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Energy Spectrum Resulting from Electron Slowing Down

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Energy Spectrum Resulting from Electron Slowing Down

Rosemary T. McGinnies

The process of electron slowing down is described qualitatively and the resulting energy spectrum defined in terms of differential track length. A method developed by Spencer and Fano for the calculation of the electron spectra has been applied extensively by means of an automatic computer. The accumulation of secondary knock-on electrons is included in the calculations and bremsstrahlung losses are neglected. Tabulations of differential track length are given for aluminum, copper, tin, lead, air, water, bone, muscle, and polyethylene (C_2H_4), with up to 17 source energies each. Source energies range from 10.46 million electron volts to 6.438 kilo electron volts. A tabulation of the probability for the production of knock-on secondary electrons is also given. These data are an abstract of a much larger tabulation, printed by the computing machine, which is available on loan from the library of the National Bureau of Standards.

1. Introduction

This Circular belongs to the series of surveys and tabulations of data on radiation physics that is being published with the support of the biophysics branch of the Atomic Energy Commission.

The main object of this Circular is to present the energy spectrum resulting from the slowing down of electrons in various materials; that is, the energy distribution of the electrons that traverse any small volume of material when they are produced with a given energy by sources distributed through the material. This energy spectrum is of interest for the analysis of biological and chemical effects of β-rays, of electron beams, and also of the X- and γ-rays which act through secondary electrons. It is also of interest for the theory of cavity ionization chambers [1].[1]

Extensive computations of the electron slowing down spectra, applying the method of Spencer and Fano [2], have been made with an automatic computer. Selected results are presented here. The full set of tabulations is available on loan from the library of the National Bureau of Standards and is described in section 4.

Section 2 defines the energy spectrum. Section 3 contains a qualitative description of the process of electron slowing down and the procedure for calculating the spectrum. The various illustrative figures and tables of data are described in section 4 and their accuracy and range of applicability discussed in section 5. The method of calculation is described in some detail in section 6.

2. Energy Spectrum Defined as a Track Length

Consider a source of electrons that is distributed uniformly throughout an infinite, homogeneous material and emits electrons of kinetic energy E_0 at a constant rate of N_0 per unit volume and per unit time. An ideal small electron detector of spherical shape and radius R scores with equal probability electrons arriving from any direction within the material. Call $N(E)\,dE$ the number of electrons with energy in the interval between E and $E+dE$, which traverse the detector in a unit time when it is placed at an arbitrary point within the material. The number $N(E)$ divided by the cross-sectional area πR^2 of the detector represents the *differential spectrum of the electron flux* at the point of observation,

$$\phi(E)=N(E)/\pi R^2 \qquad (1)$$

The quantity that appropriately represents the energy spectrum of electrons is the differential spectrum of the electron flux per unit source strength, which is called

$$y(E_0,E)=\phi(E)/N_0, \qquad (2)$$

where the initial energy E_0 is indicated as an explicit variable.

Notice the dimensional characteristics of eq (1) and (2). The differential spectrum $N(E)$ is a number per unit-time and per unit-energy interval; the differential flux $\phi(E)$ is a number per unit-area, per unit-time, and per unit-energy interval. Division of the differential flux by the source strength N_0, number per unit volume, and per unit time, causes the flux per unit source strength $y(E_0,E)$ to have the dimensions of a *distance per unit-energy interval.* Indeed $y(E_0,E)\,dE$ represents the average *track length* traveled by an electron while its energy lies between E and $E+dE$ in the course of its slowing-down process.

The integral $\int_{E'}^{E_0} y(E_0,E)dE$ gives the mean distance traveled by an electron before its energy drops below E'. In particular,

$$R(E_0)=\int_0^{E_0} y(E_0,E)dE$$

represents the mean distance traveled by an electron before it comes to rest; that is, its "range".

[1] Figures in brackets indicate the literature references on page 8.

The usual definition of the range given in [3, eq (15)]; namely, $R(E_0)=\int_0^{E_0}(-dE/dx)^{-1}dE$, corresponds to an approximate evaluation of $y(E_0,E)$, as discussed in section 3.

The differential track length $y(E_0,E)$ can be defined directly, without reference to a source constant in time and flux distributed uniformly throughout a material. One may call N_0 the *total* number of electrons of energy E_0 emitted anywhere within a material at any time during an experiment, and call $\phi(E)dE$ the integral of the flux observed by ideal detectors at *all* points of the material over the *whole* duration of the experiment. The ratio of the quantities thus defined is independent of the space and time distribution of the electron emission and thus coincides with the ratio (2) which pertains to the special case of a uniform distribution.

The average time spent by an electron within the energy interval from E to $E+dE$ is $y(E_0,E)dE/v(E)$, where $v(E)$ is the velocity of an electron with kinetic energy E. For a uniformly distributed source of N_0 electrons per unit volume per unit time, $N_0y(E_0,E)dE/v(E)=\phi(E)dE/v(E)$ is the space *density* of electrons in the stated energy interval.

If the electrons are produced with nonuniform initial energy, i. e., with an initial spectral distribution numbering $\mathcal{N}(E_0)dE_0$ in the interval from E_0 to E_0+dE_0, their differential track length at the energy E is given by

$$y(E)=\frac{\int_0^\infty \mathcal{N}(E_0)y(E_0,E)dE_0}{\int_0^\infty \mathcal{N}(E_0)dE_0}. \tag{3}$$

This formula is normally required in the application to the secondary electrons produced by X- or γ-rays.

