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THE PREDICTION OF SOLAR ACTIVITY AS A BASIS FOR THE PREDICTION OF RADIO PROPAGATION PHENOMENA

I. Introduction

The close correlation between variations of ionospheric phenomena and the variation of solar activity, which became apparent within a few years after the first measurements of ionospheric critical frequencies were made,[1] enables the prediction of ionospheric phenomena, upon which high-frequency radio propagation depends, if first a prediction of solar activity is made.

Solar activity, as manifest in the variations of solar diameter, corona, prominences, faculae, flocculi, sunspots, light emission, and a large number of dependent terrestrial phenomena, exhibits a pulsating behavior, comparatively feeble, but in many respects similar to the pulsations of other variable stars. Its prediction is based upon study of the frequency, intensity, and duration of these pulsations, as observed over a long period of time.

Of all the solar phenomena which exhibit such pulsating variations, sunspots are those for which fairly accurate measurements have been made over the longest period of time. The study of their behavior, therefore, forms, at present, the best basis for the prediction of solar activity.

As early as 1849, R. Wolf began the study of sunspots as an index of solar activity, and expressed their variation for this purpose in terms of "relative sunspot number" - the sum of the total number of sunspots observed plus ten times the number of spot groups, this sum being multiplied by a factor depending upon seeing conditions and observing apparatus - an arbitrary measure which has since proved, by its excellent correlation with a wide variety of phenomena, to have been a very fortunate choice. By correlation of his own extensive observations with those of other investigators, Wolf[2] extended the period over which monthly estimates of relative sunspot number might be made to the year 1749. This series of relative sunspot numbers, extended to the present time, is presented in the report IRPL-R23, "Solar-Cycle Data for Correlation with Radio Propagation Phenomena." By investigation of all previous records, times of all maxima and minima were established with an accuracy of about ±2 years as far back as 1610, and occasional times of maxima were determined for even earlier times, the most conspicuous, as selected by Wolf, being those of the years 372, 840, 1078, and 1372. From these Wolf determined the cycle duration of relative sunspot number to be slightly greater than eleven years.

Following the discovery by Hale[3] of the magnetic fields in sunspots, and later observation at Mt. Wilson Observatory[4] which showed that the direction of such magnetic fields reversed for alternate eleven-year sunspot cycles, it became apparent that the principal period of pulsation of solar activity should be approximately twice that of the sunspot-number cycle. Additional evidence for this is afforded by the variation in relative sunspot number; also, for recent years where the observations are likely to be of greater accuracy; these show alternate high and low maxima, with characteristically different variations of the sunspot number with time from the maximum, in each of the two types. (Cf. Figs. 11 through 21, IRPL-R23, "Solar-Cycle Data for Correlation with Radio Propagation Phenomena").

Beyond the outstanding evidence of an approximately twenty-two or twenty-three year period of pulsation in solar activity, the series of sunspot numbers has afforded a somewhat ambiguous basis for accurate knowledge of future sunspot numbers. Inference of all available solar data concerning the basic nature and causes of cyclic solar activity, of which the roughly determined empirical laws of sunspot behavior are a part, is, of course, the ultimate means of predicting solar activity. Unfortunately, however, although several plausible theories of sunspot formation have been proffered, lack of sufficient solar data prevents verification of any of them.

It seems possible that the cyclic behavior of solar activity, although feeble, may be akin to the cyclic behavior of many other stars, such as those of the Cepheid type, and those, particularly, of longer-period variability, since, as for the latter, the periodicity of solar variation is very long, and far from precisely established. It is of particular interest in this respect that the shape of the curve of sunspot number versus time bears a striking resemblance to that of the curve of light emission with time for stars of both the Cepheid type and for long-period variables (compare Figs. of IRPL-R23, "Solar-Cycle Data for Correlation with Radio Propagation Phenomena," and Figs. 258, 259, 262, 277, "Astronomy. II. Astrophysics and Stellar Astronomy", H. N. Russell, R. S. Dugan, J. Q. Stewart; Ginn and Co.), having a steep slope before maximum, followed by a gentler slope from maximum to the following minimum. The pronounced variations exhibited by many other variable stars are thought to be dependent upon naturally periodic expansions and contractions conditioned by gravitation and the elastic properties of the gaseous material composing the star. However, there also seems some reason to believe that the feeble periodicity of solar activity could possibly result from a solar state of comparative instability, where variations might easily follow the imposed periodicity of even a relatively weak force.

II. Theories of Solar-Activity Variation

Wolf,[5] noting the depression in sunspot number occurring in 1863, at the time of Jupiter's aphelion, and similar anomalies at the times of earlier aphelia, first proposed that the cause of the cyclic variation of solar activity lay in the action of the planets, particularly Jupiter, upon the non-rigid matter of the sun.

The consideration of planetary action as a cause of solar-activity variation has been the subject of numerous later investigations by W. De la Rue, B. Stewart and B. Loewy,[6] H. Fritz,[7] K. Birkeland,[8] E. W. Brown,[9] S. Newcomb,[10] A. Schuster,[11] E. Frankel,[12] D. Alter,[13] and W. A. Luby[14,15,16]. Since the period of Jupiter (11.86 years) is not far different from the length of the sunspot-number cycle, and Jupiter is the most massive of the planets, the action of Jupiter is considered chiefly responsible for the variation of solar activity by advocates of its planetary origin.

E. W. Brown[9], among others, has considered simply the gravitational attraction of planets in causing solar tides. He gives the tide-raising force of each planet, in terms of that of the earth, as follows:

Mercury	8.6	Jupiter	2.24
Venus	2.06	Saturn	0.11
Earth	1.00	Uranus	0.002
Mars	0.03	Neptune	0.0006

Neglecting the effect of the inner planets as probably contributing little to the long-period variation of solar activity, this variation is chiefly ascribed to the tidal variation resulting from the eccentricity of Jupiter's orbit, because of which the tidal force will vary by ±0.33, and to the action of Saturn on this variation. By combining two simple variations having Jupiter's orbital period (11.86 years) and half that of Saturn relative to Jupiter (9.93 years), Brown obtained a fairly good representation of the series of sunspot numbers. It is difficult, however, to reconcile this explanation of solar activity variation with the relative amplitudes of the planetary tidal forces, particularly the lack of any pronounced periodicity related to the motion of Mercury.

A more reasonable explanation is given by W. A. Luby,[14,15,16] who has considered the action resulting from the precessional couple of each planet upon the equatorial bulge of the sun. Showing that Poincaré's condition for the precession of a fluid body in the manner of a rigid body is not fulfilled in the case of the sun and its planets, and that their precessional couples will result in variations in the sun's equatorial bulge, he gives the relative values of the mean couples and their extreme values as:

	Mean Couple	Extreme Values of Couple
Mercury	0.3250	0.59, 0.17
Venus	1.138	
Earth	1.00	1.051, 0.951
Mars	0.024	
Jupiter	1.89	2.18, 1.71
Saturn	0.083	0.0978, 0.0668
Uranus	0.00186	
Neptune	0.0054	

Here the couple due to Jupiter is seen to be the greatest. The couple for each planet will vary through zero at the nodal points of the planet's orbit with respect to the solar equator. Comparison of sunspot numbers, grouped according to the various orbital periods, with the variations in precessional couple shows fairly good correlation in each case, with a decrease of sunspot number generally following nodal positions. The degree of correlation in each case, and the amount of lag, however, are rather variable, and, as for other planetary explanations of the variation of solar activity, no good explanation is given for a 22-23 year periodicity, which seems to be the outstanding observed fact.

Other possible evidence for the action of planets on solar activity (although later shown to be better evidence for Bjerknes' hydrodynamic theory) is afforded by the observations of Mrs. A.S.D. Maunder,[17] corroborated by R. J. Pocock[18], that more sunspots are apparently formed on the sun's eastern hemisphere as viewed from the earth, than on the sun's western hemisphere. That this result might be only apparent, depending upon the tilt of the sunspot cross-section was suggested by Mrs. Maunder in the initial presentation of these data, and has later been discussed by W. Gleissberg,[19,20] who has shown that an average tilt of only 0.6° would be sufficient to explain the observed effect. Following Mrs. Maunder's observations, A. Schuster[11] showed that the distribution of sunspots was related to the zenith position, relative to the sun, of the various planets, to a greater degree than that attributable to chance.

Consideration of solar hydrodynamic vortices as a cause of sunspots followed Emden's[21] investigations of the hydrodynamics of gaseous spheres.

Although largely qualitative in nature, the hydrodynamic theory of solar activity variation proposed by V. Bjerknes[22] is outstanding in that a simple, plausible explanation is thereby afforded for a great many solar phenomena. Bjerknes shows that the conditions of thermodynamic equilibrium in the solar atmosphere are favorable for the formation of stratified circulation, which may entail the existence of zonal vortex rings. Observed sunspot phenomena may be explained on the assumption of a pair of such zonal vortex rings in each hemisphere, north and south, each pair participating in the general circulation, and each vortex ring of each pair, in turn, rising to cut the surface of the photosphere in limited regions above or below the sun's equator. If waves exist in each vortex ring, the intersections of the vortex ring with the photosphere surface will result in pairs of sunspots, each sunspot corresponding to a cross-sectional area of intersection.

The circulating motion of the vortices explains Hale's observation[3] of a magnetic field in every sunspot. The occurrence of sunspots in pairs, each member of the pair having opposite magnetic polarity, ensues from the assumption of a continuous, wavy, zonal vortex, since it must always be cut doubly by the surface of the photosphere. The Evershead effect follows from the centrifugal pumping action of the vortex. The darkness of sunspots with respect to the rest of the solar disc, indicating lower temperatures in their upper surfaces, may be ascribed to the dip caused by the vortex in the photosphere surface, and its corresponding core of cooled gases from upper regions. The spiral structure, observed in the high layers of the solar atmosphere near sunspots, follows from the radial inflow of gases in compensation for the sink caused by the vortex motion; the spiral character of the inflow depends upon solar rotation only, and therefore, as observed, does not change with the magnetic polarity of the sunspots.

