THEORY AND INTERPRETATION OF EXPERIMENTS
ON THE TRANSMISSION OF SOUND THROUGH PAR-
TION WALLS

By Edgar Buckingham

ABSTRACT

As a preliminary, the principles brought out by W. C. Sabine's work on the reverberation of closed rooms are discussed, and the equations needed for interpreting the observed facts are deduced for further reference.

The theory of reverberation suggests experimental methods for measuring the acoustic transmittance of panels by means of observations in two closed rooms which are in acoustic communication only through the panel, which is set up as a part of the partition wall between the rooms. The necessary equations are developed for interpreting such experiments and expressing the results in absolute units, so as to give values which are characteristic of the panel alone and independent of the peculiarities of the laboratory where the tests are conducted.

The proposed methods are then compared with those employed by Paul E. Sabine and F. R. Watson and it is shown that the quantities measured by the different methods are not physically identical. Hence discrepancies are to be expected among the values obtained, although it seems probable that all the methods would arrange any given set of panels in the same order.

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I. INTRODUCTION

Our scientific knowledge of the acoustic properties of closed rooms is due almost entirely to the work of W. C. Sabine, who was the first to apply trained common sense to the practical problems of architectural acoustics. In the course of 20 years of indefatigable labor, he developed methods of experimentation and the mathematical theory needed for interpreting the observations, beside obtaining the greater part of the quantitative data now available to architects for their guidance in securing acoustically satisfactory results in the design of theatres, lecture rooms, etc., or in correcting the defects of existing structures.

Unfortunately, the papers published up to Sabine's untimely death give only a very incomplete account of his work; and, as related by Professor Lyman in his preface to Sabine's Collected Papers on Acoustics (Harvard University Press, 1922), much that had already been accomplished was apparently left unrecorded or has been lost, so that while the fundamental work on reverberation can be followed in considerable detail, the later experiments on the measurement of sound transmission are barely outlined and the mathematical treatment of them is not given at all.

So far as can be judged from his published papers, Sabine developed his equations step by step, as he needed them for coordinating his experimental results and interpreting them to an audience not composed of professional physicists; and in the form in which he left it, the treatment seems somewhat cumbersome. But having the complete experimental investigation before us, it is possible to present the mathematical aspect of the subject of reverberation in more compact form by deduction from certain simple assumptions, the approximate truth of which may be regarded as having been established by the experiments.

The equations needed for interpreting experiments on the transmission of sound through partition walls depend upon and follow naturally from the theory of reverberation. Sabine doubtless developed this part of the theory, but he did not publish it and, so far as the writer has discovered, no one else has done so. The primary purpose of the present paper is therefore to discuss the principles of certain methods of measuring transmission, and to give the appropriate equations. But since frequent reference to the equations for reverberation is unavoidable, they will be given first, although the excellent paper by E. A. Eckhardt in the Journal of the Franklin Institute for June, 1923, makes it unnecessary to discuss them at length.
II. THE FUNDAMENTAL IDEA OF THE THEORY OF REVERBERATION

In the paper entitled "On the Absorbing Power of Wall Surfaces," Sabine states the following three general propositions regarding the dying out of sound in closed rooms after the source has ceased to emit sound:

(a) The duration of audibility of the residual sound is nearly the same in all parts of an auditorium.

(b) The duration of audibility is nearly independent of the position of the source.

(c) The efficiency of an absorbent in reducing the duration of the residual sound is, under ordinary circumstances, nearly independent of its position.

These three propositions contain the gist of the theory. In connection with the known fact that the loudness of a sound of given pitch and quality increases with the rate at which sound energy reaches the observer's ears, the first two show that, wherever the source may have been, the sound waves are very soon so distributed through the room by successive reflections from the walls—including in this term the floor, ceiling, and all exposed surfaces of furniture, etc.—that the amount of sound energy falling on any small object in the room is nearly the same from all directions and for all locations. The third proposition corroborates this conclusion by reference to an independent receiver—the absorbent. Since the ear can not perceive very rapid variations of intensity, the conclusion of course refers only to time averages over a finite though short interval, and is not valid for each separate instant.

If the sound emitted by the source were instantaneously so effectively diffused and mixed up by reflection that the room at once became uniformly filled with sound energy and that the flow of energy at any point was the same in all directions, the three propositions quoted would be not "nearly" but exactly true. And they can not be "nearly" true, as experiment showed them to be, unless the actual state of affairs, so far as it affects the ear, is an approximation to the ideal just mentioned.

The assumption of complete uniformity and diffuseness is therefore a safe starting point for an approximate theory, no exact theory of such excessively complex phenomena being at all possible of attainment.

III. REMARKS ON THE FUNDAMENTAL ASSUMPTION

Before proceeding to mathematical developments, it is well to consider the physical meaning of the assumption and to inquire how reality differs from the ideal and what circumstances may be expected
to make the approximation better or worse. Echoes and interference due to regular reflection will be discussed later, but at first we shall proceed as if sound were reflected diffusely, as light is from a perfectly matt surface.

Let us suppose, to begin with, that the whole internal surface of the room in question is perfectly nonabsorbent, so that if the source keeps on sounding, the total amount of sound energy in the air of the room continually increases. At any instant the volume density of energy will evidently be, on the whole, greater near the source than farther away, and all directions will not be quite equivalent.

After a certain amount of sound energy has thus been given out, let the emission of the source be cut off. Since the sound waves already started are not weakened by absorption, they continue indefinitely to be reflected back and forth, and become more and more mixed up and diffused. The initial influence of the positions of the source and the point of observation is gradually obliterated by the successive reflections, and the state of affairs approaches the ideal, perfectly uniform and diffuse distribution of energy.

Since all real surfaces absorb sound to some extent, the theoretically infinite number of reflections needed for perfect attainment of the ideal state can not take place. But smooth rigid surfaces reflect sound more completely than even the best mirrors reflect light; and in a room with such walls, a great many reflections do occur before a moderately loud sound dies down so far as to be inaudible, so that the approach to the ideal state is closer than might be expected at first sight. Moreover, the speed of sound is so high that in a room of moderate size a great many reflections occur in a short time, and the approach to the ideal state is rapid.

If a continuous source of sound is started in any real, and therefore absorbent, room, it gradually fills the room with sound of increasing intensity until the increasing absorption by the walls just balances the emission of the source and a steady state is established. The energy within any volume element of the room may then be regarded as the sum of a nearly uniform and diffuse part due to sound waves which were emitted some time ago and have already been reflected a great many times, and a nonuniform component due to waves which have been emitted recently and have suffered only a few reflections or none at all. This latter part will, on the whole, be directed away from the source.

