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# TABLES FOR THE CALCULATION OF THE MUTUAL INDUCTANCE OF CIRCUITS WITH CIRCULAR SYMMETRY ABOUT A COMMON AXIS

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## ABSTRACT

In the calculation of the inductance of circuits which have circular symmetry about an axis the case of two circular coaxial filaments is of fundamental importance. By integration of formulas for this simple case formulas for the inductance of coils of finite cross section may be derived, and even in cases where it is impossible to perform the integration, numerical values of the inductance may be obtained from the simple case by mechanical quadrature.

Many different formulas have been derived for the calculation of the mutual inductance of coaxial circular filaments, but the wealth of available formulas is a source of difficulty to the computer who must select that formula which is best adapted to each individual case. Furthermore, if many computations have to be made the labor involved is considerable.

In the present paper tables are given which do away with the necessity of selecting a formula, and the inductance is obtained by taking the product of the geometric mean of the radii and a factor taken by interpolation from a table. An accuracy of 1 part in 10,000 in the value of the mutual inductance is readily attained.

## CONTENTS

## I. INTRODUCTION

Formulas for the mutual inductance of two coaxial circles are of great importance in the calculation of the inductance of some of the most common forms of circuits. The mutual inductance of coaxial coils or solenoids, the self-inductance of circular coils and rings, the mutual inductance of a coil and the disk of a Lorenz apparatus, all these may be obtained by integration or mechanical quadrature from the mutual inductance of two coaxial circular filaments. Two elliptic integral formulas for this case were given by Maxwell,[1] which furnish values of the mutual inductance whose accuracy is limited only by the accuracy with which the elliptic integrals may be obtained. In certain cases, however, series expansions are more convenient, and this fact, together with the mathematical interest of the problem, has led to the development by various authors of series formulas, some of which converge well for circles near together and others for more distant circles. The lists of inductance formulas published by the Bureau of Standards includes 29 formulas for this case alone. This wealth of formulas, while it attests the thoroughness with which the problem has been treated, is a source of embarrassment to the computer, who must select the most suitable formula or formulas for each new individual problem. In addition, the computations, if many have to be made, demand a considerable expenditure of time and labor.

For these reasons tables for simplifying the calculations and removing the necessity of making a choice of appropriate formulas in any given case should prove useful.

A short table by the author of the present paper appeared in 1917.[2] This gave values of the factor $F$ in the simple formula $M=F\sqrt{Aa}$ for the mutual inductance of two coaxial circles of radii $A$ and $a$. This table was based on the ratio between the smallest and largest distance between the circumferences of the two circles. Formulas (1), (6), and (7), and Table 1 of Scientific Paper 169 of the Bureau of Standards were employed in calculating the table.

Very recently a paper has appeared by Curtis and Sparks,[3] in which new formulas for mutual inductance of the coaxial circles are derived and certain of the older formulas are simplified. Tables and charts are given, not only for calculating the inductance but for the determination of the dimensions of pairs of circles which shall have a desired mutual inductance. These tables, which are very complete, allow the necessary constants to be obtained by simple interpolation with an accuracy which suffices to give the mutual

[1] Electricity and Magnetism, 2, § 701.

[2] B. S. Circular 74, Table 16, p. 286.

[3] Formulas, Tables, and Curves for Computing the Mutual Inductance of Two Coaxial Circles, B. S. Sci. Paper No. 492.

inductance accurate to one-tenth of 1 per cent even in unfavorable cases, and considerably more accurate in favorable cases. The method of calculation adopted differs from that of the table referred to in the preceding paragraph.

The tables of the present paper furnish an amplification of the earlier table of the author, an accuracy, at worst, of 1 part in 10,000 having been aimed at. This has been made possible without undue labor by the recent publication of the table of elliptic functions of Nagaoka and Sakurai.[4]

Of the various possible forms of series expansion of the elliptic integral formulas for the mutual inductance of coaxial circles that in $q$ functions gives the best convergence. Two such formulas were given by Nagaoka[5] in 1903, together with tables of certain of the quantities appearing in the formulas. In spite, however, of their very rapid convergence, the necessity of performing a rather tedious calculation to obtain the principal quantities $q$ and $q_1$ has hindered the extended use which these formulas deserve. This difficulty is removed by the elliptic function tables just referred to, which give values of $q$ and $q_1$ as functions of the moduli $k$ and $k'$ on which they depend. Thus it has been possible to base the present tables upon the Nagaoka formulas using other formulas as a check.

## II. DESCRIPTION OF THE TABLES

Let

$a$ = the radius of the smaller circle,
$A$ = the radius of the larger circle,
$D$ = the distance between their planes,
$r_1$ = the longest distance between their circumferences,
$r_2$ = the shortest distance between their circumferences,
$M$ = the mutual inductance of the two circles,

Then the mutual inductance will be expressed, as was also done in the earlier table,[6] as

$$M=f\sqrt{Aa}=fA\sqrt{\frac{a}{A}} \qquad (1)$$

The mutual inductance is given in microhenrys, and the dimensions in centimeters. Of course, when (as will be the case with $\delta$ and $\Delta$ in the following) a ratio of two dimensions is to be calculated, either system may be used, provided both dimensions are expressed in the same system. The factor $f$ depends only upon the relative values of the quantities $A$, $a$, and $D$, and not upon their absolute values.

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[4] Institute of Phys. and Chem. Research, 2, pp. 1–67; Tokyo; 1922.
[5] Phil. Mag. 6, p. 19; 1903. B. S. Sci. Paper 169, pp. 11 and 215–220.
[6] See footnote 2, p. 2.

In the earlier table, $F$ was tabulated for different values of the parameter $k' = \frac{r_2}{r_1}$, and the square of this

$$k'^2 = \frac{\left(1-\frac{a}{A}\right)^2 + \frac{D^2}{A^2}}{\left(1+\frac{a}{A}\right)^2 + \frac{D^2}{A^2}} = \frac{(A-a)^2+D^2}{(A+a)^2+D^2} \tag{2}$$

is chosen here for the nearer circles. For the more distant circles the square of the complementary modulus

$$k^2 = 1-k'^2 = \frac{4\frac{a}{A}}{\left(1+\frac{a}{A}\right)^2 + \frac{D^2}{A^2}} = \frac{4aA}{(A+a)^2+D^2} \tag{3}$$

may be used. These two quantities are the square of the moduli of the elliptic integrals which occur in Maxwell's formula for the mutual inductance.

The following is a list of the tables:

Table 1. Values of $f$ and $\log_{10} f$ for values of $k'^2$ in steps of 0.005 from zero up to 1.000, and for values of $k^2$ in steps of 0.005 from 1.000 down to zero.

Table 2. Auxiliary table giving values of $f$ for values of $\log_{10} k'^2$ in steps of 0.1 between $-6$ and $-1$. Very close circles.

Table 3. Auxiliary table giving values of $\log_{10} f$ for values of $\log_{10} k^2$ in steps of 0.1 from $-1$ to $-4$. (Very distant circles.)

Table 4. Equal circles. Values of $f$ and $\log_{10} f$ for values of the parameter $\delta = \frac{\text{distance}}{\text{diameter}} = \frac{D}{2A}$ in steps of 0.01 from zero to 1. (Circles near together.)

Table 5. Auxiliary table for equal circles very near together. Values of $f$ for values of $\log_{10} \delta$ in steps of 0.1 from $-4$ to $-0.7$.

Table 6. Equal circles far apart. Values of $f$ and $\log_{10} f$ in steps of 0.01 from 1.00 to zero, of the parameter $\Delta = \frac{\text{diameter}}{\text{distance}} = \frac{1}{\delta}$.

Table 7. Auxiliary table for equal circles very far apart. Values of $\log_{10} f$ for values of $\log_{10} \Delta$ in steps of 0.1 from $-0.7$ to $-2.3$.

## III. METHODS OF COMPUTING THE TABLES

Tables 1 to 3, inclusive, the squares of the moduli $k$ and $k'$ are chosen as parameters, rather than the moduli themselves, because the relation $k^2 = 1-k'^2$ or $k'^2 = 1-k^2$ allows of an easy passage from one parameter to the other. This is not true of the moduli themselves.

This fact is pointed out by Nagaoka and Sakurai.[7] Either parameter may be chosen in any individual case.

In the calculation of Tables 1 to 3 Nagaoka's formula (8), Scientific Paper No. 169, was used except for values of $k'^2$ less than 0.15, where Nagaoka's formula (9) (same reference) was employed. In both cases the quantities $q$ and $q_1'$ were taken directly from the tables of Nagaoka and Sakurai, and the quantities $\epsilon$ and $\epsilon_1$ from Tables 15 and 16, Scientific Paper No. 169.

In Table 1 interpolation is not so favorable for small values of $k'^2$ as for the larger values. In this region the auxiliary Table 2 is useful. This was calculated by means of formula (4A), Scientific Paper No. 320. The limiting value of $f$ as $k'^2$ approaches zero is seen from this formula to be

$$f=0.002\,\pi\left(\log_e\frac{16}{k'^2}-4\right)$$

which shows that for small values of $k'^2$, the relation between $f$ and $\log_{10} k'^2$ is nearly linear.

Likewise for values of $k^2$ near zero the interpolation in Table 1 is difficult. The auxiliary Table 3 takes care of this case. It was calculated by formula (5), Scientific Paper No. 169. This formula becomes, for very small values of $k^2$.

$$\log_{10} f=\bar{3}.392240+\frac{3}{2}\log_{10} k^2$$

which shows that the relation between $\log_{10} f$ and $\log_{10} k^2$ is, for small values of $k^2$, very nearly linear.

Tables 1 to 3 include, of course, also the special case of circles of equal radii. Because of the great importance of this latter case, it has seemed well to prepare additional tables for *equal* circles, using the very convenient parameters $\delta=\frac{\text{distance}}{\text{diameter}}=\frac{d}{2A}$, for nearer circles, and, for ease of interpolation, $\Delta=\frac{\text{diameter}}{\text{distance}}$ for distant circles. That parameter is to be used in any given case which is less than unity.

