The Effect of Fading on the Desired-to-Undesired Signal Ratio Required to Provide a Specified Grade of Service For A Given Percentage of A Specified Period of Time

by

Kenneth A. Norton
National Bureau of Standards
Central Radio Propagation Laboratory
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It is the purpose of this memorandum to outline the factors involved in determining the allowance for fading which must be made in the allocation of two stations operating on the same or adjacent radio frequencies in order to provide a specified grade of service for a given percentage of a specified period of time. For example, consider two double-sideband High Frequency broadcast stations operating on adjacent radio frequencies f_a and f_b in different countries and assume that we wish to determine the separation $(f_a - f_b)$ required to permit station (a) to cover its service area (e.g., to the borders of the country in which station (a) is located) without serious interference from station (b). Let d_a be the distance from station (a) to the point on the border at which the interference from station (b) is expected to be a maximum and let d_f be the distance from station (b) to this same point on the border. Let E_a (50%) and E_b (50%) denote the expected median carrier intensities of stations (a) and (b) at the specified point on the border, expressed in microvolts across the receiver terminals. The median values are to be determined for the period of time during which the allocation is expected to apply; for example, if changes in allocation are contemplated for each month of each year, the median values correspond to the expected 50% values for the month in question. Thus, in general E_a and E_b will de-

pend upon the propagation paths of lengths, d_a and d_b, the powers, p_a and p_b, of the two transmitters, the antenna directivities, D_a and D_b, and the period of time, T, for which the allocation is to apply. If there were no fading of either the desired or the undesired carriers, then a satisfactory allocation can be determined by satisfying the following inequality:

$$E_a(d_a, p_a, D_a) / E_b(d_b, p_b, D_b) \geq R(f_a - f_b) \qquad (1)$$

(Case with no fading)

where $R(f_a - f_b)$ denotes the minimum ratio of the desired-to-undesired signal voltages at the receiver input required to ensure service of the specified grade. Thus $R(f_a - f_b)$ is determined by the grade of service desired. R can, of course, also be determined for two stations operating on nearly the same frequency; for example, two standard broadcast stations assigned to the same carrier frequency will usually maintain an instantaneous difference in their carriers of less than 20 cycles per second and in this case R(0) is usually taken to be equal to 20, (i.e., 26 db) for a barely satisfactory grade of broadcast service. Much larger values of R(0) are required when the tolerance becomes greater than, say, 100 cycles per second and $R(f_a - f_b)$ is also much greater than 20 when $f_a - f_b$ is less than, say, 10 kc/s; in these two cases the two carriers beat with each other resulting in a strong objectionable tone in the receiver output. However, for still larger carrier separations, $R(f_a - f_b)$ will decrease monotonically with an increase in the frequency separation, $f_a - f_b$.

In general, with High Frequency broadcast stations, both E_a and E_b will vary rapidly with time and an additional factor, F, must be introduced on the right of (1) in order to ensure that the specified grade of service will be available for a given percentage, P, of the specified period of time, T. Experience has shown that E_a and E_b vary, for High Frequency stations, in accordance with the Rayleigh distribution when the period of time, T, is of the order of one hour or less. Thus

$$p = 100e^{-0.693\left[E(p)/\,E(50\%)\right]^2} \tag{2}$$

where E(p) denotes the signal voltage exceeded for the percentage of time, p. It may also be shown that the instantaneous ratio E_a/E_b of two voltages E_a and E_b, each of which varies in accordance with the Rayleigh distribution, will be larger than the ratio $E_a(50\%)/E_b(50\%)$ of their median values by a factor, F_r, for a percentage of the time, P, given by:

$$P = \frac{100\,F_r^2}{1 + F_r^2} \qquad \text{(T less than about one hour)} \tag{3}$$

Thus, when the specified period of time is an hour or less and $f_a - f_b$ is in the range such that the interference is due to a beat between the two carriers, (1) becomes:

$$\frac{E_a\,(d_a,\,p_a,\,D_a,\,50\%)}{E_b\,(d_b,\,p_b,\,D_b,\,50\%)} \geq F_r\,R(f_a - f_b) \tag{4}$$

(Service of grade R will be available for a percentage, P, of time T when T is less than about one hour and $f_a - f_b$ is in the audio range)

Thus, in order to ensure that service of grade R be available for 99% of the time, F_r will be equal to 10 and the ratio of the hourly median field must be greater than 10 times R in order to provide a service of that grade for 99% of the time. As was pointed out in the Provisional Frequency Board United States Preparatory Team, Report of Committee A, the allowance for fading determined by (3) is appropriate for automatic telegraph transmission regardless of the value of $f_a - f_b$. In this same report it was also pointed out that no such allowance for fading is necessary for broadcast transmission when $f_a - f_b$ lies outside the audio band of the receiver. The audio pass band selectivity characteristic determines the nature of the transition between these large and small values of $f_a - f_b$.