The relative importance of these two parts depends on the absorbing power of the room. If the walls were perfectly absorbent and did not reflect at all, there would be no uniform part; all the sound received at any place in the room would come from the direction of the source, and its intensity would fall off with the inverse square of the distance, just as if there were no walls and the sound were being emitted in the
open air. The higher the reflecting power of the interior surface of the room, the less important is the nonuniform component in relation to the whole, and the more nearly will the ideal state be approached.

If the emission of the source ceases after a steady state has been set up, the nonuniform component, due to recent emission and early reflection, automatically passes over into the other; and as the residual sound dies down, it becomes more and more nearly uniform and perfectly diffuse.

These elementary conclusions have next to be somewhat modified by the consideration of regular reflection. If the interior surface of a room were, in whole or in part, of highly polished silver, a source of light in the room would produce very different degrees of illumination on a white screen placed at various points and turned in various directions. Reflection from the walls would produce bright and dark regions, which would shift about if the source were moved.

Something analogous is often observed with sound; and in large empty rooms with highly reflecting walls, the loudness with which a given source of sound is heard may vary considerably when either the source or the observer changes position. The analogy of light suggests the examination of such cases by means of the conception of sound rays and a law of reflection like that for light rays, and this method may be very useful in some cases; but it is liable to be misleading and should be used with caution.

Completely regular reflection, whether of sound or of light, occurs only from surfaces which are large compared to the wave length, in both linear dimensions and radius of curvature. As the surface is made smaller and smaller, a larger and larger fraction of the incident energy is scattered, or reflected diffusely, until, when the dimensions of the surface are of the same order of magnitude as the wave length, most of the incident energy which is not absorbed is scattered, and very little is sent back according to the law of regular reflection.

Now, the note an octave above middle C, which is somewhere near the average of the pitches that are of most practical interest and importance, has a wave length of about 2 feet. Hence, although notes of this pitch may be reflected quite regularly from a large flat wall or ceiling, small isolated surfaces, such as the back of a chair, a column, or the narrow front of a balcony, will not give much regular reflection but will scatter the sound waves that strike them and tend to make the distribution of sound through the room more uniform. Decorative elements, such as pilasters, cornices, and coffered ceilings, tend to produce a similar scattering and reduce the amount of regular reflection, so that it is usually much less disturbing than might be expected offhand. Nevertheless, the effects of regular reflection are often quite appreciable, so that the loudness with which a constant source is heard and the duration of the residual
sound, are not quite the same in all parts of the room but only "nearly" so.

The foregoing leads directly to the consideration of interference. With a constant source of fixed pitch and quality, regular reflection results in interferences and gives rise to maxima and minima of intensity distributed in a fixed pattern throughout the room. The sound impulses which produce the resultant effect at a particular point in the room will have arrived at that point after different numbers of reflections and some will have been more weakened by absorption than others, because the different parts of the interior surface of the room do not all reflect equally well. Hence, if the emission is stopped and the residual sound is left to die out, the interference pattern changes and shifts about in a very complicated and quite unpredictable manner, and the decay of sound intensity at any one point is not regular but rapidly fluctuating. The same sort of thing happens during the period when the source has been started but has not yet built up the steady state in which absorption just balances emission.

For this reason, a receiving instrument which responded instantaneously to variations of intensity within a very small region would give records so complicated as to be unintelligible. Observations on residual sound have, therefore, to be made by a slower instrument which will average a good many fluctuations without showing their details, or else by the ear which has this same power of averaging by reason of the persistence of sensation.

During the steady state, the difficulty of the fixed interference pattern remains, and observations at a fixed point by an instrument of small dimensions, even if equally sensitive in all directions, can not be relied on to tell anything about the average intensity of the sound throughout any large region. The advantages of ear observations, in which the observer gets an average impression from two points and can readily move his head about so as to get a space average, led Sabine very early in his investigations to abandon the use of artificial receiving instruments and plan the experiments so that direct measurements of intensity were unnecessary and time measurements of the duration of audibility were sufficient. Now that the initial obscurities of the subject have been cleared up, more attention may profitably be devoted to perfecting purely instrumental methods of observation which, if they can be made satisfactory and reliable, will immensely decrease the effort demanded of the experimenter and save a great deal of time.

It is evident from the foregoing discussion that the assumption that the sound energy in a closed room is always uniformly distributed in space and perfectly diffuse as regards direction of propagation, is very far from being true in instantaneous detail. But
Sabine's investigations showed that with sufficient skill and patience it is possible to obtain average results over small regions and short intervals of time, which differ but little from those that would be obtained if the assumption were strictly true; and his theoretical treatment of reverberation while different in appearance is the same in substance as the following deductive treatment which starts from the assumption of perfect uniformity and diffuseness.

IV. NOTATION, DEFINITIONS, AND ASSUMPTIONS

The following notation and nomenclature will be adopted:

- \( C \) = the speed of sound at the temperature of the air in the room, which is assumed to be uniform.
- \( E \) = the power of the source, or the rate at which it gives out energy in the form of sound waves.
- \( V \) = the volume of the room.
- \( S \) = the internal exposed area of the room and its contents.
- \( \alpha \) = the absorption coefficient or absorptivity of any element \( dS \).
  It is defined as the fraction of the energy falling on \( dS \) which is not thrown back into the room but is either dissipated into heat in the substance of the wall or transmitted and given out elsewhere, outside the room.
- \( a \) = the absorbing power or absorptance of the whole room, with its contents, defined by the equation
  \[
  a = \int_0^S \alpha \, dS = \bar{\alpha} S 
  \]

where

- \( \bar{\alpha} \) = the average absorptivity of the interior exposed surface.
- \( \rho \) = the volume density of the sound energy at any time \( t \); by the fundamental assumption discussed above, \( \rho \) is the same everywhere.
- \( \rho_o \) = the steady value of \( \rho \) when the source has already been emitting at the constant rate \( E \) for a long time.
- \( \rho_m \) = the least density that produces any sensation in the ear or the density at the limit of audibility.
- \( r \) = the reverberation time of the room. Following Sabine, it is defined as the time required for the residual sound, remaining after the emission of the source has been cut off, to decrease to one millionth of its initial intensity, or the time required for the energy density to decrease in the ratio \( 10^{-6} \).

- \( b = \frac{Ca}{4V} \); Sabine calls this the "rate of decay of the sound."

- \( \epsilon = \frac{\rho C}{4} \); it is the rate at which sound energy strikes unit area of the walls.

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The absorptivity of a solid surface depends on the pitch of the sound and is usually greater for high than for low notes, although resonance may cause marked exceptions to this general rule. In order, therefore, to give \( \alpha \), \( a \), and \( \overline{a} \) definite meanings, it will be supposed that the source emits a pure musical note of fixed pitch.