3. Slowing-Down Process

An electron traversing a material dissipates its energy in a very large number of inelastic collisions with atoms of the material. In the great majority of these collisions, the energy lost by the electron is of the order of 10 ev per collision. The aggregate effect of a large number of such collisions yields a nearly continuous process of slowing down. At some stage during the process the electron energy will fall within any given moderately small interval below the initial energy, for example, between 48,600 and 48,700 ev. The electron will then travel on, remaining in that energy interval for the average distance required to dissipate an energy amount equal to the width of the interval, namely 100 ev in this example. This distance equals the interval width divided by the average rate of energy dissipation ("stopping power"), the value of which is given in [3]. According to this schematization of the slowing-down process, the differential track length would be simply the reciprocal of the stopping power $(-dE/dx)^{-1}$. This result is implied by the representation of the range of an electron according to

$$R(E_0)=\int_0^{E_0}(-dE/dx)^{-1}dE.$$

However, the model of continuous slowing down fails to treat adequately the statistical effects of the minor fraction of collisions in which an electron loses an unusually large amount of energy. Some of these collisions result from interaction with the internal electrons of atoms, whose disturbance requires a large energy transfer ranging up to 100,000 ev for very heavy elements. Other collisions are of the "knock-on" type, in which a large amount of momentum is transferred to a single atomic electron and this electron recoils out of its atom with correspondingly high energy, irrespective of its initial binding within the atom. In an extreme type of knock-on collision, the incident electron and the one ejected from the atom emerge from the collision with comparable energies; since the two electrons are indistinguishable, one calls "primary" the electron that emerges with the higher energy, and "secondary" the one with the lower energy.

One effect of occasional large energy losses is that an electron may skip a moderately small energy interval jumping directly from a higher to a lower energy; e. g., from 48,900 to 47,900 ev in the example mentioned above. The differential track length at any given energy is thereby reduced, though only by a few percent. A much larger effect is the production of secondary electrons which are numerous even though mostly of low energy and whose own track length must be added to that of the primary electron. The differential track length at energy values $E \ll E_0$ is thereby raised by a very large factor. Finally, since large energy losses are infrequent, none of them will usually occur until a substantial energy loss has been accumulated through small-loss collisions; therefore, the possibility of large losses does not actually influence $y(E_0,E)$ so long as E is still close to E_0, whereas the model of continuous slowing down makes allowance for large losses independently of the difference E_0-E.

The Spencer-Fano method of calculation utilizes as an initial approximation of the track length of primary electrons a convenient analytical expression, which is akin to the reciprocal stopping power, but discounts the possibility of large energy losses when $E \sim E_0$. The initial estimate serves as a basis for an estimate of the production and cumulative track length of secondary electrons. The estimated differential track lengths of primaries and secondaries are then added. Their sum departs from the actual differential track length $y(E_0,E)$ by an amount that is not large and obeys an equation amenable to numerical solution.

4. Description of Data

For each stopping material the only input data needed for the calculation are values for the average excitation energy I and the ratio, Z/A, of the atomic number and weight for the material. The materials for which the program was run together with values for I are listed in table 1. Data on the first 9 materials are given in this Circular. Some of the values of I were measured by Bakker and Segrè [4]; others are those recommended by Caldwell [5], while a few of those for compounds and mixtures were computed using the expression $I=13.5\ Z$ of Bloch [6]. The code may easily be modified and run for other materials. Seventeen spectra were computed for each material with source energies E_0 ranging from 10.46 Mev to 6,440 ev. The source energies are distributed evenly on a logarithmic scale with succeeding values in the ratio $2^{2/3}=1.59$. For each spectrum the energy ranges from E_0 down to 402 ev.

Differential track lengths are expressed in this Circular in mass units g cm^{-2} $(mc^2)^{-1}$, so as to be independent of the density of the stopping material. They are given in table 2 for nine different materials. The source energy is given at the top of each column, and the degraded energies, succeeding values of which are in the ratio $2^{1/2}=1.414$, are listed in the columns to the left of each table in both mc^2 and Mev units. To convert the differential track length from g cm^{-2} $(mc^2)^{-1}$ to g cm^{-2} (Mev)$^{-1}$, multiply the number given in the table by 1.957.

A convenient unit of stopping power $2Cmc^2/\beta^2$, which depends on the energy through β, is used to "scale" the functions involved in the calculation; i. e., to reduce them to a dimensionless form. Its reciprocal is then a useful unit for differential track length in mass units. The quantity $C=N_A\pi r_0^2 Z/A=0.150\ Z/A$ cm^2/g is an atomic constant that may be interpreted as the total "area" covered by the electrons of radius r_0 contained in 1 g of material; N_A is Avogadro's number, $r_0=e^2/mc^2$, e and m are the charge and mass of the electron. For purposes of conversion, table 3 gives the value of the ratio $(2Cmc^2/\beta^2)/(Z/A)=0.300\ mc^2/\beta^2$ as a function of energy, and table 1 gives Z/A for different materials.

