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PREDICTION OF ANNUAL SUNSPOT NUMBERS

BY A. G. McNISH AND J. V. LINCOLN

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1. Purpose

Since discovery of the sunspot cycle by Schwabe in 1851, considerable interest has been exhibited by a number of investigators in forecasting the trend of future cycles. The methods of prediction previously used have covered a wide range, including harmonic analysis, assumption that cycles repeat after a number of years, and numerous empirical relationships between heights of maxima, rate of rise, etc. Until recently these predictions were largely of academic interest, but with the discovery of the close relationships which exist between radio propagation conditions and sunspot activity the prediction of sunspot numbers has assumed great practical importance.

It would be desirable to predict the ionospheric conditions directly, but that is not yet possible since ionospheric measurements do not extend much beyond one cycle of solar activity---not enough to be regarded as a fair sample of cyclic phenomena in the statistical sense. However, the Zürich relative sunspot numbers constitute a highly homogeneous series extending back to 1849, while less reliable data extend the series back to 1749. Since the correlation is very high between ionospheric phenomena, for the interval during which they have been observed, and relative sunspot numbers for the same interval, conclusions reached from study of the sunspot data may be assumed to apply to the ionospheric phenomena.

The various formulas which have been developed for prediction of the sunspot cycle lead to widely different predictions. The originators of the various prediction methods do not give satisfactory estimates of the reliability of their formulas for the future.

2. Method

To develop a satisfactory sunspot prediction formula for use in calculating radio frequencies for future use the following assumptions were made at this laboratory, following a method developed by Wiener: (1) In a time series exhibiting cyclic tendencies an estimate, to a first approximation, of a future value in the series is the mean of all past values for the same part of the cycle, and (2) this estimate can be improved by adding to the mean a correction proportional to the departures of earlier values of the same cycle from their respective means, the factors of proportionality being determined by the method of least squares. The prediction formula then becomes

$$R'_n = \bar{R}_n + \Delta R'_n = \bar{R}_n + k_{n-1}\,\Delta R_{n-1} + k_{n-2}\,\Delta R_{n-2} \cdots \quad (1)$$

in which R'_n is the value to be predicted in a particular cycle, $\bar{R}_n$ the mean of all the nth values in preceding cycles, ΔR_{n-1} the departure of the particular R_{n-i} from $\bar{R}_{n-i}$, and k_{n-i} is a prediction coefficient to be calculated. The value of the k_{n-i}'s are adjusted so that

$$\sum_{1}^{N} (\Delta R_n - \Delta R'_n)^2_N$$

is a minimum when the summation is taken over N cycles of past data.

Substituting for the value of $\Delta R'_n$ in the above expression its value as given in (1), differentiating with respect to each k_{n-i} and equating to zero, we derive the array of symmetrical simultaneous equations which must be solved to obtain the best value of each k_{n-i}. They are

$$\begin{aligned} S_{n-1,\, n-1}\, k_{n-1} + S_{n-1,\, n-2}\, k_{n-2} + \cdots S_{n-1,\, n-i}\, k_{n-i} &= S_{n,\, n-1} \\ + S_{n-2,\, n-2}\, k_{n-2} + \cdots S_{n-2,\, n-i}\, k_{n-i} &= S_{n,\, n-2} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ S_{n-i,\, n-i}\, k_{n-i} &= S_{n,\, n-i} \end{aligned}$$

in which $S_{n-j,\, n-i} = \sum_{1}^{N} (\Delta R_{n-j}\ \Delta R_{n-i})_N$

An array of equations is formed for each year of the cycle and as many coefficients as are desired may be calculated. By arbitrarily setting any or several k_{n-i}'s equal to zero, coefficients may be obtained which apply for predictions from particular years or any combination of them. Thus, if it is desired to predict, say, ΔR for the fourth year of the cycle, the coefficients may be derived for a prediction based on ΔR's for the first, second and third years, the third year alone, the second year alone or any other combination. Naturally the coefficient for predicting from any given year will be different if other years are also used in the prediction.

Calculation of several k_{n-i}'s was performed for each year of the sunspot cycle except the zero year. Data employed for the computations were the smoothed 12-month means of Zürich relative sunspot numbers from 1834 to 1943. For reasons to be set forth later, data prior to 1834 were rejected. First, the beginning of each cycle to the nearest tenth of year was selected, each cycle being assumed to begin at a point of minimum sunspot number obtained by passing a smooth curve through the twelve-month running averages. The times of minimum so selected for the entire series back to 1755 are shown in Table 1 compared with times given by Zürich, selected for being the lowest twelve-month average value for each cycle. The differences, where they exist, are slight.

Having selected the minimum, the twelve-month means were then ordered for the successive years after minimum for all the cycles and their averages for the last 10 cycles were obtained. Departures of the twelve-month averages from corresponding values for the mean cycle were then derived for the first, second, third year, etc., after the minimum for each cycle. These departures are shown in Table 2, together with the departures for the earlier cycles from the same mean cycle. These are the values of ΔR_n.

Values calculated for $S_{n-i,\ n-j}$ are given in Table 3. Since $(n-1)-i = n-(i + 1)$ etc. the number of values of S required for computing a given number of k's is relatively small. From the values in Table 3, 130 k's were computed on the assumptions that $\Delta R_n'$ would be calculated (1) from the immediately preceding value of ΔR, (2) from several successive preceding values of ΔR, and (3) any combination of preceding values of ΔR from which certain values of ΔR were neglected. In the case of the fifth year of the cycle values of k_{n-i} were computed for all values of i up to and including 5. By arbitrarily setting certain k_{n-i}'s equal to zero, alternate values of the other k_{n-i}'s were computed which were applicable for computing ΔR_n from specific preceding values of ΔR. Thus for the fifth year of the cycle, fifteen prediction formulas were obtained based on prediction from the preceding year, the two preceding years, up to the five preceding years as well as prediction from the fifth, fourth, or any other preceding year alone or any combination of these years. The coefficients are given in Table 4.