In computing these tables, which cover the case of circles of equal radii, use was made of the tables of Nagaoka and Sakurai, but it was necessary to interpolate to obtain the values of $q$ and $q_1$ using the relations which exist for *equal* circles

$$k'^2=\frac{\delta^2}{1+\delta^2}=\frac{1}{1+\Delta^2},\quad k^2=\frac{\Delta^2}{1+\Delta^2}=\frac{1}{1+\delta^2} \tag{6}$$

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[7] See footnote 4, p. 3.

It is to be noted that the same relation exists between $k^2$ and $\Delta$ as exists between $k'^2$ and $\delta$, so that the calculation has to be made for only one of the quantities $k^2$ and $k'^2$.

The auxiliary Tables 5 and 7 supplement Tables 4 and 6, in just the way that Tables 2 and 3 supplement Table 1. The limiting formulas are, for Table 5

$$f = 0.004\pi\left(\log_e \frac{4}{\delta} - 2\right) \tag{7}$$

and for Table 7

$$\log_{10} f = \bar{3}.392240 + 3 \log_{10} \Delta \tag{8}$$

For many cases simple interpolation from all the tables will give sufficient accuracy. When, however, the full accuracy of the tables is necessary, second, and, in rarer cases, third differences need to be taken into account. For this purpose the following interpolation formula may be used

$$F(x) = F_0 + K_1 d_1 + K_2 d_2 + K_3 d_3 \tag{9}$$

in which $F(x)$ is the desired value to be interpolated, $F_0$ is the tabulated value for the value of the argument next smaller than the desired value, $K_1$ is the fractional part of the tabular interval in question, $d_1$, $d_2$, and $d_3$ are the first, second, and third differences given in the table, and $K_2$ and $K_3$ are further binomial coefficients in the Newton interpolation formula. For convenience these coefficients may be taken from Table 8, in which is an extension of Table 14, Scientific Paper No. 169. The values in Table 8 are calculated from the defining equations

$$K_2 = \frac{K_1(K_1-1)}{2!} \text{ and } K_3 = \frac{K_1(K_1-1)\ (K_1-2)}{3!}$$

## IV. EXAMPLES

### EXAMPLE 1

As an example of the use of the tables, let us take the case of two circles of radii 20 and 25 cm whose planes are distant 10 cm.

Here $a = 20$, $A = 25$, $D = 10$, and $k'^2 = \frac{25+100}{45^2+100} = 0.0588236$.

The interpolation from Table 1 is as follows:

$$\begin{aligned} \text{for } k'^2 = 0.055 \qquad f &= 0.011497 \\ d_1 &= -480 \\ d_2 &= 43 \\ d_3 &= -7 \end{aligned}$$

From Table 8

$$K_1=0.7647,\ K_2=0.090,\ K_3=0.037$$

and thus

$$K_1d_1=\quad -367.1$$
$$K_2d_2=\quad -3.9$$
$$K_3d_3=\quad -0.2 \qquad \text{Sum}=-371$$

Thus the interpolated value is

$$f=0.011497-0.000371=0.011126$$

Table 2 may also be used for this case. We have $\log_{10} k'^2=\bar{2}.769551$, and the interpolation from Table 2 gives

$$\text{for } \log_{10}F=\bar{2}.7\ \ f \text{ is } 0.012013 \qquad K_1=0.6955$$
$$K_1d_1=\ -884 \qquad K_2=0.104$$
$$K_2d_2=\ -3$$

$$\therefore f=0.011126, \text{ as before.}$$

The value of $\log_{10} f$ interpolated from Table 1 is $\bar{2}.060585-14155-83-5=\bar{2}.046342$, to which corresponds $f=0.0111261$. To this last value corresponds the value $M=\sqrt{500}\ (0.0111261)=0.248787$ microhenrys. The value calculated directly for this case in example 4, Scientific Paper No. 169, is 0.2487875.

The accurate interpolation has been made in this case to show that the tables are capable of great precision. If, instead, simple interpolation only had been employed, the values given above would be replaced by 0.011130, 0.011129, and 0.0111283, respectively, which would be near enough for most purposes. With Table 8 available, however, it is well to take *second* differences into account, since the additional labor is negligible. Only rarely will *third* differences be important.

EXAMPLE 2

As an example of circles very near together, we may take $a=A=25$, $D=1$. Thus $k'^2=\frac{1}{2501}$, $\log_{10} k'^2=\bar{4}.601886$. Table 2 gives by simple interpolation $f=0.041467$. Since the circles are equal Table 4 may be used. The quantity $\delta=\frac{d}{2A}=\frac{1}{50}=0.02$, so that for this case interpolation is not necessary. The value in Table 4 is $f=0.041467$ as before. Using the value $\log_{10}\delta=\bar{2}.301030$, Table 5 gives $f=0.041466$. The value of the mutual inductance corresponding to the last is $M=25\ (0.041466)=1.03665\ \mu h$. The more accurate value calculated in example 2, Scientific Paper No. 169, is 1.036664.

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EXAMPLE 3

The circles $a = A = 10$, $D = 100$, may be regarded as illustrating the case of circles far apart.

$$k^2 = \frac{400}{400 + 10{,}000} = \frac{1}{26}, \quad \log_{10} k^2 = \bar{2}.585027$$

also $\Delta = 0.2$.

Table 6 gives for $\Delta = 0.2$, $f = 0.0000191650$. Table 7 yields $\log_{10} f = \bar{5}.282509$, to which corresponds the same value of $f$. This is an unfavorable case for Table 1, but taking account of third differences the resulting value of $f$ is in agreement exactly with the foregoing. The mutual inductance is thus 0.000191650 microhenrys, which agrees closely with the value 0.0001916496 calculated in example 6, Scientific Paper No. 169.

No application of the formulas to actual coils will here be made. For coils of rectangular cross section the mutual inductance of the circles passing through the centers of the coils would be found and then multiplied by the product of the number of turns on the two coils. This would give an approximation to the mutual inductance of the coils, but the result would have to be corrected to take into account the actual distribution of the current over cross sections of finite area. This matter is treated in Scientific Paper No. 169, pages 33 to 52.

TABLE 1.—*Values of the factor f in the formula* $M = f\sqrt{Aa}$

$$k'^2 = \frac{(A-a)^2 + D^2}{(A+a)^2 + D^2}$$

$$k^2 = 1 - k'^2 = \frac{4aA}{(A+a)^2 + D^2}$$

[Units: The values of $f$ are such that, when $A$ and $a$ are in centimeters, $M$ is in microhenrys]

| $k'^2$ | $f$ | $d_1$ | $d_2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ | $k^2$ |
|---|---|---|---|---|---|---|---|---|
| 0.005 | 0.025722 | −4,248 | ---------- | $\bar{2}$.410300 | −78,387 | ---------- | ---------- | 0.995 |
| .010 | .021474 | 2,442 | ---------- | .331913 | 52,468 | ---------- | ---------- | .990 |
| .015 | .019031 | 1,716 | ---------- | .279445 | 41,023 | ---------- | ---------- | .985 |
| .020 | .017315 | 1,315 | ---------- | .238423 | 34,310 | ---------- | ---------- | .980 |
| .025 | .016000 | 1,063 | ---------- | .204113 | 29,856 | ---------- | ---------- | .975 |
| 0.030 | 0.014937 | −890 | +127 | $\bar{2}$.174257 | −26,671 | ---------- | ---------- | 0.970 |
| .035 | .014047 | 763 | 96 | .147586 | 24,255 | ---------- | ---------- | .965 |
| .040 | .013284 | 667 | 76 | .123331 | 22,363 | ---------- | ---------- | .960 |
| .045 | .012617 | 591 | 62 | .100968 | 20,830 | ---------- | ---------- | .955 |
| .050 | .012026 | 529 | 49 | .080138 | 19,555 | ---------- | ---------- | .950 |
| 0.055 | 0.011497 | −480 | 43 | $\bar{2}$.060585 | −18,510 | 925 | ---------- | 0.945 |
| .060 | .011017 | 437 | 36 | .042075 | 17,585 | 790 | ---------- | .940 |
| .065 | .010580 | 401 | 30 | .024490 | 16,795 | 676 | ---------- | .935 |
| .070 | .010179 | 371 | 26 | $\bar{2}$.007695 | 16,119 | 611 | ---------- | .930 |
| .075 | .0098079 | −3,439 | 233 | $\bar{3}$.991576 | 15,501 | 532 | ---------- | .925 |
| 0.080 | 0.0094640 | −3,206 | 206 | $\bar{3}$.976075 | −14,969 | 480 | ---------- | 0.920 |
| .085 | 91434 | 3,000 | 184 | .961106 | 14,489 | 435 | ---------- | .915 |
| .090 | 88434 | 2,816 | 165 | .946617 | 14,054 | 396 | ---------- | .910 |
| .095 | 85618 | 2,651 | 149 | .932563 | 13,658 | 359 | ---------- | .905 |
| .100 | 82967 | 2,502 | 133 | .918905 | 13,299 | 324 | ---------- | .900 |
| 0.105 | 0.0080465 | −2,369 | 122 | $\bar{3}$.905606 | −12,975 | 296 | ---------- | 0.895 |
| .110 | 78096 | 2,247 | 111 | .892631 | 12,679 | 271 | ---------- | .890 |
| .115 | 75849 | 2,136 | 102 | .879952 | 12,408 | 252 | ---------- | .885 |
| .120 | 73713 | 2,034 | 94 | .867544 | 12,154 | 236 | ---------- | .880 |
| .125 | 71679 | 1,940 | 86 | .855388 | 11,918 | 219 | ---------- | .875 |

