THE DETERMINATION OF THE RATIO OF TRANSFORMATION AND OF THE PHASE RELATIONS IN TRANSFORMERS.

By E. B. Rosa and M. G. Lloyd.

Alternating-current transformers are so useful in the measurement of current and potential, by reducing the current or potential that must be applied directly to the instruments, that they have been extensively used in engineering work for the measurement of current, voltage, and power, not only for the heaviest currents and highest potentials, but also for currents and potentials of moderate values. Such transformers, when properly constructed, can safely be employed in connection with precision voltmeters, ammeters, and wattmeters for measurements of considerable accuracy. Indeed, if the constants of the transformers have been accurately measured, the precision of the results will depend chiefly on the indicating instruments, for the transformers themselves are more permanent and less liable to injury than the more delicate instruments used with them.

Transformer losses have been an object of much study, and their determination has become a familiar test; the measurement of ratios is one which may be carried out without complicated apparatus and is easily accomplished; but the question of phase relations seems to have remained a subject of theoretical study principally, and to have received scant experimental attention.[1] It is of importance, however, not merely as a matter of general interest or in the design of transformers, but also in the measurement of power. For measurements of voltage or of current, it is necessary to know only the ratio of transformation involved, but

[1] Since the above was written, a comprehensive article on the subject has appeared by L. T. Robinson, Proc. A. I. E. E., 28, p. 981; 1909.

for measurements of power with a wattmeter the phase relations are also involved, and accuracy can not be assured unless these are known. Usually the phase of the secondary is so nearly the reverse of the primary phase that the error with high power-factors would be insignificant, but with low power-factors a large error might be introduced, as may be seen by reference to the numerical examples given below. In view of these facts, it is thought desirable to publish the methods of measurement of these quantities in use at the Bureau of Standards.[2]

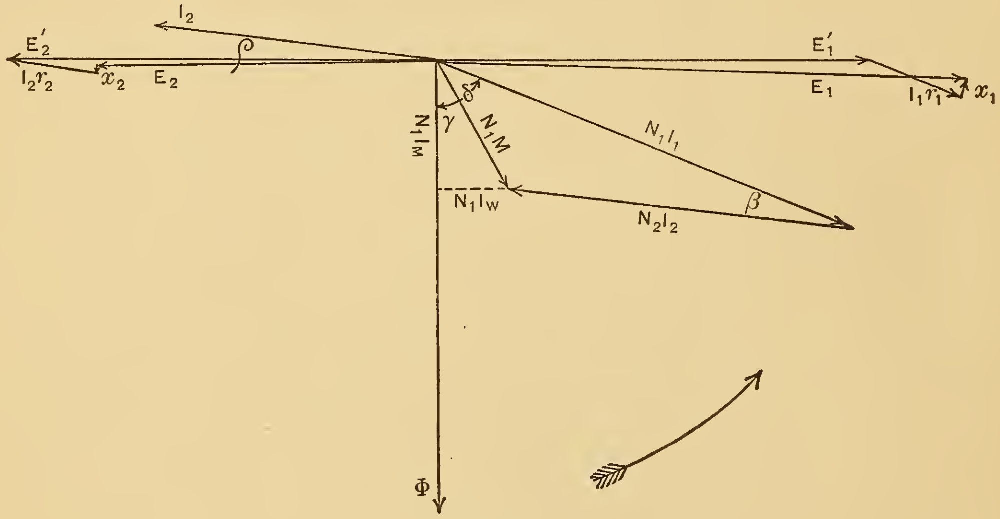

Fig. 1.

In the vector diagram, Fig. 1, are shown the various quantities which go to determine the ratio of the transformer. The length of the vector is proportional to the maximum value of the quantity, and it is considered to be rotating uniformly in a counter clockwise direction. Its projection on a fixed diameter will then represent the instantaneous value of the quantity, assuming it to be sinusoidal. The angles between the vectors represent the phase-angles of the corresponding quantities.

Φ represents the magnetic flux linking both primary and secondary windings. It induces in the secondary winding an electromotive force E_2' and in the primary winding an electromotive force in the same direction, but of different magnitude, fixed by the number of turns. If we call the exciting current M, the cur-

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[2] Since this was written, other methods have been developed; they are described by Agnew and Fitch, Phys. Rev. **28**, p. 473, 1909, and will appear in a later number of this Bulletin.

rent-turns necessary to produce the flux Φ is represented by N_1M, and it is made up of two components N_1I_1 and N_2I_2. The electromotive force applied to the primary terminals may be separated into three components; the first balances the induced emf. due to the flux Φ, and is represented by E_1'; the second balances the emf. (of self-induction) x_1 due to any leakage flux which links with the primary, but not the secondary; the remainder sends current through the primary and is represented by I_1r_1. The vector sum E_1 of these three components must be the terminal emf. of the primary winding.

The secondary terminal electromotive force, E_2, represents what is left after deducting the ohmic drop I_2r_2 and the induced emf. x_2 (due to leakage flux linking the secondary alone) from the total emf. E_2'.

The ratio of E_1 to E_2 is known as the ratio of a potential transformer, and in general it differs appreciably from the ratio of E_1' to E_2', which is the ratio of primary to secondary turns. The phase-angle between E_1 and E_2 reversed is the angle which it is proposed to measure. This may be either positive or negative.

If no current be taken from the secondary, E_2 becomes identical with E_2'; N_1I_1 becomes identical with N_1M; I_1r_1 becomes smaller and farther from E_1' in phase; the ratio is more nearly the ratio of turns; the secondary emf. is in advance of the primary. As the secondary current is increased the ohmic drop in the secondary increases and E_2 decreases; I_1 must increase to maintain M; I_1r_1 increases and comes more nearly into phase with E_1'; hence E_1 must be increased to maintain the same magnetic conditions, or if E_1 be maintained constant the flux decreases and E_2 suffers further contraction, so that the ratio is increased; x_1 will in general increase and the phase angle will approach zero and finally become negative.

If the applied voltage E_1 be altered, the same diagram will still represent the quantities to a different scale, provided the external secondary impedance be unchanged, so that I_2 retains its proportion to E_2. The ratio and phase relations thus remain unchanged. This is strictly true of a transformer with air core, and with an iron core the deviation of the ratio from constancy becomes appreciable only when saturation is approached, so that x_1 and x_2 no longer remain proportional, and the permeability of the iron falls off to such an extent that N_1M must be increased out of all proportion to E_1. But within the working range of the transformer we may say that the ratio and phase angle are independent of the voltage.

A change of frequency involves a change of flux, which, in turn, requires a change in the magnetizing current. If the frequency be increased, the magnetizing current is decreased, but at the same time is thrown more nearly in phase with the emf., so that the change in ratio is slight. If a very large increase be made in the frequency, so that the flux is very low, x_1 and x_2 will be increased also, the ratio will be appreciably raised, and the phase angle decreased.

If, however, the voltage be changed in proportion to the frequency, so that the same magnetic flux is maintained, the conditions are little altered, and for the same impedance in secondary circuit the ratio is little affected. For the same secondary *current*, the ohmic drop is proportionally decreased for an increased frequency and the ratio also decreased. The effect here will depend upon the load and increase with it.

This diagram and the discussion have been based upon the supposition that all the quantities concerned follow a sinusoidal variation. In a transformer containing iron this is never realized, for if the applied electromotive force is sinusoidal, the current will not be, owing to the varying permeability of the iron. The discussion remains substantially valid, however, if we let the current vectors in the diagram represent the equivalent sine waves.

If the applied electromotive force is not sinusoidal, then neither the magnetic flux nor the other electromotive forces will follow the sine law of variation. Since the two induced electromotive forces have the same wave form, the terminal electromotive forces can differ from them, and the ratio can be affected, only in so far as the leakage and the resistance drop in the windings influence them. The qualitative effect may be determined from theoretical considerations.

