NBS CIRCULAR 598

Techniques for Accurate Measurement of Antenna Gain

UNITED STATES DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

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Techniques for Accurate Measurement of Antenna Gain

H. V. Cottony

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Preface

The subjects of antennas and the measurement of their performance have been of steadily growing importance in recent years. Although the principles of measurements are well known, in carrying out the measurements one encounters many practical difficulties which are likely to introduce errors in the results unless suitable precautions are observed. In describing the results of their antenna measurements most authors mention their techniques very briefly, if at all. As a result, the accuracy of published figures may sometimes be in doubt.

Considerable experimental work on antennas has been carried on at the National Bureau of Standards, particularly in connection with high-gain antennas for ionospheric scatter applications. During this work, in order to improve the reliability of the measurements, techniques for minimizing the experimental error were developed. The results of measurements using these techniques were found to be consistent and accurate. This Circular describes these techniques.

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A. V. Astin, *Director.*

Contents

Techniques for Accurate Measurement of Antenna Gain

H. V. Cottony

Comparison of published results of experimental antenna measurements, particularly gains, reveals apparent discrepancies of the order of one or more decibels. Experimental work at the National Bureau of Standards on scaled model antennas for long-range VHF communication via ionospheric scatter revealed some sources of difficulties and led to the adoption of special precautions resulting in significantly more consistent and, it is believed, more accurate results. The procedures are based on the comparison method [1],[1] but include, in addition to the standard antenna, the use of a third antenna here designated as the reference antenna. To obtain more accurate measurements it was found essential to correct for the standing wave pattern in the field set up, presumably, by reflections from the irregularities in the terrain. Special features of instrumentation, including methods for minimizing and measuring matching losses, are described. The accuracy of the techniques has been verified by measuring the gain of an antenna, the value of which could be accurately calculated.

1. Introduction

The experimental methods and techniques suitable for work on antennas vary considerably, depending on the frequency at which the measurements are made. The methods described here have been employed primarily in the band of 300 to 400 Mc. The frequency of 400 Mc has been found convenient. Wherever the discussion is illustrated by numerical figures, the frequency of 400 Mc, or a wavelength of 75 cm, is understood. However, with minor modifications such as the use of lumped-constant matching networks instead of tuning stubs, the methods can be and have been employed at frequencies as low as 50 Mc. The normal frequency band for ionospheric scatter communication is approximately 30 to 50 Mc. Antenna measurements at 400 Mc were made possible by recourse to model techniques.

Except for certain applications, such as LF antennas and ship-borne and air-borne antennas, in which the terrain or the body of the ship or plane form an integral part of the radiating system, it is convenient and customary to consider the "free-space" performance of the antenna. Then the effect of the ground can be superimposed upon the performance of the antenna itself.

Because the physical standard of gain for these measurements has been a center-fed, half-wave dipole, the gain figures cited are relative to such a dipole rather than an omnidirectional radiator used in the present IRE definition of gain. The difference is, of course, 2.15 db.

The principle of reciprocity makes it possible to test an antenna as receiving or as transmitting. Both ways have been tried and each found to have its advantages. The method described here employs the antenna as a receiving antenna.

2. Principal Problems

In the interests of simplicity, the principal problems of experimental measurement of antennas may be grouped in five categories:

1. Problems of site selection.
2. Problems created by proximity to the ground.
3. Errors caused by terrain irregularities.
4. Time drift in equipment performance.
5. Other problems.

2.1. Problems of Site Selection

An ideal site for an antenna range should be large, smooth, and free from all obstructions likely to cause serious interfering reflections. The length of the antenna range is dictated by the gain of the antenna to be measured, its configuration, and the accuracy expected for the measurements. As is well known, the wave front should be plane over the effective aperture of the antenna. In case of antennas deriving their gain from end-fire effect such as rhombics or Yagis, the decay of field due to inverse distance effect should be small over the distance occupied by the antenna. In the light of experience in operating the NBS antenna ranges, the latter requirement appears normally to be the more exacting. The longest antenna measured on any of these ranges was a dual rhombic antenna having an over-all length of approximately 80 wavelengths. If the contributions of the forward and rear halves of the rhombic are to be within 0.1 db of each other, the separation between the antenna and the target transmitter (antenna) is to be at least as large as S, given by the relationship:

$$10 \log \left(\frac{S+40\lambda}{S}\right)^4 = 0.1$$

In this instance $S/\lambda \simeq 7{,}000$. The exponent 4 appears because, for grazing angles of arrival, at a constant height above ground, the field intensity varies inversely as the fourth power of the distance to the radiating source.

[1] Figures in brackets indicate the literature references on page 8 of this Circular.

2.2. Problems Created by Proximity to the Ground

When measuring the gains of microwave antennas a common practice is to employ a directional antenna in conjunction with the target transmitter and to raise the antenna being tested sufficiently high off the ground to avoid the interfering ground reflection. This procedure is less practical at the VHF and the lower UHF bands because the antennas are, in general, less directive and the antenna elevations above ground necessary to minimize the reflections from the ground would have to be prohibitively great. A common practice when measuring the gains of an antenna at these lower frequencies is to measure its response to the radiation from a distant target transmitter and to compare it with the response obtained from a half-wave dipole located at the same height above ground as the antenna being measured. Gains obtained in this manner are sometimes referred to as "HDSH" (relative-to-half-wave-dipole-at-same-height). The figures for gain obtained in this manner can be numerically equal to the free-space gain provided certain precautions are observed.