3. Reliability

Predictions were made for each of the years of the ten cycles in question using all of the combinations of prediction formulas given in Table 4. Differences between observed and predicted values were then formed to test the reliability of the predictions. An estimate of reliability for predictions beyond the observed data requires that the standard deviation $(\sigma = \sqrt{\sum (R' - R)^2/N})$ be multiplied by $\sqrt{(N/\ N-M)}$ in which N is the number of cases and M is the number of parameters used in the prediction. When the prediction is made from the mean alone, that is, by assigning to R_n' the value obtained for the nth year by averaging all of the cycles, $M = 1$; when the value of R_n' is predicted by correcting this value by the departure for the preceding year, $M = 2$; and if the prediction is from the five preceding years (as it may be when $n = 5$), $M = 6$. These estimates of the values of σ for each year of the cycle are given in Table 5.

When these values of σ are examined, it is apparent that for most of the years of the cycle the "best" prediction is obtained by correcting the mean value by the departure from mean of the preceding year. However, there are a number of cases in which the "best" prediction is obtained by using special combinations of years. One might naively assume that improved predictions would result by selecting for each year of the cycle that particular group of coefficients which gave rise to the lowest value of σ in the past cycles. This would be fallacious, because, owing to statistical considerations, it is likely that some particular system may "work best" for a limited number of cases. The system which "works best" on the average should be adopted.

From the foregoing considerations it was decided to employ only the mean, corrected by the departure of the preceding year, for prediction of future sunspot numbers. For most practical considerations this is satisfactory for forecasts of propagation conditions. If more advanced predictions are desired, the prediction may be based on the mean, corrected for the departure two years preceding the year to be predicted.

Since the prediction coefficients are subject to statistical fluctuations, it was decided to plot them against the year of the cycle for which they apply and to adopt prediction coefficients from a smoothed curve through the values. By interpolation along this smoothed curve, prediction coefficients may be obtained for non-integral times after the beginning of a cycle, Fig. 1.

4. Auto-correlation in the Data

The calculations performed for obtaining the prediction coefficients may be employed for studying statistical characteristics of sunspot cycle. A somewhat idealized representation of the data is shown in Fig. 2A obtained by spacing the minima shown in Table 1 at 11-year intervals and plotting the observed annual value at these minima and for the 10 succeeding years. Deviations of these observed annual values from the "mean cycle" obtained from the data of the last 10 cycles are shown in Fig. 2B. The conservatism of the series is conspicuous from the tendency of adjacent annual deviations to have the same sign and similar magnitudes, the tendency upon which this system of prediction is based. It is also clear that this tendency decreases as the separation in time of the deviations in any given cycle is increased, so that the further in advance a prediction is made the greater is its uncertainty. Prediction of the later years of a cycle from the early years seems unwarranted.

These tendencies find numerical expression in the mean auto-correlation coefficients of the series which may be calculated from the S-values by the formula

$$r_i = \sum_{1}^{N-i} \frac{S_{n-o,\, n-i}}{\sqrt{S_{n-o,\, n-o}\, S_{n-j,\, n-i}}} \Big/ (N-i) \qquad (j = i)$$

These values are plotted in Fig. 3 for values of i up to 3. It is interesting to note that if the correlation coefficients between each pair of adjacent years in the cycle are multiplied by the ratios of the sunspot numbers of the later year to the earlier year in the same pair, a series of prediction coefficients is obtained which agree closely with those computed above from the ΔR's alone.

The foregoing remarks apply in particular to the series in Fig. 2 subsequent to 1834 which were used in the calculations. It may be noted that the deviations as shown in Fig. 2B prior to 1834 are somewhat greater than those subsequent to 1834. There is good reason for believing that the earlier observations are subject to considerable uncertainty because of absence of a uniform system of observations and reductions from one observer to another. Also the data subsequent to 1834 are substantiated by regular measurements of geomagnetic fluctuations, initiated by Gauss and others at that time, which show a high correlation with sunspots.

5. Test for Validity of Data

It is of interest to inquire whether any statistical basis for rejecting the earlier observations can be established. In order to increase the size of the working sample all 11 years of each of the later 10 cycles were normalized by expressing the deviations from the mean cycle in terms of the standard deviation for that year, thus supplying a sample of 11 x 10 = 110 individuals. Frequencies of occurrence of these deviations in 6 class intervals were tested by the chi-squared test, to ascertain the probability that they arose from a normal distribution (degrees of freedom 6-3 = 3). Neglecting the effect of auto-correlation, there was only 1 chance in 1000 that the normalized deviations came from a normal distribution. (The distribution could not be expected to be normal since the sunspot numbers have a lower bound of zero.)

The deviations were then fitted by inspection to a type VI Pearsonian curve (see Fig. 4), and the chi-squared test (chi-squared = 3.37) showed a probability of 0.19 that the hypothesis of a VI Pearsonian distribution was valid, effects of auto-correlation again being neglected (in this case the degrees of freedom were 6-4 = 2). The observations prior to 1834 were normalized, using the σ from the later series, and tested to ascertain the probability that they came from the same set as the later observations. Chi-squared in this case was 117. In this case the degrees of freedom were 6-1 = 5, the mean, standard deviation, and skewness having been determined by the data from the later series. Values of chi-squared as great as 117 for 5 degrees of freedom do not appear in the tables; there is practically zero probability that the deviation obtained for the earlier years of the series came from the same set as those of the later years.

The above calculations were performed on the assumption that there are 110 <u>independent</u> observations in the later series and 77 independent observations in the earlier series. A difficulty arises in taking into account the equivalent number of independent observations; certainly the numbers cannot be as great as 110 and 77, nor can they be less than 10 and 7. Fortunately it is not necessary to decide on the exact equivalent number of independent observations. Assuming the extreme case that there were 10 and 7 independent observations, the probability that the later series comes from a VI Pearsonian distribution becomes about 0.85 while the probability that the earlier series comes from the same distribution lies between 0.01 and 0.001. This is equivalent to saying that if we observe sunspot numbers for the next 10,000 years, we would not expect to encounter a series of 7 cycles which differed as much from the last 10 cycles as the 7 preceding them did. Since the first auto-correlation coefficient is 0.73, we may assume that about $\frac{1}{4}$ of each observation is independent, and that therefore the number of independent observations in the later and earlier series is 110/4 and 77/4 respectively. These give values of chi-squared equal to 0.34 and 29 which indicate probabilities of about 0.65 and less than 0.001 for the two hypotheses.

