METHODS, FORMULAS, AND TABLES FOR THE CALCULATION OF ANTENNA CAPACITY

By Frederick W. Grover

ABSTRACT

To calculate the capacity of an antenna a certain charge is assumed upon the antenna and the resulting potential is calculated.

In carrying out this method the difficulty is met that, in general, the law of distribution of the charge is not known. Howe made the assumption that sufficient accuracy is attained if first a uniform distribution of charge is supposed to exist and the potential calculated at various points of the antenna, the average of these potentials being taken as the final equilibrium potential. Howe called attention to discrepancies between the values obtained by his method and published values for the same antennas by the inductance method. The present paper shows that the two methods agree if appropriate inductance formulas are employed. The Howe method is more general, and it is believed to give sufficient accuracy for engineering requirements.

Formulas are given for the common types of single and multiple wire antennas in a form convenient for numerical computation, together with tables of constants which will be found useful in such calculations. In addition, tables of the capacities of both horizontal and vertical single-wire antennas and horizontal two-wire antennas have been included, which should render all calculation unnecessary in many inportant practical cases.

CONTENTS

I. INTRODUCTION

Formulas for the calculation of the capacity between two parallel wires of infinite length have long been known. The case of a single wire of infinite length, stretched parallel with the surface of the earth at a distance which is small, compared with its length, may be treated by the same formulas, since, by the theory of electric images, the effect of the induced charge on the earth may be taken into account by supposing the earth to be replaced by a wire of the same dimensions as the given wire and carrying a charge of opposite sign. This image wire is assumed to lie as far below the surface of the earth as the actual wire is above the surface of the earth. A very complete treatment of the whole subject for wires of infinite length, together with a bibliography, is given in a paper by Kennelly [1] who has also in a later article [2] published curves for simplifying the calculations.

[1] Proc. Am. Phil. Soc., 48, p. 142; 1909.

[2] Elec. World, Oct. 27, 1910.

The formulas for wires of infinite length suffice for calculations with transmission lines, but in the very important case of an antenna, the wires of which it is composed are seldom of such a length that their distances from one another and from the earth, can be neglected in comparison with their lengths. A correction for the lack of uniformity of distribution of the charge along the wires has to be taken into account. The capacity of a single wire has sometimes been calculated from the formula for a very long thin ellipsoid, isolated in space, but only in recent years have formulas for the more common forms of antenna been derived.

Of these, especial mention may be made of the work of Cohen,[3] who derived formulas for the capacity of an antenna consisting of a number of wires, arranged parallel to one another and to the surface of the earth (flat-top antenna) by assuming the reciprocal of the capacity per unit length to be equal to the inductance per unit length of the system of wires, assumed to be joined in parallel with the earth as return. This reciprocal relation holds exactly only when the wires are of infinite length.

The most extensive contribution is, however, furnished by the papers of Howe,[4] who has obtained formulas for a number of common forms of antenna from electrostatic considerations. The distinguishing feature of his treatment is the method used to take into account the lack of uniformity of distribution of the charge along the wires. Howe called attention to the considerable difference in the value of the capacity calculated by Cohen for a certain flat-top antenna and the value obtained by his own formula for the same case, but gave no explanation of the discrepancy.

In 1917 formulas derived by the author of the present paper were published by the Bureau of Standards.[5] In these account was taken of the finite length of the wire, but the lack of uniformity of charge distribution was only imperfectly taken into account. The method of Cohen was employed, but with a considerable simplification in the mathematical work.

An approximate formula by Austin [6] for the capacity of multiple-wire flat-top antennas has the advantage of simplicity, with an accuracy which is greater the greater the number of wires in the antenna top; that is, for close distribution of the wires.

The present paper has grown out of an examination of the difference between the results of Cohen and Howe. It will be shown that although neither method is exact, the method of Cohen leads to the same formulas as the method of Howe in those cases where the former method is applicable.

---

[3] L. Cohen, Alternating-current Problems, McGraw-Hill Publishing Co., 1913.

[4] G. W. O. Howe, Lond. Elect., **73**, p. 829; 1914. Lond. Elect., **77**, p. 761; 1916. Proc. Lond. Phys. Soc., **29**, p. 339; 1916–17.

[5] Circular No. 74, Radio Instruments and Measurements, Bureau of Standards. pp. 237–241; 1917.

[6] L. W. Austin, J. Wash. Acad. of Sci., **9**, p. 393; 1919

Finally, the Howe method of approximation has been used to derive formulas for the capacity of the more usual types of antenna. Some of these cases have already been treated by Howe, but for the most part with further simplifying assumptions. The attempt has been made here to avoid, as far as possible, approximations other than that involved in Howe's fundamental assumption. Especial attention has been paid to putting the formulas in a form convenient for numerical calculation, and to unifying the mode of expression, so that a few tables suffice for all the principal cases.

The formulas of this paper apply strictly only to the *electrostatic capacity*. For the case of alternating currents, provided that the length of the antenna is small compared with the wave length, the value of the quotient of charge by potential may be regarded as the effective capacity of the antenna.

II. GENERAL METHOD USED FOR CALCULATING THE CAPACITY

The electrostatic capacity of a conductor is defined as the quotient of its charge by its potential. The potential is the algebraic sum of the values of potential given it by its own charge and by the charges on all the other conductors of the system. The effect of the induced charges on the earth may be taken into account by including with each conductor an image conductor which is supposed to carry an equal and opposite charge to that on the conductor to which it corresponds.

Accordingly, the capacity of an infinite straight wire placed parallel to the surface of the earth is the same as that of an infinite straight wire, placed parallel to an equal wire, bearing an equal charge of the opposite sign. The exact formula for the capacity in this last case is well known. If d is the diameter of cross section of the wire, and h is its height above the earth, the capacity of the wire per unit length is

$$C_1 = \frac{1}{2\cosh^{-1}\frac{2h}{d}} = \frac{1}{2\operatorname{logn}\left[\frac{2h+\sqrt{4h^2-d^2}}{d}\right]} \tag{1}$$

For a wire 0.01 foot (0.12 inch) in diameter, placed 25 feet above the ground, the formula

$$C_1 = \frac{1}{2\operatorname{logn}\frac{4h}{d}} \tag{2}$$

differs from the exact formula (1) by only 1 part in 10^9. The exact formula takes into account the fact that the surface density of the charge is not quite uniform around the perimeter of the elements of the wire, as a result of the attraction between the charge on the wire and that on the earth. This disturbance is neglected in equation (2), and the smallness of the error thus committed in the example in question shows that this effect does not need to be taken into account with the sizes of wire and the heights common in antennas.

For an infinite wire the distribution of charge is uniform along the axis; that is, the quantity of charge is everywhere the same for portions of the cylindrical surface intercepted between planes drawn perpendicular to the axis and 1 cm apart. Neglecting the slight variation of the surface density around the perimeter of the cross section, the potential at external points and at points on the surface of the wire, is the same as would be produced by a uniform distribution of charge on the axis itself, if the quantity of charge q per centimeter, measured along the axis, be taken equal to the charge on the cylindrical surface between planes perpendicular to the axis and 1 cm apart.

III. SINGLE HORIZONTAL WIRE

For a horizontal antenna wire the height above the earth can not be regarded as negligible in comparison with the length of the wire; on the contrary, these two dimensions are, as a rule, of the same order of magnitude. No exact formula for the calculation of the capacity of such a wire is known. In fact, the simpler problem of the calculation of the capacity of an isolated cylinder has not yet been solved and seems to offer great difficulties. Fortunately, the smallness of the diameter of the wires used for antennas, as compared with their length, allows an approximation to the capacity to be obtained. The following possibilities may be considered for the case of a single horizontal wire.

We may, first, assume that the capacity per centimeter is the same as for an infinite wire of the same diameter and height above the earth, and may thus write for the capacity

$$C=\frac{l}{2\ \text{logn}\ \frac{4h}{d}} \tag{3}$$

l being the length of the wire in centimeters. This formula, however, does not take into account the fact that the charge density on the wire increases as the ends of the wire are approached, and must become infinite for an infinitesimal area of the surface at the extreme ends of the wires.

Let us suppose that a charge distribution of q units per centimeter be placed along the axis of the wire. (Fig. 1.) The potential at any point P (a, D) is readily found to be

$$v=q\left[\sinh^{-1}\frac{\frac{l}{2}+a}{D}+\sinh^{-1}\frac{\frac{l}{2}-a}{D}\right] \tag{4}$$

and taking the point P on the surface of the wire; that is, putting $D=\frac{d}{2}$, the potential of the wire due to the charge is

$$v_1=q\left[\sinh^{-1}\frac{\frac{l}{2}+a}{d/2}+\sinh^{-1}\frac{\frac{l}{2}-a}{d/2}\right] \tag{5}$$

The similar distribution of charge of $-q$ per centimeter on the image wire will give to the wire at the point P the potential

$$v_2=-q\left[\sinh^{-1}\frac{\frac{l}{2}+a}{2h}+\sinh^{-1}\frac{\frac{l}{2}-a}{2h}\right] \tag{6}$$

The potential values calculated from equations (5) and (6) are plotted in Figure 2 for a wire 50 feet long, 0.01 foot in diameter,

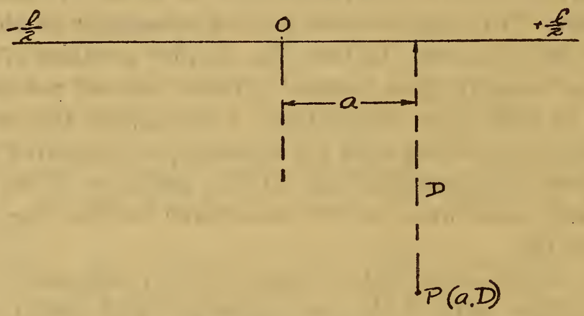

FIG. 1.—*Potential due to a uniform line charge at an external point*

placed 25 feet above the ground. Curve A shows the potential variation along the wire, due to the assumed uniform distribution of charge on the wire. The potential has a maximum value of 18.420 q at the center, and decreases only very slightly until the ends are nearly reached, when the value drops suddenly to 9.903 q. The potential due to the charge on the image wire, curve B, is also largest at the center, and is quite uniform over the wire, decreasing somewhat toward the ends. The value at the center is -0.962 q, and at the ends -0.881 q. The resultant potential, curve C, which is the sum of the curves A and B, has, therefore, a maximum value of 17.458 q at the center and 9.022 q at the ends, but the changes are small, excepting within the last few inches of wire at the ends.

Thus, the value of potential at the center may be taken as representing approximately the actual potential of the wire, and this approximation is a closer one than the value 18.420 q given by the infinite wire formula for in the latter is included the potential which would be contributed by uniform axial charges of density q per centimeter reaching from the ends of the wire to infinity.

IV. HOWE'S APPROXIMATION

The assumed uniformly distributed charge on the wire would not be in equilibrium. The potential variation shown in curve *C* (fig. 2) would cause charge to move from the center toward the ends. Thus, the potential at the middle would fall, and that at the ends would rise, until the whole wire would reach a uniform potential. It is thus evident that the potential value calculated for the middle of the wire on the assumption of uniform charge density is too large, and the value similarly calculated for the ends is too small. As an approximation to the true equilibrium potential, Howe calculated the average of the potentials taken over the length of the wire. The expression for this average is readily obtained by integrating equations (5) and (6) over the wire, and dividing the result by the length of the wire. The average potential due to the charge on the wire is thus found to be 17.807 q (DD', fig. 2), and that due to the image charge $-0.934\,q$. Their sum is 16.874 q (EE', fig. 2). If, instead of supposing an axial distribution of charge, the distribution were supposed uniform over the cylindrical surface of the wire, and the average of the potentials taken over the surface of the wire, the result found would differ in this example by only 5 parts in 100,000. Thus, for the dimensions common with antenna wires, the assumption of an axial distribution in carrying out the Howe method of approximation is justified.

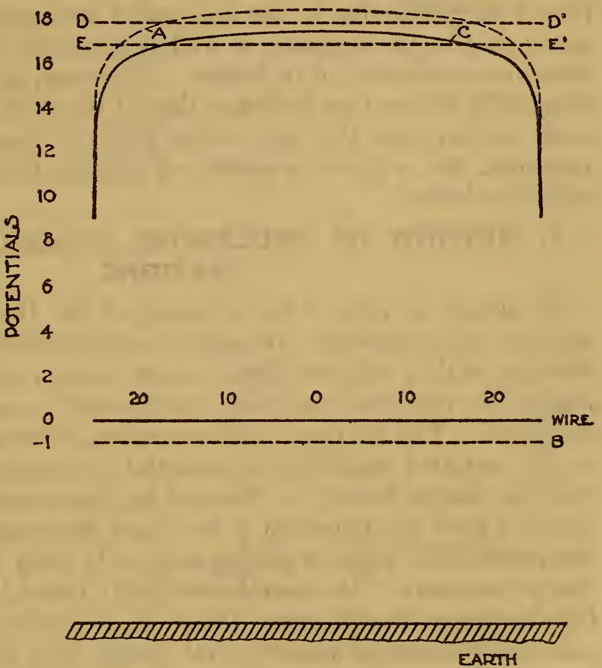

FIG. 2.—*Potential curves of uniformly charged horizontal wire*

The different approximations to the potential of the wire considered may be summarized as follows:

Infinite wire formula, $v = 18.420\ q$
Uniform axial distribution (middle point), $v = 17.458\ q$
Howe's approximation, $v = 16.874\ q$

The first and second of these are known to be too large. A confirmation of this fact is furnished if we consider the wire to be compared with a charged isolated ellipsoidal conductor, having its greatest dimension the same as that of the wire, and the same central cross section as that of the wire. Using the previous nomenclature, the potential of such a conductor is known to be $v_0 = \frac{2ql}{\sqrt{l^2-d^2}} \cosh^{-1}\frac{l}{d}$ which, for an ellipsoid equivalent to the wire previously considered, gives a value (18.420 q) which differs by a negligible amount from the potential produced at its middle point by the same charge uniformly distributed on an isolated cylindrical wire. This fact is not surprising, since it is known that on the ellipsoid the charge included between planes perpendicular to the axis and 1 cm apart is everywhere the same; that is, the ellipsoid, as well as the wire, has a distribution of q units per centimeter of its length. However, the cross section of the ellipsoid is everywhere less than that of the wire, except at the central cross section, and this fact would lead one to expect for it a higher potential, for a given quantity of charge, than for the equivalent cylindrical wire.

V. METHOD OF SUCCESSIVE NUMERICAL APPROXIMATIONS

To obtain an idea of the accuracy of the Howe approximation, a method of successive numerical approximations was employed. Starting with a uniform linear charge density upon the wire and its image, the potential variation over the wire is calculated, as already described. The average, or Howe value, gives a first approximation to the required equilibrium potential. A second approximation to the true charge density is obtained by increasing the values at those points where the potential is low, and decreasing the values where the potential is high, in such a way as to keep the total quantity of charge constant. As a simple method of doing this, the values of the linear charge density were taken as inversely proportional to the calculated potential values (total charge kept constant). With this new distribution of charge the corresponding new potential distribution has to be calculated, and the average taken. This gives a second approximation to the equilibrium potential. Correcting the charge distribution again in the inverse ratio to the potential variation of the potential distribution last found, potential values are again calculated and averaged, and thus a third approximation to the equilibrium potential found, and so on.

The chief difficulty with the method lies in the fact that the expressions for the calculation of the charge density and potentials of the second and higher approximations can not be integrated directly, but methods of mechanical integration and averaging have to be employed, thus requiring the evaluation of many ordinates of the curves of these quantities. Thus the work is very laborious and time consuming.

The only case for which the author has obtained more than the second approximation to the potential is that of a single vertical wire 50 feet long, 0.01 foot in diameter, with its lower end only 1 foot from the ground. The effect of the earth is in this case very marked, so that this may be regarded as rather a severe test of the simple Howe approximation. The curves of the successive approximations to the equilibrium calculated for this case by the method just outlined are shown in Figure 3. The curve A is for uniform linear charge density on the wire and its image. The average or Howe approximation to the potential is 16.563 q. Curve B shows that the potential corresponding to the corrected charge density (second approximation) is very uniform over the wire except for the last 0.2 foot at each end. The average of this curve gives 16.42 q as the second approximation to the equilibrium potential. Again correcting the charge densities, the potential distribution resulting hardly varies in the fourth figure, except at the extreme end of the wire. The third approximation to the equilibrium potential is 16.41 q, or about 1 per cent less than the Howe approximation. The convergence of the method appears to be entirely satisfactory. It should be noted that in making these calculations the effect of the charges on the end faces of the cylindrical wire were taken into account. Their effect is found to be inappreciable at distances greater than about the diameter of the cross section of the wire.

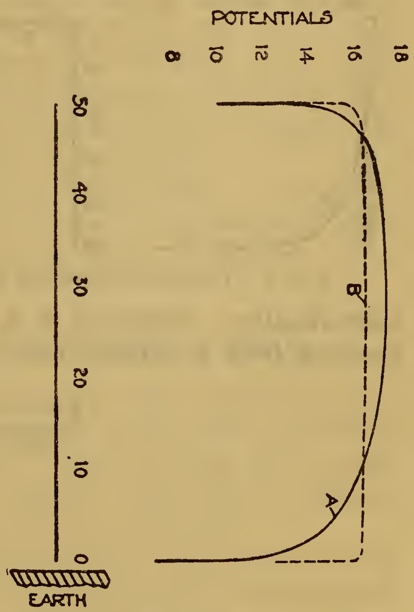

Fig. 3.—*Successive approximations to the potential of a single vertical wire*

A second approximation to the potential of the same wire isolated in space has been calculated. The first or Howe approximation, as has already been mentioned, is 17.807 q. The second approximation is only 2 to 3 parts in 1,000 less, which makes it probable that the value 17.75 q is correct to perhaps a unit in the last place.

From the nature of the case it is apparent that the method of successive numerical approximations can not give a general estimate of the accuracy of Howe's approximation. Each wire or combination of wires has to be treated as a special case. The fact that the error in the case of a single wire or a vertical wire and its image is small is to be expected, since the cases which they resemble (ellipsoid and hyperboloid of revolution, respectively) are known to be in equilibrium with equal charges between equally spaced planes drawn perpendicular to the axis. However, in the case of more complicated wire systems no evidence is yet available as to the error of the Howe

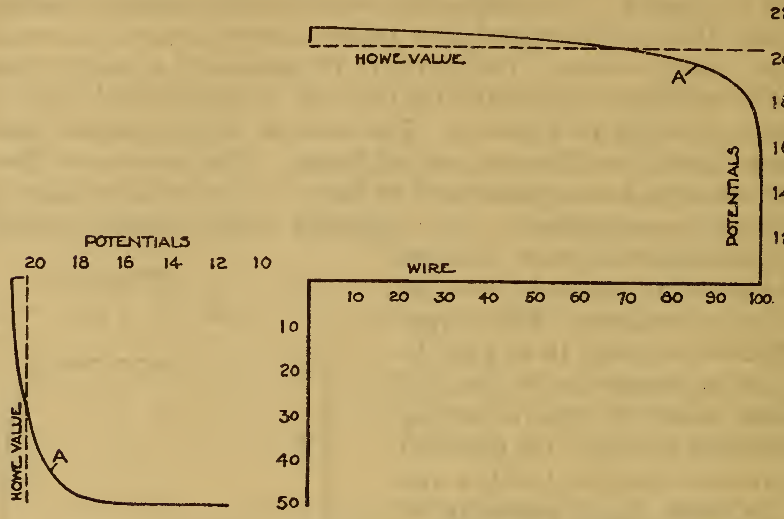

FIG. 4.—*Potential variation of uniformly charged isolated* L *antenna*

approximation. Figures 4, 5, 6, and 7 show the potential variation resulting from a uniform linear charge density on two important

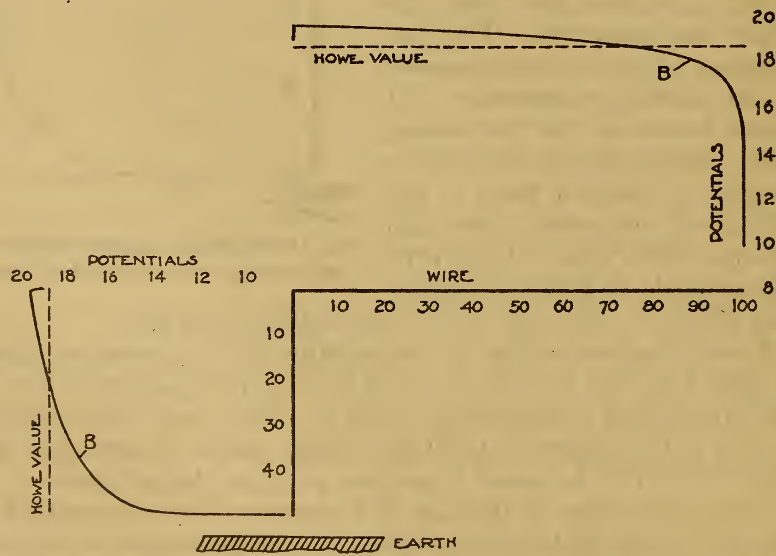

FIG. 5.—*Potential variations of uniformly charged* L *antenna*

cases, viz, two wires each 0.01 foot in diameter and of lengths 50 and 100 feet, respectively, joined in the one case to form an inverted L and in the other to form a T antenna. The shorter wire is placed vertical with its lower end 5 feet from the ground and the image wires are included. Curve *A* shows in the case of each combination the potential distribution resulting from the charge of the wires, while curve *B* in each case shows the potentials as modified by the earth. The Howe values are also indicated in each case. From the potential curve for the L antenna it appears that this case is little different from what would be found for the same wires arranged to form a single vertical antenna. Thus, the Howe approximation is probably as accurate in this case as has been found for a vertical wire by the method of successive approximation. For the T antenna, and still more so in other more complicated antenna systems, nothing can be stated as to the accuracy of Howe's approximation. The author believes it probable that for such long thin wires as are found in practice the error is not important in the light of the uncertainties introduced by errors in the measurements of the dimensions, irregularities in the earth's surface, disturbances due to neighboring objects, effect of imperfect dielectrics, and the like.

Accordingly, the formulas of this paper are based on the Howe approximation, and it remains to show how it may be extended to the treatment of the more complicated forms of antenna. Two general methods are available.

VI. TREATMENT OF COMBINATIONS OF WIRES

First, we may follow Howe, assuming, first, that a uniform linear charge density q be imparted to the whole system and calculating the potential distribution which would result, average these values over the whole length of wire in the system. This value is taken as an approximation to the true or equilibrium potential.

Thus, denoting by l and m the lengths of two wires joined to form a system, and by v_1 and v_2 the values of the potentials at any points of the wires, due to a uniform linear charge density upon the wires and their images, then the approximation to the equilibrium potential is to be taken as equal to

$$\begin{aligned} v &= \frac{1}{l+m}\left[\int_0^l v_1 dl + \int_0^m v_2 dm\right] \\ &= \frac{l}{l+m}\left[\frac{1}{l}\int_0^l v_1 dl\right] + \frac{m}{l+m}\left[\frac{1}{m}\int_0^m v_2 dm\right] \end{aligned} \tag{7}$$

which indicates that the mean is to be taken of the Howe potentials for the two wires, weighting them according to their lengths.