The response of an antenna to radiation from a distant source when measured in proximity to the ground is a vector sum of the free-space responses of the antenna to the direct radiation and to that reflected by the ground. If the direct radiation arrives at an angle to the horizontal which is not insignificant compared to the vertical beam width of the antenna pattern, the response of the antenna to the reflected radiation may be quite different from that arriving directly. If the radiation arrives at a very low (i. e., grazing) angle, then the phase of the two components will differ by very nearly 180 degrees. In such case the net response of the antenna will be equal to the vector sum of the component responses and will be very small compared to each of the latter. Again, even a small change in either the amplitude or the phase of one of the component responses may introduce a significant change in the net response.

Figure 1 presents the relationship existing between the antenna being measured, A, the target transmitter antenna, T, and the ground. The heights of the two antennas above ground are H_A and H_T, respectively. A–O is a horizontal reference line in the plane of the two antennas. The ground at which the reflection takes place is assumed to be plane and smooth so that the reflected radiation may be represented by direct radiation coming from an image of the transmitting antenna, T'. The ground constants are assumed such that, for either plane of polarization, with the angles of reflection involved, the amplitude of the reflected wave is equal to that of the incident and its phase is reversed. The free-space response of the antenna in the direction ω, i. e., the open-terminal voltage developed in response to a unit electric field arriving from the direction ω, is denoted by $R(\omega)$. If the direction A–T is denoted as Ω, and the direction A–T,′ by $\Omega+\Delta\Omega$, the net response at the terminals of the antenna may be denoted by V_A given by the relationship:

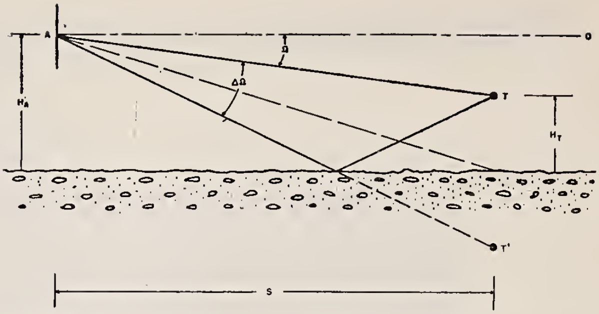

FIGURE 1. *Relationship between the antenna, target transmitter, and the ground.*

A, Antenna being measured; T, target transmitter; T', image of target transmitter; $\Delta\Omega$, angular separation of target transmitter and image at antenna under measurement.

$$V_A=R(\Omega)Ee^{j\delta/2}-R(\Omega+\Delta\Omega)Ee^{-j\delta/2}, \qquad (1)$$

where $Ee^{j\delta/2}$ is the direct radiation and $-Ee^{-j\delta/2}$ is the radiation arriving after reflection by the ground. The phase difference, δ, is due to the pathlength difference for direct and reflected radiation and, for antenna separation, S, very large compared to the antenna heights, it is given by the expression: $4\pi(H_AH_T/S\lambda)$. The negative sign in front of the reflected component holds for both the horizonal and the vertical polarization provided the ground is not a "perfect conductor" and the angle of arrival approaches a grazing value. By conventional manipulation, the relationship (1) can be transformed into:

$$V_A=4\pi R(\Omega)\,\frac{H_AH_T}{S\lambda}\,E \cdot\left[1-\frac{\lambda}{4\pi H_A}\cdot\frac{\frac{d}{d\Omega}R(\Omega)}{R(\Omega)}\,e^{-j(\pi+\delta)/2}\right]. \qquad (2)$$

If a half-wave dipole is substituted for the antenna, its free space response may be denoted $D(\Omega)$ and its response when placed in the position of antenna A, by V_D.

$$V_D=4\pi D(\Omega)\,\frac{H_AH_T}{S\lambda}\,E. \qquad (3)\ [2]$$

Comparison of responses yields:

$$\frac{V_A}{V_D}=\frac{R(\Omega)}{D(\Omega)}\left[1-\frac{\lambda}{4\pi H_A}\cdot\frac{\frac{d}{d\Omega}R(\Omega)}{R(\Omega)}\cdot e^{-j(\pi+\delta)/2}\right]. \qquad (4)$$

[2] This relationship is fully correct for a horizontal dipole facing the target transmitter and very nearly so for a vertical dipole.

But the gain of the antenna in direction Ω, relative to a half-wave dipole, denoted as $G(\Omega)$, is given by:

$$G(\Omega)=\left|\frac{R(\Omega)}{D(\Omega)}\right|^2 \frac{r_D}{r_A}, \tag{5}$$

where r_A is the resistive component of the antenna impedance and r_D is that of the dipole. Because the impedances of both the unknown and the standard antennas are matched to the same equipment, the ratio r_D/r_A is equal to unity. Therefore:

$$G(\Omega)=\left|\frac{V_A}{V_D}\right|^2 \left|\frac{1}{1-\frac{\lambda}{2\pi H_A}\cdot\frac{\frac{d}{d\Omega}R(\Omega)}{R(\Omega)}\cdot e^{-j\left(\frac{\pi+\delta}{2}\right)}}\right|^2. \tag{6}$$

The gain of the antenna relative to a dipole is seen to be equal to the square of the ratio of the amplitudes of the two responses multiplied by a correction factor within bars, also squared. In order that this factor be equal to unity, the second term in its denominator must equal zero. While the correction factor may be made to approach unity by increasing the height of the antenna above ground, it can only be made unity by orienting the antenna in such a way that the derivative of its gain with respect to the vertical angle is equal to zero. This condition is satisfied completely only when the radiation pattern is symmetrical in the vertical plane and the plane of symmetry passes through the base of the target transmitter antenna, i. e., when it forms an angle $\Omega+(\Delta\Omega/2)$ with the horizontal. With the antenna at heights not exceeding a few wavelengths and the separation of approximately 1,000 wavelengths, no detectable errors in measurements have been observed if the plane of symmetry, instead of being inclined at an angle $\Omega+(\Delta\Omega/2)$, is maintained horizontal. However, if the antenna is tilted to any significant degree, say in an attempt to measure the level of secondary lobes, a considerable error (e. g., of several decibels) may be introduced. This has been confirmed experimentally.

