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A SHIELDED RESISTOR FOR VOLTAGE TRANSFORMER TESTING

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ABSTRACT

Voltage transformers are widely used in measuring electric power and energy, and since their accuracy enters directly into the charges for this commodity the need for testing them is apparent. This paper describes the equipment and method used at the Bureau of Standards for testing such transformers over the range from 220 to 30,000 volts. The null method employed is capable of a precision of 0.01 per cent in ratio and 0.3 minute in phase angle at 60 cycles. The 500,000-ohm working resistor connected across the high voltage is subdivided into sections of 20,000 ohms, each of which is inclosed in a metal shield which in turn is connected to a tap at the proper point of an auxiliary guard circuit. The charging currents from these shields to ground, therefore, flow in the guard circuit only and do not directly affect the working circuit. A theoretical discussion of subdivided shielded resistors of this type and also of the uniformly shielded type shows that the error arising from capacitance to ground, though greatly reduced by the use of the guard circuit, is not completely eliminated and increases in importance very rapidly as the working voltage of the resistor is increased.

CONTENTS

I. INTRODUCTION

The steady growth in the size and operating voltage of electric power systems and the constant rise in the standards of accuracy required in electrical measurements have directed more and more attention to the accuracy of the voltage transformers which are used in practically all commercially important measurements of electrical energy. The errors introduced into power measurements by voltage transformers of good design are small, but cases often arise in which they can not be neglected, and the transformer must be tested to determine accurately the ratio of primary to secondary voltage and the phase angle between these vector quantities.

Relative methods of test, in which the transformer under test is compared with a standard transformer of the same nominal ratio, of which the true ratio and phase angle are known, are quite simple and easy to apply.[1] Convenient portable apparatus [2] is on the market for making such tests and the precision with which such comparisons can be made is quite high.

Primary testing laboratories, such as the United States Bureau of Standards, are, however, called upon to test these standard transformers for others to use and must, therefore, apply some absolute method of measurement. The purpose of the present paper is to describe the equipment which has been developed at the Bureau of Standards for this purpose, and which has been in satisfactory operation there for the past seven years. Some space will also be devoted to the general theory of apparatus of this type to show its possibilities and limitations, and to indicate other possible applications as a guide to other laboratory workers.

Since both ratio and phase angle are physically dimensionless being merely ratios of the in-phase and quadrature components, respectively, of a pair of alternating vector quantities, no measurement of any electrical quantity in international units is essential. The test, therefore, readily lends itself to null methods in which the secondary voltage is opposed to a known fraction of the primary voltage. This fraction (of known vector value) can be obtained by connecting two impedances in series across the primary terminals of the transformer. While these impedances might conceivably be resistors, inductors, or condensers, the only type which has thus far been used in precise measurements is that in which resistance predominates and the current through the two impedances is substantially in phase with the primary voltage. The phase angle of the voltage transformer may be compensated for by suitably associating either a condenser or a self or mutual inductor with the low-side impedance.

Somewhat similar conditions are met with in other measurements at high voltages as, for example, in the measurement of the power loss in condensers or cables. If this measurement is made by wattmeter or bridge methods an accurately known impedance is needed to serve either as a series resistor for the wattmeter or as one arm of the bridge, and resistors of the type here described may be useful.

In constructing resistance coils of moderate range (1 to 1,000 ohms) it is customary to wind the wire bifilarly on spools. In coils of low value, 1 to 10 ohms, the magnetic field close to the wires is not entirely eliminated and gives rise to a small but appreciable residual

[1] H. B. Brooks, B. S. Sci. Paper No. 217; P. G. Agnew, B. S., Sci. Paper No. 233.

[2] For example, the "Comparator voltmeter" of the Weston Electrical Instrument Corp., and the "Potential transformer testing set" of the Leeds & Northrup Co.

inductance. There is, however, also a certain electrostatic capacitance between adjacent turns and adjacent layers which tends to neutralize this inductive effect and in 100-ohm coils this capacitance just about balances the effect of the self-inductance and the coil is very closely nonreactive. In coils of 1,000 ohms, and higher the capacitance effects predominate. The resulting phase displacement in 1,000-ohm coils is usually only about 0.1′ at 60 cycles and by suitable construction [3] it can be kept negligible even in 10,000-ohm coils. In voltage transformer testing, however, resistances ranging from 20,000 to 500,000 ohms are involved and capacitance presents a more serious problem. Winding the resistance wire on thin mica cards reduces the capacitance trouble by separating the ends of the wire, which differ most in potential, without introducing any appreciable magnetic field. Such cards, however, have capacitance to their neighbors and to surrounding objects so that in the construction of resistors for high voltage the combination of even nonreactive units may result in a circuit having an appreciable phase angle. Furthermore, any capacitance to other objects in the laboratory will make the phase angle of the circuit change with changes in location or potential of such objects, so that consistent results may be expected only with equipment in which the measuring circuit is almost completely shielded by metal surfaces maintained at definite potentials.[4] Because of these considerations the equipment used at the Bureau of Standards for voltage transformer testing is constructed with its working circuit entirely shielded. It is subdivided into a number of sections and the shield surrounding each section is connected to the proper point of a "guard" circuit which is connected at its ends in parallel with the working circuit. Resistors of this type have been described by Orlich,[5] Kouwenhoven,[6] and Hiecke [7] and seem to be the most desirable type of impedance for the range between 20,000 and 500,000 ohms.

II. DESCRIPTION OF APPARATUS

The working circuit has a total resistance of about 520,000 ohms and is formed of 27 units. Two of these have resistances of 10,000 ohms each, while the others have 20,000 ohms each. The manganin resistance wire is wound on cards of built-up mica 14 by 5.6 cm. The resistance of each card is about 1,250 ohms and each unit (except the first two) contains 16 cards. Each set of cards is hung vertically in its shield on two glass rods which pass through holes in the upper

[3] H. L. Curtis and F. W. Grover, B. S. Sci. Papers Nos. 175 and 177; 1911.
[4] Campbell, G. A., The shielded balance, Elect. Wld., 43, p. 647; 1904.
[5] Orlich, E., and Schultze, H., Archiv. fur Elek., 1, pp. 1, 88, 232; 1912.
[6] Kouwenhoven, W. B., Diss. Elektrotech. Institute Karlsruhe, 3; 1913.
[7] Hiecke, R., Electrotech. u. Masch., 33, p. 505; 1915.

corners of the cards and rest in notches in the side of the shields. The shields are formed by brass boxes 20 by 20 by 13 cm identified by numbers in Figure 1. The boxes are filled with oil which serves to decrease the temperature rise of the working circuit particularly on short overloads. The terminals of the resistor are brought out through fluted hard-rubber bushings to binding posts on the outside of the box.

The guard circuit has the same resistance as the working circuit and is made of cards of the same kind which, however, are not inclosed but are open to the air. The guard cards are hung on glass rods on