Since the general circulation in the stratum where the spots are formed may be assumed to consist of motion toward the poles in the lower part, upward motion near the poles, and motion toward the equator in the upper portion, the greatest temperature contrast occurs in the regions just above and below the equator, where masses of gas, cooled by radiation, descend and come into contact with the hot ascending masses of the next lower circulating stratum. This results in concentration of the circulation in regions just above and below the equator, with greater likelihood for the formation of the vortices causing sunspots in these regions.

If the assumed pairs of wavy zonal vortices in each solar hemisphere participate, as they must, in the general solar circulation, one will rise to the surface of the photosphere at a comparatively high latitude, and progress slowly, with an increased number of intersections, toward the equator, then, with a decreasing number of intersections, descend, as the second vortex rises in high latitudes. This process explains both the latitude change of the sunspot belts with the solar cycle, and the reversal of magnetic polarities with each eleven-year half-cycle.

Variation of observed solar rotation with latitude also ensues from the above assumptions of general circulation. Both polar and equatorial regions, participating least in the general circulation, will rotate more nearly in the manner of a rigid body than regions in intervening latitudes, where a lag will occur in the outer regions of the photosphere. Lack of evidence, so far, for a shorter rotation period in polar regions, as well as for the higher temperatures in polar regions predicted by this theory, is possibly because of difficulty of good observation in polar regions. The east current in the lower photosphere surface where the zonal vortices give rise to sunspots may account for the asymmetric properties of sunspot binaries. W. Gleissberg[20] has shown that the observed excess of sunspots in the sun's eastern hemisphere, as viewed from the earth,[17,18] previously mentioned as possible evidence for the planetary theories of sunspot origin, is thus accounted for by Bjerknes' theory.

Another explanation of many observed properties of sunspots, together with some quantitative check with solar observations, has been developed on the basis of a system of solar magneto-hydrodynamic waves, in a recent series of papers by H. Alfvén[23,24,25,27], and by C. Walén.[26] Because of the difficulty in explaining the strength of the observed magnetic fields associated with sunspots, on the basis of previous theories, this theory postulates the observed fields, and shows that both the relative darkness of sunspots and the Evershead effect follow from decreases in hydrostatic pressure, with consequent cooling of the solar gases, depending upon magnetostatic pressure.

Genesis of the magnetic fields in sunspots is assumed to occur near the sun's center, where an initial hydrodynamic ring-whirl, split into two parts, each starting off in opposite directions as a magneto-hydrodynamic wave along the magnetic lines of force forming the sun's general magnetic field, slowly travels, until in about forty years, it reaches the solar surface, and causes strong magnetic fields there. The wave front first intersects the solar surface at high latitudes, and gradually progresses toward the equator. In this manner the opposed polarities of sunspot binaries in the sun's northern and southern hemispheres, as well as the variation in latitude of the sunspot zones, are explained.

Assuming current ideas concerning the change of solar density with radius, as given by Waldmeier[28] and by G. Blanch, A. N. Lowan, R. E. Marshak, and H. A. Bethe,[29] Alfvén[27] obtains mathematical expression for the latitude variation of the sunspot zone with time, which is in good agreement with observation if the wave front causes sunspots upon a solar surface generated by a radius between 7.0×10^{10} cm and 6.5×10^{10} cm in length, on the assumption of a homogeneous solar magnetic field within a region distant 1.8×10^{10} cm to 2.4×10^{10} cm from the sun's center, the sun's magnetic dipole moment being between 1.5×10^{33} and 6.2×10^{33} gauss cm^3, these latter values being in good agreement with Zeeman-effect observations.

III. Empirical Laws of Solar-Activity Variation.

A. General Investigations Concerning Variation of Solar Activity.

So far, lack of sufficient experimental data prevents both choice among these theories and their quantitative adaptation for prediction purposes. The practical prediction of solar activity is therefore based upon empirical laws of behavior, for which the same lack of sufficient experimental data renders both determination of the laws and consequent prediction of future solar activity grossly inexact.

From Wolf's earliest researches, as noted before, an approximately eleven-year periodicity was noted in the series of relative sunspot numbers, as well as asymmetry about the maximum value in each period, the time of approach from minimum to maximum being considerably shorter than the time from maximum to the following minimum. This asymmetry has been the subject of considerable investigation by W. De la Rue, B. Stewart and B. Loewy[30]. For the past century, where measurements have been more exact, alternate eleven-year sunspot-number cycles have borne similarity to each other, and sunspot-number maxima have been alternately high and low, as shown in the following table (from R. C. Linder[31]):

Cycle	Yearly Peak Maximum	Month	Monthly Peak Maximum
1843-1856	1848 - 124.3	Oct.	1847 - 130.4
1856-1857	1860 - 95.7	July	1860 - 116.7
1867-1879	1870 - 139.1	May	1870 - 176.0
1879-1889	1883 - 63.7	April	1882 - 95.8
1889-1901	1893 - 84.9	Aug.	1893 - 129.2
1901-1913	1905 - 63.5	Feb.	1902 - 108.2
1913-1923	1917 - 103.9	Aug.	1917 - 154.5
1923-1934	1928 - 77.8	Dec.	1929 - 108.0
1934-1943	1937 - 118.8	July	1938 - 165.3

If this behavior is maintained, which seems a reasonable assumption on the basis of the approximately 22-23-year periodicity also indicated by the magnetic polarities of sunspots, the next sunspot maximum should be fairly low, as indicated by the extreme values of 95.7 and 63.5 for low maxima in the above tables, and the time variation of sunspot number that of a typical low-maximum cycle. Figs. 1, 2, and 3 present the averaged sunspot numbers of the last three or four low cycles, taken for coincident times of minimum, maximum, and first appearance of a spot belonging to the cycle.

Search for empirical laws governing solar activity have proceeded generally according to two different schools of thought, one holding that solar activity is a periodic phenomenon, the other considering each solar-activity cycle as an independent outburst.

B. Investigations Concerning Periodicity of Solar Activity.

From considerations of the solar cycle as a periodic phenomenon, prediction is based upon harmonic analysis of a number of previous cycles, and synthesis of the results of this analysis to extend the values to future times. If only the recent, more accurate, data are used in this analysis, there is risk of inaccuracy in prediction should any pronounced long-period cyclic variation exist. If the analysis is extended to include the data of remote times, their inclusion will add inaccuracy should there be no very long periodicity. Analyses of sunspot numbers on the basis of periodicity have been made by the following: R. Wolf,[2] H. Fritz,[7] A. Schuster,[32] S. Oppenheim,[33] A. Michelson,[34] S. Hirayama,[35] H. Kimura,[36] H. Turner,[37,38] Larmor and Yamaga,[39] G. Yule,[40] D. Alter,[13] A. Douglass,[41] H. Clayton,[42] and C. Anderson.[43]

Probably the best argument for the validity of this method of analysis lies in the good agreement found between the periodicity of the principal component determined from inclusion of the very oldest observations, 11.1 years, by Wolf and by Fritz, and that determined from analysis of only the relatively precisely known data since 1793 by Clayton, 11.17 years. Reasonably good agreement has also been found for other periods as determined by various investigators,[32,42] where periodicities have been found within the following limits by two or more different researches:

Years
4.8- 4.9
5.6- 5.6
7.3- 7.6
8.1- 8.4
8.5- 8.9
9.9-10.0
11.2-11.4
11.9-13.5
14.0-14.9
19.9-20.5

Of particular interest is the 33-year recurrence of high maxima noted by A. Schuster[32] which he traces back to Chinese records near the beginning of our era, the unusually high maximum in 1837 being ascribed to coincident phase of the partials of periods 4.38, 4.80, 8.38, and 11.125 years, that in 1870 to coincident phase of all of these as well as the partial having a 13.5-year period. It is of interest in this respect that the last maximum, in 1937, which occurred after Schuster made his analysis, was one of the highest on record.

Alter[13] obtained a fundamental period of 252 years for the series of sunspot numbers, and pointed out that this was very nearly equal to 21 orbital periods of rotation for Jupiter, $8\frac{1}{2}$ for Saturn, 3 for Uranus, and $1\frac{1}{2}$ for Neptune, in possible substantiation of a planetary theory for the origin of solar-activity variation. Clayton[42] obtained a fundamental sunspot periodicity of 89.36 years; beginning with the minimum occurring in the year 1798, which Wolfer weighted as 8 on a scale of 10, and advancing all values by 89.36 years, fairly good representation was obtained for later cycles.

Anderson's analysis[43] resulted in the determination of a fundamental period of 312 years. His analysis in one respect possessed an important advantage over preceding analyses in that it was based upon reversal of alternate sunspot-number cycles; analysis of sunspot data since 1749 resulted, upon synthesis and extension to earlier times, in remarkably accurate location of maxima and minima between 1610 and 1749. It is unfortunate for purposes of such analyses that accurate separation of the sunspots near times of minima into their proper half-cycle is only possible for recent years, since the ensuing ambiguity may well affect the accuracy of such analyses to a considerable degree.

Although, as noted above, fairly good agreement exists among a number of investigators for several periodicities, all sunspot-number cycle periodicities save the one of approximately eleven years are characterized by relatively low amplitudes, and poorly defined phase relations. According to the particularly thorough analysis of Yule,[40] the phase of even the pronounced eleven-year periodicity is rather vague; he considers other periodicities so poorly established as to be insignificant, and the best representation probably that of an approximately eleven-year sinusoidal function if multiplied by another function of the time, governing its amplitude, and representative of its secular change. In substantial agreement with these views are the reports of Michelson[34] and Larmor and Yamaga.[39] In substantiation of Yule's remarks concerning the indefiniteness of phase associated with even the approximately eleven-year periodicity. W. Gleissberg[44] has noted a progressive decrease, during the past three hundred years, in the ratio of the duration of ascending activity to that of descending activity during the approximately eleven-year sunspot-number cycle.