It will also be assumed that absorptivity does not depend on the intensity of the incident sound, so that \( \alpha \) is a constant for sounds of a given pitch. It is quite conceivable that this assumption may not be very accurately true, especially for soft, highly absorbent surfaces; but no variation with intensity has been noted and, to the degree of accuracy hitherto attained in the experiments which the theory undertakes to represent, any such effect may safely be neglected.

The air in the room has certain natural frequencies of free vibration, and the same is true of the walls, furniture, etc., though these latter vibrations will usually be strongly damped. The room and the air constitute an imperfectly elastic connected system, capable of being set into very complicated states of vibration by suitably timed sound impulses from the source. Such a system may respond strongly, by resonance, to notes near certain frequencies, and when this happens, the system may react on the source to change its frequency. Moreover, vibrations of other frequencies than that of the exciting impulses may be set up in the system, and energy transferred back and forth among the various modes of vibration. The result is that though the source may give out a pure note, the sound in the room may be a confused mixture of notes of various pitches, which are not simply related and are differently absorbed by the walls.

In theory, such a state of affairs will always exist if the frequency of the source is close to any one of the numerous natural frequencies of the system; but usually there are only a very few notes, within the practically important range of pitch, to which the system responds strongly enough to make these resonance effects seriously disturbing. It will be postulated that the notes to which the theory refers are not within the narrow critical regions of pitch where appreciable resonance occurs; and upon this condition, each element of a compound sound may be regarded as separately subject to the theory, with different values of \( E \) and \( \alpha \) for each component, so that the total result is the summation of the results for the separate elements.
V. THE RATE AT WHICH SOUND ENERGY STRIKES THE WALLS OF THE ROOM

In Figure 1, let $dS$ be an element of area of the wall of a room in which the energy density $\rho$ is uniform and perfectly diffuse; and let $dV$ be a volume element, at the distance $r$ from $dS$, in a direction which makes the angle $\phi$ with the interior normal $n$.

![Diagram](image)

**Fig. 1**

Of the energy $\rho dV$ which is contained within $dV$ at any instant, that portion will ultimately strike $dS$ which is moving in directions included within the solid angle $d\omega = dS \cos \phi/r^2$ subtended by $dS$ at $dV$. This fraction is $d\omega/4\pi$, so that the amount of energy inside $dV$ which will ultimately strike $dS$ is

$$\frac{\rho dV dS \cos \phi}{4\pi r^2}$$

(2)

Sound waves leaving $dV$ toward $dS$ will reach $dS$ within one second, if the distance $r$ to be traversed is not greater than the speed of propagation $C$. Hence the total amount of energy that falls on $dS$ in one second from all possible directions is the sum of the values of the expression (2) for all the volume elements of a hemisphere of radius $C$ described about $dS$ as a center.

Setting

$$dV = 2\pi r^2 \sin \phi \, dr \, d\phi$$

(3)

substituting in (2), and integrating, we have

$$\frac{\rho ds}{2} \int_0^C dr \int_0^{\pi/2} \sin \phi \cos \phi d\phi = \frac{\rho C dS}{4}$$

(4)

and the rate at which energy of sound waves falls on unit area of the walls is

$$\frac{\rho C}{4} = \epsilon$$

(5)
VI. THE GROWTH AND DECAY OF SOUND IN A CLOSED ROOM

The whole amount of sound energy in the room at any instant is $V\rho$, and the rate at which it increases is the difference between the rate of supply from the source and the rate of absorption by the walls. By (1) and (5) the rate of absorption is

$$\int_0^s \alpha dS = \epsilon a = \frac{C}{4} a = b V \rho$$

so that we have the equation

$$V \frac{d\rho}{dt} = E - b V \rho$$

in which $V$ and $b$ are constants for a given room, pitch, and temperature. It is assumed here that the dissipation of sound energy into heat in the air, by viscosity, conduction, and radiation, is negligible, as Sabine showed it to be in practice.

If the emission $E$ is constant, the result of integrating (7) from $t_i$ to $t$ is

$$\log \frac{E - b V \rho_t}{E - b V \rho_i} = b(t - t_i)$$

where $\rho_i$ is the value of $\rho$ at the time $t_i$.

The energy density $t$ seconds after the source has started emitting in the previously quiet room, is found from the general equation (8) by setting $\rho_i = t_i = 0$ and solving for $\rho$; and setting $t = \infty$ in the resulting expression gives the final steady value $\rho_0$. The residual density $t$ seconds after the emission has ceased, is found by setting $E = 0$, $\rho_i = \rho_0$, $t_i = 0$ and solving for $\rho$. The results are as follows:

During the initial period of growth

$$\frac{\rho}{\rho_0} = 1 - e^{-bt}$$

in the steady state

$$\rho_0 = \frac{E}{b V} = \frac{4 E}{\alpha C}$$

during the decay of the residual sound

$$\frac{\rho}{\rho_0} = e^{-bt}$$
Since \( b = aC/4V \), equation (9) shows that the larger the room and the smaller its absorbing power, the slower it is in filling up with sound; and (11) shows a similar slowness in the decay of the residual sound after the emission has been stopped. Equation (10) shows that the steady intensity is proportional to \( E/\rho \) and independent of the volume, except in so far as an increase of volume usually increases the absorbing area.

VII. THE MEASUREMENT OF ABSORPTION

Substituting \( b = aC/4V \) in (11) and solving for \( a \) gives the equation

\[
a = \frac{4V}{tC} \log \frac{\rho_o}{\rho}
\]

(12)

but this suggestion for determining \( a \) from simultaneous values of \( t \) and \( \rho/\rho_o \) is worthless because of the lack of a satisfactory method of measuring \( \rho/\rho_o \) directly. The suggestion contained in equation (10) or

\[
a = \frac{4E}{C\rho_o}
\]

(13)

suffers from the same defect; what is needed is a method which does not require a measurement of \( \rho/\rho_o \) or \( E/\rho_o \), and Sabine's ingenuity provided this method. It presupposes only that the power of the source \( E \) can be varied in a known ratio, and this requirement was met by using, as a source, different combinations of identical organ pipes, placed far enough apart in the room that it could be assumed that they emitted independently and that the total power of any combination was the sum of the powers of the separate pipes used in that combination.

