VISIBILITY OF RADIANT ENERGY.[1]

By K. S. Gibson and E. P. T. Tyndall.

ABSTRACT.

In cooperation with the Nela Research Laboratories a new determination of the visibility of radiant energy has been made by the step-by-step method, an equality-of-brightness method with little or no hue difference in the two parts of the photometric field. Between 490 and 680 mμ, inclusive, measurements were made upon 52 observers, some of them common to previous investigations; for 38 of these observers measurements were continued to include 430 and 740 mμ.

Energy values were based upon radiometric and spectrophotometric measurements made at the bureau, checked by an independent color temperature measurement at the Nela Research Laboratories. Luminosity values were obtained with a Brace spectrophotometer. The step was made by moving the collimator slit. The ratios of luminosities were measured with a Brodhun variable sector. The photometric field was the divided circle type subtending an angle of 3°. Brightnesses were safely above the Purkinje region except, perhaps, at the ends of the spectrum. Detailed comparisons are made between the individual and average results of this investigation and those of previous investigations.

There is no certain difference between the average values of visibility obtained by the flicker and equality-of-brightness methods, provided the former is used under the experimental conditions recommended by Ives and the latter does not depart too widely from these conditions.

A revision of the I. E. S. mean curve is proposed which results in better agreement with the average experimental visibility data and still gives the same wave length center of gravity for light of a color temperature of 2,077° K. as is given by Ives's physical photometer solution.

CONTENTS.

[1] This paper was presented, in part, at the Washington meeting of the Optical Society of America, October 27, 1922.

I. INTRODUCTION.

By the *visibility of radiant energy* is meant the ratio of luminosity to radiant power—the luminous efficiency—at the various wave lengths (or frequencies) in the spectrum; that is, $V_\lambda = L_\lambda / E_\lambda$. This may be expressed in absolute units—for example, in lumens per watt—but is more often considered from the relative standpoint with reference to unity at the maximum. This *relative visibility* has been measured by several investigators with a total number of observers that is now more than 250, and the number and extent of these investigations show the importance which has been attached to an accurate determination of this function.

It is obvious from the meaning of the term that the measurement of relative visibility necessitates the measurement of relative spectral luminosity and relative spectral radiant power. Since the measurement of relative luminosity can be made, directly or indirectly, only by some photometric method, any peculiarity of this method should be intimately connected with the form of the visibility curve.

There are but two methods of photometry which are in common use to-day. These are known as the equality-of-brightness method (simultaneous comparison) and the flicker method (alternate comparison). The equality-of-brightness method is the older, and for lights of the same quality of color (hue and saturation) is capable of greater precision than the other. The precision of measurement by the equality-of-brightness method, however, decreases as the difference in the color quality of the two lights increases, and for most observers the attempt to photometer lights of widely different quality—for example, white with red, green, or blue—is subject to enormous uncertainty. On the other hand, the precision by the flicker method is affected but little by differences in quality of color.

The measurement of relative spectral luminosity consists, essentially, in the determination of relative brightness at the various wave lengths of the spectrum. Because of the difficulty of making such measurements by the simple equality-of-brightness method relatively little information of present value has been thus obtained except at low brightnesses where the Purkinje phenomenon appears; nor does it seem possible to obtain any extensive reliable data by this method where large differences of hue exist in the two parts of the photometric field. Other methods have also been tried, known as the threshold-of-vision, the acuity-of-vision, and the critical-frequency methods, but for one reason or another these were not found suitable for such investigations as have been conducted during the last 10 years. Descriptions of these methods, with references to the original work, may be found in papers by Ives (1)[2] and by Coblentz and Emerson (2).

By means of the flicker photometer, however, several reliable investigations of the visibility of radiant energy have been made, the work of Ives (3), Nutting (4), Coblentz and Emerson (2), Reeves (5), and So (6) being the most notable; but in spite of these extensive data the nature of the visibility function was still somewhat indefinite. This was not only a result of the rather large differences obtained for the average visibility by these several investigators, but was also a consequence of the general question as to whether or not for lights of different color quality the flicker photometer gives measurements comparable with those obtained by the widely used equality-of-brightness method. That it does not, under certain circumstances, is readily granted by all. That it does, under certain specified conditions of field size and brightness and possibly other factors, is one of the points at issue.

Ives, who was the first to make an extensive investigation of visibility by the flicker method, prefaced his measurements by a study (1) of the effect of the field size and brightness upon the results obtained by the two methods. He came to the conclusion that a field size of approximately 2° and a brightness of approximately 2.5 millilamberts (7) (1) (3) were the essential conditions which brought the two methods into agreement. All the subsequent flicker measurements of visibility have been made with this size of field. Brightnesses have varied from that used by Nutting, somewhat greater than Ives's, to that used by Coblentz, about one-fourth of Ives's. As noted above, the differences between the results obtained in the several investigations are rather large, and it is not readily apparent from these data that these differences bear any relation to the brightnesses used.

The 2° field thus recommended by Ives is considerably smaller than the usual Lummer-Brodhun field so widely used in photometric measurements. It was therefore questionable whether the flicker visibility data could be legitimately employed in computations based upon photometric data obtained with the larger fields. Furthermore, granted it were correct to use data obtained by a different method and under different experimental conditions, the discrepancies among the various flicker data made it very difficult to decide upon any average visibility curve. For these and other

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[2] The figures given in parentheses here and throughout the text relate to the reference numbers in the Bibliography given at the end of the paper.

reasons discussed in detail in their paper, Hyde, Forsythe, and Cady made a new statistical determination of relative visibility. They used the step-by-step, or cascade, method, which is an equality-of-brightness method with little or no hue difference in the two parts of the photometric field, and chose their conditions—field size and brightness—similar to those used with the ordinary Lummer-Brodhun field. The average curve they obtained came within the extreme average values which previous investigators had obtained by the flicker method, though in the orange and red it was lower than all but one of the flicker curves. Their investigation showed the entire feasibility of the step-by-step method as a means of obtaining reliable average visibility data by equality-of-brightness photometry.

In connection with a contemplated committee report on heterochromatic photometry and related subjects, and in view of the fact that only the one reliable and extensive investigation of visibility had been made by the equality-of-brightness method as against several by the flicker method, Dr. Hyde, of the Nela Research Laboratories, as president of the U. S. National Committee of the International Commission on Illumination, requested the Bureau of Standards to make this new measurement of visibility by the step-by-step method, and the National Lamp Works of the General Electric Co. has contributed very generously to the financial support of the investigation.

The authors have had the advice of a committee appointed by the director of the bureau, consisting of Messrs. Skinner, Crittenden, and Priest. They are also indebted to M. C. Malamphy for assistance in the operation of the apparatus for the luminosity measurements and in the extensive computations connected with the investigation.[3] They would also thank all those who so readily gave of their time to the measurements.

The *step-by-step method* of measuring visibility is so called from the fact that the luminosity for an observer's eye under given conditions at any wave length is measured simultaneously relative to that at a slightly different wave length, this latter relative to that at a third wave length, and so on throughout the spectrum, the luminosity at each wave length being compared with that at a closely adjacent wave length and the size of step between wave lengths being so chosen that little or no hue difference is perceptible between any two wave lengths being compared. With these successive ratios of luminosity it is then possible to compute the

[3] They are further indebted to H. J. McNicholas for advice in connection with the study of precision; to C. L. Snow, who made most of the illustrations; and to Messrs. Keegan, Cromer, and Durr for assistance in the computations.

relative luminosity curve for the region of the spectrum studied. Reduction of this curve to an equal energy basis gives the visibility curve as usual.

It might at first seem that this method would require a prohibitive number of photometric settings, but as noted by other inves-

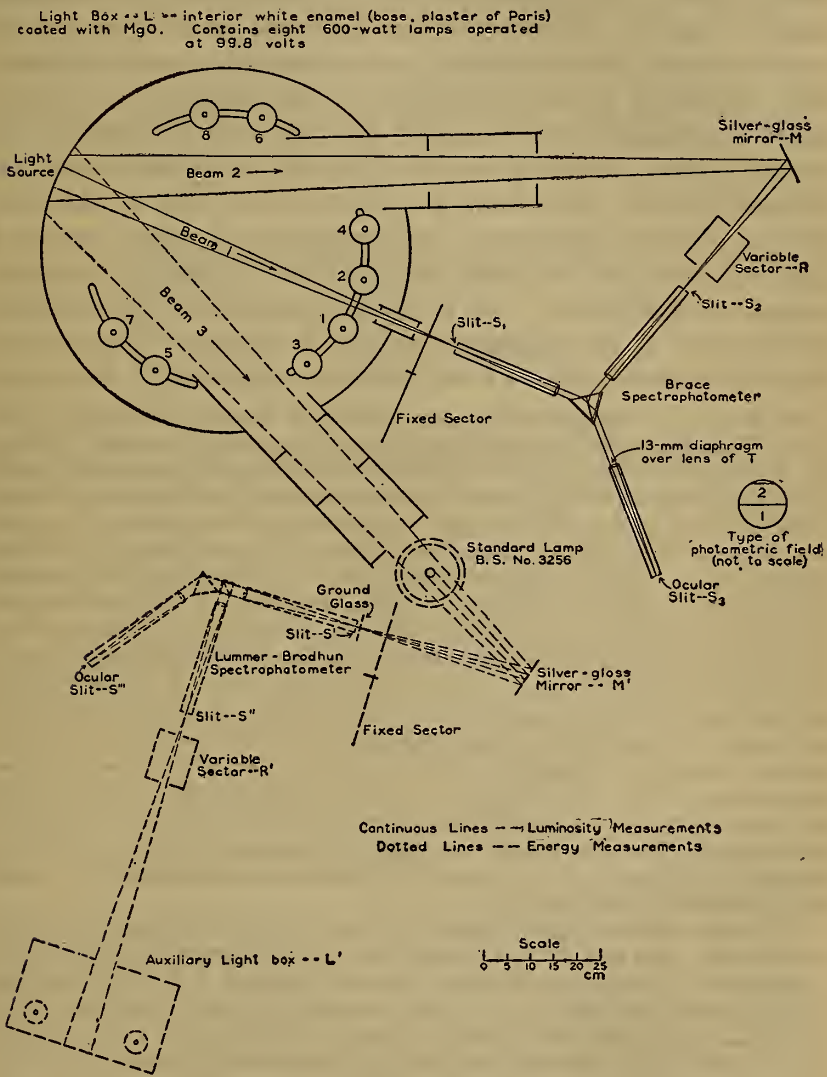

FIG. 1.—*Outline of apparatus used for the luminosity and energy measurements.*

tigators it is possible to take readings at a comparatively few wave lengths, plot a curve, and obtain the values of the ratios of luminosity from this curve. Various details of this method are discussed later.

II. AUXILIARY WORK.

1. APPARATUS.

The arrangement of the apparatus is outlined in Figure 1. As noted in the diagram, the continuous lines refer to the luminosity measurements, the dotted lines to the determination of radiant power or energy. Luminosity was measured on the Brace spectrophotometer, energy on the Lummer-Brodhun spectrophotometer. Variation in the brightness of the photometric field of either instrument was obtained by the rotating-rhomb-and-variable-sector device *R* or *R'*. The source of light was the diffusely reflected light from the interior back wall of the light box *L*. In the luminosity measurements the same source of light was thus used for both beams 1 and 2, a simple voltmeter control being sufficient. Various details concerning the construction and use of the apparatus, as well as methods of calibration and measurement, will be found below, or elsewhere in the paper.

The Brace (15) spectrophotometer was a Schmidt & Haensch instrument, kindly loaned to the bureau for this investigation by the University of Nebraska. Slits S_2 and S_3 were bilateral and accurately graduated in hundredths of millimeters. Slit S_1 was made of constant width, 0.20 mm, and was adjustable at right angles to its length in the focal plane of the collimator lens. This adjustment was controlled by a screw of ½-mm pitch with a drum divided into 100 parts. The focal length of the lenses was about 25 cm. The prism was of the usual type, two 30° flint-glass prisms cemented together with α-monobromonaphthalene, with a silver strip 6 mm wide along the middle from base to apex. It was set for minimum deviation at wave length 578 mμ. The index of refraction for the sodium D lines was 1.6492. The total length of spectrum between the He lines 447.1 and 706.5 mμ was about 21 mm. By raising the prism and putting a 13-mm diaphragm over the telescope lens a photometric field of the sort shown was obtained with a single dividing line between the parts. Its angular aperture was about 3°. Diaphragms in both collimators and in the telescope helped reduce the stray light. In addition, the prism was repolished and a new silver strip put in especially for this investigation, and the lenses and prism were kept carefully cleaned. The unavoidable stray light still remaining was accounted for as explained in Section II, 5. The use of the instrument in the luminosity measurements is described in Section III, 1.

The Lummer-Brodhun spectrophotometer (16) was used in the ordinary way with the contrast field. The slits were all bilateral and accurately graduated in hundredths of millimeters. The focal length of the lenses and dispersion of the prism were practically identical with those of the Brace spectrophotometer. Diaphragms in the collimators and telescope and a careful cleaning of all the optical surfaces reduced the stray light so far as possible; that remaining was eliminated by means of selective ray filters placed before the ocular slit. This was necessary only at wave lengths beyond 660 mμ and 480 mμ, approximately. The method of use of the instrument for the energy measurements will be understood by reference to Figure 1 and Section II, 2. It was also used to measure the spectral transmission of the Brace spectrophotometer, as described in Section II, 3.

The wave length calibration of the arbitrary scale of each instrument was made by means of various Hg, He, Ne, and Cu spectral lines. The equation of each calibration curve was obtained in the modified form of Hartmann's (17) empirical formula:

$$s=s_0+\frac{c}{\lambda-\lambda_0}$$

where s=scale reading, λ=wave length, and s_0, c, and λ_0 are constants. The constants were obtained by use of the wave lengths 706.5, 546.1, and 435.85 mμ and the corresponding scale readings. (Values of s computed at every 20 mμ for the Brace spectrophotometer checked the graphical values of s within 1 mμ between 390 and 750 mμ and over most of the range much closer than that.) The constants being obtained, the values of $d\lambda/ds$ were then computed at every 20 mμ and the resulting values plotted against λ. The curve for the Brace spectrophotometer is given in Figure 3. Values of $d\lambda/ds$ for the two instruments bore a constant ratio to each other within a few thou-

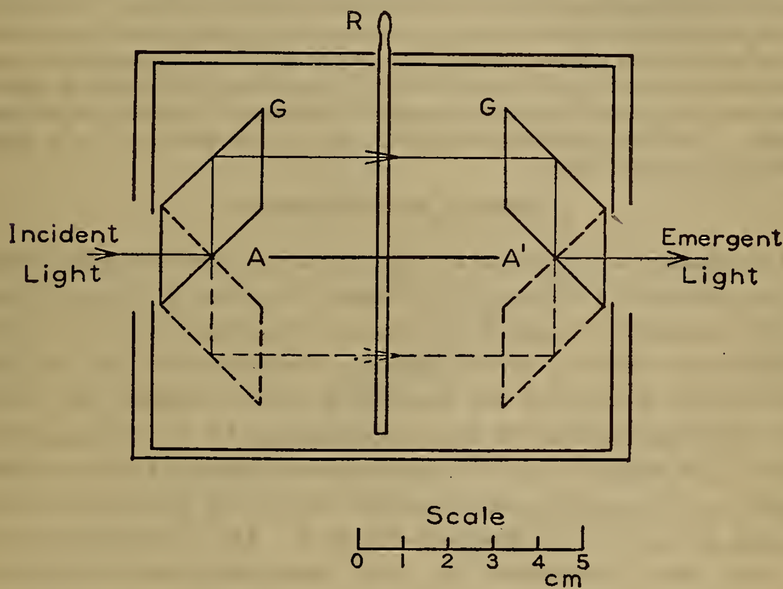

FIG. 2.—*The Brodhun variable sector.*

sandths of 1 per cent. The amount of spectrum included by the various slit widths used in the investigation may be obtained from Figure 6, where the change in wave length (millimicrons) is given for a 0.25-mm displacement of S_1. Practically the same values hold on the Lummer-Brodhun spectrophotometer.

The rotating-rhomb-and-variable-sector device, which, for short, will be spoken of merely as the variable sector, was after the design by Brodhun (18). The essential parts (fig. 2) are a pair of glass rhombs *G*, which are rotated rapidly as a unit by a motor about the axis *A A′* (which is the direction of the incident and emergent light), and a variable sector *R* whose angular aperture is controlled by the operator. The relative values of the sector openings are read from a scale attached to the instrument. Thus the sector is fixed in position and the beam of light rotates across it, instead of the light being fixed and the sector rotating as is the case with most sectored disks. Inasmuch as not only the luminosity measurements but also the measurements of the energy of the source, the transmission of the spectrophotometer, and the determination of stray light were all to be obtained by means of this variable sector, it was subjected to very careful tests as to the relation between the scale reading and the light transmitted. By a careful comparison with standard rotating sectored disks calibrated by the photometry section of the bureau this relation was found to be strictly linear between about 10 and 105 on the scale. (The maximum sector opening possible is somewhat larger than 180°, and 100 on the scale corresponds to an opening of 180°.) This was further tested by measuring the transmission of standard glasses (19) with both the Lummer-Brodhun and Brace spectrophotometers, and transmissions as low as one-half of 1 per cent were obtained with surprising accuracy. Both these tests showed also that the zero of the scale corresponded to zero transmission of light so far as could be detected, probably at least within one or two tenths of a division. As a precaution, however, in the energy, transmission-of-spectrophotometer, and luminosity measurements, the scale reading was kept as high as was feasible.

The light box L (fig. 1) was of sheet steel, the inner side and top covered with white enamel and the base with plaster of Paris; and the whole interior was finally coated heavily with magnesium oxide put on by burning magnesium ribbon. Slots in the bottom where the lights were fastened, the emergent-light tubes in the side, and the conical top with tall chimney combined to furnish sufficient self-ventilation to prevent undue heating. Eight 600-watt Mazda C lamps were placed inside as shown. These could be burned in pairs (symmetrical with respect to beam 1) or all eight together. They were operated at 99.8 volts, as it was not feasible to use a higher voltage with all eight lamps.

2. ENERGY MEASUREMENTS.

The *fundamental standard of radiant energy* used for many years in the colorimetry section of the bureau has been a 500-watt gas-filled incandescent lamp (B. S. No. 1717) operated at 118.0 volts. Its luminous efficiency for this voltage, determined in 1917 by the photometry section of the bureau, was 15.6 lumens per watt. This was redetermined in 1921 and found to be 15.75 lumens per watt. The relative spectral distribution of energy for the visible region was measured radiometrically by W. W. Coblentz, of the bureau, in 1917. It is shown in Figure 3. The color temperature of the lamp, computed by the luminosity-center-of-gravity method, was found to be 2,848° K. Details of this method and other information relative to the measurements upon the standard lamps have been published in a paper by Priest (20).

Four other gas-filled Mazda C incandescent lamps (B. S. Nos. 3254, 3255, 3256, and 3257) have been selected (1921) to serve as *secondary standards* in order that lamp 1717 might be subjected to less use. The voltages at which these lamps color matched lamp 1717 was determined by four different observers. The average voltage so determined for three of the four lamps was 101.0 volts (lamp 3254, 101.1 volts). Lamps 3256 and 3257 at 101.0 volts were then each measured against lamp 1717 at 118.0 volts on the Lummer-Brodhun spectrophotometer. The ratios of the energies of these two lamps to that of lamp 1717 under these conditions was found to be constant within ±1.1 per cent from 500 to

660 mμ, inclusive, and within ±1.9 per cent from 429 to 740 mμ, inclusive. (These slight deviations seem from the data to be entirely observational and give no indications of any consistent change in ratio from one end of the spectrum to the other.)

Lamp 3257 was then sent to the Nela Research Laboratory with a request for the color temperature at 101.0 volts. The value found there for this voltage was 2,848° K, the same value as that computed for lamp 1717 at 118.0 volts, with which it had been color matched as already noted.

