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NATIONAL BUREAU OF STANDARDS
CENTRAL RADIO PROPAGATION LABORATORY
WASHINGTON, D. C.

THE COMPARATIVE ACCURACY OF VARIOUS EXISTING AND PROPOSED RADIO NAVIGATION SYSTEMS

BY W.Q. CRICHLOW

NATIONAL BUREAU OF STANDARDS
CENTRAL RADIO PROPAGATION LABORATORY
WASHINGTON 25, D.C.

Subject: Correction Sheet

To: Recipients of Report No. CRPL-4-1

Page 3 - Eq(1) should read as follows:

$$d_{rms}^2 = \frac{k_X^2 \sigma_X^2 + k_Y^2 \sigma_Y^2 + 2r_{XY} \cos\theta \, k_X k_Y \sigma_X \sigma_Y}{\sin^2\theta} \qquad (1)$$

Page 7 - Ninth line from bottom - 0.018 B σ should read 0.01745 B σ, and on Figure 5, the corresponding number should have been rounded off to 0.017.

Page 17 - TABLE II - Omit line reading as follows:

Aximuthal	HF D/F (3 sta)	67 mi**	60°	mi

Page 17 - TABLE II - In column under d_{rms}, for Hyperbolic, L. F. Loran, (3 sta.), change 0.51 mi to 0.81 mi.

Page 18 - TABLE III - After the column heading, BASELINE ANGLE, add ****.

The following acknowledgments should be added after Page 14:

The preparation of this report was begun in the Radio Propagation Section, Communication Liaison Branch, Office of the Chief Signal Officer, The Pentagon, Washington, D. C. Acknowledgment of the assistance of that group during the early phases of the study is hereby made.

Acknowledgment is also made of the useful advice obtained from Messrs. Kenneth A. Norton and R. Silberstein. The assistance of Mr. John Harman in designing the figures and developing appropriate methods of presentation is also acknowledged. Messrs. Norton, Silberstein and Harman are with the Central Radio Propagation Laboratory.

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THE COMPARATIVE ACCURACY OF VARIOUS EXISTING AND PROPOSED RADIO NAVIGATION SYSTEMS

BY WILLIAM Q. CRICHLOW

Contents

I. SUMMARY

The concept of the root mean square distance error as a means of evaluating the accuracy of a navigational fix was introduced by the Operational Research Staff in the Office of the Chief Signal Officer. This concept is further expanded here to apply to the three basic types of navigation systems, namely: range, azimuthal, and hyperbolic. The factors affecting the accuracy of each type of system are analyzed in a manner such that quantitative comparisons can be made of various proposed and existing navigation schemes. It is shown that, with the range and hyperbolic type systems, the base line should be as long as possible in order to obtain the greatest accuracy within a given area; but, with the azimuthal-type systems there is a particular base-line length which yields the maximum accuracy within a given area. It was also found that there is an optimum base-line angle which, in the case of hyperbolic systems, produces the maximum accuracy for a given area of coverage or conversely, a maximum area within which a specified accuracy will be obtained. A base-line angle of 60° gives the maximum accuracy with azimuthal-type systems when an optimum base-line length is used.

Using estimated values of the standard errors of position readings, the accuracies of several navigation systems have been calculated. For short-range coverage the Decca System appears to be most accurate. An rms distance error of less than 0.0082 mile or 43 feet is expected within an area of 10,000 square miles. L. F. Loran has the best estimated accuracy for consistent day and night long-range coverage. The expected rms distance errors are

(over)

3.7 miles and 5.4 miles for day and night respectively, within an area of 1,500,000 square miles.

In addition to the size of the area that can be served with a given accuracy, the shape of the service area is also of importance. Contours of constant accuracy have been plotted for the three types of systems and a direct comparison of the shapes can be readily made.

In order to evaluate the service areas completely it is necessary to determine accurately how the standard error of a navigation reading depends upon the position of the observer. To do this requires the collection of much further experimental data on radio propagation than is now available. The conclusions reached in this paper relative to the optimum systems of navigation for various purposes are based on the very meagre amount of operational propagation data now available and will probably require modification when further data become available. The results obtained in this report are presented in such a form that it is an easy matter to introduce future data and thus arrive at such modified conclusions.

II. INTRODUCTION

During the past few years a number of radio navigation systems have been proposed, each system claiming certain advantages over the others. To date, however, no comparative tests or analyses have been made which show quantitatively the relative merits of each. It is very desirable, particularly in the case of long-range navigation, to adopt a particular system for world-wide use so that equipment carried in ships and airplanes can be standardized; but before a final decision can be reached it is desirable to compare the various systems in terms of a common set of performance standards.

The purpose of this report is to present the geometric considerations which are involved in determining the navigational accuracies to be expected with the various types of systems. This material, when supplemented by experimental radio wave propagation data, should be of assistance in the choice of the optimum navigation system.

III. GENERAL

In a report prepared by the Operational Research Staff,[1] a general formula was presented for determining the root mean square distance error of a navigational fix.

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[1] W. Q. Crichlow, J. W. Herbstreit, E. M. Johnson, K. A. Norton, C. E. Smith, "The Range Reliability and Accuracy of a Low Frequency Loran System," Report No. ORS-P-23, Operational Research Staff, Office of the Chief Signal Officer, The Pentagon, Washington 25, D.C.

The formula is:

$$d_{rms} = \frac{k_X^2 \sigma_X^2 + k_Y^2 \sigma_Y^2 + 2r_{XY} \cos\theta \, k_X k_Y \sigma_X \sigma_Y}{\sin^2\theta} \qquad (1)$$

where:

d_{rms} = Root mean square distance error (approximately 65% of the fix distance errors will be less than d_{rms} and approximately 96% less than $2d_{rms}$ in the usual case where the random errors of measurement are normally distributed; see Fig. 26, page 46, of above report).

σ_X, σ_Y = Standard error of X and Y position reading in microseconds or degrees azimuth (depending on type of system used).

k_X, k_Y = miles per unit X or Y reading.

r_{XY} = correlation coefficient between the X and Y readings.

θ = angle of cut between position lines.

This formula was originally derived for Loran, but is applicable to any navigation system where the following conditions are fulfilled:

(1) The distance error must be sufficiently small so that within any region of fix uncertainty a group of position lines may be considered essentially straight and parallel.

(2) Systematic errors of measurement can be eliminated by calibration leaving only random errors.

In the experimental L.F. Loran System[2] there was no apparent relationship of σ_X or σ_Y to distance from the transmitters. This condition was probably due to the fact that long base-lines were used between the transmitters, and at all receiving locations there was at least one ionospheric path. It could, in that case, be assumed that the average value of σ throughout the coverage area would be sufficiently accurate for prediction purposes. The expected distance errors that are given in this report are expressed in terms of a σ which is assumed to be constant throughout the service area. In order to determine the absolute magnitude of the errors, the appropriate value of σ must be determined experimentally for each type of system. If σ is found to vary systematically with distance, the contours of constant accuracy must be modified accordingly; the contour shapes shown later in this report must thus be considered to be a first approximation to the correct shape based on the assumption of constant σ throughout the service area. When the propagation to a particular receiving point from all transmitting stations involves ground waves only, the value of σ may be expected

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[2] See footnote 1, page 2.

to be much smaller than for the case when one or more of the propagation paths involves transmission through the ionosphere. Thus short base-line systems may be expected to be characterized by one value of σ at short distances and by a much larger value of σ at the larger distances at which ionospheric transmission is involved. This limitation on the validity of the following treatment, which is based on constant σ, must be kept constantly in mind for a proper appreciation of the significance of the expected service areas shown.

Since we have assumed that both σ_X and σ_Y are independent of position, it follows that $\sigma_X = \sigma_Y = \sigma$, and (1) becomes:

$$d_{rms} = \frac{\sigma}{\sin \theta} \sqrt{k_X^2 + k_Y^2 + 2r_{XY} k_X k_Y \cos \theta} \qquad (2)$$

Radio navigation systems may be divided into three basic classifications, namely: range, azimuthal, and hyperbolic. These three types are analyzed separately in this report to show how the navigational accuracy of each depends upon various geometrical parameters.

The shapes of the areas, within which the accuracy of fix is as good as or better than the value on the contour, vary from one type of system to the next and for the different values of accuracy assumed. The shapes of these regions, together with their areas, are the two most important characteristics to be considered in the selection of a navigational system designed to fulfill a particular operational requirement.

IV. RANGE-TYPE NAVIGATIONAL SYSTEMS

Possibly the simplest type of navigation system to evaluate is one in which a measurement is made of the distance to two fixed stations. Examples of this type are radar and responder beacons in which the distance is indicated by the elapsed time between a transmitted pulse and a received echo.

Lines of position are concentric circles about the fixed stations as shown in Fig. 1, each circle representing a constant delay time in microseconds between the transmitted pulse and received echo. The intersection of position lines from two fixed stations then gives a positional fix.

The expected distance error at any point in such a system may be determined by evaluating (2). k_X and k_Y are the miles per microsecond corresponding to small changes in the X and Y position lines. A pulse travels from the observer to the fixed station and back again at the speed of light. The distance to a position line, represented by a delay time of t microseconds is given by:

$$r = \frac{tc}{2} \qquad (3)$$

Where:

c = Velocity of light = 0.186 miles per microsecond.

Then:

$$k_X = k_Y = \frac{dr}{dt} = \frac{c}{2} = 0.093 \text{ miles per microsecond.} \tag{4}$$

Since the pulses from each fixed station travel over different paths, the correlation coefficient is likely to be small and will be assumed to be zero. Thus (2) may be written:

$$d_{rms} = \frac{0.093\,\sigma\sqrt{2}}{\sin\theta} = \frac{0.1315\,\sigma}{\sin\theta} \text{ miles} \tag{5}$$

where:

σ is the standard error in microseconds with which the position line may be read.

Since the position lines are concentric circles about the fixed stations, the angle of intersection, θ, is equal to the angle measured between the fixed stations.

Contours of constant accuracy may then be determined by evaluating (5). For each value of d_{rms}/σ, θ is constant and thus the contours become arcs of circles which pass tarough the fixed stations as shown on Fig. 2.

Of considerable significance is the area within which the rms distance error is less than a specified amount. In Fig. 2 the area, within which the distance error is less than $0.14\,\sigma$ mile, has been cross-hatched in order to show the shape of the area. It may be noted that there are two identical crescent-shaped regions, one on each side of the base line, B, within each of which d_{rms} will be less than $0.14\,\sigma$ mile. Outside the cross-hatched regions the error will be greater and the contours will inclose a larger area. On the base line and its extensions, d_{rms} actually becomes infinite.

By reference to equation (5) it may be seen that the minimum possible error occurs for $\theta = 90°$, in which case d_{rms} is equal to $0.1315\,\sigma$ mile. The contour representing this error is a single circle having a diameter equal to and centered in the middle of B. The area representing regions where the error is less than a specified value decreases as this specified error decreases, approaching zero as the value of $0.1315\,\sigma$ is approached, so that the 0.1315 contour is a line to which the cross-hatched area degenerates.

Since the contours are arcs of circles passing through the fixed stations, the area contained within each contour (i.e., the area within which d_{rms} will be less than that shown on the contour) can be expressed as follows:

$$A = \frac{B^2}{\sin^2 \theta} \left[\frac{\pi}{2} - \theta + \sin \theta \cos \theta \right] \text{ square miles} \qquad (6)$$

where:

A = Area within a constant accuracy contour in square miles.

B = Spacing between fixed stations in miles.

θ = Angle between the fixed stations as measured at the intersection of the two position lines.

The above equation has been evaluated in terms of σ / d_{rms} (i.e., (5) and (6) are solved simultaneously) in order to show how the coverage area depends upon the required accuracy (i.e., d_{rms}) and upon σ. This relation is shown in Fig. 3 and it can be seen that the coverage area decreases rapidly as the required accuracy is increased (i.e., d_{rms} decreased) or as the standard deviation of the position reading, σ, increases. The area goes to zero at the value of $(\sigma / d_{rms}) = 7.6$, where $\theta = \pi/2$ as explained above. On the other hand, as σ / d_{rms} approaches zero, θ approaches zero and we see by (6) that the area approaches the asymptote,

$$A \doteq \frac{90.84 \, B^2}{(\sigma / d_{rms})^2}$$

This comes about since the shapes of the contours approach perfect circles for small values of σ / d_{rms}. It should be pointed out that the coverage area, for a fixed value of σ / d_{rms}, is proportional to the square of the spacing between the fixed stations and therefore, the spacing should be as wide as possible. The limiting factor in this case would be the power output required to provide sufficient field intensity over the area. In general, with this type of system, transmitting equipment would be required in the airplane and the feasible power ratings for such equipment, rather than accuracy considerations, will probably always determine the effective coverage areas.

V. AZIMUTHAL-TYPE NAVIGATION SYSTEMS

The most common azimuthal type of navigation system is one in which a D/F bearing is taken with a directional antenna. There are other azimuthal systems such as POPI[3] and Consol[3] in which an electrical phase or amplitude measurement is converted to a D/F bearing. All of these systems depend upon bearing measurements and thus can be analyzed together since the accuracy contours will have the same shape.

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[3] "Electronic Navigation Systems," Final Report-Part II on Contract OEMsr - 1441, Central Communications Research, Harvard University, Cambridge, Mass, and
"Report of Electronic Subdivision Advisory Group on Air Navigation," Report No. TSELC-SP 2, February 1946, Air Technical Service Command, Wright Field, Dayton, Ohio.

The two basic assumptions with this type of system are that the position lines from each fixed station are straight radial lines and have a uniform spacing in azimuth as shown in Fig. 4. Although this condition is not met exactly with a system such as POPI, it is approximated sufficiently closely by the usual system in which a single station consists of three or more closely spaced antennas at the corners of an equilateral polygon with transmission on each in succession.

The expected distance error of a fix, consisting of the intersection of two azimuthal position lines, may be obtained by evaluating (2). With this type system k_X and k_Y are in miles per degree azimuth. For small deviation angles the miles error for a given bearing error will equal the distance from the transmitter, D_X, times the angle in radians so that

$$k_X = \frac{\pi}{180} D_X = 0.01745\ D_X \text{ miles per degree} \qquad (7)$$

When the X and Y propagation paths are different, the correlation coefficient is likely to be small and will be assumed to be zero and (2) becomes:

$$d_{rms} = \frac{0.01745\ \sigma}{\sin \theta} \sqrt{D_X^2 + D_Y^2} \text{ miles} \qquad (8)$$

where:

D_X and D_Y = Distance in miles from the X and Y fixed stations, respectively, to the receiving point.