C. Statistical Investigations of Solar-Activity Variation.

Analyses of the series of sunspot numbers on the basis that each approximately eleven-year cycle represents a relatively independent outburst of solar activity have been made by J. Halm,[45] H. Ludendorff,[46] M. Waldmeier,[47,53] A. Durkee,[48] J. Stewart and H. Panofsky,[49] B. Thüring,[50] J. Stewart and F. Eggleston,[51,52] W. Gleissberg,[54,55] and W. Brunner.[56]

Waldmeier's extensive studies of the behavior of sunspot-number cycles have resulted in the following empirical laws.[53]

If R_M = maximum smoothed monthly sunspot number of a cycle,
T = time, in years, from minimum to maximum,
Θ = time, in years, from maximum to a sunspot number of 7.5 (an average minimum value),
R_5 = sunspot number five years before the maximum,
S_1 = sum of the smoothed monthly average sunspot numbers from minimum to maximum,
S_2 = sum of the smoothed monthly average sunspot numbers from maximum to minimum,
Q = T/Θ,

then

$$\log R_M = \underset{\pm 0.09}{2.69} - \underset{\pm 0.02}{0.17}T \qquad \text{for even cycles}$$

$$\log R_M = \underset{\pm 0.10}{2.48} - \underset{\pm 0.02}{0.10}T \qquad \text{for odd cycles}$$

$$\Theta = \underset{\pm 0.6}{3.0} + \underset{\pm 0.006}{0.030}\, R_M \qquad \text{for both odd and even cycles}$$

$$R_5 = \underset{\pm 6.7}{-11.4} + \underset{\pm 0.06}{0.29}\, R_M$$

$$S_1 = \underset{\pm 3.2}{0.4}\, R_M + \underset{\pm 340}{2538}$$

$$S_2 = \underset{\pm 600}{-572} + \underset{\pm 5.9}{40.6}\, R_M$$

$$Q = \frac{15.64 - 581 \log R_M}{3.0 + 0.030\, R_M} \qquad \text{for even cycles}$$

$$Q = \frac{24.8 - 10.00 \log R_M}{3.0 + 0.030\, R_M} \qquad \text{for odd cycles}$$

Values found for Q lie between the limits 0.37 and 1.72, with an average value of 0.7.

Brunner[56] has obtained the following laws:

If T_1 = time, in years, between time of beginning of cycle (appearance of first spot belonging to the cycle) and maximum,
T_2 = time, in years, between maximum and preceding minimum,

then

$$\log R_M = 2.44 - 0.082T_1 \qquad \text{Eq. (a)}$$

$$= 2.74 - 0.18T_2 \qquad \text{Eq. (b)}$$

Using these relationships, together with an estimated R_M based on the average ratio of high to immediately succeeding low maxima for the last four low cycles, A. Shapley[57] has predicted values of $T_1 = 6.6$ years and $T_2 = 4.7$ years for the next sunspot cycle, giving times of minimum and maximum at 1944.9 and 1949.6. His estimated value of 80 for R_M, although in reasonably good agreement with the low values indicated by Figs. 1, 2, and 3 as representative of recent low cycles, and with the predictions of Clayton and Anderson on the basis of harmonic analysis, is one of the lowest values currently predicted for the next sunspot maximum. If the abnormally high ratio of 1.87 between the maxima occurring in 1870.6 and 1883.9 were omitted in averaging the ratios between alternate high and low maxima, and the more probable value 1.35 adopted instead of the average used by Shapley, R_M would be predicted as 88. If the time of inception of the new cycle were taken as that of first appearance of the high latitude spot appearing April 2 and 3, 1942, at latitude 34°N,[58] instead of the later date, Dec. 20, 1942, used by Shapley, the time of the next maximum would be predicted according to the method used above, as 1948.3, and the time of the preceding minimum would be 1943.9. The latter is in fairly good agreement with the recent minimum near the beginning of 1944.[59]

An idea of the precision inherent in this method may be obtained by inspection of Figs. 4 and 5, which show graphically the relationships of Eqs. (a) and (b) above, for which the coefficients of correlation are, respectively, -0.86 and -0.93, and of the following table which gives the values of R_M, T_1 and T_2 predicted in the same manner for the past two cycles (the data for these cycles, however, being used in the prediction), in comparison with observed values:

R_M		T_1		T_2	
Observed	Computed	Observed	Computed	Observed	Computed
78	71	6.6	7.2	4.8	4.9
119	111	5.3	4.8	3.6	3.9

The empirical laws obtained by W. Gleissberg[54,55] lead to a predicted value of sunspot number for the next maximum far in excess of those obtained by other methods. Their determination is based upon the application of probability laws for a Gaussian error distribution to the deviations from average found for certain approximately constant quantities determined from sunspot data. If, in each sunspot-number cycle,

t_r = the time, in months, during which the smoothed relative numbers increase from $1/4\ R_M$ to R_M

t_f = the time, in months, during which the smoothed relative numbers decrease from R_M to $1/4R_M$

t_ℓ = the period of low activity, in months, between $1/4R_M$ on a waning cycle and $1/4R_M$ on the following increasing cycle

$R_M^{(4)}$, $t_\ell^{(4)}$, $t_r^{(4)}$, $t_f^{(4)}$ being the averages of every four successive values of R_M, t_ℓ, t_r, t_f, respectively, in the sunspot-number series,

$A = t_r^{(4)} + 0.2R_M^{(4)}$

$B = t_r^{(4)} - 0.4t_\ell^{(4)}$

$C = t_r^{(4)} + 0.8t_f^{(4)}$,

each of the quantities A, B, and C are found to be nearly constant for the entire series, being approximately 55.5, 16.5, and 77.5, respectively, their distribution being approximately Gaussian, with nearly identical standard deviations (σ = approximately 1.95) in each case. Using this value in the expression

$$h = \frac{1}{\sigma\sqrt{2}}$$

where h is the constant of Gauss's law of errors, h is determined as being 0.36.

Thus the following probability laws are determined:

1. The probability that A lies, in any set of four successive cycles, between $55.5 - \delta$ and $55.5 + \delta$ is equal to $\mathrm{erf}(0.36\,\delta)$.

2. The probability that B lies, in any set of four successive cycles, between $16.5 - \delta$ and $16.5 + \delta$ is equal to $\mathrm{erf}(0.36\,\delta)$.

3. The probability that C lies, in any set of four successive cycles, between $77.5 - \delta$ and $77.5 + \delta$ is equal to $\mathrm{erf}(0.36\,\delta)$ where erf denotes the error function.

In order to effect the prediction of sunspot number by means of these laws, changes in $t_r^{(4)}$ are also assumed to have a Gaussian distribution. On the basis of this assumption, Gleissberg similarly obtains the following "law":

4. The probability of $t_r^{(4)}$ changing from one set of cycles to the next by not more than δ is equal to $\mathrm{erf}\ (0.16\,\delta)$.

Although the assumption involved in this "law" at first may seem reasonable, inspection of the accompanying histogram, Fig. 6, shows that the actual distribution of the changes in $t_r^{(4)}$ is far from that of a Gaussian distribution. Whereas application of a χ^2 test to the deviations of A, B, and C about their mean values give values, respectively, of $\chi^2 = 2.26$, 6.06, and 15.8, N, the number of class intervals, = 14, 14, 13, with consequent values of p, the probability of obtaining, by chance, a fit to the theoretical distribution as poor as or worse than the one obtained being, respectively, (A) better than 0.99, (B) better than 0.05 and slightly worse than 0.99, and (C) better than 0.05, all of which exceed usually acceptable fiducial limits, values of χ^2 obtained for the deviations of $t_r^{(4)}$ are so great that neither they nor their correspondingly low values of p are even considered in any ordinarily used probability tables. Moreover, the changes in $t_r^{(4)}$ are such as to manifest a somewhat periodic time variation, and therefore should not be treated in this manner. Predictions based upon the above laws, therefore, since all involve the quantity $t_r^{(4)}$, are not likely to have great accuracy.

Gleissberg, on the basis of the relations given above, makes the following predictions for the next sunspot cycle:

1. The probability that the highest smoothed relative number of the coming cycle will exceed 145 is 86%.

2. The probability that in the coming cycle the reduced period of rising will be shorter than 32 months is 91%.

3. It may be expected with a probability of 93% that the period of low activity which will precede the next spot cycle will be shorter than 40 months.

4. The probability that the interval from the last sunspot maximum to the next will be shorter than 11.1 years, thus occurring before May 1948, is 19 to 1.

It may be noted that the value of R_M implied by (1), above, is abnormally high. If the "most probable" value of R_M, corresponding to a probability value of 50%, is computed, this value for the coming cycle is 216, a value for smoothed relative sunspot number which has never before been attained.

(The writer is unable to check Gleissberg's numerical values for "law" 4, and obtains an average value of $\Delta\ t_r^{(4)}$ of 3.08, with σ, the standard deviation = 2.55, and h = 0.278. Using these values, but otherwise following Gleissberg's procedure, the "most probable" value of R_M for the next cycle is given as 182 in October 1945, $\frac{R_M}{4}$ being attained in July 1943).

Had Gleissberg's method been used for the values of R_M for the past two sunspot cycles, values in excess of 122.3 and 92.5 would have been predicted, taking advantage of the data of these cycles, with a probability of 86%; the "most probable" values would have been, respectively, predicted as 191 and 161 for the last and next-to-last cycles. Observed values were, respectively, 119.2 and 78.1.