In view of the above evidence lamp 3256 at 101.0 volts was taken as the *standard of radiant energy for the visibility investigation*, and Coblentz's original values for lamp 1717 at 118.0 volts have been

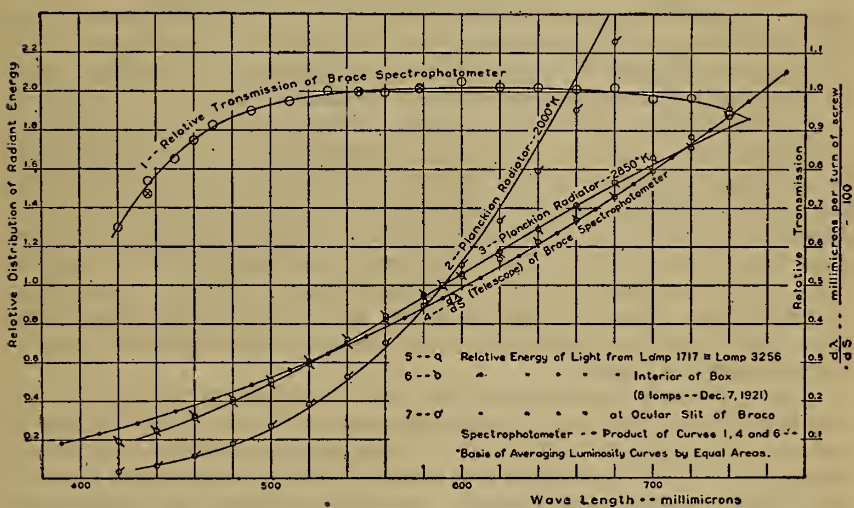

FIG. 3.—*Transmission, dispersion, and energy characteristics of apparatus.*

taken as the values for lamp 3256 at 101.0 volts and used in all computations based thereupon.

The radiant energy from the interior of the light box *L* was measured in terms of that of lamp 3256 at frequent intervals. The energy curve with eight lamps (put in December 7, 1921) is shown in Figure 3. It would have a color temperature of nearly 3,000° K. The energy curves for the several pairs of lamps differed slightly, as a rule, from this average eight-lamp curve, and these differences were taken into account in the computations.

Since the essential thing to know is the relative energy distribution at the ocular slit of the Brace spectrophotometer, the values of the relative energy of the light from the interior of the box must be multiplied at each wave length by $d\lambda/ds$ and by the spectral transmission of the Brace instrument. The resulting energy distribution is shown in Figure 3. Its color temperature would be very close to 2,077° K., that of the carbon lamps adopted as the standards of luminous intensity.

The color matching of the four secondary standard lamps with lamp 1717 was done as follows: By means of a Martens photometer a comparison lamp was color matched with lamp 1717 at 118.0 volts, the voltage on the comparison lamp being varied to match the color, while brightness match was maintained at all times. Four observers each made several trials. Then lamp 3256 (or one of the others) was substituted for lamp 1717, and with the comparison voltage set to the average of the previous readings for a given observer that observer then matched the two lamps in color as before, this time varying the voltage upon the standard lamp.

The spectrophotometric comparison of lamp 3256 (and 3257) with lamp 1717 was by the usual substitution method. Lamp 3256 was placed approximately as shown in Figure 1. A block of magnesium carbonate replaced the silver-glass mirror M' and the ground glass before S' was removed. This lamp and the auxiliary light box L' were operated in parallel on a storage battery with a voltmeter and rheostat control. The procedure was to take readings of R' for a photometric match at 540 mμ, then at two other wave lengths, and then at 540 again. Voltages were never adjusted except between any two consecutive 540 readings. Similar readings of R' were then taken with lamp 1717 replacing lamp 3256, aud finally with lamp 3257. When at any wave length the sector readings for lamp 3256 (and 3257), corrected slightly if need be for slight changes in the 540 readings, were divided by those for lamp 1717, the resulting ratios were constant within the limits already mentioned. Slits of 0.20 mm ($S'=S''=S'''$) were used from 460 to 700 mμ, inclusive, increasing to 0.40 mm at 420 and 740 mμ. Stray light was eliminated by the use of colored glasses before S''' at wave lengths 420–460 and 700–740 mμ, inclusive, the transmission curves of which are shown in Figure 5.

The energy from the interior of the light box L was measured in terms of that of lamp 3256 in exactly the same way as that was compared to lamp 1717, except that the apparatus was as represented in Figure 1. Light from the interior of the box along beam 3 was reflected by the silver-glass mirror M' onto the ground glass before S', lamp 3256 being, of course, out of the way. After a set of readings was made throughout the spectrum, lamp 3256 was inserted as shown, a rotating sector of fixed aperture used to reduce the brightness properly, beam 3 cut off, and with no other change of conditions, readings of R' were taken as before. From these the energy of the light from the interior of the box was computed relative to that of lamp 3256, the resulting energy values being as illustrated in Figure 3.

3. SPECTRAL TRANSMISSION OF THE BRACE SPECTROPHOTOMETER.

Inasmuch as the other factors entering into the determination of visibility are relative, no attempt was made to measure the absolute transmission of the Brace spectrophotometer. Its relative transmission as a unit was measured under conditions approximating very closely those actually experienced in the luminosity measurements. The values so obtained are plotted in Figure 3. These values are the average of from one to four determinations always made relative to the transmission at 546.1 mμ, which value is arbitrarily taken as unity. The curve so obtained is what one would expect from the color of the prism, which is slightly yellowish.

The relative spectral transmission along the path S_3 S_1 (fig. 1) was determined with the Lummer-Brodhun spectrophotometer with both heterogeneous and homogeneous light. Several arrangements of apparatus were tried, the one finally adopted being shown in Figure 4. The light source L, a 300-watt gas-filled lamp, and the magnesium carbonate block B were fixed to the telescope T of the Brace instrument, so that there was no relative motion of T, L, and B when the telescope was rotated to change the wave length of light used. Slit S_3 was 0.3 mm in width. Slit S_1 was removed and the spectrum was formed in the plane of slit S', which was also 0.3 mm. (Slit S_1 was 0.2 mm for most of the luminosity measurements. See fig.

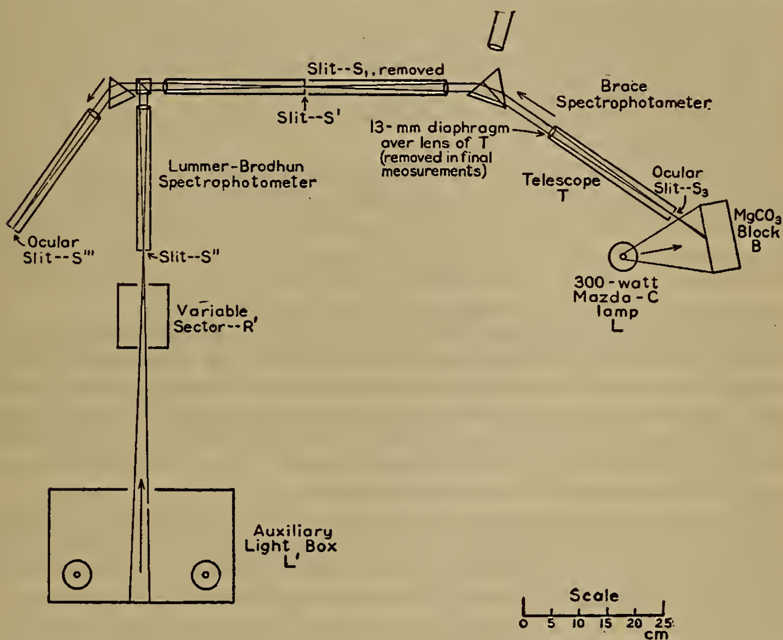

FIG. 4.—*Outline of apparatus used for the spectral-transmission measurement of the Brace spectrophotometer.*

22.) The ocular slit S''' was kept at 0.5 mm, a width which would more than include the image of S_3. This image is formed by light whose homogeneity is controlled mainly by the width of S', but its width depends upon that of S_3. Great care was taken that this image was kept in the center of the ocular slit, its position being observed by using the eyepiece before S'''. A small shift of this image to one side or the other of the ocular slit was found to produce an appreciable error. The position of the image could be controlled by a minute lateral shifting of S', which was accomplished by a small rotation of one leveling screw. Illumination for slit S'' was provided by the light box L', and the brightness was varied by the rotating sector R' for the photometric balance. The width of S'' was 0.3 mm. L' and L were connected in parallel to a storage battery and the voltage kept constant by voltmeter and rheostat control.

With each spectrophotometer set at 546.1 mμ, the instruments were adjusted until the spectral image of S_3 subtended by S' was sharply focused and centered in the ocular slit S'''. After removing the eyepiece 10 readings of R' were taken and the spectrophotometers then set for a different wave length. If necessary, the slight leveling adjustment was made, and then 10 readings taken at this wave length. Similar readings were taken at one or two more wave lengths and then another set at 546.1 mμ. In this manner readings were made throughout the region desired, a check reading at 546.1 mμ being taken after every two or three determinations at other wave lengths. With proper care the readings at 546.1 mμ were found to be constant within the usual photometric precision.

The Brace spectrophotometer was then removed and a set of readings taken at the same wave lengths with the carbonate block and lamp similarly placed before S'. At each wave length the average sector reading with the Brace spectrophotometer before S' was divided by the average reading with it removed. The average value of this ratio at 546.1 mμ was arbitrarily taken as unity, and the ratios at the other wave lengths were then reduced to this arbitrary scale by dividing by the average ratio at 546.1 mμ. The resulting values give the spectral transmission relative to that at 546.1 mμ. No correction for the dispersion of the instruments had to be made because of the constant ratio found to exist at all wave lengths between the values of $d\lambda/ds$ for the two instruments (Sec. II, 1).

It was at first attempted to make the transmission measurements under the actual conditions obtaining in the visibility investigation; that is, with the Brace prism in the same position and with the 13-mm diaphragm over the lens of T. Under these circumstances, however, only half the Lummer-Brodhun field could be filled (the silver strip of the Brace prism blocking off the lower half), and the precision was rather poor. Several sets of measurements were made, however.

In later trials the Brace prism was raised so as to take the obstructing silver strip out of the field and the diaphragm was removed from T. In this way practically the whole of the Lummer-Brodhun field was filled and the contrast method of matching used. This resulted in much better agreements among repeated measurements of the transmission at the various wave lengths. As there seemed to be no consistent difference between these values and those obtained earlier, these latter were used entirely to obtain the curve shown in Figure 3.

By substituting a Hanovia 220-volt quartz-mercury arc for the lamp L, it was found possible to obtain check readings by means of homogeneous light at wave lengths 435.85, 546.1, and 578.0 mμ. Because of the unsteadiness of the mercury arc it was necessary to alternate sets of readings at the wave length desired and at 546.1 mμ. The averages of five or six of such sets gave a determination of the transmission relative to that at 546.1 mμ, and the averages of several determinations are plotted in Figure 3.

4. FIELD BRIGHTNESS.

The average relative field brightnesses as viewed by the observer under the various experimental conditions are shown in Figure 22 in connection with the discussion of precision given in Section V, 2. The actual relative brightness would, of course, vary according to each individual's luminosity, and the results shown are those taken from the final average luminosity curve for all the observers, the breaks in the curve occurring when either the number of lamps, the ocular slit width, or the rotating sector before S_1 (fig. 1) was changed.

Actual average brightnesses may be computed from this relative brightness curve and the measured value of brightness obtained at the wave length of maximum luminosity. This numerical value of brightness at approximately 580 mμ is given in Figure 22, expressed in the various units which will enable ready comparison with the brightnesses used in other investigations.

To measure the brightness of the field, a calibrated screen was obtained from the photometry section. This screen was of milk glass and was used with a standard carbon lamp (B. S. 3350, at 106 volts, 34.7 candles). The transmitted brightness for 1 candle-cm was 240 millilamberts, giving a transmitted brightness of 0.0066 candles /cm^2 at a distance of 20 cm between lamp and screen. This brightness was reflected into the telescope lens by a right-angled prism, filling part of the field usually filled by light from S_1. The brightness of the light from S_1 and the known brightness of the calibrated screen could then, in turn, be matched with the light from S_2 by means of the variable sector and the brightness of the light from S_1 found in terms of the known brightness of the screen. This known brightness was corrected for the reflections at the surfaces of the right-angled prism and telescope lens (transmission of prism and lens assumed equal to $0.828=0.91^2$). The quality of color of the carbon lamp was not greatly different from the yellow at 580 mμ, and the brightness determination was therefore made at this wave length. Measurements were made by two observers, the agreement being good, and the average value thus obtained was 0.017 candles /cm.^{2}

5. STRAY LIGHT.

All spectrophotometers have a certain amount of stray light present at any region of the spectrum, as may be noted if the light passing through the ocular slit is examined with a small hand spectroscope. Throughout the brighter part, however—viz, from about 510 to about 660 mμ for a prism instrument and the usual incandescent sources—the relative value of this stray light is too small to introduce appreciable error; but beyond these limits the amount present increases more and more rapidly and becomes 100 per cent when the ultra-violet and infra-red regions are reached. Since the percentage of stray light varies with the distribution of energy in the source used and may be influenced by other factors, it was studied as far as possible under the actual conditions obtaining in the luminosity measurements.

It is understood, of course, that the correction for stray light can be made only approximately, since it would vary somewhat with each individual depending upon his visibility. For this reason it is advisable to eliminate stray light when possible rather than to measure it and make the corrections. Certain precautions taken have already been mentioned, viz, the diaphragms and the special attention to the optical surfaces. In addition, certain glasses were available which absorb the stray light in the red very efficiently. Therefore, but few measurements were made in the red, and these largely for the purpose of determining where the stray light began to be appreciable. In the blue such efficient filters are not available as for the red. The stray light was therefore measured more carefully than in the red and correction made over that part of the range where a blue filter was not used.

In Figure 5 are shown the stray-light curves obtained in the red and blue and the transmission curves of the glasses used to eliminate this stray light, together with the range of wave lengths over which they were used. Methods of making corrections arising because of the stray light or the varying transmission of the filters are noted in Section III, 2.

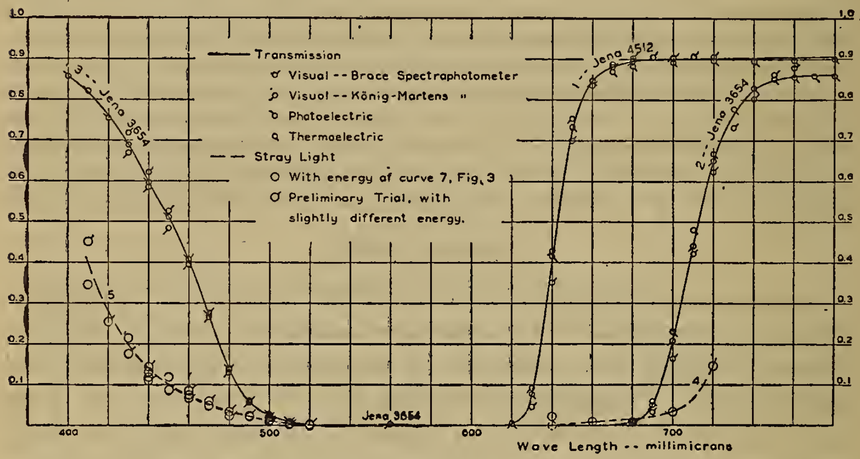

FIG. 5.—*Stray light and filters used for its elimination.*

Stray light is the fraction of the total brightness resulting from the stray light present. Filters used as follows:
Jena 3654—430–450 mμ, inclusive.
Jena 4512—660–700 mμ, inclusive.
Above two combined—720–740 mμ, inclusive.

The brightness B of a spectrophotometric field is the sum of the brightnesses of the spectral light B_λ and the stray light B_s. When the two halves of the field of the Brace spectrophotometer (fig. 1) are matched for brightness, the following equation may be written:

$$B_\lambda + B_s = (B'_\lambda + B'_s) K_\lambda M \tag{1}$$

where the primes have reference to the light in part 2 of the field—viz, that passing through the variable sector and through S_2—M is the sector reading and K_λ is a constant at any given wave length. If a specimen of glass is placed before S_1 and a new match is obtained at this wave length with the sector reading N, the following equation is true:

$$B_\lambda T_\lambda + B_s T_s = (B'_\lambda + B'_s) K_\lambda N \tag{2}$$

where T refers to the transmission of the glass.

Equation (2) divided by equation (1) gives:

$$\frac{B_\lambda T_\lambda + B_s T_s}{B_\lambda + B_s} = \frac{N}{M} \tag{3}$$

The quantity $B_s/(B_\lambda+B_s)$ is the fraction of the total brightness resulting from the stray light present.

When T_λ equals zero,

$$\frac{B_s}{B_\lambda+B_s}=\frac{N}{T_s M} \tag{4}$$

when T_s equals zero,

$$\frac{B_\lambda}{B_\lambda+B_s}=\frac{N}{T_\lambda M}=1-\frac{B_s}{B_\lambda+B_s}$$

and

$$\frac{B_s}{B_\lambda+B_s}=1-\frac{N}{T_\lambda M} \tag{5}$$

Therefore, if at any wave length either T_s or T_λ is known for any glass, the other being zero, the quantity $B_s/(B_\lambda+B_s)$ may be obtained by taking readings of the sector for a match first when the filter is before the slit S_1 and then when it is removed. From the nature of the transmission of the glasses which seemed most suitable the method of equation (4) was used in the blue and that of equation (5) in the red.

As already noted, glasses were available which absorb practically all the stray light in the red while transmitting the spectral light very highly. Therefore, but few values were obtained in the red and these largely for the purpose of finding out at what wave length the stray light became sufficient to necessitate elimination. A glass of the type of curve 1, Figure 5, was used, T_λ being obtained simultaneously with the other readings by means of a similar glass placed over the ocular slit. Results are shown by curve 4, Figure 5. It was decided from this study that the stray light present was inappreciable at wave lengths less than 660 mμ.

The method of equation (4), used in the blue, necessitates the use of glasses having high total light transmission and having a sharp transition between high transmission and high absorption. Orange and yellow glasses of the Noviol type (19) were used. Their total transmissions for light of the energy distribution used in the luminosity measurements were measured directly by two observers by means of a Martens photometer. The assumption was made that this total light transmission was the same for the stray light as for the original light. This seemed justifiable, since the quality of color of the stray light transmitted by the filter was always that of the filter as viewed by the original light. At those wave lengths closely adjacent to that at which the spectral transmission becomes inappreciable (known by previous measurements by the photographic or photo-electric method, and verified here) readings of the sector were made for a match of the two halves of the field when the glass was before S_1 and when removed. This, of course, necessitated matching spectral (bluish) light against stray (yellowish) light when the glass was before S_1. This was not very difficult at the low brightnesses resulting. Measurements were made by two observers (one having practically normal visibility, the other low in the blue) and the average taken. Great care was taken that the measurements should be made with the eye fixed steadily on the dividing line of the field. If it were allowed to wander to the side, the blue immediately became relatively much brighter (the Purkinje effect).

With these data and equation (4) values of the fraction of the total brightness resulting from the stray light were computed for those wave lengths from 5 to 25 mμ less than that at which the spectral transmission becomes inappreciable, the most weight being given to the values nearest this wave length. Results are shown by curve 5, Figure 5.

6. DETERMINATION OF THE STEP.

The step-by-step method necessitates accurate information as to what shift in the wave length of the light entering the ocular slit is caused by a shift of slit S_1 (fig. 1) through a definite distance

s. That is to say, for every position of the telescope one must know the $d\lambda/ds$ of S_1 expressed in millimicrons per screw turn. (As noted in Section II, 1, S_1 was made of constant width, 0.20 mm, and could be moved at right angles to its length in the focal plane of the collimator lens, this being accomplished by a screw of ¼-mm pitch with a drum divided into 100 parts.) The most rigorous way of getting this relation seemed to be the following: The telescope was set for some wave length λ'. A suitable source of homogeneous light was placed in front of slit S_1 and the screw then turned to bring three lines of wave lengths λ_1, λ_2, and λ_3 (differing but a few millimicrons from each other) in turn into the ocular slit of the telescope. The reading of S_1 was taken for each line. These are, therefore, three points on a curve between wave length and screw reading of S_1 for one particular position of the telescope, wave length λ'. In effect the spectrophotometer was reversed using collimator for telescope. The slope of this slit calibration curve was desired at λ'. It was readily obtained by using the modified Hartmann formula (17)

$$s = s_0 + \frac{c}{\lambda - \lambda_0}$$

the three values of λ and S_1 determining the constants. By differentiation the value of $d\lambda/ds$ was found for $\lambda = \lambda'$, which is the only position of the telescope at which this particular equation is valid. Values of $d\lambda/ds$ for S_1 were thus obtained for five values of λ, namely, 460, 490, 520, 580, and 650 mμ. As $d\lambda/ds$ for the telescope at these five wave lengths was already accurately known (Sec. II, 1, and figs. 3 and 6), the ratio of $d\lambda/ds$ for slit S_1 to $d\lambda/ds$ for the telescope was then computed. These ratios are plotted against scale reading of the telescope in Figure 6. It was then an easy matter to read the value of this ratio for any wave length, and with the known value of $d\lambda/ds$ for the telescope compute $d\lambda/ds$ for slit S_1 at any position of the telescope. The resulting values are plotted in Figure 6, $d\lambda/ds$ of slit S_1 in millimicrons per turn of screw against λ of the telescope.