σ = Standard error in degrees with which the position line may be determined.

Putting D_X and D_Y in terms of the base-line length, B, (8) becomes:

$$d_{rms} = \frac{0.01745\ B\ \sigma}{\sin \theta} \sqrt{\left(\frac{D_X}{B}\right)^2 + \left(\frac{D_Y}{B}\right)^2} \text{ miles} \qquad (9)$$

Equation (9) has been evaluated and accuracy contours have been plotted in Fig. 5. It may be noted that all contours having a value of d_{rms} equal to or greater than 0.018 B σ pass through the fixed stations, while those having smaller errors do not. The smallest value of d_{rms} attainable with the system is equal to 0.01605 B σ and occurs at the two points shown on the perpendicular bisector of the base line. It is of interest to note that, although the maximum accuracy with an azimuthal type system occurs at only two points, it occurs at all points along a circle when the range-type system is used. Here, as with the range-type systems, d_{rms} is infinite on the base line and base-line extensions.

The area inclosed within each contour of Fig. 5 was measured with a planimeter and expressed in terms of square base-line units. The area thus obtained was plotted in Fig. 6 as a function of the accuracy attainable on the contour (i.e., $B\,\sigma/d_{rms}$). From this curve it may be seen that with a fixed base line, B, the area decreases as σ/d_{rms} is increased, and goes to zero when $B\,\sigma/d_{rms} = 62.23$, 62.23 being the reciprocal of 0.01605.

When D_X and D_Y are large with respect to the base line, B, (8) can be expressed by:

$$D^2 = \frac{B\,d_{rms}\sin\alpha}{0.01745\,\sigma\sqrt{2}} \tag{10}$$

where:

D = Distance from center of base line to the observer ($D \gg B$)

α = Angle between observer and base line with vertex at center of B.

Equation (10) is an expression for the distance to contours having poor accuracy and the area within these contours can be determined by integration as follows:

$$A = 2\int_0^{\pi} \tfrac{1}{2}D^2\,d\alpha = \frac{\sqrt{2}\,B\,d_{rms}}{0.01745\,\sigma} = 81.04\,B\,d_{rms}/\sigma \tag{11}$$

Equation (11) has also been plotted on Fig. 6 and it may be seen that the measured area approaches (11) asymptotically for small values of $B\,\sigma/d_{rms}$.

In order to show the effect of varying the base line, Fig. 6 was replotted in the form shown in Fig. 7. This was accomplished by multiplying the ordinate values in Fig. 6 by the square of the corresponding abscissa values.

Here it can be seen that for any contour of a given accuracy ($\frac{\sigma}{d_{rms}}$ = constant) there is a base-line length that will give the maximum area within the contour. The optimum value of B is equal to $41d_{rms}/\sigma$. It should be pointed out that the shape of the contour is determined by the value of $B\,\sigma/d_{rms}$ as shown in Fig. 5 and where a certain shape of coverage area is required it may not be possible to use the optimum value of B.

The preceding discussion applies to positional fixes involving only two fixed stations. However, it is possible to more than double the coverage area within a contour of given accuracy by adding a third station. Figs. 8 through 11 show the combined coverage obtained by using three stations, two at a time. Each figure is for a different base-line angle using the same base-line lengths between the center and two outside stations. The composite contour results from choosing portions of the separate two-station contours which give the greatest inclosed area. Some improvement in accuracy could have been obtained by calculating on the basis of all three fixes but the improvement would not have been sufficient to alter the conclusions reached here. A further discussion of this subject is contained in a report by N.D.R.C.[5] describing an electric D/F evaluator which is capable of determining the most probable position of a fix when a large number of stations are used.

By measuring the areas within each contour of Figs. 8 through 11, Fig. 12 was obtained. The separate curves are for different base-line angles and it may be seen that there is an optimum value of B for each value of base-line angle. The curve for a base-line angle of zero degrees is simply the two-station curve, since the first and third stations coincide when the base-line angle is reduced to zero. By holding σ/d_{rms} constant and choosing the optimum value of B, the maximum area within the contour occurs when the base-line angle is 60° and $(B\sigma/d_{rms}) = 45$. Under these conditions the area within this optimum contour is more than twice the area possible within a contour of the same accuracy using only two stations.

When the required coverage area is given, the optimum base-line length for maximum accuracy throughout that coverage area can be determined from Fig. 12. Thus, assuming that a base-line angle of 60° is feasible, we see by Fig. 12 that $A(\sigma/d_{rms})^2 = 4530$ when $(B\sigma/d_{rms}) = 45$. For example, if the required area, A = 10,000 square miles, (σ/d_{rms}) on the contour enclosing that area will be equal to $\sqrt{4530/10000} = 0.672$ and the optimum value of the base-line length $B = 45/0.672 = 67$ miles.

VI - HYPERBOLIC TYPE NAVIGATION SYSTEMS

A typical example of a hyperbolic navigation system is Loran. With this system a position line is determined by the difference in time of arrival of pulses from two transmitters. Another set of hyperbolic position lines is obtained by comparing arrival times of pulses from a second pair of stations. The intersection of two of these position lines then gives a navigational fix as shown on Fig. 13.

Other systems, such as Decca, compare the relative phases of two c.w. signals rather than the arrival time of pulses; thus in this case also, the position lines will be hyperbolic. When the distance to the receiver is large compared to the spacing between transmitters, it is true that the hyperbolas approach straight radial lines; however, even at these large distances it is not permissible to use the azimuthal system theory since the radials are not uniformly spaced in azimuth except in the sectors adjacent to the perpendicular bisectors of the base-lines.

In the following discussion, σ is expressed in microseconds delay between

[5]Allison, Lewis & Fryer, "Electrical Direction Finder Evaluator," Final Report on Project 13-121, N.D.R.C., Division 13.

pulses and when the analysis is being applied to phase measuring systems a factor must be applied to convert electrical degrees to time in microseconds.

Equation (2) was applied to a Loran system in the Operational Research Staff report cited. In that report it was shown that:

$$d_{rms} = \frac{0.093\ \sigma}{\sin \theta} \sqrt{\frac{1}{\sin^2 \phi_1} + \frac{1}{\sin^2 \phi_2} + \frac{2r_{XY} \cos \theta}{\sin \phi_1 \sin \phi_2}} \quad (12)$$

where:

σ = Standard error of delay reading in microseconds.

$2\ \phi_1$ = Angle at receiving point between one pair of transmitters.

$2\ \phi_2$ = Angle at receiving point between second pair of transmitters.

θ = Angle of cut between position lines = angle between the bisectors of $2\ \phi_1$ and $2\ \phi_2$. (For a three station system $\theta = \phi_1 + \phi_2$).

r_{XY} = Correlation coefficient between the X and Y position lines.

Equation (12) was first evaluated for a three-station system for an assumed value of r_{XY} equal to 0.309 since this was the value determined for the experimental L. F. Loran system. Figs. 14 and 17 were then obtained. Each figure shows the contours of constant accuracy for a different angle between the base-lines. It may be noted that the accuracy in regions of the base-line extensions is very poor and, in fact, d_{rms} becomes infinite there.

The area within each contour was measured with a planimeter and plotted in Fig. 18. It may be seen that for a given value of σ/d_{rms} there is a particular base-line angle that will give the maximum area. Where a high accuracy is required, a base-line angle of about 90° will give the greatest coverage area and where a greater distance error can be tolerated, angles approaching 180° are more desirable. The asymptote shown on Fig. 18 was determined for a base-line angle of 180°. It is valid at distances from the base-line where θ is very small and the derivation is similar to that of (11). The limiting area then, which is inclosed within contours of large d_{rms} is given by:

$$A = 5.223\ B^2\ \sigma / d_{rms} \quad (13)$$

where:

Base-line angle = 180° and, $\theta < 6^\circ$

The area given by (13) in this limiting case has the shape of two identical circles, one on each side of the baseline.

The composite curve of Fig. 19 was obtained by choosing the optimum base-line angle for each value of σ/d_{rms} and thereby represents the maximum enclosed area that can be obtained for each value of σ/d_{rms}. It should be kept in mind that the shapes of the contours change with base-line angle and it may not always be advantageous to adjust the angle for maximum area.

A four station system was also evaluated with the stations located at the corners of a square. Under these conditions, the accuracy in the center is greater than that possible with any other arrangement; having a value of $d_{rms} = 0.1315\ \sigma$ miles at the center. With this arrangement the stations on the diagonals are paired together and, since there is no common path involved in obtaining a fix, the correlation coefficient will be very low and has been assumed equal to zero. The accuracy contours have been plotted in Fig. 20. Using the diagonal pairs of stations for a fix there is a region of poor accuracy on the extensions of the diagonals. The accuracy in this region can be improved by using the opposite three stations for a fix and disregarding the nearest transmitter. The contours thus obtained have also been shown in Fig. 20.

The area within each composite contour was determined and has been plotted as a function of σ/d_{rms} in Fig. 19. This may be compared with the curve for a three station system. Here it may be seen that the four station coverage area is greater, particularly in the regions of high accuracy.

With any hyperbolic system, just as with range type systems, the coverage area for a given accuracy is proportional to B^2 and so the base-line should be made as large as possible consistent with the power requirements. When low frequencies are used, ground wave synchronization is possible with base-lines up to approximately 1000 miles.

VII - COMPOSITE RANGE AND AZIMUTHAL SYSTEMS

The navigation system proposed by the Civil Aeronautics Administration for short range coverage incorporates both range and azimuthal measurements for a navigational fix. Because of the simplicity of operation and the ease of interpreting the data obtained, a brief treatment of the expected accuracy of such a composite system seems desirable.

With this type system, a single fixed location is required with the range navigation lines appearing as concentric circles about the fixed station as shown in Fig. 1 and the azimuthal navigation lines appearing as radials from the fixed station as shown in Fig. 4. A fix then consists of the intersection of a circle and a radial.

In this case the standard errors for the two navigation lines will not be equal, the range error, σ_X , being expressed in microseconds and the azimuthal error, σ_Y , being expressed in degrees. k_X and k_Y apply to the range and azimuthal navigation lines respectively and are obtained as in Sections IV and V. Since the navigation lines always intersect at right angles, θ is always equal to 90°, which considerably simplifies the determination of the accuracy.

From (2) the expected distance error may be expressed by:

$$d_{rms} = \sqrt{K_X^2 \ \sigma_X^2 + K_Y^2 \ \sigma_Y^2} \tag{14}$$

where:

K_X = 0.093 miles per microsecond

K_Y = 0.01745 D miles per degree azimuth

σ_X = Standard error of range measurement in microseconds

σ_Y = Standard error of azimuth measurement in degrees

D = Distance from fixed station to observer in miles

Substituting the values for K_X and K_Y, (14) becomes:

$$d_{rms} = \sqrt{0.008649 \ \sigma_X^2 + 0.0003045 \ D^2 \ \sigma_Y^2} \tag{15}$$

When σ_X and σ_Y are constant throughout the coverage area, as would be expected in practice for short range systems, the contours of constant d_{rms} are concentric circles centered on the fixed station with the greatest accuracy at the center. The area within a contour of given accuracy is expressed by:

$$A = \frac{\pi \ (d_{rms}^2 - 0.008649 \ \sigma_X^2)}{0.0003045 \ \sigma_Y^2} \ \text{square mi.} \tag{16}$$

VIII - ESTIMATED ERRORS TO BE EXPECTED WITH PROPOSED NAVIGATION SYSTEMS

In order to show the application of the results obtained in this report, estimates have been made of the rms distance errors to be expected from a few of the proposed navigation systems. It should be emphasized that the standard errors of position lines that are given here may be subject to considerable error since in most cases there have been no reliable measurements made. The primary purpose in presenting these estimates is to show examples of how systems may be evaluated when extensive experimental data do become available. The estimates made here are for the systems as they were originally proposed and no attempt has been made to modify the initial proposal to obtain greater accuracy. Comparisons between systems on this basis, therefore, may be misleading. For example, with the POPI System, the proposed separation between antennas at each transmitting station (not to be confused with base-line length between transmitting stations) is a half wavelength. If greater separation is used, greater accuracy will be indicated but there will be sector ambiguities. It is possible to increase the antenna spacing to such an extent that the accuracy is equal to or better than that attainable with Decca. Of course, in this case where the spacing is comparable to the distance to the receiver, the position lines must be treated as hyperbolas instead of straight radial lines as has been done in this paper.

In Table I, estimated standard errors of the position lines are given for the different classes of propagation conditions which may be encountered and for several proposed systems grouped according to type, i.e., range, azimuthal, or hyperbolic. The estimates given are intended to represent average values over the parts of the coverage area served by the given types of propagation. Thus, the standard errors given are tabulated according to whether the propagation is by means of ground wave, daytime ionospheric wave, or nighttime ionospheric wave. In making the estimates it has been assumed that any systematic propagation or equipment errors may be eliminated by chart corrections.

In the case of the Decca system, which depends on a measurement of electrical phase, it was necessary to use a conversion factor in order to obtain a value of σ in microseconds. This was done by assuming a standard phase deviation of 2° at 200 kilocycles.

The various systems were first evaluated in terms of the accuracies attainable with short range coverage in which the propagation was by means of the ground wave. It was assumed that the radiated power was adequate for a distance range of 100 to 150 miles and that a coverage area of 10,000 square miles was required. The rms distance errors which would be expected on the contours enclosing 10,000 square miles are given in Table II. With the range and hyperbolic type systems the accuracy within a given area increases with base-line length. For this reason the base-lines were made as long as possible without exceeding the range requirement. A length of 100 miles was chosen as being suitable. With the azimuthal systems the optimum base-line length for 10,000 square miles coverage as determined from Fig. 12 was found to be 67 miles. In all cases the base-line angle that was used provides maximum accuracy on the 10,000 square mile contour.

The Decca system has the greatest estimated accuracy for short range coverage, the rms distance error being less than 0.0082 miles or 43 feet within an area of 10,000 square miles. It should be remembered, however, that there are a large number of closely spaced sectors in which the same navigation reading will be obtained with this system and these must be carefully resolved. The Shoran system comes next in accuracy with a value of d_{rms} equal to 0.015 miles or 79 feet within the same coverage area. Although the accuracy is somewhat less than with Decca, there are no difficulties with sector identification with the Shoran system. The CAA system probably provides the simplest type of information and requires less skill on the part of the operator. Even though the expected distance error is 1.1 miles for a coverage area of 10,000 square miles it may be more suitable due to the ease of operation.