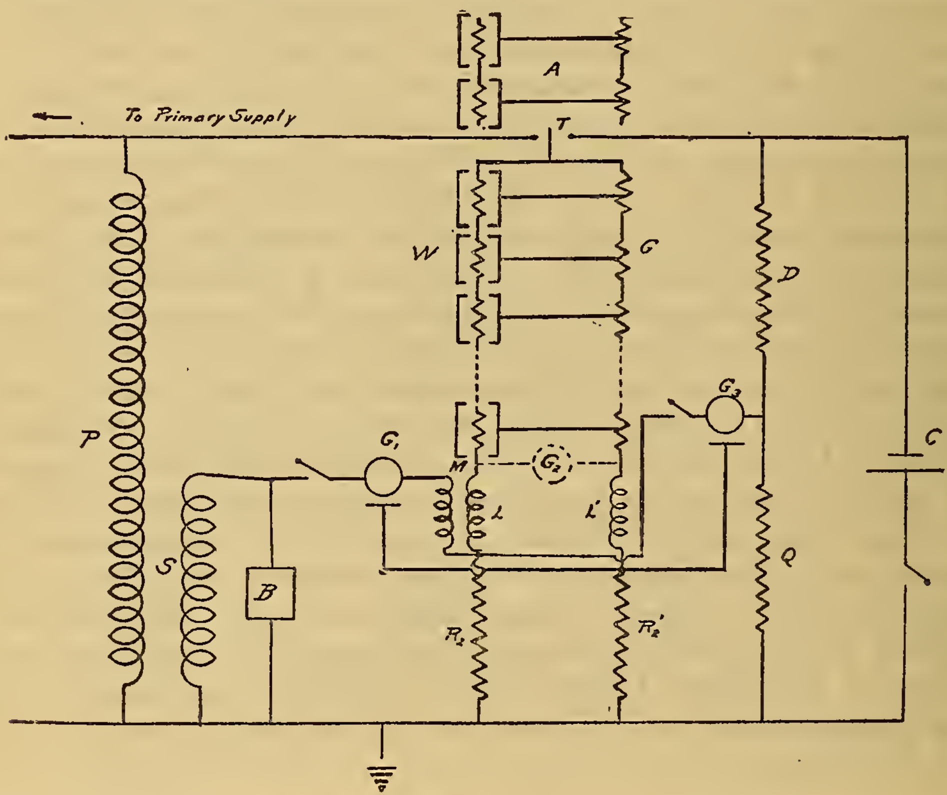

Fig. 2.—*Circuits used in the precise testing of voltage transformers for ratio and phase angle*

racks, each of which corresponds to two adjacent boxes of the working circuit. A tap at the center of each rack has the same potential as the lead joining the two boxes. Two other taps at the one-fourth and three-fourth points have the same potential as the mid-points of the two working circuit boxes and each of these latter taps is connected permanently to the corresponding shield.

The equipment is mounted as shown in Figure 1 on a rack made of wooden bars spaced by porcelain bus-bar supports. Four boxes and the corresponding two guard racks rest on each shelf.

The connections used for voltage transformer testing are shown schematically in Figure 2. Here *W* and *G* are the working and

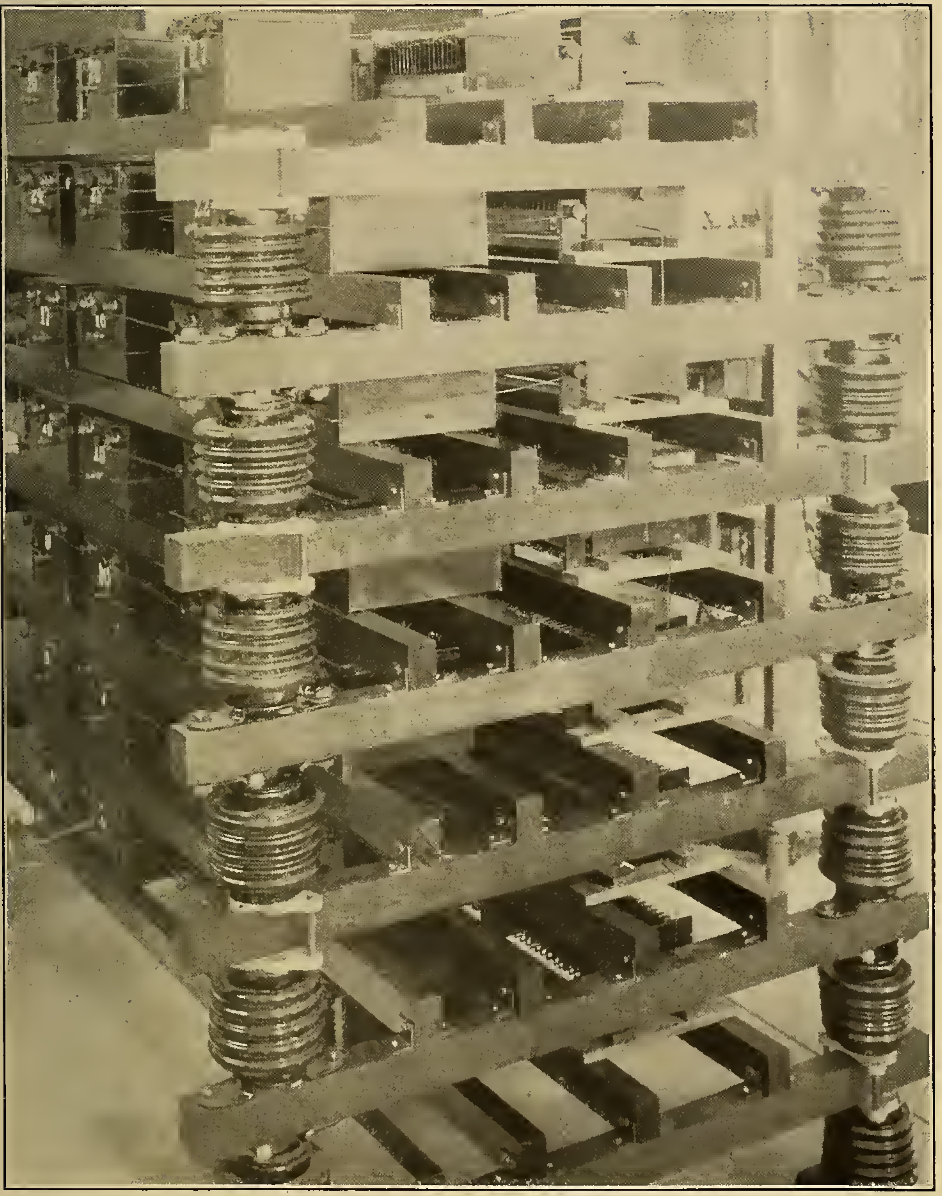

FIG. 1.—*General view of shielded resistor*

guard circuits, respectively. When switch T is thrown to the left they are connected in parallel with the primary P of the transformer under test. At A are shown two idle units which are left in position on the rack, but are disconnected from the circuit when not in use. The voltage at the terminals of the secondary S of the transformer under test is opposed to the drop on the resistor R_2. This consists of a 6-decade resistance box, the 1,000-ohm units of which consist of three separate "Curtis type" coils wound on porcelain spools. In most work the number of units is chosen to give a current of about 0.05 amperes in both the working and guard circuits and R_2 is about 2,000 ohms.

The mutual inductance M, the primary of which is in series with the working circuit, serves to introduce into the circuit of the ac. galvanometer, G_1 a small emf in quadrature with the drop in R_2, thus compensating for the phase angle of the transformer under test. The inductor L' in the guard circuit has the same value as the self-inductance of the primary L of the mutual inductor. All parts of the wiring, galvanometer support, etc., are shielded by lead sheaths or metal plates which are connected to suitable points on the guard circuit.

III. PROCEDURE

The procedure in testing a voltage transformer with this apparatus is to connect between T and ground as many units as are needed to give a current of about 0.05 ampere. Switch T is closed to the left and the high-voltage circuit is energized. The frequency, the secondary voltage, and the burden B are adjusted to the desired values. A preliminary balance is made by adjusting R_2 and M using the ac. galvanometer G_1 as a detector. This galvanometer is then shifted by suitable switches to the position shown at G_2 by the dotted lines and R_2' and L' in the guard circuit are adjusted for a balance. The galvanometer is again connected at G_1 and the final balance of R_2 and M is made. The high-voltage supply is then cut off, switch T is immediately thrown to the right and the dc. galvanometer G_3 is used to balance the Wheatstone bridge formed by the arms W, R_2, Q, and D. Q is a standard resistor of 10, 100, or 1,000 ohms, and D is a precision decade resistor. The I^2R loss in the working circuit at 30,000 volts amounts to about 1.8 kw and the temperature rise is sufficient to cause changes in the resistance of W which somewhat exceed the limit of precision of 0.01 per cent which is aimed at. By means of this dc. bridge measurement, the current for which is supplied by a 20-volt battery C, any appreciable error caused by the heating of W or R_2 by the working current is eliminated. It may be noted that the guard wire functions on the dc. balance to prevent leakage errors.