The size of the step finally chosen was one-half turn (of the screw controlling the position of S_1) from 740 to 510 mμ, inclusive, and a whole turn from 510 to 430 mμ. In computing the wave lengths of the successive steps it must be remembered that the values of $d\lambda/ds$ given by the curve of Figure 6 for S_1 are merely the *rates* at which λ is varying with s and do not remain constant even over the length of one step. Slightly different values will therefore be obtained, depending upon which way the curve is worked through the spectrum. To illustrate with the actual figures used: Starting at 510.000 mμ a total of 70 steps takes one to 740.779, but upon returning by 70 steps the value of 507.825 mμ is reached; that is, in one case the steps have all been a trifle small, in the other a trifle large, but the *average wave lengths* for corresponding steps will be true for all practical purposes. In addition to these 70 steps there are 31 in the blue (510–430), making about 100 steps for the complete range. This, of course, applies to the computations, not the observations.

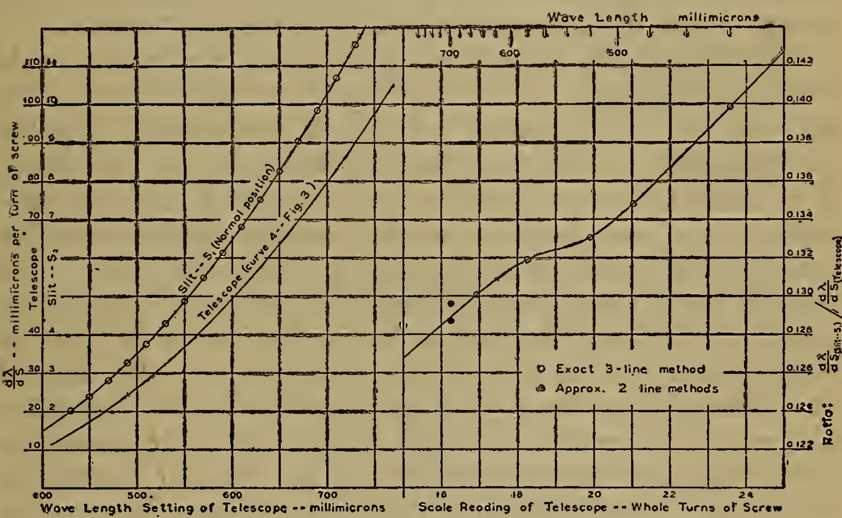

FIG. 6.—*Dispersion characteristics used in computation of wave length interval of step.*

An approximate and much simpler method of obtaining the values for the ratio curve of Figure 6 is available by the use of pairs of closely adjacent lines instead of three lines as in the exact method already described. If Δs_t is the difference in telescope readings for these lines with S_1 normal and Δs_{S1} is the difference in the positions of S_1 when the telescope is set at the mean reading for S_1 normal, then

$$\frac{\Delta s}{\Delta s_{S1}} = \frac{d\lambda/ds_{S1}}{d\lambda/ds_t},$$

approximately. Considerable preliminary work showed this two-line method to be less reliable than the three-line method. It is not only inexact theoretically, but the determination of Δs_t is relatively coarse for two lines near together.

A second two-line method is available from the fact that

$$\frac{\Delta\lambda/\ \Delta s_{S1}}{d\lambda/ds_t} = \frac{d\lambda/ds_{S1}}{d\lambda/ds_t}$$

approximately.

For wave length about 700 mμ, where three suitable lines were not available, a pair of lines was used with results by the two approximate methods as shown. The curve as drawn is similar to the one actually used.

III. LUMINOSITY AND VISIBILITY.

1. PROCEDURE.

In determining the size of step to use one must compromise between the desire to eliminate hue difference at all wave lengths and the desire to minimize the effect of observational errors and the number of computations. The eye is extremely sensitive to small differences of wave length in the yellow and blue-green regions, a difference of the order of magnitude of 1 mμ being perceptible (21). It was therefore found entirely impracticable to eliminate all hue difference in the yellow and orange. On the other hand, if the step is chosen too large, the resulting hue difference makes the photometric settings erratic and unsafe.

The scheme finally adopted was to make the step from one side of the normal position of S_1 to an equal mechanical distance on the other side of the normal. If λ is the wave length at which the telescope is set and $\Delta\lambda$ is the total step, then the luminosities at $\lambda - \frac{1}{2}\Delta\lambda$ and at $\lambda + \frac{1}{2}\Delta\lambda$ are each compared with the light through S_2 of wave length λ, and the resulting readings will give a ratio of the luminosity at $\lambda - \frac{1}{2}\Delta\lambda$ to that at $\lambda + \frac{1}{2}\Delta\lambda$. (The two half steps, being equal mechanically, are not quite equal in terms of wave length because of the varying dispersion; but the ratio of luminosities in the two cases is that for the exact interval $\Delta\lambda$.) There is thus for a given size step but half the hue difference present in the field that there would be if the luminosity at λ were compared directly to that at $\lambda \pm \Delta\lambda$. As mentioned in the last section, the size of step $\Delta\lambda$ finally decided upon was one-half turn of the screw (one-fourth turn each side of the normal) from 510 to 740 mμ and one turn (one-half turn each side) from 510 to 430 mμ. One turn equals 0.25 mm. In terms of wave lengths this amounts to approximately 4.9 mμ at 710, 3.4 mμ at 610, and 1.9 mμ at 510 mμ for the one-half turn, and approximately 3.8 mμ at 510 and 2.0 mμ at 430 mμ for the whole turn. (See fig. 6.)

The hue difference in the yellow and orange under these conditions was perceptible to most observers and often caused trouble with the settings. In the blue-green with the whole-turn step there were traces of hue difference to most observers.

The conditions and procedure for taking the observations were, in general, as follows: The complete range from 430 to 740 mμ was divided into two parts, one part comprising the wave lengths from 510 to 660 mμ, which will be called the main region, the other parts from 510 to 430 and 660 to 740 mμ, the end regions.

It was never attempted to take a complete run in a half day, but either the main or the end regions could be easily finished in a half day (one to two and one-half hours) without undue haste or fatigue. Two or more complete runs were finally made on each observer, with three exceptions. In one of the runs through the main region readings were taken from the red toward the blue, in the other run from the blue toward the red. In the end regions readings were taken from the brighter toward the dimmer regions, in one set the red end first, in the other the blue end. From 660 to 510 mμ readings were taken at every 15 mμ, from 660 to 740 mμ at every 20 mμ, from 510 to 430 mμ at every 10 mμ. (This procedure was varied in the case of two observers who took readings at every 10 mμ throughout the spectrum.)

The observations were taken in groups as follows: Between 660 and 510 mμ in three groups: 660, 645, 630, and 615 mμ; 600, 585, and 570 mμ; 555, 540, 525, and 510 mμ. From 660 to 740 mμ in two groups: 660, 680, and 700 mμ; 720 and 740 mμ. From 510 to 430 mμ in three groups: 510, 500, and 490 mμ; 480, 470, and 460 mμ; 450, 440, and 430 mμ.

For any one group, for example, 600–570 mμ, S_1 was displaced from its normal position by half the total step, as already explained, and four readings of R taken at each of the three wave lengths; then S_1 was shifted the whole step to the other side of its normal position and eight readings of R taken at each wave length, four readings at each wave length in the reverse order and four in the original order; then S_1 was shifted back to its first position and four more readings of R taken at each wave length in the reverse order. Thus any possible change of conditions in apparatus or observer would tend to average out, if it were not detected, and these wave lengths would be completed before another set was begun. The various experimental conditions under which these observations were made—brightness, slit width, filter, etc.—will be found in Figures 5, 7, and 22.

2. COMPUTATIONS.

At each position of the telescope the average reading of R to match with S_1 at $\lambda - \frac{1}{2}\Delta\lambda$ was divided by the reading at $\lambda + \frac{1}{2}\Delta\lambda$, this result thus giving the ratio of the luminosity at $\lambda - \frac{1}{2}\Delta\lambda$ to that at $\lambda + \frac{1}{2}\Delta\lambda$. When a filter was used over the ocular slit to eliminate stray light, this ratio was divided by the ratio of the transmissions at these wave lengths—that is, by $\frac{T_{(\lambda - \frac{1}{2}\Delta\lambda)}}{T_{(\lambda + \frac{1}{2}\Delta\lambda)}}$. When a filter was not used and stray light was present, this ratio was multiplied by the ratio of the percentages of spectral brightness to total brightness at these wave lengths.

The resulting true values of the ratio of luminosities at $\lambda - \frac{1}{2}\Delta\lambda$ and $\lambda + \frac{1}{2}\Delta\lambda$—that is, $\frac{L_{(\lambda - \frac{1}{2}\Delta\lambda)}}{L_{(\lambda + \frac{1}{2}\Delta\lambda)}}$—were then plotted at $\lambda + \frac{1}{2}\Delta\lambda$, λ being the position of the telescope at the various wave lengths at which measurements of the ratio were made, and $\frac{1}{2}\Delta\lambda$ being known at all wave lengths for the size of step used. Values read from the curve drawn through these points now gave at any wave length λ the ratio of the luminosity at $\lambda - \Delta\lambda$ to that at λ.

Values of the ratio were then read off at the successive wave lengths corresponding to the size of step. Assuming an arbitrary value at some one wave length, 659.3 mμ in this investigation, relative values of the luminosity L were computed by multiplying this arbitrary value by the ratio at 659.3, the result being L for 655.0; this value was then multiplied by the ratio at 655.0, giving L at 650.85; and so on to 430 mμ. Dividing the value of L at 659.3 by the ratio at 663.65 gave the value of L at 663.65, and so on to 740 mμ. Machine computation was used, enough significant figures being carried as to make rejection errors entirely negligible.

The resulting luminosity curve was then changed slightly if necessary (because of small differences in the energy distribution of the various pairs of lamps or because of small changes with age) to reduce it to that corresponding to the energy distribution at which the luminosity curves were averaged. The area was then measured with a planimeter and the factor found to reduce the values to an arbitrary equal-area basis, the same area for all curves.

The luminosity as thus finally obtained was divided by the energy at the ocular slit (curve 7, fig. 3) to obtain the visibility, which could then be reduced to unit maximum.

The above procedure of computation was that carried through for each individual. The reduction of the luminosity curves to an equal-area basis (which in no way changes the individual's relative visibility values) is an essential prerequisite to the averaging of the luminosity or visibility of any two or more observers, following the reasoning and procedure of Hyde, et al., which assumes that the total luminous sensation is the same for all observers for a source having a color temperature close to that of the standard carbon lamps, 2,077° K.

The final average visibility curves and those for the various groups of individuals have been, therefore, unless otherwise noted, obtained by this method of averaging the individual luminosity values after reduction to an equal-area basis.

The various steps in the process of computing the final visibility curve from an original ratio curve are illustrated in Figure 7. While the individual luminosity values to equal area were averaged as a basis for the final visibility curve, it was also of interest and value to average the individual ratio values and compute a final visibility curve from this average ratio curve. The graphical steps in this process are those actually shown in Figure 7.

Another factor entering into the averaging of the final visibility curve should be mentioned. An examination of the original data immediately shows that some observers were able to duplicate their ratio values very closely on different days, but that others could not do this so well. This was, in fact, the criterion as to whether or not a given observer should make more than the two sets of observations. Often a third complete set through the main region was taken, while it was the rule to take sufficient check points after the second or third run to eliminate any large uncertainty still remaining at any wave length. This inability of some observers to repeat the measurements with the desired precision was apparently a result of one or more of three factors: (*a*) General inability to make precise photometric settings, (*b*) inability to make precise settings in the yellow and orange because of the small hue difference existing in the two parts of the field in that region, and (*c*) actual changes in visibility on different days.

A detailed discussion of the precision is given later in Section V, 2. It is sufficient to state here that on the basis of ability to duplicate the ratio values in the main spectral region on different days, the observers were arbitrarily divided into two groups of 26 each, called "good" and "poor," and in the final average adopted as expressing the results of this investigation the values obtained by the good observers have been given double weight.

3. RESULTS.

The average visibilities for the two groups of good and poor observers and the 2/1 weighted average for the total number of observers are given in Table 1 and Figure 8. This weighted average is that used in all graphs unless otherwise noted. The

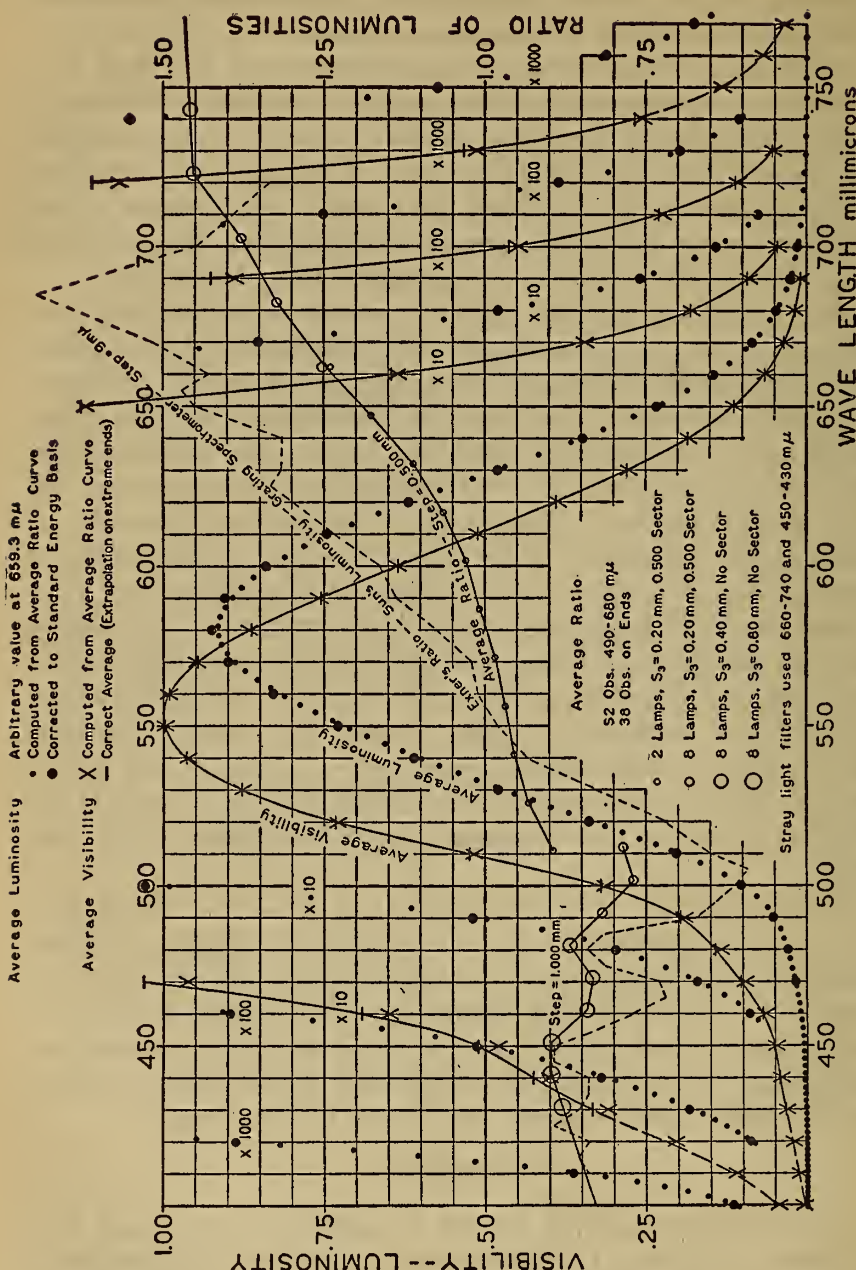

Fig. 7.—Average luminosity characteristics for the present investigation.

For further explanation, see Section III of paper.

difference in values between the two groups is unimportant in the blue, but in the orange region centering at 600 mμ there is a very decided difference. The weighted average, however, is but slightly different from the straight average.

Slit-width corrections were made by means of Hyde's (22) equation

$$\phi(\theta)=\frac{1}{A}\left[F(\theta)-K\Delta^2F(\theta)+\cdot\cdot\cdot\cdot\cdot\right]$$

where

$\phi(\theta)$ = the luminosity in the pure spectrum,

$F(\theta)$ = the luminosity in the impure spectrum (as observed),

$\Delta^2F(\theta)=F(\theta+c)+F(\theta-c)-2F(\theta)$, c being half the sum of the

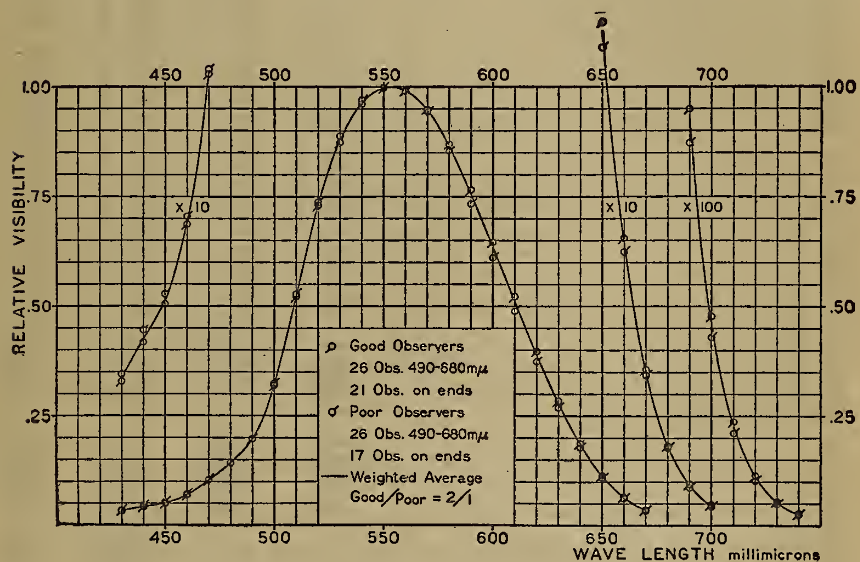

FIG. 8.—*Average values of relative visibility—present investigation.*

collimater and telescope slit-widths expressed in θ units. (Higher terms in the series were not used.)

A, K = constants depending on the slit widths, A, however, not entering into the correction.

These corrections as thus computed were very small, except in the far red where they became as large as 3 per cent. Throughout the spectral region 520 to 640 mμ they were less than 0.1 per cent, and less than 0.5 per cent throughout most of the blue. All the curves as plotted are uncorrected for slit widths. Differences are inappreciable in the graphs, except on the third magnification in the red. Slit-width corrections have been applied only to the final average data as given in Tables 1 and 3.

TABLE 1.—Average Values of Relative Visibility—Present Investigation.

Wave length in millimicrons.	Values of relative visibility for— Good observers (26 obs., 490–680mμ; 21 obs., on ends).	Poor observers (26 obs., 490–680 mμ; 17 obs., on ends).	Total number of observers (weighted average, $\frac{\text{good}}{\text{poor}}=\frac{2}{1}$)	Final values of relative visibility (weighted average with slit-width corrections).
400	....	....	[1] 0.005	0.005
410	....	....	[1] .012	.012
420	....	....	[1] .022	.022
430	0.033	0.034	.033	.033
440	.042	.045	.043	.043
450	.050	.053	.051	.051
460	.069	.070	.069	.069
470	.103	.104	.103	.103
480	.143	.143	.143	.143
490	.197	.197	.197	.196
500	.319	.324	.320	.318
510	.522	.528	.524	.523
520	.730	.737	.732	.732
530	.873	.888	.878	.878
540	.961	.970	.964	.964
550	.998	.999	.998	.998
560	.991	.990	.991	.991
570	.949	.943	.947	.947
580	.869	.852	.863	.863
590	.765	.733	.754	.754
600	.646	.610	.634	.634
610	.522	.489	.511	.511
620	.397	.374	.389	.389
630	.284	.269	.279	.279
640	.187	.178	.184	.184
650	.1147	.1089	.1127	.1125
660	.0656	.0624	.0645	.0642
670	.0356	.0343	.0351	.0349
680	.0182	.0177	.0180	.0178
690	.0095	.0087	.0093	.0092
700	.0048	.0043	.0046	.0045
710	.0024	.0021	.0023	.0022
720	.00114	.00102	.00111	.00108
730	.00055	.00050	.00053	.00051
740	.00028	.00025	.00027	.00026
750	....	....	[1] .00014	.00014
760	....	....	[1] .00007	.00007
770	....	....	[1] .00004	.00004

[1] Extrapolated from average ratio curve.