For the reasons stated above the earlier data were not employed in deriving the prediction formulas. However, if they had been included, no outstanding change in the coefficients would have resulted but the estimate of reliability of the predictions would have been decreased.

6. Reliability of Empirical Methods

One of the faults of many of the methods for predicting the sunspot cycle is that no reliability estimate of the accuracy of the predictions is supplied, a criticism which, it is felt, cannot be made against the method set forth here. Estimation of reliability for certain types of formulas is very difficult. Undoubtedly many formulas can be derived which, when applied to past cycles, give satisfactory predictions, for it is always possible with a sufficiently large number of degrees of freedom to fit satisfactorily any finite group of events to any desired accuracy. In estimating the reliability of future predictions due allowance must be made for the degrees of freedom involved in the prediction formula.

A more subtile consideration is involved in estimating the reliability of a prediction based on an "empirical" formula. Empirical formulas are usually obtained by trying various relationships and selecting those which "worked" best in the past. If a large number of relationships were tried it would not be surprising to find that some of them worked very well but allowance must be made for the number of relationships examined. The number implicitly examined but rejected by simple inspection of the data is ordinarily much greater than the number explicitly examined and consequently is difficult to take into account in assessing reliability. Such considerations do not apply if an *a priori* prediction formula is assumed, even though the parameters of the formula are obtained empirically from past data. But in estimating the reliability of the prediction, allowance must be made for the number of parameters in the formula as was done in computing the standard deviations of the predictions given in Table 5.

7. Application to Present Cycle

These prediction methods have been applied to the first part of cycle 18 as shown in Fig. 5 in which the mean curve for the last 10 cycles, the observed curve to date, and the predicted points for each quarter year are represented. The predictions were made for one year in advance using only one coefficient, namely that to correct for the preceding year. These were interpolated from Fig. 1 for the quarter-years. Thus, the prediction for one year after the minimum was obtained by subtracting from the observed minimum the value of the minimum for the average cycle (7.5 - 5.0 = 2.5). This quantity was multiplied by the coefficient relating the zero year to the first year (2.5 X 1.2 = 3.0, see Fig. 1) and added to the value for the first year in the average cycle giving the predicted value (3.0 + 15.0 = 18.0) etc. Agreement with the observed curve was extremely satisfactory for the first six quarterly predictions, but rapid increases in the sunspot numbers during the last few months have thrown the predictions into disagreement for the last available quarter-year. However, the difference between observed and predicted values is only 17.5 units while the standard deviation of the predictions is 11.8 units. There is one chance in six (approximately) that a value will exceed the standard deviation by this amount.

The highest point predicted by this method has the value 121. The limits of its reliability may be stated as follows: Probability that difference (X) of the observed and predicted values lies within the ranges $X > +20$ units, 0.08; $+20 > X > +10$, 0.12; $+10 > X > 0$, 0.23; $0 > X > -10$, 0.32; $-10 > X > -20$, 0.23; $X > -20$, 0.03. Two points are plotted on the curve for more than a year in advance; these indicate that the maximum should occur during the latter half of 1947.

8. Comparison with Stewart's Method

Plotted in Fig. 5 are also shown a series of predictions (interpolated in part) by Prof. John Q. Stewart, communicated to this laboratory in a note dated June 14, 1946. Prof. Stewart's explanation of the method of prediction is as follows: "A formula for representing smoothed spot numbers during a single cycle has been described in previous papers (J. Q. Stewart and H. A. Panofsky, Ap. J. 88, pp. 385-407, 1938; J. Q. Stewart and F. C. Eggleston, Ap. J. 91, pp. 72-82, 1940.). When the last previous cycle, number 17 had just reached maximum, at the end of 1937, Stewart and Eggleston (J. Q. Stewart and F. C. Eggleston, Physical Review 55, 1102, 1939; and elsewhere.), fitted a 4-parameter curve to the smoothed monthly numbers and made a prediction of the remaining course of that cycle, 1938-1944. This prediction was successful within the announced tolerance.

"Besides describing the 4-parameter representation (parameters s, a, b, F), Stewart and Panofsky gave correlations (p.402, equations (21), (22) of b and F, respectively with a, permitting reduction to 2 parameters, namely s and a, when the data do not warrant more careful fitting. At the time of writing, monthly numbers for the present cycle are available only through October, 1946. The maximum is still a year ahead, and the limited part of the curve yet observed is not enough to establish a really reliable prediction. Nevertheless, it is of interest to make the best one we can at so early a stage in the cycle. The solution will improve with every added month.

"Mr. George S. Baldwin, Jr., of Brookline, Massachusetts, a Princeton undergraduate (class of 1949), has computed and plotted the family sunspot curves for different values of a, which follow when the corresponding values of b and F given by the correlations mentioned above are adopted. To the same scale monthly numbers since 1943 were plotted on tracing paper. By sliding the observed curve in the time coordinate, a preliminary value of a for a fit was found, and also of s, the time of start of cycle 18. Then the fit was improved by a more detailed comparison with carefully smoothed monthly numbers.

"The result --- which it must be emphasized again is only a tentative fit for cycle 18 --- is s = 1943.9, time of start;

$\underline{a} = 4.2$;
$\underline{y} = 1948.0$, time of maximum;
$\underline{V} = 133$, height of maximum

The computed curve reached spot number 50 at 1945.8, and is due to reach 100 at 1946.7 and 125 at 1947.3. The time of maximum is subject to less uncertainty than its height V. The predicted 133 would mean the highest sunspot activity since 1870. If spots during 1946 increase faster than the prediction, the maximum will be higher and will come a tenth-year or more earlier; if the spots increase more slowly, the maximum will be a little later and lower.

"The values of $\underline{b}$ and $\underline{F}$ indicated by the correlation already mentioned and used in the prediction are: $\underline{b} = 1.027$, $\log \underline{F} = 1.375$.

"For convenience the formula for a cycle is repeated here:

$$R = F(r - s)^a \, e^{-b(r - s)}$$

where $\underline{r}$ is the date in years and $\underline{R}$ is the computed (smoothed) Wolf number, while $\underline{e}$ is the base of natural logarithms.