From the foregoing it follows that measurements of free-space radiation pattern or the gain of an antenna in an arbitrary plane relative to the axis of the antenna, while in proximity to the ground are likely to contain serious inaccuracies. It may be possible, under some conditions, to correct for these by computations. Both the gain and the radiation pattern can, however, be measured directly in the planes of symmetry. Most conventional antennas are symmetrical about their principal E and H planes.

2.3. Errors Caused by Terrain Irregularities

It has been found experimentally that an antenna in the field of a distant target transmitter will give a varying response when its position is changed by small increments even though the height above ground is maintained constant. In a given location, using a specific antenna, the varying response forms a pattern which appears to change only slowly with time. Figure 2 illustrates a type of pattern formed by the response of a half-wave dipole to the radiation of a distant target transmitter arriving at a grazing angle. The figure also indicates that the pattern retains its principal characteristics over a period of days. If a directive antenna is substituted for the dipole the character of the response is changed. The rapid variations with displacement disappear, but the more gradual fluctuations remain. These variations in response appear to be attributable to minor irregularities or inhomogeneities in the terrain. The pattern presented in figure 2 was measured on the antenna range now in use, located on Table Mesa, near Boulder, Colo. The terrain of the range is naturally very smooth. The technique for correcting for terrain irregularities was developed a number of years ago at an antenna range at Sterling, Va., in use between 1951 and 1954. The terrain at that range was not nearly so smooth and the variations in re-

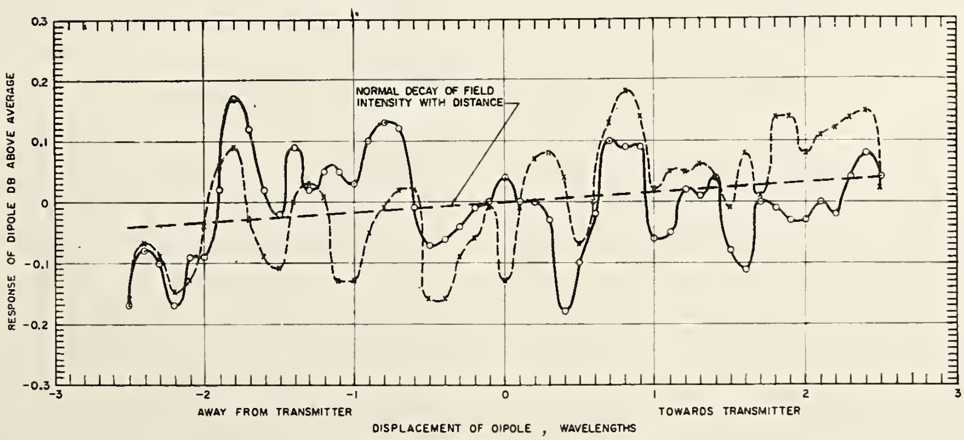

Figure 2. *The response of a half-wave dipole as a function of its position.*
The two curves were recorded with dipoles in identical positions, 4 days apart.

sponse were considerably greater (of the order of ±0.7 db). If the gain measurement is made by comparisons of the response of the unknown antenna with that of a dipole placed at random in the immediate neighborhood of the antenna, an uncertainty of several tenths of a decibel is introduced. It was found that best results were obtained by measuring the response of the unknown antenna, removing it, and by substituting in its place a dipole or some other standard antenna. The dipole or the standard antenna is then moved over the area occupied by the antenna being measured and the average of its response in that area is employed for comparison with that of the unknown antenna.

2.4. Time Drift in Equipment Performance

The procedure for comparing the responses of the unknown and the standard antennas, described in the preceding section, requires a lapse of time while one antenna is being substituted for another. To minimize the possibility of error through changes in the output of the transmitter or the sensitivity of the receiver it was found desirable to introduce a third antenna designated as the "reference antenna." The reference antenna was placed to one side of the unknown (or standard) antenna site, sufficiently far from it (25 wavelengths was found to be sufficient for most cases) so as not to interfere with the measurement of the response of the unknown (or standard) antenna. When measuring the response of the unknown antenna the receiving equipment is switched between the unknown and the reference antennas and the response of the unknown antenna is expressed in decibels above the reference. When measuring the response of the standard antenna the receiving equipment is likewise switched between the standard and the reference antenna which is in identical position as before. The response of the standard antenna is expressed in decibels below the reference. The sum of the two responses in decibels is the gain of the unknown antenna above the standard, also in decibels.

2.5. Other Problems

When an antenna is used as a receiving antenna the available signal power at its terminals is proportional to its gain. The comparison of the available signal powers of two antennas is most conveniently carried out by matching the terminal impedance of each to that of the receiving equipment. Although this is well known, in practice, nevertheless, it is sometimes neglected or carried out without sufficient care. Errors of several decibels in gain measurement may readily result from a difference in the degree of mismatch between the unknown and standard antennas and the remainder of the receiving equipment. The value for the gain of the unknown antenna obtained by the comparison method refers to the point at which the cable, common to both measurements, is connected to the unknown antenna. This value involves the losses within the matching networks, balun transformers, etc., of both the unknown and the standard antennas. By the use of low-loss components these are generally kept at a sufficiently low value to be neglected (less than 0.1 db). If this is not practical, the losses are measured and the gain is corrected by adding the loss in the connections to the unknown antenna to the uncorrected value and by subtracting the loss in the connections to the standard antenna. The principle of the method employed for the measurement of losses is described in reference [2].

