2E. Probability and Likelihood Ratios

Introduction

Statistics and probability are both useful tools for interpreting data and observations. They have some overlapping features, but they are conceptually different. Those who support the use of probability and likelihood ratios (LRs) in forensic science opinions see them as an application of logical reasoning with uncertainty. Opposition to using probability and LRs in forensic science opinions focuses on the subjectiveness (variability) in the LRs that are provided, the use of LRs that do not align with performance data and misunderstanding of the expression.

Statistics Versus Probability

Statistics are derived and extracted from the empirical world, whereas probability projects itself onto the empirical world.[1]

Statistics
- Applied Science
- Frequency of past events
- Collects and summarizes empirical data from observed events
- Estimates a characteristic of a population based on a sample
- Given an observation, what processes/models would explain the observation
- Inductive
- Uncertainty is expressed by a level of confidence

Probability
- Theoretical Science
- Assessing the probability of future events or past events in doubt
- Begins with general assumptions and quantifies outcomes
- Given a process/model with stated assumptions that align with the individual's beliefs, what are the probabilities of possible outcomes
- Deductive
- An individual's uncertainty is expressed in the probability value itself on the continuum between zero and one

Probabilities and Likelihood Ratios

Although probability is often thought to be an objective measure because it is expressed as a number, it is in fact subjective. Probability measures the strength of an individual's belief that the event will occur or did occur and is subjective because it relies on differing information available and assumptions made to arrive at the probability number. There is no single, correct probability for the possibility that an event will occur. In theory, an infinite number of combinations of assumptions can be made that would result in an infinite number of probability values. No probability is right or wrong, although some probability assignments may be considered more reasonable than others.

A likelihood ratio (LR), a ratio of two mutually exclusive probabilities, is also subjective.

Figure 1. Calculation for likelihood ratios

$$Likelihood\ Ratio = \frac{Probability\ 1}{Probability\ 2}$$

In forensic science, the LR is typically the ratio of the probabilities of the evidence under two alternative propositions.

Figure 2. Calculation

$$R = \frac{Probability\ of\ the\ Evidence\ |Prosecution\ Hypothesis\ (H_1)}{Probability\ of\ the\ Evidence|Defense\ Hypothesis\ (H_2)}$$

A probability is always a non-negative number between zero (a certainty of *non*occurrence) and one (a certainty of occurrence). Values between zero and one represent the uncertainty in the individual's belief of a particular event occurring. Probability can also be expressed as a percentage between 0% and 100%.

Since probabilities are a non-negative number, an LR will also be a non-negative number. An LR will have a value between zero and infinity (see Figures 1 and 2).
- If Probability 1 approaches zero, the LR will approach zero
- If Probability 2 approaches zero, the LR will approach infinity
- If Probability 1 and 2 are equal, the LR will equal 1

LR – Frequentist and Bayesian Approaches

Although sometimes referred to as the Bayes factor, an LR can be assigned using either a frequentist or Bayesian approach. Regardless of how it is assigned, an officer of the court will need to make their own evaluation of any LR provided.

A frequentist approach relates probability to the frequency of observing an event in a large number of experiments where the event may or may not occur. Inferences are based on random sampling. Frequentist statistics focus on the probability (Pr) of the evidence (E) given a hypothesis (H), and is expressed as Pr(E|H).

Under a Bayesian approach, the probability of an event occurring is defined as the degree of one's belief in the truth of the proposition that asserts that the event will occur. Inferences can be made even if the information is incomplete using probability distribution models to describe a belief. The Bayesian approach helps an individual quantify their personal probability assessment of a hypothesis given the evidence, Pr(H|E).

For both, any single LR is looking at two mutually exclusive hypotheses (H_1 and H_2) but to be exhaustive, within the context of the case, additional LRs (H_1 vs H_3, H_1 vs H_4, and so on) may need to be assessed.

What Adds Subjectivity?

Combinations of different assumptions or different levels of belief can result in the same probability or LR (e.g., one person's probability of 80% may not mean the same as another person's probability of 80%). Conversely, two people with the exact same information may not provide the same probability or LR.

Footnote:
1. RSS: Fundamentals of Probability and Statistical Evidence in Criminal Proceedings, section 1.5.

In forensic science, some aspects can vary between forensic science practitioners (FSPs) within a single forensic science service provider (FSSP) or between FSSPs that will result in variation in the LR produced (see Table 1). Standardization may help reduce the variability of the LRs produced.