Table III was obtained by assuming a required coverage area of 1,500,000 square miles and shows the distance errors to be expected from each system when used for long-range coverage, involving now in every case the use of ionospheric waves. The tabulated values of d_{rms} would be expected on the contours enclosing 1,500,000 square miles.

The systems were adjusted for maximum accuracy on the contour enclosing 1,500,000 square miles and it was found that this required the use of a base-line length of 820 miles and a base-line angle of 60° with the azimuthal type systems. Since the accuracy of the hyperbolic systems increases with base-line length, this parameter was made as long as was permitted by the synchronization requirements of each system, and the base-line angle was adjusted to obtain maximum accuracy on the 1,500,000 square mile contour.

Range type systems were omitted from this table since the power required to cover such a large area would probably be prohibitive. For the same reason no values of distance errors were given for Standard and Sky-wave Synchronized Loran operating in the daytime. The Decca system has also been omitted from the table since no estimate of its sky-wave performance was available.

Examination of Table III reveals that L. F. Loran will give consistent 24-hour long-range service with a greater accuracy than any of the other systems which have been tabulated. An rms distance error of only 3.7 miles would be expected during the day and 5.4 miles at night on the contour enclosing 1,500,000 square miles.

TABLE I

ESTIMATED STANDARD ERROR OF POSITION LINES

TYPE	SYSTEM	σ GROUND WAVE	σ DAYTIME IONOSPHERIC WAVE	σ NIGHTTIME IONOSPHERIC WAVE
Range	Oboe (1)	0.15 μs	--------	--------
Range	Shoran (1)	0.1 μs	--------	--------
Azimuthal	POPI ($\frac{\lambda}{2}$ spacing) (2)	0.65° az.	2° az.	3° az.
Azimuthal	Consol (3λ spacing) (3)	0.17° az.	0.75° az.	1.0° az.
Azimuthal	HF D/F (4)	--------	2.5° az.	2.5° az.
Hyperbolic	Standard Loran (5)	1 μs	--------	8 μs
Hyperbolic	S. S. Loran (6)	--------	--------	9.7 μs
Hyperbolic	L. F. Loran (7)	3 μs	15. μs	22. μs
Hyperbolic	Decca (8)	0.03 μs	?	?
Composite Range & Azimuthal	CAA (9)	$\sigma_X \doteq 4$ μs $\sigma_Y \doteq 1^\circ$ az.	-------- --------	-------- --------

Footnotes for Table I

(1) The value of σ was obtained by converting to microseconds, the estimated distance error of a navigation reading which was given in "Electronic Navigation Systems," NDRC, Division 13, Final Report, Part II on Contract OEMsr-1441, Central Communications Research Laboratory, Harvard University, Cambridge, Massachusetts.

(2) The value of ground wave σ is based on a 2^{o} standard deviation of electric phase which was given in the reference of footnote (1) above. The values of σ for ionospheric propagation are rough estimates which are based on the ionospheric wave performance of Consol and L.F. Loran (see footnotes (3) and (8)).

(3) Ground wave σ was obtained from reference of footnote (1). The values of σ for ionospheric propagation were based on British measurements described in "Third Commonwealth and Empire Conference on Radio for Civil Aviation," Summer 1945, Ministry of Civil Aviation, Ariel House, Strand, W.C. 2.

(4) The values of σ were computed from data given in "Coordinated Study of Correlation of High-Frequency Direction-Finding Errors With Ionospheric Conditions," reports issued bi-monthly as the IRPL-G series by the National Bureau of Standards, July 1944 to June 1946.

(5) The groundwave value of σ was obtained from the reference of footnote (1). The value of σ for ionospheric propagation was obtained from a report by R. Naismith and E. N. Bramley, "Time Delay Measurements on Loran Transmissions", Radio Division, National Physical Laboratory, January 4, 1945.

(6) The value of σ was obtained by averaging the variance of readings at a number of monitoring stations during tests of an experimental S. S. Loran System in the United States during 1943. It is believed that in actual operation the accuracies will be somewhat better. The results of the tests are given in "Report on Combined Operational Trials of L. S. Loran System held in the United States, October 1943," prepared by R.A.F. Delegation in consultation with U.S. War and Navy Departments and the Radiation Laboratory of O.S.R.D.

(7) The ground wave value of σ was based on the value given for standard Loran and was made larger because of the longer pulse used with L.F. Loran. The ionospheric wave values for σ were obtained from "The Range, Reliability, and Accuracy of A Low Frequency Loran System," Report No. ORS-P-23, prepared by Operational Research Staff, Office of the Chief Signal Officer, The Pentagon, Washington 25, D. C.

(8) The ground wave value of σ was obtained by assuming 2^{o} standard deviation of phase at 200 kc and converting this to microseconds. The phase deviation was assumed to be equal to that obtained with POPI and will probably not be strictly correct in practice. No estimate was available for the σ to be expected with ionospheric propagation.

(9) The values given for σ_X and σ_Y are based on accuracies quoted by Mr. Donald Stuart of Civil Aeronautics Administration and were obtained from field tests. The range measurements were found to be within 2000 feet or 1% of the range, whichever was greater, and 95% of the azimuth measurements were within 2^{o}. If a normal distribution of error is assumed, the standard error is then approximately 1^{o}.

TABLE II

ACCURACIES ATTAINABLE WITH SHORT RANGE SYSTEMS EMPLOYING GROUND WAVE ONLY

(Accuracies are computed for contours enclosing service areas of 10,000 square miles.)

TYPE	SYSTEM	BASELINE LENGTH	BASELINE ANGLE***	d_{rms}
Range	Oboe (2 sta.)	100 mi. *	----	0.022 mi.
Range	Shoran (2 sta.)	100 mi. *	----	0.015 mi.
Azimuthal	POPI (3 sta.)	67 mi. **	60°	0.96 mi.
Azimuthal	Consol (3 sta.)	67 mi. **	60°	0.25 mi.
Azimuthal	HF D/F (3 sta.)	67 mi. **	60°	mi.
Hyperbolic	Standard Loran (3 sta.)	100 mi. *	85°	0.36 mi.
Hyperbolic	L.F. Loran (3 sta.)	100 mi. *	85°	0.51 mi.
Hyperbolic	L.F. Loran (4 sta.)	100 mi. *	90°	0.51 mi.
Hyperbolic	Decca (3 sta.)	100 mi. *	85°	0.0082 mi.
Composite Range & Azimuthal	CAA	--------	----	1.1 mi.

* 100 miles chosen as a convenient baseline length for short range systems using ground wave.

** 67 mile baseline gives maximum accuracy on the contour enclosing 10,000 square mile area with azimuthal type systems having a baseline angle of 60°.

*** Baseline angle chosen for maximum accuracy on contour enclosing a 10,000 square mile area.

TABLE III

ACCURACIES ATTAINABLE WITH LONG RANGE SYSTEMS EMPLOYING SKY WAVE TRANSMISSION

(Accuracies are computed for contours enclosing service areas of 1,500,000 square mile)

TYPE	SYSTEM	BASELINE LENGTH	BASELINE ANGLE	d_{rms}	
				DAY	NIGHT
Azimuthal	POPI (3 sta.)	820 mi. *	60°	[illegible] mi.	54 mi.
Azimuthal	Consol (3 sta.)	820 mi. *	60°	14 mi.	18 mi.
Azimuthal	HF D/F (3 sta.)	820 mi. *	60°	45 mi.	45 mi.
Hyperbolic	Standard Loran (3 sta.)	500 mi. **	145°	unusable	11 mi.
Hyperbolic	S.S. Loran (4 sta.)	1000 mi. ***	90°	unusable	2.4 mi
Hyperbolic	L.F. Loran (3 sta.)	1000 mi. **	95°	5.8 mi.	8.4 mi
Hyperbolic	L.F. Loran (4 sta.)	1000 mi. **	90°	3.7 mi.	5.4 mi

* 820 mile baseline gives maximum accuracy on the contour enclosing a 1,500,000 square mile area with azimuthal type systems having a baseline angle of 60°.

** Maximum baseline length for ground wave synchronization.

*** Sky-wave synchronization allows longer baseline.

**** Baseline angle chosen for maximum accuracy on the contour enclosing a 1,500,000 square mile area.

RANGE TYPE NAVIGATION SYSTEM

SHOWING POSITION LINES AS CONCENTRIC CIRCLES ABOUT S_1 AND S_2

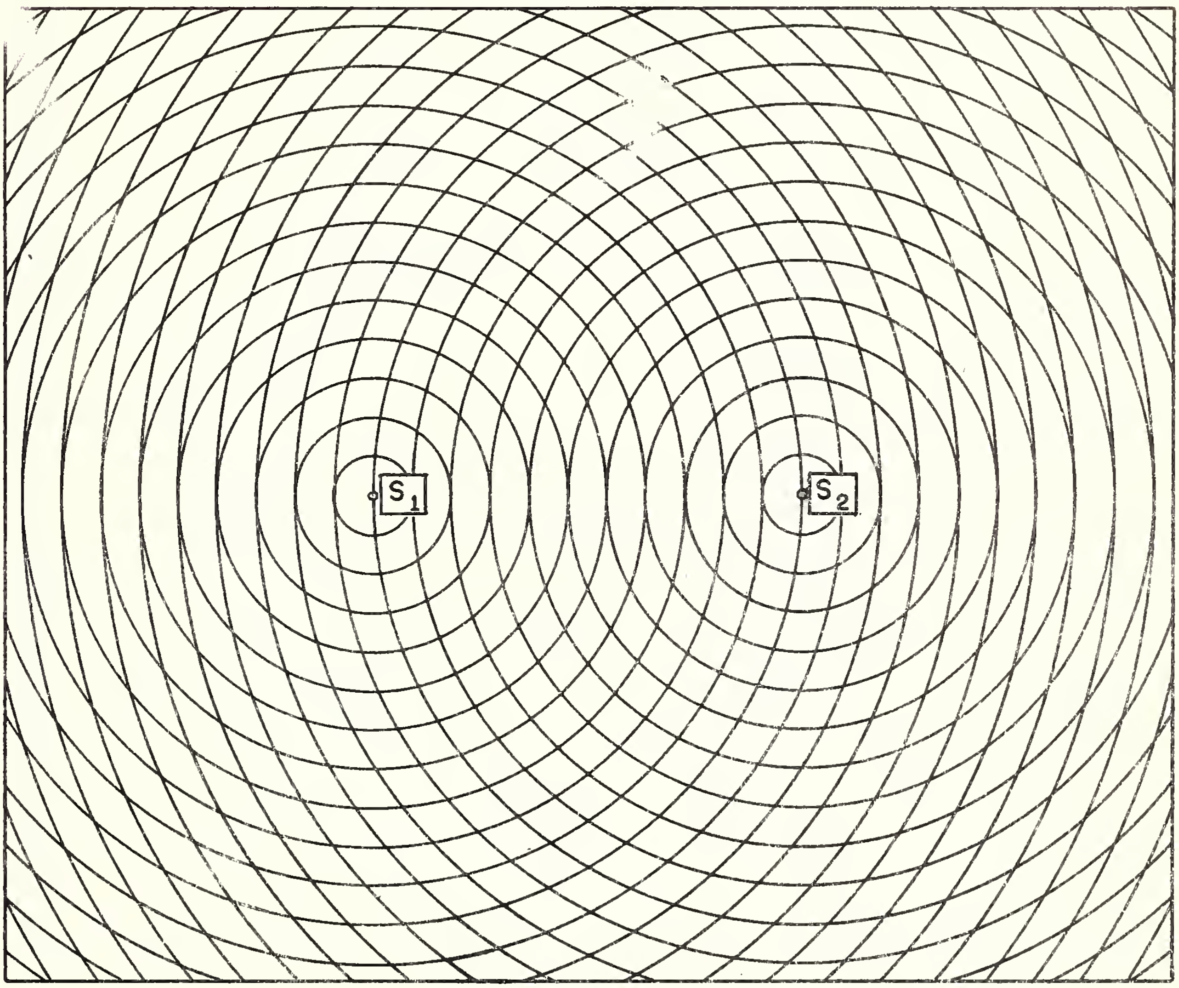

Fig. 1

RANGE TYPE NAVIGATION SYSTEM
CONTOURS OF CONSTANT ROOT MEAN SQUARE DISTANCE ERROR
(σ = STANDARD ERROR IN MICROSECONDS)