A second, and perhaps more general, treatment of complicated systems is illustrated by the following solution of the problem of a flat-top antenna consisting of four equally spaced parallel wires

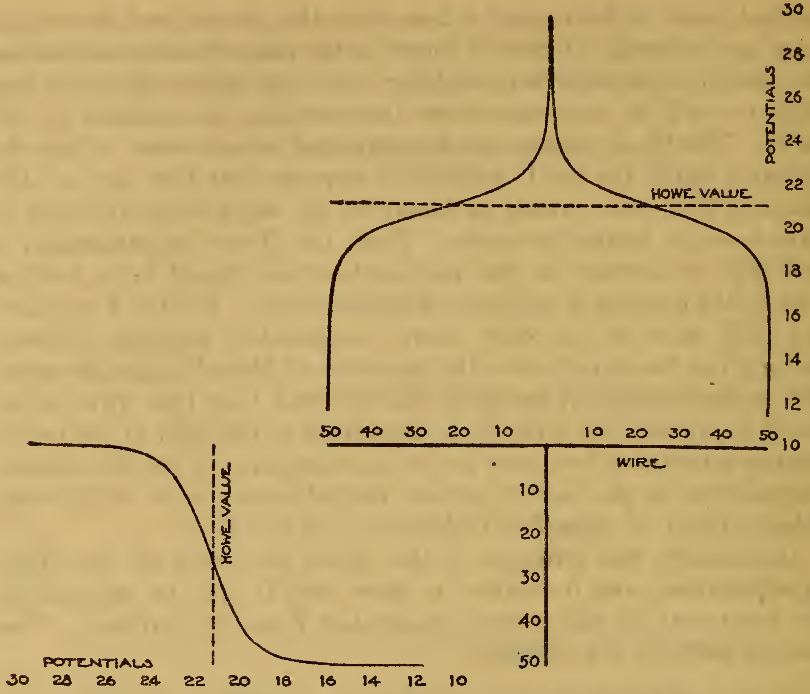

FIG. 6.—*Potentials of uniformly charged isolated* T *antenna*

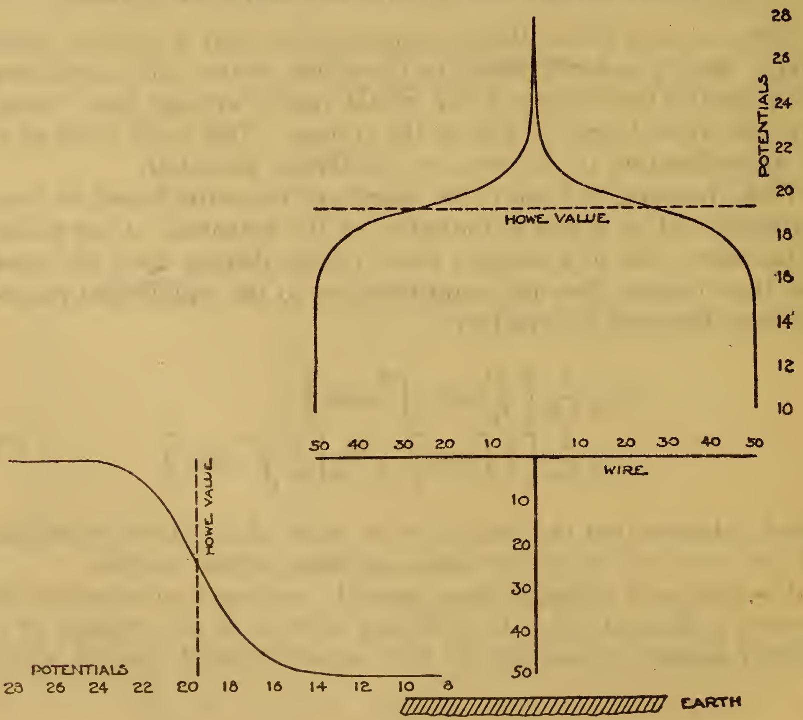

FIG. 7.—*Potentials of uniformly charged* T *antenna*

of equal length and joined at both ends, the whole being placed with the wires all at the same height above the earth.

The general Maxwell potential equations are then

$$\begin{aligned} v_1 &= Q_1p_{11}' + Q_2p_{12}' + Q_3p_{13}' + Q_4p_{14}' \\ v_2 &= Q_1p_{12}' + Q_2p_{22}' + Q_3p_{23}' + Q_4p_{24}' \\ v_3 &= Q_1p_{13}' + Q_2p_{23}' + Q_3p_{33}' + Q_4p_{34}' \\ v_4 &= Q_1p_{14}' + Q_2p_{24}' + Q_3p_{34}' + Q_4p_{44}' \end{aligned} \tag{8}$$

The potential coefficient p_{11}' refers to the potential of wire 1 due to both its own charge Q_1 and that on its image wire. Similarly, p_{12}' refers to the contribution to the potential of wire 1 by the charge Q_2 on wire 2 and the equal and opposite charge on the image of wire 2. By symmetry in this problem $Q_4 = Q_1$, and $Q_3 = Q_2$, $p_{11}' = p_{22}' = p_{33}' = p_{44}'$, $p_{23}' = p_{12}' = p_{34}'$, $p_{24}' = p_{13}'$, and the equations for v_3 and v_4 are the same as those for v_2 and v_1, respectively. To apply Howe's approximation, uniform linear charge densities are assumed to be given to the wires, q_1 to wire 1, and $-q_1$ to its image, and q_2 and $-q_2$ to wire 2 and its image wire, respectively.

Thus $Q_1 = q_1l$, $Q_2 = q_2l$, and the general equations are

$$\begin{aligned} v_1/l &= q_1(p_{11}' + p_{14}') + q_2(p_{12}' + p_{13}') \\ v_2/l &= q_1(p_{12}' + p_{13}') + q_2(p_{11}' + p_{12}') \end{aligned} \tag{9}$$

The potential coefficients must be defined in terms of the Howe method of approximation. That is, p_{12} for example, is the average of the potentials contributed to wire 1 by a unit charge distributed uniformly along the axis of wire 2, etc.

Since the wires are all at the same potential when joined together, we will have $v_2 = v_1 = v$, so that the potential equations give

$$q_1 = \frac{v}{l}\begin{vmatrix} 1 & p_{12}' + p_{13}' \\ 1 & p_{11}' + p_{12}' \end{vmatrix} \div \Delta$$

$$q_2 = \frac{v}{l}\begin{vmatrix} p_{11}' + p_{14}' & 1 \\ p_{12}' + p_{13}' & 1 \end{vmatrix} \div \Delta$$

where

$$\Delta = \begin{vmatrix} p_{11}' + p_{14}' & p_{12}' + p_{13}' \\ p_{12}' + p_{13}' & p_{11}' + p_{12}' \end{vmatrix} \tag{10}$$

The capacity is

$$\begin{aligned} C &= \frac{2(q_1 + q_2)l}{v} \\ &= \frac{4}{\Delta}\left[p_{11}' - p_{13}' - \frac{p_{12}'}{2} + \frac{p_{14}'}{2}\right] \end{aligned} \tag{11}$$

and thus

$$\frac{1}{C} = \frac{\Delta}{4\left(p_{11}' - p_{13}' - \frac{p_{12}'}{2} + \frac{p_{14}'}{2}\right)} \tag{12}$$

The solution of the problem of finding the inductance of the same four wires joined in parallel with their return circuit through the ground, the resistances of the wires being supposed negligible in comparison with their reactances, leads to the same mathematical expression for their joint inductance if, in the equation for $\frac{1}{C}$, we put for the potential coefficients the corresponding mutual inductances of the wires and their image wires. Cohen derived the expressions for the joint inductance of different numbers of parallel wires up to and including $n=6$. He assumed then that the inductance, divided by the length of the wires, is equal to the reciprocal of the capacity of the same wires, divided by their lengths, and thus obtained capacity formulas for these cases. The identity of the inductance per centimeter and the reciprocal of the capacity per centimeter is, however, exact only in the case of wires of infinite length.

Through the use of a mutual-inductance formula in which the height of the wires above the ground is neglected in comparison with their lengths, and also as the result of an error in the numerical value of a logarithm, Cohen's values for given systems of wires differed considerably from the values found by Howe for the same systems. However, using the appropriate mutual-inductance formula, this inductance method of Cohen agrees exactly with the method of using Howe's approximation sketched above, and we have the striking result that the two methods, each of them inexact, are entirely equivalent for the case of the flat-top antenna. The reason for this agreement and its limitations may be shown as follows:

The potential at any point of a wire A, of length l, due to a uniform linear charge density q_b along the axis of a wire B of length m, is $q_b \int_0^m \frac{dm}{r_{ab}}$ where r_{ab} is the distance between any two points on the two wires. The average of this potential, taken over the length of the wire A is

$$v_{ab} = \frac{q_b}{l} \int_0^l dl \int_0^m \frac{dm}{r_{ab}} = \frac{q_b}{l} N \tag{13}$$

The total charge on B is $q_b m$, so that the potential coefficient is $p_{ab} = \frac{N}{lm}$, or $\frac{N}{l^2}$ if the wires have equal length. From equation (12) it is evident that the reciprocal of the capacity per centimeter of length of the wires $\frac{l}{c}$ is a function of terms of the form lp_{ab}, or of $\frac{N}{l}$.

For the mutual inductances in the solution of the inductance problem we have to put the Neumann integral, which for the wires A and B, is $\int_0^l dl \int_0^m \frac{\cos \epsilon \, dm}{r_{ab}}$ or $N \cos \epsilon$, ϵ being the angle made by the two straight wires. Thus for the mutual inductance per centimeter of two parallel wires of equal length it becomes $\frac{N}{l}\cos\epsilon$. Thus, since the mutual inductances enter into the expression for the joint inductance in the same way that the potential coefficients do in the expression for the reciprocal of the capacity, the same expression is reached for the inductance per centimeter, as is found for the reciprocal of the capacity per centimeter. This complete equivalence of the solutions of the two problems holds for any system of parallel wires of equal length, both when they are all horizontal, or when they are all vertical. Otherwise the image wires will not be parallel to the antenna wires, and the cos ϵ factor is not unity. However, even in this case, the Howe potential coefficients may be obtained from the general Neumann integral for any two straight filaments placed in any desired position,[7] if only the cos ϵ factor be omitted from this.

It will be noticed from equation (9) that the general potential equations for a system of wires involve as coefficients the potentials per unit linear charge density, rather than the potential coefficients themselves. This quantity, which may for convenience be termed the "linear coefficient," depends only upon the ratios of the geometrical dimensions of the system. If we denote by u_{ab} the linear potential coefficients of wire A, length l, resulting from a unit linear charge density on wire B, length m, then a linear charge density q_b on B will give rise to an average potential $v_{ab}=q_b\,u_{ab}$ on A. But if $Q=q_b\,m$ denote the whole charge on B, we may write by definition for the average potential of A, $v_{ab}=Q\,p_{ab}=q_b\,m\,p_{ab}$, in which p_{ab} is the Howe potential coefficient defined on page 581. Thus, $u_{ab}=m\,p_{ab}$. It is well known that the same charge $Q=q_a\,l$ placed on A will give rise to the same potential v_{ba} on B as before. Thus, $v_{ba}=v_{ab}=q_b\,u_{ab}=q_a\,u_{ba}$. But $\frac{q_a}{q_b}=\frac{m}{l}$ and thus

$$\frac{u_{ab}}{u_{ba}}=\frac{m}{l} \tag{14}$$

In Appendix 2 are given the linear potential coefficient expressions for certain important special cases. It is to be noted that in the symbol u_{ab}, the charged body appears last in the subscript.

The expression (11) for the capacity of a four-wire flat-top antenna is not simple, and for a greater number of wires the expressions are very complicated. Fortunately a notable simplification may be attained if the division indicated in these expressions is actually carried out. For the four-wire case we find

$$\frac{1}{C}=\frac{1}{4}\left[p_{11}'+\frac{2p_{14}'+4p_{13}'+6p_{12}'}{4}\right]-\frac{1}{16}\,\frac{(p_{12}'-p_{13}')^2}{\left[p_{11}'-p_{13}'-\frac{(p_{12}'-p_{14}')}{2}\right]} \tag{15}$$

[7] G. A. Campbell, Phys. Rev.; June, 1915.

the last or remainder term being small compared with the first or principal term. If the potential coefficients be expressed in terms of one of their number the expression for n equally spaced wires is still simpler. In general, neglecting the remainder term,

$$\frac{1}{C}=\frac{p_{11}'+(n-1)\,p_{12}'}{n}-K_n \tag{16}$$

where

$$K_n=\frac{4}{n^2}\left[\begin{array}{r}\text{logn }(n-1)+2\text{ logn }(n-2)+3\text{ logn }(n-3)+\text{-------}\\ +\ (n-2)\text{ logn }2\end{array}\right]$$

Expressed in terms of the linear potential coefficients, we find for the reciprocal of the capacity per centimeter

$$\frac{l}{C}=\frac{u_{11}'+(n-1)\,u_{12}'}{n}-K_n \tag{17}$$

an expression which applies equally well when the wires are all vertical, if the appropriate linear potential coefficients be used in each case.

VII. DERIVATION OF WORKING FORMULAS FOR THE CAPACITY

The derivation of formulas for the capacity of a single horizontal or vertical wire and for systems of parallel wires in a horizontal or in a vertical plane has been fully discussed in the foregoing. Working formulas for these cases and for other types of antenna are given in section 8. All these formulas were obtained by the application of the principles already discussed. The capacities are given throughout in micromicrofarads (10^{-12} farad). To make possible the use of tables of common logarithms the denominator of each expression has been divided by 2.303. Thus, if the velocity of light be taken as 2.998×10^{10} cm per sec. there is derived the factor 0.2416 which appears in the working formulas. In most cases terms which depend upon the presence of the earth and the mutual effects of the wires have been combined and tabulated. For the usual types of antenna a few tables suffice.

The capacity formulas for the more complex forms of antenna were obtained for the most part by applying the first method of section 6; that is, supposing the system to be given a uniform distribution of charge, the potential of each wire is written down as the algebraic sum of the potentials due to its own charge and those on each of the other wires and image wires of the system. The equilibrium potential is taken as the mean of the potentials of the individual wires, each weighted in proportion to the length of the wire to which it applies. The capacity is the quotient of the total charge by the equilibrium potential. In calculating the potential the appropriate linear potential coefficients have to be used, and since natural logarithms occur in these, the factor 2.303 has to be divided out to agree with the general system of the calculations.

The effect of supporting masts and other grounded conductors near an antenna may be evaluated by the following general method. Assume that, in addition to the uniform linear density q of the antenna wires, there is a linear charge density of $-q_1$ upon the mast and $+q_1$ upon its image. The potential equations for the antenna wires thus include terms in q_1 to take account of the presence of the mast. An additional equation may be written for the potential of the mast itself, and since this is known to be zero, this last equation will be given the value of q_1 in terms of q. Thus q_1 may be eliminated from the potential equations for the wires and the capacity is then obtained by the methods already discussed.

The calculation of the added capacity given to an antenna system by the lead-in wires is a special case of the general problem of finding the joint capacity of two antenna systems when connected to form one whole, the capacity of each component being given by one of the formulas of section 8. This problem is treated separately in section 10.

VIII. WORKING FORMULAS FOR CALCULATING CAPACITY OF VARIOUS PRACTICAL FORMS OF ANTENNAS

In the following formulas the capacity is in micromicrofarads (10^{-12} farad); logarithms are to the base 10. Ample accuracy in the values of the logarithms will be attained by the use of a four-place table, although if a five-place table be employed interpolations will be unnecessary. Linear dimensions are given both in centimeters and in feet. The use of subscripts, as explained below, will make clear which system is meant in all cases. However, where the ratio of two dimensions is used as a parameter, either system may be used, as long as both dimensions are expressed in the same system; and this fact will be indicated by the omission of subscripts.

The following nomenclature is common to practically all the formulas. Other symbols are explained where they occur.

d = diameter of wire,
D = distance between centers of parallel wires,
U = potential coefficient for unit charge density per unit length of the antenna,
C = capacity, in micromicrofarads,
l_1 = length of a horizontal wire, in centimeters,
m_1 = length of a vertical wire, in centimeters,
l_2 = length of a horizontal wire, in feet,
m_2 = length of a vertical wire, in feet,
h_1 = height of a horizontal wire above earth, in centimeters,
h_2 = height of a horizontal wire above earth, in feet,

h'_1 = height of lower end of a vertical wire above earth, in centimeters,

h'_2 = height of lower end of a vertical wire above earth, in feet,

n = number of wires joined in parallel.

1. SINGLE HORIZONTAL WIRE

$$C=\frac{0.2416 l_1}{\log \frac{4h}{d}-K}=\frac{7.36 l_2}{\log \frac{4h}{d}-K} \tag{18}$$

where K is to be taken from Table 1 for either the ratio $\frac{2h}{l}$ or $\frac{l}{2h}$, depending upon which is less than unity.

In Table 2 are given values of the capacity of single-wire horizontal antennas of various lengths and heights. This should be useful in certain practical cases.

Example 1.—For a single wire 100 feet long, stretched 50 feet above ground, and assuming the diameter of the wire to be 0.24 inch = 0.02 foot, we find $\frac{4h}{d}=\frac{200}{0.02}=10{,}000$, and thus $\log \frac{4h}{d}=4.000$. The value of $\frac{2h}{l}$ is $\frac{100}{100}=1$, and from Table 1, $K=0.336$. Thus

$$C=\frac{7.36\ (100)}{3.664}=200.9\ \mu\mu f$$

This value is in agreement with that calculated from Howe's tables.

2. SINGLE VERTICAL WIRE[8]

The wire is supposed to have a length m and its lower end is at a height h' above the surface of the earth.

$$C=\frac{0.2416\ m_1}{\log \frac{2m}{d}-k}=\frac{7.36\ m_2}{\log \frac{2m}{d}-k} \tag{19}$$

in which the constant k is to be obtained from Table 3 for the value of $\frac{h'}{m}$ or $\frac{m}{h'}$ depending upon which is less than unity.

Example 2.—Suppose a vertical wire 40 feet long, with its lower end 10 feet from the ground. The diameter of the wire will be taken as 0.24 inch as in the preceding example.

[8] *Caution.*—The formula for the capacity does not apply for the limiting case where the distance between the lower end of the vertical wire and the earth is vanishingly small, but an error of not more than a few per cent results if this distance is as small as 1 foot.

Thus in formula (19) we have $m=40$, $h'=10$, $\frac{2m}{d}=4{,}000$ $\log_{10}\frac{2m}{d}=3.602$. From Table 3, with $\frac{h'}{m}=0.25$, $k=0.291$, and thus

$$C=\frac{7.36\ (40)}{3.602-0.291}=88.9\ \mu\mu\text{f}$$

In Table 4 is given the capacity of vertical wires covering a considerable range of lengths, diameters, and heights above the ground.

3. SINGLE-WIRE INVERTED L ANTENNA[9]

Suppose the length of the horizontal portion is l, that of the vertical portion m, the height of the vertical portion from the ground h', and thus the height of the horizontal portion is $h=h'+m$. Then the capacity is given by

$$C=\frac{0.2416\ (l_1+m_1)}{U}=\frac{7.36\ (l_2+m_2)}{U}$$

in which

$$U=\frac{l}{l+m}\left[\log\frac{4h}{d}-K\right]+\frac{m}{l+m}\left[\log\frac{2m}{d}-k\right]+X \qquad (20)$$

The term X takes into account the mutual effect of the two portions of the antenna. Its value is obtained from Table 5 for different values of the ratios $\frac{l}{m}$ or $\frac{m}{l}$, and $\frac{h'}{m}$ or $\frac{m}{h'}$. The values of K and k are to be taken from Tables 1 and 3, respectively, as in the preceding examples.

As a first approximation the capacity of the inverted L may be calculated as the sum of the capacities of the component wires taken separately. This approximate method always gives a value larger than the true values.

Example 3.—Let us consider an inverted L antenna made up of the horizontal wire treated in example 1, and the vertical wire of example 2. Then $l=100$, $m=40$, $h'=10$, $d=0.24$ inch, $\frac{l}{l+m}=\frac{100}{140}=\frac{5}{7}$, $\frac{m}{l+m}=\frac{2}{7}$. With $\frac{m}{l}=0.4$, $\frac{h'}{m}=0.25$ in Table 5 we find $X=0.194$. Thus $U=\frac{5}{7}\ (3.664)+\frac{2}{7}\ (3.311)+0.194=3.757$ and

$$C=\frac{7.36\ (140)}{3.757}=274.3\ \mu\mu\text{f}$$

The simple sum of the separate capacities of the horizontal and vertical portions is 289.8, which is more than 5 per cent too large.

[9] See footnote, p. 586.

4. SINGLE-WIRE T ANTENNA

The antenna consists of a horizontal wire of length l at a height h above the ground. To the center of this is attached a vertical wire of length m. The height of the lower end of this, above ground, is denoted by h'. The capacity is calculated from the formula

$$C=\frac{0.2416\ (l_1+m_1)}{U'}=\frac{7.36\ (l_2+m_2)}{U'} \tag{21}$$

in which

$$U'=\frac{l}{l+m}(\log\frac{4h}{d}-K)+\frac{m}{l+m}(\log\frac{2m}{d}-k)+\frac{l+2m}{l+m}\cdot X_1$$

The constants K, k, and X_1 are to be taken from Tables 1, 3, and 5, respectively, the latter for the argument $\frac{m}{l}$ = ratio of m and $\frac{l}{2}$.

Example 4.—Consider a T antenna made up of the horizontal and vertical wires of examples 1 and 2. Then

$$l=100,\ m=40,\ h'=10,\ d=0.24 \text{ inch},\ \frac{l}{l+m}=\frac{5}{7},\ \frac{m}{l+m}=\frac{2}{7},\ \frac{l+2m}{l+m}=\frac{9}{7}$$

From Table 5, with the argument

$$\frac{m}{l}=0.8,\ \frac{h'}{m}=0.25,\ X_1=0.263$$

Thus $U'=3.901$, and therefore

$$C=\frac{7.36\ (140)}{3.901}=264.2\ \mu\mu\text{f}$$

If the simple sum of the separate capacities of the component wires had been used the error would have been about 10 per cent.

It is interesting to note that for the given horizontal wire joined to the given vertical wire the capacity is about 3.5 per cent smaller with the wires connected as a T antenna than as an inverted L, and for both these cases the capacity is less than the sum of the capacities of the wires taken separately. This follows from the fact that the potential of the charge of each wire is increased by the proximity of the charge on the other wire. On the average, the two wires of the T antenna are closer together than the two parts of the inverted L, and the mutual effect of the two wires is therefore greater in the former case.

5. PARALLEL HORIZONTAL WIRES IN THE SAME HORIZONTAL PLANE (FLAT-TOP ANTENNA)

The wires are supposed to have a diameter d, and to be of equal length l. They will be assumed to be n in number, arranged parallel to one another in a horizontal plane, at a height h above the surface of the earth. If the spacing is D, the width of the antenna between the extreme wires is $(n-1)D$, and this dimension is supposed to be not greater than about one-quarter of the length of the wires. The formula for the capacity is then

$$C=\frac{0.2416\ l_1}{F}=\frac{7.36\ l_2}{F}\ \mu\mu\mathrm{f} \tag{22}$$

in which

$$F=\frac{P+(n-1)Q}{n}-K_n$$

$$P=\log\frac{4h}{d}-K$$

$$Q=\log\frac{2h}{D}-K$$

The constant K_n, which depends only upon the number of the wires, may be obtained from Table 6; the constant K is to be taken from Table 1, as in preceding examples. The expression for F may also be written

$$F=\frac{\log\frac{4h}{d}+(n-1)\log\frac{2h}{D}}{n}-(K+K_n) \tag{23}$$

For the common case of a two-wire flat-top antenna, $K_n=0$, and the general formula (22) becomes

$$C=\frac{14.73\ l_2}{\log\frac{4h}{d}+\log\frac{2h}{D}-2K} \tag{24}$$

In Table 9 will be found the values of the capacity of certain two-wire antennas. This should be useful where a moderate accuracy will suffice.