A tabulation of the probability for the production of knock-on secondary electrons is given in table 4. The entries are the values of the scaled quantity $K(E,\epsilon)/(2Cmc^2/\beta^2)$, which is the same for all materials. The function $K(E,\epsilon)$, with $\epsilon<\frac{1}{2}E$, is the probability per unit track length for the production of secondary electrons of energies greater than ϵ in knock-on collisions involving primaries of energy E. The probability is calculated from the Møller relativistic cross section for electron-electron collisions (see section 6). Values of E listed to the left of the table decrease from 40.9 Mev to 1.25 kev in logarithmic steps with succeeding values in the ratio 1.414. The value of ϵ is given at the top of each column. Notice that in accordance with the Rutherford theory, which disregards exchange and relativity, values of $K(E,\epsilon)/(2Cmc^2/\beta^2)$ are approximately equal to ϵ^{-1}, provided ϵ is expressed in units of mc^2 and $\epsilon\ll E$.

Figure 1 shows how the differential track length in mass units depends on the source energy at various values of the degraded energy E. This dependence is strong when E is small because of the accumulation of secondaries.

The scaled differential track length $(2Cmc^2/\beta^2)y(E_0,E)$, which depends on the nature of the stopping material only through the average excitation energy I, is plotted in figure 2 as a function of I for given energies E_0 and E so that interpolation may be carried out as newer, and more certain data become available on the value of I. The curves for a particular degraded energy are grouped together with the one for the highest source energy at the top. None of these curves has very great curvature so that interpolation should not be difficult.

The ratio of the total electron flux to the flux of the primary electrons alone is known as the "R-factor" [7] and is approximately equal to the product of the stopping power and differential track length. A quantity $\overline{R}$ approximately equal to this factor has been obtained and is plotted in figures 3, 4, and 5. $\overline{R}$ is defined below. It is seen from these graphs that $\overline{R}$ is almost a universal function of $\ln(E_0/E)$, independent of I for the stopping material and of the source energy. Polyethylene and lead, which have respectively a very low and high value for I, are chosen as extreme examples, and the graphs show the variation of $\overline{R}$ as a function of $\ln(E_0/E)$ for a particular material for different source energies as well as the variation with I for a given source energy.

Description of the Complete Tabulation

The spectra for each material were printed out by the automatic computer in separate blocks for each source energy E_0 starting with $E_0=20.48\ mc^2$. The first column of each block is a list of the degraded energy values in units of mc^2, which are distributed evenly on a logarithmic scale with succeeding values in the ration $2^{1/6}=1.122$. For each energy the second column gives the scaled differential track length $(2Cmc^2/\beta^2)y(E_0,E)$, which depends on the nature of the material only through the average excitation energy I. The third column gives $y(E_0,E)$ in units g cm$^{-2}(mc^2)^{-1}$. The fourth column lists a dimensionless quantity $F(E_0,E)/(2Cmc^2/\beta^2)$ which is defined explicitly by eq (13) in section 6. The last column gives $\overline{R}(E)=y(E_0,E)F(E_0,E)$, the product of columns two and four.

The reciprocal of $F(E_0,E)$ constitutes a first approximation to the differential track length of the primary electrons. Since in the model of

continuous slowing down the differential track length is the reciprocal of the stopping power, $F(E_0,E)$ is closely related to the stopping power; in fact it departs from it by only a few percent for $E<\frac{1}{2}E_0$, although it becomes substantially smaller than the stopping power for E near E_0. This reduction corresponds to the discounting of large energy losses, as discussed in section 3. The dimensionless ratio $F(E_0,E)/(2Cmc^2/\beta^2)$ is accordingly analogous to the stopping number, as defined in NBS Circ. 577. To the extent that the reciprocal of $F(E_0,E)$ represents an approximation to the differential track length of the primaries, the product $\bar{R}=y(E_0,E)F(E_0,E)$ indicates approximately the ratio of the total number of electrons, primaries and secondaries, to the number of primaries alone.

Table 4 contains a more complete tabulation of the scaled integral cross section $K(E,\epsilon)/(2Cmc^2/\beta^2)$. Separate lists of values are given for 99 different values of ϵ decreasing progressively in the ratio $2^{1/6}=1.122$ and ranging from 40.9 Mev ($80.0\ mc^2$) to 393 ev ($7.689\times10^{-4}mc^2$). The entries for each value of ϵ correspond to different starting energies E; these range from $1.122\ \epsilon$ upward in steps having the ratio 1.122. For each value of ϵ, the first five entries correspond to values of $E<2\epsilon$. Accordingly, they do not represent the probability of production of secondaries with energies greater than ϵ (since by definition $\epsilon<\frac{1}{2}E$). They represent instead the probability per unit track length of a knock-on collision which reduces the energy of the primary from E to a value lower than ϵ. The sixth entry in each list, which corresponds to $\epsilon=\frac{1}{2}E$, should actually be zero. However, it is a characteristic of a floating point computation which should compute zero, that the machine does not derive zero but rather a number which is many orders of magnitude smaller than the other numbers involved in the calculation. In the present case, cross sections less than 1×10^{-3} are effectively zero.

The computing machine has printed out the numbers with the decimal point in front of the first significant figure, the exponent being indicated as follows: $0.35636105\bar{E}{-}01=0.035636105$. At most, only four of the eight figures given may be considered significant.

5. Limitations to the Accuracy of the Data

The two major limitations to the accuracy of the data presented here are the inadequacy of the collision cross sections at low energies and the neglect of bremsstrahlung losses at high energies. Both of these limitations are more severe for the heaviest materials.