The equation connecting the instantaneous electromotive forces in the primary is

$$e = N\frac{d\Phi}{dt} + ir + L\frac{di}{dt}$$

where

$$e = A \sin pt + Ah_3 \sin (3\,pt + \theta_3) + Ah_5 \sin (5\,pt + \theta_5) + \ldots\ldots$$

is the applied electromotive force.

$$i = B \sin (pt + a) + Bk_3 \sin (3\,pt + a_3) + Bk_5 \sin (5\,pt + a_5) + \ldots\ldots$$

is the primary current.

r = primary resistance

N = number of turns in the primary winding.

$L\frac{di}{dt}$ represents leakage reactance

For the secondary circuit we have a similar equation

$$e_2 = N_2 \frac{d\Phi}{dt} - i_2 r_2 - L_2 \frac{di_2}{dt}$$

Since the effect of resistance and leakage reactance is the same in both circuits, it is sufficient to consider the primary alone.

The ratio is expressed in terms of the effective terminal voltage. It is consequently necessary to express the effective voltage in terms of the above quantities.

$$E^2 = \frac{2}{T}\int_0^{\frac{T}{2}} e^2 dt = \frac{2}{T}\int_0^{\frac{T}{2}} \left(eN\frac{d\Phi}{dt} + eir + eL\frac{di}{dt}\right) dt$$

$$\frac{di}{dt} = Bp \cos (pt + a) + 3\,Bk_3 p \cos (3\,pt + a_3) + 5\,Bk_5 p \cos (5\,pt + a_5) + \ldots\ldots$$

$$\int_0^{\frac{T}{2}} eL\frac{di}{dt}dt = pLAB \int_0^{\frac{T}{2}} [\sin pt \cos (pt + a) + 3\,h_3 k_3 \sin (3\,pt + \theta_3) \cos (3\,pt + a_3) + \ldots\ldots] dt$$

$$= -LAB\left[\frac{\pi}{2}\sin a + 3\,h_3 k_3 \frac{\pi}{2} \sin (a_3 - \theta_3) + 5\,h_5 k_5 \frac{\pi}{2} \sin (a_5 - \theta_5) + \ldots\ldots\right]$$

$$\frac{2}{T}\int_0^{\frac{T}{2}} eL\frac{di}{dt}dt = -\frac{p}{2}LAB\,[\sin a + 3\,h_3 k_3 \sin (a_3 - \theta_3) + 5\,h_5 k_5 \sin (a_5 - \theta_5) + \ldots\ldots]$$

$$\int_0^{\frac{T}{2}} eirdt = rAB \int_0^{\frac{T}{2}} [\sin pt \sin (pt + a)$$

$$+ h_3 k_3 \sin (3\,pt + \theta_3) \sin (3\,pt + a_3) + \text{........}] dt$$

$$= rAB \frac{\pi}{2p} [\cos a + h_3 k_3 \cos (a_3 - \theta_3)$$

$$+ h_5 k_5 \cos (a_5 - \theta_5) + \text{........}]$$

$$\frac{2}{T} \int_0^{\frac{T}{2}} eirdt = \frac{1}{2} rAB [\cos a + h_3 k_3 \cos (a_3 - \theta_3)$$

$$+ h_5 k_5 \cos (a_5 - \theta_5) + \text{........}]$$

The two terms which have been integrated determine the difference between the terminal emf. and the induced emf., and since the induced emf. varies in the same way in both primary and secondary, any change in the ratio due to wave form will be indicated by the change in the above terms.

For sinusoidal emf., $h_3 = o = h_5 = h_7 = \text{etc.}$

$$E^2 = \frac{2}{T} \int_0^{\frac{T}{2}} eN \frac{d\Phi}{dt} dt + \frac{1}{2} A_o B_o (r \cos a - pL \sin a)$$

where A_o and B_o denote the values of A and B for this particular case. For other wave forms with the same effective voltage, A and B will have different values.

For other wave forms,

$$E^2 = \frac{2}{T} \int_0^{\frac{T}{2}} eN \frac{d\Phi}{dt} dt + \frac{1}{2} AB[\{r \cos a - pL \sin a\} + h_3 k_3 \{r \cos (a_3 - \theta_3)$$

$$- 3\,pL \sin (a_3 - \theta_3)\} + h_5 k_5 \{r \cos (a_5 - \theta_5) - 5\,pL \sin (a_5 - \theta_5)\} + \text{........}]$$

Let us first consider the effect upon the ratio at no load. For a constant effective voltage, distortion of any kind will reduce the value of A, since[3]

$$A_o^2 = A^2 (1 + h_3^2 + h_5^2 + \text{........})$$

B may be either increased or diminished, since

$$\frac{\Phi}{\Phi_o} = \frac{f_o}{f}$$

[3] See M. G. Lloyd, this Bulletin, 4, page 480, ff; 1908 (Reprint No. 88), for some of the relations here made use of.

where f is the form factor and B may be assumed to vary approximately as Φ. a will be negative and for a sine wave approximately $\frac{\pi}{3}$, making the terms in resistance and reactance both positive. It will be only slightly changed by wave distortion, and since any change will affect the sine and cosine oppositely the factor $r \cos a - pL \sin a$ may be regarded as not varying.

It has been shown by Bedell and Tuttle[4] that $a_3 - a$ must be positive and lie between 30° and 180°. For a peaked wave θ_3 is in the neighborhood of 180° and hence $a_3 - \theta_3$ will probably lie in the third quadrant, making sine and cosine both negative. The term involving the third harmonic may then be either positive or negative, according as reactance or resistance predominates. We shall not attempt to follow the terms involving higher harmonics. We can see already that ordinarily a peaked wave will decrease the ratio; for the form factor is greater than for a sine wave, and hence Φ and B are less, while A is in all cases less. Hence the applied voltage departs further from the induced voltage.

For a flat potential wave on no load, the form factor is low, Φ and B are decreased, and A as before is decreased. θ_3 is now in the neighborhood of zero, and $a_3 - \theta_3$ will be positive and small. The resistance term for the third harmonic is now positive and the reactance term negative; the ratio will ordinarily be increased.

For a full load upon the transformer, the conditions are somewhat altered. If the load be noninductive, the current will have approximately the same wave form as the applied voltage and be almost in phase with it. Consequently, A and B will both be decreased by distortion and a will approach more nearly to zero. Since a_3 lies between 30° and 180°, $a_3 - \theta_3$ will be negative for a peaked wave and will usually be positive for a flat wave. Opposing the decrease in AB is the resistance component of the harmonic term, and for a peaked wave the reactance component also. If we neglect the magnetizing current, it can be shown that the resistance components of the harmonic terms will exactly neutralize the decrease in AB. Consequently, when the leakage in a trans-

[4] F Bedell and E. B. Tuttle, Trans. Am. Inst. Elect. Engrs., **25**, p. 601; 1906.

former is small, the effect of wave form when loaded will be qualitatively the same as when unloaded, but probably less in magnitude. When the leakage is large, however, the reactance components may become important, since these have numerical coefficients equal to the order of the harmonic. With a peaked wave especially the ratio will be increased, while with a flat wave the increase will be less.

With a lagging secondary load the harmonics will be less prominent in the current than in the emf. wave. a will be negative and all the phase-angles about the same as for no load. The effect of leakage reactance will be more prominent than for non-inductive load and the effect of the resistance terms less important. Consequently, if the leakage be large, the ratio may be increased with a peaked wave and decreased with a flat wave; otherwise it will surely be decreased for a peaked wave and probably increased for a flat wave.

We see from this discussion and from the experimental results given below that the ratio of a potential transformer is quite definitely determined by given conditions, and, moreover, with a definite secondary circuit, is little affected by variation of voltage or moderate variation of frequency and wave form. Consequently, if a potential transformer be calibrated for the value of its ratio with different secondary impedances, it may be used as an instrument of precision. The phase-angle under normal conditions is so nearly zero that for most purposes the discrepancy is negligible. When used with wattmeters on low power-factors, however, this angle should be determined.