Example 5.—A flat-top antenna is composed of six wires spaced 2 feet apart, the length of the wires being 100 feet, their height 50 feet, and their diameter 0.24 inch as before.

$$D=2,\ \frac{4h}{d}=10{,}000,\ \frac{2h}{D}=50,\ \frac{2h}{l}=1$$

From Table 1, $K=0.336$, and from Table 6, $K_n=0.252$. Thus, using the last formula (23) for F, its value is obtained as follows:

$$\log\frac{4h}{d}=4.000 \qquad K+K_n=0.588$$

$$5\log\frac{2h}{D}=8.495 \qquad F\ =1.494$$

$$\text{Sum}\div 6=2.082$$

and the capacity is

$$C=\frac{7.36\ (100)}{1.494}=492.6\ \mu\mu\mathrm{f}$$

Example 6.—Attention has been called to the small gain from the use of many wires in parallel. Thus Austin states that, for parallel wires of moderate dimensions, a spacing of 1 meter is sufficient to give 90 per cent of the possible capacity. This assumes a certain width of antenna which is kept constant as more wires are added. As an illustration of this point, the capacity was calculated for different numbers of wires, each 100 feet long, 0.24 inch in diameter, placed 50 feet above the earth, the spacing being chosen in each case such that for n wires $nD = 15$ feet. The results are given in Table 10.

TABLE 10

n	C $\mu\mu f$	Per cent of maximum	n	C $\mu\mu f$	Per cent of maximum	n	C $\mu\mu f$	Per cent of maximum
2	330.5	51.0	10	577.2	89.0	18	611.8	94.4
3	408.7	62.9	11	582.7	89.8	19	613.8	94.7
4	460.6	70.9	12	588.8	90.6	20	615.9	94.9
5	495.7	76.3	13	594.0	91.5	30	626.4	96.5
6	520.5	80.2	14	598.9	92.3	40	635.0	97.9
7	539.6	83.2	15	602.8	92.9	50	637.8	98.5
8	554.2	85.4	16	606.3	93.5	100	643.9	99.3
9	565.7	87.2	17	609.8	94.0			

When these values of the capacity are plotted against the reciprocal of the number of the wires, a limiting capacity of about 650 $\mu\mu$f is indicated. This value has been used in computing the values in the column headed "per cent of maximum." A spacing of 1 meter would be obtained with five or six wires, the capacity being some 80 per cent of the maximum. Even with so few as two wires, the capacity is about 50 per cent of the maximum, and if the two wires were placed 15 feet apart instead of only 7.5, as in the table, the capacity would be 354.4, or 54 per cent of the maximum.

These conclusions were checked by deriving the formula for the capacity of a horizontal rectangular plate having a length l, a width nD, and a thickness d, situated at a height h above the earth. The treatment of this case was based upon the integration of the Howe expression for a filament of length l over the width of the rectangle. The final formula, which is long and involved, will not be given here. (See Appendix 1.) For the dimensions assumed in the previous example the limiting capacity should be 635 micromicrofarads. The difference of more than 1 per cent between this value and that given above, is probably to be explained by the fact that the ratio of width to length is rather large in this special case, and that the terms neglected in (22) are appreciable. For the more favorable case $nD = 5$, $l = 100$, formula (22) gives for a 100 wire antenna $C = 455.7$ $\mu\mu$f, while the value for the solid rectangular plate is 454.7 $\mu\mu$f.

6. ANTENNA OF PARALLEL WIRES EQUALLY SPACED IN A VERTICAL PLANE

Adopting the same nomenclature as in the preceding case, we find, for an antenna where the extreme width is not greater than, say, one-quarter of the length of the wires

$$C=\frac{0.2416 l_1}{V}=\frac{7.36 l_2}{V}$$

in which

$$V=\frac{1}{n}\log\frac{2m}{d}+\frac{n-1}{n}\log\frac{m}{D}-(k+K_n) \tag{25}$$

The quantities k and K_n are to be obtained from Tables 3 and 6, respectively.

Example 7.—For six vertical wires, each 40 feet long, arranged with a spacing of 2 feet, and with the bottom ends of the wires 10 feet above the ground, $m=40$, $D=2$, $h'=10$. Assume $d=0.02$ foot. As in example 2,

$$k=0.291,\ K_n=0.252,\ \frac{m}{D}=20,\ \log\frac{m}{D}=1.301,\ \log\frac{2m}{d}=3.602,$$

so that

$$\frac{1}{n}\log\frac{2m}{d}=0.600 \qquad k+K_n=0.543$$

$$\frac{n-1}{n}\log\frac{m}{D}=1.118 \qquad V=1.175$$

$$C=\frac{7.36\ (140)}{1.175}=250.6\ \mu\mu\text{f}.$$

7. PARALLEL WIRE INVERTED L ANTENNA

With the length of the horizontal portion equal to l, length of the vertical wires m, height of horizontal wires h, height of lower ends of the vertical wires h', spacing of wires D,

$$C=\frac{0.2416\ (l_1+m_1)}{L}=\frac{7.36\ (l_2+m_2)}{L}$$

in which

$$L=\frac{P'+(n-1)\,Q'}{n}-K_n+X$$

$$P'=\frac{l}{l+m}\left(\log\frac{4h}{d}-K\right)+\frac{m}{l+m}\left(\log\frac{2m}{d}-k\right) \tag{26}$$

$$Q'=\frac{l}{l+m}\left(\log\frac{2h}{D}-K\right)+\frac{m}{l+m}\left(\log\frac{m}{D}-k\right)$$

60750°—28——4

The constants K, k, X, and K_n are to be taken from Tables 1, 3, 5, and 6, respectively. This formula is less accurate, the wider the antenna in comparison with its length.

Example 8.—Suppose a parallel wire inverted L antenna composed of the wire systems of examples 5 and 7, joined to form one conducting system. Then

$$P'=2.617+0.946=3.563$$
$$Q'=0.974+0.289=1.263$$
$$K_n=0.252 \qquad X=0.194$$

Therefore

$$L=1.588, \text{ and } C=\frac{7.36\ (140)}{1.588}=648.9\ \mu\mu\text{f}$$

The sum of the separate capacities of the horizontal and vertical portions is 743.2, which is more than 14 per cent too large.

8. PARALLEL WIRE T ANTENNA

The antenna is supposed to be composed of n similar T's joined in parallel. Thus, the horizontal wires have the same spacing as the vertical. The total length of the horizontal portion is taken as l, the meaning of the other symbols is the same as in the preceding cases.

$$C=\frac{0.2416\ (l_1+m_1)}{T}=\frac{7.36\ (l_2+m_2)}{T}$$

$$T=\frac{P'+(n-1)\,Q'}{n}-K_n+\frac{l+2m}{l+m}\cdot X \tag{27}$$

$$P'=\frac{l}{l+m}\left(\log\frac{4h}{d}-K\right)+\frac{m}{l+m}\left(\log\frac{2m}{d}-k\right)$$

$$Q'=\frac{l}{l+m}\left(\log\frac{2h}{D}-K\right)+\frac{m}{l+m}\left(\log\frac{m}{D}-k\right)$$

The constants K, k, X, and K_n are to be obtained from Tables 1, 3, 5, and 6, respectively, using for X the ratio of $\frac{l}{2}$ to m for $\frac{l}{m}$ in Table 5.

Example 9.—A T antenna is made by joining the vertical wires of example 7 to the middle points of the horizontal wires of example 5. The constants are the same as in the preceding example, except that $X=0.263$. The value of T comes out 1.732, so that

$$C=\frac{7.36\ (140)}{1.732}=594.9$$

which is more than 8 per cent less than the value for the inverted L. The following summary presents concisely the results of the examples:

	μμf
Horizontal parallel portion alone	492. 6
Vertical parallel portion alone	250. 6
	743. 2
Same wires connected as inverted L antenna	648. 9
Same wires connected as a T antenna	594. 9

9. HORIZONTAL "CAGE" ANTENNA

The following formula supposes that the distance between the n wires is small compared with their average distance from the ground. The axis of the cage is at a height h above ground.

$$C=\frac{0.2416\, l_1 n}{U_c}=\frac{7.36\, l_2 n}{U_c} \tag{28}$$

in which

$$U_c=\log\frac{4h}{d}+\sum_{r=1}^{r=n-1}\left(\log\frac{2h}{D_r}+0.434\frac{D_r}{l}\right)-nK$$

and D_r is the distance between any given wire and another wire. If δ = the diameter of the circle on whose circumference the wires are arranged, then

$$D_r=\delta\sin r\frac{\pi}{n}$$

The quantity K is obtained from Table 1 for the given value of $\frac{l}{2h}$ or $\frac{2h}{l}$.

Example 10.—Six wires, each 100 feet long and 0.02 foot in diameter are arranged as elements of a cylinder 5 feet in diameter. The axis of the cylinder lies horizontally 50 feet above the surface of the earth.

Here $n=6$, $\delta=5$, $h=50$, $\frac{2l}{d}=10{,}000$. The distances between the wires are then $D_1=D_5=2.5$, $D_2=D_4=\frac{5}{2}\sqrt{3}$, $D_3=5$. From Table 1 for $\frac{2h}{l}=1$, $nK=2.016$

$$\log\frac{4h}{d}=4.000 \qquad 0.434\frac{D_1}{l}=0.011$$

$$\log\frac{2h}{D_1}=1.602 \qquad 0.434\frac{D_2}{l}=0.018$$

$$\log\frac{2h}{D_2}=1.364 \qquad 0.434\frac{D_3}{l}=0.022$$

$$\log \frac{2h}{D_3} = 1.301 \qquad \therefore\ U_c = 9.297$$

$$C = \frac{7.36\ (100)\ 6}{9.297} = 475.0\ \mu\mu f$$

If the same wires had been spaced at the same distance apart in the horizontal plane as the length of the chord of the circle the capacity by formula (22) would have been 520.5 $\mu\mu f$. Thus the arrangement in a cage results here in a loss of capacity of about 8 per cent. This is due to the decrease in the average distance between wires brought about by the arrangement in the cage. The advantage of this form of antenna lies, of course, in the saving of space.

Formula (28) was derived on the assumption that the effect of the earth can be evaluated with sufficient accuracy by assuming the charges on the image wires and the charges on the wires of the cage to be situated along the axis of the image and the axis of the cage, respectively. To determine the order of the error committed, an accurate evaluation was made of the effect of the image wires, using the antenna of the preceding example. It was found that, although the potentials contributed by the image wires together differed by as much as 5 per cent for the different wires of the cage, the total effect did not differ as much as 1 part in 10,000 from that calculated by the simplifying assumption used in deriving formula (28).

10. VERTICAL "CAGE" ANTENNA

The n wires of diameter d and length m are arranged as elements of a cylinder of diameter δ, whose axis is vertical, and whose lower end is at a height h' above the ground.

$$C = \frac{0.2416\, n\, m_1}{U'_c} = \frac{7.36\, n\, m_2}{U'_c} \tag{29}$$

and

$$U'_c = \log \frac{2m}{d} + \sum_{r=1}^{r=n-1} \left(\log \frac{m}{D_r} + 0.434 \frac{D_r}{m} \right) - nk$$

The value of k is obtained from Table 3, and the distance D_r between any two wires is given by

$$D_r = \delta \sin r \frac{\pi}{n}$$

Example 11.—Six vertical wires, each 100 feet long and 0.02 foot in diameter, are arranged as elements of a cylinder, 4 feet in diameter, with their lower ends 25 feet from the ground. Thus

$$m = 100,\ h' = 25,\ \delta = 4,\ d = 0.02,\ \frac{2m}{d} = 10{,}000, \text{ and } \frac{h'}{m} = \frac{1}{4}.$$

From Table 3, $k = 0.291$.

Accordingly $D_1 = D_5 = 2$, $D_2 = D_4 = 2\sqrt{3}$, $D_3 = 4$.

$$\log \frac{2m}{d} - nk = 4.000 - 1.748 = 2.252,$$

$$\sum \log \frac{m}{D_r} = 2\ (1.699 + 1.460) + 1.398 = 7.716,$$

$$0.434 \sum \frac{D_r}{m} = 0.065,$$

$$\therefore\ U'_c = 10.033$$

and

$$C = \frac{7.36\ (600)}{10.033} = 440.2\ \mu\mu f$$

11. SINGLE V ANTENNA

The antenna is supposed to consist of two wires in a horizontal plane meeting at an angle θ. The lengths of the wires are l and m, their diameters d and d' and their common height above ground h. Then

$$C = \frac{0.2416\ (l_1 + m_1)}{U_v} = \frac{7.36\ (l_2 + m_2)}{U_v} \tag{30}$$

where

$$U_v = \frac{l}{l+m}\left(\log \frac{4h}{d} - K\right) + \frac{m}{l+m}\left(\log \frac{4h}{d'} - K'\right) + Y$$

The quantities K and K' are to be obtained from Table 1 for the values of $\frac{2h}{l}$ and $\frac{2h}{m}$, respectively (or their reciprocals if the latter are less than unity).

The quantity Y is the difference of two terms Y_1 and Y_2, the first being a function of the angle θ and the ratio $\frac{m}{l}$ (supposed to be less than unity), while Y_2, which refers to the effect of the earth is a function of θ, $\frac{m}{l}$ and $\frac{2h}{l}$. Values of Y_1 and Y_2 are given in Tables 7 and 8.

If both wires have the same diameter of cross section, then

$$U_v = \log \frac{4h}{d} - \frac{l}{l+m} K - \frac{m}{l+m} K' + Y \tag{31}$$

and if, further, $m = l$ (an important case)

$$U_v = \log \frac{4h}{d} - K + Y \tag{32}$$

Since in the case of existing antennas, the distance between the free ends of the V will be readily measured, rather than the angle, the distance s thus measured may be used in the formula

$$\cos\theta=\frac{l^2+m^2-s^2}{2lm}$$

to determine the angle θ.

Example 12.—If we suppose the case $l=100$, $m=50$, $h=50$, $\theta=45°$, and $\frac{4h}{d}=10{,}000$, then $\frac{2h}{l}=1$, $\frac{m}{2h}=\frac{1}{2}$, and from Table 1 $K=0.336$, $K'=0.541$. From Table 7, for $\theta=45°$, and $\frac{m}{l}=\frac{1}{2}$, $Y_1=0.497$, and from Table 8, for $\theta=45°$, $\frac{m}{l}=\frac{1}{2}$, and $\frac{2h}{l}=1$, $Y_2=0.131$. Thus $Y=0.366$. By formula (31)

$$U_v=4.000+0.366-\frac{2}{3}(0.336)-\frac{1}{3}(0.541)=3.962$$

$$C=\frac{7.36\ (150)}{3.962}=278.6\ \mu\mu\text{f}$$

The sum of the capacities of the two wires taken singly is (see example 1) $200.9+106.4=307.3\ \mu\mu$f. Thus the mutual effect of the two wires is to reduce the capacity of the combination in a V by about 9.3 per cent.

12. TWO HORIZONTAL WIRES INCLINED TO ONE ANOTHER, BUT NOT INTERSECTING

The wires are supposed to have lengths l and m, diameters d and d', and their distances from their point of intersection, if supposed to be produced, l' and m'. (See fig. 8.)

For this case the same formula is used as in the preceding case, except that the value for Y is different. This is obtained as before as the difference of two terms, one applying to the top and the other to the image wires. Tables 7 and 8 are used, but instead of a single entry in the table for each of the two terms Y_1 and Y_2, several have to be made for each.

$$Y=\frac{l+l'+m+m'}{l+m}\cdot Y_{l+l',\,m+m'}-\frac{(l+l'+m')}{l+m}\cdot Y_{l+l',\,m'} -\frac{(l'+m+m')}{l+m}Y_{l',\,m+m'}+\frac{l'+m'}{l+m}Y_{l',\,m'} \tag{33}$$

In this the following abbreviated nomenclature is used: $Y_{l+l',\,m+m'}$ is used for the difference between the value of Y_1 for the wires $(l+l')$ and $(m+m')$ and the quantity Y_2 for the same wires, etc. For $l'=m'=n$

$$Y = \frac{l+m+2n}{l+m} Y_{l+n,\, m+n} - \frac{(l+2n)}{l+m} Y_{l+n,\, n} - \frac{(m+2n)}{l+m} Y_{n,\, m+n} + \frac{2n}{l+m} Y_{n,\, n} \tag{34}$$

and for the specially important case that $l' = m' = n$, and $l = m$,

$$Y = \frac{l+2n}{l} (Y_{n,\, n} - Y_{l+n,\, n}) \tag{35}$$

Example 13.—Two wires of equal length 100 feet make an angle of 30°, and if prolonged until they intersect, the point of intersection is 100 feet from the nearer end of each. The wires have each a diameter of cross section of 0.02 foot, and they lie in a horizontal plane 50 feet above the ground.

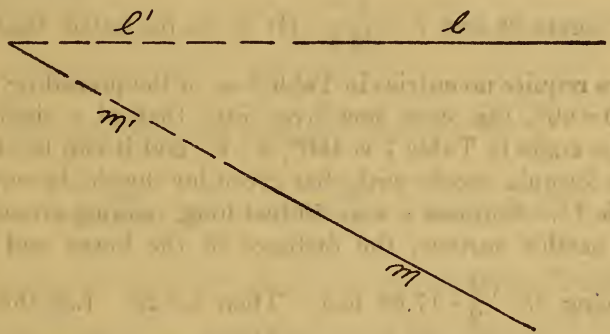

FIG. 8.—*Antenna of two nonintersecting wires*

Here $l = m = 100$, $h = 50$, $d = 0.02$, $l' = m' = n = 100$, $\theta = 30°$. Then for $Y_{n,\, n}$ we have the argument (1, 30°) in Table 7, and 1, 30°, and $\frac{2h}{l} = 1$ in Table 8. Thus $Y_{n,\, n} = 0.687 - 0.197 = 0.490$. For $Y_{l+n,\, n}$ the argument in Table 7 is $\left(\frac{1}{2}, 30°\right)$, and in Table 8 $\left(\frac{1}{2}, 30, \frac{1}{2}\right)$, so that $Y_{l+n,\, n} = 0.601 - 0.229 = 0.372$. Thus

$$Y = \frac{l+2n}{l} (Y_{n,n} - Y_{l+n,n}) = 3(0.490 - 0.372) = 0.354$$

and

$$U_v = \log \frac{4h}{d} - K + Y = 4.000 - 0.336 + 0.354 = 4.018$$

and

$$C = \frac{7.36\ (200)}{4.018} = 366.4\ \mu\mu\text{f}$$

Each of the wires alone has a capacity of 200.9 $\mu\mu$f (see example 1), so that the sum of their individual capacities is 401.8 $\mu\mu$f. Thus the mutual effect of the wires is to reduce the capacity by about 9 per cent.

If the same wires were so placed as to have their ends in contact, still keeping the angle between them equal to 30°, then $Y = 0.687 - 0.197 = 0.490$, $U_v = 4.154$, and $C = 354.4$ $\mu\mu$f. The mutual effect here is nearly 12 per cent, or 30 per cent more than in the previous case.

13. SINGLE WIRE INCLINED TO THE EARTH'S SURFACE

The wire is assumed to have a length l, a diameter of cross section d, and to make an angle θ with the earth's surface. Let the height of the lower end be h' above the ground. Then the capacity is given by

$$C = \frac{0.2416 l_1}{U_s} = \frac{7.36 l_2}{U_s} \tag{36}$$

and

$$U_s = \left(\log \frac{2l}{d} - 0.133\right) - \frac{l+2n}{l}(Y'_{n,n} - Y'_{l+n,n})$$

The quantities $Y'_{n,n}$ and $Y'_{l+n,n}$ are to be taken from Table 7, using the angle 2θ and $n = \frac{h'}{\sin\theta}$. (It is to be noted that the constants here require no entries in Table 8 as in the preceding example.)

When $\theta = 90°$, this case goes over into that of a single vertical wire. The angle in Table 7 is 180°, $n = h'$, and it can be shown that the above formula checks with that given for the single vertical wire.

Example 14.—Suppose a wire 50 feet long, making an angle of 45° with the earth's surface, the distance of the lower end from the ground being $25\frac{\sqrt{2}}{2} = 17.68$ feet. Then $n = 25$. Let the diameter of cross section be taken such that $\frac{2l}{d} = 5{,}000$. Then the first term of U_s is $(3.699 - 0.133) = 3.566$. For $Y'_{n,n}$ the argument in Table 7 is $(1, 90°)$ so that $Y'_{n,n} = 0.383$. For $Y'_{l+n,n}$, the argument is $\left(\frac{1}{3}, 90°\right)$, and the same table gives $Y'_{l+n,n} = 0.304$. Thus the second term in U_s is $2(0.383 - 0.304) = 2(0.079) = 0.158$. Therefore, $U_s = 3.408$ and $C = 108.0$ $\mu\mu$f.

If the same wire were swung about its lower end as center until it reached the vertical position, then we would have to use $\frac{h'}{l} = \frac{25}{50}\frac{\sqrt{2}}{2} = \frac{1}{4}\sqrt{2} = 0.354$, and for this Table 3 gives $k = 0.269$. The capacity comes out 107.3 $\mu\mu$f.

If the same wire were swung about the lower end into a horizontal position, $h = 25\frac{\sqrt{2}}{2}$, $\frac{2h}{l} = \frac{\sqrt{2}}{2}$, and from Table 1, $K = 0.256$, the resulting capacity being 111.8 $\mu\mu$f. Thus the capacity of the inclined wire lies between the values corresponding to the vertical and horizontal positions, as would be expected.

14. PARALLEL WIRE V ANTENNA

The antenna is supposed to consist of n' wires of equal length l' and diameter d', joined to another set of n'' wires of equal length l'' and diameter d''. Each of these sets of wires is supposed to lie in a horizontal plane at a height h above the ground and the axes of the two sets meet at an angle θ, at a point situated at distances l'_o and l''_o, respectively, from the nearer ends of the sets of wires. Let the spacing of the wires in the two sets be D' and D''.

The capacity is given by

$$C=\frac{0.2416\ (n'l_1'+n''l_1'')}{U'_v}=\frac{7.36\ (n'l_2'+n''l''_2)}{U'_v} \tag{37}$$

in which

$$U'_v=\frac{n'l'}{n'l'+n''l''}u'+\frac{n''l''}{n'l'+n''l''}u''+\frac{n'n''(l'+l'')}{n'l'+n''l''}\cdot Y$$

$$u'=\log\frac{4h}{d'}+(n'-1)\log\frac{2h}{D'}-n'\ (K'+K_n')$$

$$u''=\log\frac{4h}{d''}+(n''-1)\log\frac{2h}{D''}-n''\ (K''+K_n'')$$

The constants K' and K'' are to be obtained from Table 1 for the arguments $\frac{2h}{l'}$ and $\frac{2h}{l''}$, respectively, and the constants K'_n and K''_n from Table 6 for the values of n' and n''. The last term in U'_v takes account of the mutual effect between one set of wires and the other set and its image. This is obtained on the assumption that the effect is sensibly the same as though the two sets of wires were replaced by wires along the axes of the parallel sets and carrying the same total charges as the parallel wire sets. The error due to this simplifying assumption will not amount to as much as 1 per cent in most practical cases.