Let \( t_1 \) be the duration of audibility of the residual sound when the initial steady intensity is \( \rho_o = \rho_1 \); and let \( t_2 \) be the corresponding time when \( \rho_o = \rho_2 \), the final intensity in both experiments being that of minimum audibility or \( \rho_m \). Then by (11)

\[
\frac{\rho_m}{\rho_1} = e^{-bt_1} \quad \frac{\rho_m}{\rho_2} = e^{-bt_2}
\]

(14)

whence

\[
\frac{\rho_1}{\rho_2} = e^{b(t_1 - t_2)}
\]

(15)

or

\[
b = \frac{1}{t_1 - t_2} \log \frac{\rho_1}{\rho_2}
\]

(16)

But if, in each experiment, the source had been sounding long enough to establish a steady state, \( \rho_1/\rho_2 = E_1/E_2 \), by equation (10);
and the value of this is known, having been fixed beforehand. Hence, substituting \( b = a C/4 V \) and solving (16) for \( a \), the result is

\[
a = \frac{4 V}{C} \log \frac{E_1}{E_2} \frac{1}{t_1 - t_2}
\]

in which \( C \) is a known constant; \( V \) is obtained simply from geometrical measurements of the room; \( E_1/E_2 \) is known from the arbitrarily chosen ratio of the powers of the two sources; and only the two durations of audibility \( t_1 \) and \( t_2 \) have to be determined by the experiments.

This method of determining the absorbing power of a room dispenses with measurements of power \( E \) or energy density \( p \), and requires only time measurements by the ear and a chronograph. It assumes that the intensity for minimum audibility \( p_m \) is the same in both experiments, and Sabine's experiments showed that it was surprisingly constant for any one observer. An observer whose hearing is more acute hears the residual sound a little longer, but by the same amount in each case, so that \((t_1 - t_2)\) is unaffected and \( a \) is correctly determined without regard to the sensitiveness of the observer's ears, so long as the same observer makes both experiments.

Sabine's first attack on the problem of measuring absorption was by the much simpler and more obvious method of substitution, which can not be used for a whole room but is applicable to objects that can be brought in or carried out. If the substitution of one thing for another does not change the duration of the residual sound from a given fixed source, the two bodies are obviously equivalent as regards absorption, and the absorptivities of their surfaces are inversely proportional to their exposed areas. Comparisons of the absorptivities of different surfaces can thus be effected, and absolute values may be obtained by comparison with known areas of open window, which reflect nothing and so provide a standard of unit absorptivity.¹

The details of this and other methods of measuring absorptivity can be studied in Sabine's papers far better than anywhere else, and there is no need of discussing them here. For the present purpose of outlining the mathematical side of the subject, it suffices to note that the results confirm the assumption that, aside from the rapid fluctuations due to changing interference patterns, the sound energy is distributed nearly uniformly and diffusely through the room during the period of decay to inaudibility.

¹ Dr. Eckhardt has pointed out to me that an assumption is implied here. It is very natural to assume that the amount of sound energy which escapes through an open window, from a room in which the energy density is uniform and diffuses, is proportional to the area of the opening. But when the dimensions of the opening are comparable with the wave length, the opening has to be considered as consisting of edges as well as area. In other words, diffraction may falsify the assumption. This point requires further study.
VIII. MEASUREMENT OF THE POWER OF A SOURCE IN TERMS OF THE MINIMUM AUDIBLE INTENSITY

If \( t_a \) is the duration of audibility of the residual sound which had the initial steady intensity \( \rho_o \) due to the continued emission of a source of strength or power \( E \), equation (11) gives us

\[
o_o = \rho_m \frac{\kappa C}{4} e^{\frac{\kappa C}{4}} t_a
\]

and by (10)

\[
E = \frac{a C \rho_m}{4} \frac{\kappa C}{e^{\frac{\kappa C}{4}}} t_a
\]

Hence, if the volume of the room is known and its absorbing power \( a \) has been determined, a measurement of \( t_a \) permits of computing the value of \( E \) in terms of \( \rho_m \), though the result is very sensitive to errors in \( t_a \). To reduce the effect of a given absolute error in the determination of the time \( t_a \), it is evidently well to use a room of large volume and low absorbing power, since both these properties tend to increase the duration \( t_a \) and so diminish the error in \( E \).

IX. THE REVERBERATION TIME OF A ROOM

This is defined as the duration of audibility of a sound which had initially the standard intensity \( 10^8 \) times the minimum audible intensity. Denoting this time by \( r \) and setting \( \rho_o = 10^8 \rho_m \) in equation (11) gives us

\[
10^8 = e^{br}
\]

whence

\[
r = \frac{6 \log_a 10}{b} = \frac{13.82}{b} \text{ or } \frac{55.3V}{aC}
\]

The commonest acoustic defect of modern, hard-surfaced, lecture rooms and auditoriums is the overlapping and confusion of successive sounds, such as the syllables of a sentence, by too great reverberation. Equation (20) shows that the remedy is to increase the absorbing power of the room, and it permits of computing the reverberation time, in advance of construction, from the known absorptivities of the materials of which the exposed internal surface is to consist.

X. THE TESTING OF SOUND-INSULATING PARTITION WALLS

The obvious procedure for testing the power of a certain kind of wall to transmit sound from the air on one side to that on the other, is to set the wall up as a partition between two rooms, produce a sound in one, and compare the intensity of the sound transmitted to the second room with the intensity of the original sound. This idea leads to the type of installation adopted by W. C. Sabine for the Riverbank Laboratories and described by Paul E. Sabine in The
American Architect for July 30, 1919, in which two closed rooms are in acoustic communication through the wall under investigation but are otherwise as nearly as possible completely insulated and soundproof, both toward each other and toward their surroundings. The same general scheme has been followed in the sound laboratory of the Bureau of Standards, and it is illustrated by Figure 2.

The piece of wall upon which the experiments are to be made is set up as a panel \( P \) in the otherwise soundproof partition which separates the "sound chamber" \( I \) from the "test chamber" \( II \). The source of sound \( O \) is in room \( I \), while room \( II \) receives sound only by transmission from the air in room \( I \) through the panel; and the purpose of the observations is to determine the ratio of the intensities in the two rooms when the source is emitting at a constant rate. This ratio, however, depends on the properties of the rooms as well as those of the panel, so that a certain amount of mathematical theory is needed in interpreting the observations so as to give a result which is characteristic of the panel alone and independent of the peculiarities of the laboratory where it is tested.

**XI. SUPPLEMENTARY DEFINITIONS, ETC.**

When sound waves fall on the panel, a part of their energy is reflected and a part is taken up by the panel and disposed of in several ways. In the first place, if the panel is porous, energy is absorbed and dissipated into heat by the heavily damped waves which run into the air contained in the pores. If there are cracks or holes of appreciable size, sound waves pass through the air in these passages from one room to the other, but we shall assume that the partition is tight, so that this direct leakage of sound through air passages is negligible.