"It should be added that the writer has by a detailed statistical study established the existence of an 8-months' period in spot numbers, of amplitude sufficient to account for roughly half the departures of actual monthly numbers from a smooth curve. (This period may not be rigorously simple-harmonic, and the phase may shift at occasional intervals of a number of years.) In the present little study, therefore, the observed monthly numbers were smoothed by an 8-month chain average."

Prof. Stewart's note was in hand when the above-described investigation was begun. The essential difference between the two methods lies in the fact that his method makes no allowance for regression toward the mean cycle. To date Prof. Stewart's predictions are somewhat better than those derived by the more elaborate method described in this report but it may be expected that as the present sunspot cycle develops the law of averages will operate in favor of the statistical method.

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Table 1. Years of sunspot minima for cycles 1-18

Cycle no.	Selected for this study	Selected by Zürich
1	1755.8	1755.2
2	1766.5	1766.5
3	1775.6	1775.5
4	1784.6	1784.7
5	1798.8	1798.3
6	1810.6	1810.6
7	1823.2	1823.3
8	1834.1	1833.9
9	1843.6	1843.5
10	1856.3	1856.0
11	1867.3	1867.2
12	1878.8	1878.9
13	1889.5	1889.6
14	1901.8	1901.7
15	1913.4	1913.6
16	1923.6	1923.6
17	1933.8	1933.8
18	1944.2	1944.2

Table 2. Deviations of observed values of smoothed twelve-month running average of Zürich relative sunspot numbers from mean value of cycles 8-17

Cycle	Year after minimum (n)											
	0	1	2	3	4	5	6	7	8	9	10	11
1	4.4	-0.9	-9.4	-34.1	-32.0	-19.5	-13.2	-0.8	-4.5	-3.6	3.1	32.2
2	6.2	21.4	17.4	25.4	9.6	-4.5	-2.1	-13.4	-1.4	-15.4	5.6	76.1
3	2.7	5.8	48.0	71.1	32.4	-0.3	-3.7	-11.6	-11.1	-12.7	12.6	75.5
4	4.9	10.4	37.0	51.5	38.3	31.3	18.7	17.0	26.7	23.8	27.2	9.8
5	0.2	-8.4	-31.6	-43.1	-46.6	-43.1	-21.3	-10.6	-9.9	-14.6	-3.6	-10.3
6	-5.0	-13.1	-45.5	-67.6	-77.8	-50.6	-23.7	-8.0	-3.0	0.8	3.0	-5.8
7	-4.9	-7.8	-35.2	-53.7	-42.6	-22.7	-6.5	16.0	26.3	10.8	-0.3	-3.8
8	2.8	12.7	49.5	62.0	30.0	-6.7	10.8	-0.2	-6.7	-4.4	-0.9	18.4
9	5.6	0.7	-9.4	-18.1	10.8	37.9	22.7	17.4	30.9	30.4	25.2	8.9
10	-1.1	1.8	-1.5	7.2	3.8	-3.3	-6.1	-3.8	13.3	12.6	5.8	-6.3
11	0.2	11.8	19.4	43.5	28.8	14.7	6.3	0.0	-12.8	-9.8	-0.1	-6.9
12	-2.6	-2.7	-10.6	-21.3	-35.2	-13.3	-15.7	-1.6	-16.6	-9.6	-7.0	-5.8
13	1.3	-8.0	-15.4	-9.7	-7.3	-14.5	-6.3	-6.0	-6.7	5.0	-0.6	-1.7
14	-1.4	-7.1	-20.4	-30.3	-32.1	-23.9	-19.6	4.2	2.1	-8.6	-8.9	-8.1
15	-3.4	-7.6	-4.7	-26.1	4.6	0.7	-2.5	-10.4	-6.0	-7.7	-5.9	3.5
16	0.6	1.9	-3.0	-16.4	-22.5	-9.1	-5.1	-15.4	-12.2	-11.4	-7.6	-2.1
17	-1.6	-3.2	-3.6	9.6	19.3	17.3	15.7	15.6	14.4	3.0	-0.2	0.5
Mean Cycles 8-17	5.0	15.0	50.0	80.7	91.6	86.3	69.9	49.0	33.3	22.6	12.8	11.5

Table 3. Cross product summation terms ($S_{n-i,\ n-j}$)

i ($i \geq j$)	n-j (= n when j = 0)											
	0	1	2	3	4	5	6	7	8	9	10	11
0	65.34	497.57	3726.95	86[illegible]3.90	5083.16	3049.22	1657.92	962.12	2057.49	1587.69	890.68	624.12
1	-	78.52	1186.49	5202.83	5172.10	2502.20	1985.58	708.26	1038.96	1598.14	1116.31	280.79
2	-	-	147.53	1787.71	3053.28	959.46	2361.47	1083.98	1032.27	733.64	1089.90	293.17
3	-	-	-	221.75	977.12	415.79	1777.87	732.52	1607.63	818.23	569.32	258.74
4	-	-	-	-	213.01	352.59	-	-	-	-	-	-
5	-	-	-	-	-	213.72	-	-	-	-	-	-

Table 4. Prediction coefficients, (k_{n-i}), for values of annual sunspot numbers for the nth year of the cycle based on years 1843-1944.

$(R_n = \bar{R}_n + k_{n-1}\ \Delta R_{n-1} + k_{n-2}\ \Delta R_{n-2} + \dots)$ (coefficients in each column constitute a set)

i	n = 1	n = 2			n = 3						n = 4									
1	1.2	0.0*	2.5	2.4	0.0*	0.0*	1.0	1.0	0.0*	1.4	0.0*	0.0*	0.0*	0.8	0.7	0.7	0.6	0.0*	0.0*	0.0*
2		2.3	-0.8	0.0*	0.0*	3.8	1.2	1.1	3.6	0.0*	0.0*	0.0*	0.9	0.1	0.0	-0.1	0.0*	0.8	0.0*	0.8
3					3.4	-1.1	-0.4	0.0*	0.0*	0.0*	0.0*	1.8	-0.4	-1.3	-0.8	0.0*	0.0*	0.0	2.0	0.0*
4											3.3	1.1	1.8	2.1	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*