In the series transformer we are concerned, not with the electromotive forces, but with the ratio of the primary and secondary currents. This ratio depends upon the exciting current and the power factor of the load, as well as upon the ratio of turns.

Let ρ be the angle by which the secondary current lags behind the induced emf., and resolve each side of the triangle of current-turns into components parallel and normal to the direction of Φ. Let δ be the angle between the primary current and this direction and let I_W and I_M be the two components of M. Then, referring to the vector diagram, Fig. 1,

$$N_1I_1 \cos \delta = N_2I_2 \sin \rho + N_1I_M$$

$$N_1I_1 \sin \delta = N_2I_2 \cos \rho + N_1I_W$$

Squaring and adding the two equations, we have

$$N_1^2I_1^2 = N_2^2I_2^2 + N_1^2M^2 + 2N_2I_2N_1 (I_M \sin \rho + I_W \cos \rho)$$

or

$$\left(\frac{I_1}{I_2}\right)^2 = \left(\frac{N_2}{N_1}\right)^2 + \left(\frac{M}{I_2}\right)^2 + 2 \frac{N_2}{N_1I_2} (I_M \sin \rho + I_W \cos \rho)$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}\sqrt{1 + \left(\frac{N_1 M}{N_2 I_2}\right)^2 + 2\frac{N_1}{N_2I_2} (I_M \sin \rho + I_W \cos \rho)}$$

$$= \frac{N_2}{N_1}\left[1 + \frac{1}{2}\left(\frac{N_1 M}{N_2 I_2}\right)^2 + \frac{N_1}{N_2I_2} (I_M \sin \rho + I_W \cos \rho)\right]$$

approximately.

The smaller M is with respect to I_2, the more nearly the ratio becomes simply the inverse ratio of the numbers of turns. $\frac{M}{I_2}$ may be small from three causes. The iron of the core may be of high permeability, so that only a low magnetomotive force is needed. Secondly, the impedance of the secondary circuit may be low, permitting the necessary current to flow with a low magnetic flux. Finally, the load on the transformer may be large.

The deviation from ratio of turns increases with the angle ρ; that is, with the reactance of the secondary circuit. The ratio of currents can only equal the ratio of turns by having a large negative value of ρ; that is, a leading current in the secondary. The value of ρ necessary is determined by the relation

$$\cos (\delta - \gamma) = \frac{N_1M}{2I_2N_2}$$

where

$$-\rho = 90^\circ + 2\gamma - \delta$$

for in this case

$$I_M \sin \rho + I_W \cos \rho = -\frac{1}{2}\frac{N_1 M^2}{N_2 I_2}$$

and

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

The angle β, denoting the phase difference between primary and secondary currents, is decreased by reactance in the secondary circuit and is increased for a leading current in secondary. The latter condition is scarcely one which would be attained in practice.

With a definite secondary circuit, and increasing current, Φ increases at the same rate, but at low flux densities M does not increase so fast, and the ratio is diminished. When the maximum permeability is passed, however, M increases faster than the secondary current and the ratio will begin to rise again. The phase-angle is also diminished rapidly at first, then becomes nearly constant, and finally increases at high flux densities. The larger the secondary resistance, the sooner this turning point is reached, but it will ordinarily lie beyond the full load of the transformer.

We see from the above that the ratio will depart least from the ratio of turns when the secondary circuit has low impedance and the core has a generous section of high permeability. For constancy of ratio, it is necessary to have constant permeability in the core with different inductions. This condition has been more nearly realized since the advent of silicon-alloy-steel, which has a high and slowly changing permeability at low inductions.

The effect of wave form upon the ratio of currents is in altering the necessary induction in the core and thereby the exciting current. The emf. induced in the secondary circuit must be proportional to the current in it, and its effective value is also proportional to the product of its form factor and the maximum flux in the core. If the form factor be increased, the maximum flux will be diminished and the exciting current likewise diminished. The numerical relations will depend upon the form of permeability curve of the core, but the direction of the effect will be that stated, the assurance of this being greater since high flux densities are never used in series transformers. Since a lower exciting current means a lower ratio, we may say in general that a peaked wave of current will give a lower ratio, and a flat wave will give a higher ratio. As the exciting current enters merely as a correction to the ratio, and as the wave form only slightly alters the exciting current, the effect of wave form will necessarily be slight.

POTENTIAL TRANSFORMERS.

The ratio of a potential transformer is determined by means of a differential dynamometer voltmeter. In this instrument the torque due to one set of coils is balanced against the torque due to the other set. Each set of coils consists of a pair of fixed coils and one moving coil between them. The two moving coils are rigidly connected, one above the other, but have separate leading-in wires. In determining the ratio of the primary and secondary voltages of a transformer the coils of each set are connected in series with each other and with a large noninductive resistance. One pair is supplied with current from the primary terminals, the other from the secondary terminals. Then, if each pair has the

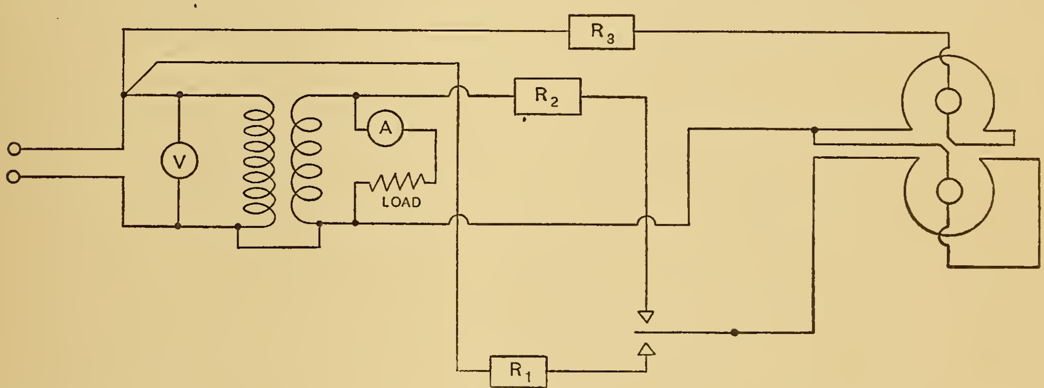

Fig. 2.

same constant (that is, if the torques are equal and opposite for equal currents in the coils), a balance is obtained when the resistances of the two circuits are proportional to the respective electromotive forces. To prevent interaction between the two sets of coils, the moving coil of one set is in the plane of the fixed coils of the other set. In other words, the two moving coils are mounted on the suspended system at right angles to each other. A current of about 0.025 ampere in each system gives sufficient sensibility so that a change of one part in five thousand may be detected.

In using the instrument, the primary emf. is applied through a suitable resistance to one set of coils; to the other set first the primary and then the secondary emf. is applied; the resistance being adjusted each time for a balance (see Fig. 2). The ratio of the two latter resistances is the ratio of the electromotive forces at the terminals of the transformer.

For, let

$k_1 k_2$ be the constants of the two sets of coils;

$R_1 R_2 R_3$ the total resistances of the circuits in the respective cases;

$E_1 E_2$ the terminal electromotive forces acting simultaneously upon the two sets of coils;

E the emf. for the auxiliary measurement, which may or may not be the same as E_1.

Then

$$\left(\frac{E}{R_3}\right)^2 k_1 = \left(\frac{E}{R_1}\right)^2 k_2$$ when the same emf. is applied to both.

$$\left(\frac{E_1}{R_3}\right)^2 k_1 = \left(\frac{E_2}{R_2}\right)^2 k_2$$ when primary emf. is applied to one and secondary emf. to the other.