A second part of the energy received by the panel goes to setting up true sound waves; that is, longitudinal elastic vibrations, in the solid material. These are propagated perpendicularly through the panel and affect the air on the farther side; but on account of the great difference of density of the air and the panel, the energy of these true sound waves in the panel must always be extremely small,
and its influence on the transmitting power of the panel must be absolutely negligible under all ordinary circumstances.

The third thing to be considered is bodily motion of the central parts of the panel, which is forced, by the varying air pressure on one side, to bend back and forth and so sets up vibrations of the same frequency in the air on the other side. The panel acts as an imperfectly elastic plate, which is more or less rigidly supported at its edges and is set into forced vibration. Energy is dissipated in the plate and at the imperfectly supported edges, and some is conducted away laterally, but a certain residue remains and is transmitted and given out as sound to the air in the test chamber. It is this transmitted residue of the energy absorbed by the panel with which we are now concerned.

The fraction of the energy falling on the panel which is thus transmitted will be called the transmissivity of the panel and denoted by $\tau$; and the quantity

$$T = \tau S$$

where $S$ is the area of one face of the panel, will be called the transmitting power or transmittance of the panel.

Unless the panel is perfectly inelastic, it will have certain natural frequencies of free vibration and it will respond to sounds of these frequencies and transmit them more freely than sounds of other frequencies. The transmittance thus varies with the pitch of the sound, and to make $T$ and $\tau$ definite it will be supposed, as in the consideration of reverberation, that the source employed emits a single pure note of fixed pitch.

It is quite conceivable that the transmittance of a panel may vary with the intensity of the sound incident upon it, faint sounds being more perfectly transmitted than loud ones of the same pitch, or vice versa; and it seems by no means safe to assume that $\tau$ and $T$ will always be independent of the intensity of the sound; that is, of the amplitude of the motion of the panel. For the present, however, we shall assume that there is no such variation with intensity, and that $\tau$ and $T$ are constant for any one pitch.

The two rooms, with their inclosed masses of air and the panel between them, form an even more complicated vibrating system than a single room. Two different and nearly independent kinds of resonance are possible in this system—resonance of the panel, which would be nearly the same in one laboratory as in another, and resonances of the two rooms, which are peculiar to the laboratory and are not greatly affected by the properties of the panel. It will be stipulated that the pitch of the source used shall not be such as to excite strong resonance in either of the two rooms; but no restriction is placed on the pitch, as regards the free periods of the panel.
The symbols $V_1$, $S_1$, $a_1$, $b_1$, etc., shall have, for room $I$, the meanings explained in section 4; and $V_2$, $S_2$, etc., shall denote the corresponding quantities for room $II$. The steady energy densities, after the source in room $I$ has been emitting at the constant rate $E$ for a long time shall be denoted by $\rho_{01}$ and $\rho_{02}$.

The absorbing powers $a_1$ and $a_2$ and the reverberation times $r_1$ and $r_2$ are to be understood as referring to the properties of the two rooms with the panel in place, $S_1$ and $S_2$ including the area $S$ of one face of the panel.

XII. STATIC DETERMINATION OF TRANSMITTANCE

If the source has been sounding long enough to establish a steady state in both rooms, the time rate at which sound energy strikes the face of the panel in the sound chamber is $S\rho_{01}C/4$ (see equation 5); and the rate at which the panel gives out energy to the air of the test chamber is

$$\frac{7S\rho_{01}C}{4} = \frac{\rho_{02}CT}{4}$$

The rate of absorption by the walls of room $II$ is

$$\frac{1}{4} S_2\rho_{02}Ca_2 - \frac{1}{4} \rho_{02}Ca_2$$

and since transmission and absorption are equal, in the steady state, we have by (22) and (23)

$$\frac{T}{a_2} = \frac{\rho_{02}}{\rho_{01}}$$

Instrumental devices for measuring the ratio of the two steady intensities $\rho_{01}$ and $\rho_{02}$ need not be discussed here; but if such devices can be perfected, equation (24) offers the simplest imaginable means for computing the transmittance of the panel from the absorbing power of the test room, which may be assumed to be already known. Experiments by this method with different absolute values of the intensity $\rho_{01}$ would answer the question whether the transmittance did or did not vary appreciably with the intensity of the sound.

If quantitative receiving instruments can not be employed in such a way as to give satisfactory mean values of $\rho_{02}/\rho_{01}$ which are independent of interference patterns and loud spots due to regular reflection, the instrumental measurement has to be replaced by Sabine's procedure of measuring the duration of audibility of residual sound, and it becomes necessary to consider how the intensity of the sound in the two rooms dies down after the source in room $I$ has ceased to emit.
XIII. THE DECAY OF THE RESIDUAL SOUNDS

If the emission of the source is cut off after a steady state has been established, the rate of decrease of the total energy in either room is equal to the rate of absorption by that room minus the rate at which energy is being received from the other room by transmission through the panel. We therefore have the simultaneous equations

\[ -V_1 \frac{d\rho_1}{dt} = C \rho_1 \, a_1 - \frac{C \rho_2}{4} \, T \]  
(25)

\[ -V_2 \frac{d\rho_2}{dt} = C \rho_2 \, a_2 - \frac{C \rho_1}{4} \, T \]  
(26)

or

\[ \frac{d\rho_1}{dt} = \frac{b_1 T}{a_1} \rho_2 - b_1 \rho_1 \]  
(27)

\[ \frac{d\rho_2}{dt} = \frac{b_2 T}{a_2} \rho_1 - b_2 \rho_2 \]  
(28)

which are to be solved subject to the condition that at \( t=0 \) (see equation 24),

\[ \left( \frac{\rho_2}{\rho_1} \right)_{t=0} = \frac{\rho_{02}}{\rho_{01}} = \frac{T}{a_2} \]  
(29)

In the practical testing of sound-insulating partitions, the transmittance is small and the amount of energy transmitted from the sound chamber to the test chamber and then back again will be too small to have any appreciable effect on the intensity in the sound chamber, or on the duration of audibility of the residual sound in it. This means that the term \( \frac{b_1 T \rho_2}{a_1} \) may be omitted from equation (27) and that, to a very close approximation, we have

\[ \frac{d\rho_1}{dt} = -b_1 \rho_1 \]  
(30)

or, after integrating from \( o \) to \( t \),

\[ \rho_1 = \rho_{01} e^{-b_1 t} \]  
(31)

Substituting this value of \( \rho_1 \) in (28) we have

\[ \frac{d\rho_2}{dt} = m e^{-b_1 t} - b_2 \rho_2 \]  
(32)

where

\[ m = \frac{b_2 T}{a_2} \rho_{01} \]  
(33)
and upon substituting the new variable

\[ y = e^{-bt} \rho_2 \]  

(34)
equation (32) may be thrown into the form

\[ \frac{dy}{y(b_1 - b_2 + my)} = -dt \]  

(35)

Integrating from \( o \) to \( t \), eliminating \( y \) by means of (34), solving for \( \rho_2 \), and setting \( \rho_{02} = \rho_{01} T/a_2 \) (equation 29) gives us

\[ \rho_2 = \left( \rho_{01} \frac{T}{a_2} + \frac{m}{b_1 - b_2} \right) e^{-bt} - \frac{m}{b_1 - b_2} e^{-bt} \]  

(36)

which, with (31), constitutes the solution of equations (27) and (28), on the supposition that the influence of the second room on the first is negligible.