As pointed out in section 3, the energy spectrum of electrons depends primarily on the variation with energy of the stopping power; that is, of the mean rate of energy loss in all collisions. The calculation takes this mean rate into account accurately, to within the uncertainty on the value of the constant I (see below). The probability distribution of the energy losses in individual collisions has a subsidiary, but appreciable, effect on the energy spectrum to be calculated. This effect stems particularly from collisions in which an electron loses a substantial fraction ($\geq10\%$) of its energy. The probability of these collisions was taken into account through a formula (the relativistic Møller cross section for knock-on collisions) which disregards the binding of electrons within atoms. Therefore, this probability was estimated accurately only for collisions with an energy loss much larger than the binding energy of the atomic electron which is ejected. The error thus incurred has severe influence on the results of the calculation which pertain to electrons of energies comparable to the binding energies of atomic electrons. In the materials for which calculations were made, the highest binding energy of atomic electrons is of the order of 100 kev for the K electrons of Pb. The K electrons in a heavy element constitute a small fraction of all atomic electrons. For this reason, the error incurred by the use of inaccurate probabilities of energy losses is estimated to be small until the energy of the electrons being slowed down falls below half of the binding energy of the L-shell electrons in heavy elements. In light elements, the binding energies are low, and the error is expected to be small down to the lower limit of 0.404 kev.

The machine calculation proceeded to 0.404 kev for all materials. In the heavier ones it was found that when the stopping number is of the order of unity, the numerical procedure breaks down to give negative values for the flux. This is due to an inaccurate evaluation of the integral of $[y(E_0,E')K(E',E)-yE_0,E)\bar{K}(E',E)]$ for $E'\sim E$ (see section 6).

Spencer and Fano have shown in pilot calculations that bremsstrahlung emission is quite overwhelming at high energies in Pb but entirely negligible at low energies in Al. It is the low-Z materials and lower energies that are generally important for biological applications so that for these purposes bremsstrahlung may reasonably be neglected. Following the same criterion as in [3], the tabulations presented in this Circular are limited to the energy range where radiation effects contribute less than 5 percent to the total energy loss.

At present there is uncertainty regarding the value of the average excitation energy I for each material. The dependence on I occurs in a logarithmic term so that small errors in its value are of little importance. However, there is a large discrepancy between the experimental values of Bakker and Segrè [4] and those recommended by Caldwell [5]. Consequently, the automatic

computations were carried out for both sets of values of I. The evaluation of I is discussed in NBS Circ. 577.

At high energies the effect of the density of the material traversed by a charged particle becomes important. This effect may be represented as an exponential variation of I; i. e., $I=I_0\exp(\delta/2)$. That is, the density effect is given by a correction δ to the stopping number. Sternheimer [8] has derived an equation for δ which has been applied by Nelms [9] to a variety of materials using both sets of values for I. Making this correction would have the effect of increasing the differential track length approximately in the ratio $\delta(2Cmc^2/\beta^2)/F(E_0,E)$. This amounts to less than a few percent for heavy materials for which the high-energy cutoff is rather low. For low-Z materials the density effect increases so that for water and polystyrene, for example, the correction is of the order of 2 percent at 1 Mev and 10 percent at 8 Mev. These data could be used to correct I for the density effect, but no such correction was made in the present calculations.

The code that was written for the solution of this problem was tested using the same input data for Al and Pb as Spencer and Fano used for their pilot calculations. The two sets of numbers agree to within 2 percent. This discrepancy is attributable to the fact that a finer mesh and some slightly different approximations were used in the present calculation. In addition, two sets of calculations were done on a desk computer using some of the same input data as were used with the automatic computer. Agreement was obtained to five significant figures.[2]

6. Method of Calculation

The differential track length $y(E_0,E)$ was obtained by solving an equation that expresses the balance between the production of electrons with energies above a certain limit E and the disappearance of such electrons through processes that reduce their energy below this limit. An electron of energy E' drops below the energy limit E when it produces a secondary electron with energy larger than $E'-E$. The probability of this process per unit track length has been indicated in section 4 as $K(E',E'-E)$. Owing to the identity of the electrons in a collision, the mathematical function that represents the probability of energy losses has the property $K(E',E'-E)=K(E',E)$. Therefore, if $K(E',E)$ represents the probability per unit path that an electron's kinetic energy drops from E' to a value *below* E, and $y(E_0,E')\,dE'$ represents the track length of electrons with energies between E' and $E'+dE'$, the mean number of electrons dropping from this infinitesimal energy range to the range below E is $dE'\,y(E_0,E')\,K(E',E)$. If the production of electrons in the infinitesimal energy range is $\mathcal{N}(E')\,dE'$, the equality of the production and disappearance of electrons with energies greater than E requires that

$$\int_E^\infty dE'y(E_0,E')K(E',E)=\int_E^\infty \mathcal{N}(E')dE'. \quad (4)$$

The probability $K(E',E)$ is an integral over the more elementary probability $k(E',\epsilon)$ of energy losses of specified magnitude ϵ; namely

$$K(E',E)=K(E',E'-E)=\int_{E'-E}^{\frac{1}{2}E'} k(E',\epsilon)d\epsilon. \quad (5)$$