The constants of the transformer may then be computed from the following formulas:

$$N = \left(1 + \frac{D}{Q}\right) \cos \theta \tag{1}$$

$$\theta = \frac{3438\,\omega}{R_2} \left(M - \frac{L_w}{N}\right) \text{ minutes} \tag{2}$$

where N = true ratio of primary to secondary voltage
θ = phase angle of transformer in minutes (positive when the reversed secondary voltage leads the primary voltage)
L_w = inductance of working circuit (that is, sum of L and effective inductance of shielded section) in henries
$\omega = 2\pi \times$ frequency

IV. EXPERIMENTAL WORK

Before putting the equipment into commission for testing work certain special measurements were made in order (1) to determine the necessary corrections for the effective resistance and inductance of the working circuit on ac. and (2) to verify the theoretical relations (given in the following section) on which the design had been based. As explained below, the ac. performance of such an assemblage of units depends upon both the effective vector impedance z_k of each unit and upon the shunting constant s_k which fixes the interchange of charging current between the guard and working circuits.

The measurements of effective impedance z_k were made with the ac. bridge circuit shown in Figure 3, using a substitution method. The unit under test was inserted at X in series with an adjustable resistor R_4 and an inductor L_4. This combination constituted one arm of a bridge of which the other three arms were the shielded units R_1, R_2, and R_3. The potential of the shield of X was adjusted to any desired value by connecting it to a tap on the auxiliary circuit R_8, R_7, R_6, R_5. Shield potentials out of phase with the bridge voltage could be obtained by inserting an inductance at R_7 or R_6. The bridge was supplied through a transformer with 900-cycle current at about 150 volts, and balanced with the telephone receiver T_1 by adjustment of R_4 and L_4. The mid-point of the transformer was grounded and the impedances Z' were adjusted so as to give a balance when the telephone was connected at T_2. All connecting leads were run in grounded metal sheaths, and an outer grounded shield surrounded the shield of the unit under test.

Measurements were made by inserting in arm 4 first a resistor S of known impedance, and then the resistor X under test. The change in the settings of R_4 and L_4 then gave directly the difference in the ac. resistance and inductance of the two units. The standard available for this work was a 10,000-ohm coil kindly loaned by Dr. H. L. Curtis, who had previously compared it carefully with a 10,000-ohm circuit formed by two long straight parallel manganin wires the inductance of which could be computed from their dimensions. With this as a basis and with $R_3=10{,}000$ ohms, values were obtained on (*a*) 10,000-ohm unit No. 1, (*b*) 10,000-ohm unit No. 2[8], and (*c*) on the combination of two 20,000-ohm units Nos. 11 and 24 in parallel. R_3 was then changed to 20,000 ohms and comparisons again made among (*a*) unit No. 11, (*b*) unit No. 24, and (*c*) units Nos. 1 and 2 in series. This procedure gave two independent values for the

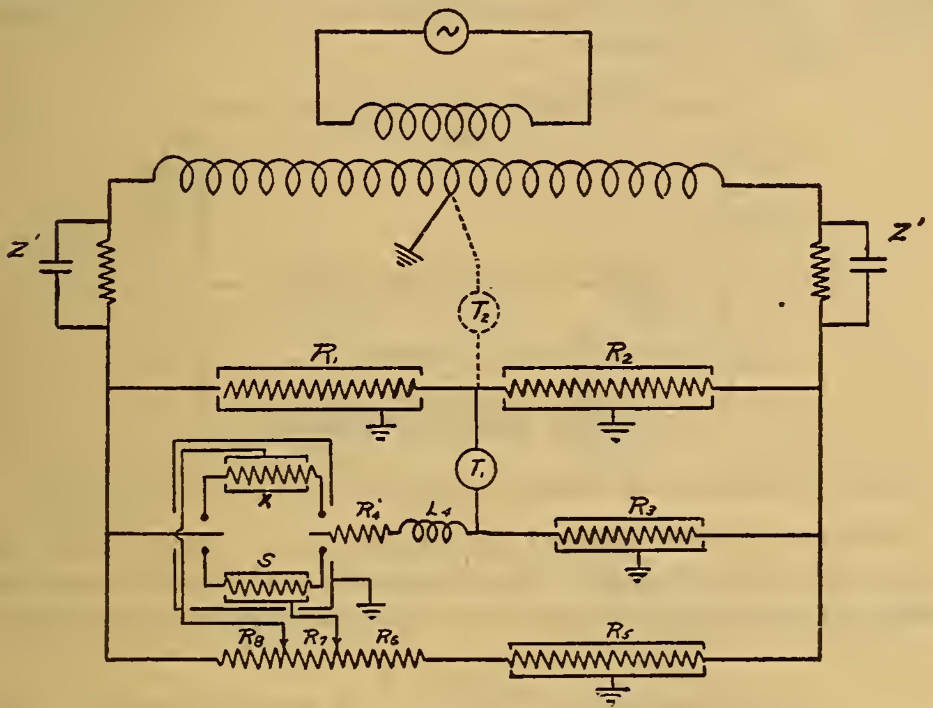

FIG. 3.—*Bridge circuit used in measuring effective impedance of individual units*

effective inductances of the 20,000-ohm units which were found to agree to 0.6 millihenries.

The other 20,000-ohm units were then compared with No. 24 and all were found to have very nearly the same inductance. Measurements were also made of the effect of changes in shield potential p_k on the impedance z_k. These confirmed the approximate relations which are theoretically predicted by equation (42) below.

(1). The effective resistance of a unit is a linear function of the quadrature component of p_k.

(2). The effective inductance of a unit is a linear function of the real component of p_k.

[8] Later Dr. C. Moon kindly compared this unit directly against another parallel-wire standard which he had set up and obtained concordant results.

(3). The slopes of these lines are (for symmetrical constructions) $\frac{\omega r^2 C}{2}$ and $\frac{r^2 C}{2}$, respectively. Where p_k is defined as the excess of the potential of the shield over the potential of the ground end of the unit, expressed in terms of the voltage across the unit.

Measurements on s_k, the factor which gives the charging current to the shield in terms of the current through the resistor, were made with the circuit shown in Figure 4. As before X is the unit under test. For values of $p_k = \frac{1}{2}$, R_1 and R_3 were 20,000 ohms and R_4 and R_5 were 10,000 ohm units, R_2 and R'_2 were small adjustable resistors. Other values of p_k could be obtained by suitable changes in the various resistances.

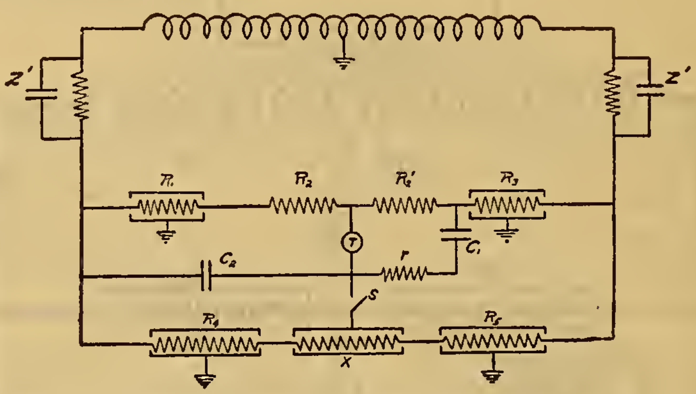

FIG. 4.—*Circuit used in measuring shunting, constant s_k of individual units*

Condenser C_2 was inserted as a ballast and a zero setting was made with switch S open. The change ΔC_1 in C_1 when S was closed gave a measure of s_k as shown by the following approximate equation:

$$s_k = j\omega \Delta C_1 R'_2 \frac{R_4 + R_5 + X}{R_1 + R_2 + R'_2 + R_3} \text{ approximately by} \qquad (3)$$

The results obtained on several units confirmed the approximate theoretical relations (equation (43)) as follows:

(1). The imaginary component of s_k is a linear function of the real component of p_k.