It may be noted here that the \( T \) which appears in (28), (29), (33), and (36) is the transmittance \( T_{12} \) from room \( I \) to room \( II \). The transmittance \( T_{21} \), in the opposite direction disappeared when the term \( (b_1 T \rho_2/a_1) \) was dropped from equation (27), and it has therefore not been assumed that the transmittance is the same in both directions, but only that it is small.

**XIV. THE RELATION OF TRANSMITTANCE TO DURATION OF AUDIBILITY**

Let \( t_1 \) and \( t_2 \) be the durations of audibility of the residual sounds left in rooms \( I \) and \( II \), respectively, after the emission of the source in room \( I \) has ceased, so that we have

\[ \begin{align*}
\rho_1 &= \rho_m \text{ at } t = t_1 \\
\rho_2 &= \rho_m \text{ at } t = t_2
\end{align*} \]  

(37)

Substituting these values in (31) and (36) and equating the results gives us the equation

\[ \rho_{01} e^{-bt_1} = \left( \rho_{01} \frac{T}{a_2} + \frac{m}{b_1 - b_2} \right) e^{-bt_2} - \frac{m}{b_1 - b_2} e^{-bt_2} \]  

(38)

and, after eliminating \( m \) by means of (33) and solving for \( T/a_2 \), we have

\[ \frac{T}{a_2} = \frac{(b_1 - b_2) e^{-bt_1}}{b_1 e^{-bt_2} - b_2 e^{-bt_2}} \]  

(39)
If the two rooms have the same reverberation time, so that \( b_1 = b_2 \), equation (39) becomes indeterminate; but the difficulty may be avoided by returning to (35), which now reduces to the form

\[
\frac{dy}{y^2} = m dt
\]  

(40)

Integrating from 0 to \( t \) and eliminating \( y, y_o, \) and \( m \), we have

\[
\rho_2 = \rho_{01} \frac{T}{a_2} (1 + b_2t) e^{-b_1t}
\]

(41)
in place of (36).

Setting \( b_1 = b_2 = b \), we have by (31) and (41)

\[
\rho_m = \rho_{01} e^{-bt} = \rho_{01} \frac{T}{a_2} (1 + bt) e^{-bt}
\]

(42)

whence

\[
\frac{T}{a_2} = \frac{e^{-b(t_1 - t_2)}}{1 + bt_2}
\]

(43)

which may be compared with the more general equation (39) which is applicable when the reverberation times of the rooms are different.

In the case of a very weak or flexible panel, the transmittance may be so high that the influence of room II on room I is no longer negligible. Equations (27) and (28) in the forms

\[
\frac{d\rho_1}{dt_1} = \frac{b_1 T_{21}}{a_1} \left( \rho_2 - \rho_1 \right)
\]

\[
\frac{d\rho_2}{dt_2} = \frac{b_2 T_{12}}{a_2} \left( \rho_1 - \rho_2 \right)
\]

(44)
must then be solved generally, subject to the condition

\[
\frac{\rho_{02}}{\rho_{01}} = \frac{T_{12}}{a_2}
\]

(45)

and the result which corresponds to equation (38) is

\[
e^{-bt_1} \left( D_1 K_2 e^{D_1 t_1} - D_2 K_2 e^{D_2 t_1} \right) = e^{-bt_2} \left( K_1 e^{D_1 t_2} - K_2 e^{D_2 t_2} \right)
\]

(46)

where

\[
B = \frac{b_2 T_{12}}{a_2} = \frac{CT_{12}}{4 V_2}
\]

(47)

\[
D_1, D_2 = \frac{b_1 - b_2}{2B} \pm \sqrt{\left( \frac{b_1 - b_2}{2B} \right)^2 + \frac{V_2 T_{21}}{V_1 T_{12}}}
\]

(48)

\[
K_1 = \rho_{01} \left( 1 + D_1 \frac{T_{12}}{a_2} \right)
\]

(49)

\[
K_2 = \rho_{01} \left( 1 + D_2 \frac{T_{12}}{a_2} \right)
\]
When the substitutions indicated by equations (47) to (49) are made in (46), the resulting equation is so complicated that it appears impossible to obtain a solution corresponding to (39) or (43). While it would be interesting to have this solution for the case of high transmittance, the matter is fortunately of no great importance, because in the practical testing of sound-insulating partitions, the solutions (39) and (43) are sufficiently approximate.

XV. THE DETERMINATION OF TRANSMITTANCE FROM MEASUREMENTS OF DURATION OF AUDIBILITY

The values of the quantities

\[ b_1 = \frac{C_{a_1}}{4V_1} = \frac{13.82}{r_1} \]
\[ b_2 = \frac{C_{a_2}}{4V_2} = \frac{13.82}{r_2} \]  

may be computed from the dimensions of the rooms, if the absorptivities of their surfaces are known; or they may be found from measurement of the reverberation times \( r_1 \) and \( r_2 \) by using a source of known power (see equations 10 and 16). Assuming the values of \( b_1 \) and \( b_2 \) to be known, equations (39) and (43) are available for computing the value of \( T/a_2 \) from the observed values of \( t_1 \) and \( t_2 \).

1. ROOMS WITH EQUAL REVERBERATION TIMES

If the reverberation times are equal, the results may be reduced by means of (43) which is much the simpler of the two, and which may also be written in the form

\[ T = \frac{e^{-13.82\frac{t_1}{r}}}{1 + 13.82 \frac{t_2}{r}} \]  

The condition is satisfied if the two rooms are identical and the absorptivity of the panel is the same on both sides; but even if the rooms are somewhat different, the longer reverberation time may be brought down to equality with the shorter by introducing absorbent materials, and the condition for the validity of (43) or (51) may thus be fulfilled.