Stewart and Panofsky[49] have shown that the relative sunspot number, at any time, R, may be expressed as

$$R = F(r-s)^a e^{-b(r-s)}$$

where

s = the time of outburst of a new cycle
r = the time, in years, thereafter
F, a, and b are constants for any one cycle.

This relationship, which is that of a Pearsonian Type III distribution curve, gives:

$$\text{Time at which maximum occurs} = v = s + \frac{a}{b}$$

$$R_M = \frac{Fa^a e^{-a}}{b^a}$$

If a cycle has already passed maximum, therefore, the quantities a, b, and F may be determined and the remainder of the cycle predicted. If s is known, and the quantities R_M, v, and any other value of R for a given time, are estimated by other means, or, in case of the last quantity, possibly known, the shape of the cycle can also be estimated.

Figs. 7 through 11 present comparison of twelve-month running-average observed sunspot numbers for the last two cycles with the predictions of Clayton, and of Anderson, values obtained for these cycles by the Brunner-Shapley method, and "most probable" values obtained by Gleissberg's method. Similarly, predictions by these various methods are given for the next two cycles.

D. Heliographic-Latitude Variations of Solar Activity.

For conditions influencing radio transmission which may be considered as dependent upon particle emission, rather than light emission, from the sun, knowledge of the heliographic latitude of the spots is important.

Carrington[60] discovered the dependence of sunspot latitude on the phase of the cycle in 1855. Notable later investigations concerning their latitude variation were made by Spörer[61], W. Maunder[62], and Waldmeier.[63,64]

Waldmeier has given the following empirical laws relating the average smoothed heliographic latitude of the sunspots to the average smoothed relative sunspot number:

If, as before,

R_M = the average smoothed maximum sunspot number for the cycle

and

ϕ_{-50} = the average smoothed heliographic latitude of the sunspot zone 50 solar rotation periods before sunspot maximum

ϕ_M = the average smoothed heliographic latitude of the sunspot zone at sunspot maximum

ϕ_{+50} = the average smoothed heliographic latitude of the sunspot zone 50 solar rotation periods after sunspot maximum,

then

$$\phi_{-50} = (17.58 \pm 1.74) + (0.0339 \pm 0.0189) . R_M$$

$$\phi_M = (8.19 \pm 1.36) + (0.0699 \pm 0.0143) . R_M$$

$$\phi_{+50} = (5.44 \pm 0.85) + (0.0427 \pm 0.0089) . R_M$$

An interesting relation between the sunspot latitude and the speed of solar rotation has been noted in a recent paper by W. Gleissberg.[65] If Δt represents the time during which the average heliographic latitude of the spots, ϕ, changes by 1°, it may be shown that the values of $\Delta t \sin 2\phi$, as given by Waldmeier's empirical laws, approximate a constant value of 3. Since it has been previously shown by Newton[66] that the dependence of the sun's sidereal daily motion, ξ, on the heliographic latitude, ϕ, can be represented by

$$\xi = a - b \sin^2 \phi,$$

where a and b are positive constants, and the variation of ξ for one degree of latitude may be taken as approximately proportional to $\sin 2\phi$, one may conclude that the speed of shifting of the sunspot belts toward the equator is nearly proportional to the gradient of the sun's rotational velocity, provided that the two phenomena are associated with each other.

Moreover, if Δt may be considered proportional to the differential $\frac{dt}{d\phi}$, then, from the constancy of $\Delta t \sin 2\phi$, by integration,

$$\log \tan \phi = -\alpha t - \tfrac{1}{2} \log 3,$$

where t is the time from the moment when $\phi = 30°$. Using logarithms to the base 10, and substituting the value $\Delta t \sin 2\phi = 3$, the constant α is determined as 0.05, and the above equation may also be used for the prediction of sunspot latitudes.

IV. Conclusions

Survey of the numerous empirical laws for the prediction of relative sunspot number as a measure of solar activity, and of average heliographic latitude of the sunspots as a measure of the location of this activity, indicates that at present there are insufficient solar data to permit any but rough estimates concerning future solar activity for long-distant future times. Practical prediction, therefore, must at present consider of estimating the future course of solar activity as well as possible, preferably by several of the independent methods indicated in this report, (the methods of Anderson,[43] Waldmeier,[53] Brunner,[56] and Stewart and his colleagues[49,51,52] being especially recommended), with revisions of initial predictions based upon the accumulation of more recent data, made as frequently, and continued for as long as is practicable for the purpose for which they are desired.

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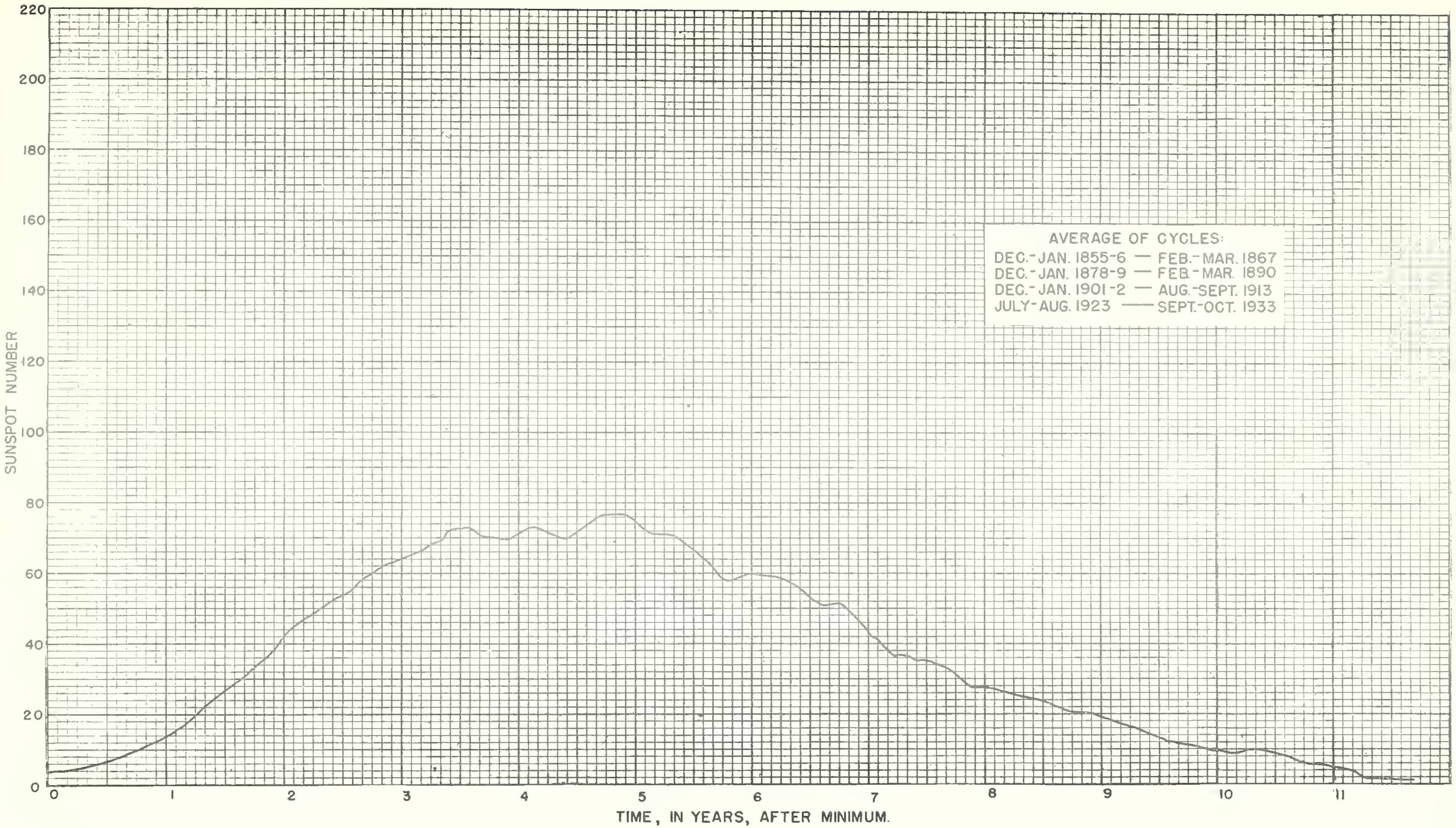

Fig. 1. AVERAGE SUNSPOT NUMBER FOR PAST FOUR LOW SOLAR-ACTIVITY CYCLES, TAKEN WITH COINCIDENT TIMES OF MINIMA.