The functions $k(E',\epsilon)$ and $K(E',E)$ are not well known, but their application in eq (4) is such as to require only a partial information, which is readily available in approximate form, as shown by the following discussion. Since the probability of small energy losses ϵ is very large, even the integral probability $K(E',E)$ decreases rapidly as $E'-E$ increases. Therefore, the major contribution to the integral $\int_E^\infty dE'y(E_0,E')K(E',E)$ arises from the values of E' very close to E; i. e., most electrons drop out of the energy range $E'>E$ only after their energy has fallen close to the limit E. Over the range of values of E' from E to $E+\delta$, where δ amounts to a small fraction of E, the function $y(E_0,E')$ varies but little; so that the major contribution to the integral on the left of (4) can be estimated in the form

$$\int_E^{E+\delta} dE'y(E_0,E')K(E',E)\sim y(E_0,E) \times\int_E^{E+\delta} K(E',E)dE'. \quad (6)$$

Therefore, it is essential for our purpose to know the integral $\int_E^{E+\delta} K(E',E)dE'$, but the detailed values of $K(E',E)$ for E' near to E are not required. As shown in [2] this integral is approximately equal to $\int_0^\delta \epsilon k(E,\epsilon)d\epsilon$; i. e., to the average rate of energy dissipation due to collisions with energy loss no larger than δ. The stopping power is the limiting value of this integral at $\delta=\infty$, but the limiting value is already approached when δ is only a fraction of E, provided E is of the order of at least 100 kev. Therefore, data from the theory of stopping power can be utilized to evaluate the contribution from $E'\sim E$ to $\int_E^\infty dE'y(E_0,E') \times K(E',E)$; this is done below by a suitable transformation of the basic eq (4).

To calculate the contribution to $\int_E^\infty dE'y(E_0,E')$

[2] The differential track lengths obtained by Spencer and Fano are in error by something less than 1 percent due to an error in the numerical evaluation of eq (41).

$\times K(E',E)$ from values of E' not very close to E, one requires a good estimate of $K(E',E)$ in this range and, therefore, an estimate of the probability $k(E,\epsilon)$ of energy losses ϵ not much smaller than E. The probability of a collision with such a large energy loss can be calculated on the assumption that the collision "knocks on" an atomic electron so violently that the initial bonds of this electron within matter may be disregarded. The error incurred by this assumption was discussed in section 5. The Møller formula for the probability $k(E',\epsilon)$ of collisions with an energy transfer ϵ from and incident electron to a "free" electron initially at rest was entered in eq (5) to calculate $K(E',E)$. However, the upper limit of the integral was set at $\epsilon=\frac{1}{2}E'$, rather than at $\epsilon=\infty$, because in a collision between "free" electrons one of them always emerges with more than half of the initial energy and this electron is regarded as the "primary" (see section 3). The result is

$$K(E',E)=\int_{E'-E}^{\frac{1}{2}E'} k(E',\epsilon)d\epsilon = \frac{2Cmc^2}{\beta'^2}\left\{\frac{1}{E'-E}-\frac{1}{E}+\frac{E-\frac{1}{2}E'}{(E'+1)^2} + \frac{2E'+1}{E'(E'+1)^2}\ln\frac{E}{E'-E}\right\}, \quad \text{for } E\geqq\tfrac{1}{2}E', \tag{7}$$

where $2Cmc^2/\beta'^2$ indicates the function of E' and of the electron density discussed in section 4, and all energies are expressed in units mc^2. The probability $K(E',E)$ is understood to vanish for $E\leqq\frac{1}{2}E'$, so that the upper limit of the integral on the left of (4) is, in effect, $E'=2E$ rather than $E'=\infty$. Since eq (7) is unrealistic for small values of $E'-E$, but only the correctness of $\int_E^{\delta} K(E',E)\,dE'$ matters in this range for the solution of our problem, the desired result is obtained by assuming eq (7) to hold to the lower limit

$$E'-E=\overline{Q}=\left(\frac{I}{mc^2}\right)^2\frac{\exp(\beta^2)}{2E(E+2)}, \tag{8}$$

where all energies are expressed in units of mc^2 and I indicates the average excitation energy (in ev), to be taken from stopping-power experiments or theory [3]. In the insignificantly small range of values of $E'-E<\overline{Q}$, $K(E',E)$ was assumed for convenience to have the constant value $K(E+\overline{Q}, E)$, as given by (7).

The following data on the production of electrons were entered in the integral $\int_E^{\infty}\mathcal{N}(E')dE'$ on the right of eq (4). Primary electrons are produced by the source, and secondary electrons by knock-on collisions; we write $\mathcal{N}(E')=\mathcal{N}_p(E')+\mathcal{N}_s(E')$. The source of electrons of energy E_0 was assumed to be of unit strength. It is accordingly represented by the Dirac function $\mathcal{N}_p(E')=\delta(E'-E_0)$, whose integral over any interval including E_0 yields 1. The production of secondary electrons of energy larger than E per unit track length of an electron of energy E' is given again by an integral over the Møller cross section and is expressed in terms of the function (7); namely, by

$$\int_E^{\frac{1}{2}E'} k(E',\epsilon)d\epsilon=K(E',E'-E) =-K(E',E), \quad \text{for } E\leqq\tfrac{1}{2}E'. \tag{9}$$