To minimize the effect of a given absolute error in the times, \( t_1 \), \( t_2 \), and \( r \) should evidently be made as long as possible, which means that the rooms should be large and have highly reflecting walls, and that the initial steady intensity \( p_{01} \) should be made large by using a powerful source.
There then remains the final question whether the panel should be large or small, and to this there is no definite answer, because there are conflicting requirements. In the steady state, and to some extent after the emission of the source has ceased, the panel acts as a source for room $II$; and in order to satisfy the condition assumed in the theory, viz., that the sound shall be uniform and diffuse throughout the room, the area of the panel should evidently be as small as practicable in relation to the whole wall area. But on the other hand, if $S$, and therefore $\tau S = T$, is very small, the intensity in the test chamber will always be small, the duration of audibility will be short, and the percentage error in $t_2$ will be large. Some sort of compromise is necessary but there does not seem to be any a priori method for selecting a best value of $S/S_2$.

There are, however, quite other considerations which make it apparent that measurements on very small panels are of no practical value, no matter how accurate they may be; but since they are equally applicable to all methods of measuring transmittance, and are, moreover, sufficiently obvious, they need not be discussed here.

2. ROOMS WITH UNEQUAL REVERBERATION TIMES

If the two rooms are so different that their reverberation times cannot be made equal without both being short, it is necessary to revert to (39) which may also be put into the form

$$T = \frac{(r_1 - r_2)e^{-13.8S_1/r_1}}{a_2 - e^{-13.8S_2/r_1} - r_1 e^{-13.8S_1/r_1}}$$

(52)

And since this expression approaches o/o as $r_1$ and $r_2$ approach equality, the reverberation times should evidently be made very different in order to minimize the effects of errors in the times.

The question then arises whether the smaller or the larger room should be used as the test chamber, and a little consideration shows that the test chamber should be the one with the long reverberation time. For the initial steady intensity $\rho_{02}$ will always be small compared to $\rho_{01}$, and unless $r_1$ is long, the duration of audibility of this initially weak sound in the test chamber will be so short that its value $t_2$ is liable to a very large percentage error, whereas the duration $t_1$ in room $I$ will be of the same order of magnitude as $r_1$ and can be measured with the same accuracy.

Another line of reasoning also points to the desirability of putting the source in the smaller rather than in the larger room. In room $I$, the absorptivity of the panel will usually not be very different from that of the remainder of the walls and the presence of the panel will not cause any serious departure from the uniformity of energy distribution assumed in the theory. But in room $II$, the
panel alone is acting as a source and, as remarked above, the uniformity of distribution will be improved if the area of the walls of the room is made large in comparison with the area of the panel.

On the whole, the conditions of dissimilarity which require the use of equation (39) or (52) do not appear favorable to accuracy of determinations by the present method. If the only two rooms available are of very different sizes, the larger should be used as the test chamber; but in building a new laboratory for testing transmission, the rooms should be designed for equal reverberation times and should be as large and nonabsorbent as practicable, under the imposed limitations of cost of construction.

XVI. THE SOUND-RAY METHOD OF MEASURING TRANSMISSIVITY

A method of measuring transmissivity which does not depend on the theory discussed in this paper has been employed by Prof. F. R. Watson and is described in his book on Acoustics of Buildings (Wiley, 1923). A beam of sound from a source at the focus of a parabolic mirror is directed through a doorway between two rooms, and a Rayleigh disk resonator is placed on the axis of the mirror in the second room. The intensity is measured by the Rayleigh disk, first with the doorway open, and second when it is closed by the panel under test; and the ratio of the second intensity to the first is taken as the transmissivity of the panel.

While definite numerical values may be obtained in this way with a particular arrangement of the apparatus, their practical value seems questionable. With mirrors, panels, and openings, of which the linear dimensions are, at most, only a very few wave lengths, it seems that diffraction and scattering may so affect the results obtained as to make the analogy with optics altogether misleading and the interpretation of the results very uncertain. Aside from this, and assuming that the apparatus is all large enough, in terms of wave lengths, to invalidate the objection just mentioned, the question remains whether the transmissivity of a panel for waves which all come from one direction and continue in that same direction on the other side, is the same as or bears any simple relation to the ratio in which the panel reduces the intensity of sound striking it from all directions, as heard by an observer in a closed room beyond the panel.

It appears that both the practice of this method and the significance of the results obtained by it require further study.
XVII. THE RIVERBANK LABORATORIES METHOD FOR TRANSMISSIVITY

The greater part of the published data on transmissivity are from the experiments of Paul E. Sabine, at the Riverbank Laboratories, by one of the methods devised by W. C. Sabine. No clear description of the procedure has been published, so that comments on it are somewhat hazardous; but it appears to be a modification of the method, discussed in Section XV of the present paper, in which the experiment consists in the measurement of the duration of audibility of sound in the two rooms. The following remarks, offered with some hesitation, refer to the writer's conception of the method as obtained from the papers referred to above.

The modification, which eliminates the properties of the test chamber from the reasoning and greatly simplifies the computations, consists in the fact that the listening in the test chamber is done close to the panel, and that no attempt is made to measure the average duration of audibility throughout the room. The fundamental idea is to listen to the sound existing in the sound chamber directly through the panel, without being disturbed by, or having to take account of, the reflection from the walls of the test chamber, which serve merely as a shield against extraneous noises but do not otherwise affect the results obtained.

To make the principle of this method clearer, let us suppose that the observer remains in the sound room, and that after he has measured the duration of audibility of the sound, he covers his ears with two identical air-tight caps and again measures the duration of audibility of a sound of the same initial intensity as before. The ear caps reduce the intensity of the sound that reaches the ears in some ratio $r^1$, so that at the instant $t_2$ when the sound becomes inaudible, the sound outside must be $1/r^1 = K$ times as intense as the minimum of audibility. Since the sound decays at the same rate $b_1$ in both cases, we have from the first measurement

$$\rho_m = \rho_0 e^{-b_1 t_t}$$

and from the second

$$K \rho_m = \rho_0 e^{-b_1 t_2}$$

whence

$$K = e^{b_1 (t_1 - t_2)}$$

and the two experiments determine the value of $K$, the "reduction factor" of the earcaps.

In the experiments as actually conducted, the one test chamber incloses both the observer's ears, and its walls, with the panel, take

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1 See The American Architect: (a) July 30, 1919; (b) July 28, 1920; (c) Sept. 28 and Oct. 12, 1921; (d) July 4, 1923. These papers will be referred to by letter.
the place of the earcaps. Communication with the sound chamber occurs only through the panel, and since the rest of the walls is soundproof, the effects produced in the test chamber are exactly the same as if the whole test chamber were placed inside the sound chamber if that could be done without changing the acoustic properties of the sound chamber. Equation (55) therefore determines the reduction factor of the test room with the given panel in place, and it involves explicitly only $b_1$. But the question remains whether the $K$ which is computed in this way does not, in fact, depend on the properties of the test chamber; that is, whether $t_2$ does not depend on them as well as on the transmitting power of the panel.