From which

$$\left(\frac{k_1}{k_2}\right) = \left(\frac{R_3}{R_1}\right)^2 = \left(\frac{E_2}{E_1}\right)^2 \left(\frac{R_3}{R_2}\right)^2$$

and

$$\frac{E_1}{E_2} = \frac{R_1}{R_2}.$$

It is to be noticed that E need not be the same as E_1; that is, any change in the voltage produced by altering the load on the transformer or by fluctuation of the supply does not affect the result. For convenience in computation the resistance R_2 should be a round number, such as 1000, 5000, or 10000. R_3 is adjusted for balance with the switch in upper position, and R_1 is adjusted for balance with the switch down.

To determine the phase-angle between primary and secondary, the fixed coils are supplied as before, while the two moving coils are successively connected in series with a condenser and supplied with current from the primary terminals. This current is nearly in quadrature with the current in the fixed coils, and will produce very little deflection. Its value is afterwards determined by sending it through one pair of coils and noting the deflection.

It is to be remembered that the phase-angle is usually very small, and only one or two significant figures are necessary in the result.

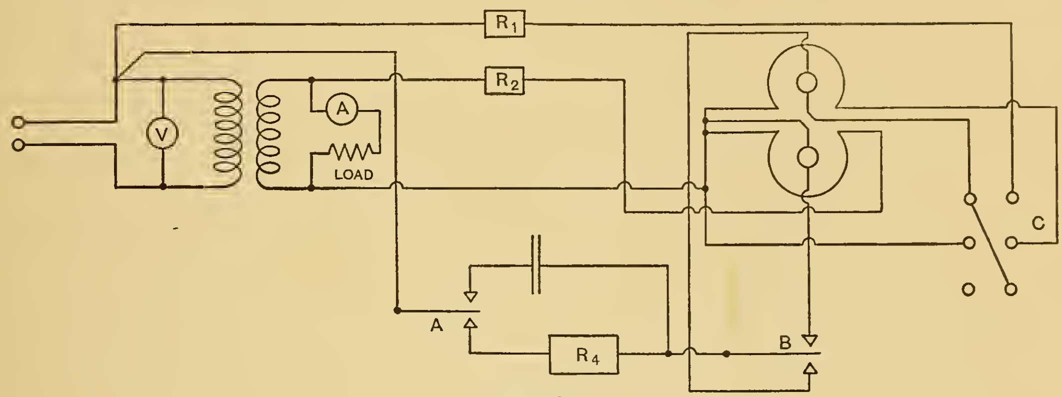

Fig. 3.

The connections are shown in Fig. 3. With switches *A* and *B* thrown down and switch *C* thrown up, the deflection is noted. Switch *B* is then thrown up and the resistance R_2 adjusted to give the same deflection as before. This makes the two fields of equal deflecting strength. Switch *A* is then thrown up, connecting the condenser in series with the moving coils, and the deflections D_1 and D_2 are read for the two positions of switch *B*. Each deflection

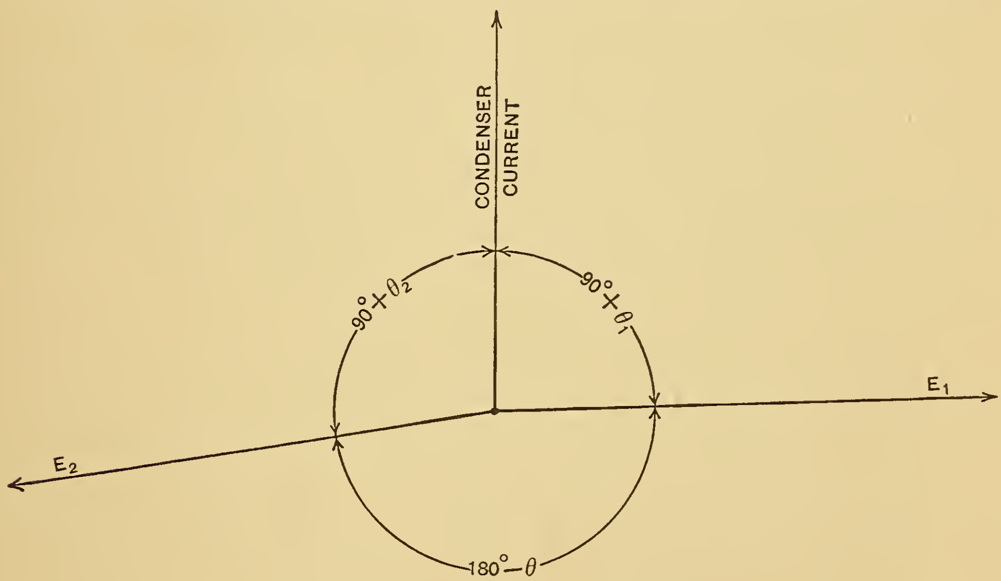

Fig. 4.

is a measure of the phase-angle between the condenser current and one of the terminal electromotive forces. Switches *C* and *B* are then thrown down, and the deflection D_3 noted with switch *A* thrown up. Finally the instrumental constant is determined by throwing switch A down and C up, and observing the deflection D_4. This deflection should be made about the same as D_3, by adjusting the resistance R_4, whose value must be known.

Let

k_1 k_2 = constants of instrument as before.

$90° + \theta_1, 90° + \theta_2$ = angles between condenser current and terminal electromotive forces.

$180° - \theta$ = lag of secondary behind primary emf., so that $\theta = \theta_1 + \theta_2$

D_1 D_2 D_3 D_4 = deflections.

R_1 R_2 R_3 = resistances as before.

R_4 = resistance whose admittance is approximately same as that of condenser.

I_3 = condenser current.

$I_4 = \frac{E_1}{R_4}$ = current for calibrating.

With the condenser current, I_3, in each moving coil in turn we have

$$D_1 = k_1 I_3 \frac{E_1}{R_1} \cos (90° + \theta_1) = -k_1 I_3 \frac{E_1}{R_1} \sin \theta_1$$

$$D_2 = -k_2 I_3 \frac{E_2}{R_2} \sin \theta_2 = -k_1 I_3 \frac{E_1}{R_1} \sin \theta_2$$

if $k_2 \frac{E_2}{R_2}$ be made equal to $k_1 \frac{E_1}{R_1}$ as mentioned above.

Then

$$-\sin \theta = -\sin \theta_1 - \sin \theta_2 = +\frac{R_1}{k_1 I_3 E_1} (D_1 + D_2)$$

$$D_3 = k_1 I_3^2$$

$$D_4 = k_1 I_4 \frac{E_1}{R_1} = k_1 \frac{E_1^2}{R_1 R_4}$$

Hence

$$\sqrt{D_3 D_4} = k_1 \frac{E_1 I_3}{\sqrt{R_1 R_4}}$$

and

$$\sin\theta = -\sqrt{\frac{R_1}{R_4}}\frac{(D_1+D_2)}{\sqrt{D_3D_4}}$$

If $D = D_1 + D_2$ and $D_4 = D_3$ then

$$\sin\theta = -\sqrt{\frac{R_1 D}{R_4 D_3}}$$

For the highest accuracy k_1 should be determined separately for the deflections D_1 D_2 and the larger deflections D_3 D_4. Ordinarily it may be taken as the same in both cases.

We see then that the phase-angle may be determined by four observed deflections if the two resistances be known and a steady voltage is available. If the voltage is not maintained at a constant value throughout the observations, it should be observed at the time of each reading and corrections made for it. Small fluctuations, however, would make no appreciable error.

TABLE I.

Transformer D.—120/120 volts, 60 cycles, 500 watts. Primary resistance 0.39 ohm. Secondary resistance 0.68 ohm.

[Tested July 21, 27, 1905, at 110 volts, 60 cycles; exciting current=0.52 ampere.]