To calculate Y we have the equation

$$Y=\frac{l'+l'_o+l''+l''_o}{l'+l''}Y\ (l'+l'_o, l''+l''_o)-\frac{(l'+l'_o+l''_o)}{l'+l''}Y\ (l'+l'_o, l''_o)$$
$$-\frac{(l'_o+l''+l''_o)}{l'+l''}Y\ (l'_o, l''+l''_o)+\frac{l'_o+l''_o}{l'+l''}Y\ (l'_o, l''_o) \tag{38}$$

each of the terms being the difference of two quantities Y_1 and Y_2 taken from Tables 7 and 8, respectively, for the arguments θ, and the ratio of the lengths which appear in the parentheses which follow the symbol Y in the formula, and for the value of $2h$ divided by the greater of the two lengths in each case.

In most practical cases, however, there will be simplifying conditions. The following are the most important of these special cases.

When the wires have all the same diameter, the same spacing, and the number in each leg of the V is the same, then $d'=d''=d$, $D'=D''=D$, $n'=n''=n$. The capacity is now

$$C=\frac{0.2416\ (l'_1+l''_1)}{U'}=\frac{7.36\ (l'_2+l''_2)}{U'} \tag{39}$$

with

$$U'=\frac{\log\frac{4h}{d}+(n-1)\log\frac{2h}{D}}{n}-\frac{l'}{l'+l''}K_1-\frac{l''}{l'+l''}K_2-K_n+Y$$

As the simplest cases of all, we may assume, in addition, that the wires of the two legs of the V have the same length l, and that their ends have the same distance from the intersection of their axes; that is, $l'=l''=l$, $l'_0=l''_0=l_0$. Then in formula (39)

$$U'=\frac{\log\frac{4h}{d}+(n-1)\log\frac{2h}{D}}{n}-(K+K_n)+Y \tag{40}$$

and

$$Y=\frac{l+2l_0}{l}\Big[Y\,(l_0,\,l_0)-Y\,(l+l_0,\,l_0)\Big]$$

Example 15.—As an illustration of the preceding formulas we may take the case of a parallel-wire V antenna, each leg consisting of six wires of diameter 0.02 foot, spaced 2 feet apart; the whole antenna is supposed to be in a plane 50 feet above the ground. Suppose further, that the length of the wires in one set is 100 feet and those of the other set 50 feet, and that the point of the intersection of the axes of the two sets is, respectively, 50 and 25 feet from the nearer ends of the two legs, their angle being 45°.

Then $l'=100$, $l'_0=50$, $l''=50$, $l''_0=25$, $D=2$, $n=6$, $d=0.02$, $\theta=45°$.

From Table 1, for $\frac{2h}{l}=1$, $K'=0.336$, and for $\frac{2h}{l}=2$, $K''=0.541$, so that

$$U'=\frac{4.000+5\ (1.699)}{6}-\frac{2}{3}\ (0.336)-\frac{1}{3}\ (0.541)-0.252+Y.$$

To calculate Y we have

$$\frac{l''+l''_0}{l'+l'_0}=\frac{75}{150}=\frac{1}{2}\qquad \frac{l''_0}{l'+l'_0}=\frac{25}{150}=\frac{1}{6}\qquad \frac{l'_0}{l''+l''_0}=\frac{50}{75}=\frac{2}{3}$$

$$\frac{l''_0}{l'_0}=\frac{25}{50}=\frac{1}{2}\qquad \frac{2h}{l'+l'_0}=\frac{100}{150}=\frac{2}{3}\qquad \frac{2h}{l''+l''_0}=\frac{100}{75}=\frac{4}{3}\qquad \frac{2h}{l'_0}=\frac{100}{50}$$

$Y_1=$	0.497	0.286	0.535	0.497
$Y_2=$	.150	.050	.117	.070
Diff.=	.347	.236	.418	.427

$$\frac{l'+l'_0+l''+l''_0}{l'+l''}=\frac{225}{150}=\frac{3}{2} \qquad \frac{l'+l'_0+l''_0}{l'+l''}=\frac{175}{150}=\frac{7}{6}$$

$$\frac{l'_0+l''+l''_0}{l'+l''}=\frac{125}{150}=\frac{5}{6} \qquad \frac{l'_0+l''_0}{l'+l''}=\frac{75}{150}=\frac{1}{2}$$

$$Y=\frac{3}{2}\ (0.347)-\frac{7}{6}\ (0.275)-(0.418)\ \frac{5}{6}+\frac{1}{2}\ (0.427)=0.112$$

and finally

$$U'=1.538 \qquad C=\frac{7.36\ (150)}{1.538}=717.3\ \mu\mu f$$

The capacity calculated by simply adding the capacities of the legs taken separately is 778.1 $\mu\mu f$, so that the mutual effect of the two legs is to reduce the capacity by about 8 per cent. If the ends of the two legs came together, so that $l'_0=l''_0=0$, and we would find from Tables 7 and 8 that $Y_1=0.497$, $Y_2=0.131$, so that $Y=0.366$, and the capacity comes out 616.1 $\mu\mu f$.

15. ANTENNA OF PARALLEL WIRES IN A PLANE INCLINED TO GROUND

The antenna is supposed to consist of n wires spaced a distance D apart, the whole set being situated in a plane making an angle θ with the surface of the earth. The wires have each a diamater d and are of equal length l, while the lower ends of the wires are at a height h' above the ground.

The capacity is calculated by the formula

$$C=\frac{0.24167 l_1}{U_1}=\frac{7.36 l_2}{U_1} \tag{41}$$

in which

$$U_1=\frac{\log\frac{2l}{d}+(n-1)\log\frac{l}{D}}{n}-(0.133+K_n)-Y'$$

The term Y', which takes into account the effect of the charges upon the earth, is calculated on the assumption that its value is closely given by supposing the wire charges and image charges to be concentrated along the axes of the set of wires and the images, respectively. Its value is obtained by formula (35), using Table 7.

Example 16.—Suppose six wires spaced 2 feet apart, in a plane inclined 45° to the ground. The wires are of diameter 0.02 foot, and 50 feet long, and their lower ends are 50 feet above the ground. Then $\frac{2l}{d}=5{,}000$, $\frac{l}{D}=25$, $K_n=0.252$. To calculate Y' we use formula (35). The distance $l'=50\sqrt{2}$, $\frac{l'}{l+l'}=0.586$. Then from Table 7 for the argument (1, 90°) we find $Y\ (l', l') = 0.383$, and for the argument (0.586, 90°) we find $Y\ (l', l+l') = 0.361$, so that

$$Y' = \frac{(50+100\sqrt{2})}{50}(0.022) = 0.084$$

Then

$$U_1 = \frac{3.699+5\ (1.398)}{6} - (0.133+0.252) - 0.084 = 1.313$$

and the capacity is

$$C = \frac{7.36\ (50)}{1.313} = 280.3\ \mu\mu\text{f}$$

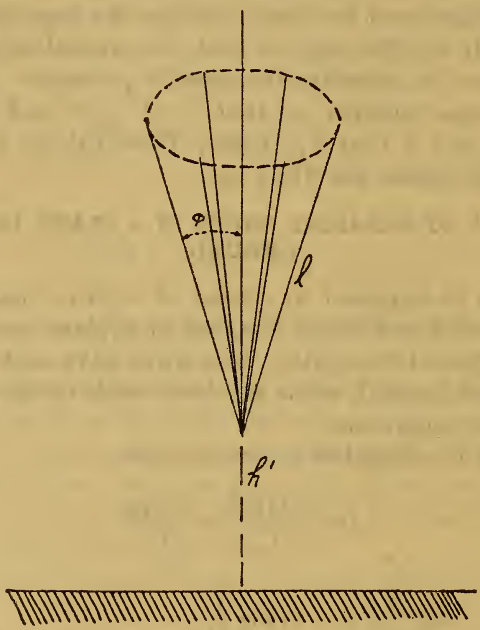

FIG. 9.—*Conical form of antenna*

If the same wires were placed in a horizontal plane 50 feet above the ground, the capacity would be 285.5 $\mu\mu$f, and if they were placed vertical with their lower ends 50 feet above the ground, the capacity would be 278.2 $\mu\mu$f.

16. CONICAL ANTENNA

Let the conical antenna consist of n wires of length l, and of diameter of cross section d, spaced at equal angles as elements of a cone whose half angle is φ, and whose point is a distance of h' from the ground, the axis of the cone extending vertically above the apex. (See fig. 9.)

The capacity is found by the formula

$$C = \frac{0.2416\,l_1 n}{U_k} = \frac{7.36\,l_2 n}{U_k} \tag{42}$$

Here

$$U_k = \log \frac{2l}{d} - 0.133 + \Sigma Y' - \frac{n}{\cos \varphi}(k - 0.133) \tag{43}$$

the constant k being obtained from Table 3 for the argument $\frac{h}{l \cos \varphi}$. The term $\Sigma Y'$ is the sum of the Y_1 terms for any wire and the remaining wires, the values being taken from Table 7 for arguments θ given by the angles between the wires. The lengths of the wires being the same, the length ratio in Table 7 is unity in each case.

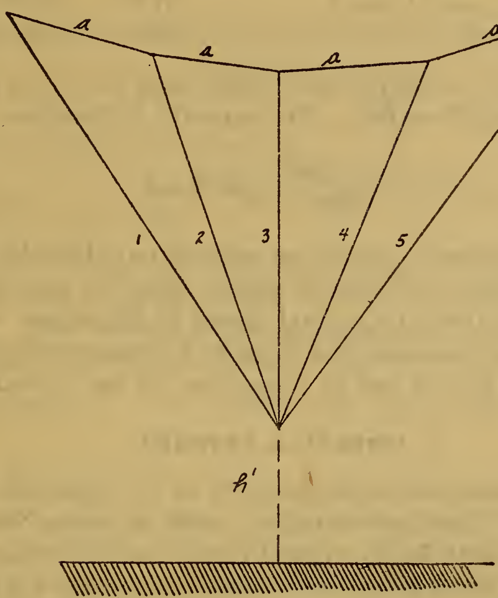

Fig. 10.—*Fan form of antenna*

The last term in U_k takes into account the effect of the earth. The expression for this is not exact, but the error in practical cases will be only a few per cent in the value of this term which is not more than about one-tenth of the whole quantity U_k. To calculate exactly the effect of the earth necessitates in this case the use of complicated formulas. The approximation here employed in obtaining formula (43) is to replace the antenna and image wires each by a single wire along the axis of the cone, the distance between the nearer ends of these two vertical wires being $2h'$, their lengths each being $l \cos \varphi$, and the linear charge density on each being taken as $\frac{q}{\cos \varphi}$, where q is the actual linear density originally assumed upon each wire in obtaining the capacity formula.

Example 17.—Suppose the wires composing a conical antenna are six in number, spaced at equal intervals upon a cone of half angle

30°, with its apex 50 feet above the ground. Each wire has a length of 100 feet, and a diameter of 0.02 foot. Then $\frac{2l}{d}=10{,}000$, $\varphi=30°$, $h'=50$, $l\cos\varphi=86.6$, and thus in Table 3 we have to take the value of k corresponding to the argument $\frac{50}{86.6}=0.583$; that is, $k=0.238$. To obtain the second term in U_k we find that the angles between the wires are as follows:

Wires 1 and 2, and 1 and 6	28.96°	$Y'=0.705$
Wires 1 and 3, and 1 and 5	51.3°	$Y'=.520$
Wires 1 and 4	60°	$Y'=.477$

Thus $\Sigma Y'=2\,(0.705+0.520)+0.477=2.927$, and for U_k we have $4.000-0.133+2.927-0.727=6.066$. The capacity is therefore

$$C=\frac{7.36\ (600)}{6.066}=728.0\ \mu\mu\text{f}.$$

With such an antenna it should be pointed out that the gain from increasing the number of wires is largely offset by the reduction in capacity resulting from the mutual effects of the wires. Compared with a vertical cage antenna, the capacity is greater with the conical antenna, but the gain is not in proportion to the amount of space occupied.

17. UMBRELLA ANTENNA

The same formula for the capacity is to be employed as for the conical antenna. The approximation made in taking into account the effect of the earth in the conical case is not so accurate for the umbrella type. However, the error in practical cases will not exceed 1 per cent in the value of the capacity.

18. FAN OR HARP ANTENNA

This is made up of n wires joined together at their lower ends. From this junction the wires are carried upward and are attached at equal intervals along a horizontal guy rope. (See fig. 10.)

The dimensions required are the lengths of the different wires l_s, the distances between their points of attachment on the guy rope a, and the distance of the point of junction from the ground h'. Then the angles between the various wires are to be calculated. Thus for any two consecutive wires of lengths l_r and l_s, whose points of attachment are separated by a distance a, the angle θ is found from the relation $\cos\theta=\frac{l_r^2+l_s^2-a^2}{2\,l_r l_s}$.

The potential of any one of the wires is that due to its own charge, plus that due to the charges on each of the other wires of the fan, minus that due to the image charges. The effects of the other wires are evaluated by the constants given in Table 7 which hold for wires intersecting at an angle θ. The effect of the image may be accurately enough taken into account by supposing both antenna top and images to be replaced by vertical wires having a length equal to the average vertical component of the lengths of the wires, the charge upon these equivalent wires being taken to have a density equal to the sum of the densities on the individual wires. The equivalent wires are supposed to lie in the same straight line with their nearer ends separated by twice the height of the lowest point of the fan from the ground.

Carrying through these operations we find for the capacity formula

$$C=\frac{0.2416\,\Sigma l_1}{U_t}=\frac{7.36\,\Sigma l_2}{U_t} \tag{44}$$

in which the quantity U_t is given by

$$U_t=\frac{1}{\Sigma l}\left[\sum_s l_s \log\frac{l_s}{d}+\sum_{r,s}\frac{(l_r+l_s)}{2}\,Y'_{rs}\right]-0.133-\frac{\Sigma l_s}{\lambda}\,(k-0.133) \tag{45}$$

The constants Y'_{rs} are to be taken from Table 7 for the angle between the pairs of wires r and s, and for the ratio of the smaller length to the greater. The constant λ is the average of the vertical components of the lengths of the wires of the fan, and k is to be taken from Table 3 for the argument $\frac{h}{\lambda}$. An example will make clear the use of the formula.

Example 18.—Suppose in Figure 10 that there are five wires meeting at a point 50 feet above the ground, and that the wires are fastened to a horizontal wire at points 10 feet apart. The middle wire is vertical and has a length of 100 feet and the diameter of the wires is 0.02 foot.

The lengths of the wires are then found to be $l_1=l_5=102.5$, $l_2=l_4=100.5$, and $l_3=100$, so that $\Sigma l=505$. The mean value of $\log_{10}\frac{l_s}{d}$, weighted according to the lengths of the wires, is 4.004. The angles between the wires are as follows:

1 and 2 = 4 and 5	5.60°	
2 and 3 = 3 and 4	5.71°	
1 and 3 = 3 and 5	11.31°	2 and 4, 11.42°
1 and 4 = 2 and 5	17.02°	
1 and 5	22.62°	

By rather rough interpolation from Table 7 the constants, corresponding to these angles and the ratios of the lengths of the wires of the pairs, were found to be

Pair	Y′	Pair	Y′	Pair	Y′
1, 2 or 4, 5	1.323	2, 5 or 1, 4	0.882	2, 3 or 3, 4	1.314
3, 5 or 1, 3	1.034	1, 5	.785	2, 4	1.043

Thus, the terms $\sum_{r,s} \frac{l_r+l_s}{2} Y'_{rs}$ give a final result of 4.372. The average vertical component of the lengths of the wires is $\lambda = 100$, and from Table 3 for $\frac{h}{\lambda} = \frac{50}{100} = 0.5$, $k = 0.247$, so that the image term is $\frac{505}{100} \times 0.114 = 0.576$, and the capacity is $C = \frac{7.36\ (505)}{7.667} = 484.6\ \mu\mu f$.

IX. USE OF TABLES FOR THREE COMMON FORMS OF ANTENNAS

The capacities of certain common forms of antennas may be obtained without calculation from Tables 2, 4, and 9. These tables are, respectively, for single-wire horizontal, single-wire vertical, and two-wire horizontal antennas. The length of each horizontal wire is denoted by l and of the vertical wire by m.

In each case there is given the capacity, C, in micromicrofarads, and the capacity per unit length in micromicrofarads per foot. It will be noted that the capacity per unit length varies but slowly with the length and the height above the ground. Thus, accurate interpolation of the capacity per unit length may be made for antennas not included in the tables, and so by multiplication by the length, the capacity of the antenna.

In addition to the capacity and linear capacity, the tables include the potential coefficient of the antenna for unit charge density per unit length of the antenna. This quantity U is useful in calculating the capacity of combinations of horizontal and vertical wires, such as, for example, the calculation of the effect of lead-in wires. This point will be illustrated in the succeeding section.

Four different sizes of wires (0.005, 0.01, 0.015, and 0.02 foot in diameter) are included in each table. These sizes, it is believed, cover the majority of antennas met in practice. Interpolation in the tables may be made for sizes lying between these values. The table for two-wire horizontal antennas covers three spacings of the wires, viz, 1 foot, 2 feet, and 3 feet. It is hoped that the range here covered will suffice for the large majority of practical cases.

Example 19.—Suppose the capacity of a two-wire antenna of 0.0125-foot wires, 125 feet long, spaced 2 feet apart, at a distance of 75 feet from the ground, is to be found from Table 9.

The following values of the linear capacity were interpolated from the table, using second differences, for the required length of 125 feet:

h	d=0.01	0.015	0.02
60	2.675	2.764	2.831
80	2.629	2.714	2.778
100	2.601	2.684	2.746

Interpolating from these for the length 75 feet the resulting values are

$d = 0.01$	0.015	0.02
2.638	2.724	2.788

and interpolating, finally, for the given diameter of wire we find for the capacity per foot, 2.684, so that the capacity of the antenna is 335 $\mu\mu$f. The complicated case just considered is purposely taken to illustrate what is possible with the table. In general, it will be well to use formula (24) directly, when more than two interpolations have to be made in the table. For this case the formula gives 334.9 $\mu\mu$f.

X. CALCULATION OF CAPACITY OF LEAD-IN WIRES

A problem frequently met is the calculation of the capacity of the combination of two sets of wires joined together, each of the elements being readily calculable singly by methods and formulas already given. A first approximation is to add the capacities of the elements, but this takes no account of the mutual effect of the elements upon one another and gives a value which is too high.

In general, the simplest method for obtaining the accurate value of the capacity of the combination is to obtain the unit potential coefficients of the separate elements either from Tables 2, 4, or 9. Likewise the mutual unit potential coefficients are to be obtained. With these values the linear charge density ratios for the elements are to be so determined as to make the potentials equal for all the elements. The process is illustrated in the next example, which has been so chosen as to employ the tabulated values of linear potential coefficients in Tables 2, 4, and 9.

Example 20.—A two-wire horizontal antenna, 200 feet long and 80 feet above the earth, has joined to one of its ends as a lead-in a single vertical wire 60 feet long. The diameter of all the wires will be assumed as 0.02 foot and the spacing of the wires of the horizontal portion as 2 feet.

From Table 9 the linear potential coefficient of the horizontal portion is found to be 2.771, and from Table 4, with $m = 60$ and $h' = 20$, we find $U_{22} = 3.505$. The mutual potential coefficient will be obtained with sufficient accuracy if we assume that the charge upon the horizontal portion is concentrated upon a single wire halfway between the actual wires. Then from Table 5, for the argument

$$\frac{m}{l} = \frac{60}{200} = 0.3 \text{ and for } \frac{h'}{m} = \frac{1}{3}, \text{ there is found } X = 0.168$$

To obtain the linear mutual potential coefficients from the constant X which is useful in calculations with wires at right angles, we note that we may write

$$X=\frac{x}{2.303\,(l+m)},\ U_{12}=\frac{x}{4.605\,l},\ U_{21}=\frac{x}{4.605\,m}$$

so that

$$U_{12}=\frac{l+m}{2l}\cdot X,\ U_{21}=\frac{l+m}{2m}\cdot X$$

are the general relations connecting these linear potential coefficients and the tabulated quantity. Thus for the present case

$$U_{12}=0.109,\ U_{21}=0.364$$

If we assume the linear charge densities upon the horizontal and vertical portions as q_1 and q_2, respectively, then we may write for the potentials of the two portions

$$v_1=q_1 U_{11}+q_2 U_{12}$$

$$v_2=q_1 U_{21}+q_2 U_{22}$$

and the condition that these may be equal is

$$\frac{q_2}{q_1}=\frac{U_{11}-U_{21}}{U_{22}-U_{12}}=\frac{2.407}{3.396}=0.709$$

Thus the common potential is

$$v=q_1\,[2.771+0.709\,(0.109)]=2.848\,q_1$$

the total charge

$$Q=q_1\,[200+60\,(0.709)]=242.5\,q_1$$

and the capacity

$$C=\frac{7.36\,(242.5)}{2.848}=625.2\ \mu\mu\text{f}$$

The sum of the capacities of the separate portions is from Tables 4 and 9, 531.1 + 126.0 = 657.1 $\mu\mu$f, which is thus seen to be about 5 per cent too large.

It is evident that this method is applicable in the general case of the calculation of the capacity of lead-in wires. To go into all the types of lead-in wires which occur in practice would take us too far afield. It is clear, however, that the formulas already given should cover usual arrangements in so far as the capacity of the lead-in wires in themselves are concerned. To take into account their effect upon the other parts of the antenna system it will usually suffice to make some simplifying assumption as was done in the preceding example, since this mutual effect is a relatively small part of the whole capacity.

XI. TABLES FOR ANTENNA CAPACITY CALCULATION

Three of the following tables give directly the capacities of three simple forms of antennas; they are Tables 2, 4, and 9. The other tables are auxiliary to certain of the formulas.