Since the walls of the test chamber are not perfectly absorbent, there is necessarily some reverberation; and what the observer hears is due partly to reflection from the walls and not solely to sound waves coming directly from the panel to his ears. Hence, in order that $t_2$ and the resulting value of $K$ may be characteristic of the panel alone and sensibly independent of the properties of the room, it is necessary to reduce the intensity of the diffuse reflected sound to a negligible fraction of the intensity received by the observer directly from the panel; and if this is done, the value of $K$ obtained from the observations by means of (55) will be truly representative of a property of the panel alone.

The obvious procedure is, therefore, to listen close to the panel where the sound is loudest, and also to decrease the intensity of the diffuse reflected sound by increasing the absorbing power of the test chamber until a further increase has no further perceptible effect on the value of $t_2$. This seems to be the method adopted, the increase of absorption being obtained by introducing absorbent materials or by opening a door so as to enlarge the test chamber and so increase the area of its absorbing surface. (See reference b.)

The observations having been made in this way so that $t_2$ depends only on the panel, for a given initial intensity in the sound room, the reduction factor $K$ is definite and we have by (55)

$$\log_a K = b_1(t_1 - t_2) = \frac{Ca_1}{4V_1}(t_1 - t_2)$$

or

$$\log_{10} K = \frac{0.4343 C}{4V_1} a_1(t_1 - t_2)$$

From the description of the Riverbank Laboratories (reference a) it appears that the volume of the sound chamber is about $V_1 = 288m^3$; and taking the speed of sound at room temperature to be about $C = 340 \text{ m/sec.}$, reduces equation (57) to the form

$$\log_{10} K = 0.128 a_1(t_1 - t_2)$$
where the absorbing power $a_1$ is expressed in square meters. This is, perhaps, as nearly identical as could be expected with the equation given by Paul E. Sabine (reference $C$, equation 2) in which the coefficient is 0.126 instead of 0.128.

XVIII. REMARKS ON THE VARIOUS METHODS OF MEASURING TRANSMISSIVITY

Upon reviewing the methods which have been considered, it is readily seen that the quantities measured are not physically identical, and that while each method determines a property of the panel which may be called its transmissivity, the values obtained for the same panel by different methods may be quite different without our having to attribute the discrepancy to experimental errors. This has already been pointed out in connection with the "Sound ray method" (Section XVI), which need not be further discussed here, but it will be well to consider the relation of the method employed at the Riverbank Laboratories to the methods outlined in Sections XII and XV of the present paper, which are suggested immediately by the theory of reverberation.

In the static method of Section XII and in the residual sound method of Section XV, neither of which has yet been put into practice, so far as the writer knows, what is determined is the effect of the panel on the average intensity of sound in a closed room which is protected by the panel from a general diffuse sound outside. This seems, on the whole, to be the most useful form in which to have information about the sound transmitting, or sound insulating, properties of walls or panels. These methods depend on the fundamental assumption of the theory of reverberation, that the sound in each of the two rooms is nearly uniform and diffuse; and a close approximation to this condition for the validity of the methods requires that both rooms be large, if the panels are to be large enough to be of practical interest.

The quantity that appears in the equations is $T/a_2$; but since both $a_2$ and the area of the panel $S$ may be taken as known, the transmissivity of the panel or

$$\tau = \frac{T}{S} = \left( \frac{T}{a_2} \right) \frac{a_2}{S}$$

may be computed for comparison with values found by other methods.

The procedure followed at the Riverbank Laboratories makes use of the theory of reverberation as regards the sound chamber, and the sound chamber is fairly large; but the method has the great advantage of requiring only a small test chamber, because the proper-
ties of the test chamber and the nature of the distribution of sound in it are eliminated from consideration, their effects being overpowered and drowned out by the expedient of listening close to the panel. The question now arises, how the reduction factor $K$, or its reciprocal $1/K = \tau'$, is related to the transmissivity $\tau$, which has been defined, for the purposes of this paper, as the fraction of the sound energy striking one face of the panel which is given out as sound to the air in contact with the other face.

To answer this question would require an investigation of the effect of direction on the sensitiveness of the ear, and the details of the experimental procedure would have to be known. In the absence of this information no definite answer is possible, but a few further comments may be worth while.

In treating reverberation, the impossibility of analyzing a mass of diffuse reflected sound into separate wave trains and considering these in detail, obliges us to introduce the conception of sound as merely a form of radiant energy, to which the same methods of reasoning are applicable as to thermal radiation. This necessitates the assumption that, for a given frequency, the strength of the auditory sensation is uniquely determined by the rate at which energy reaches the ear; or that the "intensity" of a diffuse sound—in the objective sense of the physical stimulus—is the same thing as, or is proportional to, the energy density in the space outside the ear. And in accordance with this assumption, the term "intensity" has been used interchangeably with "energy density." The assumption seems to be perfectly justified by the fact that the theory based on it does furnish a satisfactory representation of the phenomena of reverberation investigated by W. C. Sabine.

But when we turn to the consideration of sound which is not diffuse but comes from a single direction, the assumption needs further examination. When sound waves reach the observer's ear from a single direction it is still true that the loudness of a pure note of given pitch is uniquely determined by the rate at which energy reaches the ear or, since the speed of propagation is constant, by the energy density. But it can by no means be assumed without proof that a given density in a directed beam will have the same effect on the ear as the same density of diffused sound energy inside a reflecting inclosure. In particular, it cannot be assumed that the minimum density, which is just audible, is the same for directed as for diffuse sound. On the contrary, it seems very probable that it is different. But we do not know what the relation is.

Now, at a point close to the panel and not too near its edge, the sound waves are nearly plane and are moving approximately along the normal to the panel. Hence, when the listening in the test
chamber is done close to the panel, the observations are made in a directed beam and not in a region of diffuse sound as they are, approximately, in the sound chamber. The conclusion is that while the method adopted at the Riverbank Laboratories and the methods outlined in Sections XII and XV of the present paper may very possibly arrange any given series of panels in the same order, the absolute values obtained for the transmissivity of a particular panel by the different methods are not at present directly comparable. Before the relation between the two kinds of result can be established, it will be necessary to carry out an extensive investigation of the relation of the minimum audible energy density to the direction, or directions, from which the sound reaches the observer.

WASHINGTON, March 11, 1925.