TABLE 1.—*Values of the factor f in the formula* $M=f\sqrt{Aa}$—Continued

| $k'^2$ | | $d_1$ | $d_2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ | $k^2$ |
|---|---|---|---|---|---|---|---|---|
| 0.130 | 0.0069739 | −1,854 | 79 | $\bar{3}$.843470 | −11,703 | 199 | ---------- | 0.870 |
| .135 | 67885 | 1,775 | 74 | .831767 | 11,504 | 187 | ---------- | .865 |
| .140 | 66110 | 1,701 | 69 | .820263 | 11,317 | 174 | ---------- | .860 |
| .145 | 64409 | 1,632 | 65 | .808946 | 11,142 | 161 | ---------- | .855 |
| .150 | 62777 | 1,567 | 59 | .797804 | 10,981 | 152 | ---------- | .850 |
| 0.155 | 0.0061210 | −1,508 | 57 | $\bar{3}$.786822 | −10,829 | 142 | ---------- | 0.845 |
| .160 | 59702 | 1,451 | 53 | .775993 | 10,687 | 133 | ---------- | .840 |
| .165 | 58251 | 1,398 | 49 | .765306 | 10,554 | 124 | ---------- | .835 |
| .170 | 56853 | 1,349 | 46 | .754752 | 10,430 | 118 | ---------- | .830 |
| .175 | 55504 | 1,303 | 45 | .744322 | 10,312 | 112 | ---------- | .825 |
| 0.180 | 0.0054201 | −1,258 | 41 | $\bar{3}$.734010 | −10,203 | 104 | ---------- | 0.820 |
| .185 | 52943 | 1,217 | 39 | .723807 | 10,099 | 98 | ---------- | .815 |
| .190 | 51276 | 1,178 | 38 | .713708 | 10,001 | 90 | ---------- | .810 |
| .195 | 50548 | 1,140 | 35 | .703707 | 9,991 | 87 | ---------- | .805 |
| .200 | 49408 | 1,105 | 33 | .693796 | 9,824 | 81 | ---------- | .800 |
| 0.205 | 0.0048303 | −1,072 | 33 | $\bar{3}$.683972 | −9,743 | 77 | ---------- | 0.795 |
| .210 | 47231 | 1,039 | 29 | .674229 | 9,666 | 71 | ---------- | .790 |
| .215 | 46192 | 1,010 | 30 | .664563 | 9,595 | 68 | ---------- | .785 |
| .220 | 45182 | 980 | 27 | .654968 | 9,527 | 64 | ---------- | .780 |
| 225 | 44202 | −953 | 27 | .645441 | 9,463 | 59 | ---------- | 0.775 |
| 0.230 | 0.0043249 | 926 | 25 | $\bar{3}$.635978 | −9,404 | +57 | ---------- | .770 |
| .235 | 42323 | 901 | 24 | .626574 | 9,347 | 53 | ---------- | .765 |
| .240 | 41422 | 878 | 23 | .617227 | 9,294 | 48 | ---------- | .760 |
| .245 | 40544 | 854 | 23 | .607933 | 9,246 | 49 | ---------- | .755 |
| .250 | 39690 | 831 | 20 | .598687 | 9,197 | 43 | ---------- | .750 |
| 0.255 | 0.0038859 | −811 | 21 | $\bar{3}$.589490 | −9,154 | +41 | ---------- | 0.745 |
| .260 | 38048 | 790 | 20 | .580336 | 9,113 | 37 | ---------- | .740 |
| .265 | 37258 | 770 | 18 | .571223 | 9,076 | 36 | ---------- | .735 |
| .270 | 36488 | 752 | 19 | .562147 | 9,040 | 33 | ---------- | .730 |
| .275 | 35736 | 733 | 17 | .553107 | 9,007 | 30 | ---------- | .725 |
| 0.280 | 0.0035003 | −716 | 17 | $\bar{3}$.544100 | −8,977 | +29 | ---------- | 0.720 |
| .285 | 34286 | 700 | 17 | .535123 | 8,948 | 25 | ---------- | .715 |
| .290 | 33587 | 683 | 16 | .526175 | 8,923 | 24 | ---------- | .710 |
| .295 | 32904 | 667 | 15 | .517252 | 8,899 | 22 | ---------- | .705 |
| .300 | 32237 | 652 | 14 | .508353 | 8,877 | 20 | ---------- | .700 |
| 0.305 | 0.0031585 | −638 | 14 | $\bar{3}$.499476 | −8,857 | +17 | ---------- | 0.695 |
| .310 | 30947 | 624 | 14 | .490619 | 8,840 | 16 | ---------- | .690 |
| .315 | 30324 | 610 | 13 | .481779 | 8,824 | 15 | ---------- | .685 |
| .320 | 29714 | 597 | 13 | .472954 | 8,809 | 10 | ---------- | .680 |
| .325 | 29117 | 584 | 12 | .464145 | 8,799 | 11 | ---------- | .675 |
| 0.330 | 0.0028533 | −572 | 13 | $\bar{3}$.455346 | −8,788 | +8 | ---------- | 0.670 |
| .335 | 27961 | 559 | 11 | .446558 | 8,780 | 6 | ---------- | .665 |
| .340 | 27402 | 548 | 11 | .437778 | 8,773 | 4 | ---------- | .660 |
| .345 | 26854 | 537 | 12 | .429005 | 8,769 | 4 | ---------- | .655 |
| .350 | 26317 | 525 | 9 | .420236 | 8,765 | 3 | ---------- | .650 |
| 0.355 | 0.0025792 | −516 | 11 | $\bar{3}$.411471 | −8,762 | ---------- | ---------- | 0.645 |
| .360 | 25276 | 505 | 10 | .402709 | 8,764 | ---------- | ---------- | .640 |
| .365 | 24771 | 495 | 10 | .393945 | 8,764 | −3 | ---------- | .635 |
| .370 | 24276 | 485 | 9 | .385181 | 8,767 | −5 | ---------- | .630 |
| .375 | 23791 | 476 | 10 | .376414 | 8,772 | −7 | ---------- | .625 |
| 0.380 | 0.0023315 | −466 | 8 | $\bar{3}$.367642 | −8,779 | −6 | ---------- | 0.620 |
| .385 | 22849 | 458 | 9 | .358863 | 8,785 | −10 | ---------- | .615 |
| .390 | 22391 | 449 | 9 | .350078 | 8,795 | −10 | ---------- | .610 |
| .395 | 21942 | 440 | 8 | .341283 | 8,805 | −14 | ---------- | .605 |
| .400 | 21502 | 432 | 8 | .332478 | 8,818 | −13 | ---------- | .600 |
| 0.405 | 0.0021070 | −424 | 7 | $\bar{3}$.323660 | −8,832 | −17 | ---------- | 0.595 |
| .410 | 20646 | 417 | 9 | .314828 | 8,845 | 19 | ---------- | .590 |
| .415 | 20229 | 408 | 7 | .305983 | 8,862 | 19 | ---------- | .585 |
| .420 | 19821 | 401 | 7 | .297121 | 8,881 | 19 | ---------- | .580 |
| .425 | 19420 | 394 | 7 | .288240 | 8,900 | 21 | ---------- | .575 |
| 0.430 | 0.0019026 | −387 | 7 | $\bar{3}$.279340 | −8,921 | −22 | ---------- | 0.570 |
| .435 | 18639 | 380 | 7 | .270419 | 8,943 | 25 | ---------- | .565 |
| .440 | 18259 | 373 | 6 | .261476 | 8,968 | 24 | ---------- | .560 |
| .445 | 17886 | 367 | 7 | .252508 | 8,992 | 25 | ---------- | .555 |
| .450 | 17519 | 360 | 8 | .243516 | 9,017 | 31 | ---------- | .550 |

TABLE 1.—*Values of the factor f in the formula* $M=f\sqrt{Aa}$—Continued.