Secondary Current*	Ratio	R_1	R_2
0	1.001	5117	5111
1	1.010	5163	5111
2	1.019	5209	5111
3	1.028	5139	5000
4	1.037	5186	5000
5	1.047	5234	5000
6	1.057	5285	5000
7	1.067	5337	5000
8	1.078	5388	5000

*Current taken by instrument neglected.

Great care must be used in giving the proper algebraic sign to the deflections D_1 and D_2. Ordinarily θ_1 will be negative and θ_2

positive. If the connections to the instrument be such as to make the deflection D_4 in the same direction for both coils, this means that D_1 and D_2 will be in opposite directions. D_1 will have the direction of D_4, and should be considered positive, while D_2 is considered negative. This makes θ a positive angle when D_2 is numerically greater.

If this precaution be taken when connecting the instrument, the deflections D_1 and D_2 may be mechanically combined in the instrument by sending the condenser current through both moving coils in series. A single reading then gives $D_1 + D_2$.

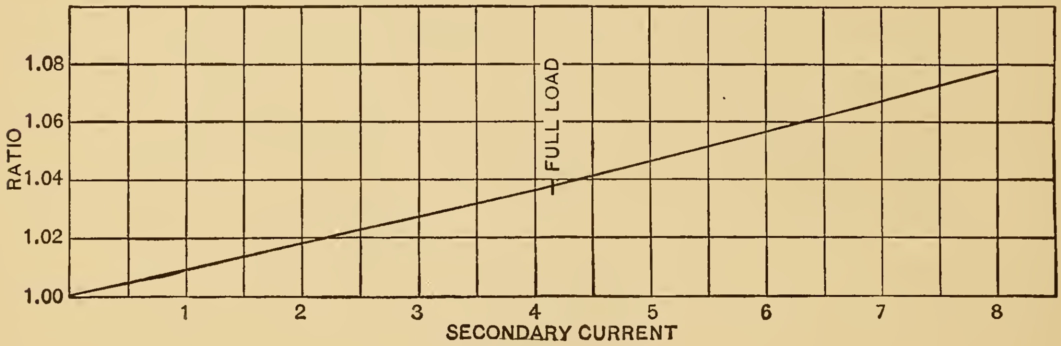

Fig. 5.

It has been assumed in the above measurement that a sine wave of electromotive force was used. If a distorted wave be used, the condenser current will have the harmonics magnified, and will not have the same wave form as the other currents in the apparatus. Since the secondary electromotive force has approximately the same wave form as the primary, the phase angle has still a very definite meaning, but it would be better to replace the condenser current by another whose phase is displaced in some other way. In the experiments given below a sine wave was used.

It may also be mentioned here that the noninductive resistances used in series with the dynamometer coils should be large enough to make the inductance of these coils negligible at the frequency used. Since different multipliers are used with the two sets of coils (except for ratio 1 : 1) the lag would, otherwise, be different in the two field coils and would introduce an equal error in the measurement of phase-angle. In getting the ratio it would be sufficient to use the impedance in place of the resistance of the instrument coils.

TABLE II.

Transformer G.—1100/110 volts, 60–125 cycles, 50 watts.

[Tested April 11, 1905. Secondary resistance constant. Slight overload.]

Cycles	E_2	Ratio
59.7	60	10.17
59.0	70	10.17
59.0	80	10.17
59.0	90	10.17
59.0	100	10.17
59.0	110	10.17
59.0	120	10.17
59.0	130	10.175
44.5	90	10.18
55.	90	10.18
60.	90	10.17
No load on secondary		
40	93	9.925
45	100	9.917
55	100	9.904
60	100	9.904

Table I gives the readings and results of a set of observations upon a 1:1 transformer to determine the variation of ratio with load. It is to be noted that the ratio changes almost 4 per cent between no load and full load. These values are plotted in Fig. 5.

Table II shows the effect of changes in voltage and frequency with constant secondary resistance. The ratio decreases slightly as frequency rises, but the change with voltage is less than 0.1 per cent.

Table III shows the changes with voltage and with secondary resistance in another transformer.

TABLE III.

Transformer H.—3000/120 volts, 60 cycles.

[Tested April 29, 1905. Stepping up, 60 cycles. Similar transformer connected to secondary and resistance varied in *its* secondary.]

E	Resistance of Secondary Circuit of Aux. Transformer	Inverse Ratio
80	9090	21.54
98	9090	21.58
120	9090	21.58_5
126	9090	21.58_5
120	1000	21.11
120	1500	21.26_5
120	2000	21.35
120	3000	21.44
120	5000	21.50_5
120	∞	21.61

In Tables IV and V results are given for two step-down transformers with secondary capacity of 500 and 400 amperes, respectively. The ratios rise rapidly with the load and at 200 amperes have changed 2.6 per cent and 3.6 per cent, respectively. Transformer E was tested also at 180 cycles, and the ratio in this case changes even more rapidly. A great difference is noticed also in the phase-angle. The ratios were also determined at 30 cycles and 55 volts. The ratios are plotted in the curves of Fig. 6 and the phase-angles in Fig. 7. The phase-angle under normal conditions is at first positive and decreases with the load. The ratios are also plotted in terms of secondary resistance in Fig. 8 for transformer E.

In Tables VI and VII are given the results of varying the wave form. The form of wave was varied by connecting two generators in series, the two being mounted on a single shaft and giving frequencies of 60 and 180 cycles, respectively. Each generator alone gives an approximate sine wave. One connection of the generators gives a peaked wave; by reversing the terminals of one machine this is changed into a flat or dimpled wave. The wave forms were determined on the oscillograph.

It will be seen that on no load the variation of ratio is less than 0.1 per cent. With the transformer loaded the effect is less, the ratio being less for a peaked wave, thus indicating that the ohmic drop is the determining factor and that the leakage effect is only apparent in decreasing the effect at full load.

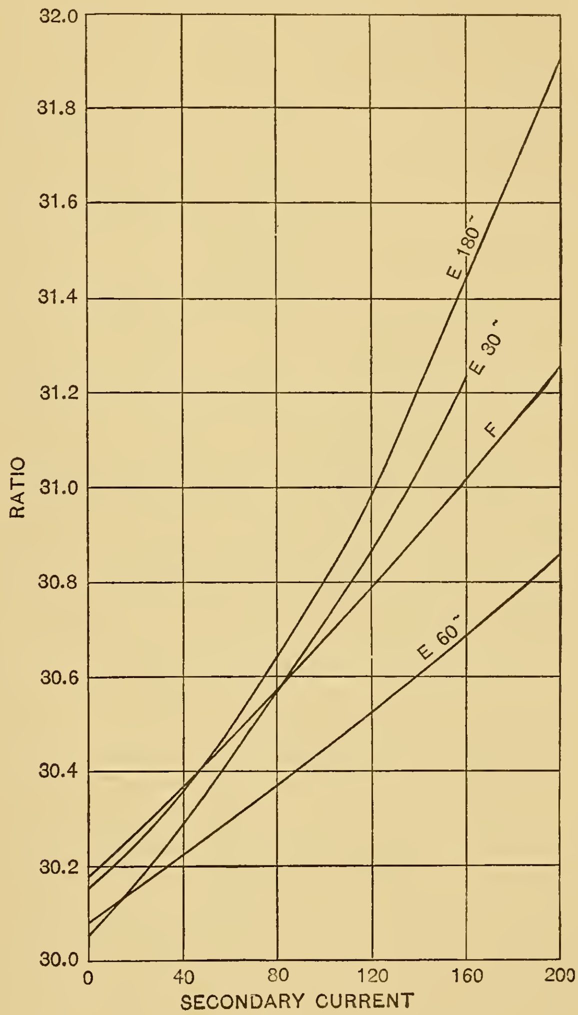

Fig. 6.

To make the effect of leakage apparent, a transformer was improvised by winding two coils upon opposite sides of a core of laminated iron. The results are given in Table VIII and show a very manifest increase in ratio with peaked wave, when the secondary was loaded.