In addition to instability with time, the electronic equipment is subject to some degree of nonlinearity of response. If the comparison of antenna gains is made on the basis of relative magnitudes of receiver outputs, precautions must be taken either to assure the linearity of response or to calibrate the response over the range of levels employed in the measurements. An arrangement found best by far, from the standpoint of accuracy of final results, is to insert an attenuator in front of the IF amplifier and to adjust its attenuation to maintain the same signal level while connected to either antenna. The difference in the attenuator settings is the gain of one antenna relative to the other.

3. Description of the NBS Antenna Range

Figure 3 presents a schematic diagram showing the layout and the interconnection of equipment at the NBS antenna range. The separation, S, between the target transmitter and the receiving antennas is approximately one-half mile (~1,000 wavelengths). The target transmitter has normally been a 60-degree corner-reflector antenna with a single half-wave dipole as the driven element. It is located approximately three wavelengths above the ground. The reference antenna is identical to the target transmitter antenna in construction and position above ground. The height of the antenna under test varies with the requirement; it is not necessary to have it at the same height as the reference antenna. A balun transformer of low-loss construction transforms the usual balanced termination of antenna to unbalanced coaxial lines. The impedance of each antenna is matched to its length of a 50-ohm coaxial transmission cable using a stub-tuner. The two lengths of the transmission cable are made equal in order that the variations in losses in the course of the day's measurements be very nearly the same. Such losses might presumably take place due to the heating of the cable by the sun. A 50-ohm coaxial switch permits rapid switching of the equipment from one antenna to the other. The 50-ohm coaxial attenuator serves

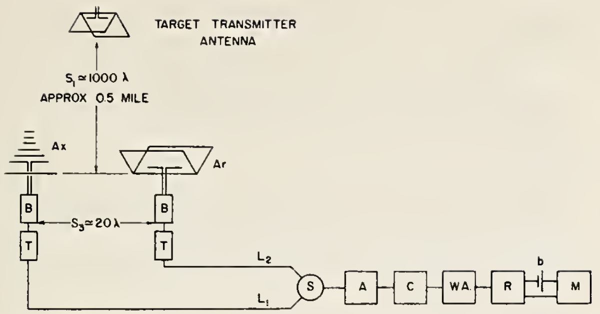

FIGURE 3. *The layout and the interconnections of the equipment at the NBS antenna range for gain measurements.*

Ax, Antenna being measured; *Ar*, reference; *B*, balun transformers (2); *T*, matching units (2); L_1 and L_2, 50-ohm coaxial transmission cables of equal length connecting unknown and reference antennas to receiving equipment; *S*, 50-ohm coaxial switch; *A*, 50-ohm, 10-db coaxial attenuator; *C*, preamplifier-converter; *W. A.*, waveguide-below-cutoff attenuator; *R*, 30-Mc IF amplifier-detector; *b*, battery bucking rectified output of receiver; *M*, 0–30μA indicating meter.

to make the input impedance of the converter more nearly 50 ohms, thus minimizing the standing wave ratio on the transmission cable. The converter changes the frequency to the 30 Mc of the IF amplifier. The converter is a laboratory-constructed device, but converter heads of an APR–4 receiver can be and have been used satisfactorily. The waveguide below the cut-off attenuator between the converter and the IF amplifier is employed to maintain the input to the amplifier at a constant level. It has been designed and constructed at the National Bureau of Standards. A convenient feature of this attenuator is a revolution counter geared to indicate relative attenuation in hundredths of a decibel. The accuracy with which the attenuation can be read is a function of the machining accuracy and is substantially better than the accuracy of measurement. A description of its design is contained in reference [3].

The IF amplifier is likewise laboratory-constructed equipment. It was built with emphasis on stability and linearity. Because the amplification takes place at a constant level, the linearity is unimportant for gain measurements. However, it becomes important when measurements of radiation patterns are made. With this arrangement of equipment the frequency conversion takes place at a varying level. However, the linearity of conversion has been demonstrated to be linear over a range of 80 to 100 db [4, 5].

In order to attain accuracy in the final results of the order of 0.1 db, it has been necessary to refine the techniques so that the precision and reproducibility of the measurements be, if possible, an order of magnitude higher. In order to match the antennas to within 0.01 or 0.02 db of maximum response and in order to maintain the output within the same tolerance, it has been found essential to make the output indication quite sensitive. This is accomplished by the use of a potentiometer technique. The output of the receiver is balanced out by a single dry cell battery, a microammeter indicating the difference between the two. Sensitivity of 0.02 db per division of a large-scale meter has been found satisfactory.

Two-stub matching units are generally used for matching antennas to the transmission lines. It is necessary to design the antenna termination in such a way that the impedances are within the matching range of the units. The three-stub matching units, which can match all impedances, have not been in general use because when making measurements of matching loss it is possible to attain an infinite number of settings giving a 50-ohm match. Some of these settings can have very high losses.

The procedure for measuring losses in the balun and matching unit consists of adjusting the impedance of the output of the matching unit to 50 ohms, resistive. The terminals of the antenna are then shorted and the impedance is measured again without disturbing the positions of the tuning stubs. The loss can be obtained directly from the standing wave ratio. The theory is outlined in reference [2].