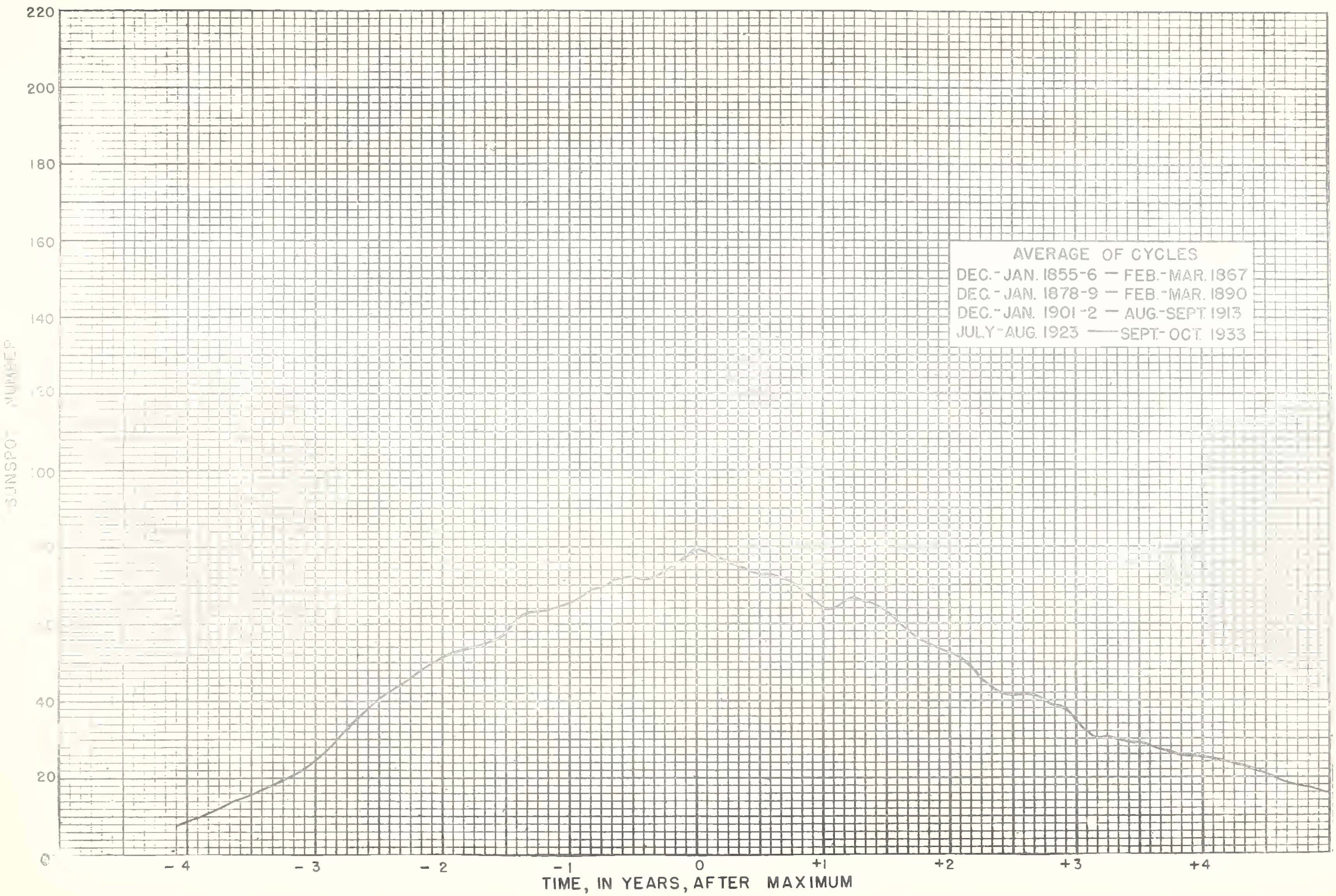

Fig.2. AVERAGE SUNSPOT NUMBER FOR PAST FOUR LOW SOLAR-ACTIVITY CYCLES, TAKEN WITH COINCIDENT TIMES OF MAXIMA.

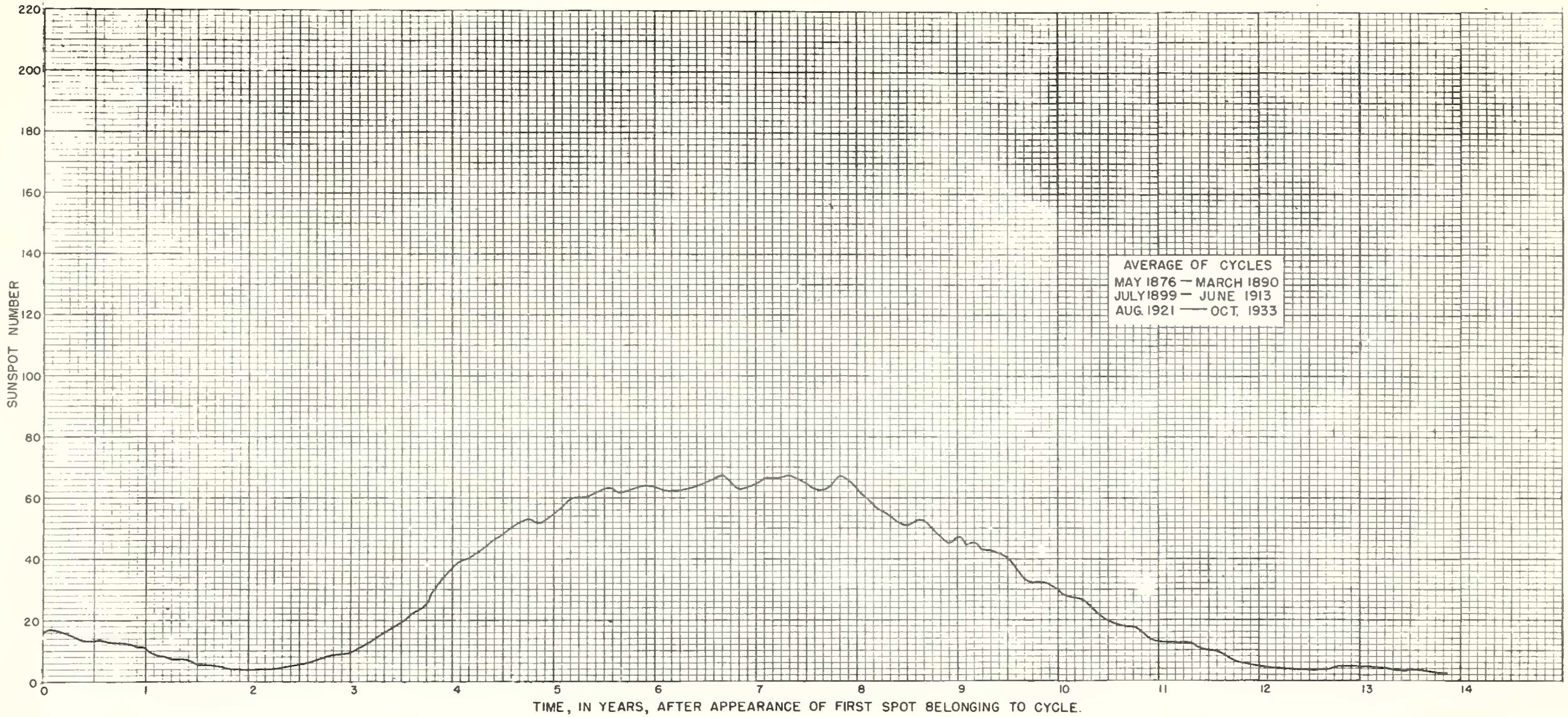

Fig. 3. AVERAGE SUNSPOT NUMBER FOR PAST THREE LOW SOLAR-ACTIVITY CYCLES, TAKEN WITH COINCIDENT TIMES OF APPEARANCE OF FIRST SUNSPOT BELONGING TO CYCLE

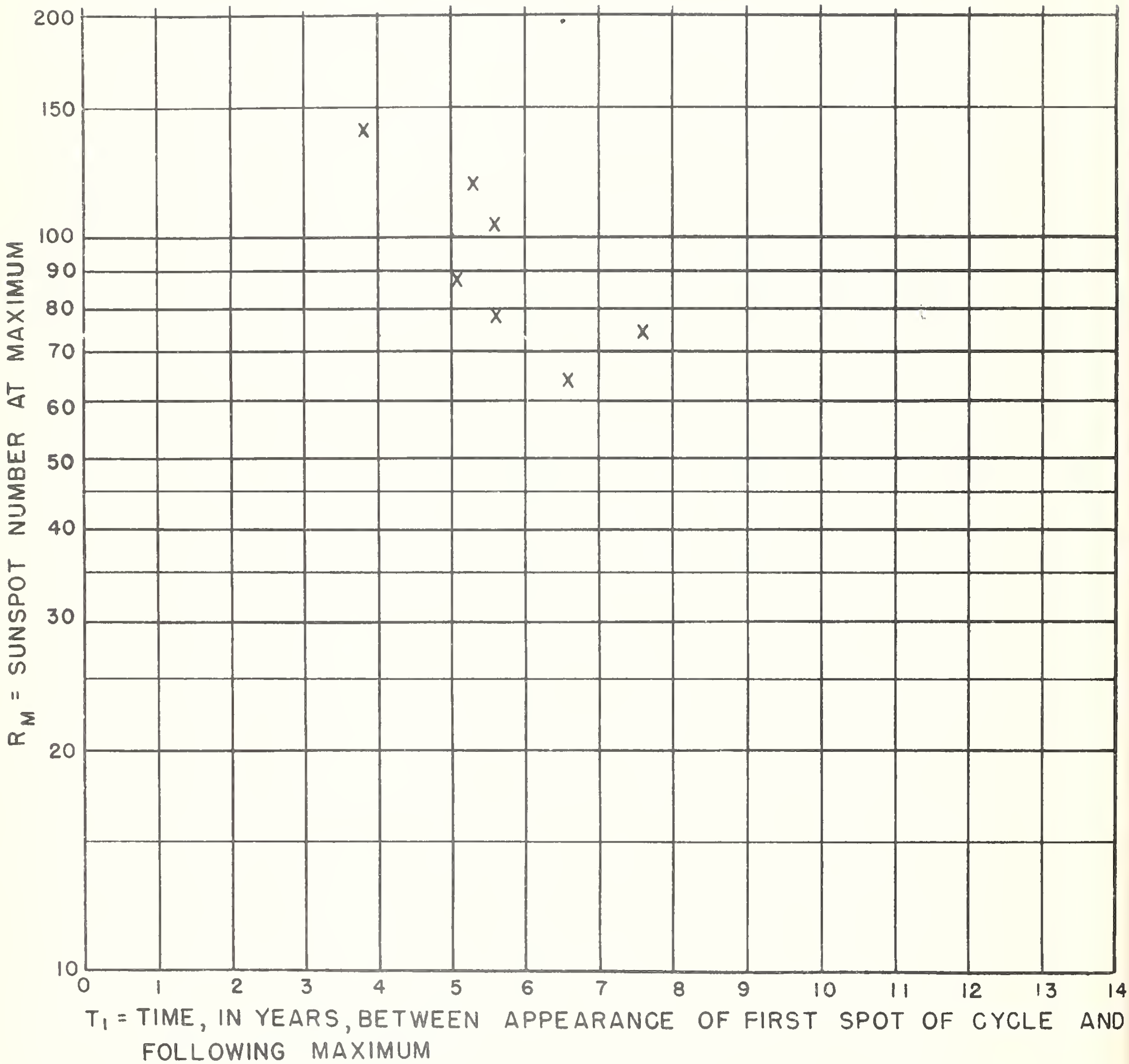

Fig. 4. VARIATION OF MAXIMUM SMOOTHED SUNSPOT NUMBER WITH TIME OF MAXIMUM AFTER APPEARANCE OF FIRST SPOT OF CYCLE.