i	n = 5														
1	0.0*	0.0*	0.0*	0.0*	1.0	1.1	1.0	1.0	0.5	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*
2	0.0*	0.0*	0.0*	0.2	-0.6	-0.7	-0.4	-0.5	0.0*	0.1	0.0*	0.3	0.0*	0.0*	0.1
3	0.0*	0.0*	-0.4	-0.5	-0.6	-0.6	-0.2	0.0*	0.0*	-0.6	-0.5	-0.3	0.0*	0.1	0.0*
4	0.0*	0.2	1.1	0.9	2.3	2.5	0.0*	0.0*	0.0*	1.7	1.8	0.0*	0.7	0.0*	0.0*
5	3.3	3.0	2.7	2.8	0.6	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*

i	n = 6						n = 7						n = 8					
1	0.0*	0.0*	0.5	0.5	0.0*	0.7	0.0*	0.0*	0.2	0.0	0.0*	0.4	0.0*	0.0*	0.8	0.9	0.0*	1.1
2	0.0*	0.3	0.2	0.2	0.5	0.0*	0.0*	0.4	0.3	0.4	0.4	0.0*	0.0*	0.0	0.0	0.2	0.6	0.0*
3	0.2	-0.2	0.0	0.0*	0.0*	0.0*	0.1	-0.1	-0.1	0.0*	0.0*	0.0*	0.5	0.6	0.3	0.0*	0.0*	0.0*

i	n = 9						n = 10						n = 11					
1	0.0*	0.0*	0.8	0.9	0.0*	0.8	0.0*	0.0*	0.8	0.8	0.0*	0.7	0.0*	0.0*	0.7	0.7	0.0*	0.3
2	0.0*	0.5	-0.2	-0.2	0.8	0.0*	0.0*	0.5	-0.2	-0.1	0.5	0.0*	0.0*	0.3	-0.3	-0.3	0.2	0.0*
3	0.6	0.3	0.1	0.0*	0.0*	0.0*	0.6	0.0	0.2	0.0*	0.0*	0.0*	0.1	-0.1	0.0	0.0*	0.0*	0.0*

* coefficient set equal to zero in computation.

Table 5. Standard deviations of predictions for cycles 8-17 using coefficients, k_{n-1}, of Table 4

$$\sigma = \sqrt{\frac{\Sigma(\Delta R'_n - \Delta R_n)^2}{N - m}}$$

	Years used in prediction						Year predicted following observed minimum (n)										
	Mean	n-1	n-2	n-3	n-4	n-5	1	2	3	4	5	6	7	8	9	10	11
1	x	-	-	-	-	-	7.4	20.4	31.0	23.8	18.4	13.6	10.3	15.1	13.3	9.9	8.3
2	x	x	-	-	-	-	7.1	10.6	13.0	15.7	15.1	6.8	8.9	10.8	6.6	3.6	8.2
3	x	-	x	-	-	-		20.6	16.6	18.0	19.2	8.5	8.6	13.3	11.3	6.3	8.4
4	x	-	-	x	-	-		-	31.3	19.9	19.4	4.0	10.4	12.3	11.6	8.3	8.7
5	x	x	x	-	-	-		11.1	13.2	16.9	12.4	5.3	9.2	11.3	6.9	3.8	8.5
6	x	-	x	x	-	-			17.4	19.0	20.3	12.7	9.1	13.2	11.4	6.7	9.0
7	x	x	x	x	-	-			14.3	21.2	13.3	5.7	9.5	11.7	7.3	3.8	9.3
8	x	-	-	-	x	-				23.4	18.7						
9	x	-	-	-	-	x				-	17.1						
10	x	-	-	x	x	-				21.0	19.3						
11	x	-	-	-	x	x				-	18.2						
12	x	-	x	x	x	-				20.1	20.7						
13	x	-	-	x	x	x				-	19.3						
14	x	x	x	x	x	-				18.5	8.8						
15	x	-	x	x	x	x					21.0						
16	x	x	x	x	x	x					9.6						

