SOME EFFECTS OF THE DISTRIBUTED CAPACITY BETWEEN INDUCTANCE COILS AND THE GROUND

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ABSTRACT

It is well known that the effective capacity of two condensers which are connected in series is \( \frac{C_1 C_2}{C_1 + C_2} \) if \( C_1 \) and \( C_2 \) are the capacities of the two condensers. In the paper here presented it is shown that this rule does not always hold, and it is found that another more general relation is true. This relation depends upon the method of measuring the effective capacity. If the method used is that of resonance adjustment and if a coil symmetrical as to its two terminals is used in the resonance experiment, then it is shown mathematically and verified experimentally that \( \frac{C_1 C_2}{C_1 + C_2} \) is a linear function of \( \frac{1}{C_1 + C_2} \).

When an inductance coil is used in electrical circuits, its distributed capacity affects its behavior in various ways. Most of these are due to the capacity between various parts of the coil as well as to the capacity of the coil to ground. The two are usually indistinguishable. Thus, the effective capacity of a coil depends both on the capacity between parts of the coil and on the capacity of parts of the coil to ground. The same is true for the resistance. There appears to be one effect, however, which depends on the effective capacity of the coil to ground, primarily. This effect is observed when two condensers connected in series are connected across the coil terminals and when the common terminal of the two condensers is grounded.

Let \( C_1 \), \( C_2 \) be the capacities of the two condensers. Vary either \( C_1 \) or \( C_2 \) until resonance is obtained. If the coil had no capacity to ground, resonance would be obtained whenever \( \frac{C_1 C_2}{C_1 + C_2} \) had a proper value, depending on the frequency and on the inductance of the coil, but independent of the particular values of \( C_1 \) and \( C_2 \). If the coil has capacity to ground, this is no longer true. In this case another more complicated relation takes the place of the relation

\[ \frac{C_1 C_2}{C_1 + C_2} = \text{const.} \]

The nature of this relation may be found by representing the distributed capacity to ground by a series of capacities, as in Fig. 1. The whole coil is here divided into \( n \) sections, and the capacity of each to ground is represented by a lumped capacity.
at its end, as e. g., by C₂. The line GH represents the ground. The inductances of the various sections are written, respectively, as \(L_1, L_2, \ldots L_n\). The resistances of the same sections are denoted by \(R_1, R_2, \ldots R_n\). The emfs induced in the sections are written as \(E_1e^{j\omega t}, E_2e^{j\omega t}, \ldots E_ne^{j\omega t}\), where \(E_1, E_2 \ldots E_n\) are supposed to be independent of the time, where \(e\) is the base of natural logarithms and where \(j = \sqrt{-1}\). It is understood in writing the emfs in the complex form just used that the real part of the complex quantity written is considered only, and that in all the equations that follow the real part only of both sides will be considered. The mutual inductance between the sections having inductances \(L_1, L_2\) will be written \(M_{12}\) and a similar notation will be used for all other sections.

The currents in \(L_1, L_2, \ldots L_n\) are written
\[I_1e^{j\omega t}, I_2e^{j\omega t}, \ldots I_ne^{j\omega t}\]
The currents in \(C_1, C_2, \ldots C_{n+1}\) are written as
\[I_1'e^{j\omega t}, I_2'e^{j\omega t}, \ldots I_{n+1}'e^{j\omega t}\]
Kirchhoff's laws, when applied to the network of Fig. 1, give

\[
\begin{align*}
I_1' &= I_1 \\
I_2' + I_1 &= I_2 \\
I_3' + I_2 &= I_3 \\
&\vdots \\
I_n' + I_{n-1} &= I_n \\
I_n &= -I_{n+1}
\end{align*}
\] (1)

**Fig. 1.—Equivalent circuit for coil and its distributed capacity to ground**
and also

\[
\begin{align*}
(L_1 j\omega + R_1) I_1 + M_{12} j\omega I_2 + M_{13} j\omega I_3 + \cdots + M_{1n} j\omega I_n & \\
- \frac{I'_1}{jC_1 \omega} + \frac{I'_1}{jC_1 \omega} = E_1 \\
M_{12} j\omega I_2 + (L_2 j\omega + R_2) I_2 + M_{23} j\omega I_3 + \cdots + M_{2n} j\omega I_n & \\
- \frac{I'_2}{jC_2 \omega} + \frac{I'_2}{jC_2 \omega} = E_2 \\
M_{n} j\omega I_1 + \cdots + \cdots + (L_n j\omega + R_n) I_n & \\
- \frac{I'_{n+1}}{jC_{n+1} \omega} + \frac{I'_n}{jC_n \omega E_n} = E_n
\end{align*}
\]