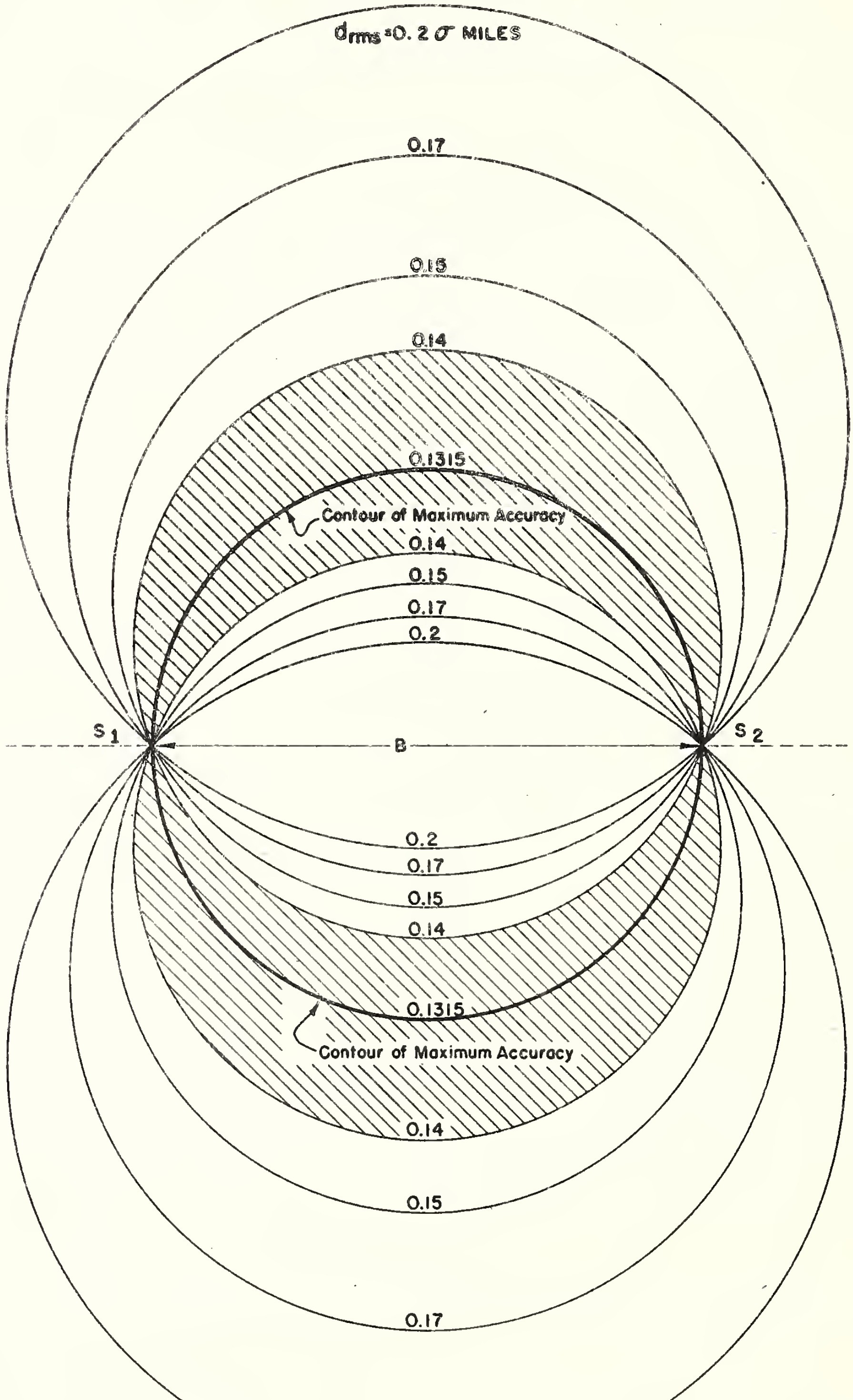

Fig. 2

RANGE TYPE NAVIGATION SYSTEM

AREA IN SQUARE BASELINE UNITS WITHIN CONSTANT ACCURACY CONTOURS
VERSUS
ACCURACY IN TERMS OF $\frac{\sigma}{d_{rms}}$

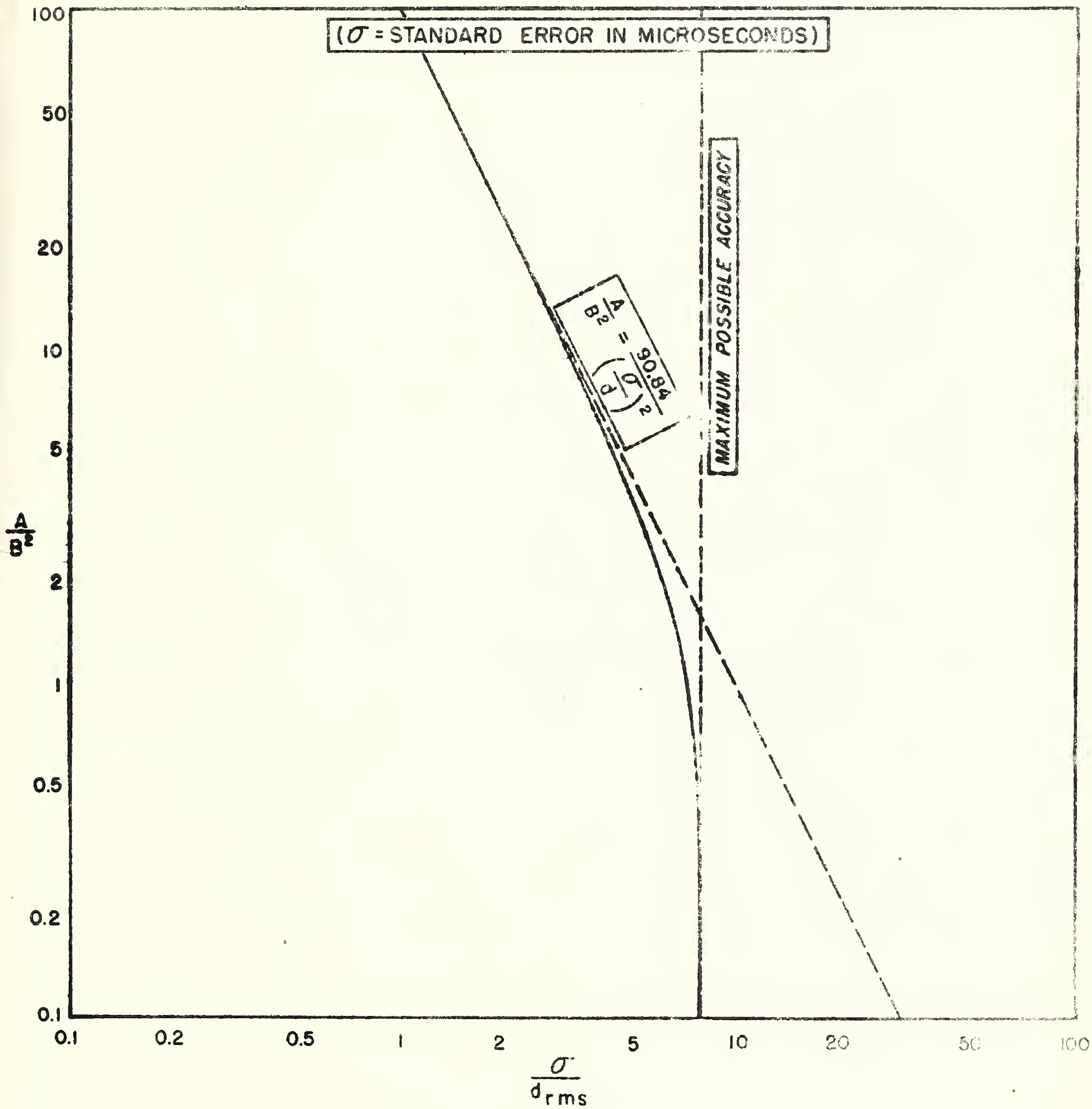

Fig. 3

AZIMUTHAL TYPE NAVIGATION SYSTEM

SHOWING POSITION LINES AS RADIALS FROM STATIONS S_1 AND S_2

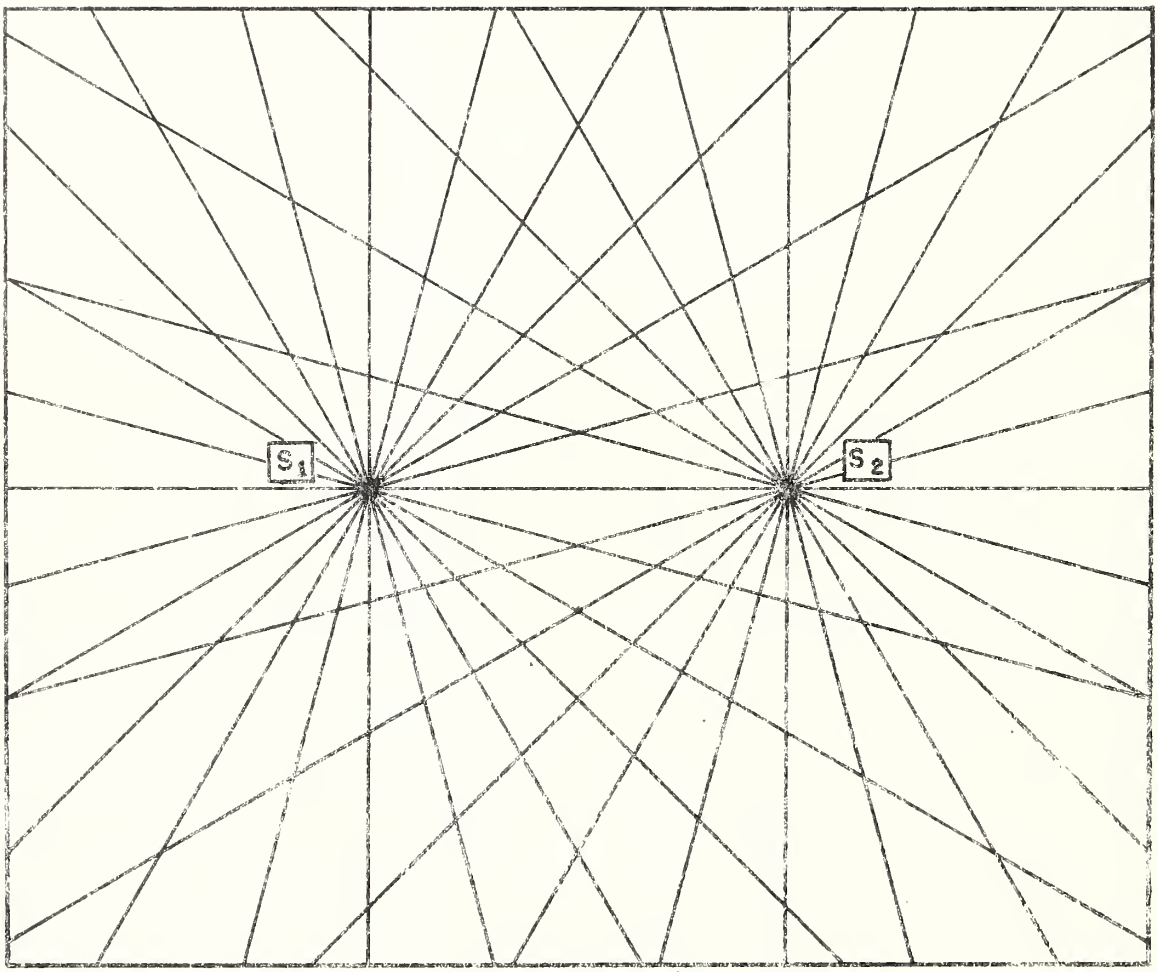

Fig. 4

AZIMUTHAL TYPE NAVIGATION SYSTEM
USING TWO FIXED STATIONS S_1 AND S_2

CONTOURS OF CONSTANT ROOT MEAN SQUARE DISTANCE ERROR
ARE SHOWN IN TERMS OF σ AND BASELINE LENGTH B

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

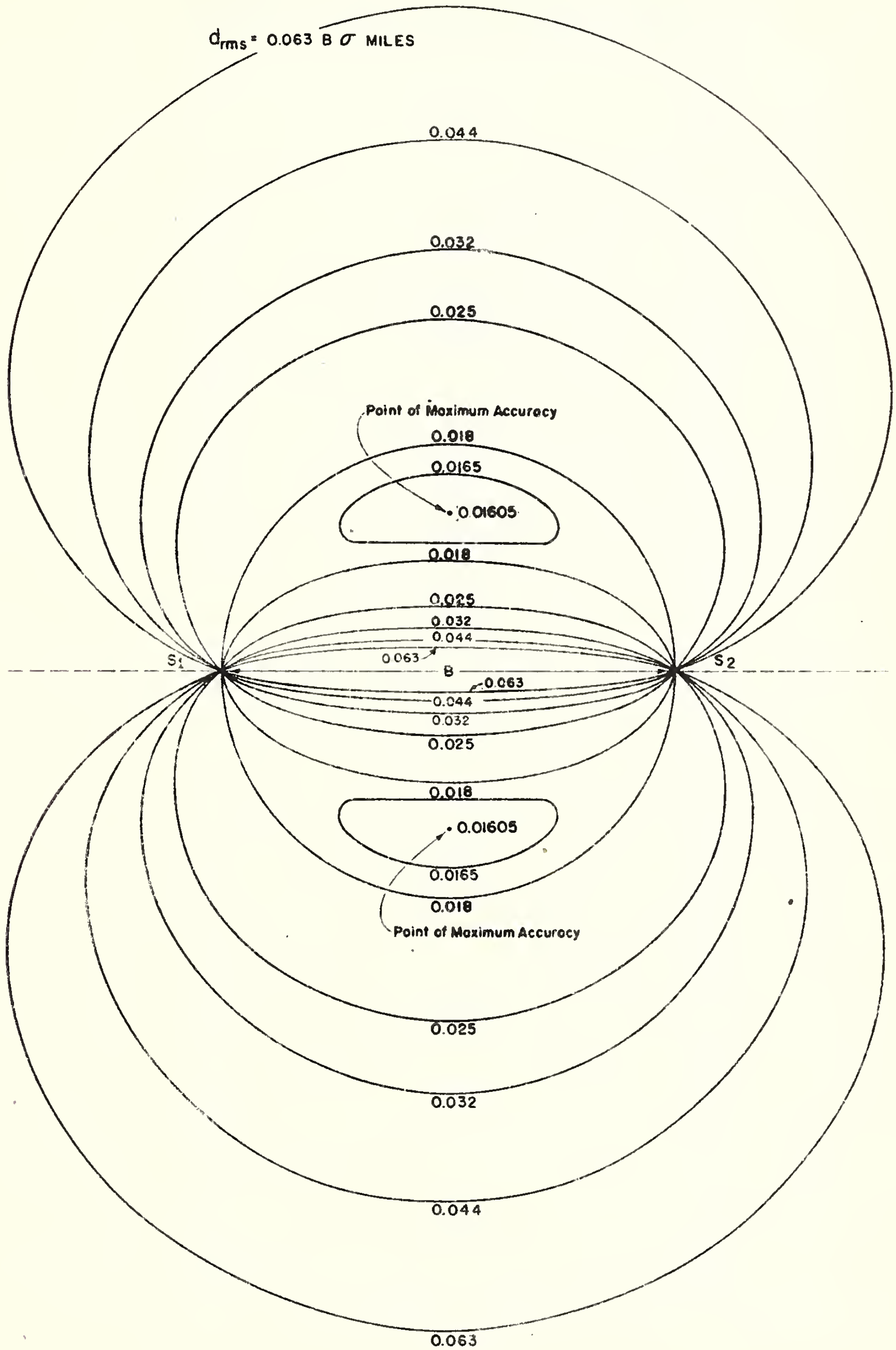

Fig. 5

AZIMUTHAL TYPE NAVIGATION SYSTEM USING TWO FIXED STATIONS

AREA IN SQUARE BASELINE UNITS WITHIN CONSTANT ACCURACY CONTOURS VERSUS ACCURACY IN TERMS OF σ AND BASELINE LENGTH

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