(2). The imaginary component of s_k is equal to $\omega r C\left(\frac{1}{2} - p_k\right)$.

As will be seen from relation (2) s_k becomes zero if $p_k = \frac{1}{2}$ when the unit is symmetrical. This fact makes it quite desirable to operate the circuit with this value of p_k.

The validity of the equations given below connecting the constants of the series combination of units with those of a single unit was also tested by connecting the two 10,000 ohm units in series and directly measuring the impedance of the combination for various values of the shield potentials. The results are given in Table 1.

TABLE 1

p_1	p_2	L_1	L_2	s_1	L_n comp.	L_n obs.
0.0	0.0	+1.0	+1.0	$+13.1\times10^{-4}j$	+4.3	+4.0
.2	.2	+ .3	+ .3	+7.9	+2.0	+2.1
.5	.5	− .3	− .3	0.0	− .6	− .6
.8	.8	− .9	− .9	−7.9	−3.2	−3.5
1.0	1.0	−1.4	−1.4	−13.1	−5.1	−5.3
.0	1.0	+1.0	−1.4	+13.1	+1.7	+1.8

p_1 and p_2 are the respective values of p_k at which units No. 1 and 2, respectively, were operating. L_1 and L_2 are the values of inductance (in millihenrys) and s_1 is the value of charging current ratio found experimentally on unit No. 1 at those particular values of p. The sixth column gives the effective inductance of the combination as computed from equation (49) while the last column gives the effective inductance actually observed.

Further tests were made by deliberately making various unusual adjustments of the guard system and noting the resulting change in the apparent phase angle and ratio of the transformer which was being tested. Since the transformer was actually unaffected by these changes the observed differences are the result of the changes thus introduced into the measuring network. A comparison of the observed differences in the case of a 25,000-volt transformer with those computed by the theoretical equations given below is contained in Table 2.

TABLE 2

Maladjustment	Shift in phase angle		Shift in ratio factor	
	Observed	Computed	Observed	Computed
	Minutes	*Minutes*		
500 ohms added to ground side of guard circuit	1.2	1.05	0.0000_2	0
2,000 ohms added to ground side of guard circuit	4.9	4.2	$.0001_3$	0
1 henry plus 98 ohms added to ground side of guard circuit	+.2	−.2	$.0002_3$	0.00022
Shield No. 12 connected to tap No. 13	−1.2	−.95	$.0000_2$	0
Shield No. 3 connected to tap No. 4	−2.0	−1.5	$.0000_2$	0
$p_k=1$ for all boxes	−6.2	−7.	.00015	0

The agreement between computed and observed values given in the two preceding tables, while closer than 0.8′ and hence satisfactory, does not suffice to prove the validity of the method since only the effects of changes in p_k are there considered. The question of whether or not the guard system in normal adjustment does actually give the intended values of p_k remains unanswered. The resistances of the several sections of the guard wire can be easily adjusted to the nominally correct values, but the capacitance from the shields and from the guard wire itself to ground and to surrounding objects introduces a disturbing influence of indeterminate magnitude, as will be shown in the following section. The error introduced into the working circuit by a given disturbance in the shield potentials is usually much less than the error which the same capacitance would produce in an unshielded resistor. Nevertheless this error increases rapidly with the capacitance and resistance of the circuits, and an experimental check on it is very desirable.

At least four possible ways of getting at this source of error offer themselves. The first of these is deliberately to increase the error by increasing the capacitance involved. This can be done either by connecting additional dummy shields to the tap points of the guard circuit or more simply by placing a metal plate near the guard circuit and connecting it to ground or to the live terminal of the circuit. In the apparatus here described the presence of a plate which must have increased the capacitance manyfold changed the impedance of the working circuit by less than 2 parts in 10,000.

The second test is deliberately to decrease the disturbance in the shield potentials by reinforcing it when possible by direct connection to the step-up transformer. In the present case the transformer secondary had four sections connected in series, and tests (by substitution) showed that these sections were very closely identical. When connections were made from the three intermediate taps to three points spaced equidistantly along the resistance of the guard circuit, the working circuit was affected by less than 0.5′ in phase angle and thirty millionths in resistance.

The third and most direct test for the accuracy of the guard potentials is to measure the difference in potential between the guard and working circuits at various points along their length. This can be done with an accuracy of a few volts and without seriously disturbing the potential distribution by using a small gold-leaf electroscope. In the equipment here described the observed potential difference had a maximum value of 24 volts at a point about two-thirds of the total resistance above ground. The total applied voltage was 24,000 volts in this case. Such a disturbance of the potential distribution could be caused by a uniformly distributed capacitance to ground of about 100 micromicrofarads, and introduces an error of only 80 parts per million in the resistance of the working circuit.

A fourth but inferior basis for estimating guard circuit errors can be obtained by observing the ac. impedance of the guard circuit itself. This is most readily done by comparison with the working circuit, using the ac. galvanometer connected at G_2 in Figure 2. The observed impedance is the result of the effects on the guard resistor of two classes of capacitance (*a*) the capacitance between adjacent shields, and (*b*) the capacitance from shield to ground. The former has very little effect on the potential distribution, but tends to make the current through the guard circuit lead the voltage.

The latter capacitance shunts current off to ground and makes the delivered current lag. The amount of effect (a) can be estimated from measurements of the capacitance between adjacent boxes and the residual phase angle of the guard circuit after this has been allowed for serves to give an estimate of the capacitance to ground.

Table 3 gives a general idea of the precision with which measurements made with this equipment may be duplicated. The data tabulated are the ratio and phase angle at no load, 60 cycles, on three ranges of a transformer which had its primary winding formed by four very closely equal coils, and hence should have shown very nearly identical values for ratio-factor and phase angle on all ranges.

TABLE 3

Primary range of transformer in volts	Number of resistor units used	Ratio factor	Phase angle
			Minutes
25,000	24	1.0001_3	−1.5
25,000	24	1.0002_1	−1.3
25,000	24	1.0002_1	−1.4
12,500	24	1.0001_6	−1.1
12,500	12	1.0001_0	−1.2
6,250	12	1.0000_8	−.8
6,250	6	1.0000_8	−1.0

Some of the observed differences may be the result of capacitance effects in the transformer itself.

V. THEORETICAL RELATIONS

To gain a more definite idea of these capacitance effects mentioned above, let us consider some particular cases as follows.

If an inductive resistance R has a capacitance C across its terminals, the impedance of the circuit thus formed is given by:

$$Z=\frac{R}{1+\omega^2C^2R^2}+\frac{j\omega(L-CR^2)}{1+\omega^2C^2R^2} \tag{4}$$

if $\omega^2 LC$ is small compared to 1. Hence the ac. resistance is given by

$$R_{ac}=R\ (1-\omega^2C^2R^2)\ \text{approximately} \tag{5}$$

and the phase angle is

$$\theta=\tan^{-1}\omega\left(\frac{L}{R}-CR\right) \tag{6}$$

or if L is small

$$\theta=\tan^{-1}(-\omega CR)\quad \text{radians} \tag{7}$$

These relations show that for the case of a single circuit the phase angle is nearly proportional to the frequency, the capacitance and the resistance, and that the fractional error in resistance is of the same order of magnitude as the square of the phase angle (in radians). It is also evident that the current leads the applied voltage and that the current is the same at the two ends of the working circuit.

Let us next consider a nonreactive resistor R which has a condenser C connected between its mid-point and ground (fig. 5). The current in the lower part of the circuit at B now differs from that at A and the impedance of the circuit requires a more specific definition than merely "the vector quotient of applied voltage divided by the resulting current." If this resistor is to be used on the primary side of an equipment for voltage transformer testing, we are primarily interested in the current which it delivers at the grounded end B, and will, therefore, define the impedance in this and similar cases as "the vector quotient of applied voltage divided by the current delivered at the terminal nearest to ground potential." This works out to be approximately

$$Z=\frac{V_n}{I_B}=R\left(1+j\frac{\omega CR}{4}\right) \tag{8}$$

and we see that the delivered current lags behind the applied voltage; that is, the circuit has an apparent positive inductance, although no magnetic field has been considered.