Roessler[5] found the ratio larger with a peaked wave, indicating large leakage in his transformer. This is explained by the fact that his coils were wound side by side and not one over the other. His results (as regards ratio) do not apply to good transformers, where the effect will usually be in the opposite direction and negligible in amount.

TABLE IV.

Transformer E.—120/4 volts, 60 cycles, 2000 watts. Primary resistance = 0.076 ohm. Secondary resistance = 0.00027 ohm.

[Tested July 17, 1905, at 110 volts; exciting current at 60 cycles = 1.3 amperes; exciting current at 180 cycles = 0.45 ampere.]

Secondary Current	Ratio		Phase Angle		Ratio at 30 cycles 55 volts
	60 cycles	180 cycles	60 cycles	180 cycles	
0	30.08	30.15	+0°39′	−3°35′	30.05
40	30.22	30.35	+0°29′	−4°18′	30.28
80	30.37	30.65	+0°18′	−4°50′	30.57
120	30.52	30.98	+0° 7′	−5°20′	30.87
160	30.68	31.44	−0° 4′	−5°30′	31.24
200	30.86	31.90	−0°15′	−5°36′	

TABLE V.

Transformer F.—120/4 volts, 50 cycles, 1600 watts. Primary resistance = 0.17 ohm. Secondary resistance = 0.00027 ohm.

[Tested July 7, 1905, at 110 volts, 60 cycles; exciting current = 0.65 ampere.]

Secondary Current	Ratio	Phase Angle
0	30.18	+0° 54′
40	30.37	+0° 24′
80.5	30.57	0′
121	30.79	−0° 26′
160	31.02	−0° 52′
200	31.26	−1° 22′

[5] G. Roessler, Electrician, **36**, p. 151; 1895.

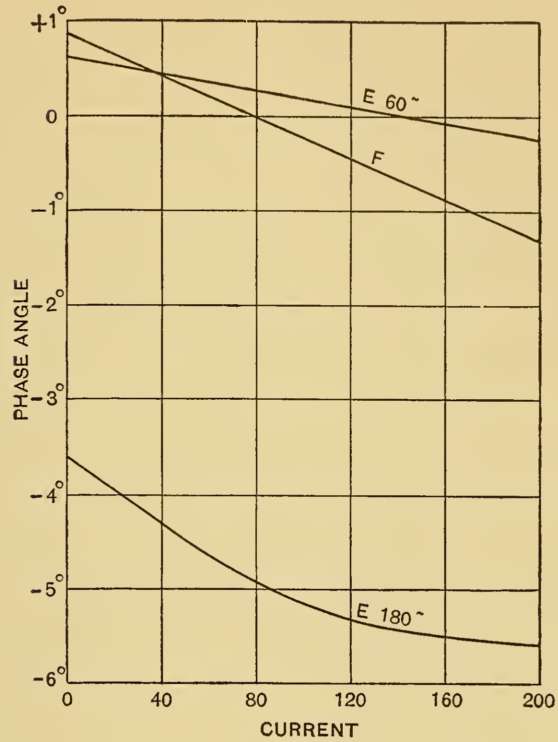
+1°
0
-1°
-2°
-3°
-4°
-5°
-6°
PHASE ANGLE
E 60~
F
E 180~
0
40
80
120
160
200
CURRENT

Fig. 7.

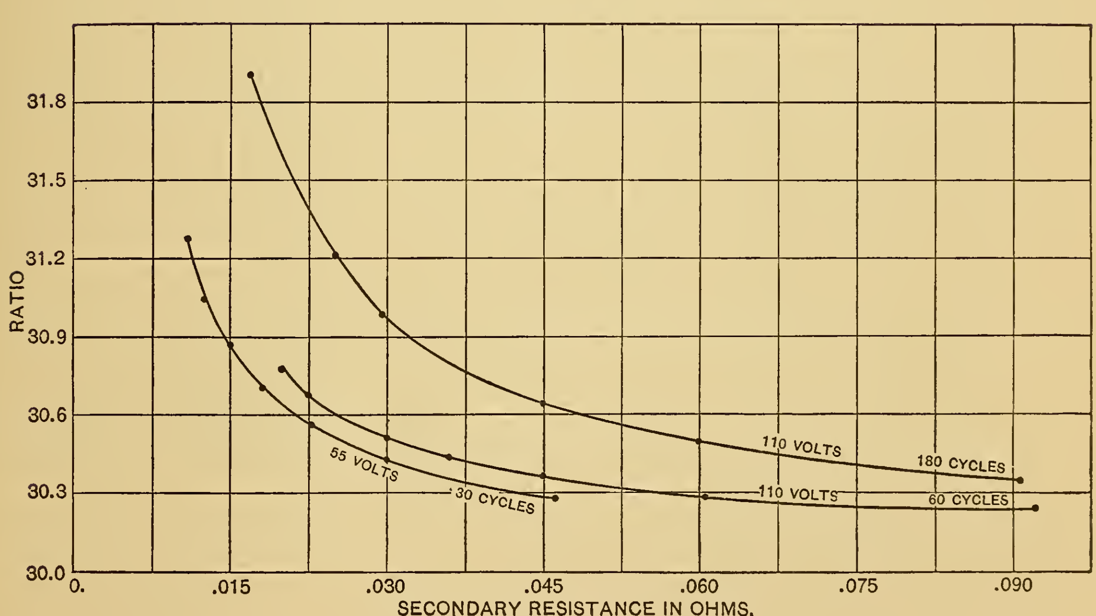
31.8
31.5
31.2
30.9
30.6
30.3
30.0
RATIO
110 VOLTS
180 CYCLES
55 VOLTS
30 CYCLES
110 VOLTS
60 CYCLES
0.
.015
.030
.045
.060
.075
.090
SECONDARY RESISTANCE IN OHMS.

Fig. 8.

TABLE VI.

Transformer G.—1100/110 volts, 60–125 cycles, 50 watts.

[Tested February 17, 1908, at 60 cycles, 1100 volts, 30 per cent of third harmonic.]

Wave Form	Load	R_1	R_2	Ratio
Sine	None	40103	4058	9.880
Peak	None	40103	4061	9.873
Sine	Full	40103	3983	10.070
Dimple	Full	40103	3980	10.078
Peak	Full	40103	3984	10.068

TABLE VII.

Transformer D.—480/120 volts, 60 cycles, 500 watts.

[Tested February 17, 1908, at 60 cycles, 480 volts, 17 per cent of third harmonic.]

Wave Form	Load	R_1	R_2	Ratio
Sine	None	20116	5016	4.010
Peak	None	20116	5020	4.007
Flat	None	20116	5014	4.012
Flat	Full	20116	4833	4.161
Peak	Full	20116	4836	4.159
Sine	Full	20116	4834	4.160

TABLE VIII.

Special transformer.—60/60 volts.

[Tested February 19, 1908, at 60 cycles, 60 volts, 24 per cent of third harmonic.]

Wave Form	Load	R_1	R_2	Ratio
Sine	None	1943.5	1612	1.205
Peak	None	1935	1612	1.200
Flat	None	1984	1612	1.230
Flat	1.5 ampere	4203	1612	2.605
Peak	1.5 ampere	4243	1612	2.630
Sine	1.5 ampere	4113	1612	2.548

CURRENT TRANSFORMERS.

The currents in the primary and secondary of a series transformer may be determined by a dynamometer in each circuit, of the type already described [6] in this bulletin. They are astatic, wound on frames of mahogany, have field coils which are wound with stranded wire (for the higher ranges), air damping, and the deflections are read with telescope and circular scale. As shown in the article cited, after being calibrated on direct current these instruments are correct for alternating currents of a wide range of frequency and any wave form.