In Figure 9 are shown results obtained on two selected groups of observers—the observers over 40 years of age and the women observers. No observer was common to both groups. Both groups are red sensitive, but the observers over 40 do not average low in the blue as might be expected and as has been found previously. A study of the individual data shows that 4 of these observers over 40 were low in the blue, 4 normal, and 3 high. Only 1 of the 7 women examined could be called blue sensitive.[4]

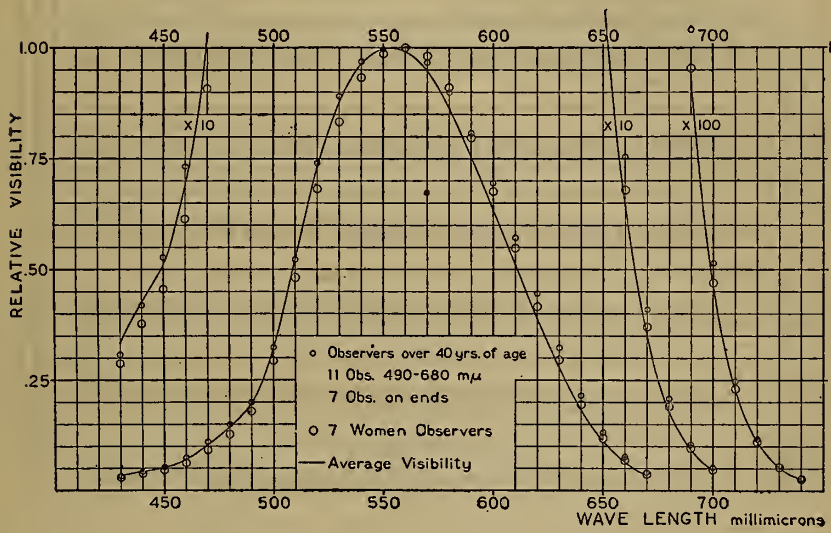

FIG. 9.—*Average values of relative visibility for two groups of observers.*

The individual visibility values for the 52 observers are listed in Table 2. These are all to unit maxima. If it is desired to obtain the average data for two or more of these observers on the basis of equal areas of luminosity curves, the visibility values should first be multiplied by the factor given in the table for each observer. In addition to the visibility values the wave length centers of gravity ($\lambda_c = \Sigma V\lambda / \Sigma V$) and the relative areas of the curves to unit maxima are given for each observer. The wave lengths of maximum visibility may be estimated from the data.

[4] The terms red sensitive, blue sensitive, etc., are very indefinite and unsatisfactory and depend upon the reference point—in this case unit maximum of the visibility curves. If the curves had been plotted to a common value at 580 mμ, the wave length of maximum luminosity, or on the basis of equal areas of the luminosity curves, as they were computed, then the observers over 40 would have been subnormal in the blue, the women more so, and neither group would appear so highly red sensitive.

TABLE 2.—Individual Values of Relative Visibility [1] (to Unit Maxima).

[To average on basis of equal areas of luminosities at 2,077° K., values should first be multiplied by the multiplying factors [6] given for each observer.]

Wave length in millimicrons.	Values of relative visibility for the following observers:													
	Final [1] average.	[2] 1 E.P.T.T.[3] M.[4]	2 K. S. G. M.	3 I. G. P. M.	4 H. J. M. M.	5 M. C. M. M.	6 G. E. M. M.	7 F. P. P. M.	8 C. O. F. M.	9 C. L. C. M.	10 P. V. W. M.	11 A. H. B. M.	12 A. F. E. M.	13 R. F. J. M.
		[5] 29	31	36	29	21	31	33	31	27	32	26	27	41
	1.097	[6] 0.971	1.054	1.222	1.073	1.079	1.095	1.289	1.074	1.200	0.926	1.198	1.117	1.032
	560.1	[7] 567.2	561.9	557.8	560.4	558.0	557.2	554.5	562.9	554.9	563.3	555.8	559.6	563.1
	1.000	[8] 1.027	1.006	0.914	1.004	1.031	1.025	0.912	0.980	0.965	1.121	0.957	0.980	1.013
430	0.033	0.018	0.024	0.030	0.029	0.043	0.048	0.039	0.030	0.038	0.046	0.032	0.030	0.022
440	.043	.020	.032	.040	.040	.054	.056	.048	.036	.050	.057	.041	.039	.032
450	.051	.021	.040	.045	.050	.067	.068	.055	.041	.061	.067	.053	.046	.039
460	.069	.030	.056	.059	.069	.093	.092	.069	.054	.086	.085	.069	.062	.053
470	.103	.049	.088	.089	.106	.138	.138	.101	.083	.128	.119	.105	.094	.078
480	.143	.070	.125	.127	.150	.186	.191	.138	.112	.162	.154	.150	.134	.110
490	.197	.108	.174	.186	.210	.247	.259	.191	.155	.214	.206	.210	.185	.160
500	.320	.216	.289	.299	.321	.330	.391	.317	.268	.347	.338	.345	.308	.283
510	.524	.443	.504	.494	.507	.574	.604	.562	.476	.608	.566	.585	.531	.493
520	.732	.691	.718	.698	.709	.760	.770	.786	.692	.822	.792	.796	.730	.720
530	.878	.873	.868	.832	.854	.908	.893	.924	.844	.942	.949	.925	.867	.874
540	.964	.952	.959	.944	.946	.972	.971	.988	.945	.993	.999	.989	.957	.956
550	.998	.973	.997	.999	.996	.990	.999	.998	.996	.998	.998	.998	.998	.992
560	.991	.998	.992	.961	.992	.999	.984	.960	.984	.961	.995	.970	.992	.998
570	.947	.994	.966	.888	.954	.953	.930	.864	.928	.881	.994	.892	.953	.971
580	.863	.952	.895	.796	.884	.862	.843	.732	.832	.771	.951	.778	.871	.900
590	.754	.870	.794	.685	.788	.754	.743	.592	.729	.643	.858	.657	.755	.797
600	.634	.762	.680	.532	.664	.636	.628	.464	.632	.527	.771	.537	.610	.684
610	.511	.635	.551	.392	.528	.507	.504	.353	.534	.416	.668	.420	.472	.568
620	.389	.497	.416	.287	.396	.377	.377	.262	.431	.308	.537	.309	.354	.439
630	.279	.365	.295	.206	.281	.268	.265	.184	.319	.214	.400	.210	.253	.315
640	.184	.241	.190	.140	.180	.183	.172	.120	.215	.135	.274	.135	.167	.210
650	.1127	.152	.113	.0862	.106	.115	.102	.0732	.134	.0790	.169	.0820	.101	.129
660	.0645	.0890	.0631	.0490	.0591	.0662	.0569	.0418	.0784	.0443	.0986	.0456	.0580	.0752
670	.0351	.0488	.0337	.0260	.0320	.0358	.0306	.0224	.0427	.0266	.0536	.0247	.0315	.0413
680	.0180	.0248	.0169	.0132	.0165	.0189	.0158	.0116	.0223	.0138	.0280	.0126	.0166	.0214
690	.0093	.0129	.0084	.0065	.0085	.0096	.0079	.0058	.0112	.0069	.0142	.0065	.0085	.0107
700	.0046	.0065	.0041	.0032	.0042	.0048	.0041	.0028	.0056	.0035	.0069	.0034	.0044	.0053
710	.0023	.0032	.0020	.0016	.0022	.0024	.0021	.00139	.0027	.0017	.0034	.0017	.0022	.0026
720	.00111	.0016	.00097	.00074	.00103	.00121	.00101	.00069	.00137	.00083	.0016	.00082	.00113	.00125
730	.00053	.00077	.00047	.00036	.00050	.00059	.00051	.00034	.00069	.00039	.00080	.00039	.00061	.00061
740	.00027	.00039	.00023	.00018	.00025	.00029	.00028	.00017	.00034	.00018	.00040	.00020	.00037	.00031

[1] Without slit-width corrections.
[2] Number of the observer.
[3] Initials of the observer.
[4] Sex of the observer.
[5] Age of the observer.
[6] Multiplying factor.
[7] Wave length center of gravity, λc, 450–670 mμ.
[8] Relative areas of curves to unit maxima, 450–670 mμ.

Wave length in millimicrons.	Values of relative visibility for the following observers: Final average. 1.097 560.1 1.000	14 A. Q. T. M. 44 0.988 560.4 1.092	15 C. G. P. M. 34 1.306 549.9 0.944	16 C. C. K. M. 34 1.260 556.6 0.904	17 M. K. F. F. 35 0.956 568.8 1.026	18 W. H. H. M. 31 1.205 555.5 0.955	19 W. F. M. M. 33 0.896 569.0 1.081	20 C. F. S. M. 31 1.246 554.3 0.940	21 M. D. S. M. 27 1.170 559.5 0.940	22 U. M. S. M. 28 1.042 562.2 1.019	23 J. F. S. M. 41 0.975 564.2 1.055	24 M. M. F. 38 1.023 566.6 0.984	25 R. M. C. F. 25 1.055 562.0 1.008	26 C. G. S. F. 28 1.068 559.9 1.013
430	0.033	0.031	0.049	0.029	0.014	0.035	0.040	0.055	0.024	0.030	0.025	0.022	0.034	0.028
440	.043	.044	.061	.039	.018	.046	.047	.064	.029	.041	.036	.026	.043	.040
450	.051	.056	.072	.051	.023	.056	.051	.072	.033	.047	.051	.028	.053	.054
460	.069	.083	.101	.069	.033	.075	.063	.090	.043	.062	.073	.037	.068	.080
470	.103	.128	.154	.096	.053	.112	.086	.124	.073	.090	.101	.061	.097	.119
480	.143	.181	.215	.138	.077	.156	.110	.168	.114	.121	.138	.090	.138	.155
490	.197	.253	.288	.197	.114	.218	.144	.230	.168	.166	.195	.131	.192	.212
500	.320	.400	.438	.311	.211	.350	.243	.357	.283	.294	.310	.230	.301	.327
510	.524	.623	.674	.499	.394	.584	.436	.566	.490	.517	.498	.411	.475	.519
520	.732	.796	.884	.690	.624	.794	.636	.745	.740	.712	.680	.614	.670	.728
530	.878	.901	.961	.840	.812	.919	.797	.870	.910	.858	.818	.769	.817	.880
540	.964	.959	.999	.950	.938	.990	.926	.964	.986	.950	.919	.880	.934	.950
550	.998	.985	.978	.999	.998	.992	.996	1.000	.998	.998	.976	.955	.999	.978
560	.991	1.000	.901	.960	.992	.934	.994	.974	.969	.997	1.000	.996	.991	.999
570	.947	.985	.794	.872	.978	.866	.970	.900	.904	.984	.984	.985	.983	.988
580	.863	.931	.674	.751	.933	.777	.934	.784	.798	.926	.938	.920	.931	.903
590	.754	.851	.562	.623	.862	.668	.888	.650	.672	.828	.877	.834	.826	.766
600	.634	.739	.449	.501	.774	.558	.819	.510	.547	.703	.774	.734	.699	.643
610	.511	.607	.341	.391	.665	.437	.728	.384	.433	.562	.637	.612	.560	.518
620	.389	.467	.243	.286	.536	.302	.608	.275	.328	.422	.500	.474	.414	.388
630	.279	.333	.170	.199	.404	.203	.465	.185	.244	.298	.361	.338	.289	.277
640	.184	.216	.110	.130	.277	.127	.324	.116	.169	.194	.236	.221	.186	.185
650	.1127	.128	.0682	.0802	.171	.0750	.206	.0688	.107	.116	.142	.134	.112	.116
660	.0645	.0722	.0396	.0468	.0988	.0424	.121	.0392	.0591	.0648	.0790	.0762	.0627	.0666
670	.0351	.0380	.0223	.0265	.0533	.0232	.0673	.0212	.0312	.0348	.0420	.0410	.0340	.0364
680	.0180	.0190	.0120	.0139	.0274	.0118	.0351	.0112	.0157	.0175	.0220	.0213	.0177	.0179
690	.0093	.0092	.0061	.0071	.0141	.0058	.0180	.0058	.0080	.0086	.0114	.0107	.0089	.0088
700	.0046	.0044	.0031	.0036	.0072	.0028	.0094	.0031	.0042	.0041	.0058	.0052	.0043	.0043
710	.0023	.0021	.0015	.0018	.0035	.00135	.0048	.0016	.0022	.0019	.0029	.0025	.0021	.0022
720	.00111	.00104	.00076	.00092	.0017	.00065	.0024	.00078	.00112	.00089	.00136	.00112	.00102	.00102
730	.00053	.00052	.00038	.00046	.00082	.00032	.00123	.00036	.00057	.00043	.00064	.00051	.00049	.00049
740	.00027	.00025	.00019	.00024	.00042	.00016	.00062	.00017	.00029	.00022	.00032	.00024	.00024	.00023

TABLE 2.—Individual Values of Relative Visibility (to Unit Maxima)—Continued.

Wave length in millimicrons.	Values of relative visibility for the following observers:													
	Final average. 1.097 560.1 1.000	27 B. S. W. M. 27 1.128 554.9 1.020	28 H. B. W. F. 26 1.178 559.0 0.938	29 F. H. E. M. 22 1.049 563.5 0.989	30 W. E. M. M. 42 1.108 559.7 0.983	31 F. J. S. S. F. 28 1.045 563.1 0.997	32 W. E. F. M. 41 1.144 [9]560.9 [9]0.945	33 E. P. H. M. 43 1.051 560.6 1.020	34 A. G. W. M. 41 0.906 565.9 1.063	35 A. H. T. M. 36 1.134 560.1 0.949	36 A. F. B. M. 33 1.118 557.0 0.999	37 B. E. J. F. 33 1.098 557.8 1.014	38 F. L. M. M. 28 1.018 562.0 1.025	39 W. F. A. M. 29 1.085 562.1 0.973
430	0.033	0.040	0.032	0.021	0.032	0.032	...	...	...	...	0.064	0.038	...	0.018
440	.043	.056	.045	.034	.044	.038	...	...	...	...	.067	.048	...	.037
450	.051	.064	.052	.045	.046	.044	...	...	...	...	.072	.059	...	.046
460	.069	.088	.066	.062	.062	.058	...	...	...	...	.090	.081	...	.065
470	.103	.139	.091	.092	.099	.086	...	...	...	...	.123	.119	...	.100
480	.143	.198	.127	.134	.137	.121	...	...	...	...	.163	.165	...	.132
490	.197	.276	.183	.197	.187	.174	0.138	0.180	0.153	0.168	0.240	.232	0.173	.183
500	.320	.434	.304	.317	.309	.291	.236	.303	.252	.287	.373	.373	.294	.290
510	.524	.662	.488	.462	.498	.467	.418	.522	.446	.475	.577	.584	.480	.462
520	.732	.849	.664	.628	.748	.665	.674	.768	.667	.665	.763	.772	.727	.628
530	.878	.958	.818	.761	.900	.811	.871	.929	.834	.856	.877	.892	.884	.786
540	.964	.998	.926	.852	.963	.914	.978	.986	.942	.972	.953	.952	.972	.907
550	.998	.986	.986	.924	.985	.966	.998	.996	.991	.998	.992	.986	1.000	.978
560	.991	.948	.996	.984	1.000	.986	.962	.998	.997	.964	.997	.999	.992	1.000
570	.947	.892	.942	.993	.971	1.000	.912	.950	.972	.902	.952	.965	.970	.966
580	.863	.814	.836	.921	.866	.948	.830	.863	.929	.814	.853	.881	.908	.872
590	.754	.715	.710	.794	.717	.808	.728	.767	.877	.704	.720	.766	.815	.756
600	.634	.574	.574	.686	.581	.672	.608	.647	.817	.580	.597	.634	.684	.648
610	.511	.440	.449	.565	.462	.542	.479	.534	.713	.464	.473	.498	.564	.532
620	.389	.342	.332	.428	.362	.416	.353	.427	.578	.363	.346	.360	.439	.406
630	.279	.242	.227	.305	.271	.300	.246	.310	.442	.264	.241	.248	.328	.292
640	.184	.160	.144	.208	.185	.202	.160	.200	.309	.176	.155	.157	.218	.193
650	.1127	.0964	.0864	.136	.112	.127	.100	.116	.194	.110	.0927	.0930	.131	.114
660	.0645	.0559	.0484	.0838	.0640	.0739	.0581	.0646	.113	.0638	.0537	.0529	.0732	.0625
670	.0351	.0313	.0267	.0474	.0349	.0408	.0312	.0342	.0622	.0351	.0300	.0289	.0396	.0320
680	.0180	.0168	.0134	.0248	.0179	.0216	.0160	.0170	.0315	.0180	.0153	.0150	.0203	.0156
690	.0093	.0089	.0066	.0127	.0089	.0108	...	...	...	...	.0074	.0075	...	.0076
700	.0046	.0046	.0031	.0064	.0044	.0053	...	...	...	...	.0035	.0039	...	.0037
710	.0023	.0024	.00139	.0033	.0021	.0026	...	...	...	...	.0016	.0020	...	.0019
720	.00111	.00113	.00067	.0016	.00098	.00130	...	...	...	...	.00076	.00097	...	.00089
730	.00053	.00053	.00033	.00074	.00047	.00062	...	...	...	...	.00035	.00046	...	.00046
740	.00027	.00028	.00017	.00038	.00022	.00029	...	...	...	...	.00017	.00022	...	.00026

[9] 490—630 mμ. λc (average) for this range equals 559.9 instead of 560.1. λc (individual), in this and the other similar cases, was therefore increased by 0.2 to make comparable with other values.