The measurement of losses is a clumsy, awkward, and time-consuming operation. The precision is of the order of 0.1 db, significantly below that for gain measurements. Efforts have been made to minimize the matching losses to a point where they can be neglected. A step in that direction has been the elimination of the solid dielectric coaxial cable between the terminals of the antenna and the matching unit and the use of an air-dielectric balun. Figure 4 shows the construction of such a balun. It consists of a rigid, air-dielectric 50-ohm coaxial line made of large diameter, silver-plated tubing. One end is terminated in a coaxial fitting, the other is adapted for balanced connection. The adaption consists of a quarter-wave slot in the outer conductor. The inner conductor

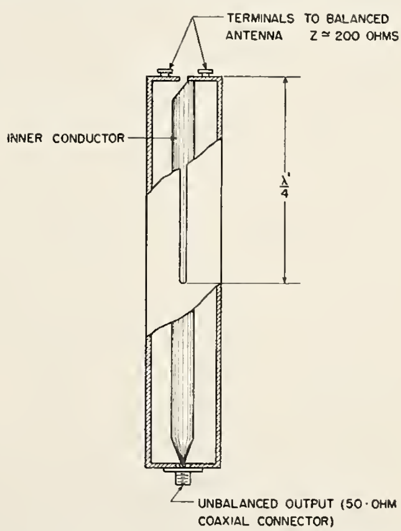

FIGURE 4. *Construction of low-loss, air-dielectric balun transformer.*

is terminated to one side of the slot. The balanced load is placed across the slot. Besides low loss, this balun has the advantage of having very good balance. There is a 4 to 1 impedance transformation in the balun so that the balanced load should have a design impedance of approximately 200 ohms. This type of balun has been employed in radar equipment AN/TPS–3. Tests over a wide range of load impedances using such a balun indicated losses below the limit of measurement (less than 0.05 db).

It has been found convenient to begin the day's measurement by calibrating the reference antenna. This is normally done by erecting a standard antenna, commonly a half-wave dipole, at the exact location and height at which the unknown antenna is to be erected, matching it to the cable, and then comparing it with the reference antenna for response. The standard antenna is moved over the area to be occupied by the unknown antenna in steps of 0.1 wavelength. The differences in responses of the two antennas are noted and recorded for each position. Following completion of the calibration the differences are averaged out. The result, denoted $(\overline{A_r - A_s})$, is referred to as the calibration constant. The standard antenna is then replaced by the unknown antenna. This is matched and the difference in the settings of the attenuator required to maintain the same output with the unknown and the reference antennas, denoted $(A'_A - A'_r)$, is referred to as the response of the antenna above the reference. The gain of the unknown antenna, G_A, is then given by:

$$G_A = G_s + (A'_A - A'_r) + (\overline{A_r - A_s}) + L_{mA} - L_{ms}$$

where

G_s = Gain of the standard antenna, in decibels,

L_{mA} = matching losses to the antenna being tested, in decibels,

L_{ms} = matching losses to the standard antenna, in decibels.

If the antenna under test is relatively small and has low gain, it has been found advantageous to compare its response with that of the reference antenna while changing its position by increments of 0.1 wavelength over the same area over which the standard antenna was moved during the calibration. The response above the reference is then averaged, and the average, $(\overline{A'_A - A'_r})$ is then used in computing the gain. The responses of larger antennas having higher gains appear to be unaffected by changes in position smaller than the dimensions of the antenna. The losses in the cable up to the matching unit cancel out as do the losses to the reference antenna. A more detailed justification of the manipulation is presented in appendix 1.

4. Experimental Results

The procedures described in this paper have been employed for measuring gains of a variety of antenna types, particularly those which appeared promising for long-range VHF scatter communication. Reference [6] enumerates the more important ones. The accuracy realized is difficult to evaluate. In the case of rhombic antennas, the measured gains have been within about 1 db of the computed values. It should be realized, however, that computations of power gain of rhombic antennas involve certain approximations, e. g., that the current remains constant throughout the length of the wires. It is also necessary to know the terminal impedance of the rhombic. It is believed that the measured gain figures are within 1 db of the correct values.

In the case of corner-reflector antennas it is likewise difficult to secure a known standard of comparison. Theoretical computations of gains for corner-reflector antennas invariably assume reflecting surfaces infinite in extent. Recent measurements at the National Bureau of Standards indicate that even with widths and lengths of five wavelengths, the gain of the corner-reflector antenna fails to reach completely its asymptotic value. The measured values are generally within 1 db of the computed and are estimated to be within 0.2 db of the true values.

In case of Yagi antennas, there are no theoretical standards of performance, except possibly for very small ones with no more than three parasitic elements. However, there are a number of papers presenting the results of experimental measurements. Fishenden and Wiblin [7] is one of the more familiar. The results of NBS measurements were compared with those by Fishenden and Wiblin in reference [6]. It has been found that the gains measured at NBS ranges were higher by approximately 1.5 db. Recently measurements of the gain of antennas which were accurate reproductions of the Fishenden and Wiblin, in respect to lengths and diameters of the elements and the spacing between the elements, were carried out at the NBS antenna range. The gains obtained were consistently closer to the NBS values when using equal-length directors. The difference of 0.2 to 0.4 db between the NBS and the Fishenden and Wiblin designs could be accounted for by the closer element spacing employed in the NBS Yagis (0.2 versus 0.34 wavelength). Most of the work on Yagi antennas was carried out at the antenna range at Sterling, Va. In 1954 the Central Radio Propagation Laboratory of NBS was moved to Boulder, Colo., and the antenna range was re-established on Table Mesa, Colo., in the vicinity of Boulder. The gain measurements of Yagi antennas of various lengths were rechecked and found to be quite close to the values measured at the Sterling, Va., range.