The total production of primary and secondary electrons with energies larger than E is, then,

$$\int_E^{\infty}\mathcal{N}(E')dE'=\int_E^{\infty}\mathcal{N}_p(E')dE'+\int_E^{\infty}\mathcal{N}_s(E')dE' =1-\int_{2E}^{E_0}dE'y(E_0,E')K(E',E), \quad \text{for } E\leqq E_0. \tag{10}$$

We have indicated the upper limit of the last integral as E_0 instead of ∞, because $y(E_0,E')$ vanishes for $E'\geq E_0$. Having thus specified the collision probability and the source function, we obtain the basic equation

$$\int_E^{2E} dE'y(E_0,E')K(E',E) =1-\int_{2E}^{E_0}dE'y(E_0,E')K(E',E), \tag{11}$$

where $K(E',E)$ is given by (7) for $E'-E\geq\overline{Q}$ and is constant for $E'-E\leqq\overline{Q}$.

The numerical solution of this equation can be simplified by a transformation that permits one, in effect, to evaluate analytically the integral over values of E' near E, where $K(E',E)$ is large and varies rapidly. To this end one adds and subtracts on the left side of (11) the expression $y(E_0,E)\int_E^{2E}\overline{K}(E',E)\,dE'$, where $\overline{K}(E',E)$ is chosen so that it is nearly equal to $K(E',E)$ for $E'\sim E$ but can be integrated analytically. This expression constitutes an initial estimate of the left side of (11); the difference $\int_E^{2E}dE'[y(E_0,E')K(E',E)-y(E_0,E)\,\overline{K}(E',E)]$ which remains to be evaluated is small and, above all, receives only a minor contribution from the interval of integration $E'\sim E$. The choice of the function $\overline{K}(E',E)$ was

$$\overline{K}(E',E)=\frac{2Cmc^2}{\beta^2}\left\{\frac{1}{E'-E}-\frac{1}{E}+\frac{E-\frac{1}{2}E'}{(E+1)^2} -\frac{2E+1}{E(E+1)^2}\ln\frac{E}{E'-E}\right\}, \quad \text{for } E'\geqq E+\overline{Q}, \tag{12}$$

$$\overline{K}(E',E)=\overline{K}(E+\overline{Q},E), \quad \text{for } E\leqq E'\leqq E+\overline{Q}.$$

We define then the integral

$$F(E_0,E)=\int_E^{E+\Delta} dE'\overline{K}(E',E)=\frac{2Cmc^2}{\beta^2}\left\{1+\ln\frac{\Delta}{\overline{Q}}\right.$$

$$\left.-\frac{\Delta}{E}-\frac{2E+1}{E(E+1)^2}\Delta\left(1+\ln\frac{E}{\Delta}\right)+\frac{1}{4}\Delta\frac{2E-\Delta}{(E+1)^2}\right\}, \quad (13)$$

where

$$\Delta=\begin{cases}E, & \text{for } E<E_0,\\ E_0-E, & \text{for } E>\frac{1}{2}E_0,\end{cases}$$

and where terms of the order of $\overline{Q}$ have been disregarded. We add on the left of (11) the product $y(E_0,E)\ F(E_0,E)$ and compensate this addition by subtracting $y(E_0,E)\ \overline{K}(E',E)$ from the integrand. The equation can then be written in the form

$$y(E_0,E)=\frac{1}{F(E_0,E)}\left\{1-\int_{E+\Delta}^{E_0} dE'y(E_0,E')\right.$$

$$\times K(E',E)-\int_E^{E+\Delta} dE'[y(E_0,E')K(E',E)$$

$$\left.-y(E_0,E)\overline{K}(E',E)]\right\}. \quad (14)$$

By disregarding the integrals in this formula, one obtains $y(E_0,E)=1/F(E_0,E)$ which represents an initial approximation to the solution of our problem. Comparison of eq (13) with the stopping-power formulas of [3] shows that $F(E_0,E)$ is approximately equal to the stopping power, calculated by excluding all energy losses $\epsilon>\Delta$. Therefore, our initial approximation is essentially equivalent to the solution obtained from the model of continuous slowing down. The exclusion of losses $\epsilon>\Delta$ has an appreciable effect when E is not much smaller than E_0 so that Δ becomes small. In this range $F(E_0,E)$ becomes substantially smaller than the stopping power, and the approximation $y(E_0E)\sim 1/F(E_0,E)$ is superior to the approximation of continuous slowing down, as indicated in section 5.

The actual calculations were carried out by solving numerically the complete eq (14). The solution utilizes the fact that, for any given value of E, the integrals in (14) depend primarily on values of $y(E_0,E')$ for E' substantially larger than E, and can, therefore, be evaluated accurately if the solution has previously been obtained at a sufficient number of values of E' between E and E_0. One thus starts solving at $E=E_0$, where the integrals vanish, and obtains $y(E_0,E)$ in succession for lower values of E.