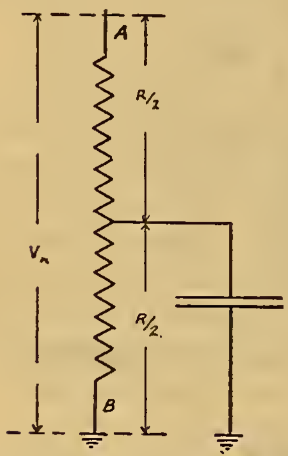

Fig. 5.—*Resistor with capacitance to ground concentrated at center*

If we define K as a quantity such that at any point of the circuit it is equal to the quotient of the resistance from that point to ground divided by the total resistance, then the potential V at any point along the wire is given by

$$V=V_n\ K \tag{9}$$

when the condenser is absent. If we let

$$a^2=j\omega R\ C \tag{10}$$

then, with the condenser in place, we have for points below the tap

$$V=\frac{V_n K}{1+\frac{a^2}{4}} \tag{11}$$

and for points above the tap

$$V=\frac{V_n\left(K\left(1+\frac{a^2}{2}\right)-\frac{a^2}{4}\right)}{1+\frac{a^2}{4}} \tag{12}$$

In practice the stray capacitances are not usually concentrated at certain points, but distributed along the length of the circuit. Therefore, let us next take the case of a circuit of resistance R which has its total capacitance to ground C distributed uniformly along it, so that any section of resistance $R\,dK$ has a capacitance $C\,dK$ to ground and potential $V\,(K)$ above ground.

This system is, of course, a special case of a transmission line. The current passing off to ground from any element is

$$dI = j\,\omega V\,C\,dK \tag{13}$$

and the drop in voltage in the element is

$$dV = I\,R\,dK \tag{14}$$

combining these equations gives

$$\frac{d^2V}{dK^2} - a^2V = 0 \tag{15}$$

as the differential equation for V in terms of K, when $a^2 = j\omega CR$ The solution is

$$V = \frac{V_n \sinh\ aK}{\sinh\ a} \tag{16}$$

The current at the base $(K=0)$ is

$$I_o = \frac{1}{R}\left(\frac{dV}{dK}\right)_o = \frac{aV_n}{R}\left(\frac{\cosh\ aK}{\sinh\ a}\right)_o = \frac{V_n\ a}{R \sinh a} \tag{17}$$

and the apparent impedance is

$$Z_o = \frac{V_n}{I_o} = \frac{R \sinh a}{a} \tag{18}$$

The hyperbolic sine may be expanded in a power series giving

$$\begin{aligned} Z_o &= R(1 + \frac{a^2}{\lfloor 3} + \frac{a^4}{\lfloor 5} - - - -) \\ &= R(1 + \frac{j\omega RC}{6} - \frac{\omega^2R^2C^2}{120} - - -) \end{aligned} \tag{19}$$

It will be noted that this differs from the preceding case which assumed a lumped capacitance (equation (8)) in the numercial coefficient of the $j\omega CR$ term and in the appearance of a term of higher order corresponding to a change in resistance.

Since in almost any practical form of construction an increase in R is accompanied by an increase in C the phase angle of the system will increase nearly in proportion to R^2. Thus for a 10,000-ohm resistor for which $C=20$ micromicrofarads the phase angle is only

0.04′, but for 1 megohm, with $C = 212 \times 10^{-12}$ the angle becomes 46′, which is actually larger than that of many of the voltage transformers which the equipment might be required to test. The direct reduction of these errors by using low values of R requires at high voltages the dissipation of inconveniently large amounts of power. This is unsatisfactory both because of temperature errors and because the large areas required for heat dissipation increase C and tend to offset the gain resulting from the reduction in R.

The alternative remedy is that described above of subdividing the resistor and surrounding each section by a shield which is maintained at a definite potential which approximates that of the resistor section within it.

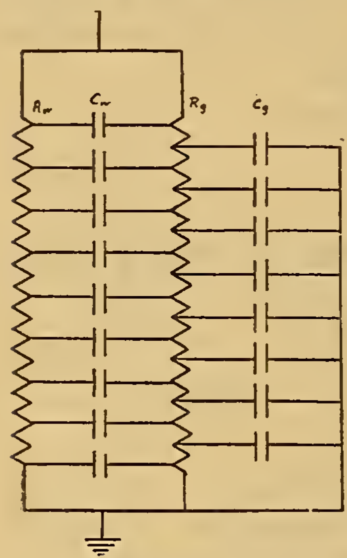

FIG. 6.—*Uniformly shielded resistor with uniformly distributed capacitance to ground*

In what might be termed the ideal form of such a shielded resistor the subdivision may be considered as carried to the limit of having each differential element of the wire shielded by a corresponding element of the guard wire.

The equivalent circuits for such a structure are indicated in Figure 6. Here R_w and R_g are the total resistances of the "working" and "guard" circuits, respectively, and C_g is the total capacitance from the guard circuit to ground (assumed uniformly distributed) C_w is the capacitance (also assumed uniform) between the working circuit and the shields which cover it.

As before we have for any infinitesimal section of the working circuit

$$dI_w = j\omega C_w (V_w - V_g)\, dK \tag{20}$$

$$dV_w = R_w I_w dK \tag{21}$$

and for each element of the guard circuit

$$dI_g = j\omega C_g V_g dK + j\omega C_w (V_g - V_w)\, dK \tag{22}$$

$$dV_g = R_g I_g dK \tag{23}$$

Combining these gives

$$\frac{d^4 V_w}{dK^4} - a'^2 \frac{d^2 V_w}{dK^2} - b^2 V_w = 0 \tag{24}$$

as the differential equation for V_w, the potential of any point of the working circuit. Here

$$a'^2 = j\omega (C_g R_g + C_w R_w + C_w R_g) \tag{25}$$

$$b^2 = \omega^2 C_w C_g R_w R_g \tag{26}$$

The solution of this equation for the case of $V_w = V_g = V_n$ at $K = 1$ and $V_w = V_g = 0$ at $K = 0$ is

$$V_w = \frac{V_n}{(1-2\eta)} \left[(1-\eta) \frac{\sinh m_3 K}{\sinh m_3} - \eta \frac{\sinh m_1 K}{\sinh m_1} \right] \tag{27}$$

η is a constant of the circuit defined by

$$\eta = \frac{1-\sqrt{1+\frac{4b^2}{a'^4}}}{2} \tag{28}$$

and m_1 and m_3 are defined by

$$m^2{}_1 = a'^2\ (1-\eta) \tag{29}$$

$$m^2{}_3 = a'^2\eta \tag{30}$$

The corresponding solution for the potential V_g of the guard wire is

$$V_g = \frac{V_n}{(1-2\eta)}\left[(1-\eta)\ (1-\eta\rho)\ \frac{\sinh m_3 K}{\sinh m_3} - \eta\ (1-\rho\ (1-\eta))\ \frac{\sinh m_1 K}{\sinh m_1}\right] \tag{31}$$

where ρ is defined by

$$\rho = \left(1 + \frac{R_g}{R_w} + \frac{R_g\ C_g}{R_w C_w}\right) \tag{32}$$

and the effective impedance of the working circuit is

$$Z_{ow} = \frac{R_w\ (1-2\eta)}{\frac{(1-\eta)\ m_3}{\sinh m_3} - \frac{m_1\eta}{\sinh m_1}} \tag{33}$$

While this rigorous expression appears rather formidable, an exoan-sion in power series reduces it to

$$Z_{ow} = R_w\left[1 - \frac{7}{360}\omega^2 C_g C_w R_g R_w + \frac{31}{15,120}\ j\omega^3 C_g C_w R_g R_w\ (C_g R_g + C_w R_w + C_w R_g)\right] \tag{34}$$

For small values of C_w (the capacitance which couples the two circuits together) a'^2 becomes approximately $j\omega C_g R_g$ and b^2 and η become very small. The second term (equation (33)) in the denominator drops out and the first term approaches unity so that Z_{ow} approaches R_w as C_w is decreased.