| $k'^2$ | $f$ | $d_1$ | $d_2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ | $k^2$ |
|---|---|---|---|---|---|---|---|---|
| 0.455 | 0.0017159 | 352 | 5 | $\bar{3}$.234497 | 9,048 | −29 | ----- | 0.545 |
| .460 | 16805 | −347 | 5 | .225449 | −9,077 | 32 | ----- | .540 |
| .465 | 16458 | 342 | 6 | .216372 | 9,109 | 34 | ----- | .535 |
| .470 | 16116 | 336 | 6 | .207263 | 9,143 | 33 | ----- | .530 |
| .475 | 15780 | 329 | 4 | .198120 | 9,176 | 37 | ----- | .525 |
| 0.480 | 0.0015451 | −325 | 7 | $\bar{3}$.188944 | −9,213 | −38 | ----- | 0.520 |
| .485 | 15126 | 318 | 4 | .179731 | 9,251 | 38 | ----- | .515 |
| .490 | 14808 | 314 | 6 | .170480 | 9,289 | −42 | ----- | .510 |
| .495 | 14494 | −308 | ----- | .161191 | −9,331 | ----- | ----- | .505 |
| 0.500 | 0.00141860 | −3,029 | +51 | $\bar{3}$.151860 | −9,374 | −44 | ----- | 0.500 |
| .505 | 138831 | 2,978 | 49 | .142486 | 9,418 | 46 | ----- | .495 |
| .510 | 135853 | 2,929 | 49 | .133068 | 9,464 | 48 | ----- | .490 |
| .515 | 132924 | 2,880 | 49 | .123604 | 9,512 | 50 | ----- | .485 |
| .520 | 130044 | 2,831 | 46 | .114092 | 9,562 | 52 | ----- | .480 |
| 0.525 | 0.00127213 | −2,785 | 45 | $\bar{3}$.104530 | −9,614 | −53 | ----- | 0.475 |
| .530 | 124428 | 2,740 | 47 | .094916 | 9,667 | 55 | ----- | .470 |
| .535 | 121688 | 2,693 | 43 | .085249 | 9,722 | 57 | ----- | .465 |
| .540 | 118995 | 2,650 | 44 | .075527 | 9,779 | 61 | ----- | .460 |
| .545 | 116345 | 2,606 | 42 | .065748 | 9,840 | 61 | ----- | .455 |
| 0.550 | 0.00113739 | −2,564 | 42 | $\bar{3}$.055908 | −9,901 | −63 | ----- | 0.450 |
| .555 | 111175 | 2,522 | 42 | .046007 | 9,964 | 65 | ----- | .445 |
| .560 | 108653 | 2,480 | 40 | .036043 | 10,029 | 70 | ----- | .440 |
| .565 | 106173 | 2,440 | 39 | .026014 | 10,099 | 69 | ----- | .435 |
| .570 | 103733 | 2,401 | 39 | .015915 | 10,168 | 73 | ----- | .430 |
| 0.575 | 0.00101332 | −2,362 | 40 | $\bar{3}$.005747 | −10,241 | −74 | ----- | 0.425 |
| .580 | .00098970 | 2,322 | 36 | $\bar{4}$.995506 | 10,315 | 78 | ----- | .420 |
| .585 | 96648 | 2,286 | 37 | .985191 | 10,393 | 82 | ----- | .415 |
| .590 | 94362 | 2,249 | 38 | .974798 | 10,475 | 81 | ----- | .410 |
| .595 | 92113 | 2,211 | 34 | .964323 | 10,556 | 86 | ----- | .405 |
| 0.600 | 0.00089902 | −2,177 | 36 | $\bar{4}$.953767 | −10,642 | −88 | ----- | 0.400 |
| .605 | 87725 | 2,141 | 35 | .943125 | 10,730 | 91 | ----- | .395 |
| .610 | 85584 | 2,106 | 34 | .932395 | 10,821 | 95 | ----- | .390 |
| .615 | 83478 | 2,072 | 34 | .921574 | 10,916 | 97 | ----- | .385 |
| .620 | 81406 | 2,038 | 33 | .910658 | 11,013 | 101 | ----- | .380 |
| 0.625 | 0.00079368 | −2,005 | 32 | $\bar{4}$.899645 | −11,114 | −104 | ----- | 0.375 |
| .630 | 77363 | 1,973 | 32 | .888531 | 11,218 | 107 | ----- | .370 |
| .635 | 75390 | 1,941 | 32 | .877313 | 11,325 | 111 | ----- | .365 |
| .640 | 73449 | 1,908 | 31 | .865988 | 11,436 | 116 | ----- | .360 |
| .645 | 71540 | 1,878 | 30 | .854552 | 11,552 | 118 | ----- | .355 |
| 0.650 | 0.00069663 | −1,847 | 30 | $\bar{4}$.843000 | −11,670 | −123 | ----- | 0.350 |
| .655 | 67816 | 1,817 | 30 | .831330 | 11,793 | 127 | ----- | .345 |
| .660 | 65999 | 1,787 | 30 | .819537 | 11,920 | 132 | ----- | .340 |
| .665 | 64212 | 1,757 | 28 | .807617 | 12,052 | 137 | ----- | .335 |
| .670 | 62455 | 1,729 | 30 | .795565 | 12,189 | 140 | ----- | .330 |
| 0.675 | 0.00060726 | −1,700 | 28 | $\bar{4}$.783376 | −12,329 | −146 | ----- | 0.325 |
| .680 | 59026 | 1,672 | 28 | .771047 | 12,475 | 151 | ----- | .320 |
| .685 | 57355 | 1,643 | 27 | .758572 | 12,626 | 156 | ----- | .315 |
| .690 | 55712 | 1,616 | 27 | .745946 | 12,782 | 164 | ----- | .310 |
| .695 | 54096 | 1,589 | 27 | .733164 | 12,946 | 168 | ----- | .305 |
| 0.700 | 0.00052507 | −1,562 | 27 | $\bar{4}$.720218 | −13,114 | −175 | ----- | 0.300 |
| .705 | 50945 | 1,535 | 26 | .707104 | 13,289 | 182 | ----- | .295 |
| .710 | 49410 | 1,509 | 26 | .693815 | 13,471 | 188 | ----- | .290 |
| .715 | 47901 | 1,483 | 25 | .680344 | 13,659 | 196 | ----- | .285 |
| .720 | 46418 | 1,458 | 26 | .666685 | 13,855 | 203 | ----- | .280 |
| 0.725 | 0.00044960 | −1,432 | 25 | $\bar{4}$.652830 | −14,058 | −213 | ----- | 0.275 |
| .730 | 43528 | 1,407 | 25 | .638772 | 14,271 | 220 | ----- | .270 |
| .735 | 42121 | 1,382 | 24 | .624501 | 14,491 | 230 | ----- | .265 |
| .740 | 40739 | 1,358 | 25 | .610010 | 14,721 | 239 | ----- | .260 |
| .745 | 39381 | 1,333 | 23 | .595289 | 14,960 | 250 | ----- | .255 |
| 0.750 | 0.00038048 | −1,310 | +24 | $\bar{4}$.580329 | −15,210 | −261 | −10 | 0.250 |
| .755 | 36738 | 1,286 | 24 | .565119 | 15,471 | 271 | 14 | .245 |
| .760 | 35452 | 1,262 | 24 | .549648 | 15,742 | 285 | 12 | .240 |
| .765 | 34190 | 1,238 | 22 | .533906 | 16,027 | 297 | 15 | .235 |
| .770 | 32952 | 1,216 | 24 | .517879 | 16,324 | 312 | 14 | .230 |

**Table 1.**—*Values of the factor f in the formula* $M=f\sqrt{Aa}$—Continued

| $k'^2$ | $f$ | $d_1$ | $d_2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ | $k^2$ |
|---|---|---|---|---|---|---|---|---|
| 0.775 | 0.00031736 | −1,192 | 23 | $\bar{4}$.501555 | −16,636 | −326 | −16 | 0.225 |
| .780 | 30544 | 1,170 | 22 | .484919 | 16,962 | 342 | 18 | .220 |
| .785 | 29374 | 1,148 | 23 | .467957 | 17,304 | 360 | 17 | .215 |
| .790 | 28226 | 1,125 | 22 | .450653 | 17,664 | 377 | 21 | .210 |
| .795 | 27101 | 1,102 | 22 | .432989 | 18,041 | 398 | 21 | .205 |
| 0.800 | 0.00025998 | −1,081 | 22 | $\bar{4}$.414948 | −18,439 | −419 | −22 | 0.200 |
| .805 | 24918 | 1,059 | 22 | .396509 | 18,858 | 441 | 26 | .195 |
| .810 | 23859 | 1,037 | 22 | .377651 | 19,299 | 467 | 26 | .190 |
| .815 | 22822 | 1,015 | 22 | .358352 | 19,766 | 493 | 30 | .185 |
| .820 | 21806 | 994 | 22 | .338586 | 20,259 | 523 | 32 | .180 |
| 0.825 | 0.00020813 | −972 | 21 | $\bar{4}$.318327 | −20,782 | −555 | −34 | 0.175 |
| .830 | 19840 | 951 | 21 | .297545 | 21,337 | 589 | 38 | .170 |
| .835 | 18889 | 930 | 21 | .276208 | 21,926 | 627 | 43 | .165 |
| .840 | 17959 | 909 | 21 | .254282 | 22,553 | 670 | 45 | .160 |
| .845 | 17050 | 888 | 21 | .231729 | 23,223 | 715 | 51 | .155 |
| 0.850 | 0.000161624 | −8,668 | 210 | $\bar{4}$.208506 | −23,938 | −766 | −58 | 0.150 |
| .855 | 152956 | 8,458 | 210 | .184568 | 24,704 | 824 | 60 | .145 |
| .860 | 144498 | 8,248 | 209 | .159864 | 25,528 | 884 | 73 | .140 |
| .865 | 136250 | 8,039 | 208 | .134336 | 26,412 | 957 | 78 | .135 |
| .870 | 128211 | 7,831 | 210 | .107924 | 27,369 | 1,035 | 88 | .130 |
| 0.875 | 0.000120380 | −7,621 | 209 | $\bar{4}$.080555 | −28,404 | −1,123 | −102 | 0.125 |
| .880 | 112759 | 7,412 | 211 | .052151 | 29,527 | 1,225 | 114 | .120 |
| .885 | .000105347 | −7,201 | 210 | $\bar{4}$.022624 | 30,752 | 1,339 | 132 | .115 |
| .890 | .000098146 | 6,991 | 212 | $\bar{5}$.991872 | 32,091 | 1,471 | 151 | .110 |
| .895 | 91155 | 6,779 | 213 | .959781 | 33,562 | 1,622 | 178 | .105 |
| 0.900 | 0.000084376 | −6,566 | 214 | $\bar{5}$.926219 | −35,184 | −1,800 | −205 | 0.100 |
| .905 | 77810 | 6,352 | 216 | .891035 | 36,984 | 2,005 | 246 | .095 |
| .910 | 71458 | 6,136 | 219 | .854051 | 38,989 | 2,251 | 291 | .090 |
| .915 | 65322 | 5,917 | 220 | .815062 | 41,240 | 2,542 | 352 | .085 |
| .920 | 59405 | 5,697 | 224 | .773822 | 43,782 | −2,894 | −431 | .080 |
| 0.925 | 0.000053708 | −5,473 | 228 | $\bar{5}$.730040 | −46,676 | | | 0.075 |
| .930 | 48235 | 5,245 | 231 | .683364 | | | | .070 |
| .935 | 42990 | 5,014 | 236 | .633363 | | | | .065 |
| .940 | 37976 | 4,778 | 243 | .579504 | | | | .060 |
| .945 | 33198 | 4,535 | 250 | .521112 | | | | .055 |
| 0.950 | 0.000028663 | −4,285 | 257 | $\bar{5}$.457322 | | | | 0.050 |
| .955 | 24378 | 4,028 | 270 | .386992 | | | | .045 |
| .960 | 20350 | 3,758 | 282 | .308576 | | | | .040 |
| .965 | 16592 | 3,476 | 300 | .219908 | | | | .035 |
| .970 | 13116 | 3,176 | 321 | $\bar{5}$.117815 | | | | .030 |
| 0.975 | 0.000009940 | −2,855 | 354 | $\bar{6}$.997377 | | | | 0.025 |
| .980 | 7085 | 2,501 | 403 | .850353 | | | | .020 |
| .985 | 4584 | 2,098 | 488 | .661293 | | | | .015 |
| .990 | .000002486 | 1,610 | 734 | $\bar{6}$.395511 | | | | .010 |
| 0.995 | 0.000000876 | 876 | | $\bar{7}$.942327 | | | | .005 |
| 1.000 | 0 | | | | | | | 0.000 |