In order to be sure that the measurements of gain were accurate as well as consistent, an antenna, the gain of which could be depended upon to be very close to the computed value, was designed and built. Figure 5 shows the construction of this antenna. It consists of two full-wave dipoles arranged in a broadside array. The spacing between adjacent dipoles is a half-wavelength. Both dipoles are a quarter-wavelength away from a metal reflecting plane. The dipoles are connected in a center-fed arrangement, a quarter-wavelength balanced transmission line connecting each dipole to a centrally located balun. Two shorted stubs, one on each line, adjustable in length and position, are employed to match the array to give a 50-ohm terminal impedance. Because the arrangement of dipoles is symmetrical and because the matching stubs are adjusted simultaneously and symmetrically, it is possible to maintain identical currents in the two dipoles. The gain of this arrangement can be computed from mutual impedance relationships between the dipoles, and between the dipoles and their images. This is found to be 9.36 db relative to a half-wave dipole. Using the procedures described in this paper, the gain was measured a large number of times on different occasions. The average result of these measurements has been 9.31 db; the average departure of individual readings from the mean, 0.05 db; the maximum, 0.12 db. This order of accuracy, or precision, cannot be secured in all cases. Some measurements appear to be reproducible within a few hundredths of a decibel; for others care must be exercised to obtain reproducibility within about 0.2 db. In general, for antennas of moderate size, less than 5 wavelengths in the greatest dimension, the accuracy realizable with techniques described here is estimated to be within 0.2 db.

Although it is desirable to construct another standard antenna having a gain of approximately 20 db for purposes of further checking the accuracy of the techniques, no satisfactory design for this purpose has been found. To be satisfactory, the design should be such as to allow an accurate computation of gain and at the same time assure, by symmetry of construction or other means, that the current distribution is exactly that employed in the computations.

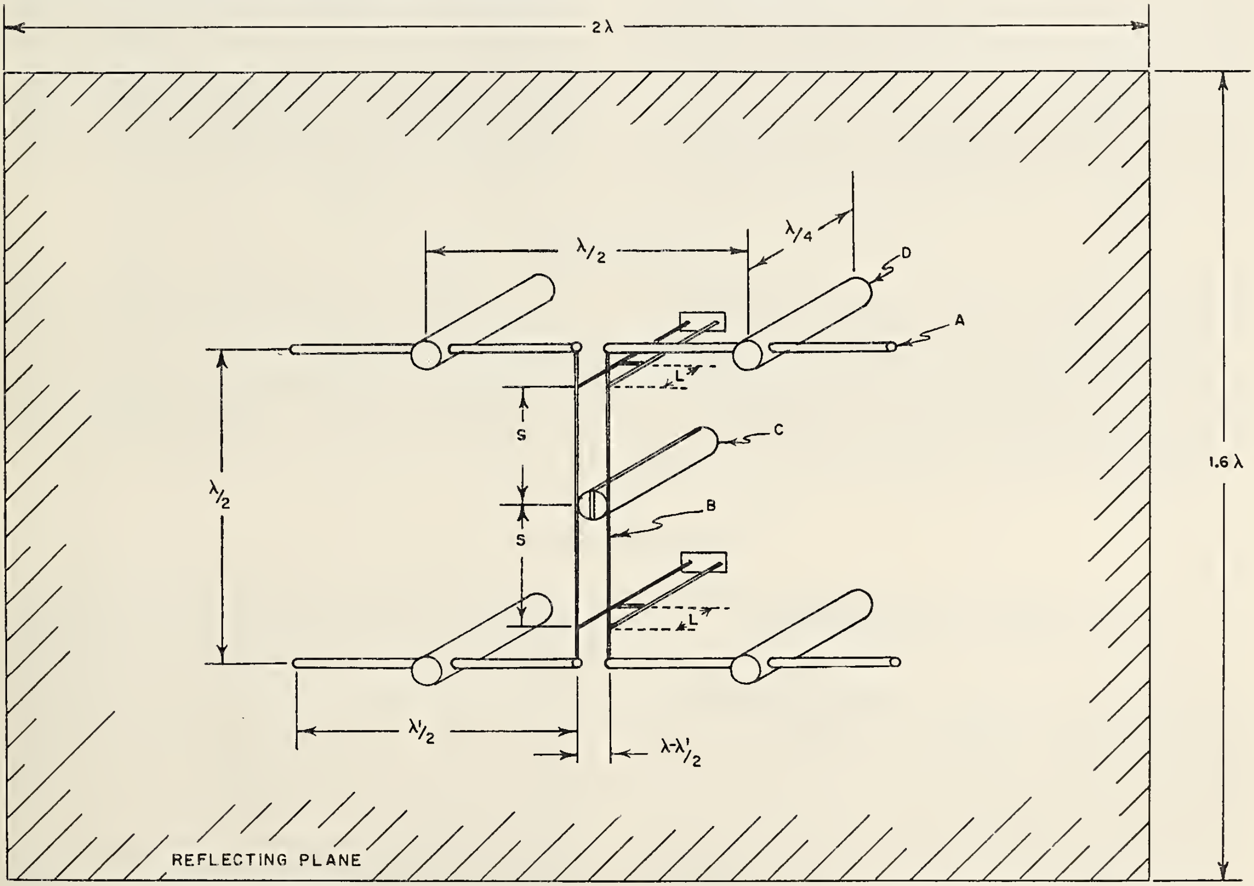

FIGURE 5. *Construction of a standard antenna for verification of accuracy of gain measurements.*

A, Half-wave dipole—one of four; *B*, open-wire transmission line—one to each pair of dipoles; *C*, slotted balun, similar to that shown in figure 4; *D*, insulating dipole support—one for each dipole; *S*, *L*, position and lengths of shorting stub required to obtain terminal impedance of 52 ohms; λ, free space wavelength; λ¹/2, length of half-wave dipole.