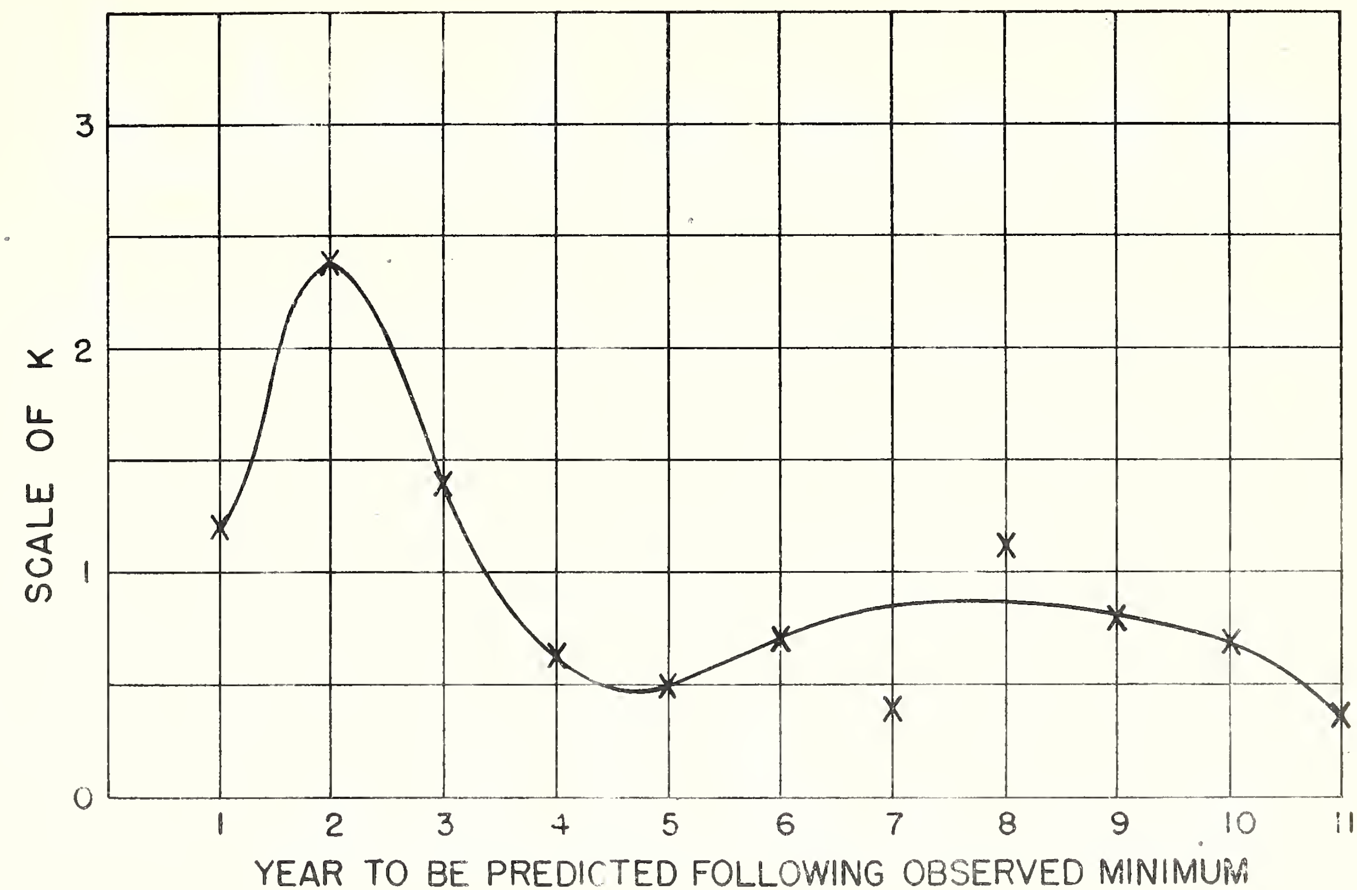


Fig. I. VALUE OF PREDICTION COEFFICIENTS, k, USING ONLY PRECEDING YEAR'S DATA FOR PREDICTING DEPARTURE ($\Delta R_n = k \Delta R_{n-1}$).

Fig. 2A. ELEVEN YEAR SERIES OF ANNUAL SUNSPOT NUMBERS 1834-1944 (A) AND DEVIATION FROM MEAN CYCLE (B)

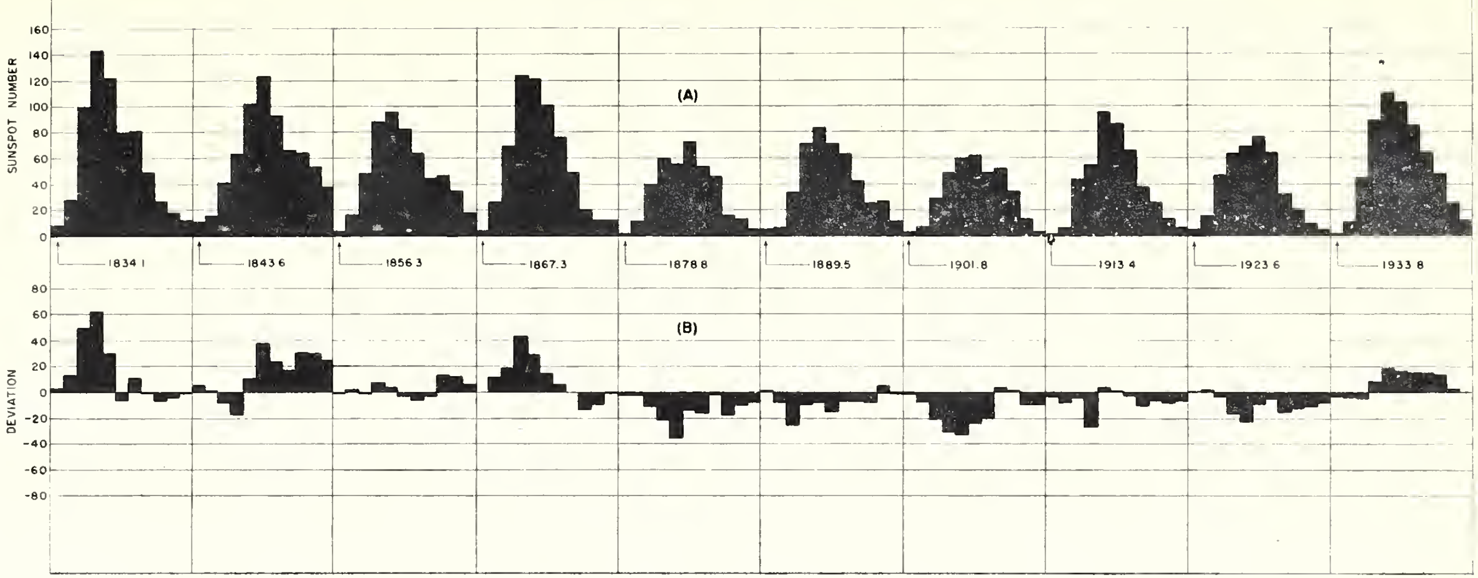
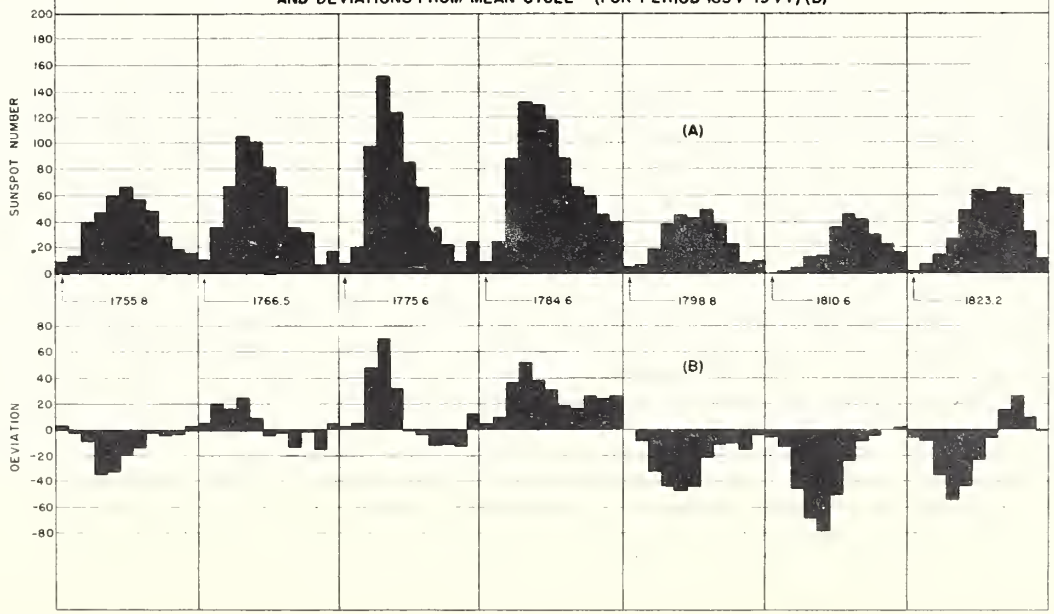