The current flows through the field coils of the dynamometer and through a standard resistance in series. The moving coil is connected through a noninductive resistance of suitable value to the terminals of the standard resistance. The deflection of the instrument is a measure of the power expended in the standard resistance, and consequently is determined by the square of the current.

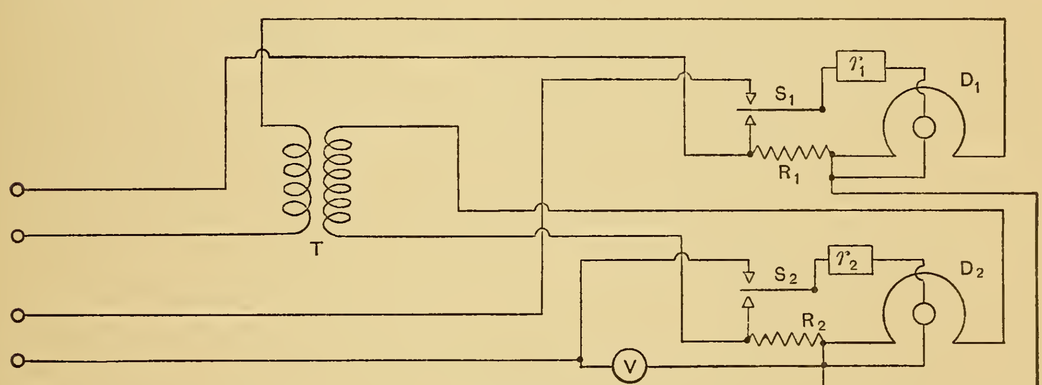

Fig. 9.

To determine the phase relation between the currents in primary and secondary, the two moving coils may be disconnected from the standard resistances and connected in series with each other. They are supplied with current exactly in quadrature with the primary current, so that there is no deflection of the dynamometer whose field coils are in the primary circuit. If the current in the secondary circuit is not exactly reversed in phase with

[6] E. B. Rosa, this Bulletin, 3, p. 43; 1907. Reprint No. 48.

respect to the primary there will be a deflection in the second dynamometer, and this serves to measure the phase difference.

Fig. 9 is a diagram of connections suitable for making both measurements by simply throwing two switches S_1 and S_2. T represents the transformer, D_1 and D_2 the dynamometers, R_1 and R_2 the standard resistances, r_1 and r_2 resistances in series with the moving coils, V a voltmeter. The switches are thrown up for phase measurement.

Let I_2 be the secondary current, i the current in moving coil. Let β be the angle by which the secondary current reversed leads the primary.

Let d_2 be the deflection of the dynamometer and k its constant. Then if the phase of the current in the moving coils has been adjusted for no deflection in the dynamometer D_1, $90° - \beta$ will be the phase angle between the two currents in D_2 and we have

$$d_2 = kI_2 i_2 \cos(90° - \beta) = kI_2 \frac{V}{r_2} \sin \beta$$

where r_2 includes the resistance of the moving coil.

After observing the deflection d_2 the switch S_2 is thrown over and r_2 is adjusted until the same deflection is again obtained. Let the new value be r_3.

Then

$$d_2 = kI_2 \frac{R_2 I_2}{r_3 + R_2} \text{ and } \sin \beta = \frac{R_2 I_2}{r_3 + R_2} \frac{r_2}{V}$$

A current in quadrature with the primary may be obtained in various ways, but most conveniently from a two-phase circuit, the second phase being applied directly to the moving coils. To have adjustment, however, an arrangement of rheostats may be used as in Fig. 10, where a is common to the two phases, and connections are made at b and d.

If only single phase be available, an air-core transformer may be used in the primary circuit, and its secondary used as a source of current for the moving coils. Since the resistance of this circuit is large, the current would be in quadrature with the primary current. Or the potential of the source may be used in conjunction with a condenser and resistances to get the necessary phase. Another well-known method is shown in Fig. 11, where *ABC* are inductive coils and *DE* noninductive resistance coils. By adjustment of these the current in *C* may be brought into exact quadrature with the supply, or with the primary current. A phase transformer with adjustable secondary may also be used.

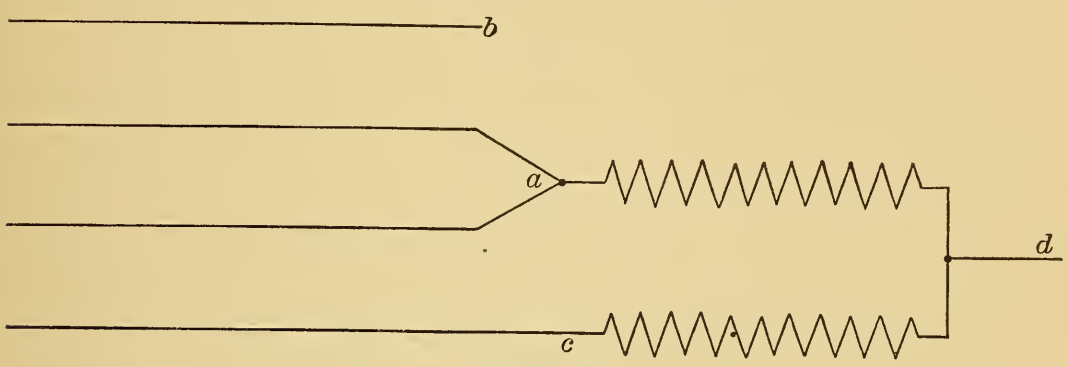

Fig. 10.

If it be not convenient to adjust for exact quadrature, the phase may be merely approximated and the deflection in D_1 observed, giving sin ν where $90° - \nu$ is the angle representing the phase relation between primary and moving coil circuits. Then the phase relation between primary and secondary is $\beta - \nu$. Care must be taken here to get the proper algebraic sign for ν.

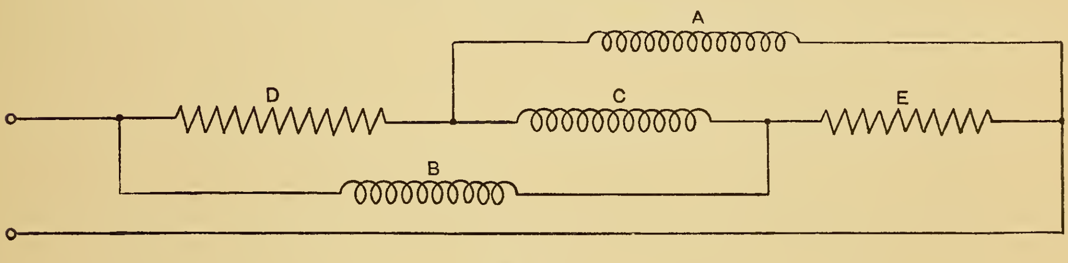

Fig. 11.

The method here indicated will give very accurate results for the conditions existing during the measurement, but as the accurate dynamometer used in the secondary circuit is of rather high resistance, the results would not be applicable to the same transformer when used with ordinary portable or switchboard instruments of low resistance. The method consequently is of little practical use. For ratio, readings taken on calibrated portable instruments of suitable type are usually sufficient, but more accurate indirect methods (i. e., not requiring the measurement of each current independently) have been used and are still undergoing development in the bureau.

Another method for determining the phase-angle of a series transformer, which is simpler and requires no computation, has been applied by making use of a special generator set. This consists of a driving motor and two generators mounted on one shaft. The generators have revolving poles and fixed armatures, and the position of one armature can be shifted circumferentially with respect to the other by means of a worm gearing. Its position is read on a graduated scale.

The current from the movable armature is sent through the fixed coil of a dynamometer (see Fig. 12). The moving coil is connected through a suitable resistance to the terminals of a small noninductive resistance standard in the primary circuit, and the position of the armature is adjusted until there is no deflection. The moving coil is next connected to a similar resistance in the secondary circuit, and the armature again shifted until there is no deflection.