On the other hand if C_w is made large compared with C_g, η again becomes small and m_1 becomes large so that the second term again drops out. $m_3{}^2$, however, approaches the value $j\ \omega\ \frac{C_g R_g R_w}{R_w + R_g}$ so that Z_{ow} approaches the value given by

$$Z_{ow} = R_w\ \frac{\sinh j\ \omega\ C_g R_p}{j\ \omega\ C_g R_p} \tag{35}$$

which is the same result as indicated by equation (18) for a single resistor of resistance R_p, where $R_p = \frac{R_w R_g}{R_w + R_g}$ is the resistance of R_w and R_g connected in parallel.

These results, therefore, indicate that if the coupling between the working and guard circuits is small enough the performance of the working circuit will be perfect even though the capacitance of the shields to ground may seriously distort the potential distribution in the guard. As the coupling is made closer, however, the disturbing capacitance begins to affect the working circuit also and in the limit of very close coupling the working circuit has the same potential distribution as the guard circuit.

To estimate the magnitude of these capacitance effects let us work out a few numerical cases.

For a single circuit, equivalent to that described above, to be used on 25,000 volts, with a working current of 0.05 amperes the heat dissipated would be 1.25 kw. and the resistance 500,000 ohms. These values are reasonable for laboratory work, and such a structure might be assumed to have a total capacitance to ground of 106 micromicrofarads. For this case at 60-cycles

$$a^2 = 0.02j$$

$$\frac{\sinh a}{a} = 1 + 0.0033j - 3.3 \times 10^{-6}$$

hence the phase shift amounts to 0.0033 radians or 11.3 minutes of arc, while the change in effective resistance is only 3 parts per million.

If a second circuit were connected directly in parallel with the first and the two used as a single circuit with no shielding, the resistance might be cut in half without greatly increasing the capacitance and the phase error would be reduced to 5.7′.

If, on the other hand, the first circuit is used to guard the second and only the latter is used as the working circuit the performance is greatly improved. If the capacitance C_w between the guard and working circuits could be kept down to the same value of 106 micromicrofarads, we would have

$$m_1^2 = 0.0524j$$
$$m_3^2 = 0.0076j$$

and $Z_{ow} = R_w \ (1 - 7.7 \times 10^{-6})$.

That is, the phase-angle error would be negligible and the resistance error would be less than 8 parts per million.

It is, however, difficult to keep the value of C_w so small, but if it was equal to 1,060 micromicrofarads, which is about the value in the present equipment; that is, 10 times the capacitance from the guard to ground, the resistance error would still be only 80 parts per million and the phase angle error less than 0.1′.

If the coupling capacitance is made still greater, however, say 10,600 micromicrofarads, the resistance error mounts to 800 parts per million and the phase angle to over 1 minute.

The errors also mount rapidly with increase in the voltage rating and hence in the resistance of the circuits. For 100,000 volts with the same current the resistance needed is 2,000,000 ohms, and for a single circuit, even if the capacitance to ground were only 212 micromicrofarads, the phase angle would be 92′ and the error in resistance 210 parts per million.

With the shielded system of 2,000,000 ohms resistance, for a coupling capacitance 10 times the ground capacitance, the phase angle error would still be 6′ and the resistance error 5,000 millionths. The shielding principle is thus seen to be applicable only within very definite limits, though it may be very effective within these limits.

While the uniformly guarded circuit discussed above lends itself to mathematical handling, the actual physical construction of such a circuit is rather difficult, and it is usually more convenient to construct a system in which the working circuit is divided into a number of units each of which is inclosed in a metal shield. Each shield is connected to a suitable point on the guard circuit, and the guard and working circuit are connected together at each end. When the number of sections is large the performance of such a shielded resistor will, of course, approach closely that of a uniformly guarded system having the same total resistance and capacitances, and the more general conclusions arrived at in the preceding paragraphs are applicable in any case. The inclosure within a single shield of parts of the working circuit which are not all at the same potential does, however, introduce a further complication which needs consideration in order that the errors introduced thereby may be evaluated. It should be noted that these capacitance effects are determinate and can be measured and corrected for, while those arising from capacitance to ground are indeterminate and remain as sources of error.

Each of the units of which the total resistor is built up may be termed a "three-terminal impedance." Two terminals, A and B, Figure 7, connect to the resistor r_k, while the third terminal C connects to the shield which has a certain capacitance to the various parts of r_k. For the *kth* unit from the ground end of the system we may denote the potential of the working circuit terminals with respect to ground by V_k and V_{k-1} (the former being the greater), and the potential of the shield by V_k'. The currents flowing through the terminals may similarly be represented by I_k, I_{k-1}, and I'_k, respectively, and are related by the equation

$$I_k = I_{k-1} + I'_k \tag{36}$$

Following the above notation and the definition already given for similar networks (equation (8)) we may define the impedance z_k of the *kth* unit by

$$z_k = \frac{V_k - V_{k-1}}{I_{k-1}} = \frac{E_k}{I_{k-1}} \tag{37}$$

This single quantity does not, in general, completely describe the function of the unit, since, in addition to delivering to the next unit below a current I_{k-1}, it also delivers to the guard circuit the current I'_k. We may, therefore, introduce the additional vector quantity s_k defined by the relation

$$s_k = \frac{I'_k}{I_{k-1}} \tag{38}$$

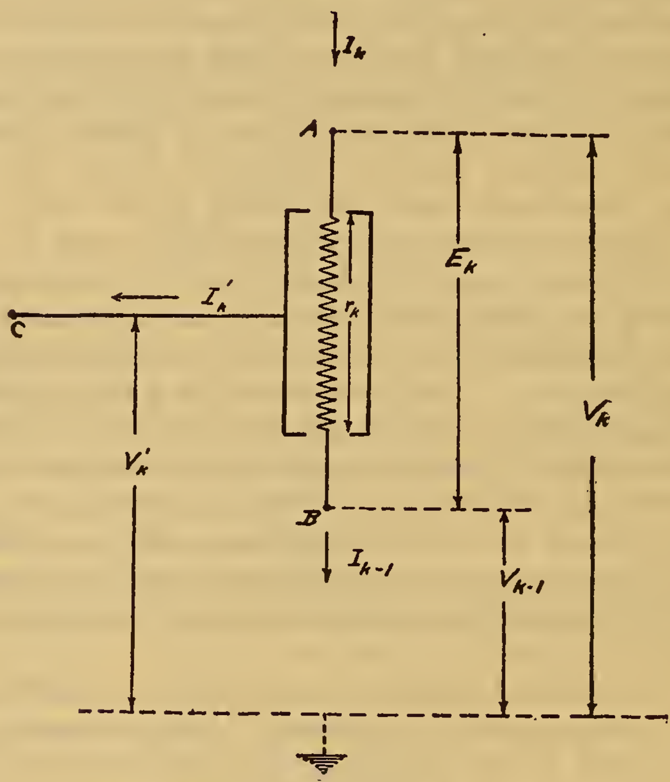

Fig. 7.—*Scheme of notation of potentials and currents in a single unit*

For any given potential distribution the two quantities z_k and s_k will completely describe the electrical behavior of the *kth* unit. Both of these quantities, however, depend upon the relative potential of the shield and its contents, so that we must introduce a new vector quantity p_k defined by the equation

$$p_k = \frac{V'_k - V_{k-1}}{V_k - V_{k-1}} \tag{39}$$

to describe this potential distribution.

z_k has the dimension of an impedance, and is only slightly affected by changes in p_k. s_k is a dimensionless ratio which is roughly proportional to p_k and is nearly a pure imaginary. In an actual apparatus it is possible and desirable to experimentally determine the functional relation between p_k and both z_k and s_k by the methods described above. There are, however, some theoretical relations between these quantities which serve as a valuable guide for the experimental work.