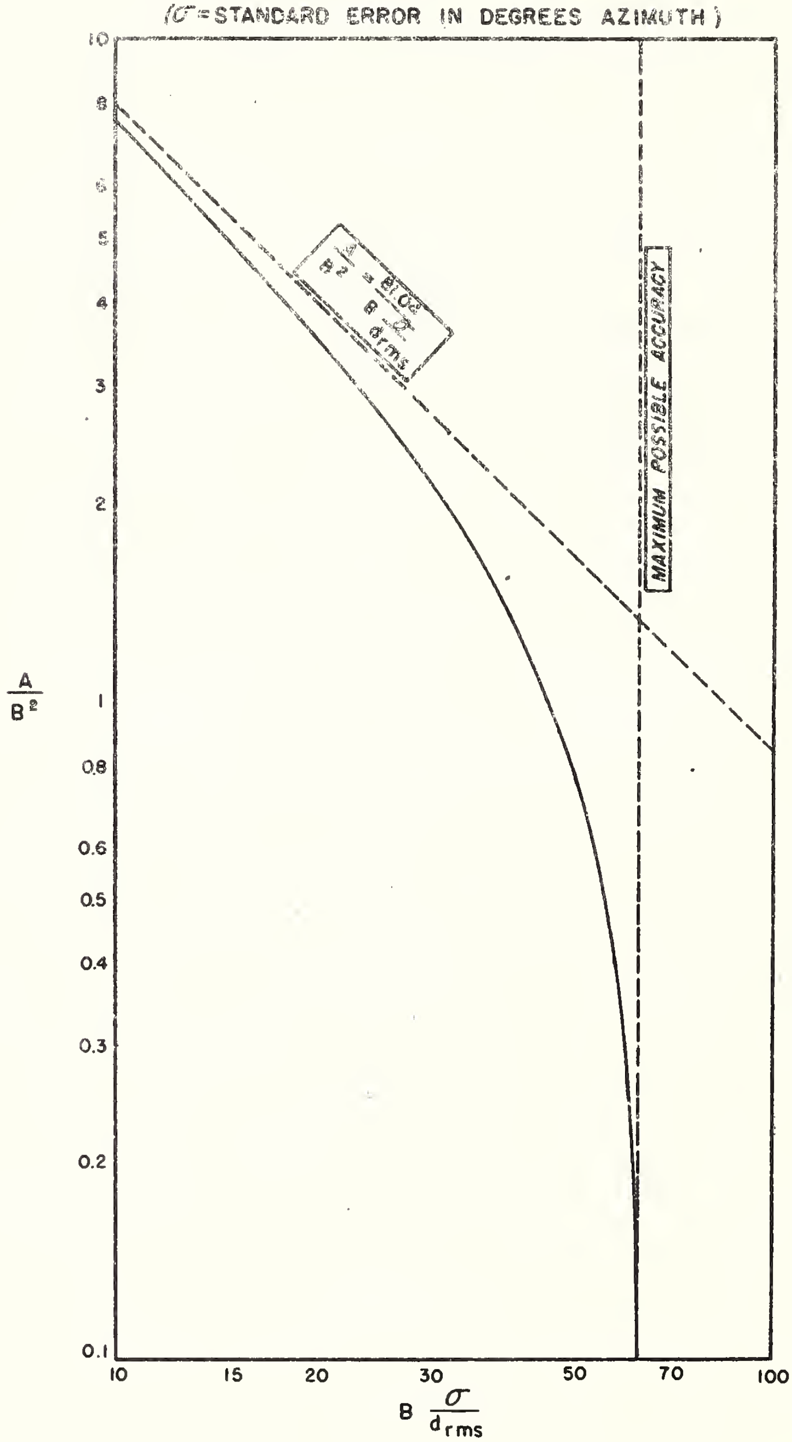

Fig. 6

AZIMUTHAL TYPE NAVIGATION SYSTEM USING TWO FIXED STATIONS

AREA WITHIN CONTOURS OF CONSTANT ACCURACY VERSUS BASELINE LENGTH KEEPING $\frac{\sigma}{d_{rms}}$ CONSTANT

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

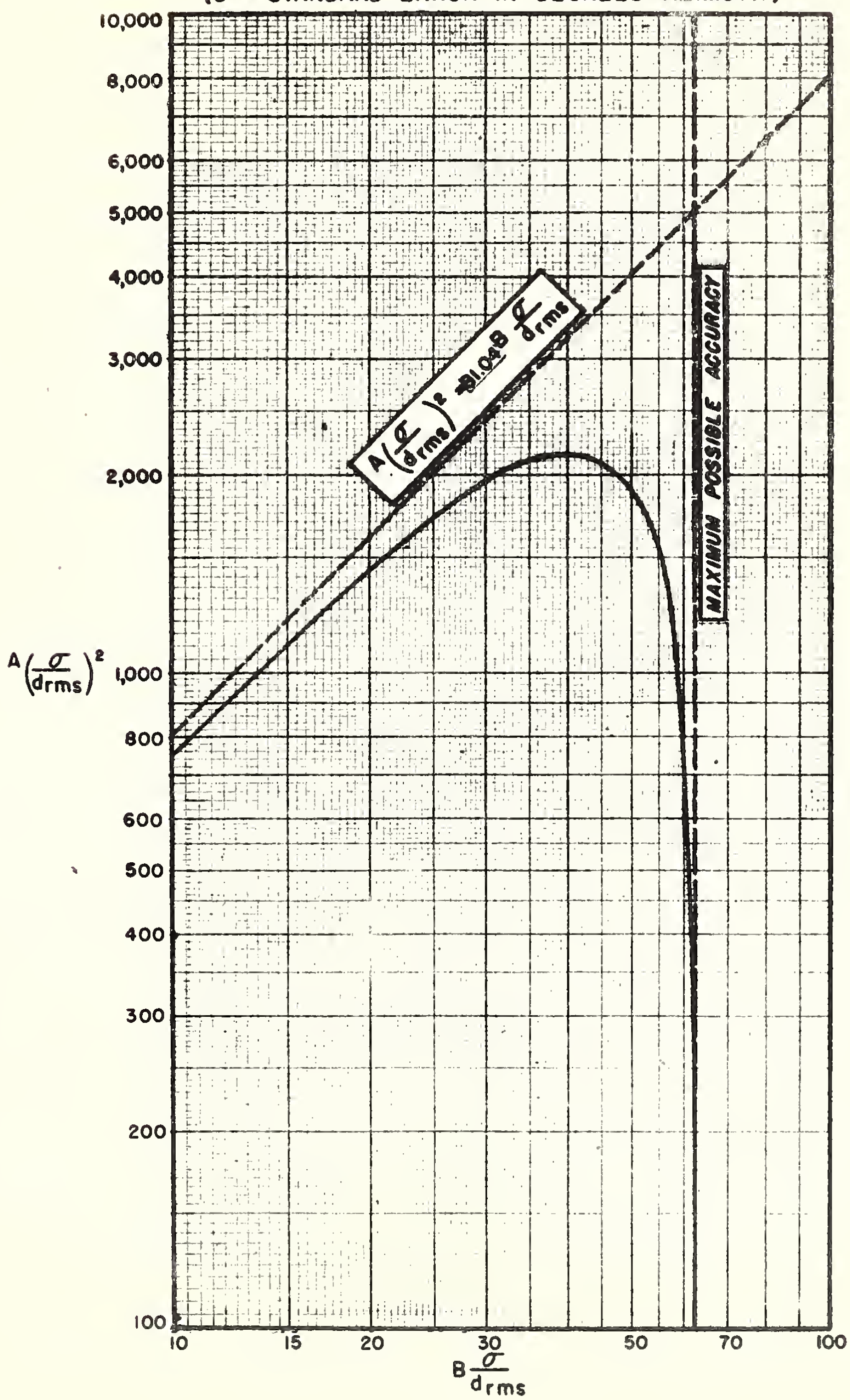

Fig. 7

AZIMUTHAL TYPE NAVIGATION SYSTEM
USING THREE FIXED STATIONS S_1, S_2 AND S_3

SHOWING CONTOURS OF CONSTANT ROOT MEAN SQUARE DISTANCE ERROR IN TERMS OF σ AND BASELINE LENGTH FOR A 60° BASELINE ANGLE

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

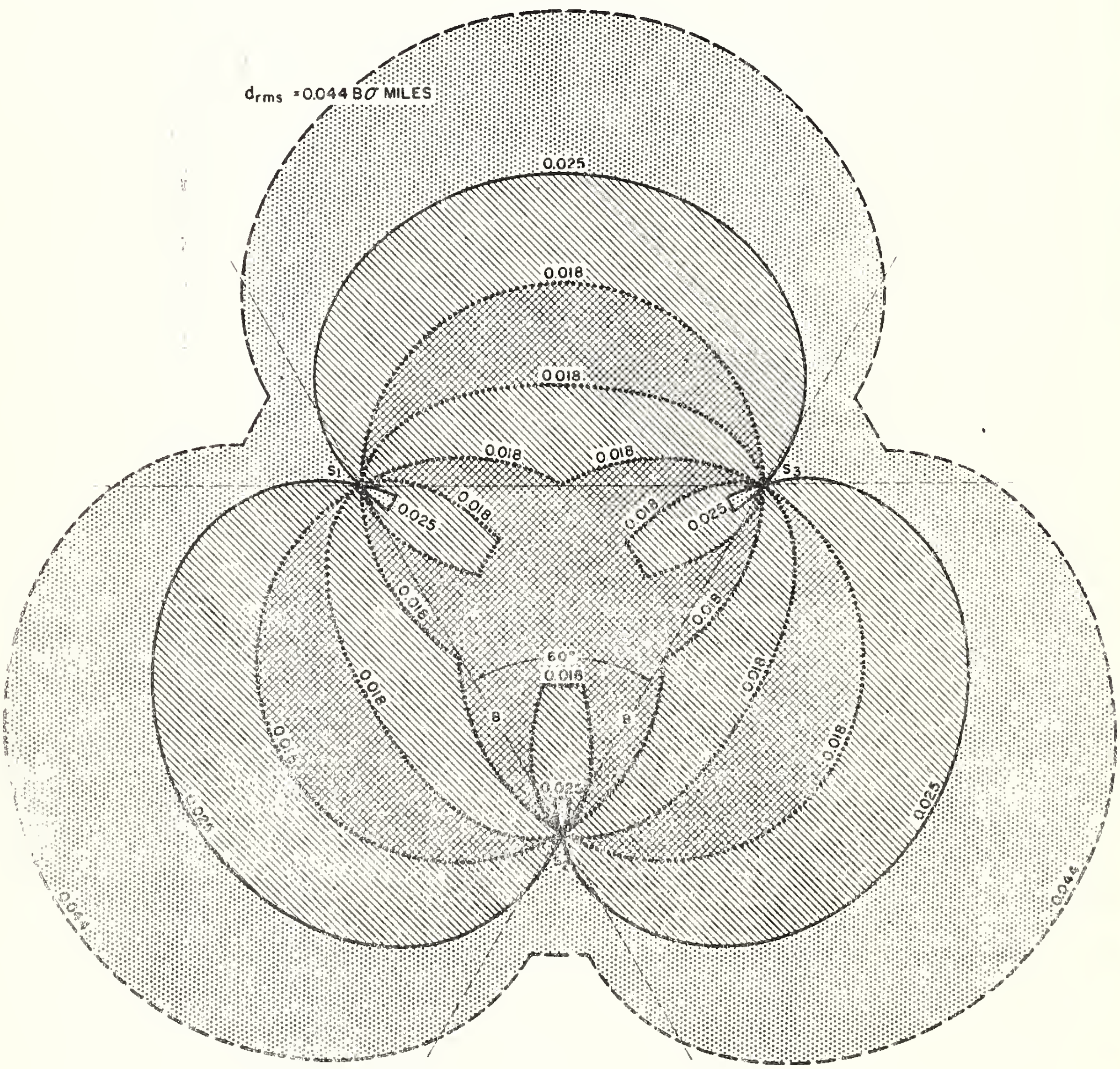

Fig. 8

AZIMUTHAL TYPE NAVIGATION SYSTEM
USING THREE FIXED STATIONS S_1 S_2 AND S_3

SHOWING CONTOURS OF CONSTANT ROOT MEAN SQUARE DISTANCE ERROR IN TERMS OF σ AND BASELINE LENGTH FOR A 90° BASELINE ANGLE

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