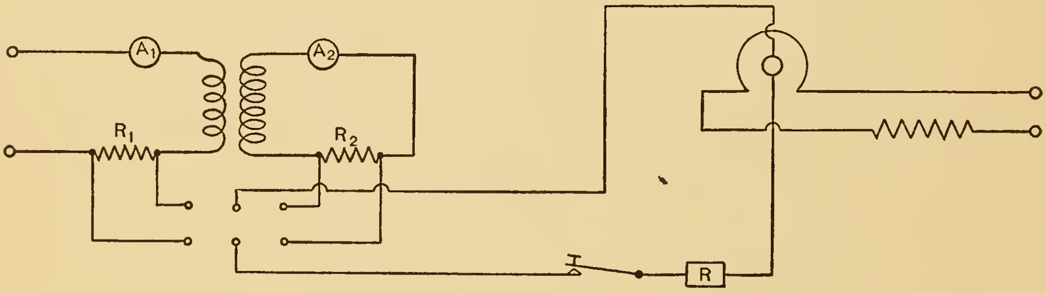

Fig. 12.

The number of electrical degrees between the two positions of the armature represents the angle β. In this case the generator has six poles, so that 120° of angular shift is equivalent to 360° electrical. The reading on the scale can be estimated to 0.1° or better, so that the uncertainty in the phase angle is not greater than 0.3° at most. When desired, this angle can be read with greater accuracy by means of finely graduated scale or vernier. The two noninductive resistances for primary and secondary circuits are chosen to give about the same drop, so that the resistance of the moving coil circuit is not altered during the measurement; any lag in this circuit is not altered on shifting from primary to secondary.

TABLE IX.

Transformer J.—25–125 cycles, 125/5 amperes, 10 watts.

[Tested March 14, 1908, at 60 cycles. 0.001 ohm in primary; 0.025 ohm in secondary; total resistance of secondary circuit, 0.054 ohm.]

Primary Current	Secondary Current	Ratio	Generator Settings		β
			Primary	Secondary	
					°
140.3	5.20	27.0	32.05	28.75	9.9
125.5	4.65	27.0	31.70	28.25	10.4
100.5	3.68	27.3	31.10	27.15	11.8
75.6	2.73	27.7	30.65	26.0	14.0
50.4	1.78	28.3	60.1	54.6	16.5
An extra ammeter inserted in the secondary circuit, doubling the impedance.					
135.3	4.72	28.6	....	....	....
125.5	4.36	28.8	31.8	28.25	10.6
100.5	3.44	29.2	....	....	....
75.6	2.50	30.2	....	....	....
50.4	1.62	31.1	30.1	24.0	18.3
Same at 30 cycles. Secondary resistance=0.054 ohm.					
140.3	4.91	28.6	32.0	27.7	12.9
125.5	4.36	28.8	31.6	27.0	13.8
100.5	3.41	29.5	31.0	25.95	15.2
75.6	2.50	30.2	30.5	24.3	18.6
50.4	1.57	32.1	29.9	22.3	22.8

This method has also the advantage that only a small resistance is required in the secondary circuit. In Tables IX, X, and XI the ratios were determined by means of calibrated portable ammeters, and the phase-angles by the method just described. Transformer J is of the type which can be slipped over a cable and the two parts of the core clamped together.

TABLE X.

Transformer K.—5/5 amperes.

[Tested March 21, 1908, at 60 cycles. 0.05 ohm in primary; 0.025 ohm in secondary; total resistance of secondary circuit, 0.054 ohm.]

Primary Current	Secondary Current	Ratio	Generator Settings		β
			Primary	Secondary	
5.00	4.99	1.002	12.95	12.90	0.1
4.04	4.03	1.0025	9.95	9.90	.1
3.02	3.01	1.003	6.8	6.75	.1
2.015	1.99	1.013	4.0	3.9	.3
1.02	0.995	1.025	1.6	1.55	.1
Same at 30 cycles.					
5.00	4.99	1.002	1.6	1.4	0.6
4.04	4.03	1.002	7.25	7.15	.3
3.03	3.01	1.007	5.0	4.8	.6
1.025	0.995	1.03	0.6	0.25	1.0
With 1 ohm additional in secondary circuit.					
5.00	4.955	1.009	1.7	1.4	0.9
Same with extra ammeter inserted in secondary circuit, doubling its impedance. 60 cycles.					
5.00	4.99	1.002	13.95	13.90	0.1
4.06	4.03	1.007	10.55	10.45	.3
3.03	3.01	1.007	7.3	7.25	.1
1.03	0.995	1.035	1.75	1.7	.1

TABLE XI.

Transformer K.—5/5 amperes.

[Tested March 21, 1908, at 60 cycles and full load.]

Extra Secondary Resistance	Ratio	β
0.0	1.003	....
.2	1.003	....
.4	1.003	....
.6	1.005	....
1.0	1.007	0.6

Transformer K has the secondary current (reversed) in almost exact phase with the primary, but the angle is not quite zero. By connecting a large impedance in the secondary circuit at full load the angle was made negative, but by an amount too small to measure though visible in the dynamometer deflection. By inserting 3 ohms in the secondary circuit with 2 amperes load β was increased to 1.8°, the maximum for this transformer under any of the conditions imposed.

The same method for phase-angles may be applied to potential transformers.

TABLE XII.

Transformer L.—5/5 amperes.

[Tested March 25, 1908, at 60 cycles. Secondary current, 4 amperes.]

Wave Form	Per cent Third Harmonic	Deflection of Primary Dynamometer	Mean.	Per cent Increase in Ratio
Peak	11	22.33 .32 .33	22.33	0.0
Flat	11	22.33 .33	22.33	0.0
Sine	....	22.33 .34	22.33_5	....
Sine	....	22.33 .32	22.32_5	....
Dimple	30	22.36 .35	22.35_5	0.05
Peak	30	22.31 .32	22.31_5	−0.03
Sine	....	22.35 .32	22.33_5	....
Dimple	30	22.36 .36	22.36	0.07
Peak	30	22.30 .29	22.29_5	−0.08
Peak	68	22.27 .25	22.26	−0.15
Dimple	68	22.41 .39	22.40	+0.17
Sine	....	22.32 .33	22.32_5	....
Dimple	68	22.38 .39	22.38_5	+0.14
Peak	68	22.25 .26	22.25_5	−0.16

Table XII shows the effect of wave form upon the ratio of a current transformer. In this case a portable dynamometer ammeter was connected in the secondary circuit, and the current adjusted until its needle coincided with a division. The error here was not greater than 0.05 per cent. In the primary circuit a mirror dynamometer was included and its reading taken at the same time. The wave form was varied in the manner already described.

The effect is seen to be inappreciable except with a large component of harmonic, and would be negligible for all practical purposes unless the component of harmonic exceed half the value of the fundamental.

To exclude the possibility of the change being in the instruments and not in the transformer, the portable dynamometer was placed in series with the mirror dynamometer in the primary circuit and readings taken with the same current in each. These showed no discrepancies beyond the error in reading for the most distorted wave.

Transformer L is a duplicate of transformer K.

For other methods of measuring transformer ratios and phase-angles, we refer the reader to the works of Robinson,[7] Drysdale,[8] Makower,[9] and Sumpner.[10]

We are indebted to Messrs. C. E. Reid, J. V. S. Fisher, and G. W. M. Vinal for assistance in taking the observations.

WASHINGTON, February 25, 1909.

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[7] L. T. Robinson, Trans. A. I. E. E., **25**, p. 727; 1906.

[8] C. V. Drysdale, Electrician, **58**, pp. 160, 199; 1906: Phil. Mag. **16**, p. 136; 1908.

[9] A. J. Makower, Electrician, **58**, p. 695; 1907.

[10] W. E. Sumpner, Phil. Mag., **9**, p. 155; 1905.