Wave length in millimicrons.	Values of relative visibility for the following observers:													
	Final average.	40 J. F. E. M. 22	41 C. A. S. M. 51	42 F. E. C. M. 46	43 A.E.O.M. M. 26	44 B. M. M. 39	45 G. E. C. M. 21	46 I. C. G. M. 33	47 E. C. C. M. 40	48 J. H. D. M. 35	49 J. G. T. M. 27	50 F. B. S. M. 32	51 P. H. W. M. 54	52 R. D. M. 34
	1.097	1.093	0.955	0.992	1.185	1.237	1.223	1.072	1.038	1.200	1.188	1.006	1.133	1.142
	560.1	563.9	564.2	566.5	556.9	558.3	555.7	560.1	556.1	558.7	556.1	561.0	556.9	561.2
	1.000	0.951	1.075	1.011	0.958	0.896	0.951	1.005	1.077	0.938	0.979	1.041	0.988	0.934
430	0.033	0.025	0.027	0.021					0.056					
440	.043	.028	.036	.025					.076					
450	.051	.031	.049	.030					.098					
460	.069	.041	.070	.043					.131					
470	.103	.063	.105	.068					.191					
480	.143	.090	.139	.096					.249					
490	.197	.124	.192	.137	0.217	0.122	0.239	0.216	.320	0.187	0.252	0.196	0.247	0.146
500	.320	.233	.314	.247	.351	.256	.373	.337	.449	.305	.380	.324	.401	.252
510	.524	.415	.495	.437	.562	.478	.573	.547	.627	.489	.575	.529	.604	.423
520	.732	.642	.716	.650	.757	.692	.780	.730	.807	.684	.796	.758	.797	.643
530	.878	.827	.860	.810	.906	.884	.920	.851	.910	.814	.931	.920	.944	.799
540	.964	.944	.936	.898	.981	.986	.987	.939	.974	.906	.990	.992	1.000	.924
550	.998	.999	.974	.937	1.000	.993	.999	.991	1.000	.965	1.000	.998	.982	.990
560	.991	.984	.994	.982	.970	.939	.982	.998	.968	.997	.987	1.000	.945	.992
570	.947	.940	.997	.999	.911	.856	.872	.955	.912	.983	.936	.986	.865	.929
580	.863	.862	.969	.958	.795	.737	.747	.864	.840	.889	.831	.908	.777	.832
590	.754	.756	.909	.855	.643	.598	.632	.758	.760	.724	.674	.792	.683	.717
600	.634	.637	.806	.739	.523	.478	.507	.640	.650	.546	.533	.667	.560	.590
610	.511	.514	.668	.611	.417	.384	.390	.518	.534	.400	.406	.549	.448	.471
620	.389	.395	.509	.477	.312	.308	.288	.400	.416	.280	.288	.437	.352	.362
630	.279	.288	.358	.353	.221	.232	.201	.286	.299	.189	.195	.332	.259	.256
640	.184	.195	.231	.244	.144	.156	.130	.186	.199	.120	.125	.228	.175	.169
650	.1127	.124	.142	.153	.0861	.0962	.0769	.110	.124	.0725	.0757	.143	.106	.104
660	.0645	.0722	.0799	.0898	.0497	.0538	.0439	.0631	.0715	.0411	.0431	.0825	.0614	.0588
670	.0351	.0407	.0436	.0499	.0275	.0294	.0247	.0335	.0386	.0223	.0237	.0441	.0343	.0318
680	.0180	.0209	.0218	.0257	.0136	.0148	.0128	.0169	.0191	.0115	.0120	.0233	.0183	.0165
690	.0093	.0105	.0108	.0128					.0095					
700	.0046	.0051	.0053	.0060					.0049					
710	.0023	.0025	.0025	.0028					.0024					
720	.00111	.00116	.00117	.00138					.00102					
730	.00053	.00054	.00054	.00068					.00045					
740	.00027	.00024	.00026	.00032					.00021					

The final average visibility curve has the same general shape as the curves by all previous investigations and detailed comparisons are made later in the paper. A peculiarity which might be noted here is the apparent irregularity in the blue. This is directly traceable to the high ratio values uniformly obtained at 480 and at 450 and 440 mμ (fig. 7). While the authors are willing to admit that the usual Purkinje effect may be part of the cause for the high values from 450 down, it does not seem to be the whole cause (note the lower ratio value at 430 mμ); while at 480 mμ, at which a higher effective brightness was used than at 460, 470, or 490 mμ and one about equal to that at 500 and 525 mμ (see fig. 22), the Purkinje effect would seem to have been entirely absent. The magnitude of this effect at 480 mμ is clearly outside of any possible error connected with stray light, size of step, or energy distribution, and such possible causes were very carefully studied. The effect of change in the size of field was not tested, but measurements made at lower brightnesses, in some cases with the blue filter, resulted in the same shape of curve with no certain change in value.

With one exception this is the first time that the step-by-step method has been used in this blue region (although Hartman used the spectral pyrometer equality-of-brightness method from 410 to 500 mμ without obtaining any such peculiar effect). This one exception came to the authors' attention at the close of the present investigation. Exner (9), in connection with a study of the primary sensation curves, obtained the luminosity of the sun's spectrum (for his own eye) by the step-by-step method with a grating dispersing system. He published his ratio values, and the reciprocals of these values (his step was taken in the opposite direction from the authors') are given by the dotted line in Figure 7. The point of immediate interest is the similarity in the shape of the two curves in the blue region—the maximum values near 480 and 445 mμ and the minimum values near 505 and 465 mμ. The experimental conditions were so entirely different in the two cases that it seems impossible to explain these variations otherwise than as real values evoked by experimental conditions as yet largely unstudied. The irregularities of Exner's data in the red were not found in the case of the two observers in the present investigation who took the ratio readings at every 10 mμ throughout.

A word might be said regarding the general shape of the ratio curve (fig. 7). If the luminosity curve became logarithmic in the end regions and a step were used constant in wave length, then the ratio curve would become horizontal in both the red and blue ends. Since a constant mechanical step was used in this investigation (except for the change at 510 mμ), then the varying dispersion, increasing the wave length of the step toward the red and decreasing it toward the blue, would prevent the ratio curve from becoming horizontal in the red, but would cause it to reach a minimum and increase again in the blue. This is what happens taking the curve as a whole, the irregularities in the blue being superposed upon this general return of the curve to higher values.

Plotting up the average ratio curve has enabled a much more accurate extrapolation to be made than would otherwise be possible. Such an extrapolation has been made from 740 to 770 and from 430 to 400 mμ on the ratio curve. After the visibility curve had been computed from this average ratio curve from 400 to 770 mμ it was compared with the correct average visibility curve, as in Figure 7, and the reduction factors at 430 and 740 mμ necessary to change to the correct values at these wave lengths were used to correct the extrapolated values out to 400 and 770 mμ. These extrapolated values are considered very good in the red but more questionable in the blue. The general direction of the extrapolation was verified with two observers, each of whom made measurements of the ratio at 750 and 420 mμ and one at 410 mμ.

IV. COMPARISONS WITH OTHER INVESTIGATIONS.

1. FLICKER DETERMINATION OF COBLENTZ AND EMERSON.

Eighteen observers were obtained for the present measurements who had taken part in the previous investigation by the flicker

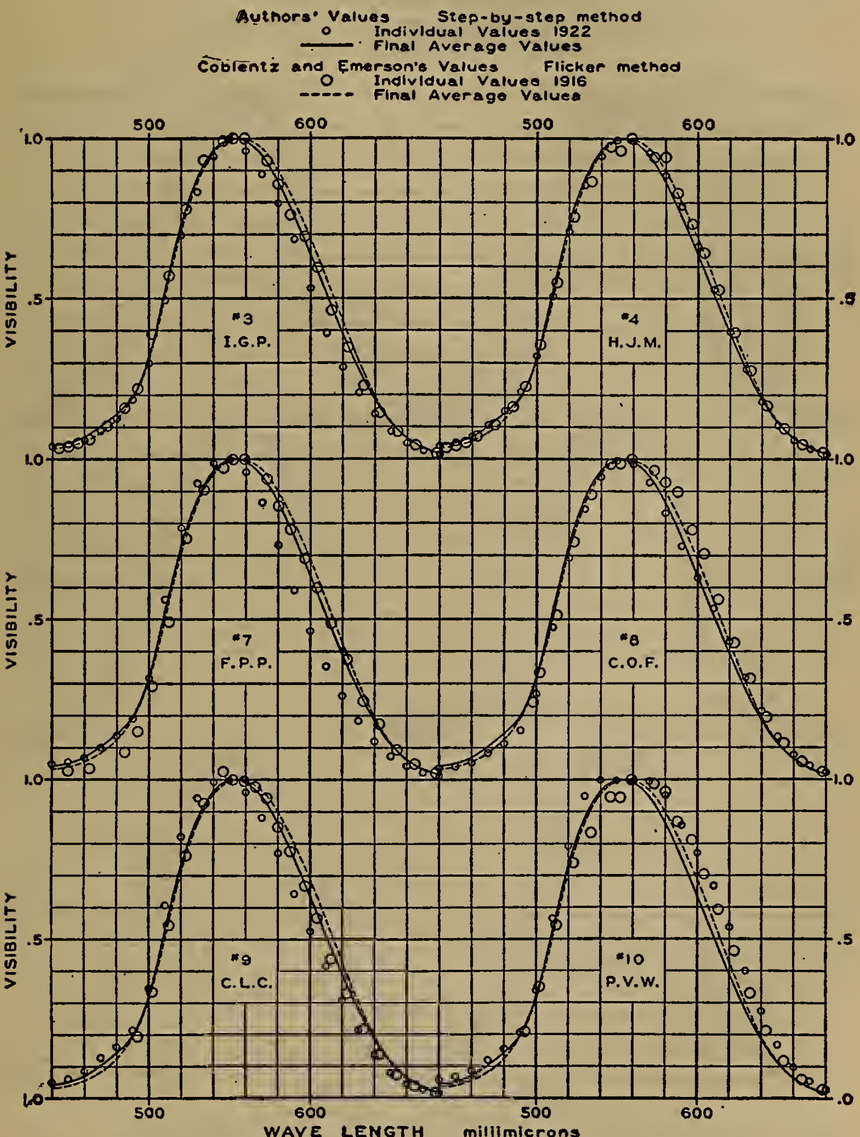

Fig. 10.—*Comparisons of individual data obtained in the present investigation, and in that of Coblentz and Emerson.*

method at the bureau (2) in 1916. All but one of these took two or more complete sets, and 12 out of the 18 were classed in the present investigation as good observers. Individual comparisons for the two investigations are made in Figures 10 to 12, comparisons between the averages of the 18 in Figure 13. In Figure 14 are shown results similar to those of Figure 13, but with four

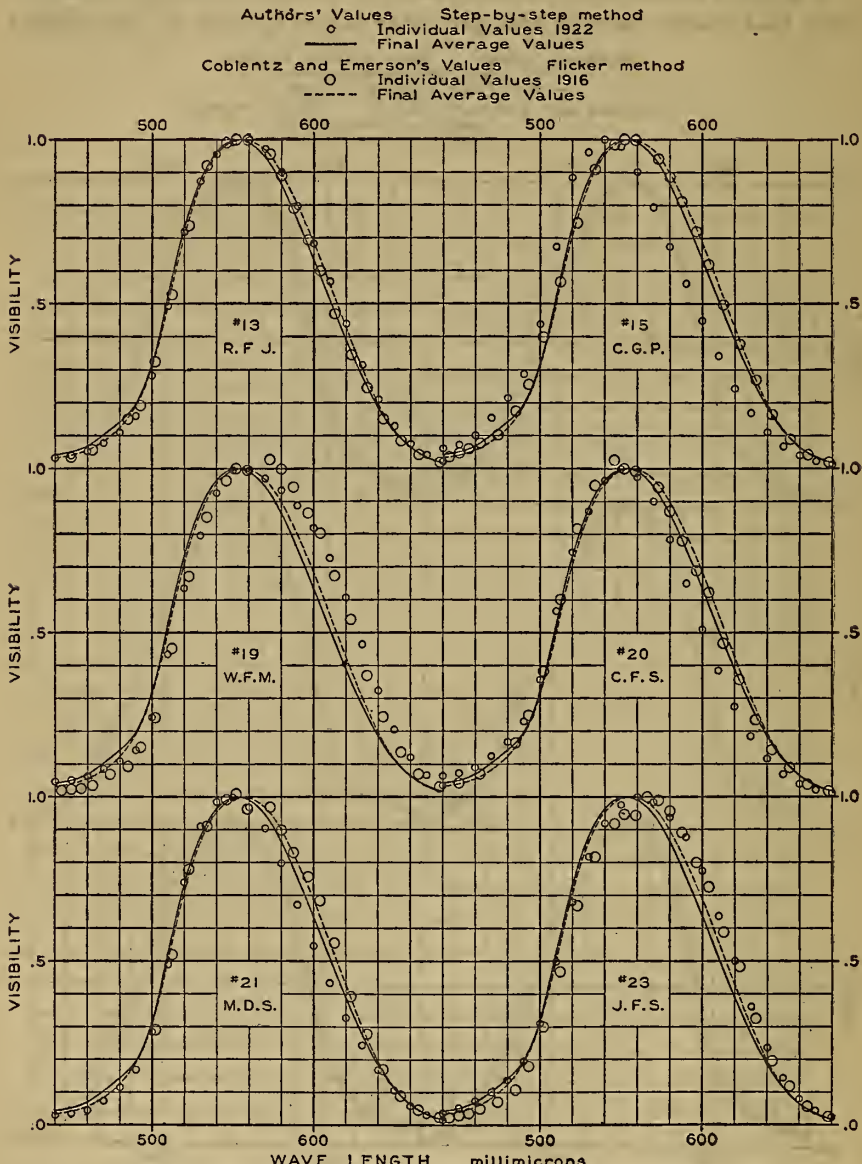

FIG. 11.—*Comparisons of individual data obtained in the present investigation, and in that of Coblentz and Emerson.*

observers omitted who showed the largest discrepancies in the two investigations. Three of these four were classed as poor observers, leaving 11 of the 14 common observers of Figure 14 classed as good observers.

Considering first the final average visibilities, the following points may be noted: (1) There is a difference in the wave length of maximum visibility—557 mμ in the former investigation, 553 mμ

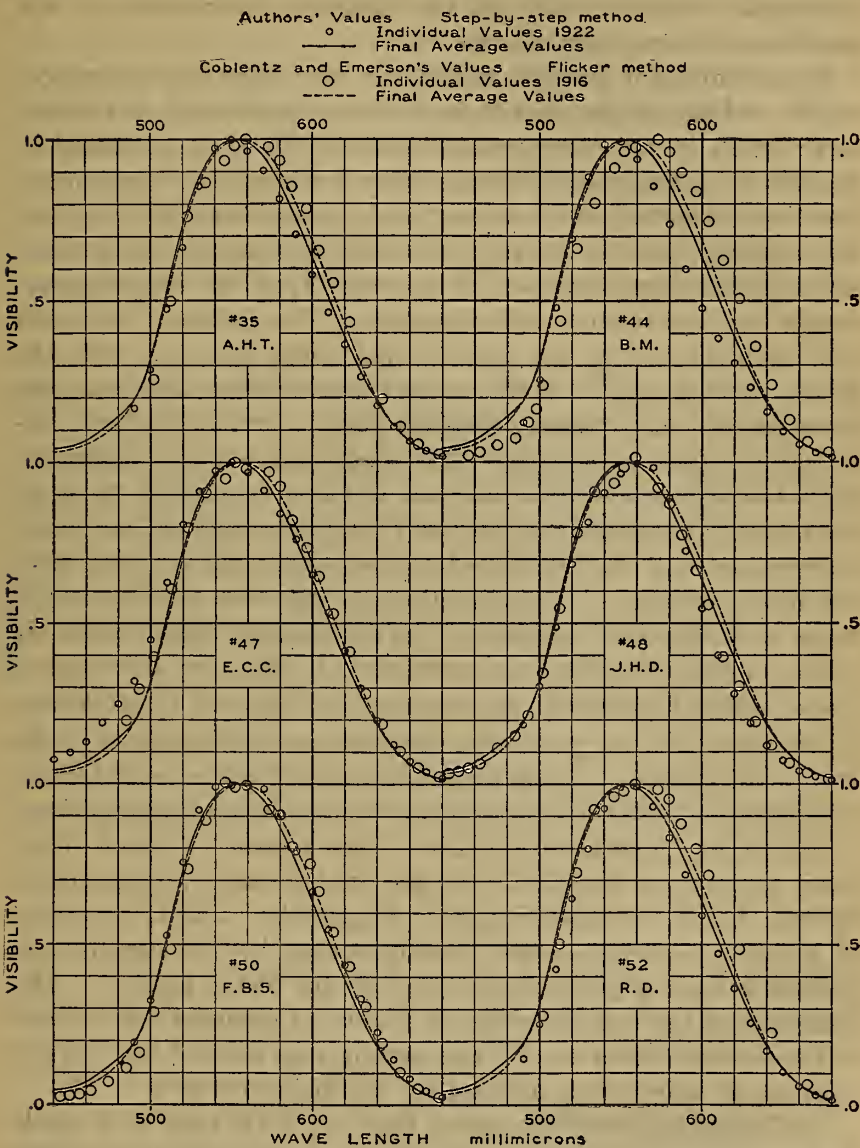

FIG. 12.—*Comparisons of individual data obtained in the present investigation, and in that of Coblentz and Emerson.*

in this one; (2) on the red side of the maximum the flicker measurements are higher than the present ones out to about 640 mμ; (3) beyond 650 mμ results are not greatly different, considering the difficulties of measurement in this region; (4) on the blue side of the maximum the two curves are nearly coincident out to 490 mμ; (5) beyond this wave length the two curves diverge, this divergence being a result of the apparent irregularity in the step-by-step curve near 480 mμ.

In searching for possible reasons for the important differences on the red side of the maximum it seeems reasonable to exclude large errors in the energy measurements, since, as explained in Section II, 2, the relative energy values for lamp 1717 were measured by Coblentz himself in 1917, only a year after his visibility investigation, and the energy values of the source in the present investigation were obtained from lamp 1717 by spectrophotometric and colorimetric measurements. The choice of observers can certainly not be the reason, since there were 125 with the flicker method and 52 in the present investigation, and since the averages of the common observers (figs. 13 and 14) show differences as great as or greater than these final average differences. Field brightnesses were of the same order of magnitude, those in the present measurements from one to two times higher than in the former, and a 3° field was used as compared with a 2° one with the flicker method. The differences are opposite in direction to those to be expected because of the difference in ages—about six years. Apparently the only conclusion is that the difference is one of method (flicker in the one case and equality of brightness in the other) for the particular field sizes and brightnesses of the two investigations. This general question will be discussed further in connection with the comparisons with still other investigations.

The 18, or 14, common observers (figs. 13 and 14) show differences in the same direction as do the final averages (except in the far red) and of the same or greater magnitude. The 14 observers of Figure 14 average almost exactly normal by the step-by-step method but are slightly red sensitive by the flicker method. The exclusion of the four observers in Figure 14 changed the average of the common observers by the step-by-step method but did not appreciably affect their average by the flicker method.

The individual visibility values (figs. 10 to 12) show as a whole the same trend of differences as do the averages, but of widely varying magnitude. In the majority of cases the agreement on the blue side of the maximum is good, while on the red side it is poor, this resulting, of course, in the final average differences as shown. Only three or four observers obtained higher values in the orange by the step-by-step method than by the flicker method, and these differences were slight.

It may be noted that these 18 common observers include those who show the most extreme variations from the normal, and the curves thus give a good idea of the variations to be expected among a large group of observers. In this connection reference may also be made to Figure 13 of Coblentz and Emerson's paper, which gives all 125 curves plotted in the same figure.

A matter of interest in these comparisons is the method of averaging. Hyde, Forsythe, and Cady state that they averaged Coblentz and Emerson's 125 individual values by the method of equal areas of the luminosity curves and obtained essentially the

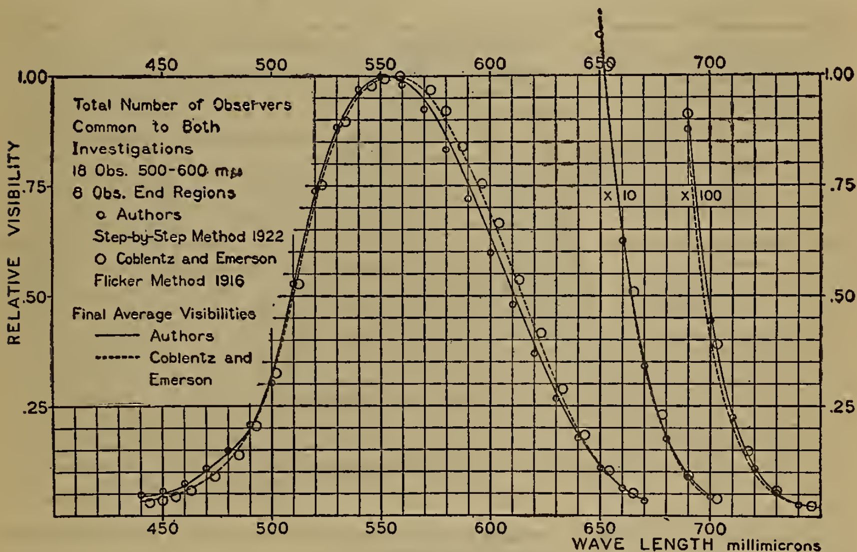

FIG. 13.—*Comparisons of average data obtained in the present investigation, and in that of Coblentz and Emerson.*

same results as the latter did averaging on the basis of unit maxima of the visibility curves. They caution, however, on the differences to be expected by the two methods of averaging if but a few individuals' values are being averaged.

The authors averaged their 52 observers by the method of unit maxima of the visibility curves and obtained essentially the same results as were obtained on the basis of equal areas of the luminosity curves. In Figures 13 and 14 the values of the observers by the flicker method were taken from the published data, unity at the maxima, and were averaged on that basis, while the same observers in this investigation were averaged on the usual equal-areas-of-luminosity basis. It seemed desirable, therefore, to average the 14 observers of Figure 14 by the step-by-step method on the unit-maxima-of-visibility basis, to afford a more accurate comparison of the two methods. This was done, and the results are shown by the small horizontal dashes in Figure 14. Results by the two methods are brought into slightly better agreement, therefore, for the 14 (and probably 18) observers, but are unaffected in the case of the total number of observers, when the same method of averaging is employed.