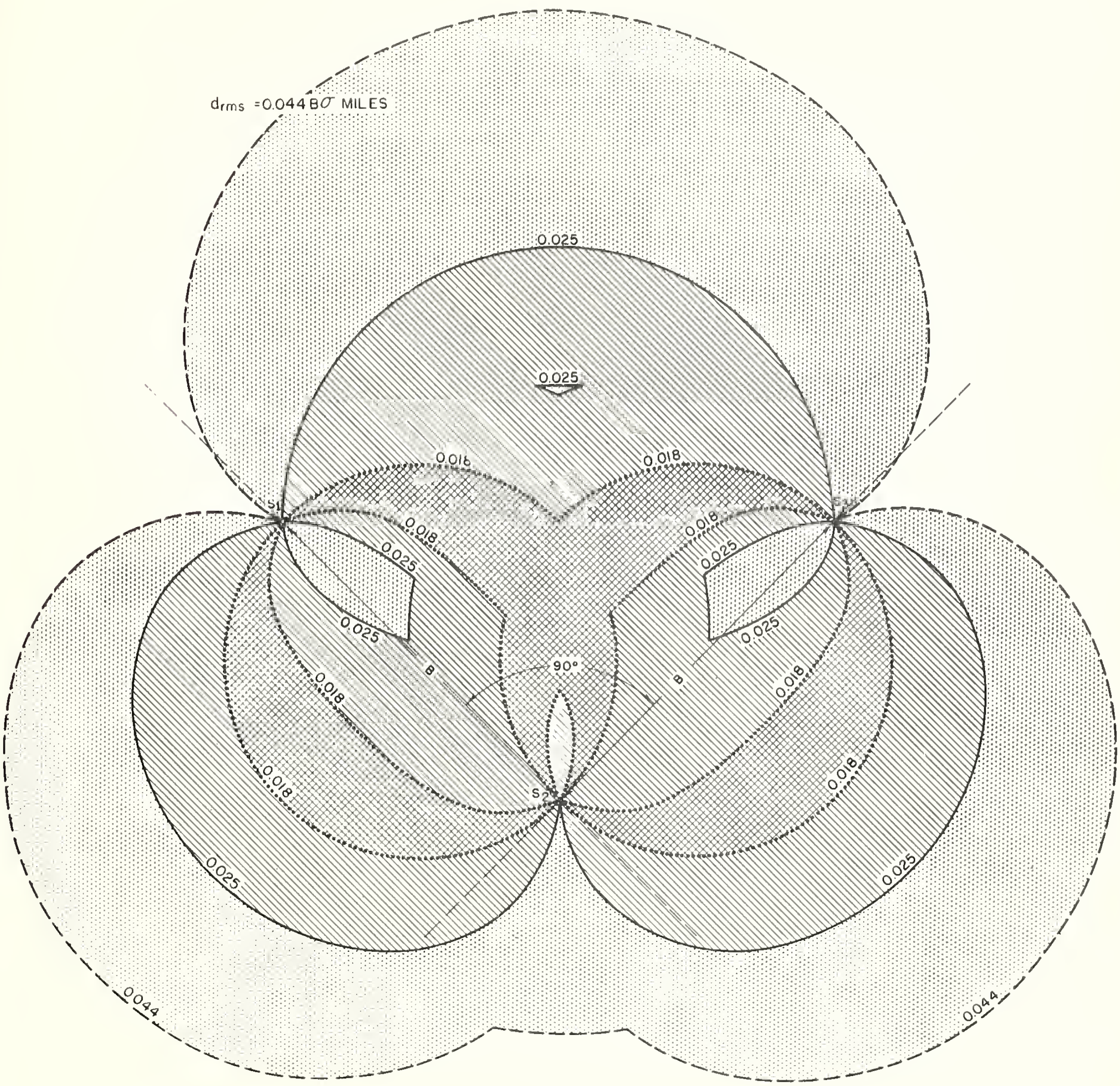

Fig 9

AZIMUTHAL TYPE NAVIGATION SYSTEM USING THREE FIXED STATIONS S_1, S_2 AND S_3

SHOWING CONTOURS OF CONSTANT ROOT MEAN SQUARE DISTANCE ERROR IN TERMS OF σ AND BASELINE LENGTH FOR A 135° BASELINE ANGLE

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

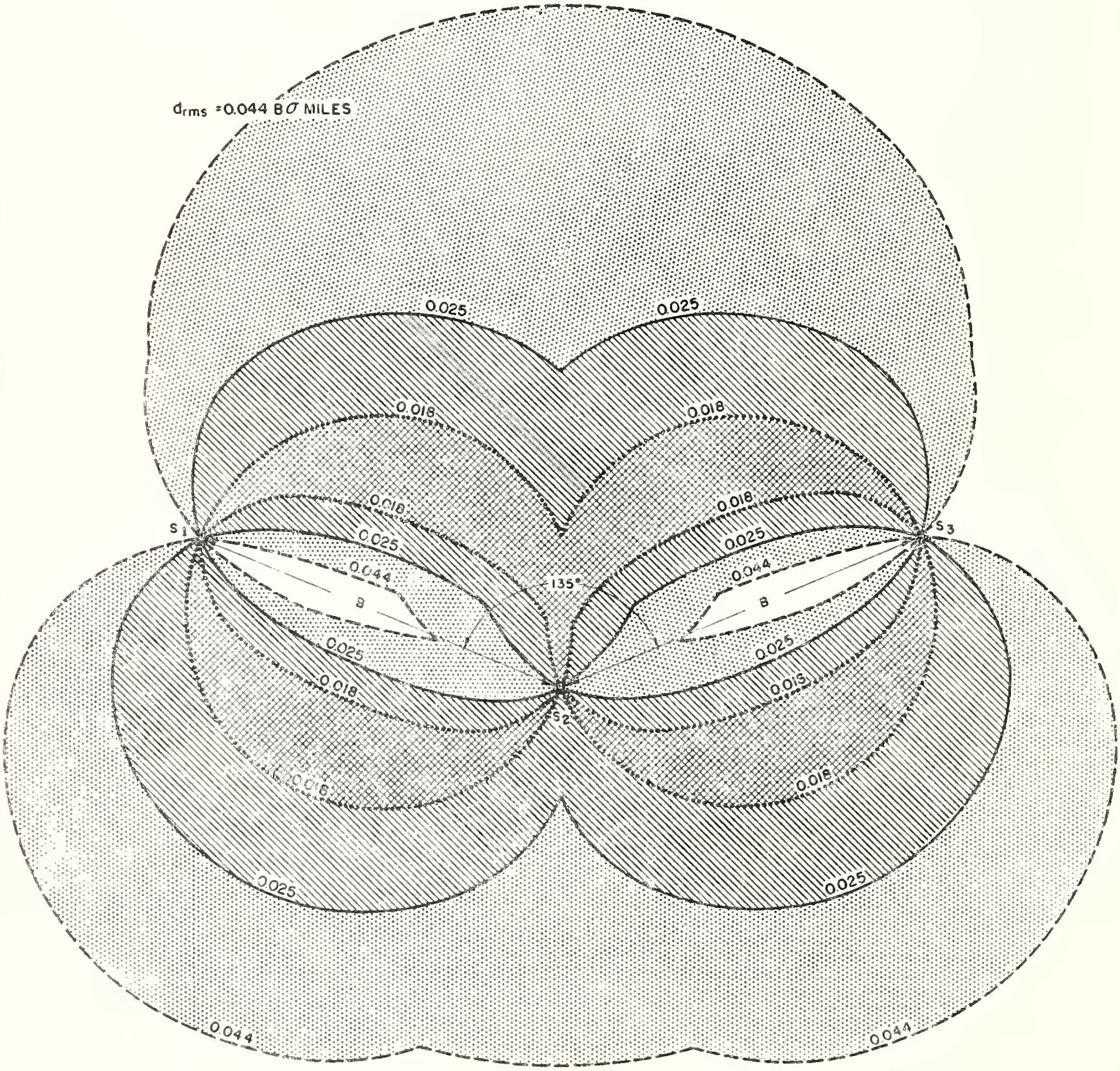

Fig.10

AZIMUTHAL TYPE NAVIGATION SYSTEM
USING THREE FIXED STATIONS S_1, S_2 AND S_3

SHOWING CONTOURS OF CONSTANT ROOT MEAN SQUARE DISTANCE ERROR IN TERMS OF σ AND BASELINE LENGTH FOR A 180° BASELINE ANGLE

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

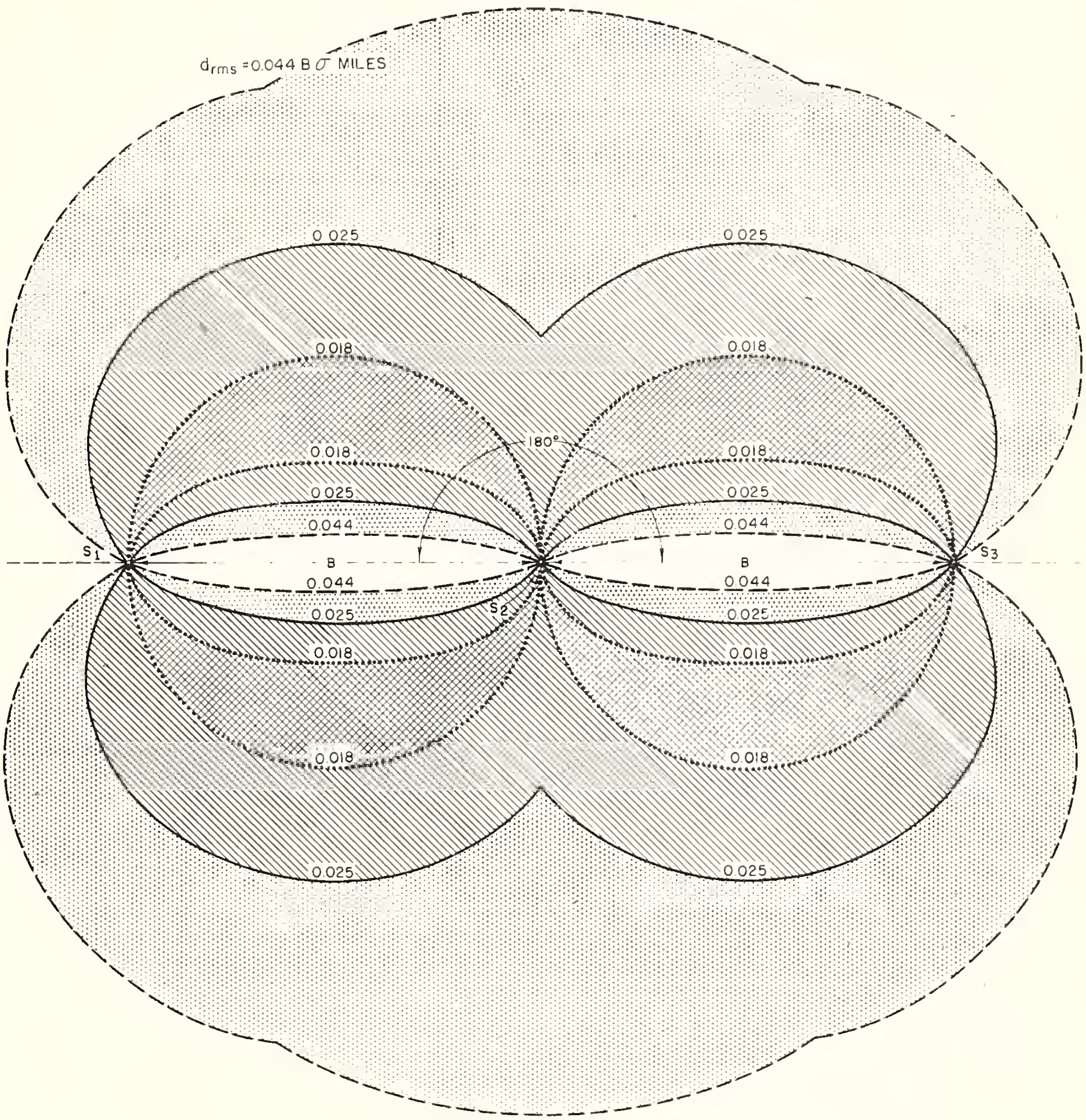

Fig 11

AZIMUTHAL TYPE NAVIGATION SYSTEM USING THREE FIXED STATIONS

AREA WITHIN CONTOURS OF CONSTANT ACCURACY VERSUS BASELINE LENGTH KEEPING $\frac{\sigma}{d_{rms}}$ CONSTANT

CURVES ARE FOR BASELINE ANGLES AS LABELED

(σ = STANDARD ERROR IN DEGREES AZIMUTH)