TABLE 1.—*Values of the constant K for use in formula (18) and for horizontal wires in general*

[The argument to be used is either $\frac{2h}{l}$ or $\frac{l}{2h}$, according to which is less than unity]

$\frac{2h}{l}$	K	$\frac{l}{2h}$	K	$\frac{l}{2h}$	K
0	0	1.00	0.336	0.50	0.541
0.1	0.042	.95	.350	.45	.576
.2	.082	.90	.364	.40	.617
.3	.121	.85	.379	.35	.664
.4	.157	.80	.396	.30	.721
.5	.191	.75	.414	.25	.790
.6	.223	.70	.435	.20	.874
.7	.254	.65	.457	.15	.990
.8	.283	.60	.482	.10	1.155
.9	.310	.55	.510	.05	1.445
1.0	.336	.50	.541		

Further values may be calculated from the formulas

$$K=\frac{\frac{2h}{l}-\left(\sqrt{1+\left(\frac{2h}{l}\right)^2}-1\right)}{2.303}+\log_{10}\frac{1+\sqrt{1+\left(\frac{2h}{l}\right)^2}}{2}\quad\text{for }\frac{2h}{l}\leqq 1$$

and

$$K=\frac{0.3069-\frac{2h}{l}\left(\sqrt{1+\left(\frac{l}{2h}\right)^2}-1\right)}{2.303}+\log_{10}\frac{\frac{l}{2h}+\sqrt{1+\left(\frac{l}{2h}\right)^2}}{\frac{l}{2h}}\quad\text{for }\frac{l}{2h}\leqq 1$$

TABLE 2.—*Constants for single-wire horizontal antennas*

[Symbols used above columns, etc., are defined on p. 606]

DIAMETER OF WIRE=0.06 INCH=0.005 FOOT

l feet	h=10 feet			h=15 feet			h=20 feet			h=25 feet			h=30 feet		
	U	C	C/l	U	C	C/l	U	C	C/l	U	C	C/l	U	C	C/l
10	3.362	21.89	2.189	3.397	21.67	2.167	3.414	22.57	2.257	3.427	21.48	2.148	3.434	21.30	2.130
20	3.567	41.27	2.064	3.629	40.56	2.028	3.663	40.18	2.009	3.684	39.96	1.998	3.698	39.81	1.990
30	3.660	60.30	2.011	3.743	58.99	1.966	3.790	58.26	1.942	3.819	57.82	1.927	3.839	57.57	1.919
40	3.712	79.31	1.983	3.810	77.27	1.932	3.868	76.11	1.903	3.905	75.39	1.885	3.930	74.91	1.873
50	3.746	98.24	1.965	3.856	95.44	1.909	3.921	93.85	1.877	3.965	92.88	1.858	3.995	92.1	1.842
60	3.770	117.1	1.952	3.888	113.6	1.893	3.961	111.5	1.858	4.009	110.2	1.836	4.044	109.2	1.820
80	3.801	154.9	1.936	3.931	149.8	1.872	4.013	146.7	1.834	4.070	144.7	1.809	4.111	143.2	1.790
100	3.821	192.6	1.926	3.958	186.0	1.860	4.047	181.9	1.819	4.110	179.1	1.791	4.157	177.0	1.770
150	3.848	286.9	1.913	3.997	276.2	1.841	4.096	269.5	1.797	4.168	264.9	1.766	4.223	261.4	1.743
200	3.861	381.3	1.906	4.017	366.4	1.832	4.122	357.1	1.786	4.199	350.6	1.753	4.259	345.6	1.728

l feet	h=40 feet			h=50 feet			h=60 feet			h=80 feet			h=100 feet		
	U	C	C/l	U	C	C/l	U	C	C/l	U	C	C/l	U	C	C/l
10	3.442	21.38	2.138	3.447	21.35	2.135	3.451	21.3	2.135	3.455	21.3	2.130	3.458	21.3	2.128
20	3.715	39.64	1.982	3.728	39.5	1.974	3.735	39.4	1.970	3.743	39.3	1.966	3.748	39.3	1.964
30	3.862	57.17	1.906	3.881	56.9	1.896	3.891	56.75	1.892	3.905	56.5	1.885	3.913	56.4	1.881
40	3.964	74.27	1.856	3.985	73.9	1.847	3.999	73.6	1.840	4.016	73.3	1.833	4.029	73.1	1.827
50	4.036	91.2	1.824	4.061	90.6	1.812	4.078	90.2	1.805	4.100	89.8	1.795	4.113	89.5	1.789
60	4.091	107.9	1.799	4.120	107.2	1.787	4.140	106.7	1.778	4.167	106.0	1.767	4.182	105.6	1.760
80	4.169	141.2	1.765	4.206	140.0	1.750	4.232	139.1	1.739	4.265	138.0	1.726	4.286	137.4	1.718
100	4.222	174.3	1.743	4.266	172.5	1.725	4.296	171.3	1.713	4.337	169.7	1.697	4.362	168.7	1.687
150	4.303	256.6	1.711	4.358	253.3	1.687	4.398	251.0	1.673	4.453	247.9	1.653	4.489	245.9	1.639
200	4.348	338.5	1.692	4.411	333.7	1.668	4.458	330.2	1.651	4.523	325.4	1.627	4.567	322.3	1.612

TABLE 2.—*Constants for single-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.12 INCH=0.01 FOOT

l feet	*h*=10 feet			*h*=15 feet			*h*=20 feet			*h*=25 feet			*h*=30 feet		
	U	*C*	*C/l*	*U*	*C*	*C/l*	*U*	*C*	*C/l*	*U*	*C*	*C/l*	*U*	*C*	*C/l*
10	3.061	24.0	2.404	3.096	23.8	2.377	3.113	23.6	2.364	3.126	23.5	2.354	3.133	23.5	2.349
20	3.266	45.1	2.254	3.328	44.2	2.212	3.362	43.8	2.189	3.383	43.5	2.176	3.397	43.3	2.166
30	3.359	65.7	2.191	3.442	64.2	2.138	3.489	63.3	2.109	3.518	62.8	2.091	3.536	62.4	2.080
40	3.411	86.3	2.158	3.509	83.9	2.098	3.567	82.5	2.064	3.604	81.7	2.042	3.629	81.1	2.028
50	3.445	106.8	2.136	3.555	103.5	2.070	3.620	101.6	2.033	3.684	100.4	2.008	3.694	99.6	1.992
60	3.469	127.3	2.122	3.587	123.1	2.052	3.660	120.7	2.012	3.708	119.1	1.985	3.743	118.0	1.967
80	3.500	168.2	2.102	3.630	162.2	2.028	3.712	158.6	1.982	3.769	156.2	1.952	3.810	154.5	1.931
100	3.520	209.1	2.091	3.657	201.3	2.013	3.754	196.1	1.961	3.809	193.2	1.932	3.856	190.9	1.909
150	3.547	311.2	2.075	3.659	298.7	1.991	3.795	290.9	1.939	3.867	285.5	1.903	3.922	281.5	1.877
200	3.560	413.5	2.068	3.716	396.1	1.980	3.821	385.3	1.926	3.898	377.6	1.888	3.958	371.9	1.860

l feet	*h*=40 feet			*h*=50 feet			*h*=60 feet			*h*=80 feet			*h*=100 feet		
10	3.141	23.4	2.343	3.146	23.4	2.339	3.150	23.4	2.336	3.154	23.3	2.334	3.157	23.3	2.330
20	3.414	43.1	2.156	3.427	43.0	2.148	3.434	42.9	2.144	3.442	42.8	2.138	3.447	42.7	2.135
30	3.561	62.0	2.067	3.580	61.8	2.059	3.590	61.5	2.050	3.604	61.3	2.042	3.612	61.1	2.038
40	3.663	80.4	2.009	3.684	79.9	1.998	3.698	79.6	1.990	3.715	79.2	1.981	3.728	79.0	1.974
50	3.735	98.5	1.971	3.760	97.8	1.957	3.777	97.4	1.949	3.801	96.8	1.936	3.812	96.5	1.931
60	3.790	116.5	1.942	3.819	115.6	1.927	3.839	115.0	1.917	3.866	114.2	1.903	3.881	113.8	1.897
80	3.868	152.2	1.902	3.905	150.8	1.885	3.931	149.8	1.872	3.964	148.5	1.856	3.985	147.8	1.847
100	3.921	187.7	1.877	3.965	185.6	1.856	3.995	184.2	1.842	4.036	182.4	1.824	4.061	181.2	1.812
150	4.002	275.9	1.839	4.057	272.1	1.814	4.097	269.5	1.797	4.152	265.9	1.773	4.188	263.6	1.757
200	4.047	363.7	1.818	4.110	358.2	1.791	4.157	354.1	1.770	4.220	348.6	1.743	4.266	345.1	1.726

DIAMETER OF WIRE=0.18 INCH=0.015 FOOT

l feet	*h*=10 feet			*h*=15 feet			*h*=20 feet			*h*=25 feet			*h*=30 feet		
10	2.885	25.5	2.549	2.920	25.2	2.521	2.937	25.1	2.506	2.950	24.9	2.495	2.957	24.9	2.489
20	3.090	47.6	2.382	3.152	46.7	2.335	3.186	46.2	2.310	3.207	45.9	2.295	3.221	45.7	2.285
30	3.183	69.4	2.312	3.266	67.6	2.253	3.313	66.6	2.222	3.342	66.1	2.202	3.362	65.7	2.189
40	3.235	91.0	2.277	3.333	88.3	2.208	3.391	86.8	2.170	3.428	85.9	2.147	3.453	85.3	2.132
50	3.269	112.6	2.252	3.379	108.9	2.178	3.444	106.8	2.137	3.488	105.5	2.110	3.518	104.6	2.092
60	3.293	134.1	2.235	3.411	129.5	2.158	3.484	126.8	2.112	3.532	125.0	2.083	3.567	123.8	2.063
80	3.324	177.1	2.214	3.451	201.4	2.131	3.536	166.5	2.081	3.592	163.9	2.049	3.634	162.0	2.025
100	3.344	220.1	2.201	3.481	211.4	2.114	3.570	206.2	2.062	3.633	202.6	2.026	3.680	200.0	2.000
150	3.369	327.7	2.185	3.520	313.6	2.091	3.619	305.1	2.034	3.691	299.1	1.994	3.746	294.7	1.965
200	3.384	435.0	2.175	3.540	415.0	2.075	3.645	403.8	2.019	3.722	395.5	1.978	3.782	389.2	1.946

l feet	*h*=40 feet			*h*=50 feet			*h*=60 feet			*h*=80 feet			*h*=100 feet		
10	2.965	24.8	2.482	2.970	24.8	2.476	2.974	24.8	2.475	2.978	24.7	2.472	2.981	24.7	2.469
20	3.238	45.5	2.273	3.251	45.3	2.264	3.258	45.2	2.259	3.266	45.1	2.254	3.271	45.0	2.250
30	3.385	65.2	2.174	3.404	64.9	2.162	3.414	64.7	2.150	3.428	64.4	2.147	3.436	64.3	2.142
40	3.487	84.4	2.111	3.512	83.8	2.096	3.522	83.6	2.090	3.559	82.7	2.068	3.552	82.9	2.072
50	3.559	103.4	2.068	3.584	102.7	2.054	3.601	102.2	2.044	3.623	101.6	2.032	3.636	101.2	2.024
60	3.614	122.2	2.037	3.643	121.2	2.002	3.663	120.6	2.010	3.690	119.7	1.995	3.705	119.2	1.987
80	3.688	159.7	1.996	3.729	157.9	1.974	3.755	156.8	1.960	3.788	155.4	1.942	3.811	154.5	1.932
100	3.745	196.5	1.965	3.789	194.2	1.942	3.819	192.7	1.927	3.860	190.7	1.907	3.885	189.4	1.894
150	3.826	288.6	1.924	3.881	284.5	1.897	3.921	281.6	1.877	3.976	277.7	1.851	4.012	275.2	1.835
200	3.871	380.3	1.902	3.934	374.2	1.871	3.981	369.8	1.849	4.046	363.8	1.819	4.110	358.2	1.791

TABLE 2.—*Constants for single-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.24 INCH=0.02 FOOT

l feet	h=10 feet			h=15 feet			h=20 feet			h=25 feet			h=30 feet		
	U	C	C/l	U	C	C/l	U	C	C/l	U	C	C/l	U	C	C/l
10	2.760	26.7	2.667	2.795	26.3	2.634	2.812	26.2	2.617	2.825	26.0	2.605	2.832	26.0	2.599
20	2.965	49.6	2.482	3.027	48.6	2.432	3.061	48.1	2.404	3,082	47.8	2.388	3.096	47.6	2.378
30	3.058	72.2	2.407	3.141	70.3	2.343	3.188	69.3	2.309	3.217	68.6	2.288	3.237	68.2	2.274
40	3.110	94.7	2.366	3.208	91.8	2.294	3.266	90.1	2.254	3.303	89.1	2.228	3.328	88.5	2.212
50	3.144	117.0	2.341	3.254	113.1	2.262	3.319	111.2	2.224	3.363	109.4	2.188	3.393	108.5	2.170
60	3.168	139.4	2.323	3.286	134.4	2.240	3.359	131.5	2.192	3.407	129.6	2.160	3.442	128.3	2.138
80	3.199	184.1	2.301	3.329	176.9	2.212	3.411	172.6	2.158	3.468	169.8	2.122	3.509	167.8	2.098
100	3.219	228.6	2.286	3.356	219.3	2.193	3.445	213.6	2.136	3.508	209.8	2.098	3.555	207.0	2.070
150	3.246	340.1	2.274	3.395	325.2	2.168	3.494	316.0	2.107	3.566	309.6	2.064	3.621	304.9	2.033
200	3.259	451.7	2.258	3.415	431.0	2.155	3.520	418.2	2.091	3.597	409.2	2.046	3.657	402.5	2.012
	h=40 feet			h=50 feet			h=60 feet			h=80 feet			h=100 feet		
10	2.840	25.9	2.592	2.845	25.9	2.587	2.849	25.8	2.583	2.853	25.8	2.580	2.856	25.8	2.577
20	3.113	47.3	2.364	3.126	47.1	2.354	3.133	47.0	2.349	3.141	46.9	2.343	3.146	46.8	2.340
30	3.260	67.7	2.258	3.279	67.3	2.245	3.289	67.1	2.238	3.303	66.8	2.228	3.311	66.7	2.223
40	3.362	87.6	2.189	3.383	87.1	2.177	3.397	86.7	2.166	3.414	86.2	2.156	3.427	85.9	2.148
50	3.434	107.2	2.144	3.459	106.4	2.128	3.476	105.9	2.118	3.502	105.1	2.102	3.511	104.8	2.096
60	3.489	126.6	2.110	3.518	125.5	2.092	3.538	124.8	2.080	3.565	123.9	2.065	3.580	123.4	2.056
80	3.567	165.1	2.064	3.604	163.4	2.042	3.630	162.2	2.028	3.663	160.7	2.009	3.684	159,8	1.998
100	3.620	203.3	2.033	3.664	200.9	2.009	3.694	199.2	1.992	3.735	197.1	1.971	3.760	195.7	1.957
150	3.701	298.3	1.989	3.756	293.9	1.959	3.796	290.8	1.939	3.851	286.7	1.911	3.887	284.0	1.893
200	3.746	393.0	1.965	3.809	386.5	1.932	3.856	381.7	1.908	3.921	375.4	1.877	3.965	371.3	1.856

TABLE 3.—*Values of the constant k used in formula (19) and for vertical wires in general*

[The argument is $\frac{h'}{m}$ or $\frac{m}{h'}$ according to which is less than unity]

$\frac{h'}{m}$	k	$\frac{h'}{m}$	k	$\frac{m}{h'}$	k
		0.3	0.280	1.0	0.207
0.02	0.403	.4	.261	.9	.202
.04	.384	.5	.247	.8	.196
.06	.369	.6	.236	.7	.190
.08	.356	.7	.227	.6	.184
.10	.345	.8	.219	.5	.177
.15	.323	.9	.2125	.4	.170
.20	.305	1.0	.207	.3	.162
.25	.291			.2	.153
.30	.280			.1	.144
				0	.133

Further values may be calculated from the formula

$$k=0.4343+\frac{h'}{m}\log_{10}\frac{4h'}{m}+\left(1+\frac{h'}{m}\right)\log_{10}\left(1+\frac{h'}{m}\right)$$
$$-\left(1+\frac{2h'}{m}\right)\log_{10}\left(1+\frac{2h'}{m}\right)\text{ for }\frac{h'}{m}\leqq 1$$

and

$$k=0.1333+\frac{h'}{m}\left(1+\frac{m}{h'}\right)\log_{10}\left(1+\frac{m}{h'}\right)$$
$$-\frac{2h'}{m}\left(1+\frac{m}{2h'}\right)\log\left(1+\frac{m}{2h'}\right)\text{ for }\frac{m}{h'}\leqq 1$$

TABLE 4.—*Constants for single-wire vertical antennas*

[Symbols used above columns, etc., are defined on p. 606]

DIAMETER OF WIRE=0.06 INCH=0.005 FOOT

m feet	h'=5 feet			h'=10 feet			h'=20 feet			h'=50 feet		
	U	C	C/m	U	C	C/m	U	C	C/m	U	C	C/m
10	3.355	21.9	2.194	3.395	21.7	2.168	3.425	21.5	2.149	3.449	21.3	2.134
20	3.612	40.8	2.038	3.656	40.3	2.013	3.696	39.8	1.992	3.733	39.4	1.972
30	3.762	58.7	1.956	3.806	58.0	1.934	3.850	57.4	1.912	3.895	56.7	1.890
40	3.872	76.0	1.901	3.913	75.2	1.881	3.957	74.4	1.860	4.008	73.4	1.836
50	3.956	93.0	1.860	3.996	92.1	1.842	4.040	91.1	1.822	4.094	89.9	1.798
60	4.025	109.7	1.828	4.063	108.7	1.812	4.107	107.5	1.792	4.163	106.1	1.768
80	4.138	142.3	1.779	4.173	141.1	1.764	4.214	139.7	1.746	4.272	137.8	1.722
100	4.224	174.2	1.742	4.257	172.9	1.729	4.297	171.3	1.713	4.355	169.0	1.690

DIAMETER OF WIRE=0.12 INCH=0.01 FOOT

m feet	h'=0			h'=5 feet			h'=10 feet			h'=20 feet			h'=50 feet		
	U	C	C/m	U	C	C/m	U	C	C/m	U	C	C/m	U	C	C/m
10	2.867	25.7	2.567	3.054	24.1	2.410	3.094	23.8	2.379	3.124	23.6	2.356	3.148	23.4	2.338
20	3.168	46.5	2.323	3.311	44.5	2.223	3.355	43.9	2.194	3.395	43.4	2.168	3.432	42.9	2.144
30	3.344	66.0	2.201	3.461	63.8	2.127	3.505	63.0	2.100	3.549	62.2	2.074	3.594	61.4	2.048
40	3.469	84.9	2.122	3.571	82.4	2.061	3.612	81.5	2.038	3.656	80.5	2.013	3.707	79.4	1.983
50	3.566	103.2	2.064	3.665	100.7	2.014	3.695	99.6	1.992	3.739	98.4	1.968	3.793	97.0	1.940
60	3.645	121.2	2.020	3.724	118.6	1.977	3.762	117.4	1.957	3.806	116.0	1.933	3.862	114.3	1.905
80	3.770	156.2	1.952	3.837	153.5	1.919	3.872	152.1	1.901	3.913	150.5	1.881	3.970	148.3	1.854
100	3.867	190.3	1.903	3.923	187.6	1.876	3.956	186.0	1.860	3.996	184.2	1.842	4.054	181.6	1.816

DIAMETER OF WIRE=0.18 INCH=0.015 FOOT

m feet	h'=5 feet			h'=10 feet			h'=20 feet			h'=50 feet		
	U	C	C/m	U	C	C/m	U	C	C/m	U	C	C/m
10	2.878	25.6	2.557	2.918	25.2	2.522	2.948	25.0	2.497	2.972	24.8	2.476
20	3.135	46.9	2.348	3.179	46.3	2.315	3.219	45.7	2.286	3.256	45.2	2.260
30	3.285	67.2	2.240	3.329	66.3	2.211	3.373	65.5	2.182	3.418	64.6	2.153
40	3.395	86.7	2.168	3.436	85.7	2.142	3.480	84.6	2.115	3.531	83.4	2.084
50	3.479	105.8	2.116	3.519	104.6	2.092	3.563	103.3	2.066	3.617	101.7	2.034
60	3.548	124.5	2.075	3.586	123.2	2.052	3.630	121.7	2.028	3.686	119.8	1.997
80	3.661	160.8	2.010	3.696	159.3	1.981	3.737	157.6	1.970	3.794	155.2	1.940
100	3.747	196.4	1.964	3.780	194.7	1.947	3.820	192.7	1.927	3.878	189.8	1.898

DIAMETER OF WIRE=0.24 INCH=0.02 FOOT

m feet	h'=5 feet			h'=10 feet			h'=20 feet			h'=50 feet		
	U	C	C/m	U	C	C/m	U	C	C/m	U	C	C/m
10	2.753	26.7	2.673	2.793	26.4	2.635	2.823	26.1	2.607	2.847	25.8	2.585
20	3.010	48.9	2.445	3.054	48.2	2.410	3.094	47.6	2.379	3.131	47.0	2.350
30	3.160	69.8	2.329	3.204	68.9	2.297	3.248	68.0	2.266	3.293	67.0	2.235
40	3.270	90.0	2.251	3.311	88.9	2.223	3.355	87.8	2.194	3.406	86.4	2.161
50	3.354	109.7	2.194	3.394	108.4	2.168	3.438	107.0	2.141	3.492	105.4	2.108
60	3.423	129.0	2.150	3.461	127.6	2.127	3.505	126.0	2.100	3.561	124.0	2.067
80	3.536	166.5	2.081	3.571	164.9	2.061	3.612	163.0	2.038	3.670	160.4	2.005
100	3.622	203.2	2.032	3.655	201.4	2.014	3.695	199.2	1.992	3.753	196.1	1.961

TABLE 5.—*Values of the constant X for wires at right angles; formula (20)*

$\frac{m}{l}$	$\frac{h'}{m}$						$\frac{m}{h'}$				
	0	0.2	0.4	0.6	0.8	1.0	0.8	0.6	0.4	0.2	0
0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.055	0.064	0.072	0.078	0.083	0.088	0.093	0.097	0.106	0.125	0.158
0.2	.099	.116	.129	.137	.146	.155	.165	.174	.187	.207	.239
0.3	.135	.157	.173	.184	.195	.206	.214	.226	.241	.262	.291
0.4	.164	.189	.207	.222	.233	.243	.252	.263	.276	.296	.325
0.5	.186	.214	.233	.248	.260	.269	.278	.290	.305	.323	.348
0.6	.204	.233	.253	.267	.278	.286	.297	.309	.323	.340	.363
0.7	.218	.247	.267	.282	.293	.302	.311	.322	.335	.352	.373
0.8	.229	.258	.278	.292	.302	.311	.320	.330	.342	.358	.379
0.9	.237	.265	.285	.298	.308	.317	.326	.336	.347	.362	.382
1.0	.243	.271	.290	.303	.313	.321	.329	.338	.350	.365	.383

$\frac{l}{m}$	$\frac{h'}{m}$						$\frac{m}{h'}$				
	0	0.2	0.4	0.6	0.8	1.0	0.8	0.6	0.4	0.2	0
0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.130	0.137	0.141	0.144	0.146	0.147	0.147	0.150	0.153	0.155	0.159
0.2	.189	.200	.207	.213	.216	.218	.221	.224	.228	.232	.239
0.3	.222	.237	.247	.254	.260	.265	.269	.272	.275	.282	.291
0.4	.241	.259	.271	.279	.285	.290	.295	.300	.306	.314	.325
0.5	.250	.271	.285	.295	.302	.307	.312	.318	.325	.335	.348
0.6	.254	.277	.292	.303	.310	.317	.323	.330	.338	.349	.363
0.7	.254	.279	.295	.306	.314	.322	.329	.336	.346	.357	.373
0.8	.252	.278	.295	.307	.316	.324	.331	.340	.350	.362	.379
0.9	.248	.275	.293	.306	.315	.323	.330	.339	.350	.364	.382
1.0	.243	.271	.290	.303	.313	.321	.329	.338	.350	.365	.383