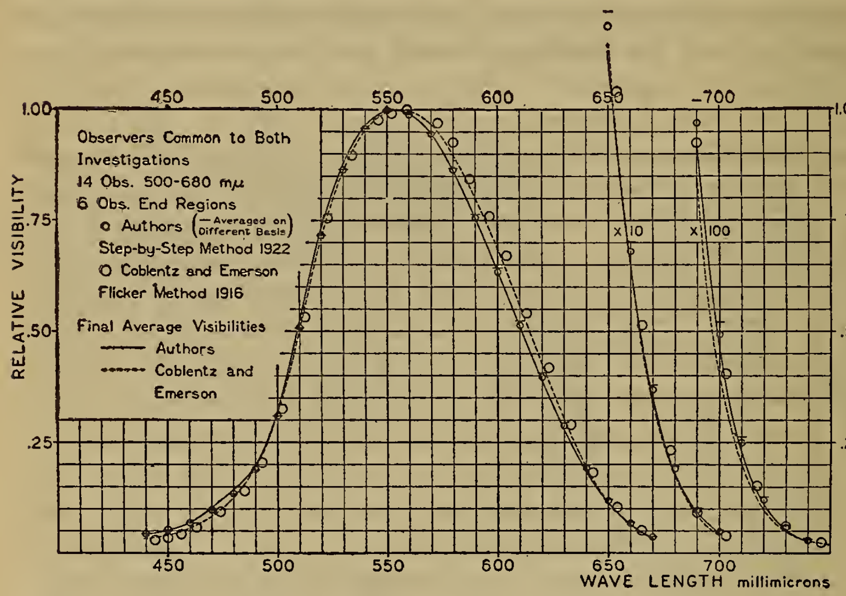

FIG. 14.—*Same as Figure 13, omitting those observers showing widest discrepancies in the two investigations.*

2. STEP-BY-STEP DETERMINATION OF HYDE, FORSYTHE, AND CADY.

During the course of the present investigation it was possible to obtain four observers from the Nela Research Laboratories whose visibility had been measured by the step-by-step method there (8) in 1918. Not as thorough a comparison could be made as was desirable, not only because of the small number but also for the reason that but two of the four had the time to take two complete sets of measurements. Nevertheless, the comparison is of interest and value, and the comparative results are shown in Figures 15 and 16. In Figure 15 the averages of the four observers are compared, in Figure 16 the individual values. In both figures the final average values for each investigation are given.

Comparing first, again, the final average visibilities for the two investigations, it will be noticed that around the maximum and on the blue side, except at 520 and 530 mμ, there is very close agreement. On the red side, however, there is an increasing divergence between the two curves as the wave length increases. It is improbable that errors in the energy measurements in either case could cause any appreciable amount of this difference. The excellent check obtained on the color temperature of a standard lamp in the two laboratories seems to preclude this. Another possible reason might be the difference in the observers. The large number of observers in the two investigations, however—29 in one case and 52 in the other—is evidence against this, as is also

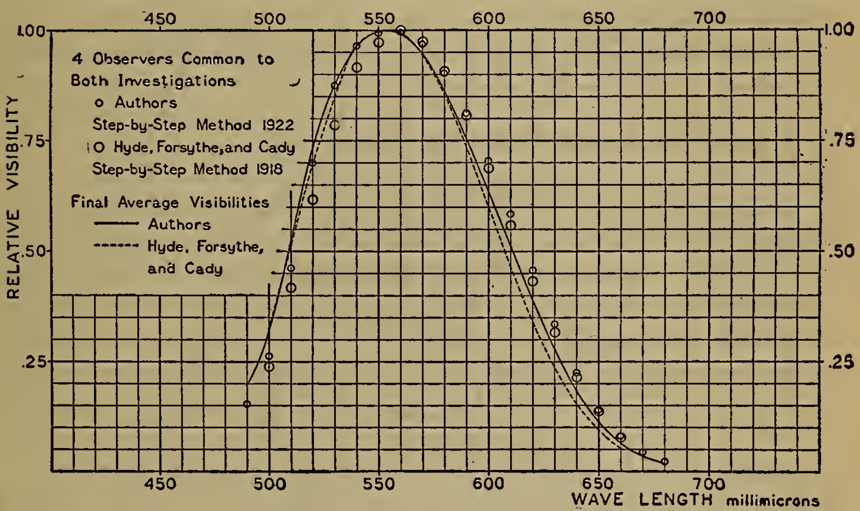

Fig. 15.—*Comparisons of average data obtained in the present investigation, and in that of Hyde, Forsythe, and Cady.*

the fact that the averages of the four common observers show differences in the same direction as do the final averages. A third possibility for explaining the divergence lies in the experimental conditions. The field brightnesses used were, however, not greatly different in the two cases, those in this investigation being slightly higher than those in the Nela measurements; but in the size of the field used there is a decided difference. In this investigation the field was 3° in diameter; at Nela the usual Lummer-Brodhun spectrophotometer field was used—the effective diameter uncertain, but less than 10°. This difference in field size may, therefore, be a real cause of the divergence in values in the orange and red regions, and the direction of this difference agrees with the conclusions reached by other investigators (1), (8), (10).

As regards the averages of the four observers common to both investigations (fig. 15), they show differences as noted above in the same direction as the final averages. The variations in magnitude of this difference are directly traceable to the large individual differences for the four observers. The average for the four is more nearly normal in this investigation than it was in the former one.

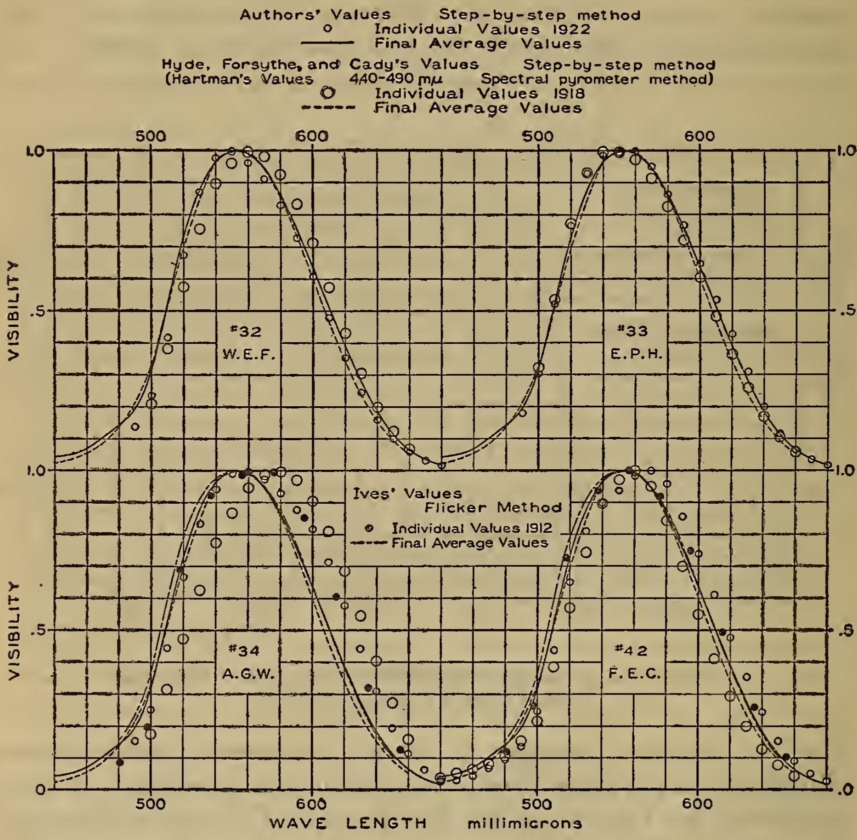

Fig. 16.—*Comparisons of individual data obtained in the present investigation, and in those of Hyde, Forsythe, and Cady, and of Ives.*

The individual visibilities (fig. 16) show differences of such a nature as to make it difficult to form any conclusion unless it is that an individual's visibility may be subject to considerable variation from time to time. In these four cases one observer gets very close agreement, if allowance is made for the difference in the final average visibilities; three get higher values in the blue in this investigation than in the former; while in the red two come higher and two lower. Further evidence concerning the variation of an observer's visibility from time to time is brought out later in the paper.

The visibility values given in Figures 16 and 19 for Hyde, Forsythe, and Cady for the end regions of the spectrum were those obtained by a spectral pyrometer method, as noted in the figures, and arbitrarily joined to the step-by-step method curve at 500 and 650 mμ. Differences in the final average values may be noted in the figures. It may be remarked that considerable of the differences in the far red would be eliminated if the spectral pyrometer values were united to the present authors' step-by-step values at 650 mμ. It is also of interest to note that the only observer of the four (No. 42) who made measurements below 490 mμ in the present investigation obtained lower values in the blue than he did in the previous investigation, whereas the average differences were the reverse of this.

3. FLICKER DETERMINATIONS OF IVES AND OF NUTTING.

Two of the four observers common to this investigation and to that of Hyde, Forsythe, and Cady were also used in the measurements made by Ives (3) in 1912 by the flicker method. Individual comparisons for the three investigations are shown in Figure 16, comparisons of the averages in Figure 17. These two observers average red sensitive in all three investigations. Relatively close agreement in the individual data may be noted between Ives's values and those of the present investigation.

The final average values of Ives are based upon 18 observers. His data were obtained under the experimental conditions which he had concluded were necessary to bring the two methods of photometry into agreement, viz, a 2° field and a brightness equivalent to approximately 250 to 300 m-candles through a 1-mm^2 slit, which he considered equal to about 2.5 millilamberts for full pupil illumination.

In 1914 Nutting (4) measured the visibility of 21 subjects by the flicker method, using a 2° field and a brightness slightly greater than that used by Ives. His results are shown in Figures 18 and 19, his revised values being given. (This revision and that of Reeves (fig. 19) were made necessary as a result of a redetermination in the energy distribution of the acetylene flame.)

Both Ives's and Nutting's average values agree closely with each other and with those of the authors on the red side of the maximum, but on the blue side show extremely wide variations from each other, the authors' values coming between the two sets. Ives (7) intimates that this difference between his and Nutting's values may be ascribed to the fact that observers of unknown color characteristics were used in the two investigations and with the numbers used, 18 and 21, the average curves might be shifted from the position which would be obtained with a larger number. Since these two investigations were made under similar conditions so far as field size and brightness are concerned, it would seem that the average of their final results with a total of 38 observers may be accepted as representative flicker values for those conditions of field size and brightness which Ives recommends as

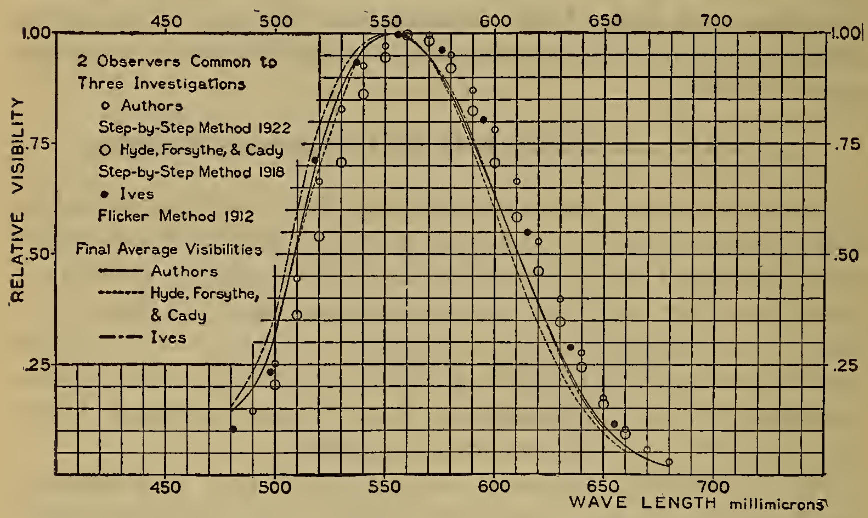

Fig. 17.—*Comparisons of average data obtained in the present investigation, and in those of Hyde, Forsythe, and Cady, and of Ives.*

standard in order to bring the flicker method and equality-of-brightness method into agreement.

In Figure 18 these average values for Ives's and Nutting's investigations, 38 observers, have been plotted and compared with the authors' values for 52 observers over as wide a wave length range as was common to the three investigations, Ives's extrapolated values not being used. The resulting agreement is close over most of the range and indicates that, if Ives's contentions are valid regarding the essential experimental conditions for making the two methods of photometry agree—viz, a 2° field and a brightness of 2.5 millilamberts—then it is more essential that the flicker method conform closely to these specifications than that the equality-of-brightness method should, the authors using a 3° field and a brightness one-half to one-fourth that used by Ives and Nutting.

This last statement will, perhaps, summarize the results of the present investigation so far as they bear upon the comparison of the flicker and equality-of-brightness methods of photometry. Ives (7) states that Coblentz and Emerson's curve is shifted to the red because the field brightness was only one-fourth that recommended by him, and that the curve of Hyde, Forsythe, and Cady is shifted toward the blue because of a similar disregard of the proper field size and brightness to be used. The results of the present investigation are not in conflict with these conclusions,

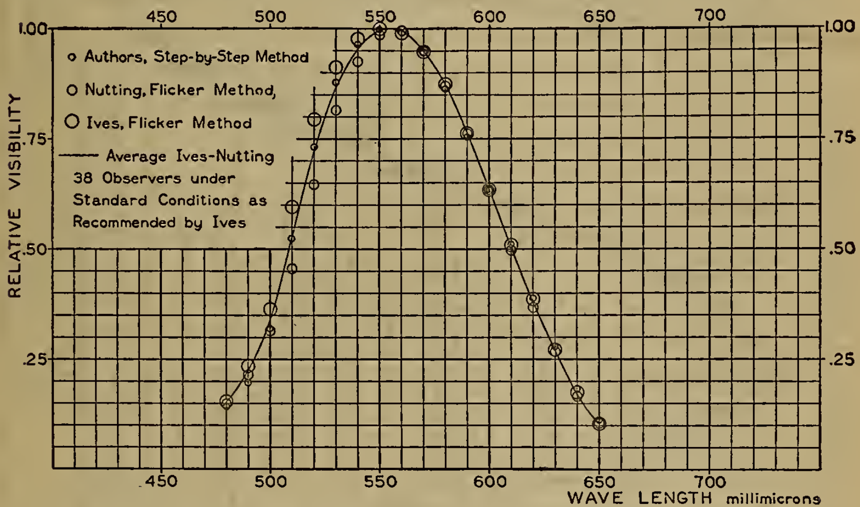

FIG. 18.—*Comparisons of average data obtained in the present investigation, and in those of Ives and of Nutting.*

except to indicate that the shift of Hyde's curve toward the blue is a result, primarily, of the large field size rather than of the brightness used.

4. GENERAL COMPILATION OF VISIBILITY DATA.

In Table 3 are given all the extensive visibility data to date. There are included, in addition to the data already discussed, the data of Reeves (5) and of So (6), experimentally obtained, the empirical data of Ives (7) on the transmission of his solution used for physical photometry, and average data recommended or adopted by Ives (7), Priest (11), the Illuminating Engineering Society (12), and the authors. All of these data (excepting the

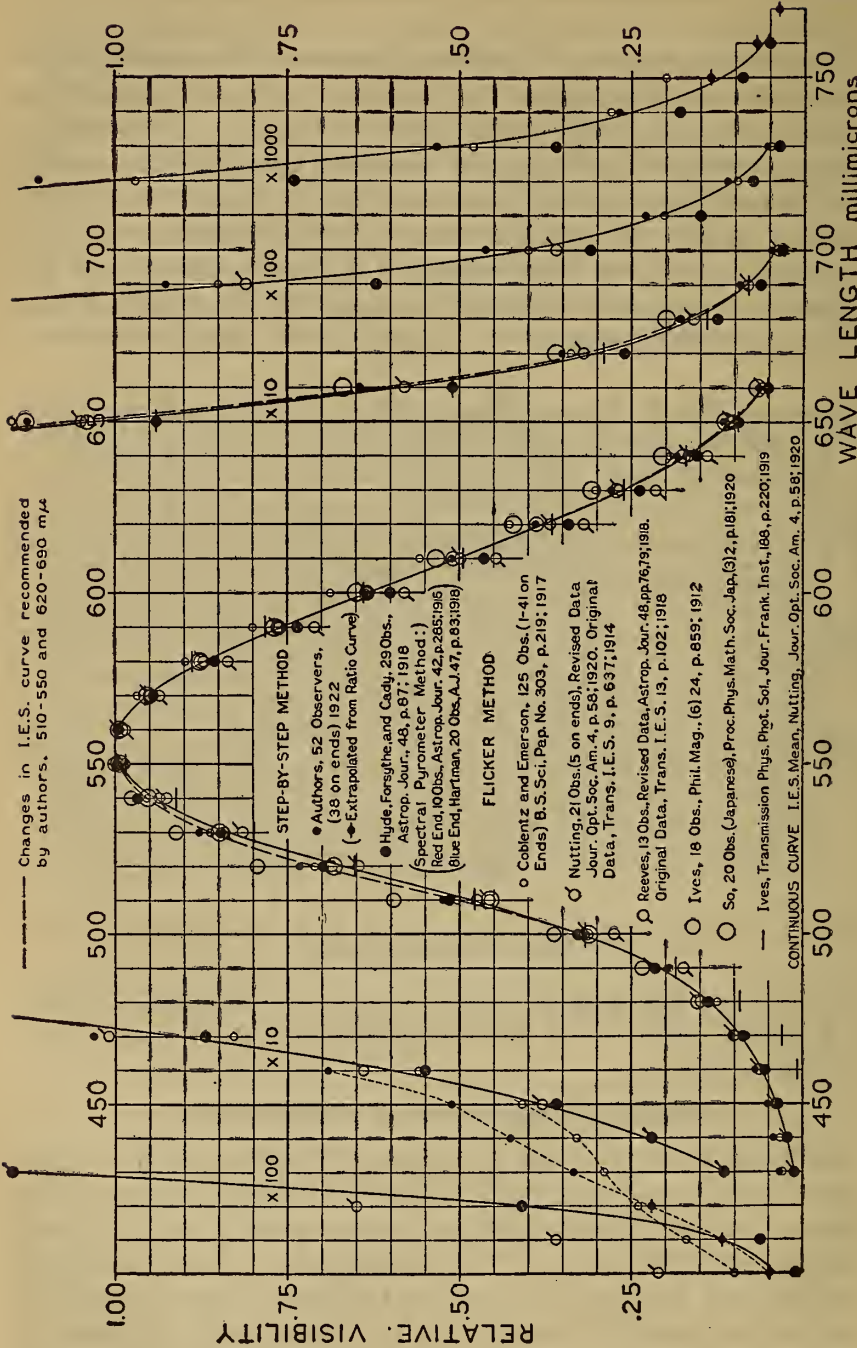

FIG. 19.—*Compilation of visibility data. See Table 3 and Figure 21.*

adopted averages of Ives and Priest) are plotted in Figure 19, and in Figure 20 the four adopted or recommended averages are given.

In addition in Table 3 are given: (*a*) The wave length centers of gravity of the visibility curves; (*b*) the areas of the visibility curves to unit maxima, relative to the authors' recommended average; (*c*) the wave length centers of gravity of the resulting luminosity curves for a Planckian radiator at 2,077° K.; and (*d*) the areas of these luminosity curves to unit maxima—all as computed by the present authors. In each of the four cases the values have been computed for two wave length ranges—(*a*) the complete range, where the data will permit (complete in the sense that inclusion of data at wave lengths outside of those

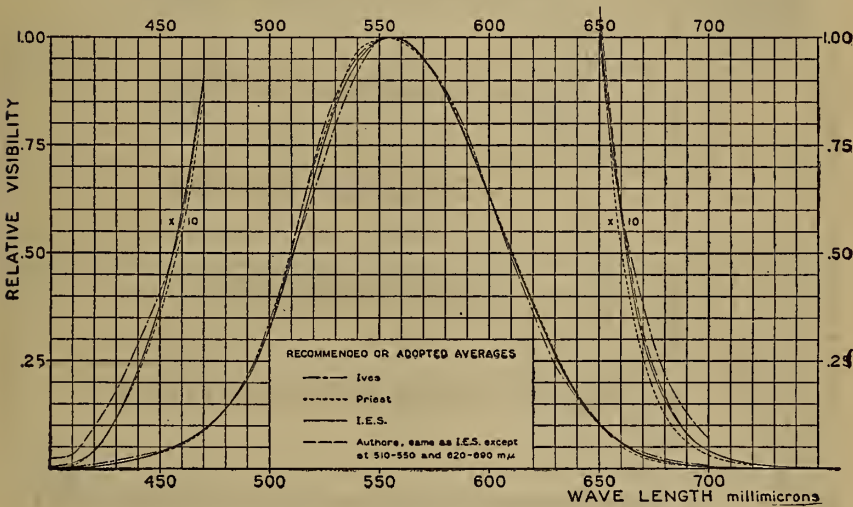

FIG. 20.—*Recommended or adopted visibility data. See Table 3.*

given will have no important effect upon the values given), and (*b*) a shorter range, complete enough to give approximate values but short enough to enable values to be computed for most of the data.