5. Conclusions

The method of gain measurement described here is, in principle, the comparison method. The more important features introduced here are the use of an intermediate standard or reference antenna, the calibration of the antenna site, and the refinements of instrumentations. Measurement of antenna gain in close proximity to the ground appears to be unavoidable for the frequency band involved. Measurements are frequently made in this way and the gains thus measured are occasionally referred to as HWSH or HDSH ("referred-to-half-wave-dipole-at-the-same-height") and are numerically equal to free-space gains.

The limitations of this method, insofar as is known, have not heretofore been discussed. One of these is that there must be symmetry in the vertical plane of the radiation pattern. As a result the method is limited to specific types of antennas. Thus far, all antennas tested at the NBS antenna range have fitted into this category. It also means that measurements of radiation pattern or gain of antennas at other than its principal plane are likely to have uncertain, but probably large errors. This, in turn, makes impractical an accurate determination of the directive gain by an integration of its radiation pattern in space.

If the procedures described here are followed with care, and if the antenna is reasonably compact, e. g., the largest dimension not exceeding 5 to 10 wavelengths, accuracies of the order of 0.2 db appear to be realizable. If the antenna structure is too bulky and heavy to be readily disassembled, it may be necessary to calibrate the site by using a directive standard antenna, the gain of which is known, instead of a dipole. The array of two full-wave dipoles described previously has been found satisfactory for this purpose. The accuracy of measurements in such cases may be somewhat lower.

The correction for matching losses has been the least satisfactory feature of the gain measurement technique. For this reason it is very desirable to take all possible precautions for minimizing the losses between the terminals of the unknown antenna and the cable leading to the receiver to a point where these can be neglected. This can normally be accomplished by the use of a large-diameter, air-dielectric balun and by eliminating the use of the solid-dielectric coaxial cable between the terminals of the antenna and the matching network.

The methods of measurement are still in the active stage of experimentation and the procedures described here will undoubtedly be modified in the light of future experience. It is hoped that this discussion will contribute to an exchange of ideas on the subject of methods of antenna measurements and lead to an improvement in accuracy, reliability, and standardization of antenna measurement techniques.

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7. Appendixes

Appendix 1. Analysis of the Calibration and Gain Measurement Procedure

The two-way comparison of antenna responses involving, as it does, losses in two cables and three matching networks, justifies some analysis to be certain that all sources of significant error are eliminated. The measurement procedure as described here depends on three assumptions, that: (1) The power output of the transmitter and the sensitivity of the receiving equipment remain constant during the time required to make the comparison measurements (of the order of 20 to 30 sec); (2) the ratio of the electric field intensity at the site of the unknown antenna to that at the site of the reference antenna remain constant during the period of calibration and measurement (of the order of several hours); (3) the changes with time of the losses in the cables leading to the reference antenna and the antenna under measurement be equal. To insure this, the cables were taken from the same roll and cut to be of equal length. It is not necessary that the losses themselves be equal.

Both the calibration and the measurements are carried out by adjusting the attenuation while connected to each antenna until the output is equal to a convenient value. These manipulations may be expressed in the form of two equations: The first relationship equates the outputs of the receiver during the calibration of the reference antenna at time t_0; the second, during the comparison of the antenna being measured with the reference at time t_1:

$$10 \log E_r^2+S+G_r-L_{mr}-L_{cr}-A_r =10 \log E_A^2+S+G_s-L_{ms}-L_{cs}-A_s,$$

$$10 \log E_r'^2+S'+G_r-L_{mr}-L_{cr}'-A_r' =10 \log E_A'^2+S'+G_A-L_{mA}-L_{cs}'-A_A,$$

where

E_r and E_r' are strengths of the electric field at the reference antenna at the times of calibration and measurement, respectively,
E_A and E_A' are the field strengths at the antenna test site at the times of calibration and measurement, respectively,
S and S' are the gains or sensitivities of receiver at the times of calibration and measurement,
G_r, G_s, and G_A are the gains of the reference, standard, and unknown antennas,
L_{mr}, L_{ms}, and L_{mA} are the matching losses at the reference, standard, and unknown antennas,
L_{cr} and L_{cA} are the cable losses at the time of calibration in the cables leading to reference and standard antennas, respectively,
L_{cr}' and L_{cA}' are the cable losses at the time of measurement in cables leading to the reference and unknown antennas. The cable leading to the unknown antenna during measurement is the same one as that connecting to the standard antenna at time of calibration,
A_r and A_r' are the attenuator settings of the receiver when connected to the reference antenna at the time of calibration and measurement, respectively,
A_s and A_A' are the attenuator settings with the standard and unknown antennas.