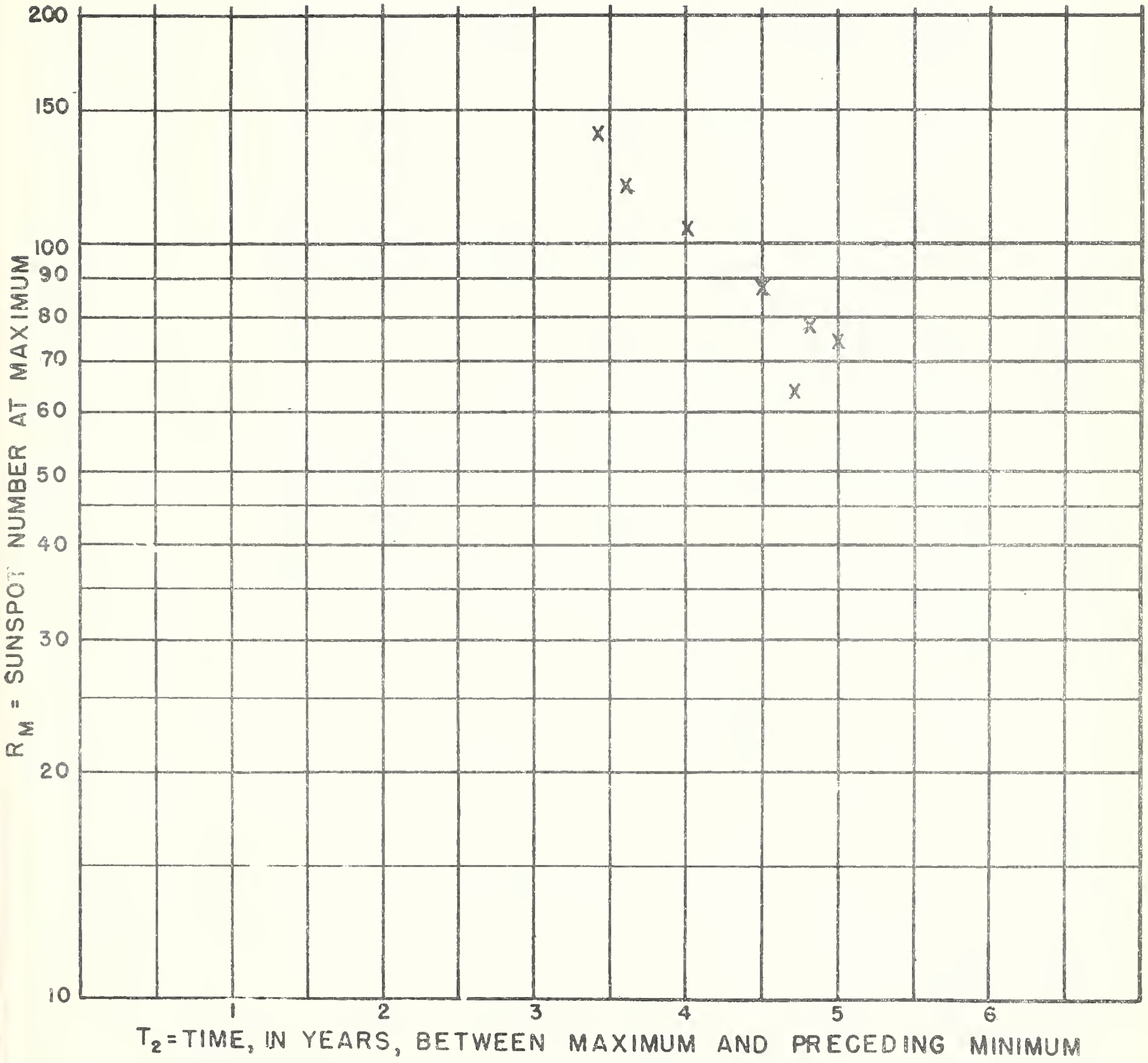

Fig.5. VARIATION OF MAXIMUM SMOOTHED SUNSPOT NUMBER WITH TIME OF MAXIMUM AFTER PRECEDING MINIMUM.

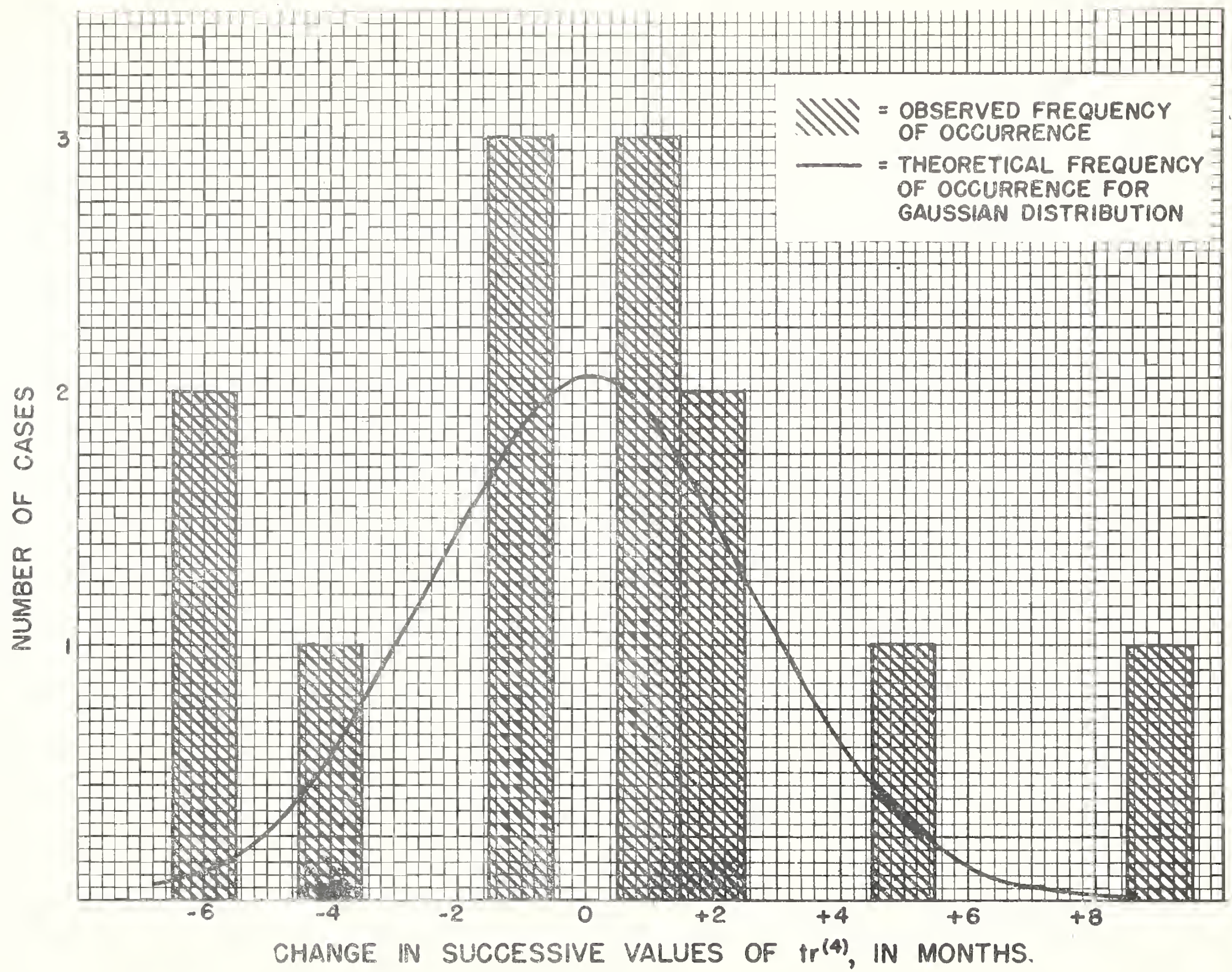

Fig. 6. COMPARISON OF OBSERVED FREQUENCY OF OCCURRENCE OF CHANGE IN SUCCESSIVE VALUES OF $\bar{t}r^{(4)}$, THE AVERAGE OVER FOUR CONSECUTIVE CYCLES OF THE TIME OF RISE IN SUNSPOT NUMBER FROM ONE-QUARTER MAXIMUM TO MAXIMUM VALUE, WITH THEORETICAL CURVE FOR NORMAL DISTRIBUTION.