The data of Reeves by the flicker method comprised only 13 observers, five of whom were common to Nutting's investigation. He also used Nutting's energy values. His field size and effective brightness are not stated. His data disagree considerably on the red side of the curve from the other flicker data, and the visibility curve is the narrowest of any so far obtained. The small number of observers could easily account for these differences, although the averages for the five observers who were common to both his and Nutting's investigations show the same sort of differences as do the final curves.

TABLE 3.—General Compilation of Visibility Data.

Wave length in millimicrons.	Experimental data.							Transmission of physical photometer solution.	Adopted or recommended averages.			
	Step-by-step method.		Flicker method.									
	Authors.	Hyde, Forsythe, and Cady.[1]	Coblentz and Emerson.[2]	Nutting.[3]	Reeves.[4]	So.[5]	Ives.[6]	Ives.[7]	Ives.[8]	Priest.[9]	I. E. S.[10]	Authors.
400	[11] 0.005	0.00009	0.010	0.0021					0.0024		0.0004	0.0004
410	[11] .012	.00062	.017	.0036					.0032	0.00062	.0012	.0012
420	[11] .022	.0041	.024	.0065					.0096	.0041	.0040	.0040
430	.033	.0115	.029	.0115					.018	.0115	.0116	.0116
440	.043	.022	.033	.022			[11] 0.029		.029	.022	.023	.023
450	.051	.036	.041	.038			[11] .047		.041	.036	.038	.038
460	.069	.055	.056	.064			[11] .073	0.007	.058	.055	.060	.060
470	.103	.087	.083	.101			[11] .107	.029	.090	.087	.091	.091
480	.143	.138	.125	.149			.154	.091	.138	.138	.139	.139
490	.196	.216	.194	.215			.235	.185	.215	.216	.208	.208
500	.318	.328	.316	.314	0.275	0.312	.363	.317	.341	.328	.323	.323
510	.523	.515	.503	.456	.474	.455	.596	.478	.493	.515	.484	.503
520	.732	.698	.710	.646	.686	.683	.794	.652	.638	.698	.670	.710
530	.878	.847	.862	.815	.841	.848	.912	.799	.795	.847	.836	.862
540	.964	.968	.954	.925	.935	.952	.977	.911	.919	.968	.942	.954
550	.998	.996	.994	.986	.993	.993	1.000	.985	.992	.996	.993	.995
560	.991	.995	.998	.995	.985	.994	.990	.999	.999	.990	.996	.995
570	.947	.944	.968	.949	.935	.953	.948	.957	.953	.948	.952	.952
580	.863	.855	.898	.871	.836	.877	.875	.888	.879	.875	.870	.870
590	.754	.735	.800	.762	.710	.769	.763	.782	.777	.763	.757	.757
600	.634	.600	.687	.634	.580	.650	.635	.639	.633	.635	.631	.631
610	.511	.464	.557	.498	.446	.534	.509	.494	.491	.509	.503	.503
620	.389	.341	.427	.368	.319	.422	.387	.365	.362	.387	.380	.381
630	.279	.238	.302	.268	.214	.308	.272	.261	.240	.272	.262	.265
640	.184	.154	.194	.166	.140	.205	.175	.170	.164	.175	.170	.175
650	.1125	.094	.115	.105		.113	.104	.094	.101	.104	.103	.107
660	.0642	.051	.0645	.058		.067	[11] .068	.051	.060	.051	.059	.061
670	.0349	.026	.0338	.032		.036	[11] .044	.029	.038	.026	.030	.032
680	.0178	.0125	.0178	.016		.020	[11] .026	.014	.022	.0125	.016	.017
690	.0092	.0062	.0085	.0081				.008	.013	.0062	.0081	.0082
700	.0045	.0031	.0040	.0036					.007	.0031	.0041	.0041
710	.0022	.0015	.00203							.0015	.0021	.0021
720	.00108	.00074	.00097							.00074	.0010	.00105
730	.00051	.00036	.00048								.00052	.00052
740	.00026	.00018	.00028								.00025	.00025
750	[11] .00014	.00009	.00020								.00012	.00012
760	[11] .00007	.00005									.00006	.00006
770	[11] .00004											

WAVE LENGTH CENTER OF GRAVITY OF VISIBILITY.

410–720 mμ.												
	559.18	558.90	561.07	....	....	....	....	....	....	560.11	560.38	560.20
450–670 mμ.												
	560.09	559.05	561.95	560.57	....	....	558.81	562.65	560.38	560.26	560.47	560.26

AREA OF VISIBILITY CURVE, UNIT MAXIMUM, RELATIVE TO AUTHORS' RECOMMENDED CURVE.

410–720 mμ.												
	1.019	0.977	1.031	....	....	....	....	....	....	1.000	0.989	1.000
450–670 mμ.												
	1.012	0.978	1.025	0.981	....	....	1.039	0.960	0.982	1.001	0.989	1.000

WAVE LENGTH CENTER OF GRAVITY OF LUMINOSITY, PLANCKIAN RADIATOR AT 2,077° K.

410–750 mμ.												
	582.13	579.65	583.05	....	....	....	....	....	....	581.05	581.58	581.57
480–690 mμ.												
	582.08	579.78	583.17	581.89	....	....	581.61	581.63	581.91	581.18	581.64	581.63

AREA OF LUMINOSITY CURVE, UNIT MAXIMUM, PLANCKIAN RADIATOR AT 2,077° K, RELATIVE TO AUTHORS' RECOMMENDED CURVE.

410–750 mμ.												
	1.022	0.976	1.009	....	....	....	....	....	....	0.991	0.991	1.000
480–690 mμ.												
	1.021	0.976	1.009	0.983	....	....	1.021	0.960	0.970	0.991	0.990	1.000

[1] Astrop. Jour., 48, p. 87; 1918.
[2] B. S. Sci. Paper No. 303, p. 219; 1917.
[3] Jour. Opt. Soc. Am., 4, p. 58; 1920. Revised data.
[4] Astrop. Jour., 48, pp. 76, 79; 1918.
[5] Proc. Phys. Math. Soc. Jap. (3), 2, p. 181; 1920.
[6] Phil. Mag. (6), 24, p. 859; 1912.
[7] Jour. Frank. Inst., 188, p. 220; 1919.
[8] Jour. Frank. Inst., 188, p. 220 (Table 1, last column); 1919.
[9] Jour. Opt. Soc. Am., 4, p. 398; 1920; Jour. Opt. Soc. Am., 4, p. 471; 1920; B. S. Sci. Papers No. 417, p. 234, footnote 10, and Figure 2; 1921; B. S. Sci. Papers No. 443; 1922.
[10] Trans. I. E. S., 13, p. 523; 1918; Jour. Opt. Soc. Am., 4, p. 58; 1920.
[11] Extrapolated.

The data of So by the flicker photometer are interesting as showing that the visibility of the average Japanese is probably not appreciably different from that of American observers. He used a field of 1.5° and an effective brightness probably not less than that recommended by Ives.

The transmission data of Ives's physical photometer solution are entirely empirical. Their greatest importance lies in the resulting value of the wave length center of gravity of light at 2,077° K., since it was with a carbon light at this color temperature that the data have been correlated with those of a large number of observers. This value, as computed by the authors, is 581.6 mμ.

Any average visibility curve should, therefore, give approximately this center of gravity for a Planckian radiator at 2,077° K. But it must not only do this; it must also represent a reasonable average of the more extensive visibility data as directly measured. Now the I. E. S. curve does give this wave length center of gravity for 2,077° K. as is shown in Table 3, but it is very obvious from Figure 19 that the curve is not a good average of the data from 510 to 550 mμ.

The authors have chosen the values of Coblentz and Emerson's data in this region for their recommended average. These values lie very close to the experimental data of Hyde, Forsythe, and Cady, to the authors' data, and to the average Ives-Nutting data shown in Figure 18.

Such a change, however, would shift the wave length center of gravity of the luminosity curve for 2,077° K. from its measured value of 581.6 mμ unless balanced by a corresponding increase in values at wave lengths greater than this. The authors believe such a change in the visibility is warranted by the experimental data. In the region from 690 to 650 mμ the majority of the data lie above the I. E. S. curve. They have, therefore, made the changes shown in Figure 19 and Table 3, continuing the changes to 620 to produce a smooth curve. These changes, although appearing slight on the visibility curve, are sufficient to bring the center of gravity for 2,077° K. back to 581.6 mμ. (Slight additional changes may be noted at 560 and 720 mμ, Table 3.)

The authors' recommended average, therefore, meets both requirements: (a) It satisfies the direct experimental measurements, and (b) it gives the same wave length center of gravity for a color temperature of 2,077° K. as that found by Ives for the average of a large number of observers.

The I. E. S. data in the violet have been accepted by the authors for lack of any good reason for changing them, but the relative as well as the absolute values are very uncertain and must be considered as tentative only.

Values of visibility around the maximum have been plotted to a magnified scale in Figure 21. The wave lengths of the maximum visibility are also given, where these have been stated by the respective authors. Attention is called to the close agreement between the two step-by-step determinations, in spite of the fact that Hyde, Forsythe, and Cady give 556 for their maxi-

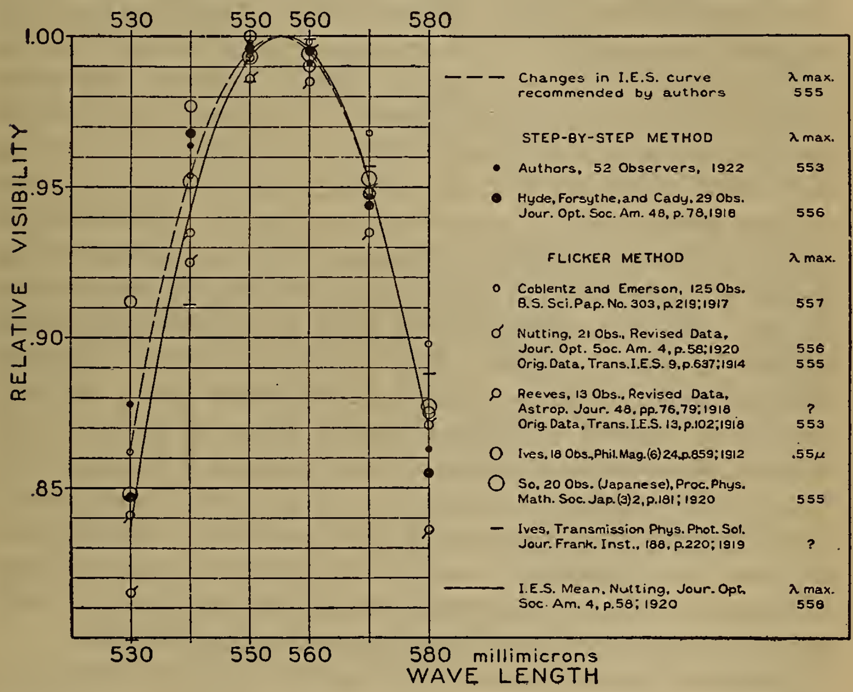

FIG. 21.—*Values of Figure 19, magnified, in region of maximum visibility.*

mum and the authors 553. Hyde, et al., state that the value they give is uncertain by 3 mμ, because of uncertainty in drawing the curve, and it may be noted that their actual values at 550 and 540 mμ are higher, respectively, than those at 560 and 570 mμ. The authors consider their value of 553 mμ graphically uncertain by not more than ± 1 mμ. It would seem from a study of all the data that 555 mμ would be a better average value for the maximum than 556 as adopted by the I. E. S., this shift in the wave length of maximum value being directly connected with the increase in values from 510 to 550 mμ, as mentioned above.

5. PECULIARITIES OF INDIVIDUAL DATA.

The question naturally arises as to what may be the reason for some of the enormous variations which have been found in some cases in the same individual's visibility curves by the different investigations. The study of the precision of the step-by-step method given in the next part of the paper and the known precision of the flicker method seem to preclude the possibility that the usual observational errors can be blamed in all cases for the variable data obtained. The following cases may be noted:

Observers Nos. 34 and 42 (Fig. 16).—These two observers have been common to three investigations. They have had long photometric experience. Ives apparently took a single run on each observer with check points. Hyde, Forsythe, and Cady took at least two runs on each observer. In the present investigation two runs with check points were made on No. 42 and one run with check points on No. 34. The agreement between Ives's and the present data is relatively close, and the differences are similar to those in the final curves and could be explained on the ground of observational error or changes in visibility with increasing age. But the values given by Hyde, et al., are clearly unexplained by these reasons. They state that some observers obtained results on the two trials agreeing within 3 or 4 per cent, while some showed differences several times as large. Here, however, are differences between their values and the nearest of the other two values as large as 25 or 50 per cent of their values. Considering the experience and known precision of measurement of these two observers, it seems impossible to explain these differences, except as actual variations in their visibilities, evoked, however, possibly by the different experimental conditions.

Observer No. 15 (Fig. 11).—This observer has had considerable photometric experience and was classed in this investigation as a good observer. He took two runs with check points. His curve is the bluest of any of the 52 observers, and yet he was found normal by the flicker method on the average for three trials. On the other hand, his Y/B ratio [5] by the flicker method was 0.936 as compared with the normal value of 0.987. He was found by Coblentz and Emerson to be variable in his readings from day to day.

Observer No. 9 (Fig. 10).—This observer took a single run early in the investigation and was found very definitely red

[5] Test ratio used in heterochromatic photometry. For explanation, see reference (10) of bibliography.

sensitive. A few months later he took a double run and was found extremely blue sensitive. The curve, as drawn, represents the average, double weight being given to the second run.

OBSERVER No. 52 (FIG. 12).—This observer is high in the red by the flicker method and low by the step-by-step method, relative to the respective averages. Objects often appear differently colored to this observer, depending upon which eye is used.

OBSERVERS Nos. 7, 21, 35, and 44 (FIGS. 10 to 12), can not be taken to prove anything in this connection. No. 35 took but a single run. No. 7 had difficulty in reproducing his results in the yellow and orange, where the color difference existed. No. 44 was extremely erratic in the yellow and green.

Contrasted with these cases are those who practically duplicated their results by a previous investigation. The data in these figures, as a whole, might be taken to indicate that conclusions of any kind based upon a study of a very few observers may be questionable.

OBSERVER No. 23 (FIG. 11) is partially color blind, and confuses reds and greens. His curve shows red sensitiveness, so called, but not so much as others who are not color blind. It has been found by others (2) that a color-blind person will have an abnormal visibility curve, but that there are many observers with abnormal visibility curves who are not color blind. This observer was the only one of the 38 examined who showed no trace of the usual irregularity in the ratio curve below 480 mμ (fig. 7). His average ratio values stay constant (0.85 to 0.865) between 480 and 430 mμ.

OBSERVER No. 19 (FIG. 11), who was found by Coblentz and Emerson to have the highest values of any observer on the red side of the maximum, in the present investigation also holds the record in this respect, although observer No. 34 was a close second, practically a duplicate. No. 34 was the "reddest" observer in the investigations of Hyde, Forsythe, and Cady, and of Ives.

V. PRECISION OF THE MEASUREMENTS.

The experimental errors entering into the determination of visibility by the step-by-step method may be classed as follows: (*a*) Those connected with the energy measurements and (*b*) those connected with the luminosity measurements, which latter may be subdivided into those connected with the photometric settings and those connected with the determination of the size of the step. In this discussion it has seemed worth while to make rather a detailed study of the general reliability of the step-by-step method.

1. ENERGY MEASUREMENTS.

Several factors are involved in the determination of the relative spectral distribution of radiant energy at the ocular slit of the spectrophotometer. These are:

1. *The relative spectral energy distribution in the light from the source used; that is, from the back of the light box L of Figure 1.*—This was measured in terms of that of the intermediate standard, lamp 3256, which was in turn measured in terms of that of the fundamental standard, lamp 1717. The uncertainties in the energy distribution of lamp 3256 were minimized by the check measurements already mentioned. The procedure was, in brief: (*a*) Color matching of lamps 3256 and 3257 (exactly similar to 3256) with lamp 1717 by four experienced observers, the same voltage, 101.0, being obtained for both lamps; (*b*) spectrophotometric comparison of lamps 3256 and 3257 at 101.0 volts with lamp 1717, the two lamps agreeing in relative energy distribution with lamp 1717 within about ±1 per cent from 500 to 660 mμ and within about ±2 per cent beyond these wave lengths (with no consistent deviations) without the aid of a potentiometer control; (*c*) the Nela research determination of the color temperature of lamp 3257 at 101.0 volts, this being 2,848° K., exactly agreeing with that computed for lamp 1717 from the original radiometric data of Coblentz; and (*d*) the use of Coblentz's original data on lamp 1717 as best expressing that of lamp 3256. It is thus believed that the uncertainty of the energy of lamp 3256 is no greater than that for lamp 1717, the reliability of which in turn was increased by the excellent independent color-temperature check from the Nela Research Laboratories.

The measurement of the energy of the light from the back of the light box *L* in terms of that of lamp 3256 should have about the same reliability as that found when the energies of lamps 3256 and 3257 were measured in terms of that of lamp 1717, viz, within ±1 to ±2 per cent. This reliability was also increased by the repeated measurements which were made, so that finally there could be little doubt as to the true shape of the curve expressing the ratio of the energies of the light from the interior of the box to that of lamp 3256. Each pair of lamps was separately measured, as well as all 8, and repeatedly remeasured to correct for the slight yellowing due to the tungsten deposit gradually forming on the bulbs. Measurements were repeated after observations were completed for 10 to 12 observers, and the measurements before and after averaged to get the best results for that group. The change would usually be from 1 to 3 per cent. New lamps were inserted once during the investigation. The shape of the curve expressing the ratio of energies of the light from the box to that from lamp 3256 showed sufficient selectivity to prove that errors of several per cent would have been introduced if the color temperature of the light from the box had been taken with Planck's equation solely as the basis of energy distribution.

The question arose as to whether the energy in beam 1 was the same as that in beam 3, Figure 1. This was accordingly tested at two different times and it was found that beam 1 was slightly but definitely bluer than beam 3. Correction was therefore made to the values as obtained from beam 3.

2. *Spectral transmission of spectrophotometer.*—As already described, this was measured as a unit, and while the experimental difficulties were rather great the measurements were repeated until the uncertainty of the curve was small. The red end is, perhaps, more uncertain than the rest. It was more difficult to get values to check on that end and no intense homogeneous source was available for check such as the Hg 435.8 line in the blue. Measurements made on other spectrometers and on prism glass give the same general and reasonable shape without, however, so much of a drop in the red.

3. *Dispersion of the spectrophotometer.*—Such measurements could be made so precisely that the errors are considered entirely negligible in comparison with the others entering into this question.

4. *Stray light or spectral transmission of filters used to eliminate stray light.*—While these corrections were actually made to the luminosity (ratio) measurements, they may well be considered here. Stray-light correction was made only from 510 to 460 mμ, inclusive, since the filters were used below 460 mμ and in the red. The stray-light curve (fig. 5) is entirely reasonable and amounts to only 7 per cent at 460 mμ, so that any error in the correction must be very small relative to the luminosity. In the case of the filters the absolute values of transmission were somewhat uncertain because of the differences obtained by the different methods, but since it is the relative values over the wave length interval corresponding to the step which are necessary, not the absolute values, and since the filters were used so far as possible where the transmission curves were not steep, the error in the luminosity must again be very small.

It would seem from the above that the uncertainty in the spectral energy distribution at the ocular slit of the spectrophotometer throughout the main part of the spectrum—that is, 500 to 660 mμ—would be within ±5 per cent, but greater, of course, for the wave lengths outside this range, perhaps reaching ±10 per cent in the ends. It is believed that this uncertainty can not be greater than that in other investigations, where the energy measurements have been based solely on radiometric or color-temperature measurements with no independent checking.