TABLE 2.—*Auxiliary table for circles very close together*

$k'^2 \leqq 0.1$

| $\log_{10} k'^2$ | $f$ | Difference | $\log_{10} k'^2$ | $f$ | Difference | |
|---|---|---|---|---|---|---|
| $\bar{6}.0$ | 0.079093 | −1,446 | $\bar{4}.5$ | 0.042938 | −1,444 | |
| $\bar{6}.1$ | 77647 | 1,447 | $\bar{4}.6$ | 41494 | 1,443 | |
| $\bar{6}.2$ | 76200 | 1,447 | $\bar{4}.7$ | 40051 | 1,443 | |
| $\bar{6}.3$ | 74753 | 1,447 | $\bar{4}.8$ | 38608 | 1,441 | |
| $\bar{6}.4$ | 73306 | 1,446 | $\bar{4}.9$ | 37167 | 1,440 | |
| $\bar{6}.5$ | 0.071860 | −1,447 | $\bar{3}.0$ | 0.035727 | −1,439 | |
| $\bar{6}.6$ | 70413 | 1,447 | $\bar{3}.1$ | 34288 | 1,437 | |
| $\bar{6}.7$ | 68966 | 1,446 | $\bar{3}.2$ | 32851 | 1,435 | |
| $\bar{6}.8$ | 67520 | 1,447 | $\bar{3}.3$ | 31416 | 1,432 | |
| $\bar{6}.9$ | 66073 | 1,446 | $\bar{3}.4$ | 29984 | 1,430 | |
| $\bar{5}.0$ | 0.064626 | −1,446 | $\bar{3}.5$ | 0.028554 | −1,426 | 4 |
| $\bar{5}.1$ | 63180 | 1,447 | $\bar{3}.6$ | 27128 | 1,422 | 5 |
| $\bar{5}.2$ | 61733 | 1,446 | $\bar{3}.7$ | 25707 | 1,416 | 6 |
| $\bar{5}.3$ | 60287 | 1,446 | $\bar{3}.8$ | 24291 | 1,410 | 7 |
| $\bar{5}.4$ | 58840 | 1,446 | $\bar{3}.9$ | 22881 | 1,403 | 9 |
| $\bar{5}.5$ | 0.057394 | −1,447 | $\bar{2}.0$ | 0.021478 | −1,394 | 10 |
| $\bar{5}.6$ | 55947 | 1,447 | $\bar{2}.1$ | 20084 | 1,384 | 12 |
| $\bar{5}.7$ | 54500 | 1,445 | $\bar{2}.2$ | 18700 | 1,371 | 14 |
| $\bar{5}.8$ | 53055 | 1,446 | $\bar{2}.3$ | 17329 | 1,357 | 17 |
| $\bar{5}.9$ | 51609 | 1,446 | $\bar{2}.4$ | 15972 | 1,340 | 19 |
| $\bar{4}.0$ | 0.050163 | −1,446 | $\bar{2}.5$ | 0.014632 | −1,321 | 23 |
| $\bar{4}.1$ | 48717 | 1,445 | $\bar{2}.6$ | 13311 | 1,298 | 27 |
| $\bar{4}.2$ | 47272 | 1,445 | $\bar{2}.7$ | 12013 | 1,271 | 31 |
| $\bar{4}.3$ | 45827 | 1,445 | $\bar{2}.8$ | 10742 | 1,240 | 35 |
| $\bar{4}.4$ | 44382 | 1,444 | $\bar{2}.9$ | 9502 | 1,205 | |
| $\bar{4}.5$ | 0.042398 | −1,444 | $\bar{1}.0$ | 0.008297 | ........ | |

For still smaller values of $k'^2$ use the formula

$$f=0.0144676\left(\log_{10}\frac{1}{k'^2}-0.533068\right)$$

TABLE 3.—*Auxiliary table for circles very far apart*

$k^2 \leqq 0.1$

| $\log_{10} k^2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ | $\log_{10} k^2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|---|---|---|---|---|
| $\bar{1}.0$ | $\bar{5}.926219$ | −157,230 | +1,581 | −365 | $\bar{3}.5$ | $\bar{7}.643272$ | −150,213 | 44 | −8 |
| $\bar{2}.9$ | .768989 | 155,649 | 1,216 | 268 | $\bar{3}.4$ | .493059 | 150,169 | 36 | 10 |
| $\bar{2}.8$ | .613340 | 154,433 | 948 | 210 | $\bar{3}.3$ | .342890 | 150,133 | 26 | 3 |
| $\bar{2}.7$ | .458907 | 153,485 | 738 | 160 | $\bar{3}.2$ | $\bar{7}.192757$ | 150,107 | 23 | 7 |
| $\bar{2}.6$ | .305422 | 152,747 | 578 | 122 | $\bar{3}.1$ | $\bar{7}.042650$ | 150,084 | 16 | 1 |
| $\bar{2}.5$ | $\bar{5}.152675$ | −152,169 | 456 | −100 | $\bar{3}.0$ | $\bar{8}.892566$ | −150,068 | 15 | ........ |
| $\bar{2}.4$ | $\bar{5}.000506$ | 151,713 | 356 | 74 | $\bar{4}.9$ | .742498 | 150,053 | 11 | ........ |
| $\bar{2}.3$ | $\bar{6}.848793$ | 151,357 | 282 | 58 | $\bar{4}.8$ | .592445 | 150,042 | 9 | ........ |
| $\bar{2}.2$ | .697436 | 151,075 | 224 | 47 | $\bar{4}.7$ | .442403 | 150,033 | 6 | ........ |
| $\bar{2}.1$ | .546361 | 150,851 | 177 | 38 | $\bar{4}.6$ | .292370 | 150,027 | 6 | ........ |
| $\bar{2}.0$ | $\bar{6}.395510$ | −150,674 | 139 | −29 | $\bar{4}.5$ | $\bar{8}.142343$ | −150,021 | 4 | ........ |
| $\bar{3}.9$ | .244836 | 150,535 | 110 | 22 | $\bar{4}.4$ | $\bar{9}.992322$ | 150,017 | 3 | ........ |
| $\bar{3}.8$ | $\bar{6}.094301$ | 150,425 | 88 | 18 | $\bar{4}.3$ | .852305 | 150,014 | 3 | ........ |
| $\bar{3}.7$ | $\bar{7}.943876$ | 150,337 | 70 | 16 | $\bar{4}.2$ | .692291 | 150,011 | 4 | ........ |
| $\bar{3}.6$ | .793539 | 150,267 | 54 | 10 | $\bar{4}.1$ | .542280 | −150,007 | ........ | ........ |
| | | | | | $\bar{4}.0$ | $\bar{9}.392273$ | ........ | ........ | ........ |