Since eq (14) may not be used at $E=E_0$, the first step in this procedure requires special attention. The numerical integration is carried out by replacing the integrals in eq (14) by finite sums and choosing a set of points E_i, one per interval of integration, at which the integrand is evaluated. Likewise, a suitable integration formula must be selected (e. g., trapezoidal rule or Simpson's rule). It is advantageous to evaluate the integrals in eq (14) on a logarithmic scale of energy values; that is, to have E_i take on values E_0, ζE_0, $\zeta^2 E_0$, etc., (where $\zeta<1$). The reasons for this are: (1) The integration can proceed owing to the smaller and smaller intervals of E even if the population of secondary electrons increases so rapidly at low energies that the solution diverges; (2) the special behavior at $E'=2E$ can be taken into account very simply by choosing ζ according to $\zeta^n=\frac{1}{2}$, where n is an integer; and (3) due to the logarithmic distribution, the same set of points may be used for different primary energies.

Rewriting eq (14) in logarithmic [3] form, we obtain

$$y(E_0,E)F(E_0,E)=1$$

$$-\int_{\ln 2}^{\ln E_0/E} d\ln(E'/E)[E'K(E',E)y(E_0,E')]$$

$$-\int_0^{\ln (1+\Delta/E)} d\ln(E'/E)[E'y(E_0,E')K(E',E)$$

$$-E'y(E_0,E)\overline{K}(E',E)], \quad (15)$$

where Δ is E_0-E or E, as in eq (13).

In their pilot calculations Spencer and Fano used $\zeta^3=1/2$, but since the present work was done on an electronic computing machine, a smaller mesh of energy values was practical so that $\zeta^6=1/2$ was chosen.

Simpson's rule, which means approximation by parabolas, was applied to both an even and an odd number of intervals of integration, spaced evenly on a logarithmic scale. In case of an odd number of intervals, the last two were fitted with a parabola which was integrated over the last interval only.

If ω_{in} and ω_{in}^* represent the Simpson's rule coefficients and the integrals in eq (15) are replaced by sums, one obtains

$$y(E_0,E_n)F(E_0,E_n)=1-\sum_{i=0}^{n-6}\omega_{in}^* y(E_0,E_i)E_iK(E_i,E_n)$$

$$-\sum_{I=n-6}^{n}\omega_{in}[E_iy(E_0,E_i)K(E_i,E_n)$$

$$-E_iy(E_0,E_n)K(E_i,E_n)]. \quad (16)$$

It is seen from eq (7) and (12) that in the limit as $E'\to E$, $K(E',E)\to 2Cmc^2/\beta'^2(E'-E)$ and $\overline{K}(E',E)\to 2Cmc^2/\beta^2(E'-E)$. Therefore, the term $i=n$ in the second sum of eq (16) approaches

$$\omega_{nn}E_n\left[\frac{d(2Cmc^2/\beta^2)\ y(E_0,E_i)}{dE_i}\right]_{E_i=E_n},$$

in the limit as $i\to n$. This term may be approxi-

[3] The symbol ln refers to the logarithm to the base $e=2.71828\ldots$.

mated by fitting a parabola to the last two intervals of integration and differentiating. Let w_i represent the coefficients in the approximate expression for the derivative and β_i be a function of energy E_i, for $i=n$, $n-1$, and $n-2$. Then

$$\omega_{nn}E_n\left[\frac{d(2Cmc^2/\beta_i^2)\,y(E_0,E_i)}{dE_i}\right]_{E_i=E_n}$$
$$\cong\omega_{nn}E_n[w_{n-2}(2Cmc^2/\beta^2{}_{n-2})y(E_0,E_{n-2})$$
$$+w_{n-1}(2Cmc^2/\beta^2{}_{n-1})y(E_0,E_{n-1})$$
$$+w_n(2Cmc^2/\beta_n^2)y(E_0E_n)]\cdot$$

One sees by inspection that each term in eq (16) is independent of the scaling factor $2Cmc^2/\beta^2$. The solution of the equation may be carried out in terms of the scaled quantities discussed in section 4; the nature of the material enters only through the dependence of $F(E_0,E_n)$ on I.

The solution of eq (16) may be expressed in terms of effective weights W_{in} according to the following iteration formula:

$$y(E_0,E_n)=\frac{1+\sum_{i=0}^{n-1}W_{in}y(E_0,E_i)}{D(E_0,E_n)}, \tag{17}$$

where

$$D(E_0,E_n)=F(E_0,E_n)-\sum_{i=n-6}^{n-1}\omega_{in}E_i\overline{K}(E_i,E_n)$$
$$+E_n\omega_{nn}w_n(2Cmc^2/\beta_n^2), \tag{18}$$

$$W_{in}=\begin{cases}-\omega_{in}E_iK(E_i,E_n), \\ \qquad\text{for } 0\leqq i\leqq n-3, \\ -\omega_{in}E_iK(E_i,E_n)-\omega_{nn}E_nw_i(2Cmc^2/\beta_i^2), \\ \qquad\text{for } i=n-2,\ n-1.\end{cases} \tag{19}$$

The fact that W_{in} seems to have two different values for $i=n-6$ is of no consequence since this is the term for which $E_n=\frac{1}{2}E_i$ and all the cross sections are identically zero.

The only remaining detail is how to start the iteration at $E_n=E_0$. Although $y(E_0,E)$ diverges sharply as E approaches E_0, this peak fortunately contributes only a negligible amount to the integral so that it may be clipped off at some convenient level such as $y(E_0,0.95E_0)$. An approximate solution,

$$y(E_0,E_n)\approx\frac{1}{F(E_0,E_n)}\left[1-\frac{\pi^2/6}{[F(E_0,E_n)/(2Cmc^2/\beta^2)]^2}\right], \tag{20}$$

which gives accurate values for $y(E_0,E_n)$ for E_n near E_0, was used for $E_n=0.95E_0$ and $E_n=E_1=\zeta E_0$. Then eq (17), (18), and (19) were used to complete the spectrum for a particular source energy E_0.