When the period of time, T, is greater than one hour, in addition to the Rayleigh fading caused by the phase interference between the various components of the received field, the hourly median values of the received fields will vary from hour to hour and from day to day due to changes in absorption along the path or to other effects causing a change of the energy in the received signals. To the extent that these longer period changes are uncorrelated for the desired and undesired stations, they will require an additional allowance, F_m, to be introduced in order to ensure service of the specified grade for a given percentage of this longer period of time. This additional factor may best be determined from an analysis of the simultaneous distribution of the hourly median intensities of the signals propagated over two transmission paths. Such studies may be made of the signals received simultaneously at a single receiving station from two transmitting stations or, in view of the

reciprocity theorem, the transmitting and receiving stations may be interchanged and we may then study the simultaneous distribution of the signals received at two receiving stations from a single transmitting station. Figure 1 gives an example of the distribution of the hourly median fields of WWV as observed simultaneously at Trinidad and at San Francisco. The 31 points represent the hourly median values of the field received at these two stations for the 31 days in July 1947. If the fading had been completely correlated on the two transmission paths, all of the points would have fallen on some diagonal line. The scattering of the individual points indicates a lack of correlation; this is measured by the ratio of the two fields which may be seen to vary from -14 to + 10 db during this period of only one month.

Figure 2 shows the cummulative distribution of the individual fields $E_a(50\%)$ and $E_b(50\%)$ and of their ratio $E_a(50\%)/E_b(50\%)$. The median values of both $E_a(50\%)$ and $E_b(50\%)$ were the same in this example, being 20.5 db above one microvolt; thus the value of F_m necessary as an additional allowance for this type of fading may be read directly off the curve for the cummulative distribution of the ratio $E_a(50\%)/E_b(50\%)$. Assuming that reception of San Francisco is desired at Washington on 15,000 kc/s with simultaneous transmission from Trinidad we find by Figure 2 that $F_m(99\%) = 6.3$ (i.e., 16 db) and $F_m(90\%) = 2.6$ (i.e., 8.5 db) in order to provide a given grade of service for 99 and 90% of the time respectively. If the desired signal were from Trinidad while the undesired signal is from San Francisco, the value of $F_m(90\%) = 2.15$ (i.e., 6.7 db) and $F_m(99\%) = 3.16$ (i.e., 10 db). In connection with this example it should be noted that the ratio of the median values

was not equal to the median value of the ratio; the latter ratio was -0.5 db in this particular example as compared to 0 db for the ratio of the medians. Differences of this type are to be expected and will be much larger than the value determined in this example when the two transmission paths are of different lengths or traverse difference latitude zones.

Other values of F_m are given in the following table:

Table of Values of F_m

Receiving Station	Desired Trans.	Dist. km	Undesired Trans.	Dist. km	Freq. kc/s	Period	F_m(99%) decibels	F_m(90%) decibels
Washington	San Fran	3912	Trinidad	3515	15,000	July 1947	16	8.5
Washington	Trinidad	3515	San Fran	3912	15,000	July 1947	10	6.7
Washington	San Fran	3912	Trinidad	3515	15,000	Jan. 1947	15	7.4
Washington	Trinidad	3515	San Fran	3912	15,000	Jan. 1947	7.6	4.4
Washington	W8XAL	591	CFRX	578		July 1947	7.9	5.5
Washington	CFRX	578	W8XAL	591		July 1947	10.3	6.1
Washington	San Fran	3912	Trinidad	3515	15,000	Sept 1947	24	8.4
Washington	Trinidad	3515	San Fran	3912	15,000	Sept 1947	20	5.6
Washington	W8XAL	591	CFRX	578		Jan. 1947	12.4	11.4
Washington	CFRX	578	W8XAL	591		Jan. 1947	19.4	7.4

In accordance with the previous discussion F_m is an appropriate allowance for fading only when the Rayleigh type of fading may be neglected. When this is not negligible it is necessary to determine the distribution

of the instantaneous ratio of the two fields. This has been done by a graphical method for the case illustrated on Figures 1 and 2 and the result is given on Figure 2. It will be seen that a substantial increase must be made in the allowance for instantaneous fading as compared to that required for changes in absorption alone.