For still smaller values of $k^2$ use the formula

$$\log_{10} f=\bar{3}.392240+\frac{3}{2}\log_{10} k^2$$

TABLE 4.—*Values of f for equal circles near together*

$$\delta=\frac{\text{distance}}{\text{diameter}}\leqq 1$$

| δ | f | $d_1$ | $d_2$ | $d_3$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.050164 | −8, 697 | | | $\bar{2}$.700389 | | | |
| .02 | 41467 | 5, 075 | | | .617698 | | | |
| .03 | 36392 | 3, 590 | | | .560999 | | | |
| .04 | 32802 | 2, 773 | | | .515900 | | | |
| 0.05 | 0.030029 | −2, 256 | + | | $\bar{2}$.477537 | | | |
| .06 | 27773 | 1, 896 | 262 | −60 | .443629 | −30, 724 | 2, 400 | −517 |
| .07 | 25876 | 1, 634 | 202 | 42 | .412905 | 28, 324 | 1, 883 | 365 |
| .08 | 24243 | 1, 432 | 160 | 30 | .384581 | 26, 441 | 1, 518 | 264 |
| .09 | 22811 | 1, 272 | 130 | 23 | .358140 | 24, 923 | 1, 254 | 203 |
| 0.10 | 0.021539 | −1, 142 | 107 | −16 | $\bar{2}$.333217 | −23, 669 | 1, 051 | −148 |
| .11 | 20396 | 1, 035 | 91 | 13 | .309548 | 22, 618 | 903 | 127 |
| .12 | 19361 | 944 | 77 | 11 | .286930 | 21, 715 | 776 | 95 |
| .13 | 18417 | 867 | 67 | 8 | .265215 | 20, 939 | 681 | 79 |
| .14 | 17550 | 800 | 59 | 7 | .244276 | 20, 258 | 602 | 70 |
| 0.15 | 0.016750 | −741 | 52 | −6 | $\bar{2}$.224018 | −19, 656 | 532 | −50 |
| .16 | 16009 | 690 | 46 | 5 | .204362 | 19, 124 | 482 | 48 |
| .17 | 15319 | 643 | 41 | 4 | .185238 | 18, 642 | 434 | 44 |
| .18 | 14676 | 603 | 37 | 3 | .166596 | 18, 208 | 390 | 28 |
| .19 | 14073 | 566 | 34 | 3 | .148388 | 17, 818 | 362 | 32 |
| 0.20 | 0.0135073 | −5, 321 | 304 | −26 | $\bar{2}$.130570 | −17, 456 | 330 | −27 |
| .21 | 129752 | 5, 017 | 278 | 24 | .113114 | 17, 126 | 303 | 23 |
| .22 | 124735 | 4, 739 | 254 | 19 | .095988 | 16, 823 | 280 | 17 |
| .23 | 119996 | 4, 485 | 235 | 18 | .079165 | 16, 543 | 263 | 18 |
| .24 | 115511 | 4, 250 | 217 | 17 | .062623 | 16, 280 | 245 | 18 |
| 0.25 | 0.0111261 | −4, 033 | 200 | −13 | $\bar{2}$.046343 | −16, 035 | 227 | −11 |
| .26 | 107228 | 3, 833 | 187 | 14 | .030308 | 15, 808 | 216 | 15 |
| .27 | .0103395 | 3, 646 | 173 | 11 | $\bar{2}$.014500 | 15, 592 | 201 | 11 |
| .28 | .0099749 | 3, 473 | 162 | 9 | $\bar{3}$.998908 | 15, 391 | 190 | 6 |
| .29 | 96276 | 3, 312 | 153 | 13 | .983517 | 15, 201 | 184 | 15 |
| 0.30 | 0.0092964 | −3, 159 | 140 | 6 | $\bar{3}$.968316 | −15, 017 | 169 | 5 |
| .31 | 89805 | 3, 019 | 134 | 9 | .953299 | 14, 848 | 164 | 8 |
| .32 | 86786 | 2, 885 | 125 | 8 | .938451 | 14, 684 | 156 | 7 |
| .33 | 83901 | 2, 760 | 117 | 5 | .923767 | 14, 528 | 149 | 6 |
| .34 | 81141 | 2, 643 | 112 | 8 | .909239 | 14, 379 | 143 | 5 |
| 0.35 | 0.0078498 | −2, 531 | 104 | 4 | $\bar{3}$.894860 | −14, 236 | 138 | 8 |
| .36 | 75967 | 2, 427 | 100 | 7 | .880624 | 14, 098 | 130 | 2 |
| .37 | 73540 | 2, 327 | 93 | 3 | .866526 | 13, 968 | 128 | 4 |
| .38 | 71213 | 2, 234 | 90 | 7 | .852558 | 13, 840 | 124 | |
| .39 | 68979 | 2, 144 | 83 | 2 | .838718 | 13, 716 | 115 | |
| 0.40 | 0.0066835 | −2, 061 | 81 | | $\bar{3}$.825002 | −13, 601 | 115 | |
| .41 | 64774 | 1, 980 | 76 | | .811401 | 13, 486 | 114 | |
| .42 | 62794 | 1, 904 | 72 | | .797915 | 13, 372 | 108 | |
| .43 | 60890 | 1, 832 | 69 | | .784543 | 13, 264 | 102 | |
| .44 | 59058 | 1, 763 | 66 | | .771279 | 13, 162 | 104 | |
| 0.45 | 0.0057295 | −1, 697 | 62 | | $\bar{3}$.758117 | −13, 058 | 99 | |
| .46 | 55598 | 1, 634 | 60 | | .745059 | 12, 959 | 97 | |
| .47 | 53964 | 1, 574 | 58 | | .732100 | 12, 862 | 95 | |
| .48 | 52389 | 1, 518 | 55 | | .719238 | 12, 767 | 92 | |
| .49 | 50871 | 1, 463 | 52 | | .706471 | 12, 675 | 92 | |
| 0.50 | 0.0049408 | −1, 411 | 49 | | $\bar{3}$.693796 | −12, 583 | 86 | |
| .51 | 47997 | 1, 362 | 49 | | .681213 | 12, 497 | 90 | |
| .52 | 46635 | 1, 313 | 45 | | .668716 | 12, 407 | 83 | |
| .53 | 45322 | 1, 268 | 44 | | .656309 | 12, 324 | 86 | |
| .54 | 44054 | 1, 224 | 42 | | .643985 | 12, 238 | 80 | |
| 0.55 | 0.0042830 | −1, 182 | 40 | | $\bar{3}$.631747 | −12, 158 | 83 | |
| .56 | 41648 | 1, 142 | 38 | | .619589 | 12, 075 | 78 | |
| .57 | 40506 | 1, 104 | 36 | | .607514 | 11, 997 | 79 | |
| .58 | 39402 | 1, 067 | 36 | | .595517 | 11, 918 | 76 | |
| .59 | 38335 | 1, 031 | 34 | | .583599 | 11, 842 | 77 | |
| 0.60 | 0.0037304 | −997 | 33 | | $\bar{3}$.571757 | −11, 765 | 75 | |
| .61 | 36307 | 964 | 31 | | .559992 | 11, 690 | 73 | |
| .62 | 35343 | 933 | 30 | | .548302 | 11, 617 | 73 | |
| .63 | 34410 | 903 | 29 | | .536685 | 11, 544 | 71 | |
| .64 | 33507 | 874 | 28 | | .525141 | 11, 473 | 72 | |

TABLE 4.—*Values of f for equal circles near together*—Continued

$$\delta = \frac{\text{distance}}{\text{diameter}} \leqq 1$$

| $\delta$ | $f$ | $d_1$ | $d_2$ | $d_3$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|---|---|---|---|
| 0.65 | 0.0032634 | −846 | 27 | ---------- | $\bar{3}$.513668 | −11,401 | 67 | ---------- |
| .66 | 31788 | 819 | 26 | ---------- | .502267 | 11,334 | 71 | ---------- |
| .67 | 30969 | 793 | 25 | ---------- | .490933 | 11,263 | 72 | ---------- |
| .68 | 30176 | 768 | 24 | ---------- | .479670 | 11,191 | 64 | ---------- |
| .69 | 29409 | 744 | 23 | ---------- | .468479 | 11,127 | 68 | ---------- |
| 0.70 | 0.0028665 | −721 | 22 | ---------- | $\bar{3}$.457352 | −11,059 | 65 | ---------- |
| .71 | 27944 | 699 | 22 | ---------- | .446293 | 10,994 | 66 | ---------- |
| .72 | 27246 | 677 | 21 | ---------- | .435299 | 10,928 | 66 | ---------- |
| .73 | 26569 | 656 | 20 | ---------- | .424371 | 10,862 | 61 | ---------- |
| .74 | 25912 | 636 | 19 | ---------- | .413509 | 10,801 | 65 | ---------- |
| 0.75 | 0.00252760 | −6,172 | 186 | ---------- | $\bar{3}$.402708 | −10,736 | 64 | ---------- |
| .76 | 246588 | 5,986 | 178 | ---------- | .391972 | 10,672 | 59 | ---------- |
| .77 | 240602 | 5,808 | 173 | ---------- | .381300 | 10,613 | 63 | ---------- |
| .78 | 234794 | 5,635 | 167 | ---------- | .370687 | 10,550 | 62 | ---------- |
| .79 | 229159 | 5,468 | 162 | ---------- | .360137 | 10,488 | 61 | ---------- |
| 0.80 | 0.00223691 | −5,306 | 153 | ---------- | $\bar{3}$.349649 | −10,427 | 56 | ---------- |
| .81 | 218385 | 5,153 | 151 | ---------- | .339222 | 10,371 | 63 | ---------- |
| .82 | 213232 | 5,002 | 144 | ---------- | .328851 | 10,308 | 56 | ---------- |
| .83 | 208230 | 4,858 | 140 | ---------- | .318543 | 10,252 | 59 | ---------- |
| .84 | 203372 | 4,718 | 136 | ---------- | .308291 | 10,193 | 57 | ---------- |
| 0.85 | 0.00198654 | −4,582 | 131 | ---------- | $\bar{3}$.298098 | −10,136 | 60 | ---------- |
| .86 | 194072 | 4,451 | 127 | ---------- | .287962 | 10,076 | 57 | ---------- |
| .87 | 189621 | 4,324 | 119 | ---------- | .277886 | 10,019 | 51 | ---------- |
| .88 | 185297 | 4,205 | 120 | ---------- | .267867 | 9,968 | 61 | ---------- |
| .89 | 181092 | 4,085 | 114 | ---------- | .257899 | 9,907 | 51 | ---------- |
| 0.90 | 0.00177007 | −3,971 | 111 | ---------- | $\bar{3}$.247992 | −9,856 | 57 | ---------- |
| .91 | 173036 | 3,860 | 106 | ---------- | .238136 | 9,799 | 55 | ---------- |
| .92 | 169176 | 3,754 | 103 | ---------- | .228337 | 9,744 | 52 | ---------- |
| .93 | 165422 | 3,651 | 101 | ---------- | .218593 | 9,692 | 55 | ---------- |
| .94 | 161771 | 3,550 | 96 | ---------- | .208901 | 9,637 | 53 | ---------- |
| 0.95 | 0.00158221 | −3,454 | 95 | ---------- | $\bar{3}$.199264 | −9,584 | 53 | ---------- |
| .96 | 154767 | 3,359 | 89 | ---------- | .189680 | 9,531 | 48 | ---------- |
| .97 | 151408 | 3,270 | 88 | ---------- | .180149 | 9,483 | 54 | ---------- |
| .98 | 148136 | 3,182 | 86 | ---------- | .170666 | 9,429 | 51 | ---------- |
| 0.99 | 144956 | 3,096 | ---------- | ---------- | .161237 | 9,378 | ---------- | ---------- |
| 1.00 | 0.00141860 | ---------- | ---------- | ---------- | $\bar{3}$.151859 | ---------- | ---------- | ---------- |