TABLE 6.—*Values of the constant K_n for use in formula (22) and other parallel wire antenna formulas*

n	K_n	n	K_n	n	K_n	n	K_n
2	0	8	0.347	14	0.550	20	0.688
3	0.067	9	.388	15	.576	30	.847
4	.135	10	.425	16	.601	40	.970
5	.197	11	.460	17	.625	50	1.063
6	.252	12	.492	18	.647	100	1.357
7	.302	13	.522	19	.668		

The general formula for K_n is

$$4.605\ K_n=\frac{4}{n^2}\left[\log n\,(n-1)+2\log n\,(n-2)+3\log n\,(n-3)+\cdots\cdots+(n-2)\log n\,2\right]$$

or

$$K_n=\frac{2}{n^2}\left[\log_{10}(n-1)+2\log_{10}(n-2)+3\log_{10}(n-3)+\cdots+(n-2)\log_{10}2\right]$$

TABLE 7.—*Values of the constant Y_1 for wires intersecting at an angle; formulas (30) ff*

θ (degrees)	$\frac{m}{l}=1.0$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
180	0.3010	0.3004	0.2983	0.2942	0.2873	0.2764	0.2598	0.2346	0.1957	0.1323
165	.3029	.3023	.3002	.2960	.2891	.2781	.2613	.2359	.1967	.1329
150	.3086	.3080	.3056	.3016	.2944	.2832	.2660	.2400	.1999	.1348
135	.3185	.3179	.3156	.3112	.3037	.2920	.2741	.2469	.2053	.1380
120	.3334	.3326	.3303	.3255	.3176	.3051	.2860	.2573	.2134	.1427
105	.3542	.3534	.3508	.3457	.3370	.3234	.3028	.2714	.2244	.1492
90	.3828	.3820	.3790	.3732	.3635	.3483	.3254	.2911	.2393	.1578
85	.3945	.3936	.3905	.3844	.3743	.3584	.3346	.2989	.2453	.1612
80	.4075	.4066	.4033	.3970	.3863	.3697	.3448	.3076	.2518	.1650
75	.4220	.4211	.4176	.4109	.3997	.3823	.3560	.3172	.2591	.1691
70	.4383	.4372	.4336	.4265	.4146	.3962	.3686	.3277	.2670	.1736
65	.4565	.4554	.4515	.4440	.4313	.4118	.3825	.3395	.2759	.1786
60	.4771	.4759	.4718	.4636	.4501	.4292	.3981	.3526	.2857	.1842
55	.5004	.4992	.4946	.4859	.4713	.4489	.4156	.3678	.2966	.1903
50	.5271	.5257	.5208	.5112	.4954	.4712	.4354	.3838	.3089	.1971
45	.5579	.5563	.5509	.5404	.5230	.4966	.4580	.4025	.3227	.2048
40	.5937	.5920	.5859	.5742	.5550	.5260	.4839	.4239	.3354	.2136
35	.6360	.6340	.6272	.6140	.5925	.5603	.5139	.4486	.3566	.2236
30	.6870	.6846	.6767	.6616	.6371	.6009	.5494	.4778	.3780	.2354
25	.7498	.7470	.7376	.7198	.6915	.6502	.5923	.5128	.4035	.2494
20	.8299	.8264	.8148	.7933	.7598	.7118	.6457	.5563	.4351	.2668
15	.9376	.9330	.9180	.8909	.8499	.7926	.7155	.6129	.4762	.2892
10	1.0960	1.0892	1.0681	1.0318	.9793	.9082	.8149	.6934	.5345	.3210
5	1.3789	1.3663	1.3314	1.2771	1.2034	1.1079	.9863	.8320	.6346	.3757

TABLE 8.—*Values of the constant Y_2 for wires in parallel planes and inclined at an angle with one another; formulas (30) ff*

$\frac{m}{l}=1$

θ (degrees)	$\frac{2h}{l}=0$	0.2	0.5	1.0	0.5	$0.2-\frac{l}{2h}$
0	∞	0.648	0.359	0.203	0.106	0.043
15	0.938	.584	.349	.202	.106	.043
30	.687	.497	.328	.197	.106	.043
45	.558	.432	.304	.191	.1045	.043
60	.477	.384	.282	.185	.103	.043
75	.422	.348	.264	.178	.102	.043
90	.383	.321	.249	.172	.101	.043
105	.354	.300	.237	.167	.099	.043
120	.333	.285	.228	.163	.098	.0425
135	.319	.274	.221	.160	.097	.0425
150	.309	.267	.216	.158	.097	.0425
165	.303	.262	.213	.156	.096	.0425
180	.301	.261	.212	.156	.096	.0425

$\frac{m}{l}=0.75$

θ (degrees)	$\frac{2h}{l}=0$	0.2	0.5	1.0	0.5	$0.2-\frac{l}{2h}$
0	∞	0.571	0.312	0.175	0.091	0.037
15	0.905	.528	.306	.174	.091	.037
30	.670	.461	.292	.171	.091	.037
45	.546	.406	.274	.167	.091	.037
60	.468	.364	.257	.163	.090	.037
75	.414	.331	.242	.158	.089	.037
90	.377	.307	.230	.154	.088	.037
105	.348	.288	.220	.150	.087	.037
120	.328	.274	.212	.147	.086	.037
135	.314	.264	.206	.144	.086	.037
150	.304	.257	.202	.142	.085	.037
165	.298	.253	.199	.141	.085	.037
180	.297	.251	.198	.141	.085	.037

TABLE 8.—*Values of the constant Y_2 for wires in parallel planes and inclined at an angle with one another; formulas (30) ff*—Continued

θ (degrees)	$\frac{2h}{l}=0$	0.2	0.5	1.0	0.5	$0.2=\frac{l}{2h}$
$\frac{m}{l}=0.5$						
0	∞	0.432	0.239	0.135	0.071	0.029
15	0.798	.414	.236	.135	.071	.029
30	.601	.379	.229	.133	.071	.029
45	.496	.343	.221	.131	.0705	.029
60	.429	.313	.210	.129	.070	.029
75	.382	.289	.200	.126	.0695	.029
90	.348	.270	.192	.124	.069	.029
105	.323	.255	.186	.121	.069	.029
120	.305	.244	.180	.1195	.068	.029
135	.292	.235	.175	.118	.068	.0285
150	.283	.230	.172	.117	.0675	.0285
165	.278	.225	.171	.116	.067	.0285
180	.276	.223	.170	.116	.067	.0285
$\frac{m}{l}=0.25$						
0	∞	0.238	0.136	0.079	0.042	0.017
15	0.550	.235	.136	.079	.042	.017
30	.432	.226	.134	.079	.042	.017
45	.366	.215	.131	.078	.042	.017
60	.322	.204	.128	.0775	.042	.017
75	.291	.194	.125	.077	.042	.017
90	.270	.185	.122	.076	.042	.017
105	.251	.178	.120	.075	.042	.017
120	.238	.172	.117	.074	.041	.017
135	.228	.167	.116	.074	.041	.017
150	.222	.164	.114	.073	.041	.017
165	.162	.162	.113	.073	.041	.017
180	.217	.161	.113	.073	.041	.017
$\frac{m}{l}=0.1$						
0	∞	0.099	0.059	0.035	0.019	0.008
15	0.289	.099	.059	.035	.019	.008
30	.235	.097	.059	.035	.019	.008
45	.205	.096	.058	.035	.019	.008
60	.184	.092	.058	.035	.019	.008
75	.169	.092	.057	.035	.019	.008
90	.158	.090	.057	.035	.019	.008
105	.149	.088	.056	.035	.019	.008
120	.143	.086	.056	.034	.019	.008
135	.138	.085	.055	.034	.019	.008
150	.135	.084	.055	.034	.019	.008
165	.133	.084	.055	.034	.019	.008
180	.132	.0835	.055	.034	.019	.008

TABLE 9.—*Constants for two-wire horizontal antennas*

[Symbols used above columns, etc., are defined on p. 606]

DIAMETER OF WIRE=0.06 INCH=0.005 FOOT

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=10 feet								
10	2.082	1.932	1.844	35.3	38.1	39.9	3.534	3.810	3.991
20	2.277	2.126	2.038	64.6	69.2	72.2	3.232	3.460	3.610
30	2.366	2.216	2.128	93.3	99.7	103.8	3.111	3.322	3.460
40	2.416	2.266	2.178	121.9	129.9	135.2	3.048	3.248	3.380
50	2.450	2.299	2.211	150.2	160.1	166.4	3.004	3.202	3.328
60	2.472	2.322	2.234	178.6	190.2	197.7	2.977	3.170	3.295
80	2.502	2.352	2.264	235.3	250.3	260.1	2.941	3.129	3.251
100	2.522	2.372	2.284	291.8	310.4	322.3	2.918	3.104	3.223
150	2.548	2.398	2.310	433.2	460.4	477.9	2.888	3.069	3.186
200	2.561	2.410	2.322	574.8	610.7	633.8	2.874	3.054	3.169
	Height=15 feet								
10	2.118	1.967	1.879	34.8	37.4	39.1	3.476	3.742	3.914
20	2.339	2.188	2.100	62.9	67.3	70.1	3.146	3.363	3.504
30	2.449	2.298	2.210	90.2	96.1	99.9	3.005	3.202	3.330
40	2.514	2.364	2.276	117.2	124.5	129.4	2.928	3.112	3.235
50	2.560	2.409	2.321	143.8	152.8	158.6	2.876	3.056	3.172
60	2.590	2.440	2.352	170.5	181.0	187.8	2.842	3.017	3.130
80	2.632	2.482	2.394	223.7	237.2	246.0	2.796	2.965	3.074
100	2.659	2.508	2.420	276.8	293.4	304.1	2.768	2.934	3.041
150	2.698	2.547	2.459	409.3	433.4	449.0	2.729	2.889	2.993
200	2.717	2.566	2.478	541.8	573.6	593.9	2.709	2.868	2.970
	Height=20 feet								
10	2.134	1.984	1.895	34.5	37.1	38.8	3.448	3.710	3.884
20	2.373	2.222	2.134	62.0	66.2	69.0	3.102	3.312	3.448
30	2.496	2.346	2.258	88.5	94.1	97.8	2.949	3.138	3.260
40	2.572	2.422	2.334	114.4	121.6	126.1	2.860	3.039	3.152
50	2.624	2.474	2.386	140.2	148.8	154.2	2.804	2.975	3.084
60	2.664	2.513	2.425	165.8	175.7	182.1	2.763	2.928	3.035
80	2.714	2.564	2.476	216.9	229.6	237.8	2.712	2.870	2.972
100	2.748	2.598	2.510	267.8	283.3	293.3	2.678	2.833	2.933
150	2.796	2.646	2.558	394.8	417.2	431.6	2.632	2.781	2.877
200	2.822	2.672	2.584	521.6	551.0	569.8	2.608	2.755	2.849
	Height=25 feet								
10	2.148	1.996	1.909	34.3	36.9	38.6	3.427	3.686	3.855
20	2.394	2.243	2.156	61.5	65.6	68.3	3.074	3.282	3.414
30	2.525	2.374	2.286	87.4	92.0	96.6	2.915	3.100	3.219
40	2.610	2.458	2.371	112.8	119.8	124.2	2.820	2.994	3.105
50	2.668	2.518	2.430	137.9	146.2	151.4	2.758	2.924	3.028
60	2.712	2.560	2.473	162.9	172.5	178.6	2.715	2.875	2.977
80	2.772	2.620	2.534	212.4	224.7	232.4	2.656	2.809	2.905
100	2.811	2.660	2.572	261.8	276.7	286.1	2.618	2.767	2.861
150	2.868	2.718	2.630	384.9	406.3	419.8	2.566	2.709	2.799
200	2.899	2.748	2.660	507.8	535.7	553.3	2.539	2.678	2.766
	Height=30 feet								
10	2.154	2.004	1.916	34.2	36.7	38.4	3.416	3.673	3.841
20	2.408	2.258	2.170	61.1	65.2	67.8	3.056	3.260	3.392
30	2.545	2.394	2.306	86.8	92.2	95.7	2.892	3.074	3.191
40	2.634	2.484	2.396	111.8	118.5	122.9	2.794	2.962	3.072
50	2.698	2.548	2.460	136.4	144.4	149.6	2.728	2.888	2.992
60	2.746	2.596	2.508	160.8	170.1	176.1	2.680	2.835	2.935
80	2.812	2.662	2.574	209.4	221.2	228.8	2.617	2.765	2.859
100	2.858	2.708	2.620	257.5	271.8	281.0	2.575	2.718	2.810
150	2.924	2.773	2.685	377.6	398.1	411.2	2.517	2.654	2.741
200	2.960	2.809	2.721	497.4	524.0	541.0	2.487	2.620	2.705

TABLE 9.—*Constants for two-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.06 INCH=0.005 FOOT—Continued

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=40 feet								
10	2.162	2.012	1.924	34.0	36.6	38.2	3.403	3.658	3.825
20	2.425	2.274	2.186	60.3	64.7	67.3	3.014	3.236	3.366
30	2.568	2.418	2.330	86.0	91.3	94.8	2.866	3.043	3.159
40	2.668	2.518	2.430	110.3	116.9	121.2	2.758	2.922	3.029
50	2.740	2.589	2.501	134.3	142.1	147.1	2.686	2.842	2.942
60	2.794	2.643	2.555	158.1	167.1	172.8	2.635	2.785	2.880
80	2.870	2.720	2.632	205.1	216.5	223.7	2.564	2.706	2.795
100	2.923	2.772	2.684	251.8	265.5	274.2	2.518	2.655	2.742
150	3.004	2.853	2.765	367.6	387.0	399.3	2.451	2.580	2.662
200	3.049	2.898	2.810	482.8	508.0	523.8	2.414	2.540	2.619
	Height=50 feet								
10	2.168	2.017	1.929	34.0	36.5	38.2	3.396	3.649	3.818
20	2.438	2.288	2.200	60.4	64.4	66.9	3.019	3.218	3.346
30	2.587	2.438	2.348	85.4	90.6	94.0	2.845	3.021	3.134
40	2.690	2.539	2.451	109.5	116.0	120.1	2.738	2.900	3.002
50	2.764	2.614	2.526	133.1	140.8	145.7	2.662	2.816	2.914
60	2.822	2.672	2.584	156.5	165.3	170.9	2.610	2.755	2.848
80	2.908	2.757	2.669	202.5	213.6	220.6	2.531	2.670	2.758
100	2.967	2.816	2.728	248.1	261.3	269.8	2.481	2.613	2.698
150	3.058	2.908	2.820	361.0	379.6	391.5	2.407	2.531	2.610
200	3.112	2.960	2.872	473.1	497.2	512.4	2.366	2.486	2.562
	Height=60 feet								
10	2.172	2.021	1.933	33.9	36.4	38.1	3.389	3.642	3.808
20	2.445	2.295	2.206	60.2	64.1	66.7	3.010	3.207	3.336
30	2.597	2.446	2.358	85.0	90.2	93.6	2.834	3.008	3.120
40	2.704	2.553	2.465	108.9	115.3	119.4	2.722	2.882	2.985
50	2.782	2.631	2.543	132.3	139.9	144.7	2.646	2.798	2.894
60	2.842	2.692	2.604	155.4	164.0	169.6	2.590	2.733	2.827
80	2.934	2.783	2.695	200.7	211.6	218.5	2.509	2.645	2.731
100	2.997	2.846	2.758	245.6	258.6	266.8	2.456	2.586	2.668
150	3.098	2.948	2.860	356.3	374.5	386.0	2.375	2.497	2.573
200	3.158	3.008	2.920	466.1	489.4	504.2	2.330	2.447	2.521
	Height=80 feet								
10	2.176	2.025	1.937	33.8	36.4	38.0	3.383	3.635	3.800
20	2.453	2.302	2.214	60.0	63.9	66.5	3.000	3.196	3.324
30	2.611	2.458	2.372	84.6	89.8	93.1	2.829	2.994	3.102
40	2.720	2.570	2.482	108.2	114.6	118.6	2.705	2.865	2.965
50	2.804	2.653	2.565	131.3	138.7	143.5	2.626	2.774	2.870
60	2.870	2.720	2.631	153.9	162.4	167.8	2.565	2.706	2.797
80	2.966	2.816	2.728	198.5	209.1	215.8	2.481	2.614	2.698
100	3.038	2.888	2.800	242.3	254.9	262.9	2.423	2.549	2.629
150	3.154	3.003	2.915	350.1	367.6	378.7	2.334	2.451	2.525
200	3.223	3.072	2.984	456.7	479.1	493.2	2.284	2.396	2.466
	Height=100 feet								
10	2.178	2.028	1.939	33.8	36.3	38.0	3.378	3.629	3.796
20	2.458	2.308	2.220	59.9	63.8	66.3	2.994	3.180	3.316
30	2.619	2.468	2.380	84.3	89.4	92.8	2.810	2.982	3.092
40	2.734	2.583	2.495	107.7	114.0	118.0	2.692	2.850	2.950
50	2.816	2.666	2.578	130.7	138.0	142.8	2.614	2.760	2.855
60	2.884	2.734	2.640	153.1	161.5	166.8	2.552	2.692	2.780
80	2.988	2.837	2.749	197.1	207.5	214.2	2.464	2.594	2.678
100	3.063	2.912	2.824	240.3	252.7	260.1	2.403	2.527	2.601
150	3.190	3.039	2.951	346.1	363.3	374.1	2.307	2.420	2.490
200	3.267	3.116	3.028	450.6	472.3	486.1	2.253	2.362	2.430

TABLE 9.—*Constants for two-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.12 INCH=0.01 FOOT

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=10 feet								
10	1. 932	1. 782	1. 694	38. 1	41. 3	43. 5	3. 810	4. 131	4. 346
20	2. 126	1. 976	1. 888	69. 2	74. 5	78. 0	3. 461	3. 724	3. 898
30	2. 216	2. 065	1. 977	99. 7	106. 9	111. 7	3. 322	3. 563	3. 723
40	2. 266	2. 116	2. 028	129. 9	139. 2	145. 2	3. 248	3. 480	3. 630
50	2. 300	2. 148	2. 060	160. 0	171. 3	178. 6	3. 200	3. 426	3. 572
60	2. 322	2. 172	2. 084	190. 2	203. 4	212. 0	3. 170	3. 390	3. 532
80	2. 352	2. 202	2. 114	250. 3	267. 5	278. 6	3. 129	3. 344	3. 482
100	2. 372	2. 221	2. 133	310. 4	331. 4	345. 1	3. 104	3. 314	3. 451
150	2. 398	2. 248	2. 160	460. 4	491. 2	511. 2	3. 069	3. 275	3. 408
200	2. 410	2. 260	2. 172	610. 7	651. 3	677. 7	3. 054	3. 256	3. 388
	Height=15 feet								
10	1. 967	1. 816	1. 728	37. 4	40. 5	42. 6	3. 742	4. 052	4. 258
20	2. 188	2. 038	1. 950	67. 3	72. 2	75. 5	3. 363	3. 612	3. 774
30	2. 298	2. 148	2. 060	96. 1	102. 8	107. 2	3. 202	3. 427	3. 573
40	2. 364	2. 214	2. 126	124. 5	133. 0	138. 5	3. 112	3. 325	3. 462
50	2. 409	2. 258	2. 170	152. 8	162. 9	169. 5	3. 056	3. 258	3. 390
60	2. 440	2. 290	2. 202	181. 0	192. 9	200. 6	3. 017	3. 215	3. 343
80	2. 482	2. 331	2. 244	237. 2	252. 5	262. 4	2. 965	3. 156	3. 281
100	2. 508	2. 358	2. 270	294. 0	312. 1	324. 2	2. 940	3. 121	3. 242
150	2. 547	2. 396	2. 308	433. 4	460. 7	478. 2	2. 890	3. 071	3. 188
200	2. 566	2. 416	2. 328	573. 6	609. 3	632. 3	2. 868	3. 046	3. 162
	Height=20 feet								
10	1. 984	1. 834	1. 746	37. 1	40. 1	42. 1	3. 709	4. 013	4. 212
20	2. 222	2. 072	1. 984	66. 2	71. 0	74. 2	3. 312	3. 552	3. 710
30	2. 346	2. 195	2. 107	94. 1	100. 6	104. 8	3. 138	3. 353	3. 493
40	2. 422	2. 272	2. 184	121. 6	129. 6	134. 8	3. 040	3. 240	3. 370
50	2. 474	2. 324	2. 236	148. 8	158. 4	164. 6	2. 975	3. 168	3. 292
60	2. 513	2. 362	2. 274	175. 7	186. 9	194. 2	2. 928	3. 115	3. 236
80	2. 564	2. 414	2. 326	229. 6	244. 0	253. 2	2. 870	3. 050	3. 165
100	2. 602	2. 451	2. 363	282. 9	300. 3	311. 5	2. 829	3. 003	3. 115
150	2. 646	2. 496	2. 408	417. 2	442. 4	458. 6	2. 781	2. 969	3. 057
200	2. 672	2. 521	2. 433	550. 7	583. 9	605. 0	2. 754	2. 920	3. 025
	Height=25 feet								
10	1. 997	1. 846	1. 758	36. 9	39. 9	41. 8	3. 686	3. 987	4. 185
20	2. 244	2. 092	2. 005	65. 6	70. 4	73. 4	3. 280	3. 518	3. 671
30	2. 374	2. 224	2. 136	93. 0	99. 3	103. 4	3. 100	3. 310	3. 447
40	2. 459	2. 308	2. 220	119. 7	127. 6	132. 6	2. 992	3. 190	3. 315
50	2. 518	2. 367	2. 280	146. 2	155. 5	161. 4	2. 923	3. 110	3. 228
60	2. 561	2. 410	2. 322	172. 4	183. 2	190. 1	2. 873	3. 053	3. 166
80	2. 621	2. 470	2. 383	224. 6	238. 4	247. 1	2. 808	2. 980	3. 089
100	2. 660	2. 510	2. 422	276. 6	293. 3	303. 9	2. 766	2. 933	3. 039
150	2. 718	2. 567	2. 480	406. 2	430. 1	445. 3	2. 708	2. 867	2. 969
200	2. 748	2. 598	2. 510	535. 6	566. 7	586. 5	2. 678	2. 834	2. 932
	Height=30 feet								
10	2. 004	1. 854	1. 766	36. 7	39. 7	41. 7	3. 673	3. 971	4. 169
20	2. 258	2. 107	2. 019	65. 2	69. 9	72. 9	3. 260	3. 493	3. 646
30	2. 394	2. 244	2. 156	92. 2	98. 4	102. 4	3. 074	3. 280	3. 413
40	2. 484	2. 334	2. 246	118. 5	126. 2	131. 1	2. 962	3. 155	3. 278
50	2. 548	2. 398	2. 310	144. 4	153. 5	159. 3	2. 888	3. 070	3. 186
60	2. 596	2. 446	2. 358	170. 1	180. 6	187. 3	2. 835	3. 010	3. 122
80	2. 662	2. 512	2. 424	221. 2	234. 4	242. 9	2. 765	2. 930	3. 036
100	2. 708	2. 557	2. 469	271. 8	287. 8	298. 1	2. 718	2. 878	2. 981
150	2. 773	2. 622	2. 534	398. 1	421. 0	435. 6	2. 654	2. 807	2. 904
200	2. 808	2. 658	2. 570	524. 1	553. 8	572. 8	2. 620	2. 769	2. 864

Table 9.—*Constants for two-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.12 inch=0.01 foot—Continued