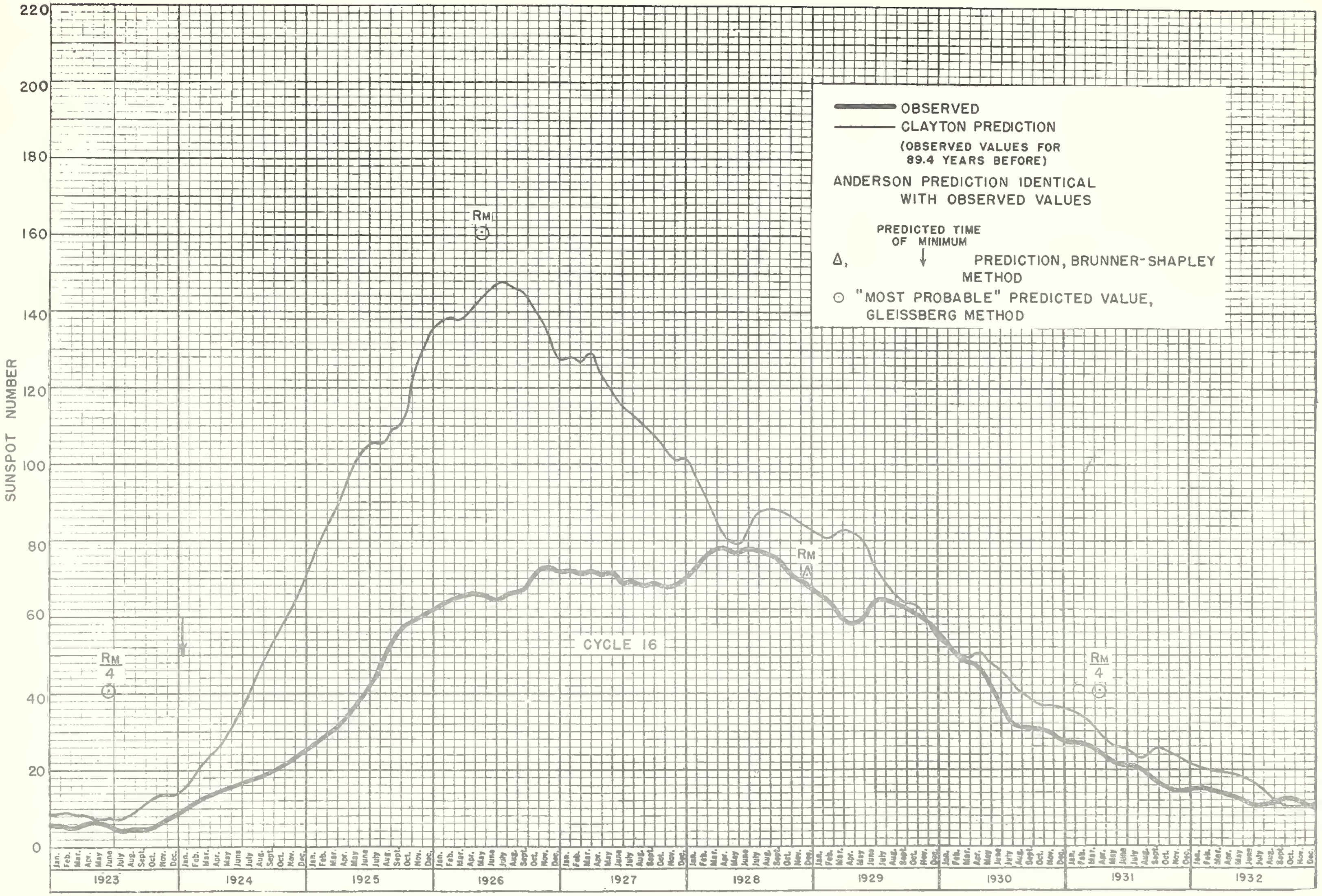

Fig. 7. COMPARISON OF TWELVE-MONTH RUNNING-AVERAGE OBSERVED SUNSPOT NUMBERS WITH VALUES PREDICTED BY VARIOUS METHODS.

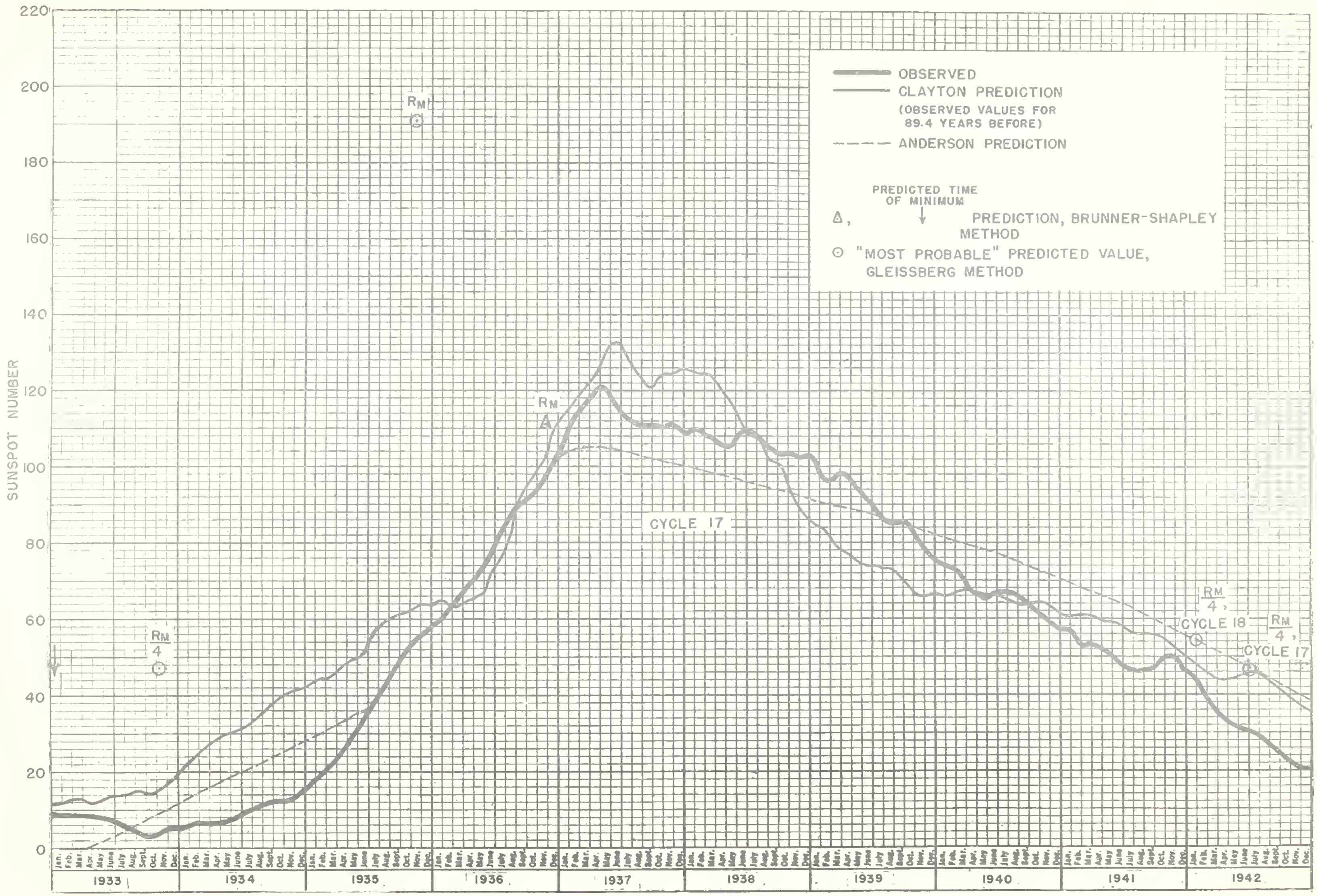

Fig. 8. COMPARISON OF TWELVE-MONTH RUNNING-AVERAGE OBSERVED SUNSPOT NUMBERS WITH VALUES PREDICTED BY VARIOUS METHODS.