2. LUMINOSITY MEASUREMENTS.

In the measurement of relative luminosity by the step-by-step method, the final result may be uncertain for two experimental reasons: (*a*) There may be errors in the values of the ratios of luminosities which the various observers obtained; and (*b*) there may be errors in the determination of the size of step or wave length interval that slit S_1 is shifted. The first uncertainty, as has been noted, was the basis of the division of the observers into two groups, designated as good and poor. It will vary with the individual. The second uncertainty—that resulting from an error in determining the step—is more nearly independent of the observer.

Both kinds of errors or uncertainties affect each value of the ratio as used in the computation. It is of interest and importance to determine the order of magnitude of these errors in the luminosity curve.

Let L_0, L_1, L_2 · · · · · L_n be the values of the luminosity at the wave lengths λ_0, λ_1, λ_2 · · · λ_n, these wave lengths being separated from each other by the exact values of the step as predetermined. Then

$$L_n = L_0 \cdot L_1/L_0 \cdot L_2/L_1 \cdot \cdot \cdot \cdot \cdot \cdot L_n/L_{n-1} \qquad (1)$$

The values L_1/L_0, L_2/L_1 · · · · · · · · L_n/L_{n-1} will be the values of the ratios read from the ratio curve at λ_0, λ_1, · · · · λ_{n-1}, respectively. L_0 will be the arbitrary value of the luminosity at λ_0.

Let $R_1 = L_1/L_0$; $R_2 = L_2/L_1$ · · · · · · · $R_n = L_n/L_{n-1}$, and let e_1, e_2, · · · · e_n be the probable errors in R_1, R_2 · · · · R_n, respectively.

By the general formula for the propagation of errors (23), we have the following equation:

$$E_n^2 = \left(\frac{dL_n}{dR_1}\right)^2 e_1^2 + \left(\frac{dL_n}{dR_2}\right)^2 e_2^2 + \cdot \cdot \cdot \cdot \cdot \cdot \cdot + \left(\frac{dL_n}{dR_n}\right)^2 e_n^2 \qquad (2)$$

where E_n represents the probable error in L_n resulting from the several probable errors e_1, e_2, · · · · e_n.

Now, from equation (1) et seq.,

$$\log L_n = \log L_0 + \log R_1 + \log R_2 + \cdots + \log R_n \tag{3}$$

and

$$\frac{dL_n}{L_n} = \frac{dR_1}{R_1} = \frac{dR_2}{R_2} = \cdots\cdots = \frac{dR_n}{R_n} \tag{4}$$

$$\therefore \frac{dL_n}{dR_1} = \frac{L_n}{R_1}, \frac{dL_n}{dR_2} = \frac{L_n}{R_2}, \cdots\cdots \frac{dL_n}{dR_n} = \frac{L_n}{R_n} \tag{5}$$

$$\therefore E_n^2 = L_n^2 \left[\frac{e_1^2}{R_1^2} + \frac{e_2^2}{R_2^2} + \cdots + \frac{e_n^2}{R_n^2} \right] \tag{6}$$

and

$$\frac{E_n}{L_n} = \pm \sqrt{\frac{e_1^2}{R_1^2} + \frac{e_2^2}{R_2^2} + \cdots\cdots + \frac{e_n^2}{R_n^2}} \tag{7}$$

The quantity 100 E_n/L_n represents the propagated probable error, in per cent, in the luminosity at λ_n which results from the various probable errors in all the ratios entering into the computations of L_n from L_0.

It will be convenient to compute each of the two classes of probable errors separately, as if the other were not present; that is, first, the errors resulting from differences in the values of R obtained by an observer at different trials, and secondly, those resulting from an error in the determination of the size of step. To simplify matters an occasional approximation has been used. Such will be obvious and are quite permissible in a discussion of this kind. The order of magnitude of the error is the important thing, not the exact numerical value.

(a) OBSERVATIONAL ERRORS.

If an observer made but a single set of observations, this error must of necessity be computed from his photometric settings. If he takes two or more sets of readings of the ratio at a given wave length, the differences in his average ratio values may be used for the computation, not using the photometric settings at all, and the computed probable error may be caused not only by photometric errors but also by possible differences in his visibility at different times; but this final uncertainty, for whatever cause, is the thing of primary interest.

The data for three good observers and three poor observers were chosen for this study. The probable error *in the final individual average ratio* was computed from the several ratio values for each observer at each wave length at which observations were made. In the case of the good observers two values of the ratio were available since, as a rule, they took but two sets of

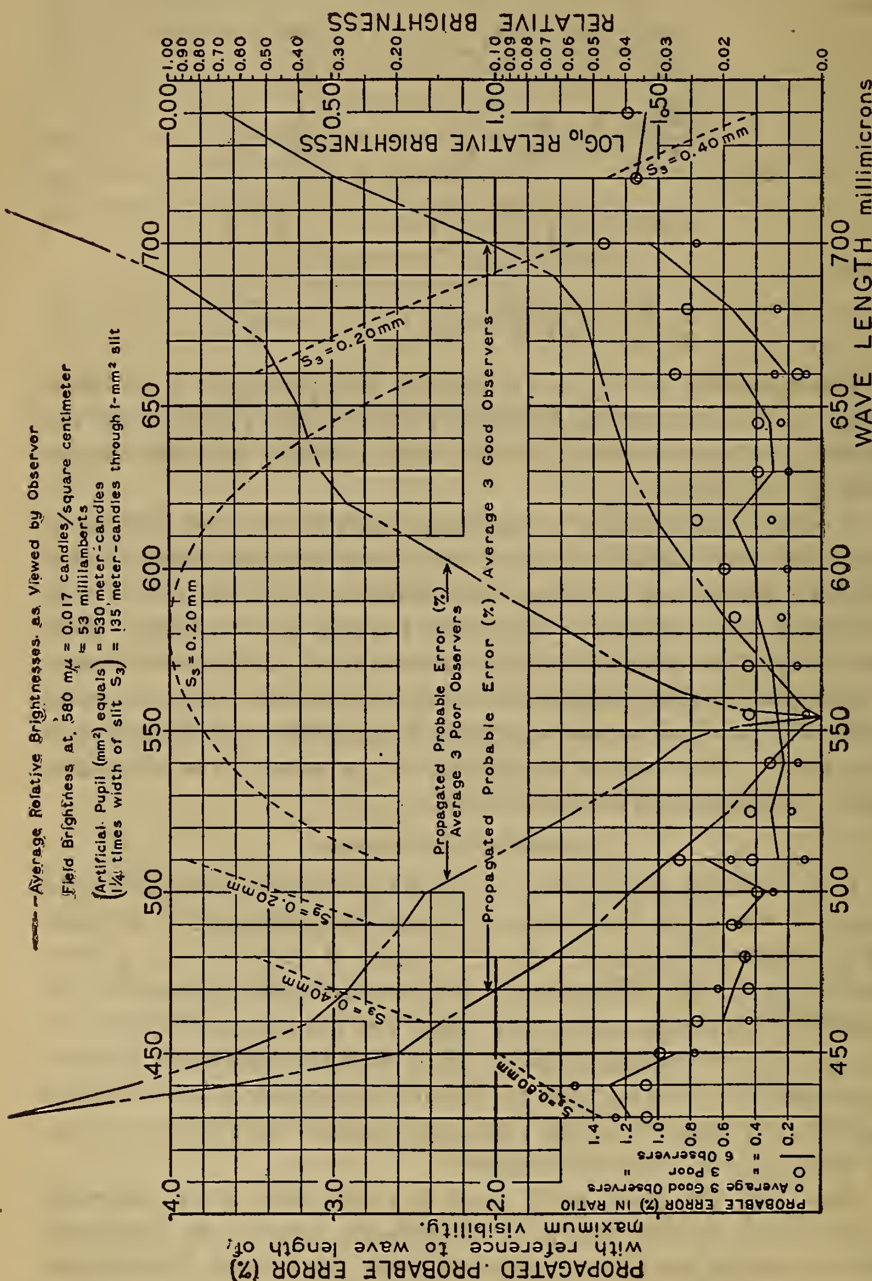

FIG. 22.—*Field brightness and observational error.*

measurements; with the poor observers three or more values of the ratio at each wave length were available for this computation.

After the probable errors had been computed at each wave length for the three good and the three poor observers, those for the three good observers were averaged together at each wave length, and likewise those for the three poor observers. The result would be, respectively, the approximate probable error at each wave length for any good observer and any poor observer, and a final average of both would be that for the average observer, so far as that can be determined with but six observers.

These average values of the probable error of the ratios (in per cent) as experimentally determined for the two classes of observers are plotted in Figure 22, large and small circles, with lines drawn through the averages. It is interesting to compare these probable errors with the brightness curves plotted in the same figure. Naturally the error increases at the extreme ends of the spectrum. Regions of maximum precision are noticeable in the green and red, but in the yellow and orange where the small hue difference was observable in the two parts of the photometric field, the probable error definitely increases.

The distinction between good and poor observers is easily apparent from 510 to 660 mμ and it was in this region only that the criterion was used. It happens that in the blue these three poor observers were as good as the three good observers.

Assuming these values of probable error constant on either side half way to the next value, the propagated probable error was computed from equation (7), relative to the wave length of maximum visibility. The number of terms thus used in the right side of equation (7) to get the propagated error at any wave length was, of course, the same as the number of steps between that wave length and the wave length of maximum visibility. The propagated probable errors thus computed for the good and the poor observers are plotted in Figure 22.

(*b*) ERRORS IN THE STEP.

Let L_3 and L_2 (fig. 23) be the luminosities at wave lengths λ_3 and λ_2, $L_3/L_2 = R_3$ being the ratio of luminosities for the step $\Delta\lambda = \lambda_2 - \lambda_3$.

Then

$$\Delta L = L_3 - L_2 = \frac{dL}{d\lambda} \Delta\lambda \tag{8}$$

and

$$\frac{L_3 - L_2}{L_2} = \frac{L_3}{L_2} - 1 = R_3 - 1 = \frac{1}{L_2} \frac{dL}{d\lambda} \Delta\lambda \tag{9}$$

Let $\delta R_3 = e_3$ be the probable error in R_3 arising from $\delta\lambda$, the probable error in $\Delta\lambda$.

Then

$$e_3 = \frac{1}{L_2}\frac{dL}{d\lambda}\delta\lambda \qquad (10)$$

and

$$\frac{e_3}{R_3} = \frac{1}{L_3}\frac{dL}{d\lambda}\delta\lambda \qquad (11)$$

If the factors in the right side of equation (11) are known for each step then e/R may be computed and E_n, the probable error in L_n, computed by equation (7).

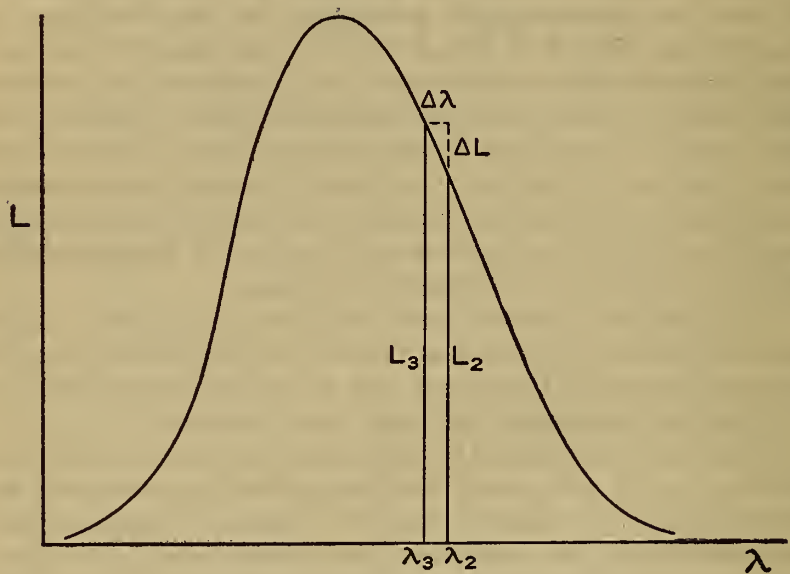

FIG. 23.—*To accompany Section V, 2 (b) of paper.*

Since the step used was variable in wave length, because of the dispersion, but constant in terms of the scale of S_1 (except for the change at 510 mμ), it has been more convenient to use the equation

$$\frac{e}{R} = \frac{1}{L}\frac{dL}{ds}\delta s \qquad (12)$$

which may be derived from equation (11), since $s = s_0 + \frac{c}{\lambda - \lambda_0}$, or from Figure 23 if L were plotted against s instead of against λ.

With equation (12) and the average data represented in Figure 7 the values of e/R were computed at each step throughout the spectrum. Instead of attempting to obtain actual values of dL/ds, the successive differences in L for each step were used.

These values of L were, of course, already available. They were multiplied by two where the step was but one-half turn. The values of δs were obtained from the original data by which the calibration of slit S_1 was made. At each of the five wave lengths (fig. 6) where measurements were made there were available three sets of readings corresponding to the three spectral lines, from which an average probable error was obtained. The probable error in the number of scale divisions of S_1 corresponding to the interval between any two spectral lines would equal the square root of the sum of the squares of the two probable errors arising in the settings corresponding to the two spectral lines. When these five average probable errors were obtained, they were, therefore, multiplied by the square root of two and the resulting values plotted as a curve. Values read from this curve were taken to represent the probable error δs in the step and were the ones used with equation (12).

The propagated probable errors thus computed, relative to the wave length of maximum visibility, are less than 0.2 per cent between wave lengths 510 and 650 mμ, less than 0.5 per cent at 430 mμ, and less than 0.7 per cent at 740 mμ. They are thus negligible in comparison with the observational errors.

3. GENERAL RELIABILITY OF THE STEP-BY-STEP METHOD.

It is, perhaps, already apparent that the step-by-step method of measuring visibility is capable of greater precision than would be expected by one who was considering the method for the first time. The usual expectation seems to be that the cumulative errors might easily become enormous and would, perhaps, be greater than would be obtained if the relative luminosity at two wave lengths were compared directly (rather than by the intermediate steps). The following facts are offered as evidence in this connection:

1. The step-by-step or cascade method has been used (13) to compare the candlepower of a lamp at 1,950° C. with that of a lamp at 1,610° C. with six intermediate steps. The errors were of the same order of magnitude whether the lamp at 1,950° was compared directly with the one at 1,610° C. or indirectly with the six intermediate steps. All the colors met with here, however, are unsaturated yellows and the quality-of-color differences very small compared to those necessarily present if spectral luminosity is measured by the equality-of-brightness method without the step-by-step feature.

2. Crittenden and Richtmyer (10) have shown that for an exact quality-of-color match the equality-of-brightness method of photometry is more precise than the flicker method, but that, even with the small differences met with in artificial light photometry, it rapidly loses precision and becomes poorer than the flicker method, the precision of which remains nearly constant. Ives (1) and Coblentz and Emerson (2) made rather extensive comparisons of the flicker and equality-of-brightness methods of measuring visibility and came to the conclusion that the flicker method was much more sensitive and reproducible than the other. In fact, the majority of observers are uncertain within wide limits as to what constitutes a photometric match where the two parts of the field differ greatly in the quality of the color. Ives (14) found, for his own eye under conditions of high illumination and small field, an exact agreement between the flicker and step-by-step luminosities, whereas the simple equality-of-brightness method gave results differing considerably from the others.

3. Data given by Coblentz and Emerson make possible a direct comparison between the flicker, the ordinary equality-of-brightness, and the step-by-step methods. At five wave lengths in the spectrum their observers made visibility measurements by the equality-of-brightness method (with large hue differences) in addition to the flicker method, and four observers made trials by both methods on different days. Probable errors of the means computed from these data taken on different days will be directly comparable with those given in Figure 22. The following are the probable errors as computed by the present authors:

1. At 493 mμ, 4 observers, 2 to 3 trials each, average probable error of the mean 8.3 per cent by equality-of-brightness, 3.0 per cent by flicker method.
2. At 523 mμ, 1 observer, 3 trials, 6.6 per cent by equality-of-brightness, 2.8 per cent by flicker method.
3. At 587.6 mμ, 4 observers, 2 to 6 trials each, 3.6 per cent by equality-of-brightness, 1.7 per cent by flicker method.
4. At 623 mμ, 3 observers, 3 to 6 trials each, 3.2 per cent by equality-of-brightness, 2.0 per cent by flicker method.
5. At 654 mμ, 2 observers, 3 trials each, 2.3 per cent by equality-of-brightness, 3.7 per cent by flicker method. (This low value of 2.3 per cent for equality-of-brightness was a result of one of the two observers getting the unusually low value of 0.2 per cent. The other got 4.4 per cent.)

A comparison of these values of the probable error of the mean of two or more trials at the various wave lengths, with the propagated errors of the mean of Figure 22, will show that the step-by-step and flicker methods of measuring visibility have the same order of magnitude of precision, and that either method is superior to the equality-of-brightness method that involves large hue differences.

4. Further evidence that the step-by-step method has a reliability of the same order of magnitude as the flicker method is indicated by the compiled data of Figure 19. Not only do the two step-by-step curves fall within the extreme flicker curves throughout the main part of the spectrum, but they also agree with each other as well as do any two flicker determinations.

5. In a search for further indications of the comparative reliability of the step-by-step method the data at 490 and 680 mμ for the 52 observers of the present investigation have been compared with the data at 492 and 678 mμ, respectively, for the first 52 observers of Coblentz and Emerson's investigation. This was done by computing the "probable error" at these wave lengths. The following values were obtained:

1. In the present investigation values of 15.4 and 18.6 per cent were obtained as the "probable errors" for the individual observer at 490 and 680 mμ, respectively, with values of 2.2 and 2.6 per cent as the "probable errors" of the means.

2. In Coblentz and Emerson's investigation values of 18.4 and 14.3 per cent were obtained as the "probable errors" of the individual observer at 492 and 678 mμ, respectively, with values of 2.8 and 2.0 per cent as the "probable errors" of the means.

These figures might at first be taken to indicate about equal precision for the two methods at these wave lengths. It is undoubtedly true, however, that the experimental errors are masked by the real differences between observers. From Figure 22 the average probable errors for the individual are about 2 per cent at 490 and 3 per cent at 680 mμ, much less than those indicated under (1) above. The known reproducibility of results by the flicker method is also much better than indicated by (2) above. The important result of these last computations is then to show that by either method the experimental errors are much less than the differences among the observers.

The authors, therefore, believe it correct to say that the step-by-step method of measuring visibility is comparable in precision to the flicker method and that both methods are considerably better than the equality-of-brightness method involving large hue differences.

VI. SUMMARY.

In cooperation with the Nela Research Laboratories visibility data for 52 observers have been obtained by the so-called step-by-step method, an equality-of-brightness method with little or no hue difference in the two parts of the photometric field. Several

of these observers were common to previous investigations by the step-by-step or flicker methods. The final average values obtained fall within the values obtained in other investigations, except in parts of the outer regions of the spectrum.

Comparisons between individuals common to other investigations show, on the average, the same sort of agreement or disagreement as do the final average curves. There is evidence that the visibility of some individuals varies considerably from time to time.

There is no certain difference between the average values of visibility obtained by the flicker and equality-of-brightness methods, provided the former is used under the experimental conditions recommended by Ives, and the latter does not depart too widely from these conditions. This does not conflict with the conclusion of Hyde, Forsythe, and Cady, and others that there is a difference between the visibility curves obtained by the two methods when the usual large Lummer-Brodhun field is used for the equality-of-brightness measurements.

A revision of the I. E. S. mean curve is proposed which results in better agreement with the average experimental visibility data and still gives the same wave length center of gravity for light of a color temperature of 2,077° K. as is given by Ives's physical photometer solution.

The step-by-step method of measuring visibility is a reliable method with a precision comparable with that of the flicker method and much superior to the simple equality-of-brightness method involving large hue differences.

Further statistical investigations would not seem to be as important as investigations into the effect of experimental conditions, such as field size and brightness, upon the visibility obtained by the two common methods of photometry. The investigation along these lines conducted by Ives should be supplemented and his conclusions verified, if correct. The step-by-step equality-of-brightness method should, however, be used, rather than the one involving large hue differences, and the investigation extended to include many more observers.

Such an investigation will probably be undertaken at the bureau in the near future, plans for which are already under way. This will include permanent apparatus for the measurement of visibility. Such apparatus will not only make possible a thorough comparison of the flicker and step-by-step methods of measuring visibility (as well as the equality-of-brightness method involving large hue differences), but will enable the visibility of any observer to be studied at various times and the data correlated with other data in physiologic optics. It will further make it possible for anyone who is particularly interested in his own visibility to obtain it at the bureau at a minimum of time and effort.

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