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=40 feet								
10	2.012	1.862	1.774	36.6	39.5	41.5	3.658	3.954	4.150
20	2.274	2.124	2.036	64.7	69.3	72.3	3.236	3.465	3.610
30	2.418	2.267	2.179	91.3	97.4	101.3	3.044	3.247	3.377
40	2.518	2.368	2.280	116.9	124.4	129.2	2.922	3.109	3.229
50	2.589	2.438	2.350	142.1	150.9	156.6	2.842	3.018	3.132
60	2.643	2.492	2.404	167.1	177.2	183.7	2.785	2.953	3.062
80	2.720	2.570	2.482	216.5	229.2	237.3	2.706	2.864	2.966
100	2.772	2.622	2.534	265.5	280.7	290.4	2.655	2.807	2.904
150	2.853	2.702	2.614	387.0	408.5	422.3	2.580	2.723	2.815
200	2.898	2.747	2.660	507.9	535.9	553.4	2.540	2.680	2.767
	Height=50 feet								
10	2.017	1.866	1.778	36.5	39.5	41.4	3.649	3.954	4.138
20	2.288	2.137	2.049	64.4	68.9	71.8	3.218	3.444	3.592
30	2.436	2.286	2.198	90.6	96.6	100.4	3.021	3.220	3.345
40	2.539	2.388	2.300	116.0	123.3	128.0	2.900	3.082	3.200
50	2.614	2.464	2.376	140.8	149.4	154.9	2.816	2.988	3.098
60	2.672	2.522	2.434	165.3	175.1	181.5	2.755	2.918	3.025
80	2.757	2.606	2.518	213.6	225.9	233.8	2.670	2.824	2.922
100	2.816	2.666	2.576	261.3	276.1	286.5	2.613	2.761	2.855
150	2.908	2.758	2.670	379.6	400.4	413.6	2.531	2.669	2.757
200	2.960	2.810	2.722	497.2	523.8	540.8	2.486	2.619	2.704
	Height=60 feet								
10	2.021	1.870	1.782	36.4	39.4	41.3	3.642	3.935	4.129
20	2.294	2.144	2.056	64.2	68.7	71.6	3.208	3.433	3.580
30	2.446	2.296	2.208	90.2	96.2	100.0	3.008	3.206	3.333
40	2.553	2.402	2.314	115.3	122.5	127.3	2.882	3.062	3.182
50	2.631	2.480	2.392	139.9	148.8	153.8	2.798	2.968	3.078
60	2.692	2.542	2.454	164.0	173.8	180.0	2.733	2.897	3.000
80	2.783	2.632	2.544	211.6	223.7	231.4	2.645	2.796	2.892
100	2.846	2.696	2.608	258.6	273.0	282.2	2.586	2.730	2.822
150	2.948	2.798	2.710	374.5	394.6	407.5	2.497	2.631	2.717
200	3.008	2.857	2.769	489.4	515.2	531.6	2.447	2.578	2.658
	Height=80 feet								
10	2.025	1.874	1.786	36.3	39.3	41.2	3.635	3.926	4.120
20	2.302	2.152	2.064	63.9	68.4	71.3	3.196	3.420	3.566
30	2.460	2.308	2.222	89.7	95.7	99.4	2.991	3.189	3.312
40	2.570	2.420	2.332	114.6	121.7	126.3	2.865	3.042	3.158
50	2.654	2.504	2.416	138.7	147.0	152.4	2.774	2.940	3.047
60	2.719	2.568	2.480	162.4	171.9	178.0	2.707	2.865	2.967
80	2.816	2.666	2.578	209.1	220.9	228.4	2.614	2.761	2.855
100	2.888	2.737	2.649	254.9	268.9	277.6	2.549	2.689	2.776
150	3.003	2.852	2.764	367.6	387.0	399.3	2.451	2.580	2.662
200	3.072	2.922	2.834	479.1	503.8	519.4	2.396	2.519	2.597
	Height=100 feet								
10	2.028	1.878	1.790	36.3	39.2	41.1	3.629	3.920	4.113
20	2.308	2.157	2.069	63.8	68.2	71.2	3.190	3.412	3.558
30	2.468	2.318	2.231	89.4	95.3	99.0	2.982	3.175	3.299
40	2.583	2.432	2.344	114.0	121.0	125.6	2.850	3.025	3.140
50	2.666	2.516	2.428	138.0	146.3	151.6	2.760	2.926	3.032
60	2.734	2.384	2.496	161.5	170.9	177.0	2.692	2.848	2.950
80	2.837	2.686	2.598	207.5	219.2	226.6	2.594	2.740	2.832
100	2.912	2.762	2.674	252.7	266.5	275.2	2.527	2.665	2.752
150	3.039	2.888	2.800	363.3	382.2	394.2	2.422	2.548	2.628
200	3.116	2.966	2.878	472.2	496.3	511.5	2.362	2.482	2.558

TABLE 9.—*Constants for two-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.18 INCH=0.015 FOOT

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=10 feet								
10	1. 844	1. 694	1. 606	39. 9	43. 5	45. 8	3. 991	4. 346	4. 584
20	2. 038	1. 888	1. 800	72. 2	78. 0	81. 8	3. 611	3. 898	4. 089
30	2. 128	1. 977	1. 889	103. 8	111. 7	116. 9	3. 460	3. 723	3. 897
40	2. 178	2. 028	1. 940	135. 2	145. 2	151. 8	3. 380	3. 630	3. 795
50	2. 211	2. 060	1. 972	166. 4	178. 6	186. 6	3. 328	3. 572	3. 732
60	2. 234	2. 084	1. 996	197. 7	212. 0	221. 3	3. 295	3. 533	3. 688
80	2. 264	2. 114	2. 206	260. 1	278. 5	290. 7	3. 251	3. 481	3. 634
100	2. 284	2. 133	2. 045	322. 3	345. 1	259. 9	3. 223	3. 451	3. 599
150	2. 309	2. 158	2. 070	478. 1	511. 5	533. 2	3. 187	3. 410	3. 555
200	2. 322	2. 172	2. 084	633. 8	677. 7	706. 3	3. 169	3. 388	3. 532
	Height=15 feet								
10	1. 879	1. 728	1. 640	39. 2	42. 6	44. 9	3. 918	4. 258	4. 486
20	2. 100	1. 950	1. 862	70. 1	75. 5	79. 0	3. 505	3. 744	3. 952
30	2. 210	2. 060	1. 972	99. 9	107. 2	112. 0	3. 330	3. 573	3. 733
40	2. 276	2. 126	2. 038	129. 4	138. 5	144. 5	3. 234	3. 462	3. 612
50	2. 321	2. 170	2. 082	158. 6	169. 6	176. 7	3. 171	3. 391	3. 534
60	2. 352	2. 202	2. 114	187. 8	200. 6	208. 9	3. 130	3. 343	3. 482
80	2. 394	2. 244	2. 156	246. 0	262. 4	273. 1	3. 074	3. 282	3. 414
100	2. 420	2. 270	2. 182	304. 1	324. 2	337. 3	3. 041	3. 241	3. 373
150	2. 459	2. 308	2. 220	449. 0	478. 2	497. 2	2. 993	3. 188	3. 315
200	2. 478	2. 328	2. 240	593. 9	632. 3	657. 1	2. 970	3. 162	3. 286
	Height=20 feet								
10	1. 896	1. 746	1. 658	38. 8	42. 2	44. 4	3. 882	4. 217	4. 440
20	2. 184	1. 984	1. 896	69. 0	74. 2	77. 6	3. 448	3. 710	3. 882
30	2. 258	2. 107	2. 019	97. 8	104. 8	109. 4	3. 260	3. 493	3. 647
40	2. 334	2. 184	2. 096	126. 1	134. 8	140. 5	3. 152	3. 370	3. 512
50	2. 386	2. 236	2. 148	154. 2	164. 6	171. 4	3. 084	3. 292	3. 428
60	2. 425	2. 274	2. 186	182. 1	194. 1	202. 0	3. 035	3. 235	3. 367
80	2. 476	2. 326	2. 238	237. 8	253. 2	263. 2	2. 972	3. 165	3. 290
100	2. 510	2. 359	2. 271	293. 3	312. 0	324. 0	2. 933	3. 120	3. 240
150	2. 559	2. 408	2. 320	431. 4	458. 6	476. 0	2. 876	3. 057	3. 173
200	2. 584	2. 433	2. 345	569. 8	605. 0	627. 7	2. 849	3. 025	3. 138
	Height=25 feet								
10	1. 909	1. 758	1. 670	38. 6	41. 9	44. 1	3. 855	4. 187	4. 406
20	2. 157	2. 006	1. 918	68. 2	73. 4	76. 7	3. 412	3. 669	3. 836
30	2. 286	2. 136	2. 048	96. 6	103. 4	107. 8	3. 219	3. 447	3. 598
40	2. 371	2. 220	2. 132	124. 2	132. 6	138. 1	3. 105	3. 315	3. 452
50	2. 430	2. 279	2. 192	151. 4	161. 5	167. 9	3. 028	3. 230	3. 358
60	2. 473	2. 322	2. 234	178. 6	190. 2	197. 6	2. 977	3. 170	3. 293
80	2. 533	2. 382	2. 295	232. 4	247. 2	256. 6	2. 906	3. 090	3. 208
100	2. 572	2. 422	2. 334	286. 1	304. 0	315. 3	2. 861	3. 040	3. 153
150	2. 630	2. 479	2. 392	419. 8	445. 3	461. 6	2. 799	2. 969	3. 077
200	2. 660	2. 510	2. 422	553. 3	586. 6	607. 8	2. 766	2. 933	3. 039
	Height=30 feet								
10	1. 916	1. 766	1. 678	38. 4	41. 7	43. 9	3. 840	4. 169	4. 388
20	2. 170	2. 019	1. 931	67. 8	72. 9	76. 2	3. 392	3. 646	3. 812
30	2. 306	2. 156	2. 068	95. 7	102. 4	106. 8	3. 191	3. 413	3. 560
40	2. 396	2. 246	2. 158	122. 9	131. 1	136. 5	3. 072	3. 278	3. 412
50	2. 460	2. 310	2. 222	149. 6	159. 3	165. 6	2. 992	3. 186	3. 312
60	2. 508	2. 358	2. 270	176. 1	187. 3	194. 6	2. 935	3. 122	3. 243
80	2. 574	2. 424	2. 336	228. 8	243. 0	252. 1	2. 859	3. 038	3. 151
100	2. 620	2. 460	2. 381	280. 8	298. 1	309. 1	2. 808	2. 981	3. 091
150	2. 685	2. 534	2. 446	411. 2	435. 6	451. 2	2. 741	2. 904	3. 008
200	2. 720	2. 570	2. 482	541. 1	572. 8	593. 1	2. 706	2. 864	2. 966

TABLE 9.—*Constants for two-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.18 INCH=0.015 FOOT—Continued

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=40 feet								
10	1.924	1.774	1.686	38.2	41.5	43.7	3.825	4.150	4.367
20	2.186	2.036	1.948	67.3	72.3	75.6	3.366	3.615	3.778
30	2.330	2.179	2.091	94.8	101.3	105.6	3.159	3.377	3.520
40	2.430	2.280	2.192	121.2	129.2	134.3	3.030	3.229	3.358
50	2.501	2.350	2.262	147.1	156.6	162.6	2.942	3.132	3.253
60	2.555	2.404	2.316	172.8	183.7	190.6	2.880	3.062	3.177
80	2.630	2.480	2.392	223.9	237.5	246.2	2.799	2.969	3.078
100	2.684	2.534	2.446	274.2	290.4	307.8	2.742	2.904	3.008
150	2.765	2.614	2.526	399.3	422.3	437.0	2.662	2.815	2.913
200	2.810	2.659	2.572	523.8	553.6	572.3	2.619	2.768	2.862
	Height=50 feet								
10	1.929	1.778	1.690	38.2	41.4	43.5	3.816	4.138	4.354
20	2.200	2.040	1.961	66.9	71.8	75.1	3.346	3.592	3.753
30	2.348	2.198	2.110	94.2	100.4	104.6	3.134	3.348	3.488
40	2.452	2.302	2.214	120.1	127.9	132.9	3.002	3.198	3.322
50	2.526	2.376	2.288	145.7	154.9	160.9	2.914	3.098	3.218
60	2.584	2.434	2.346	170.9	181.4	188.3	2.848	3.023	3.138
80	2.669	2.518	2.430	220.6	233.8	242.3	2.758	2.923	3.029
100	2.728	2.580	2.494	269.8	285.3	295.1	2.698	2.853	3.951
150	2.820	2.670	2.582	391.5	413.6	427.7	2.610	2.755	2.851
200	2.872	2.722	2.634	512.4	540.8	558.8	2.562	2.704	2.794
	Height=60 feet								
10	1.933	1.782	1.694	38.1	41.3	43.4	3.808	4.129	4.343
20	2.206	2.056	1.968	66.7	71.6	74.8	3.336	3.580	3.740
30	2.358	2.208	2.120	93.6	100.0	104.2	3.121	3.333	3.472
40	2.465	2.314	2.226	119.4	127.2	132.2	2.985	3.180	3.305
50	2.543	2.392	2.304	144.7	153.8	159.7	2.894	3.076	3.194
60	2.604	2.454	2.366	169.6	180.0	186.7	2.827	3.000	3.112
80	2.705	2.554	2.466	217.7	230.5	238.7	2.721	2.881	2.984
100	2.758	2.608	2.520	266.8	282.2	292.1	2.668	2.822	2.921
150	2.860	2.710	2.622	386.0	407.5	421.1	2.573	2.717	2.807
200	2.920	2.769	2.681	504.2	531.6	549.0	2.521	2.658	2.745
	Height=80 feet								
10	1.937	1.786	1.698	38.0	41.2	43.3	3.800	4.120	4.333
20	2.214	2.064	1.976	66.5	71.3	74.5	3.324	3.566	3.724
30	2.370	2.220	2.132	93.2	99.5	103.6	3.105	3.315	3.453
40	2.492	2.342	2.254	118.9	126.6	131.6	2.972	3.165	3.290
50	2.565	2.414	2.326	143.5	152.4	158.2	2.870	3.048	3.164
60	2.631	2.480	2.392	167.8	178.1	184.6	2.797	2.968	3.077
80	2.728	2.578	2.490	215.8	228.4	236.5	2.698	2.855	2.956
100	2.800	2.649	2.561	262.9	277.8	287.4	2.629	2.778	2.874
150	2.915	2.764	2.676	378.7	399.3	412.5	2.525	2.662	2.750
200	2.984	2.834	2.746	493.2	519.4	536.1	2.466	2.597	2.680
	Height=100 feet								
10	1.940	1.790	1.702	37.9	41.1	43.3	3.794	4.113	4.326
20	2.220	2.069	1.981	66.3	71.2	74.3	3.316	3.558	3.716
30	2.380	2.230	2.142	92.8	99.1	103.1	3.092	3.301	3.437
40	2.495	2.344	2.256	118.0	125.6	130.5	2.950	3.140	3.262
50	2.578	2.428	2.340	142.7	151.6	157.3	2.855	3.032	3.146
60	2.646	2.496	2.408	166.9	177.0	183.4	2.782	2.950	3.057
80	2.750	2.600	2.512	214.1	226.5	234.4	2.676	2.831	2.930
100	2.824	2.674	2.586	261.6	275.2	284.6	2.616	2.752	2.846
150	2.951	2.800	2.713	374.1	394.2	407.0	2.497	2.628	2.713
200	3.038	2.888	2.802	484.5	509.7	525.3	2.422	2.548	2.626

TABLE 9.—*Constants for two-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.24 INCH=0.02 FOOT

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=10 feet								
10	1.782	1.631	1.543	41.3	45.1	47.7	4.131	4.513	4.770
20	1.976	1.826	1.738	74.5	80.6	84.7	3.724	4.032	4.236
30	2.065	1.914	1.827	106.9	115.3	120.9	3.563	3.843	4.030
40	2.116	1.965	1.877	139.2	149.8	156.8	3.480	3.745	3.921
50	2.148	1.998	1.910	171.3	184.2	192.7	3.426	3.684	3.854
60	2.172	2.021	1.933	203.4	218.5	228.4	3.390	3.642	3.807
80	2.202	2.051	1.963	267.6	287.1	300.0	3.345	3.589	3.749
100	2.221	2.070	1.982	331.4	355.5	371.3	3.314	3.555	3.713
150	2.248	2.097	2.009	491.2	526.5	549.5	3.275	3.510	3.663
200	2.260	2.110	2.022	651.3	697.8	728.2	3.256	3.489	3.641
	Height=15 feet								
10	1.816	1.666	1.578	40.5	44.2	46.6	4.052	4.418	4.664
20	2.038	1.888	1.800	72.2	78.0	81.8	3.612	3.900	4.090
30	2.148	1.998	1.910	102.8	110.5	115.6	3.427	3.683	3.853
40	2.214	2.063	1.975	133.0	142.7	149.1	3.325	3.568	3.728
50	2.258	2.108	2.020	163.0	174.6	182.2	3.260	3.492	3.644
60	2.290	2.139	2.051	192.9	206.4	215.3	3.215	3.441	3.588
80	2.332	2.181	2.093	252.5	270.0	281.3	3.156	3.375	3.516
100	2.358	2.208	2.120	312.1	333.4	347.3	3.121	3.334	3.473
150	2.396	2.246	2.158	460.7	491.5	511.6	3.071	3.277	3.411
200	2.416	2.266	2.178	609.3	649.8	676.0	3.046	3.249	3.380
	Height=20 feet								
10	1.834	1.683	1.595	40.1	43.7	46.1	4.014	4.373	4.614
20	2.073	1.922	1.834	71.0	76.6	80.3	3.552	3.830	4.014
30	2.195	2.044	1.956	100.6	108.0	112.8	3.353	3.600	3.762
40	2.272	2.121	2.033	129.6	138.8	144.8	3.240	3.470	3.620
50	2.324	2.173	2.085	158.4	169.4	176.5	3.168	3.387	3.530
60	2.362	2.212	2.124	186.9	199.6	207.6	3.115	3.327	3.460
80	2.414	2.263	2.175	244.0	260.2	270.7	3.050	3.252	3.384
100	2.447	2.296	2.208	300.8	320.5	333.3	3.008	3.205	3.333
150	2.496	2.345	2.257	442.4	470.8	489.1	2.949	3.139	3.261
200	2.521	2.370	2.282	583.9	621.0	644.9	2.920	3.105	3.224
	Height=25 feet								
10	1.846	1.696	1.608	39.9	43.4	45.8	3.986	4.341	4.577
20	2.093	1.942	1.854	70.3	75.8	79.4	3.516	3.790	3.968
30	2.224	2.073	1.986	99.3	106.5	111.2	3.309	3.550	3.707
40	2.308	2.158	2.070	127.5	136.5	142.2	3.188	3.412	3.550
50	2.368	2.217	2.129	155.4	166.0	172.8	3.108	3.320	3.456
60	2.410	2.260	2.172	183.2	195.4	203.3	3.053	3.257	3.388
80	2.470	2.320	2.232	238.3	253.8	263.8	2.979	3.173	3.297
100	2.510	2.359	2.272	293.2	312.0	324.0	2.932	3.120	3.240
150	2.568	2.416	2.329	430.0	456.9	474.0	2.867	3.046	3.160
200	2.598	2.447	2.360	566.0	601.6	623.9	2.833	3.008	3.120
	Height=30 feet								
10	1.854	1.703	1.615	39.7	43.2	45.6	3.971	4.322	4.557
20	2.107	1.958	1.868	69.9	75.2	78.8	3.493	3.758	3.939
30	2.244	2.094	2.006	98.4	105.5	110.1	3.280	3.517	3.670
40	2.334	2.183	2.095	126.2	134.9	140.5	3.155	3.372	3.512
50	2.398	2.247	2.159	153.5	163.8	170.4	3.070	3.276	3.409
60	2.466	2.295	2.207	180.6	192.4	200.1	3.010	3.207	3.335
80	2.511	2.361	2.273	234.4	249.4	259.0	2.930	3.118	3.238
100	2.557	2.406	2.318	287.8	305.8	317.4	2.878	3.058	3.174
150	2.622	2.472	2.384	421.0	446.6	463.1	2.807	2.977	3.087
200	2.658	2.506	2.420	553.8	587.4	608.4	2.769	2.937	3.042

TABLE 9.—*Constants for two-wire horizontal antennas*—Continued

DIAMETER OF WIRE=0.24 INCH=0.02 FOOT—Continued

l feet	*U*			Capacity			Capacity per foot		
	D=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3	*D*=1	*D*=2	*D*=3
	Height=40 feet								
10	1.862	1.711	1.623	39.5	43.0	45.4	3.954	4.302	4.535
20	2.124	1.974	1.886	69.3	74.6	78.1	3.465	3.730	4.032
30	2.267	2.116	2.028	99.7	104.3	108.8	3.324	3.477	3.628
40	2.368	2.217	2.129	124.4	132.8	138.3	3.109	3.320	3.458
50	2.438	2.289	2.200	150.9	160.7	167.3	3.018	3.214	3.346
60	2.492	2.342	2.254	177.2	188.6	195.9	2.953	3.143	3.265
80	2.570	2.419	2.331	229.2	243.4	252.6	2.864	3.042	3.158
100	2.622	2.472	2.384	280.7	297.8	308.8	2.807	2.978	3.088
150	2.702	2.552	2.464	408.6	432.6	448.1	2.724	2.884	2.987
200	2.748	2.596	2.510	535.7	566.9	586.6	2.678	2.834	2.933
	Height=50 feet								
10	1.866	1.716	1.628	39.4	42.9	45.2	3.943	4.289	4.521
20	2.137	1.986	1.898	68.9	74.1	77.5	3.443	3.705	3.877
30	2.286	2.136	2.048	96.6	103.4	107.8	3.220	3.447	3.593
40	2.388	2.238	2.150	123.3	131.6	136.9	3.082	3.289	3.422
50	2.464	2.313	2.225	149.4	159.1	165.4	2.988	3.182	3.308
60	2.522	2.371	2.283	175.2	186.3	193.4	2.920	3.105	3.223
80	2.606	2.456	2.368	225.9	239.7	248.6	2.824	2.996	3.108
100	2.666	2.515	2.428	276.1	292.6	303.2	2.761	2.926	3.032
150	2.758	2.607	2.519	400.4	423.5	438.2	2.669	2.827	2.901
200	2.810	2.658	2.572	523.8	553.8	572.4	2.619	2.769	2.862
	Height=60 feet								
10	1.870	1.720	1.632	39.3	42.8	45.1	3.932	4.279	4.510
20	2.144	1.994	1.906	68.7	53.8	77.2	3.433	3.692	3.862
30	2.296	2.146	2.058	96.2	102.9	107.3	3.206	3.430	3.577
40	2.402	2.252	2.164	122.5	130.8	136.0	3.062	3.270	3.400
50	2.480	2.330	2.242	148.4	157.9	164.1	2.968	3.158	3.282
60	2.542	2.391	2.303	173.8	184.7	191.8	2.897	3.078	3.197
80	2.632	2.482	2.394	223.7	237.2	246.0	2.796	2.965	3.074
100	2.697	2.546	2.458	272.9	289.0	299.4	2.729	2.890	2.994
150	2.798	2.607	2.559	394.6	417.1	431.4	2.631	2.781	2.876
200	2.854	2.702	2.618	515.8	544.9	562.2	2.579	2.724	2.811
	Height=80 feet								
10	1.874	1.724	1.636	39.3	42.7	45.0	3.927	4.269	4.499
20	2.152	2.002	1.914	68.4	73.5	76.9	3.420	3.677	3.846
30	2.310	2.158	2.072	95.6	102.3	106.6	3.186	3.410	3.553
40	2.420	2.269	2.181	121.7	129.8	135.0	3.042	3.244	3.375
50	2.504	2.354	2.266	146.9	156.3	162.7	2.938	3.126	3.254
60	2.568	2.418	2.330	171.9	182.6	189.5	2.865	3.043	3.158
80	2.666	2.515	2.427	220.9	234.1	242.6	2.761	2.926	3.032
100	2.736	2.586	2.498	269.0	296.6	293.6	2.660	2.844	2.946
150	2.852	2.702	2.614	387.0	408.6	422.3	2.580	2.724	2.815
200	2.922	2.771	2.684	503.8	531.1	548.5	2.519	2.656	2.742
	Height=100 feet								
10	1.878	1.727	1.639	39.2	42.6	44.9	3.920	4.262	4.491
20	2.156	2.006	1.918	68.3	73.4	76.7	3.414	3.668	3.836
30	2.318	2.168	2.080	95.3	101.9	106.2	3.175	3.397	3.540
40	2.432	2.282	2.194	121.0	129.0	134.2	3.025	3.222	3.355
50	2.516	2.365	2.277	146.3	155.6	161.6	2.926	3.112	3.232
60	2.584	2.433	2.345	170.9	181.5	188.3	2.848	3.025	3.138
80	2.686	2.536	2.448	219.2	232.2	240.5	2.740	2.902	3.006
100	2.762	2.612	2.524	266.5	281.8	291.7	2.665	2.818	2.917
150	2.888	2.738	2.650	382.2	403.2	416.6	2.548	2.688	2.777
200	2.966	2.816	2.730	496.3	522.8	539.3	2.492	2.614	2.696