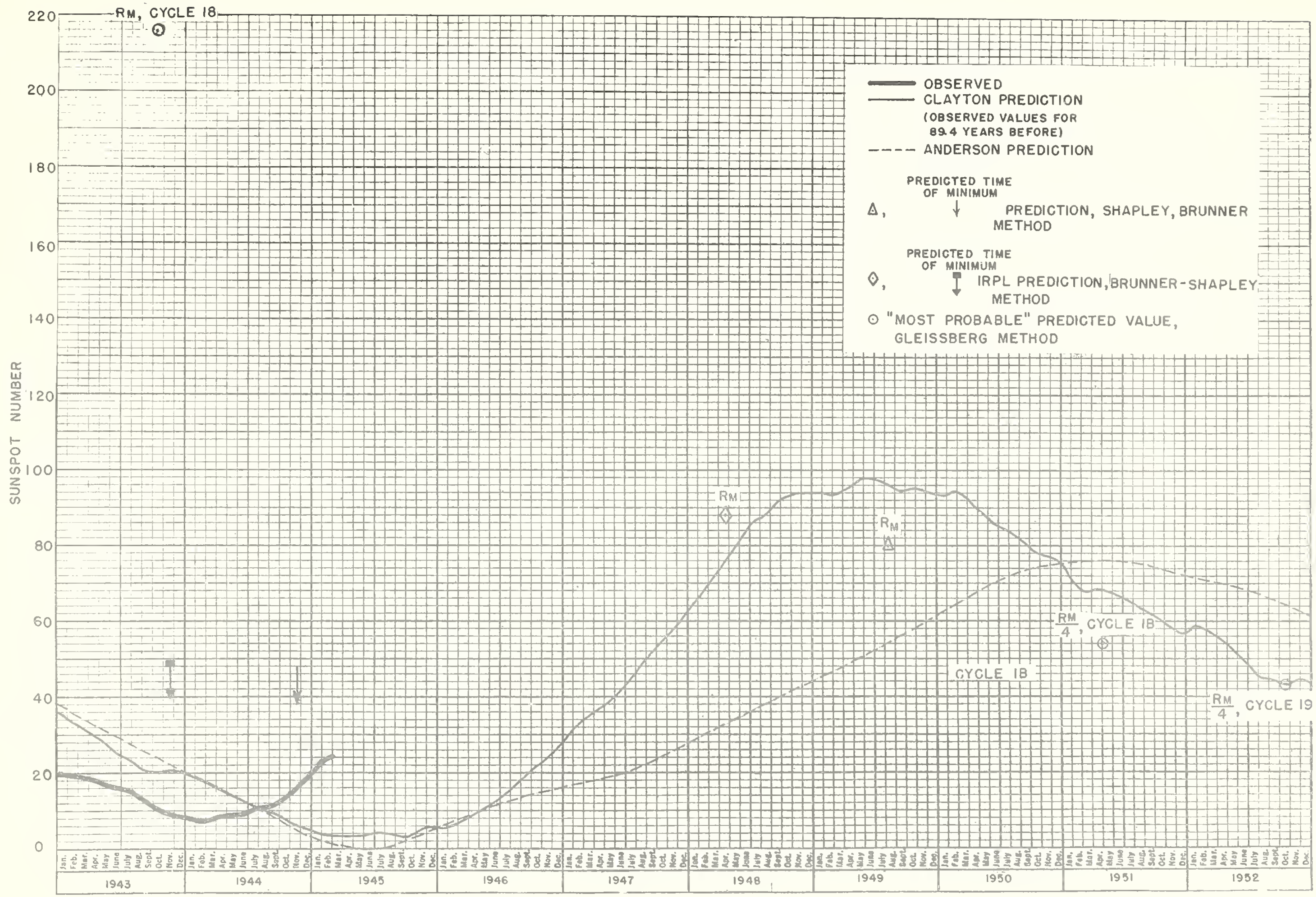

Fig.9. COMPARISON OF TWELVE-MONTH RUNNING-AVERAGE OBSERVED SUNSPOT NUMBERS WITH VALUES PREDICTED BY VARIOUS METHODS

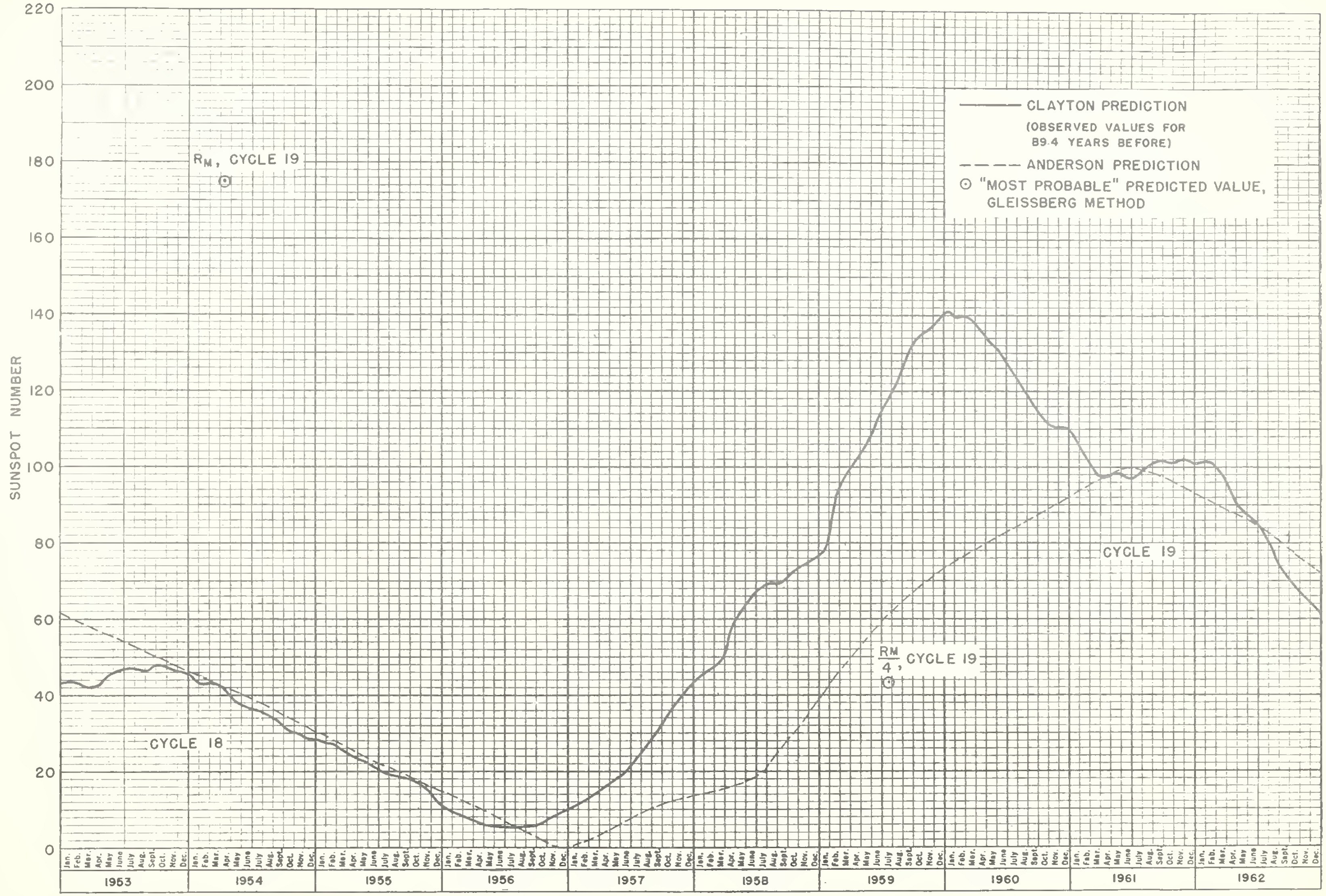

Fig.10. SUNSPOT NUMBER, PREDICTED BY VARIOUS METHODS.

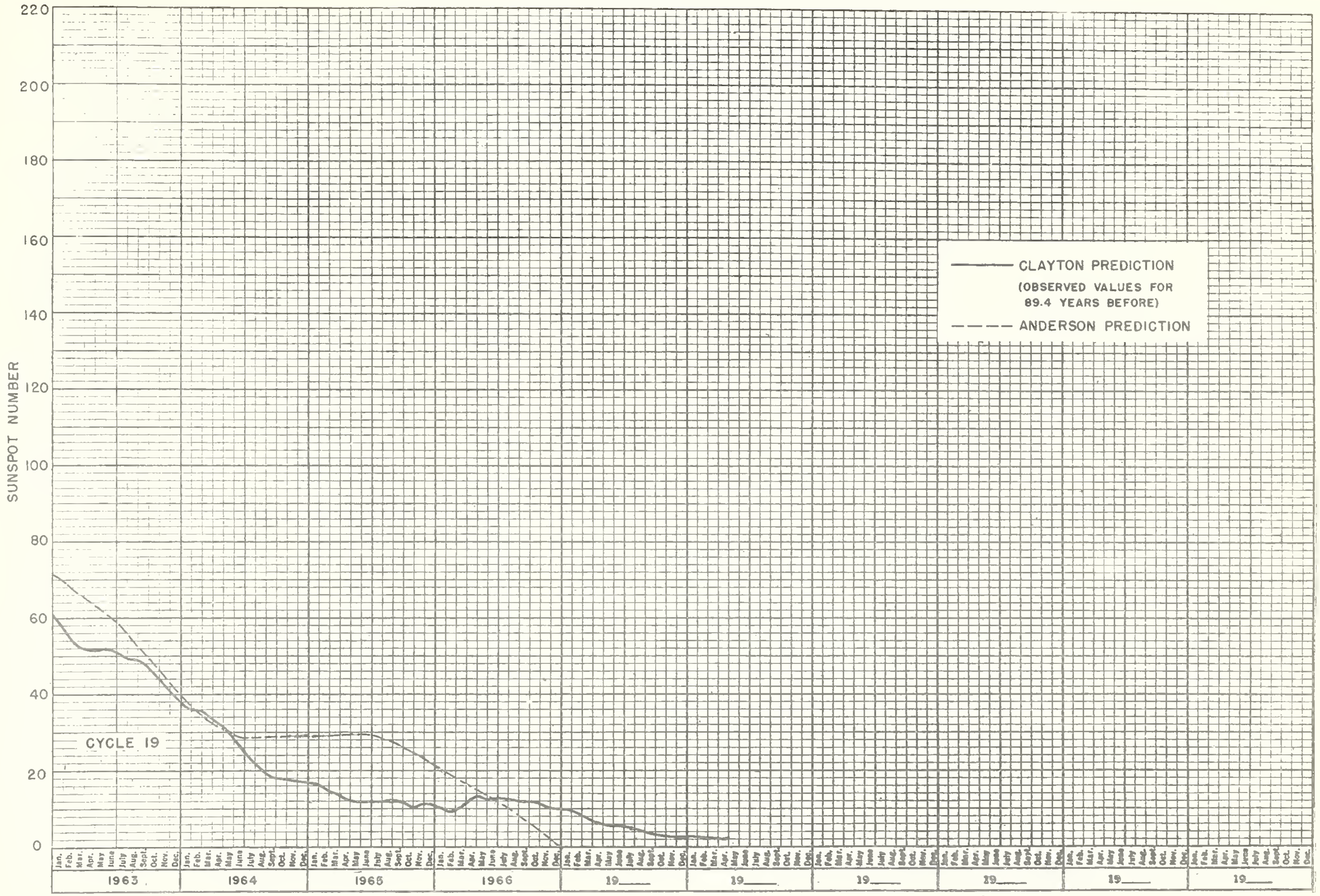

Fig. 11. SUNSPOT NUMBER, PREDICTED BY VARIOUS METHODS.