Fig. 2B. ELEVEN YEAR SERIES OF ANNUAL SUNSPOT NUMBERS 1755-1834 (A) AND DEVIATIONS FROM MEAN CYCLE (FOR PERIOD 1834-1944) (B)



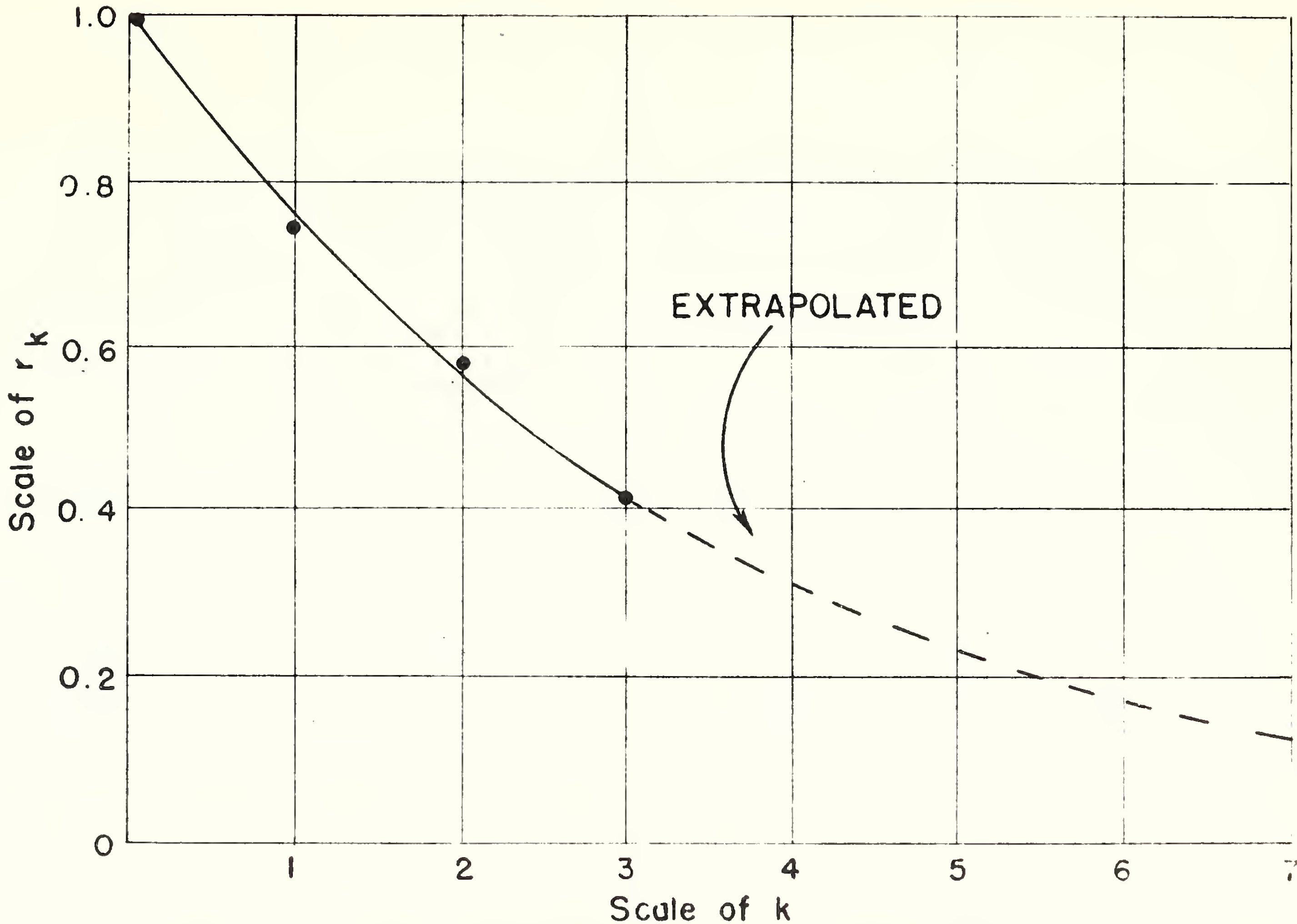


Fig. 3. CORRELOGRAM SHOWING AUTO-CORRELATION COEFFICIENTS (r_k).