XII. APPENDIXES

APPENDIX 1.—FORMULA FOR THE CAPACITY OF A SOLID-TOP ANTENNA

The antenna is supposed to consist of a rectangular metal plate of dimensions l by w, and to have a thickness of $2t$. It is placed with its plane at a height h above the ground. The capacity is derived by first assuming a uniform distribution of charge, and from the corresponding potential distribution the equilibrium potential is obtained by Howe's approximation applied to both dimensions. The capacity formula is then

$$C=\frac{16.95\ l_2 w_2}{S-S'} \tag{46}$$

in which the dimensions are supposed to be in feet, and the quantity

$$S=w\sinh^{-1}\frac{l}{\sqrt{w^2+t^2}}+l\sinh^{-1}\frac{w}{\sqrt{l^2+t^2}}+\frac{t^2}{w}\sinh^{-1}\frac{l}{t}\left(\frac{\sqrt{w^2+l^2+t^2}-\sqrt{l^2+t^2}}{\sqrt{w^2+t^2}}\right)$$

$$+\frac{t^2}{l}\sinh^{-1}\frac{w}{t}\left(\frac{\sqrt{w^2+l^2+t^2}-\sqrt{w^2+t^2}}{\sqrt{l^2+t^2}}\right)-2t\tan^{-1}\frac{wl}{t\sqrt{w^2+l^2+t^2}}$$

$$+\frac{t^2}{lw}\left[\sqrt{w^2+l^2+t^2}-\sqrt{l^2+t^2}-\sqrt{w^2+t^2}+t\right]$$

$$-\frac{1}{3lw}\left[\left(w^2+l^2+t^2\right)^{\frac{3}{2}}-\left(l^2+t^2\right)^{\frac{3}{2}}-\left(w^2+t^2\right)^{\frac{3}{2}}+t^3\right]$$

The value of S' is obtained from S by substituting $2h$ for t in the formula for S.

If the width of the antenna is small compared with its length, S and S' may be expanded in series, and we may write

$$C=\frac{0.2416\,l_1}{p_1-p_2}=\frac{7.36\,l_2}{p_1-p_2} \tag{47}$$

in which

$$2.303\ p_1=\text{logn}\ \frac{2l}{w}+\frac{1}{2}+\frac{1}{3}\frac{w}{l}-\frac{1}{24}\frac{w^2}{l^2}+\frac{1}{480}\frac{w^4}{l^4}-\ \cdot\ \cdot\ \cdot\ \cdot$$

$$2.303\ p_2=\sinh^{-1}\frac{l}{\sqrt{w^2+\beta^2}}-\left(\frac{\alpha-\beta}{l}\right)+\frac{17}{24}\frac{w^2}{\alpha^2}\cdot\frac{l}{\alpha}-\frac{1}{8}\frac{w^2}{\alpha^2}\cdot\frac{\alpha}{l}$$

$$+\frac{1}{24}\frac{w^2}{\alpha^2}\cdot\frac{\beta^2}{\alpha l}+\frac{1}{12}\frac{w^2}{\beta^2}\cdot\frac{\beta}{l}-\frac{1}{4}\frac{w^2}{\beta^2}\frac{l}{\alpha}+\frac{2}{3}\frac{w^2}{\beta^2}\cdot\frac{l^3}{\alpha^3}+\ \cdot\ \cdot\ \cdot\ \cdot$$

where

$$\beta=2h,\ \alpha=\sqrt{l^2+\beta^2}=\sqrt{l^2+4h^2}$$

APPENDIX 2.—FORMULAS FOR POTENTIAL COEFFICIENTS OF VARIOUS WIRE COMBINATIONS WITH UNIT CHARGE DENSITY

In each of the following formulas the length of the charged conductor is taken as m, and the potential of the conductor of length l, due to unit linear charge density on conductor m, is denoted by U_{lm}. The corresponding linear potential coefficient U_{ml}, supposing the conductor l to be charged, may be obtained from the general relation below. The potential coefficient $P_{lm}=P_{ml}$, due to unit charge on either conductor, is obtained from the potential coefficient per unit linear charge density by the relation $P_{lm}=P_{ml}=\frac{U_{lm}}{m}=\frac{U_{ml}}{l}$. Since U_{lm} is of zero dimensions in length, any unit of length may be employed, provided it is used consistently throughout. If U_{lm} is to be used in the denominator of the general formulas

$$C=\frac{0.2416\,l_1}{\Sigma\,U_{lm}}=\frac{7.36\,l_2}{\Sigma\,U_{lm}}$$

the value of U_{lm} found by any formula of this section must be divided by 4.605.

Single Wire.—For a single wire of diameter d and length l,

$$U_{ll}=2\left[\sinh^{-1}\frac{2l}{d}-\sqrt{1+\frac{d^2}{4h^2}}+\frac{d}{2l}\right] \tag{48}$$

$$=2\left[\sinh^{-1}\frac{2l}{d}-1\right] \text{ approx.}$$

Two Parallel Wires.—The wires are supposed to be placed as shown in Figure 11.

$$U_{lm}=\frac{\alpha}{l}\sinh^{-1}\frac{\alpha}{D}-\frac{\beta}{l}\sinh^{-1}\frac{\beta}{D}-\frac{\gamma}{l}\sinh^{-1}\frac{\gamma}{D}+\frac{2h}{l}\sinh^{-1}\frac{2h}{D}$$
$$-\frac{\sqrt{\alpha^2+D^2}}{l}+\frac{\sqrt{\beta^2+D^2}}{l}+\frac{\sqrt{\gamma^2+D^2}}{l}-\frac{\sqrt{4h^2+D^2}}{l} \tag{49}$$

$$\alpha=l+m+2h,\ \beta=l+2h,\ \gamma=m+2h$$

As special cases, we have for $D=0$ (wires in the same straight line)

$$U_{lm}=\frac{\alpha}{l}\operatorname{logn}\frac{\alpha}{l}-\frac{\beta}{l}\operatorname{logn}\frac{\beta}{l}-\frac{\gamma}{l}\operatorname{logn}\frac{\gamma}{l}+\frac{2h}{l}\operatorname{logn}\frac{2h}{l} \tag{50}$$

Wires in contact, $2h'=0$, $D\neq 0$.

$$U_{lm}=\operatorname{logn}\frac{l+m}{l}+\frac{m}{l}\operatorname{logn}\frac{l+m}{m} \tag{51}$$

Common perpendicular through the centers of the wires (fig. 12).

$$U_{lm}=\frac{l+m}{l}\sinh^{-1}\frac{l+m}{2D}-\frac{(l-m)}{l}\sinh^{-1}\frac{l-m}{2D}$$
$$-\frac{2}{l}\sqrt{\left(\frac{l+m}{2}\right)^2+D^2}+\frac{2}{l}\sqrt{\left(\frac{l-m}{2}\right)^2+D^2} \tag{52}$$

If, further, the lengths are equal, $l=m=l_1$

$$U_{ll'}=2\left[\sinh^{-1}\frac{l_1}{D}-\sqrt{1+\frac{D^2}{l_1^2}}+\frac{D}{l_1}\right] \tag{53}$$

Common perpendicular through the ends of the wires.—(a) Wires on the same side of the perpendicular:

$$U_{lm}=\sinh^{-1}\frac{l}{D}-\left(\frac{l-m}{l}\right)\sinh^{-1}\frac{l-m}{D}+\frac{m}{l}\sinh^{-1}\frac{m}{D}$$
$$-\sqrt{1+\frac{D^2}{l^2}}-\frac{\sqrt{m^2+D^2}}{l}+\frac{\sqrt{(l-m)^2+D^2}}{l}+\frac{D}{l} \tag{54}$$

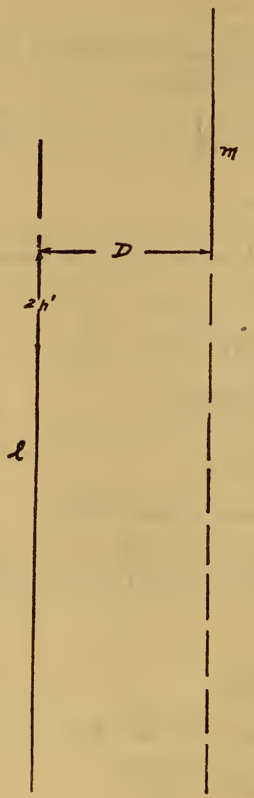

FIG. 11.—*Two parallel wires, general case*

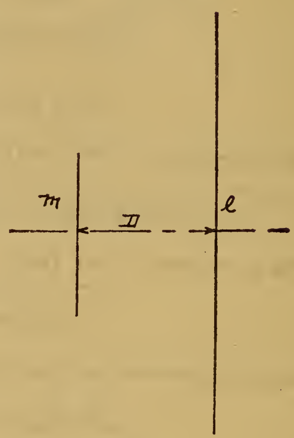

FIG. 12.—*Two parallel wires, symmetrically placed*

(*b*) Wires on opposite sides of the perpendicular:

$$U_{lm}=\frac{l+m}{l}\sinh^{-1}\frac{l+m}{D}-\sinh^{-1}\frac{l}{D}-\frac{m}{l}\sinh^{-1}\frac{m}{D}$$
$$-\frac{\sqrt{(l+m)^2+D^2}}{l}+\frac{\sqrt{l^2+D^2}}{l}+\frac{\sqrt{m^2+D^2}}{l}-\frac{D}{l} \tag{55}$$

WIRES AT RIGHT ANGLES.—Wires in the same plane intersecting at right angles:

$$U_{lm}=\sinh^{-1}\frac{m}{l}+\frac{m}{l}\sinh^{-1}\frac{l}{m} \tag{56}$$

Wires in the same plane with their nearer ends separated by a distance $2h$:

$$U_{lm}=\sinh^{-1}\frac{m+2h}{l}-\sinh^{-1}\frac{2h}{l}+\frac{2h+m}{l}\sinh^{-1}\frac{l}{2h+m}-\frac{2h}{l}\sinh^{-1}\frac{l}{2h} \tag{57}$$

Wires in parallel planes separated by a distance D and with the end of wire m separated by a distance $2h$ from the common perpendicular which runs through the end of the wire l:

$$\begin{aligned}U_{lm}=&\sinh^{-1}\frac{2h+m}{\sqrt{D^2+l^2}}-\sinh^{-1}\frac{2h}{\sqrt{D^2+l^2}}+\frac{2h+m}{l}\sinh^{-1}\frac{l}{\sqrt{D^2+(2h+m)^2}}\\&-\frac{2h}{l}\sinh^{-1}\frac{l}{\sqrt{D^2+4h^2}}-\frac{D}{l}\left(\tan^{-1}\frac{l\,(2h+m)}{D\sqrt{D^2+l^2+(2h+m)^2}}-\tan^{-1}\frac{2hl}{D\sqrt{D^2+l^2+4h^2}}\right)\end{aligned} \tag{58}$$

Wires in parallel planes separated by a distance D, the common perpendicular through the ends of both wires:

$$U_{lm}=\sinh^{-1}\frac{m}{\sqrt{l^2+D^2}}+\frac{m}{l}\sinh^{-1}\frac{l}{\sqrt{m^2+D^2}}-\frac{D}{l}\tan^{-1}\frac{lm}{D\sqrt{l^2+m^2+D^2}} \tag{59}$$

Wires Inclined to One Another.—Wires intersecting at an angle θ:

$$U_{lm}=\sinh^{-1}\frac{m-l\cos\theta}{l\sin\theta}+\frac{m}{l}\sinh^{-1}\frac{l-m\cos\theta}{m\sin\theta}+\frac{l+m}{l}\sinh^{-1}(\cot\theta) \tag{60}$$

Wires in the same plane, but not intersecting, the distances of wires l and m from the intersection of their directions being μ and ν, respectively:

$$\begin{aligned}U_{lm}=&\frac{\mu+l}{l}\left[\sinh^{-1}\frac{(\nu+m)-(\mu+l)\cos\theta}{(\mu+l)\sin\theta}-\sinh^{-1}\frac{\nu-(\mu+l)\cos\theta}{(\mu+l)\sin\theta}\right]\\&-\frac{\mu}{l}\left[\sinh^{-1}\frac{(\nu+m)-\mu\cos\theta}{\mu\sin\theta}-\sinh^{-1}\frac{\nu-\mu\cos\theta}{\mu\sin\theta}\right]\\&+\frac{(\nu+m)}{l}\left[\sinh^{-1}\frac{(\mu+l)-(\nu+m)\cos\theta}{(\nu+m)\sin\theta}-\sinh^{-1}\frac{\mu-(\nu+m)\cos\theta}{\mu\sin\theta}\right]\\&-\frac{\nu}{l}\left[\sinh^{-1}\frac{(\mu+l)-\nu\cos\theta}{\nu\sin\theta}-\sinh^{-1}\frac{\mu-\nu\cos\theta}{\nu\sin\theta}\right]\end{aligned} \tag{61}$$

Wires in parallel planes separated by a distance $2h$. The common perpendicular passes through the ends of the wires, and they are inclined at an angle θ:

$$\begin{aligned}U_{lm}=&2\tanh^{-1}\frac{m}{R_1+R_2}+2\frac{m}{l}\tanh^{-1}\frac{l}{R_1+R_4}\\&-\frac{2h}{l\sin\theta}\left[\tan^{-1}\frac{4h^2\cos\theta+lm\sin^2\theta}{2hR_1\sin\theta}-\tan^{-1}\frac{2h\cot\theta}{R_2}\right.\\&\left.-\tan^{-1}\frac{2h\cot\theta}{R_4}+\tan^{-1}(\cot\theta)\right]\end{aligned} \tag{62}$$

$$R_1^2=4h^2+l^2+m^2-2lm\cos\theta$$

$$R_2^2=4h^2+l^2,\ R_3^2=4h^2,\ R_4^2=4h^2+m^2$$

General case of any two straight wires whatever (G. A. Campbell). In Figure 13 the wires are AB and ab, their common perpendicular Pp, and the various distances are given in the figure.

$$\begin{aligned}U_{lm}=&\frac{2(\mu+l)}{l}\tanh^{-1}\frac{m}{R_1+R_2}-\frac{2\mu}{l}\tanh^{-1}\frac{m}{R_3+R_4}\\&+\frac{2(\nu+m)}{l}\tanh^{-1}\frac{l}{R_1+R_4}-\frac{2\nu}{l}\tanh^{-1}\frac{l}{R_2+R_3}-\frac{d\Omega}{l\sin\epsilon}\end{aligned}$$

where

$$\Omega = \tan^{-1}\left[\frac{d^2 \cos \epsilon + (\mu+l)(\nu+m) \sin^2 \epsilon}{dR_1 \sin \epsilon}\right] + \tan^{-1}\left[\frac{d^2 \cos \epsilon + \mu^2 \sin^2 \epsilon}{dR_3 \sin \epsilon}\right]$$

$$-\tan^{-1}\left[\frac{d^2 \cos \epsilon + (\mu+l)\,\nu \sin \epsilon}{dR_2 \sin \epsilon}\right] - \tan^{-1}\left[\frac{d^2 \cos \epsilon + \mu\,(\nu+m) \sin^2 \epsilon}{dR_4 \sin \epsilon}\right]$$

$$R_1 = \overline{Bb},\ R_2 = \overline{Ba},\ R_3 = \overline{Aa},\ R_4 = \overline{Ab}$$

$$a^2 = R_1^2 - R_2^2 + R_3^2 - R_4^2,\ \delta_{23}^2 = R_2^2 - R_3^2,\ \delta_{43}^2 = R_4^2 - R_3^2 \tag{63}$$

$$d^2 = R_3^2 - \mu^2 - \nu^2 - \frac{\mu\nu}{lm} \cdot a^2,\ \cos \epsilon = -\frac{a^2}{2lm}$$

$$\mu = \frac{2l^2(\delta_{23}^2 - l^2) - a^2(\delta_{43}^2 - m^2)}{4\,l^2 m^2 - a^4} \cdot l$$

$$\nu = \frac{2l^2(\delta_{43}^2 - m^2) - a^2(\delta_{23}^2 - l^2)}{4l^2 m^2 - a^4} \cdot m$$

The quantities l, m_1 R_1, R_2, R_3, and R_4 are obtained by direct measurement and serve to determine all the others.

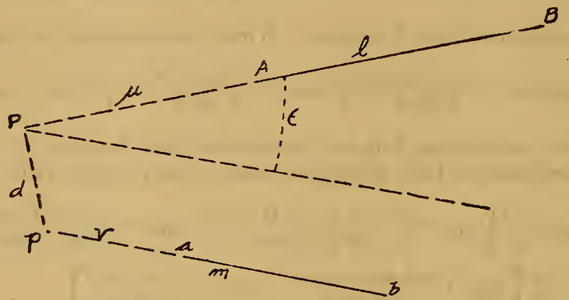

FIG. 13.—*Two inclined wires, general case*

APPENDIX 3.—AN ALTERNATIVE OR MATHEMATICAL STATEMENT OF HOWE'S ASSUMPTION

Dr. C. Snow, of this bureau, has developed the following treatment of the limitations of Howe's method. Formulating the question by means of the Newtonian integral for the electrostatic potential, and by taking the surface average and inverting the order of integration, one may arrive at the following alternative statement of Howe's assumption:

Howe's expression for the potential (and hence the capacity) is valid when, and to the same precision as, the following is valid: The sum of the deviations of linear charge density on the antenna differs from their mean value by a negligible amount.

It is obvious that this would apply very well to horizontal wires not too close together, but might not be true in the case of long, vertical wires whose lower end is close to the ground.

The proof may be outlined as follows: Let $z=0$ be the plane of the earth at zero potential, and let P_1 $(x_1y_1z_1)$ be any point on the surface of the conducting system constituting the antenna, σ $(x_1y_1z_1)$ the surface density of charge there, and dS_1 the surface element there. If P $(x\ y\ z)$ is any point in space above the ground, then the Newtonian integral for the electrostatic potential at P is

$$V(xyz) = \int\int dS_1 \sigma (x_1y_1z_1) \left\{ \frac{1}{\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}} - \frac{1}{\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z+z_1)^2}} \right\} \tag{64}$$

If $P\ (x\ y\ z)$ is any point on the surface of the conductor (antenna), the distribution of charge will be such as to give the integral on the right of (64) the same constant value, say V_1, for all positions of P on this conductor. On the other hand, if σ has the constant value σ_o at all points of this conductor (and P still refers to a point on it) the second member would not be a constant, but would be a function of the position of the point P, say $V_o\ (x\ y\ z)$ where

$$V_o\,(xyz)=\sigma_o\int\int dS_1\left\{\frac{1}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}}-\frac{1}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z+z_1)^2}}\right\} \tag{65}$$

Let S represent the entire area of the conductor (antenna). If we multiply both sides of (64) by dS and integrate over the entire surface of the conductor, the first member of the equation gives V_1S since V has the constant value V_1 over its surface. In the second member, we may interchange the order of integrations, and we obtain

$$V_1=\frac{1}{S}\int\int dS_1\sigma\,(x_1y_1z_1)\int\int dS\left\{\frac{1}{\sqrt{(x_1-x)^2+(y_1-y)^2+(z_1-z)^2}}-\frac{1}{\sqrt{(x_1-x)^2+(y_1-y)^2+(z_1+z)^2}}\right\}=\frac{1}{\sigma_oS}\int\int dS_1\sigma\,(x_1y_1z_1)\,\phi\,(x_1y_1z_1) \tag{66}$$

where

$$\phi\,(x_1y_1z_1)=\sigma_o\int\int dS\left\{\frac{1}{\sqrt{(x_1-x)^2+(y_1-y)^2+(z_1-z)^2}}-\frac{1}{\sqrt{(x_1-x)^2+(y_1-y)^2+(z_1+z)^2}}\right\} \tag{67}$$

A comparison of (67) and (65) shows that $\phi\ (x_1y_1z_1)$ is just $V_o(x_1y_1z_1)$; that is, Howe's trial potential, or the potential at any point on the antenna, assuming that its density is uniform and that it is in the presence of the earth at zero potential. Therefore (66) amounts to

$$V_1=\frac{1}{\sigma_0S}\int\int dS_1\sigma\,(x_1y_1z_1)\ V_0\,(x_1y_1z_1) \tag{68}$$

If into (68) we now introduce the explicit assumption stated above, in the form

$$\sigma\,(x_1y_1z_1)=\sigma_0+\epsilon\,(x_1y_1z_1) \tag{69}$$

where ϵ is a variable measuring the deviation from uniformity,

$$V_1=\frac{1}{S}\left\{\int\int V_0\,(x_1y_1z_1)\ dS_1+\frac{1}{\sigma_0}\int\int\epsilon\,(x_1y_1z_1)\,V_0\,(x_1y_1z_1)\ dS_1\right\} \tag{70}$$

Now, if the second integral is negligible in this expression, it reduces precisely to Howe's value for the potential. The method of passing from surface integrals to line integrals and regarding the conductors as linear is obvious. The second integral measures the error of his computation of potential V_1 $\left(\text{and hence of capacity } C=\frac{Q}{V}\right)$. It is to be noticed that this integral accounts for the peculiar form of the alternative statement of Howe's assumption given above, for his method would still be applicable even if the deviation in density ϵ became infinite at certain places (as ends of wire where it might become logarithmically infinite and still remain integrable). All that is required is that the sum (integral) $\int\int dS_1\ \epsilon\ (x_1y_1z_1)$ be small compared to σ_0.

WASHINGTON, May 9, 1926.