The above is not intended to be an exhaustive discussion of this subject but rather has as its object the exposition of the principles involved. There is some danger, too, of underestimating the importance of the fading allowance from the above discussion. For one thing, $F_m(99\%)$ will undoubtedly be much larger when the period of time is increased from one month to the much larger periods likely to be typical of actual allocation practice. Furthermore, paths with a greater total absorption than those studied will probably be characterized by larger values of $F_m(99\%)$.

SIMULTANEOUS DISTRIBUTION OF THE HOURLY MEDIAN WWV-15,000 kc/s SIGNALS RECEIVED IN JULY 1947 AT TRINIDAD, BRITISH WEST INDIES - 3515 km AND AT SAN FRANCISCO, CALIFORNIA - 3912 km

THE HOUR USED CORRESPONDED TO THE AVERAGE NOON FOR THE MIDPOINTS OF THE TWO PATHS

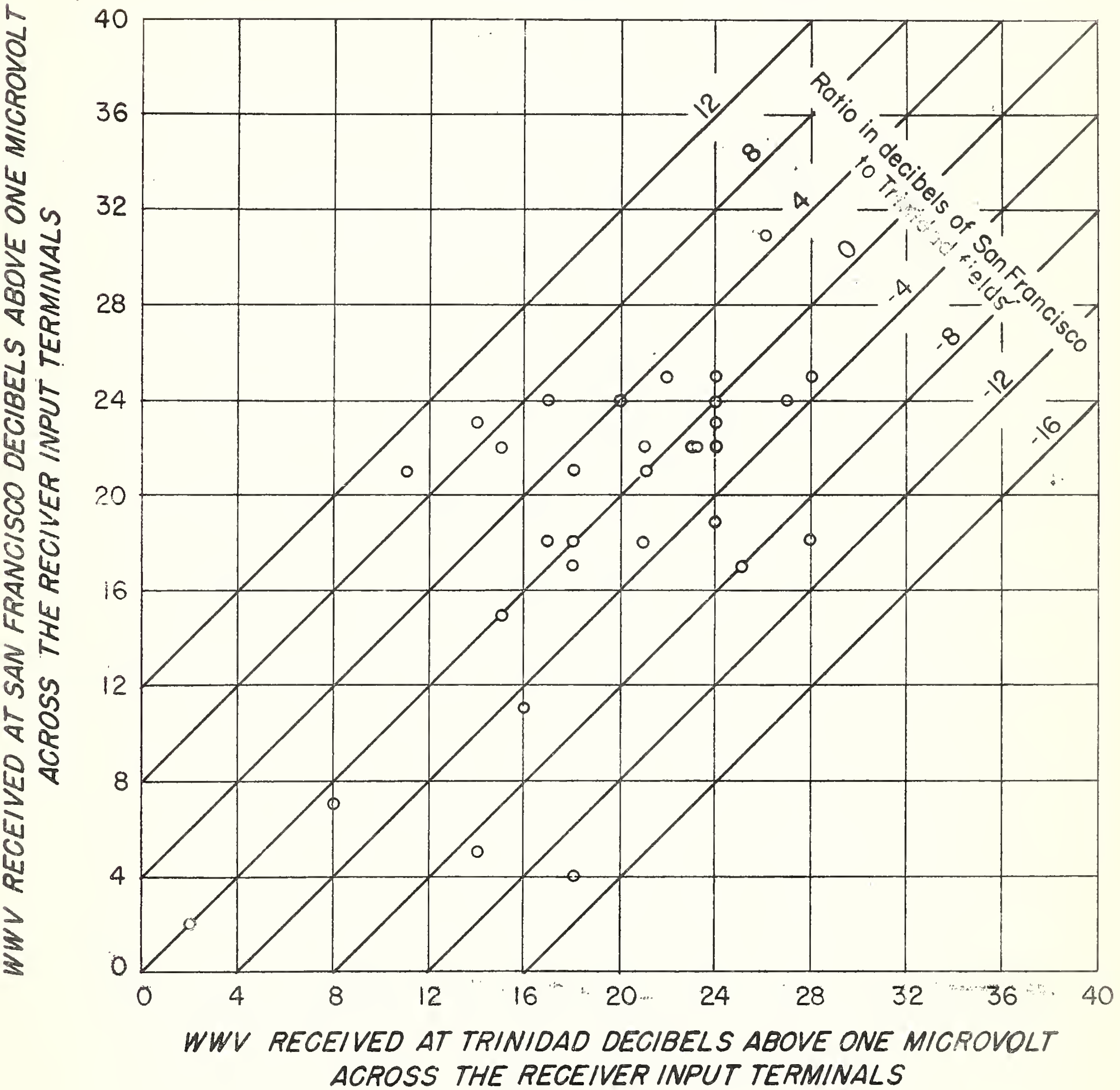

FIGURE 1

CUMULATIVE DISTRIBUTION FOR T = ONE MONTH OF THE HOURLY MEDIAN RECEIVED FIELDS E_a AND E_b AND OF THEIR RATIO E_a / E_b

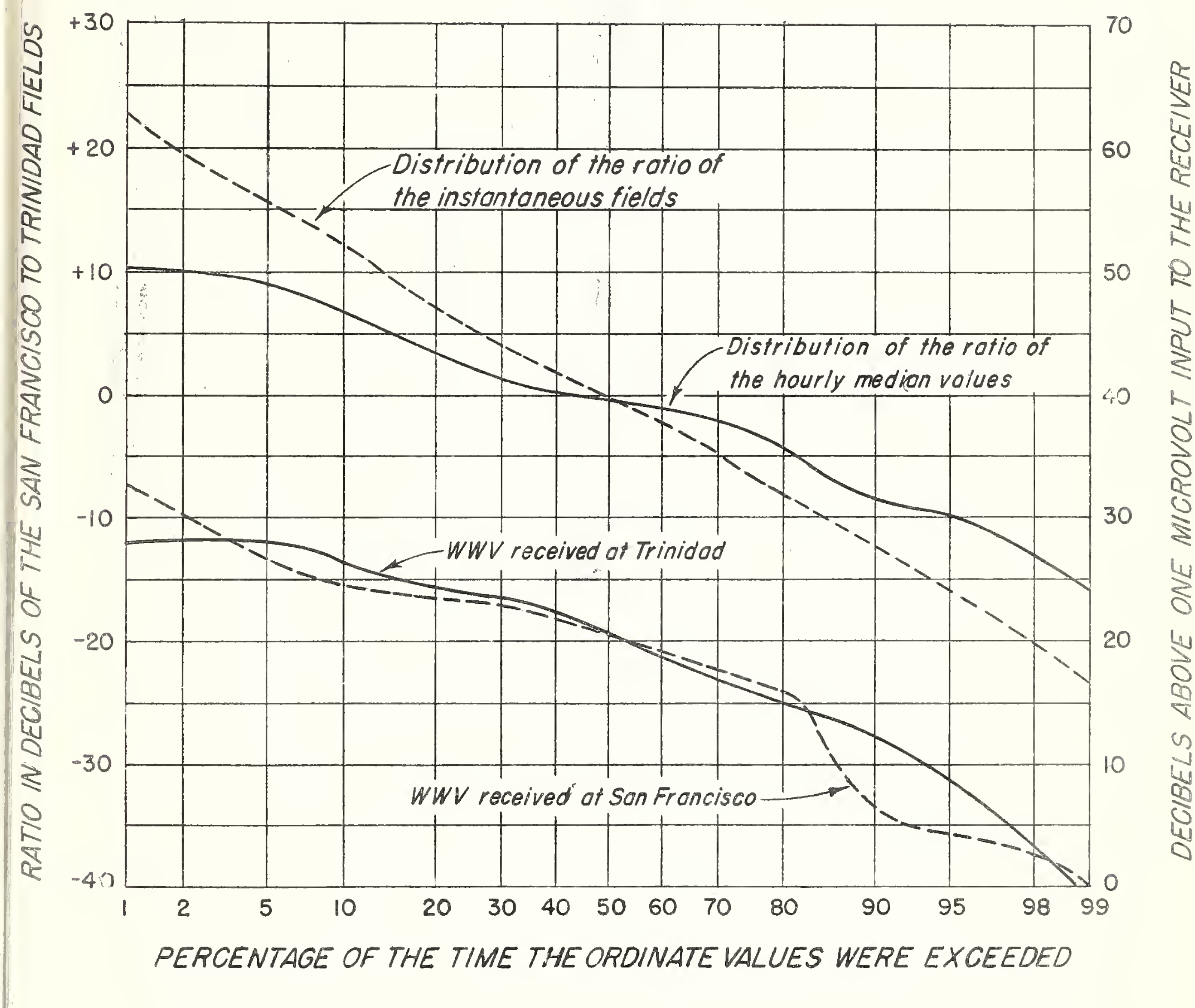

FIGURE 2