or using (1)

\[
\begin{align*}
(L_1 j\omega + R_1 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}) I_1 + (M_{12} j\omega - \frac{1}{j\omega C_2}) I_2 + M_{13} j\omega I_3 & \\
+ \cdots + M_{n} j\omega I_n = E_1 \\
(M_{12} j\omega - \frac{1}{j\omega C_2}) I_1 + (L_2 j\omega + R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_3}) I_2 & \\
+ (M_{23} j\omega - \frac{1}{j\omega C_3}) I_3 + \cdots + M_{n} j\omega I_n = E_2 \\
\cdots & \\
M_{n} j\omega I_1 + \cdots + (M_{n,n-1} j\omega - \frac{1}{j\omega C_n}) I_{n-1} & \\
+ (L_n j\omega + R_n + \frac{1}{j\omega C_n} + \frac{1}{j\omega C_{n+1}}) I_n = E_n
\end{align*}
\]

Resonance occurs when the determinant of this system of equations vanishes, provided in that determinant all the resistances are set equal to zero. This is only approximately true, but is a good approximation if the resistances are small, because for small values of the resistances the position of resonance is independent of the resistances. Thus, the condition for resonance is

\[
\begin{vmatrix}
L_1 - \left(\frac{1}{C_1} + \frac{1}{C_2}\right) \frac{1}{\omega^2} & M_{12} + \frac{1}{C_2 \omega^2} & M_{13} & \cdots & M_{1n} \\
M_{12} + \frac{1}{C_2 \omega^2} & L_2 - \left(\frac{1}{C_2} + \frac{1}{C_3}\right) \frac{1}{\omega^2} & M_{23} + \frac{1}{C_3 \omega^2} & \cdots & M_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
M_{1n} & \cdots & \cdots & \left(\frac{1}{C_n} + \frac{1}{C_{n+1}}\right) \frac{1}{\omega^2}
\end{vmatrix} = 0
\]
As an equation in \( C_1 \) and \( C_{n+1} \) it has the form

\[
\frac{A}{C_1} + \frac{B}{C_{n+1}} + \frac{D}{C_1 C_{n+1}} + F = 0
\]  

(3)

In particular, if the surroundings of the coil are symmetrical with respect to the two ends

\[ A = B, \]

and hence,

\[
A + \frac{D}{C_1 + C_{n+1}} + F \frac{C_1 C_{n+1}}{C_1 + C_{n+1}} = 0
\]  

(4)

Therefore, if for different values of \( C_1 \) proper values of \( C_{n+1} \) be found so as to secure resonance, and if \( \frac{C_1 C_{n+1}}{C_1 + C_{n+1}} \) be calculated, a constant number will not be obtained. However, if its value be plotted against the value of

\[
\frac{1}{C_1 + C_{n+1}}
\]

a straight line must result according to (4).
Fig. 3.—Observed results for variation of $\frac{C_1C_2}{C_1+C_2}$ with $\frac{1}{C_1+C_2}$

Fig. 4.—Observed results for variation of $\frac{C_1C_2}{C_1+C_2}$ with $\frac{1}{C_1+C_2}$
Curves given as $I_A$, $I_B$, $I_C$, $I_D$, $I_E$ (Figs. 3, 4, 5, and 6) give experimentally obtained graphs of $\frac{1}{C_1 + C_{n+1}}$ against $\frac{C_1}{C_1 + C_{n+1}}$. The graphs are seen to be nearly straight except at the ends,

![Graph](image)

where they have a bend. This bend is probably due to a dissymmetry in the surroundings.

By similar reasoning it is found that a relation of the form (3) is satisfied also for the more general circuit shown in Fig. 2, provided $C_o$ and $L$ are kept constant.

![Graph](image)

This seems to indicate that the curvature of the graphs observed is due to dissymmetry in the surroundings, because distributed capacity effects between different parts of the coil are probably taken account of by the above-mentioned calculation.
SUMMARY

When two condensers connected in series with common terminal grounded are connected across the terminals of an inductance coil, their effective capacity in series is not equal to their effective capacity so far as resonance of the coil is concerned. If the capacity of one condenser is $C_1$ and of the other $C_2$, the quantity

$$\frac{C_1 C_2}{C_1 + C_2}$$

is the effective capacity of the two condensers in series. As stated, it does not stay constant when $C_1$ is changed arbitrarily and $C_2$ is readjusted for resonance. However, $\frac{C_1 C_2}{C_1 + C_2}$ is linearly related to $\frac{1}{C_1 + C_2}$.

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