TABLE 5.—*Auxiliary table for equal circles very near together*

$\delta \leqq 0.2$

| $\log_{10}\delta$ | $f$ | $d_1$ | $d_2$ | $\log_{10}\delta$ | $f$ | $d_1$ | $d$ |
|---|---|---|---|---|---|---|---|
| $\bar{4}.0$ | 0.108028 | −2,894 | -------- | $\bar{2}.0$ | 0.050164 | −2,891 | 1 |
| $\bar{4}.1$ | .105135 | 2,894 | -------- | $\bar{2}.1$ | .047273 | 2,890 | 3 |
| $\bar{4}.2$ | .102241 | 2,893 | -------- | $\bar{2}.2$ | .044383 | 2,887 | 3 |
| $\bar{4}.3$ | .099348 | 2,894 | -------- | $\bar{2}.3$ | .041496 | 2,884 | 5 |
| $\bar{4}.4$ | .096454 | 2,894 | -------- | $\bar{2}.4$ | .038612 | 2,879 | 7 |
| $\bar{4}.5$ | 0.093561 | −2,893 | -------- | $\bar{2}.5$ | 0.035733 | −2,872 | 10 |
| $\bar{4}.6$ | .090667 | 2,893 | -------- | $\bar{2}.6$ | .032861 | 2,862 | 15 |
| $\bar{4}.7$ | .087774 | 2,893 | -------- | $\bar{2}.7$ | .029999 | 2,846 | 22 |
| $\bar{4}.8$ | .084881 | 2,894 | -------- | $\bar{2}.8$ | .027153 | 2,824 | 33 |
| $\bar{4}.9$ | .081987 | 2,893 | -------- | $\bar{2}.9$ | .024330 | 2,791 | 47 |
| $\bar{3}.0$ | 0.079093 | −2,893 | -------- | $\bar{1}.0$ | 0.021538 | −2,744 | 67 |
| $\bar{3}.1$ | .076200 | 2,894 | -------- | $\bar{1}.1$ | .018795 | 2,677 | 93 |
| $\bar{3}.2$ | .073306 | 2,893 | -------- | $\bar{1}.2$ | .016117 | 2,584 | -------- |
| $\bar{3}.3$ | .070413 | 2,893 | -------- | $\bar{1}.3$ | 0.013533 | ------------ | -------- |
| $\bar{3}.4$ | .067520 | 2,893 | -------- | | | | |
| $\bar{3}.5$ | 0.064627 | −2,893 | -------- | | | | |
| $\bar{3}.6$ | .061733 | 2,893 | -------- | | | | |
| $\bar{3}.7$ | .058840 | 2,892 | -------- | | | | |
| $\bar{3}.8$ | .055948 | 2,892 | -------- | | | | |
| $\bar{3}.9$ | .053055 | 2,891 | -------- | | | | |

For still smaller values of $\vartheta$, use the formula

$$=0.0299352 \left(\log_{10}\frac{1}{\delta}-0.266535\right)$$

TABLE 6.—*Values of f for equal circles far apart*

$$\Delta = \frac{\text{diameter}}{\text{distance}} \leqq 1$$

| Δ | $f$ | $d_1$ | $d_2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|---|---|---|
| 1.00 | 0.00141860 | −3,044 | 21 | $\bar{3}$.151859 | −9,420 | −140 | ---------- |
| 0.99 | 138816 | 3,023 | 23 | .142439 | 9,560 | 145 | ---------- |
| .98 | 135793 | 3,000 | 23 | .132879 | 9,705 | 141 | ---------- |
| .97 | 132793 | 2,977 | 23 | .123174 | 9,846 | 150 | ---------- |
| .96 | 129816 | 2,954 | 24 | .113328 | 9,996 | 152 | ---------- |
| 0.95 | 0.00126862 | −2,930 | 24 | $\bar{3}$.103332 | −10,148 | −158 | ---------- |
| .94 | 123932 | 2,906 | 25 | .093184 | 10,306 | 157 | ---------- |
| .93 | 121026 | 2,881 | 25 | .082878 | 10,463 | 164 | ---------- |
| .92 | 118145 | 2,856 | 25 | .072415 | 10,627 | 170 | ---------- |
| .91 | 115289 | 2,831 | 28 | .061788 | 10,797 | 166 | ---------- |
| 0.90 | 0.00112458 | −2,803 | 25 | $\bar{3}$.050991 | −10,963 | −180 | ---------- |
| .89 | 109655 | 2,778 | 29 | .040028 | 11,143 | 174 | ---------- |
| .88 | 106877 | 2,749 | 28 | .028885 | 11,317 | 185 | ---------- |
| .87 | 104128 | 2,721 | 27 | .017568 | 11,502 | 191 | ---------- |
| .86 | .00101407 | 2,694 | 29 | $\bar{3}$.006066 | 11,693 | 192 | ---------- |
| 0.85 | 0.00098713 | −2,665 | 29 | $\bar{4}$.994373 | −11,885 | −201 | ---------- |
| .84 | 96048 | 2,636 | 31 | .982488 | 12,086 | 196 | ---------- |
| .83 | 93412 | 2,605 | 30 | .970402 | 12,282 | 213 | ---------- |
| .82 | 90807 | 2,575 | 31 | .958120 | 12,495 | 210 | ---------- |
| .81 | 88232 | 2,544 | 31 | .945625 | 12,705 | 220 | ---------- |
| 0.80 | 0.00085688 | −2,513 | 33 | $\bar{4}$.932920 | −12,925 | −226 | ---------- |
| .79 | 83175 | 2,480 | 31 | .919995 | 13,151 | 232 | ---------- |
| .78 | 80695 | 2,449 | 34 | .906846 | 13,383 | 232 | ---------- |
| .77 | 78246 | 2,415 | 33 | .893463 | 13,616 | 245 | ---------- |
| .76 | 75831 | 2,382 | 34 | .879848 | 13,860 | 252 | ---------- |
| 0.75 | 0.00073449 | −2,348 | 35 | $\bar{4}$.865988 | −14,112 | −253 | ---------- |
| .74 | 71101 | 2,313 | 34 | .851876 | 14,365 | 265 | ---------- |
| .73 | 68788 | 2,279 | 36 | .837511 | 14,630 | 273 | ---------- |
| .72 | 66509 | 2,243 | 35 | .822881 | 14,903 | 279 | ---------- |
| .71 | 64266 | 2,208 | 36 | .807978 | 15,182 | 289 | ---------- |
| 0.70 | 0.00062058 | −2,172 | 37 | $\bar{4}$.792796 | −15,471 | −292 | ---------- |
| .69 | 59886 | 2,135 | 38 | .777325 | 15,763 | 305 | ---------- |
| .68 | 57751 | 2,097 | 36 | .761562 | 16,068 | 317 | ---------- |
| .67 | 55654 | 2,061 | 38 | .745494 | 16,385 | 321 | ---------- |
| .66 | 53593 | 2,022 | 38 | .729109 | 16,706 | 335 | ---------- |
| 0.65 | 0.00051570 | −1,984 | 39 | $\bar{4}$.712403 | −17,041 | −342 | ---------- |
| .64 | 49586 | 1,946 | 39 | .695362 | 17,383 | 356 | ---------- |
| .63 | 47641 | 1,907 | 39 | .677979 | 17,739 | 365 | ---------- |
| .62 | 45734 | 1,867 | 40 | .660240 | 18,104 | 377 | ---------- |
| .61 | 43887 | 1,827 | 40 | .642136 | 18,481 | 392 | ---------- |
| 0.60 | 0.00042039 | −1,788 | 40 | $\bar{4}$.623655 | −18,873 | −403 | ---------- |
| .59 | 40252 | 1,748 | 41 | .604782 | 19,276 | 418 | ---------- |
| .58 | 38504 | 1,707 | 41 | .585506 | 19,694 | 432 | ---------- |
| .57 | 36797 | 1,666 | 41 | .565812 | 20,126 | 448 | ---------- |
| .56 | 35131 | 1,625 | 41 | .545686 | 20,574 | 460 | ---------- |
| 0.55 | 0.00033505 | −1,584 | 41 | $\bar{4}$.525112 | −21,034 | −483 | ---------- |
| .54 | 31921 | 1,543 | 41 | .504078 | 21,517 | 498 | ---------- |
| .53 | 30378 | 1,502 | 41 | .482561 | 22,015 | 517 | ---------- |
| .52 | 28877 | 1,460 | 42 | .460546 | 22,532 | 534 | ---------- |
| .51 | 27417 | 1,418 | 41 | .438014 | 23,066 | 561 | ---------- |
| 0.50 | 0.000259985 | −13,766 | 418 | $\bar{4}$.414948 | −23,627 | − 578 | −26 |
| .49 | 246219 | 13,348 | 419 | .391321 | 24,205 | 604 | 24 |
| .48 | 232871 | 12,929 | 416 | .367116 | 24,809 | 628 | 29 |
| .47 | 219942 | 12,513 | 417 | .342307 | 25,437 | 657 | 33 |
| .46 | 207429 | 12,096 | 416 | .316870 | 26,094 | 684 | 31 |
| 0.45 | 0.000195333 | −11,680 | 414 | $\bar{4}$.290776 | −26,778 | − 715 | −30 |
| .44 | 183653 | 11,266 | 414 | .263998 | 27,493 | 745 | 39 |
| .43 | 172387 | 10,852 | 411 | .236505 | 28,238 | 784 | 35 |
| .42 | 161535 | 10,441 | 407 | .208267 | 29,022 | 819 | 41 |
| .41 | 151094 | 10,034 | 408 | .179245 | 29,841 | 860 | 38 |