To deduce these we may now, for an individual unit, consider K as a parameter running from 0 at the end of the circuit nearest ground to 1 at the high-potential end of the unit. At any point K of the resistor let there be a small capacitance to the shield given by $C(K)dK$ for each element. The charging current through this capacitance will contribute a change.

$$dz_k = j\omega r^2\ (1-K)\ (K-p)\ C(K)dK \tag{40}$$

to the impedance and a corresponding change

$$ds_k = j\omega r\ (K-p)\ C(K)dK \tag{41}$$

in the constant s_k. The total effective values of z_k and s_k are the combined result of the infinite number of such infinitesimal capacitances into which we may consider the total distributed capacitance between resistor and shield as being divided. Since the total effects we are here considering are small we may assume that the contribution from each element is not appreciably affected by the presence of the other elements. On this basis it can be shown that the constants of the unit are

$$z_k = r_k\ (1 + j\omega r_k\ (A - Bp_k)) \tag{42}$$

$$s_k = j\omega r_k\ (D - Cp_k) \tag{43}$$

when the coefficients A, B, C, and D are defined by the equations

$$\begin{aligned} A &= \int_0^1 K(1-K)\ C(K)dK \\ B &= \int_0^1 (1-K)\ C(K)dK \\ C &= \int_0^1 C(K)\ dK \\ D &= \int_0^1 KC(K)\ dK = C-B \end{aligned} \tag{44}$$

Since $C(K)dK$ is the capacitance of the element at K, C is the total capacitance between the resistor and shield and can be measured by the usual methods. In A, B, and D the various elements of capacitance are given different weights.

For the particular case of a symmetrical distribution of capacitance, it can be shown that

$$B = D = \frac{C}{2}$$

and hence

$$z_k = r_k(1 + j\omega r_k\left(A - \frac{C}{2} p_k\right)) \tag{45}$$

$$s_k = j\omega r_k C\left(\frac{1}{2} - p_k\right) \tag{46}$$

Having thus obtained the equations for the constants of any unit, z_k and s_k, in terms of the potential distribution p_k at which it is operating we may next consider the result of connecting a number (n) of such units in series. The total voltage applied to the group is the vector sum of the voltage across the units; that is,

$$V_n = V_o + E_1 + E_2 \text{----} E_k \text{----} E_n \tag{47}$$

which by the definition of z_k

$$= V_o + z_1 I_o + z_2 I_1 \text{----} z_k I_{k-1} \text{----} z_n I_{n-1}$$

Also by the definition of s_k

$$I_k = I_{k-1} + I'_k = I_{k-1}(1 + s_k) = I_{k-2}(1 + s_k)(1 + s_{k-1})$$

so that

$$\begin{aligned} Z_n = \frac{V_n}{I_o} = \frac{V_o}{I_o} &+ z_1 + z_2(1 + s_1) + z_3(1 + s_1)(1 + s_2) \text{----} \\ &+ z_k(1 + s_1)(1 + s_2) \text{----} (1 + s_{k-1}) + \text{----} z_n(1 + s_1) \text{----} (1 + s_{n-1}) \end{aligned} \tag{48}$$

since in practice s_k is always small compared to unity we may with good approximation neglect higher powers of s_k and obtain

$$\begin{aligned} Z_n = \frac{V_o}{I_o} + z_1 + z_2(1 + s_1) + z_3(1 + s_1 + s_2) + \text{----} + z_k(1 + s_1 + s_2 \text{----} \\ s_{k-1}) + \text{----} z_n(1 + s_1 + s_2 \text{----} s_{n-1}) \end{aligned} \tag{49}$$

If the z_k's and s_k's are all alike this reduces to

$$Z_n = \frac{V_o}{I_o} + z(n + \frac{n(n-1)}{2} s) \tag{50}$$

In terms of the p_k's the total impedance becomes in the general case where all the units have different constants

$$Z_n = \frac{V_o}{I_o} + \sum_{k=1}^{k=n} r_k + j\omega \sum_{k=1}^{k=n} r_k^2 (A_k - B_k p_k) + j\omega \sum_{l=2}^{l=n} r_l \sum_{k=1}^{k=l-1} r_k (D_k - C_k p_k) \tag{51}$$

If the units are alike but have different p_k's

$$Z_n = \frac{V_o}{I_o} + nr\left[1 + j\omega r\left\{A + \frac{n-1}{2}D - \frac{B}{n}\sum_{k=1}^{k=n} p_k - \frac{C}{n}\sum_{k=1}^{k=n-1}(n-k)p_k\right\}\right] \tag{52}$$

and if the p_k's are all equal; that is, $p_k = p$

$$Z_n = \frac{V_o}{I_o} + nr\left[1 + j\omega r\left\{A + \frac{n-1}{2}D - p\left(B + \frac{n-1}{2}C\right)\right\}\right] \tag{53}$$

or if the units are known to be symmetrical

$$Z_n = \frac{V_o}{I_o} + nr\left[1 + j\omega r\left\{A + C\left(\frac{n-1}{4} - \frac{n}{2}p\right)\right\}\right] \tag{54}$$

This equation (53) shows that the whole circuit will be nonreactive if p has the value given by

$$p = \frac{A + \frac{n-1}{2}D}{B + \frac{n-1}{2}C} + j0 \tag{55}$$

The arrangement suggested by Orlich and Kouwenhoven in which $p_1 = p_n = 5/12$ while $p_2 = p_3 \dots = p_{n-1} = 1/2$ also makes the entire circuit nonreactive if the capacitance of each unit is uniformly distributed along it in which case $A = \frac{C}{6}$

The phase angle of the circuit will be constant for all values of n if p_k has the fixed value given by

$$p_k = \frac{D}{C} + j0 \tag{56}$$

which is such as to make $s_k = 0$ so that the current is the same throughout the working circuit. For symmetrical units this gives $p_k = \frac{1}{2} + j0$.

In practice it is usually more desirable to make p_k independent of n than to have the circuit strictly nonreactive. The guard circuit can then be made up permanently with taps corresponding in potential with the mid-points of the working-circuit sections, and any desired number of the sections can be put into service without changing the guard adjustment. For this value of p_k the impedance of a series of symmetrical units is

$$Z_n = \frac{V_o}{I_o} + nr\left(1 + j\omega r\left(A - \frac{C}{4}\right)\right) \tag{57}$$

Since A and C are both real and A never exceeds $\frac{C}{4}$ the system delivers a current which leads the applied voltage by

$$\theta = \tan^{-1}\omega r\left(\frac{C}{4} - A\right) \tag{58}$$

Equation (52) can also be used to estimate the error introduced into the voltage transformer test by any of the various possible maladjustments which might occur. Thus if the impedance R_2' (fig. 2) in the ground side of the guard circuit is in error by a resistance ΔR and a reactance $\omega\Delta L$ the various values of p_k will be changed by an amount

$$\Delta p_k = \frac{\Delta R}{nr'}\left(n-k+\frac{1}{2}\right)+j\frac{\omega\Delta L}{nr'}\left(n-k+\frac{1}{2}\right) \tag{59}$$

where $r' = \frac{R_g}{n}$ is the resistance per section of the guard circuit, and the errors thus introduced into the final measurement are

$$\text{error in ratio factor} = -\omega^2\frac{Cr\Delta L}{r'}\left(\frac{n}{3}-\frac{1}{12n}\right) \tag{60}$$

$$\text{error in phase angle} = \frac{\omega Cr}{r'}\Delta R\left(\frac{n}{3}-\frac{1}{12n}\right)\text{radians} \tag{61}$$

An error of ΔR in locating the tap point on a single unit changes p_k by

$$\Delta p_k = \frac{\Delta R}{r'} \tag{62}$$

and changes the phase angle of the system by

$$\Delta\theta = \pm\frac{\omega rC}{n}\left(n-k+\frac{1}{2}\right)\frac{\Delta R}{r'}\text{ radians} \tag{63}$$

For the equipment of the Bureau of Standards using 25 units, the error in phase angle would amount to 1′ at 60 cycles if the ground side of the guard circuit were out of adjustment by 1,500 ohms or if the shield of the first unit were connected to a tap 13,000 ohms away from the correct point.