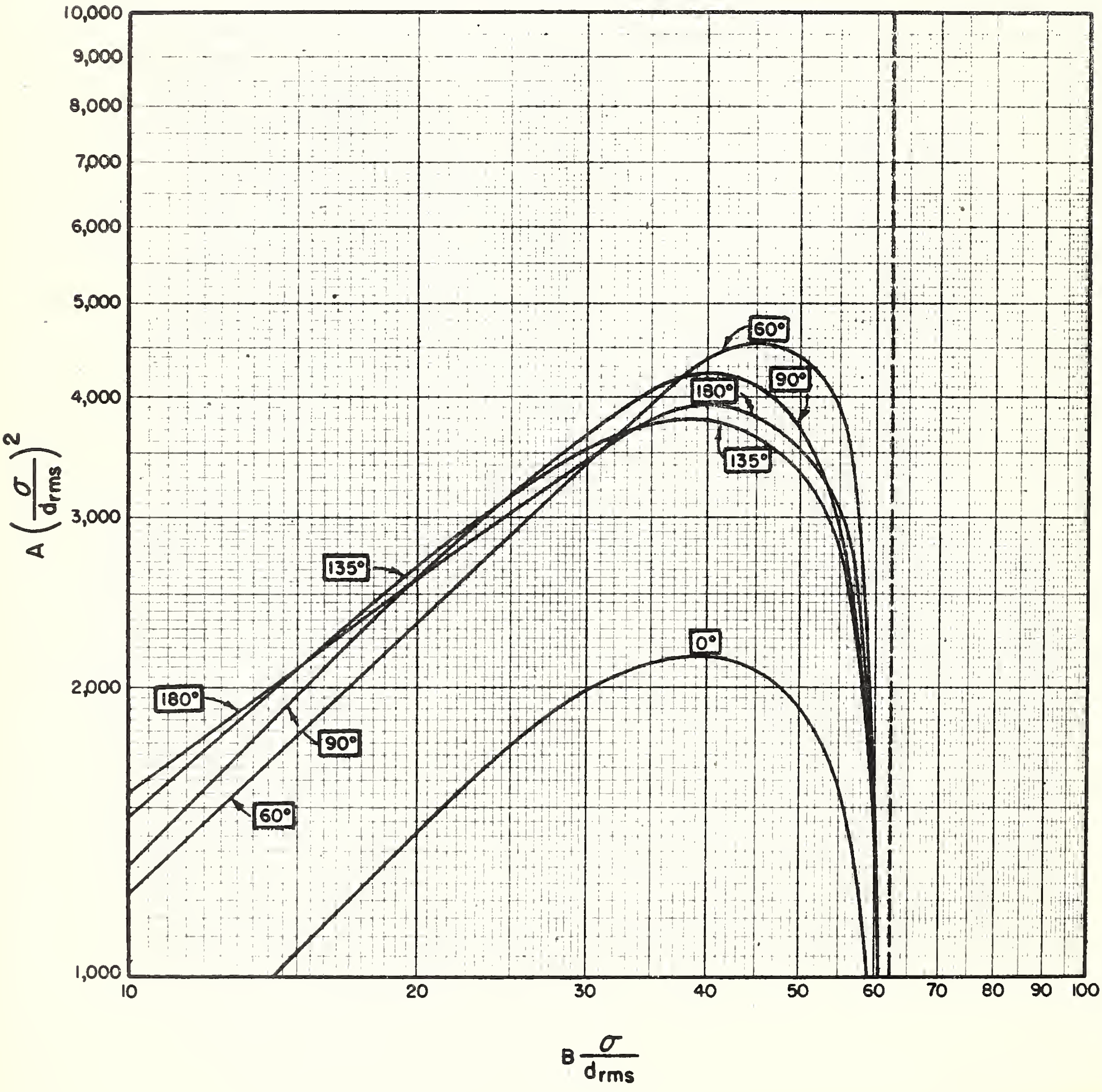

Fig.12

HYPERBOLIC TYPE NAVIGATION SYSTEM

SHOWING HYPERBOLIC POSITION LINES

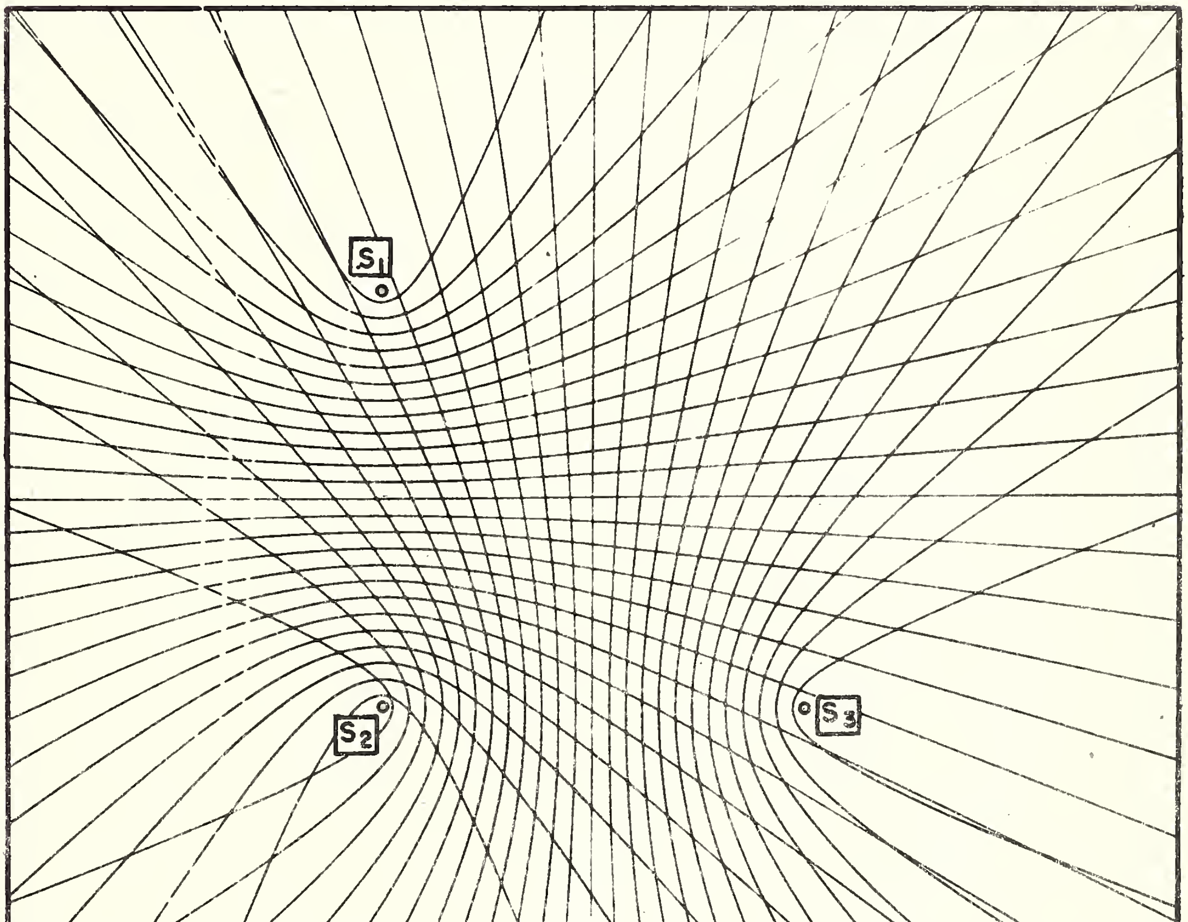

Fig. 13

HYPERBOLIC NAVIGATION SYSTEM

CONTOURS OF CONSTANT $\frac{d_{rms}}{\sigma}$

BASELINE ANGLE = 60°

(σ = STANDARD ERROR IN MICROSECONDS)

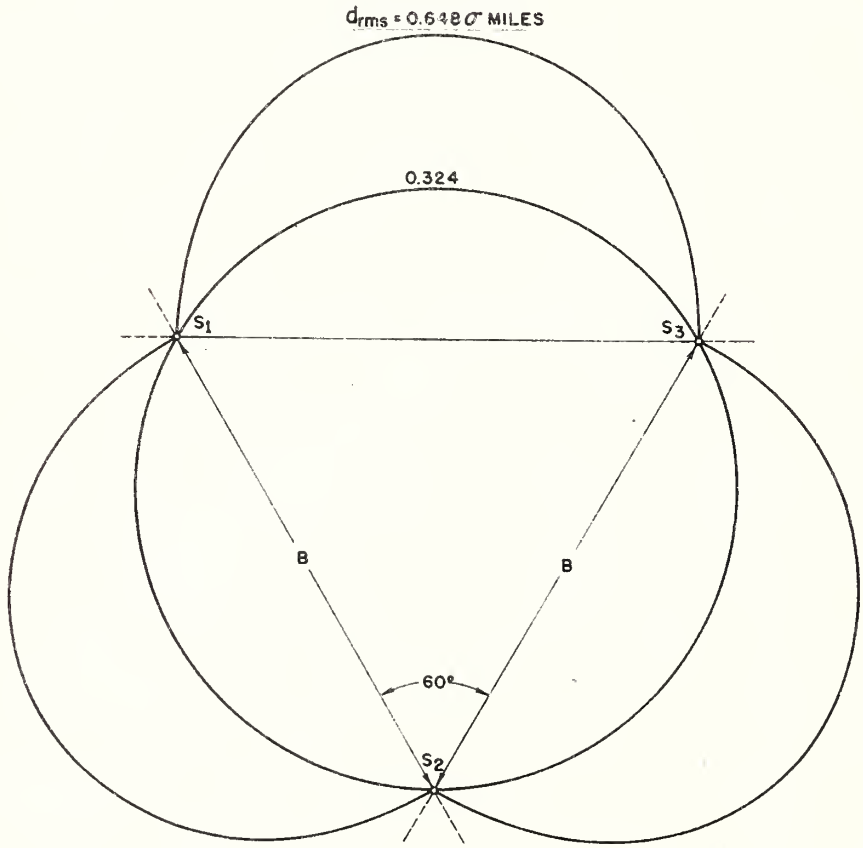

Fig. 14

HYPERBOLIC NAVIGATION SYSTEM

CONTOURS OF CONSTANT $\frac{d_{rms}}{\sigma}$

BASELINE ANGLE = 90°

(σ = STANDARD ERROR IN MICROSECONDS)

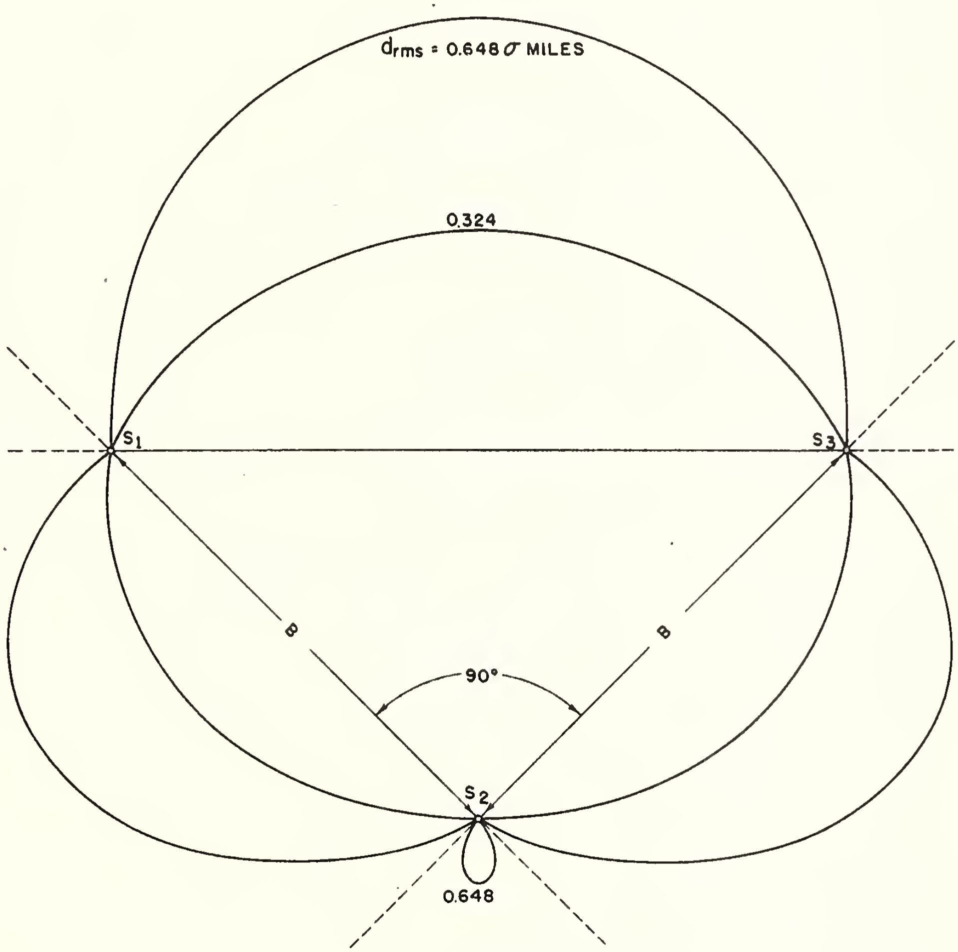

Fig. 15

HYPERBOLIC NAVIGATION SYSTEM

CONTOURS OF CONSTANT $\frac{d_{rms}}{\sigma}$

BASELINE ANGLE = 135°

(σ = STANDARD ERROR IN MICROSECONDS)

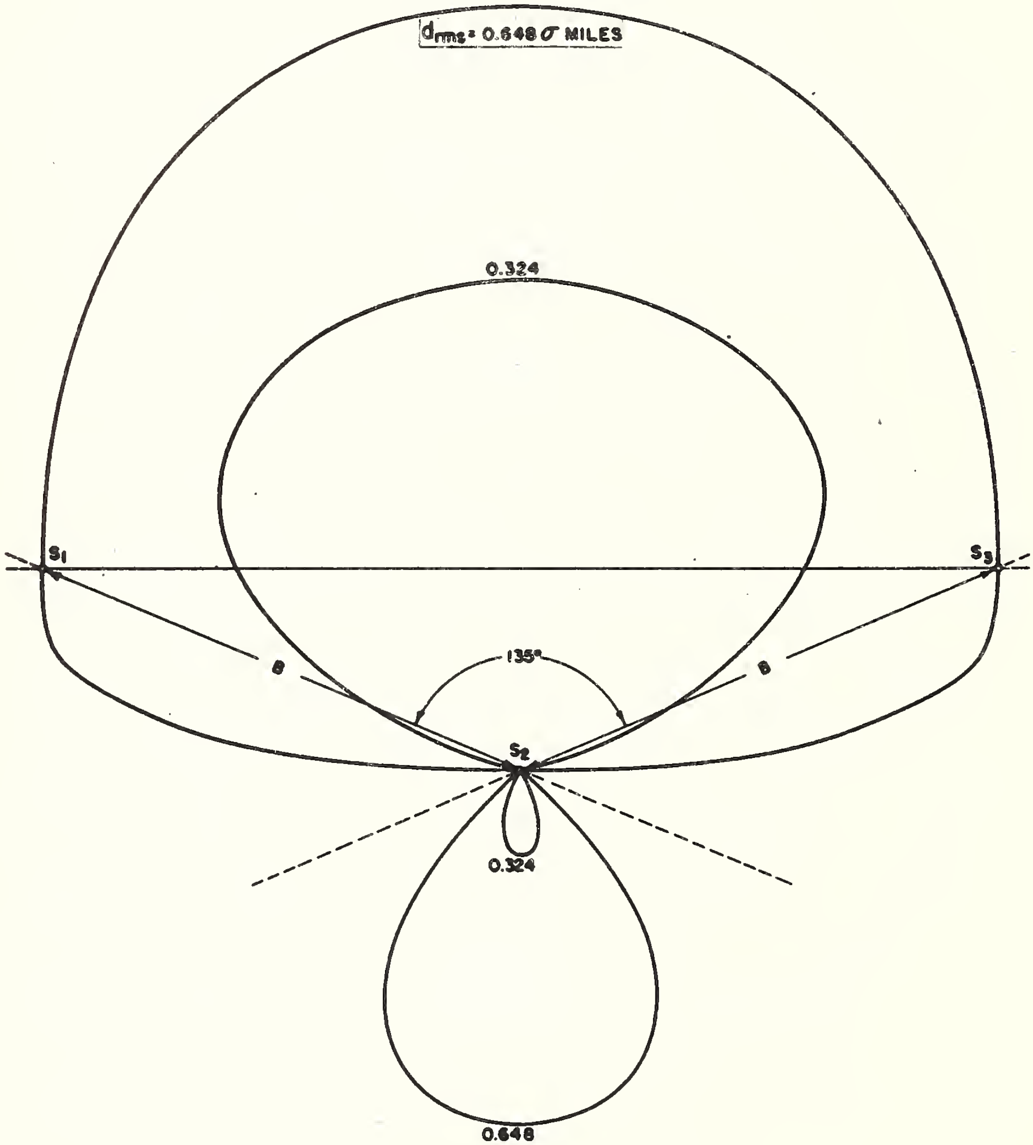

Fig. 16

HYPERBOLIC NAVIGATION SYSTEM

CONTOURS OF CONSTANT $\frac{d_{rms}}{\sigma}$

BASELINE ANGLE = 180°

(σ = STANDARD ERROR IN MICROSECONDS)

