 0° and 100° C, and the time appears nearly ripe for a similar agreement over a wider range.

NOTE II.—ON THE USE OF THE TERM "MEAN TEMPERATURE"

The "integral temperature" of a thread of mercury might be defined as the value of the expression $\dfrac{\int t\,dv}{\int dv}$ taken over the length of the thread, where t is the temperature of the volume element dv. Since this value is determined solely by the geometrical distribution of the temperature, it is independent of the nature of the material of either the thread or the tube inclosing it.   For two threads such as those in a working stem and an auxiliary stem or a fadenthermometer bulb used with it, the integral temperatures are nearly the same.   If the longitudinal distribution of temperature is the same for both tubes, as is assumed in practice, and if the cross section ratio of the tubes is constant, the integral temperatures will be exactly the same.   Such a relation of cross sections could evidently not subsist *exactly*, except with one particular distribution of temperature, unless the tubes were made of the same glass; but the error here is of a lower order of magnitude, and we may safely regard the integral temperatures as being identical for the working stem and the fadenthermometer bulb when properly placed.

The *mean* temperatures of the two threads, as defined in section 5, depend, however, on the relative expansion of mercury in glass, and will evidently not be quite identical for different kinds of glass, even though the integral temperatures be equal.   Nevertheless, to the order of accuracy required in determining a stem correction, the difference is negligible and the mean temperatures may be identified when the integral temperatures have been made sensibly equal.

The complete investigation of the subject of mean temperature, its dependence on the nature of the glass, and its relation to integral temperature requires even for an ideally perfect tube an amount of precise and detailed reasoning quite out of proportion to the value of the result obtainable and will therefore not be touched upon here.   The object of the present note is to forestall the possibility that the cautious reader may, upon recognizing the somewhat offhand way in which the term "mean temperature"

has been used in the text, be apprehensive that the difficulties of the subject have not been considered.

<div align="center">NOTE III.—THE APPARENT EXPANSION OF MERCURY IN GLASS</div>

1. Let a glass bulb be exactly filled by v_0 cm³ of mercury at $o°$. Let the expansion of the glass from $o°$ to $t°$ be g, so that at $t°$ the internal volume of the bulb is $v_0(1+g)$. If m is the expansion of the mercury in the same range, and if the bulb with the mercury which exactly filled it at $o°$ be heated to $t°$, the volume of the mercury will increase to $v_0(1+m)$ cm³, and the volume $v_0(m-g)$ cm³ must run out, m being greater than g. Since 1 cm³ of glass increases to $(1+g)$ cm³ when heated from $o°$ to $t°$, if the volume of mercury $v_0(m-g)$ cm³ at $t°$ were measured, not absolutely but in a glass vessel graduated to read true cubic centimeters at $o°$, its volume would be only $v_0\dfrac{m-g}{1+g}$ of these "glass cubic centimeters."

Hence the quantity

$$a = \frac{m-g}{1+g} \tag{1}$$

is the expansion of the mercury as it would *appear* to an observer who supposed that the glass did not expand at all; or it may also be regarded as the expansion of the mercury *relatively to the glass*, considered as of fixed volume. The quantity a is therefore known as the apparent or relative expansion of mercury in the given glass from $o°$ to $t°$.

2. Let the glass envelope consist of a capillary tube graduated with a scale of equal volumes, the volume of each scale division being ϕ_0 at $o°$ and $\phi_t = \phi_0(1+g)$ at $t°$. Let the mercury be in the form of a thread which fills N_0 divisions of the tube at $o°$, and N divisions at $t°$. The volume of the thread being $N_0\,\phi_0$ at $o°$, its volume at $t°$ is $N_0\,\phi_0(1+m)$, and the number of divisions occupied at $t°$ is

$$N = \frac{N_0\,\phi_0\,(1+m)}{\phi_t} = N_0\frac{1+m}{1+g} \tag{2}$$

and by equation (1) this may be written

$$N = N_0\,(1+a) \tag{3}$$

3. If a_f is the relative expansion of mercury in glass from $o°$ to $f°$, and a_t that from $o°$ to $t°$, a thread which occupies N_0 divisions

at $0°$ will occupy

$$N_f = N_0 \, (1 + a_f) \tag{4, a}$$

and

$$N_t = N_0 \, (1 + a_t) \tag{4, b}$$

divisions, at $f°$ and $t°$ respectively. Upon heating from $f°$ to $t°$ the number of divisions occupied will increase by the amount

$$K = N_t - N_f \tag{5}$$

By equations (4) this has the value

$$K = N_0 \, (a_t - a_f) \tag{6}$$

and if we eliminate N_0 by (4, a), it may also be written in the form

$$K = N_f \frac{a_t - a_f}{1 + a_f} \tag{7}$$

4. We now let the tube be of uniform bore, as is assumed of both the working stem and the fadenthermometer bulb, in the determination of a stem correction. The scale divisions are now of equal *length* when the tube is at an uniform temperature. Let λ be this length, in centimeters, measured when the glass is at room temperature. Let l be the length, in centimeters, of N_f scale division of the tube (the working stem), also measured with the glass at room temperature. We then have

$$N_f = \frac{l}{\lambda} \tag{8}$$

Let $n = \frac{1}{\lambda}$ be the number of scale divisions per centimeter, measured at room temperature. We then have, by equation (8)

$$N_f = n \, l \tag{9}$$

so that equation (7) reduces to

$$K = n \, l \frac{a_t - a_f}{1 + a_f} \tag{10}$$

5. Equation (10) is identical in form and practically identical in meaning with equation (2) of section 5. The actual working stem may have a nonuniform graduation, but the quantity N_f is then not the actual number of scale divisions of S opposite the fadenthermometer bulb, but the number there would be if the divisions were all of the same length as in the region of the meniscus where n is measured, and this number we get by taking the product of n and l as measured.

If the working stem and the fadenthermometer bulb are of the same glass, so that their mean linear coefficients of expansion are identical between the stem temperature $f°$ and the room temperature $r°$ at which the length measurements are made, the l which appears above in (10) is identical with the l which appears in equation (2) of section 5, and the two equations are identical in all respects.   If the two glasses are different, there is a discrepancy between the l above, measured at room temperature *on the glass tube we have been considering which corresponds to the working stem,* and the l of equation (2), section 5, which is measured *on the fadenthermometer,* these two lengths being such that they would become exactly equal if the temperature of measurement were raised to $f°$.

This discrepancy is insignificant.   The fractional error introduced by identifying these two different values of l is, very approximately,

$$\varepsilon = (f - r)\ (\beta_\mathrm{s} - \beta_f)$$

in which β_s is the mean coefficient of linear expansion of the glass of the working stem between $r°$ and $f°$, and β_f the corresponding coefficient for the glass of the fadenthermometer.   To take an extreme case, let $(f - r) = 500°$ and $\beta_s - \beta_f = 0.000\ 01$.   We then have $\varepsilon = 0.005$ or one-half of 1 per cent—an error which is always negligible.   In reality neither β_s nor β_f will ever be greater than 0.000 01 and their difference is not likely to be over 0.000 003 for thermometer glasses, so that the error is not likely ever to exceed two-tenths of 1 per cent.   The above-mentioned discrepancy between the values of l is therefore always negligible and the deduction given for equation (10) of the present note is valid for equation (2) of section 5.

WASHINGTON, July 13, 1911.

46905°—12——3