With the exception of the field strength, E, which is in volts, all parameters are logarithmic, i. e., measured in decibels. The gain of the antenna under test, G_A, may be obtained by subtracting the first equation from the second and rearranging the terms:

$$G_A=G_s-10 \log \left(\frac{E_r'}{E_A'}\cdot\frac{E_A}{E_r}\right)^2+(A_A'-A_r')+(A_r-A_s) +(L_{cs}'-L_{cs})-(L_{cr}'-(L_{cr})+(L_{mA}-L_{ms}).$$

Since the ratio of the field strengths at the test site to that at the reference antenna site is, at any one time, independent of the power output of the target transmitter, the term $10 \log (E_r'/E_A'\cdot E_A/E_r)^2$ is equal to zero. Because the cables connecting the equipment to the reference antenna and to the antenna being tested are as nearly identical as it is possible to make them, the changes in the losses in the two should be equal.
Therefore

$$(L_{cs}'-L_{cs})=(L_{cr}'-L_{cr})$$

and

$$G_A=G_s+(A_A'-A_r')+(A_r-A_s)+(L_{mA}-L_{ms}).$$

The terms $(A_A'-A_r')$ and (A_r-A_s) are the response of the antenna being tested relative to the reference antenna and the calibration of the reference antenna by the standard, respectively. These are normally averages of a large number of readings. The losses in the cables connecting the equipment to the reference and to the antenna being measured (or standard antenna) do not enter into the relationship.

Appendix 2. An Alternate Method of Gain Measurement

In the field of electrical measurements, among the most successful methods are those employing null type of indications. An attempt has been made to introduce this method to the comparison between the unknown and reference antennas and between the dipole and reference antennas. Figure 6 illustrates its principles. Each antenna is matched to and terminated in a 50-ohm load. Two commercially available waveguide-below-cutoff attenuators are tapped to each cable ahead of the load. The output of the attenuators is connected to each end of a slotted coaxial line. In this way two traveling waves directed in opposite directions are set up, creating standing waves. If the relative settings of the two attenuators are adjusted so that the magnitudes of the traveling waves are equal, the SWR will become infinite and there will be points of complete cancellation or nulls. The manipulations consist of moving the

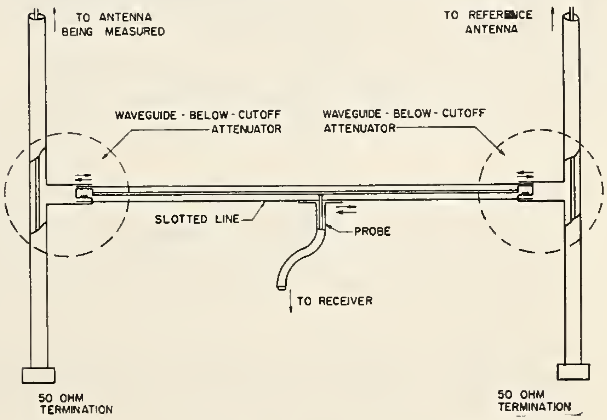

FIGURE 6. *A null method of comparison of output of antenna being measured with that of reference antenna.*

probe to locate the minimum, and adjusting the relative attenuation to obtain a null. This method has certain important advantages:

1. It is very sensitive, much more so than the comparison by switching.
2. It is independent of normal variations in the output of transmitter or sensitivity of receiver.
3. The comparison is truly instantaneous; there is no time interval in switching between antennas and adjusting attenuators.
4. There is phase information given by the position of the probe which may be of considerable interest in some cases.

On the other hand, there are certain drawbacks:
1. There are no commercially available attenuators calibrated at intervals closer than 1 db. Calibration to 0.01 db is necessary to take advantage of the sensitivity of the method.
2. It is necessary that the output connection of attenuator be exactly 50 ohms to avoid reflections of the opposing wave.
3. It is necessary to alter connections for rematching after every adjustment of antenna configuration.
4. In calibrating the site, the probe as well as attenuator has to be readjusted for each change in position.

The first of these drawbacks—the nonavailability of suitable attenuators—is the really serious one. It is believed that after suitable attenuators become available the advantages of this method of comparison may have a distinct superiority over the basic one.

Appendix 3. Directive Gain

In addition to the power gain, which has been under discussion up to this point, there is a closely related quantity known as the directive gain. The directive gain is defined as the ratio of the power radiated in the desired direction per unit solid angle to the power radiated in all directions, multiplied by 4π. Numerically the directive gain is equal to the power gain relative to an isotropic radiator plus the losses within the antenna. While many antennas, particularly those employing dipoles, have relatively low losses, others, such as rhombic antennas, may deliver a considerable portion of the total power to a terminating resistor. For such antennas the difference between the directive gain and the power gain may be several decibels. In cases where the losses are low it is theoretically possible to integrate the radiation pattern over space and to accept the directive gain as the true gain. However, the earlier discussion of the effect of the proximity to the ground on measurements has shown that it is difficult to carry out accurate measurements of the response of the antenna except in the planes of symmetry. It is, therefore, not practical to obtain an accurate measurement of gain using pattern integration.

A widely used method of estimating the gain of the antenna from the widths of its main radiation lobe is by the use of the relationship:

$$G=\frac{41{,}253}{W_\theta \cdot W_\phi},$$

where G is the directive gain, relative to an isotropic radiator and W_θ and W_ϕ are the half-power beamwidths of the main lobe, in degrees, in the two-principal planes. The number 41,253 is the area of the sphere of unit radius in "square degrees" obtained as a product of $4\cdot\pi\cdot 57.296^2$, 57.296 being the number of degrees in a radian. The formula assumes that the shape of the beam in each plane is a single lobe of a two-dimensional sinusoidal function, i. e., $E=\cos m\theta \cos n\phi$, and that the power radiated in the directions outside of the main lobe is negligible.

Experience has shown that even in the case of low-loss antennas, the gain figure obtained by the use of this relationship is considerably higher than the true gain. In general, in case of low-loss antennas, the discrepancy appears to be of the order of 1 db. This formula has, nevertheless, been found useful for checking the gain figures obtained by direct measurements, for comparison with true gain to obtain a measure of power content in the side and back lobe radiation, and for checking the self-consistency of published data on performance of antennas.

Boulder, Colo., December 17, 1957

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