TABLE 6.—*Values of f for equal circles far apart*—Continued

$$\Delta=\frac{\text{diameter}}{\text{distance}}\lesseqgtr 1$$

| Δ | f | $d_1$ | $d_2$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|---|---|---|
| 0.40 | 0.000141060 | − 9, 626 | 401 | $\bar{4}$.149404 | −30, 701 | − 898 | −58 |
| .39 | 131434 | 9, 225 | 399 | .118703 | 31, 599 | 956 | 58 |
| .38 | 122209 | − 8, 826 | 397 | .087104 | 32, 555 | 994 | 60 |
| .37 | 113383 | 8, 429 | 391 | .054549 | 33, 549 | 1, 054 | 61 |
| .36 | .000104954 | 8, 038 | 387 | $\bar{4}$.021000 | 34, 603 | 1, 115 | 59 |
| 0.35 | 0.000096916 | − 7, 651 | 381 | $\bar{5}$.986397 | −35, 718 | −1, 174 | −75 |
| .34 | 89265 | 7, 270 | 376 | .950769 | 36, 892 | 1, 249 | 71 |
| .33 | 81995 | 6, 894 | 370 | .913787 | 38, 143 | 1, 320 | 89 |
| .32 | 75101 | 6, 524 | 365 | .875644 | 39, 463 | 1, 409 | 94 |
| .31 | 68577 | 6, 159 | 356 | .836181 | 40, 872 | 1, 503 | 100 |
| 0.30 | 0.000062418 | − 5, 803 | 351 | $\bar{5}$.795309 | −42, 375 | −1, 603 | −113 |
| .29 | 56615 | 5, 452 | 342 | .752934 | 43, 978 | 1, 716 | 128 |
| .28 | 51163 | 5, 110 | 335 | .708956 | 45, 694 | 1, 844 | 141 |
| .27 | 46053 | 4, 775 | 327 | .663262 | 47, 538 | 1, 985 | 158 |
| .26 | 41278 | 4, 448 | 317 | .615724 | 49, 523 | 2, 143 | 172 |
| 0.25 | 0.000036830 | − 4, 131 | 309 | $\bar{5}$.566201 | −51, 666 | −2, 316 | −211 |
| .24 | 32699 | 3, 822 | 299 | .514535 | 53, 982 | 2, 527 | 224 |
| .23 | 28877 | 3, 523 | 289 | .460553 | 56, 509 | 2, 751 | 263 |
| .22 | 25354 | 3, 234 | 279 | .404044 | 59, 260 | 3, 014 | 312 |
| .21 | 22120 | 2, 955 | 268 | .344784 | 62, 274 | 3, 326 | 344 |
| 0.20 | 0.0000191650 | −26, 868 | 2, 574 | $\bar{5}$.282510 | −65, 600 | −3, 670 | −418 |
| .19 | 164782 | 24, 294 | 2, 460 | .216910 | 69, 270 | 4, 088 | 489 |
| .18 | 140488 | 21, 834 | 2, 342 | .147640 | 73, 358 | 4, 577 | 582 |
| .17 | .0000118654 | 19, 492 | 2, 224 | $\bar{5}$.074282 | 77, 935 | 5, 159 | 710 |
| .16 | .0000099162 | 18, 268 | 2, 099 | $\bar{6}$.996347 | 83, 094 | 5, 869 | 855 |
| 0.15 | 0.0000081894 | −15, 169 | 1, 975 | $\bar{6}$.913253 | −88, 963 | −6, 724 | −1, 073 |
| .14 | 66725 | 13, 194 | 1, 844 | $\bar{6}$.824290 | ---- | ---- | ---- |
| .13 | 53531 | 11, 350 | 1, 715 | .728603 | ---- | ---- | ---- |
| .12 | 42181 | 9, 635 | 1, 579 | .625119 | ---- | ---- | ---- |
| .11 | 32546 | 8, 056 | 1, 445 | .512496 | ---- | ---- | ---- |
| 0.10 | 0.0000024490 | −6, 611 | 1, 305 | $\bar{6}$.388997 | ---- | ---- | ---- |
| .09 | 17879 | 5, 306 | 1, 166 | .252337 | ---- | ---- | ---- |
| .08 | .0000012572 | 4, 140 | 1, 024 | $\bar{6}$.099430 | ---- | ---- | ---- |
| .07 | .00000084322 | 3, 117 | 880 | $\bar{7}$.925941 | ---- | ---- | ---- |
| .06 | 53152 | 2, 237 | 735 | .725523 | ---- | ---- | ---- |
| 0.05 | 0.00000030785 | ---- | ---- | .488336 | ---- | ---- | ---- |
| .04 | 15772 | ---- | ---- | $\bar{7}$.197900 | ---- | ---- | ---- |
| .03 | 6657 | ---- | ---- | $\bar{8}$.823311 | ---- | ---- | ---- |
| .02 | 19733 | ---- | ---- | $\bar{8}$.295199 | ---- | ---- | ---- |
| .01 | 0.000000002467 | ---- | ---- | $\bar{9}$.392207 | ---- | ---- | ---- |
| 0.00 | 0 | ---- | ---- | ---- | ---- | ---- | ---- |

TABLE 7.—*Auxiliary table for equal circles very far apart*

$$\Delta \lesseqgtr 0.2$$

| $\log_{10}\Delta$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ | $\log_{10}\Delta$ | $\log_{10} f$ | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|---|---|---|---|---|---|
| $\bar{1}$.30 | $\bar{5}$.279478 | −147, 409 | −520 | 100 | $\bar{2}$.6 | $\bar{7}$.191724 | 299, 809 | 72 | 30 |
| $\bar{1}$.25 | $\bar{5}$.132069 | 147, 929 | 420 | 89 | $\bar{2}$.5 | $\bar{8}$.891915 | −299, 881 | −42 | 12 |
| $\bar{1}$.20 | $\bar{6}$.984140 | 148, 349 | 331 | 61 | $\bar{2}$.4 | .592034 | 299, 923 | 30 | 14 |
| $\bar{1}$.15 | .835791 | 148, 680 | 270 | 56 | $\bar{2}$.3 | $\bar{8}$.292111 | 299, 953 | 16 | 4 |
| $\bar{1}$.10 | .687111 | 148, 950 | 214 | ---- | $\bar{2}$.2 | $\bar{9}$.992158 | 299, 969 | 12 | 4 |
| $\bar{1}$.05 | .538161 | 149, 164 | −175 | ---- | $\bar{2}$.1 | .692189 | 299, 981 | 8 | 4 |
| $\bar{1}$.00 | $\bar{6}$.388997 | −298, 807 | −438 | 160 | $\bar{2}$.0 | $\bar{9}$.392208 | −299, 989 | −4 | 2 |
| $\bar{2}$.9 | $\bar{6}$.090190 | 299, 245 | 278 | 103 | $\bar{3}$.9 | $\bar{9}$.092219 | 299, 993 | 2 | ---- |
| $\bar{2}$.8 | $\bar{7}$.790945 | 299, 523 | 175 | 64 | $\bar{3}$.8 | $\overline{10}$.792226 | −299, 995 | ---- | ---- |
| $\bar{2}$.7 | .491422 | 299, 698 | 111 | 39 | $\bar{3}$.7 | $\overline{10}$.492231 | ---- | ---- | ---- |

For still smaller values of Δ use the formula

$$\log_{10} f=\bar{3}.392240+3\log_{10}\Delta$$

TABLE 8.—*Coefficients for use in interpolation*

| $K_1$ | $K_2$ | $K_3$ | $K_1$ | $K_2$ | $K_3$ |
|---|---|---|---|---|---|
| 0.00 | 0 | 0 | 0.50 | −0.12500 | 0.0625 |
| .01 | −0.00495 | +0.0033 | .51 | .12495 | +0.0621 |
| .02 | .00980 | .0065 | .52 | .12480 | .0616 |
| .03 | .01455 | .0096 | .53 | .12455 | .0611 |
| .04 | .01920 | .0125 | .54 | .12420 | .0605 |
| 0.05 | 0.02375 | 0.0154 | 0.55 | −0.12375 | 0.0599 |
| .06 | .02820 | .0182 | .56 | .12320 | .0592 |
| .07 | .03255 | .0210 | .57 | .12255 | .0585 |
| .08 | .03680 | .0236 | .58 | .12180 | .0577 |
| .09 | .04095 | .0260 | .59 | .12095 | .0569 |
| 0.10 | −0.04500 | 0.0285 | 0.60 | −0.12000 | 0.0560 |
| .11 | .04895 | .0308 | .61 | .11895 | .0552 |
| .12 | .05280 | .0331 | .62 | .11780 | .0542 |
| .13 | .05655 | .0353 | .63 | .11655 | .0533 |
| .14 | .06020 | .0374 | .64 | .11520 | .0523 |
| 0.15 | −0.06375 | 0.0393 | 0.65 | −0.11375 | 0.0513 |
| .16 | .06720 | .0412 | .66 | .11220 | .0501 |
| .17 | .07055 | .0430 | .67 | .11055 | .0490 |
| .18 | .07380 | .0448 | .68 | .10880 | .0479 |
| .19 | .07695 | .0465 | .69 | .10695 | .0467 |
| 0.20 | −0.08000 | 0.0480 | 0.77 | −0.10500 | 0.0455 |
| .21 | .08295 | .0495 | .71 | .10295 | .0443 |
| .22 | .08580 | .0508 | .72 | .10080 | .0430 |
| .23 | .08855 | .0522 | .73 | .09855 | .0417 |
| .24 | .09120 | .0535 | .74 | .09620 | .0404 |
| 0.25 | −0.09375 | 0.0547 | 0.75 | −0.09375 | 0.0391 |
| .26 | .09620 | .0559 | .76 | .09120 | .0378 |
| .27 | .09855 | .0569 | .77 | .08855 | .0363 |
| .28 | .10080 | .0578 | .78 | .08580 | .0348 |
| .29 | .10295 | .0587 | .79 | .08295 | .0335 |
| 0.30 | −0.10500 | 0.0596 | 0.80 | −0.08000 | 0.0320 |
| .31 | .10695 | .0603 | .81 | .07695 | .0305 |
| .32 | .10880 | .0610 | .82 | .07380 | .0290 |
| .33 | .11055 | .0616 | .83 | .07055 | .0275 |
| .34 | .11220 | .0622 | .84 | .06720 | .0260 |
| 0.35 | −0.11375 | 0.0626 | 0.85 | −0.06375 | 0.0244 |
| .36 | .11520 | .0630 | .86 | .06020 | .0228 |
| .37 | .11655 | .0634 | .87 | .05655 | .0213 |
| .38 | .11780 | .0637 | .88 | .05280 | .0197 |
| .39 | .11895 | .0639 | .89 | .04895 | .0181 |
| 0.40 | −0.12000 | 0.0640 | 0.90 | −0.04500 | 0.0165 |
| .41 | .12095 | .0642 | .91 | .04095 | .0149 |
| .42 | .12180 | .0643 | .92 | .03680 | .0132 |
| .43 | .12255 | .0643 | .93 | .03255 | .0116 |
| .44 | .12320 | .0642 | .94 | .02820 | .0100 |
| 0.45 | −0.12375 | 0.0640 | 0.95 | −0.02375 | 0.0083 |
| .46 | .12420 | .0637 | .96 | .01920 | .0066 |
| .47 | .12455 | .0635 | .97 | .01455 | .0050 |
| .48 | .12480 | .0633 | .98 | .00980 | .0033 |
| .49 | .12495 | .0630 | 0.99 | .00495 | .0017 |
| 0.50 | −0.12500 | 0.0625 | 1.00 | 0 | 0 |

WASHINGTON, May 26, 1924.