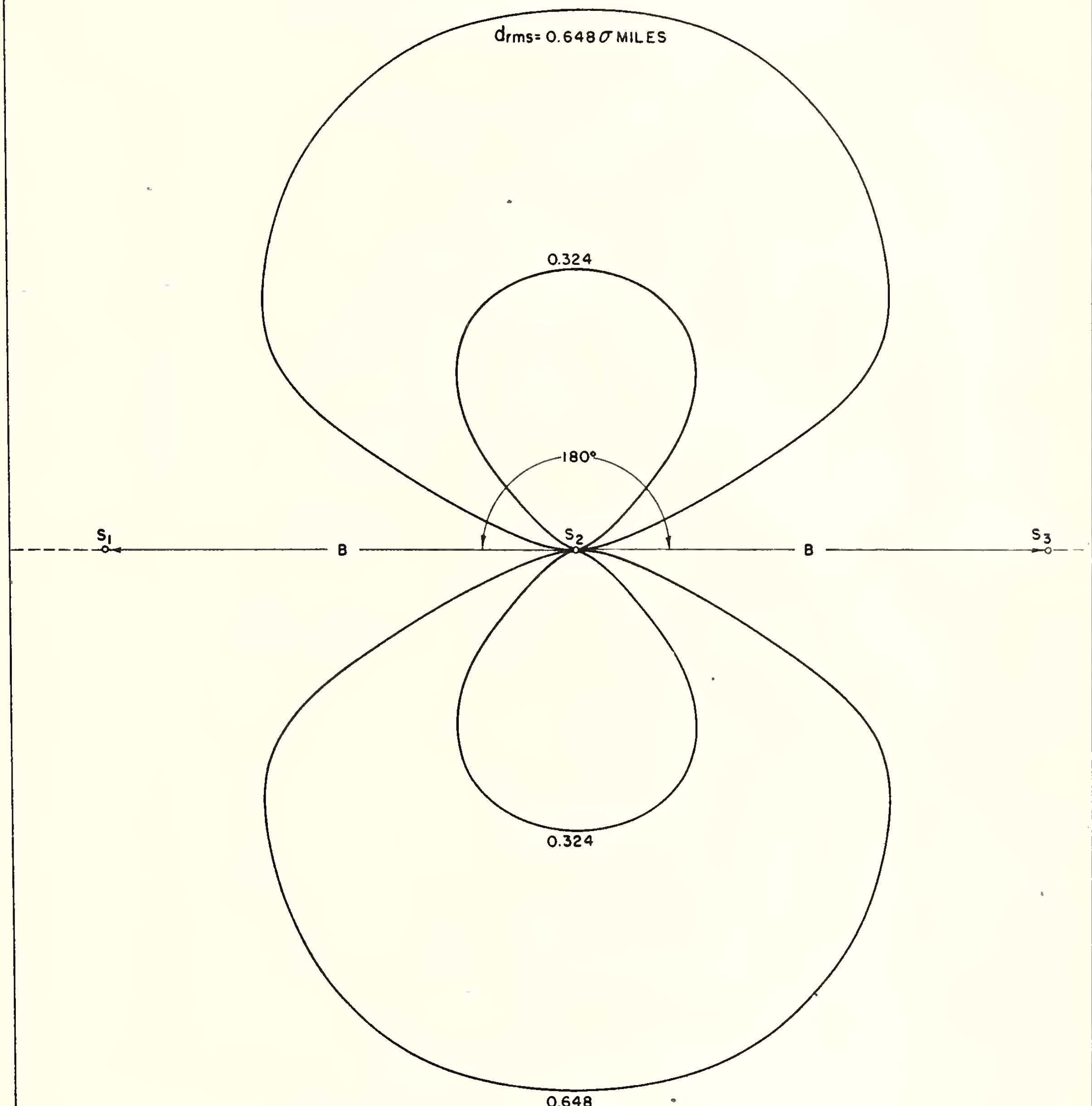

Fig. 17

HYPERBOLIC TYPE NAVIGATION SYSTEM
USING THREE FIXED STATIONS
AREA IN SQUARE BASELINE UNITS
VERSUS
$\frac{\sigma}{d_{rms}}$
FOR SEVERAL BASELINE ANGLES AS LABELED

(σ = STANDARD ERROR IN MICROSECONDS)

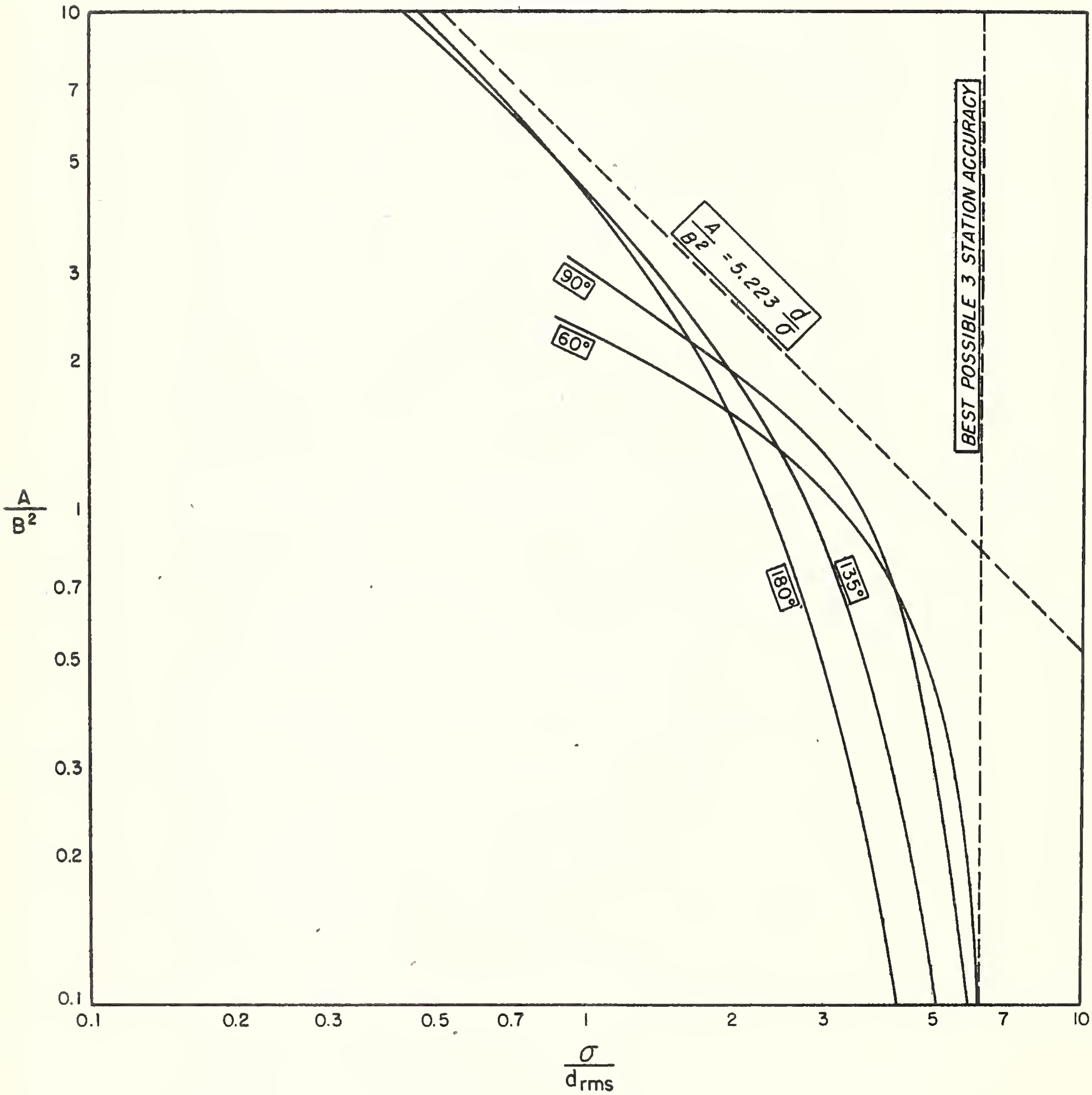

Fig.18

HYPERBOLIC NAVIGATION SYSTEM

AREA IN SQUARE BASELINE UNITS

VERSUS

$$\frac{\sigma}{d_{rms}}$$

(σ = STANDARD ERROR IN MICROSECONDS)

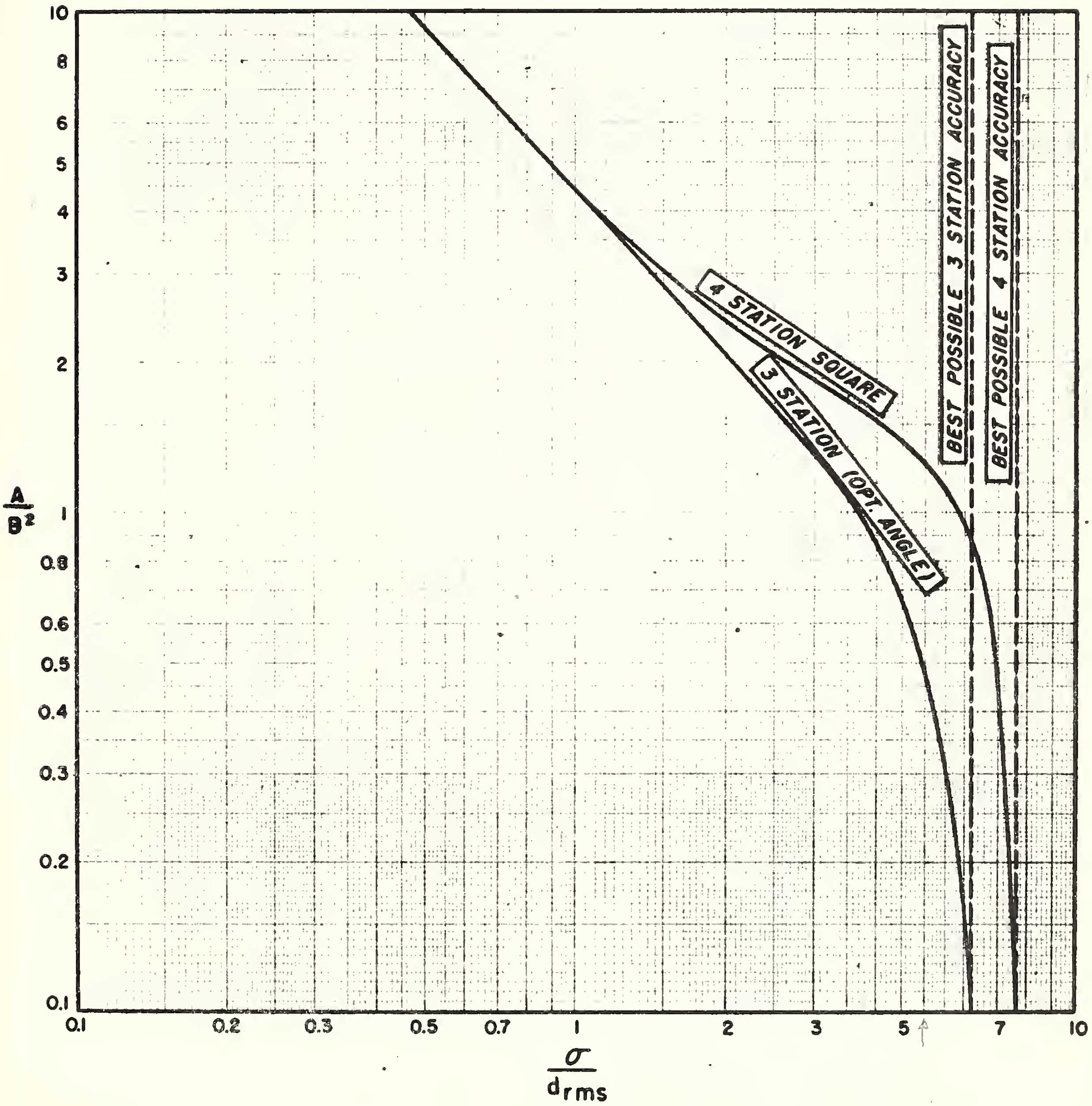

Fig. 19

CONTOURS OF CONSTANT ROOT MEAN SQUARE DISTANCE ERROR
FOR A FOUR STATION HYPERBOLIC TYPE NAVIGATION SYSTEM
WITH TRANSMITTERS LOCATED ON THE FOUR CORNERS OF A SQUARE

(σ = STANDARD ERROR IN MICROSECONDS)

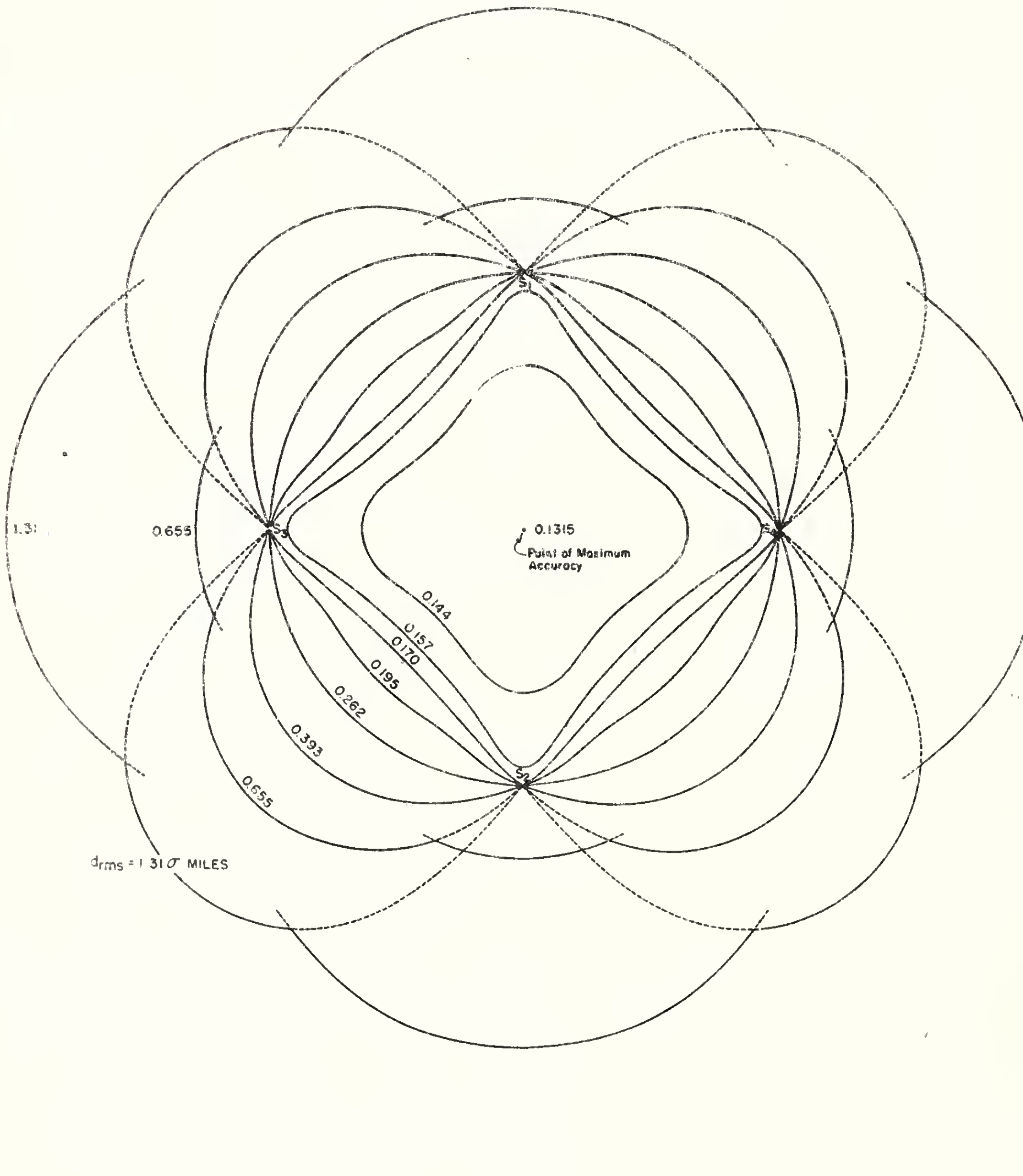

Fig. 20