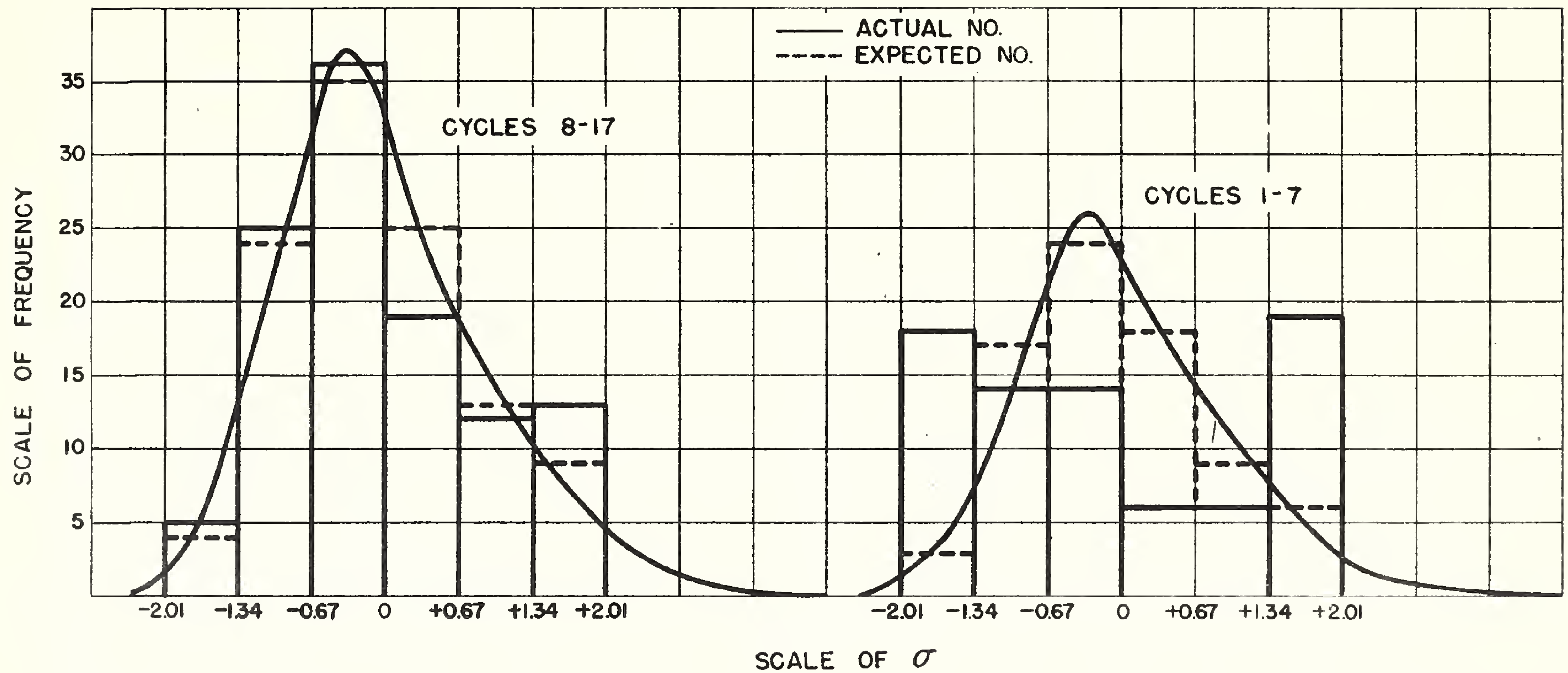


Fig. 4. PEARSON VI CURVE FIT TO YEARLY DEVIATIONS FROM MEAN CURVE FOR CYCLES 8-17 AND CYCLES 1-7.

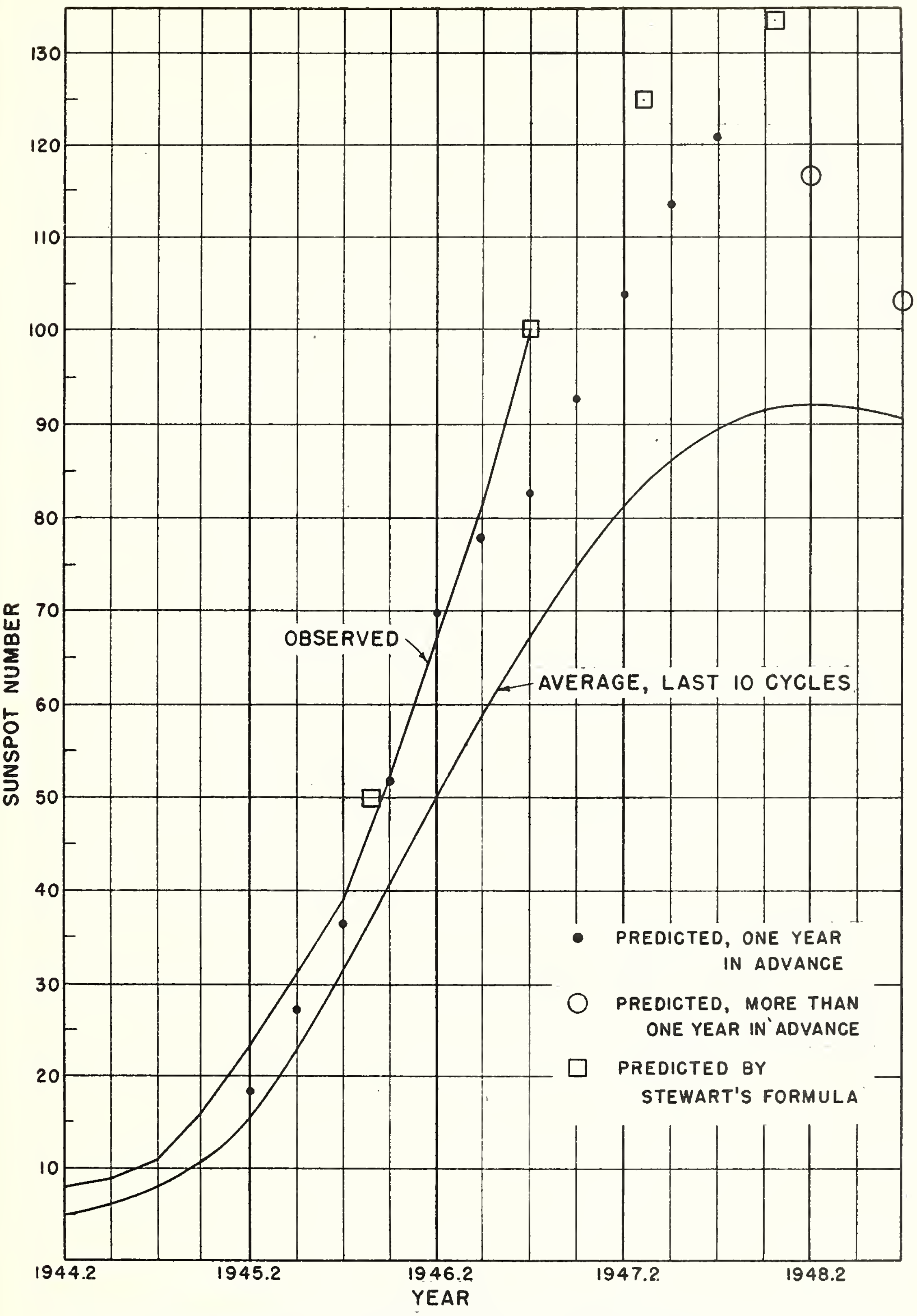


Fig. 5. COMPARISON OF OBSERVATIONS AND PREDICTIONS FOR CYCLE 18.