Table 1 Aspects that add variation to a probability, to an LR

Aspect	*Possible Variability*
Data	• Type of data (e.g., feature, attribute, characteristic, profile, spectrum) • Source of data
Method of examination	• Different technology • Different operating parameters
Method of data interpretation	• Degree of standardization between FSPs • Degree of standardization between FSSPs
Calculations performed	• Done without calculation • Calculated by a human or an algorithm • Distribution chosen to represent the data • Model used
Additional information	• Case Information available, used, not used • FSP knowledge & experience • FSSP data

Figure 3 demonstrates that multiple models can meet a set of criteria for reasonableness and produce an LR. When the evaluation of the model includes an assessment of reasonableness based on the LRs produced for known samples, the number of models that produce reasonable LRs may diminish. Models that "pass" will still produce different LRs for the same data.

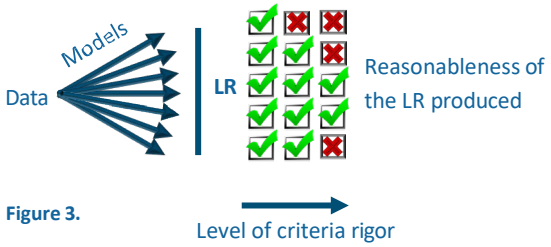

Figure 3.

Example 1: Interlaboratory study to evaluate the forensic analysis and interpretation of glass evidence: Standardized technology (LA-ICP-MS), method (ASTM E2927), case information, model, and samples. LRs: Supported the correct hypothesis with some indicating varying strength for an association.
Example 2: GHEP-ISFG collaborative exercise on mixture profiles (GHEP-MIX06): Two mixture profiles in a paper format. The data interpretation method was specific to the participant. With the same LR model, LR varied by up to 12 orders of magnitude.
Example 3: Collaborative Testing Service, Inc. DNA Interpretation Test No. 22-5881 Summary Report – LRs provided ranged from 10^9 to 10^{28} for Item 3 and 10^7 to 10^{26} for Item 4.

LR as a Numerical or Verbal Scale

Since the numerical value of an LR can range from zero to infinity, an LR is at times translated to, and accompanied by, a strong support, extremely strong support) to convey the magnitude of the LR. The inverse of the scale is used to represent the alternative proposition.

Strengths and Limitations of an LR

Strengths:

- Is a mechanism to express the relative likelihood of two competing hypotheses
- Can be assigned with or without empirical data; therefore, there is no limitation to when an LR can be provided
- Can be evaluated and calibrated, for consistency (over or understatement) with known samples
- Can be limited (e.g., a lower and upper limit) to that supported by available data
- Can be evaluated for usefulness to a specific case when context information and performance data demonstrating how often this level of LR has occurred in both H_1 and H_2 true cases is provided

Limitations:

- There is no "true value" or "correct value"
- Is subjective because of personal or organizational choices
- Can have values that vary by orders of magnitude
- Other models may be as "good" or even "better"
- Is difficult to have sufficient data from known samples to evaluate the reasonableness of the LR values produced
- An officer of the court will need to determine if an LR provided by an FSP aligns with their own evaluation of the performance data and their personal beliefs

Key Takeaways

1. Probability will have a value between 0 and 1; an LR will have a value between 0 and infinity.
2. Probabilities and LRs are personal and will vary depending on the data (empirical or personal belief), assumptions made, and calculation model used.
3. An officer of the court or the trier of fact will need to determine, either informally or formally with calculations, their own value for a probability or LR.

Related Primers

Algorithms General Statistics Population Statistics

Learn More

Likelihood Ratio as Weight of Forensic Evidence: A Closer Look & Response & Bayesian Reasoning and Evidence Communication

The Royal Society – The use of statistics in legal proceedings

The Royal Statistical Society Guides

Part of NIST SP 1500-28
https://doi.org/10.6028/NIST.SP.1500-28

Glossary: Primer 2E Probability and Likelihood Ratios

Term	Definition	Reference	Primer Nos.
Likelihood ratio	A comparison of the probabilities of the evidence under two alternative propositions	Butler, J. Advanced Topics in Forensic DNA Typing: Interpretation, 1st Edition, 2014; p226	2E Probability & Likelihood Ratios
Probability	A quantified measure between zero and one indicating how probable or likely it is that an event will or has occurred. In the frequentist interpretation, probability is based on the rates at which events occur. In the Bayesian interpretation, probability reflects a degree of belief. On this scale, zero indicates impossibility and one indicates absolute certainty	Organization of Scientific Area Committees for Forensic Sciences. (2022). Retrieved from OSAC Lexicon: https://www.nist.gov/glossary/osac-lexicon	2E Probability & Likelihood Ratios