If it is assumed that each shield has a capacitance $\frac{C_g}{n}$ to ground, in addition to the capacitance $C=\frac{C_w}{n}$ to the working circuit, and if it is assumed that the potential distribution along the guard wire is so nearly uniform that the charging current from each shield is proportional to the nominal potential of that shield above ground, then it can be deduced that with normal adjustment

$$p_k = \frac{1}{2}-j\frac{\omega C_g r'}{12n}(2n^2+1-2k^2+2k)\left(k-\frac{1}{2}\right) \tag{64}$$

and insertion of this value in equation (52) leads to

$$\frac{\Delta Z_n}{Z_n} = \frac{\omega^2 C_g C_w r' r}{12n^2}\left(\frac{7}{30}n^4+\ldots\right)\text{ approximate} \tag{65}$$

as the relative error in impedance resulting from the presence of the capacitance C_g. It will be noted that when expressed in the same form as equation (34) for the uniformly shielded case, the first term (which is the only one of importance) becomes identical, thus confirming the results of the earlier analysis.

From the foregoing equation it is possible to draw some detailed conclusions on the best way to design resistances of this type. Since the indeterminate error arising from ground capacitance is in the main proportional to the product $\omega^2 C_g\ R_g\ C_w\ R_w$ while the power consumed by the equipment is

$$W = \frac{V_n^2\,(R_g + R_w)}{R_g R_w} \tag{66}$$

it can be shown that for a given voltage and allowable error the power is least when $R_g = R_w$ and all things considered this is probably the best relation for these resistances. It is, of course, also desirable to minimize both C_g and C_w. It can be shown that if a sphere of diameter d_1 is to be surrounded by a spherical shield of diameter d_2 the product of the capacitances between the shield and ground and between the shield and its contents is least if $d_2 = 2d_1$. The actual resistors to be shielded will, of course, not be spherical in form, but this computation indicates the desirability of a fairly wide clearance on all sides between the shield and its contents. The use of oil inside the tanks as in the present construction increases C_w in direct proportion to the dielectric constant of the oil, but the gain from the increased momentary overload capacity thus obtained probably offsets this loss.

Roughly one may set the power dissipated

$$W = \frac{\cdot 2\ V_n^2}{R} = w\,A_w \tag{67}$$

where w = watts dissipated per unit area of cooling surface at rated temperature rise and A_w = area exposed to cooling medium. Also to a very rough approximation

$$C_g = K\,A_g \tag{68}$$

where K = a constant and A_g = area of shield exposed to electrostatic influence from ground. Combining these equations on the assumption that C_w is proportional to C_g, gives for the indeterminate error

$$\omega^2\,C_w\,C_g\,R^2 = K_1 \frac{\omega^2\,V_n^4\,A_g^2}{w^2\,A_w^2} \tag{69}$$

This suggests that the errors increase as the fourth power of the voltage and as the square of the frequency, while they can be reduced by increasing the heat dissipation per unit area and by increasing the ratio of cooling area to area exposed to ground. This last consideration tends to favor the use of a few units of high resistance since the inner faces of adjacent cards may contribute cooling surface without having much capacitance. On the other hand, the use of large units tends to increase the magnitude (though probably not the uncertainty) of the correction $\omega r\left(\frac{C}{4}-A\right)$ for the phase angle of the individual unit. In the case of the equipment here described the unit of 20,000 ohms was chosen as a moderate extrapolation from the 10,000-ohm coils for which the residual inductance was accurately known and with which they could be compared by a simple 2 : 1 ratio. Now that the present equipment affords a standard up to 500,000 ohms, the inductance of single units up to this magnitude could be directly measured.

VI. CONCLUSIONS

From the foregoing it may be concluded that a resistance of 500,000 ohms of the subdivided and shielded type here described can be used on alternating current of commercial frequency up to 30,000 volts with an accuracy approaching 0.01 per cent or 0.3′.

If the units are constructed symmetrically and if the shields are maintained at a potential midway between those of the terminals of the unit, then (*a*) the current is the same throughout the length of the working circuit and the constants s_k are all zero; (*b*) the phase angle of the circuit is independent of the number of units in use.

A single simple measurement of the total capacitance between the resistor and the shield of each unit gives the data needed for computing the effect on the system of any maladjustment of the guard circuit.

An additional measurement of the effective inductance of the units is necessary unless the upper limiting value of $\frac{\omega r^2 C}{4}$ is found to be negligible.

The indeterminate error arising from capacitance from shields to ground is found to increase roughly as the fourth power of the voltage rating so that the possibility of indefinitely extending the use of resistors of this type to higher voltages does not appear at all promising, though the present rating of 30 kv can be considerably exceeded by using larger units, with more effective cooling and with greater temperature rise.

The writer wishes to acknowledge the valuable assistance contributed by many members of the electrical instrument section of the Bureau of Standards during the development of this equipment, and especially to S. Isler, G. T. Morris, J. H. Pim, and W. W. La Rue, by whom most of the experimental and constructional work was done.

VII. NOTATION

A, B, C, D = coefficients dependent on the capacitance distribution in the resistance units.

A_w, A_g = areas effective in cooling, and in adding capacitance, respectively.

a, a' = dimensionless constants which measure the capacitance effects in uniform resistors (see equations (10) and (25)).

$b = \sqrt{\omega^2 C_w C_g R_w R_g}$

C = capacitance.

C_g = total capacitance between guard circuit and ground.

C_w = total capacitance between working circuit and guard circuit.

$C(K)$ = function giving the distribution of capacitance along a unit.

D = resistance of dial arm of dc. bridge.

d_1, d_2 = diameters of spherical condenser.

E_k = voltage across terminals of kth unit.

I = current.

I_k = current entering "live" terminal of kth unit.

I'_k = charging current flowing into guard circuit from shield of kth unit.

$j = \sqrt{-1}$.

K = parameter defining the location of an element along the resistor.

k = integer identifying any section of the resistor.

L = inductance.

L_w = total inductance of working circuit.

M = mutual inductance.

m_1, m_3 = roots of equation $m^4 - a^2m^2 - b^2 = 0$.

N = true ratio of voltage transformer.

n = total number of units in resistor.

p_k = relative potential of shield $\left(p_k = \dfrac{V'_k - V_{k-1}}{V_k - V_{k-1}}\right)$.

Q = resistance standard in dc. bridge.

R = resistance.

R_w, R_g = total resistances of working circuit and guard circuit, respectively.

R_2 = resistance opposing the secondary voltage of the transformer.

r_k = resistance of a single section of working circuit $= \dfrac{R_w}{n}$.

r' = resistance of a single section of guard circuit $= \dfrac{R_g}{n}$.

s_k =shunting coefficient of a unit $s_k = \frac{I'_k}{I_{k-1}}$.

V =potential.

V_w, V_g =potential of any point on working circuit or guard circuit, respectively.

V_n =potential of ungrounded terminal of complete resistor.

V_k =potential of "live" terminal of kth unit.

W =total power dissipated.

w =power dissipated per unit area of cooling surface.

Z =impedance.

Z_o =impedance of a circuit defined by current delivered at $K=0$.

Z_{ow} =impedance of working circuit defined by current delivered at $K=0$.

Z_n =impedance of circuit of n units in series.

z_k =impedance of individual unit.

θ =phase angle of impedance or of transformer.

$\eta = \frac{1}{2} - \frac{1}{2}\sqrt{1+\frac{4b^2}{a'^4}}$. See equation (28).

$\rho = 1 + \frac{R_g}{R_w} + \frac{R_g C_g}{R_w C_w}$. See equation (32).

$\omega = 2\pi \times$ frequency.

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