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7. References

[1] L. V. Spencer and F. H. Attix, Radiation Research **3,** 239 (1955).
[2] L. V. Spencer and U. Fano, Phys. Rev. **93,** 1172 (1954).
[3] A. T. Nelms, NBS Circ. 577 (1956).
[4] C. J. Bakker and E. Segrè, Phys. Rev. **81,** 489 (1951).
[5] D. O. Caldwell, Phys. Rev. **100,** 291 (1955).
[6] F. Bloch, Z. Physik **81,** 363 (1933).
[7] G. L. Hine and G. L. Brownell, editors, Radiation dosimetry (Academic Press, Inc., New York, N. Y., 1956).
[8] R. M. Sternheimer, Phys. Rev. **88,** 851 (1952).
[9] A. T. Nelms, NBS Circ. 577, Supplement (1958).

TABLE 1. *Materials for which electron spectra were computed*

Material	Z/A	I	Source of I
		ev	
Al	0.48202	163.0	Caldwell
Cu	.45619	377.0	Caldwell
Sn	.42123	709.0	Caldwell
Pb	.39571	1180.0	Caldwell
Air	.49913	94.9	Caldwell
Water	.55556	74.1	Caldwell
Bone	.53018	92.13	Bloch
Muscle	.55014	71.53	Bloch
Polyethylene (C_2H_4)	.57143	54.9	Caldwell
Polystyrene (C_2H_2)$_4$	.53846	63.8	Caldwell
AgBr	.43664	574.0	Caldwell
Al	.48202	150.0	Bakker-Segrè
Cu	.45619	276.0	Bakker-Segrè
Sn	.42123	463.0	Bakker-Segrè
Pb	.39571	705.0	Bakker-Segrè
Air	.49913	80.5	Bakker-Segrè
Water	.55556	71.25	Bloch
Bone [$Ca_3(PO_4)_2$]	.49633	171.7	Bloch
AgBr	.43664	559.5	Bloch

TABLE 2. *Electron slowing-down spectra $y(E_0,E)$ for various monoenergetic sources in terms of differential track length in mass units [$g\ cm^{-2}(mc^2)^{-1}$]*

Aluminum

I=163.0 ev, Z/A=0.48202

E_0 / E	mc^2	12.90	8.127	5.120	3.225	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.0800	0.05040	0.03175	0.02000	0.01260
	Mev	6.592	4.153	2.616	1.648	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
mc^2	*Mev*																
10.24	5.232	0.3062															
7.241	3.700	.3106	0.3268														
5.120	2.616	.3232	.3228	0.3544													
3.620	1.850	.3405	.3309	.3343													
2.560	1.308	.3616	.3422	.3354	0.3441												
1.810	0.9250	.3860	.3544	.3382	.3345	0.3521											
1.280	.6540	.4134	.3663	.3393	.3267	.3261	0.3590										
0.9051	.4625	.4434	.3772	.3372	.3154	.3065	.3094										
.6400	.3270	.4762	.3875	.3325	.3004	.2839	.2785	0.2852									
.4525	.2312	.5122	.3984	.3268	.2833	.2589	.2473	.2448	0.2568								
.3200	.1635	.5521	.4117	.3223	.2668	.2340	.2165	.2089	.2086	0.2302							
.2263	.1156	.5969	.4289	.3212	.2534	.2121	.1884	.1764	.1719	.1733							
.1600	.08176	.6475	.4516	.3253	.2449	.1949	.1652	.1488	.1409	.1384	0.1418						
.1131	.05779	.7051	.4807	.3354	.2423	.1836	.1478	.1270	.1159	.1108	.1097	0.1154					
.08000	.04088	.7708	.5168	.3521	.2460	.1784	.1364	.1111	.09683	.08942	.08627	.08612	0.09656				
.05657	.02891	.8460	.5609	.3756	.2558	.1790	.1305	.1008	.08327	.07351	.06861	.06671	.06733				
.04000	.02044	.9328	.6138	.4063	.2717	.1850	.1298	.09543	.07455	.06239	.05574	.05250	.05143	0.05296			
.02828	.01445	1.034	.6770	.4447	.2938	.1962	.1338	.09433	.06994	.05528	.04684	.04230	.04018	.03968	0.04204		
.02000	.01022	1.153	.7526	.4920	.3225	.2127	.1420	.09704	.06885	.05153	.04120	.03531	.03219	.03083	.03074	0.03545	
.01414	.007225	1.294	.8437	.5500	.3588	.2347	.1546	.1033	.07086	.05062	.03825	.03091	.02676	.02462	.02376	.02401	
.01000	.005110	1.467	.9550	.6216	.4044	.2632	.1719	.1132	.07581	.05224	.03758	.02865	.02338	.02044	.01896	.01845	0.01921
.007071	.003613	1.680	1.093	.7110	.4618	.2998	.1948	.1271	.08379	.05628	.03897	.02823	.02170	.01787	.01575	.01474	.01447
.005000	.002555	1.951	1.269	.8251	.5355	.3471	.2249	.1460	.09530	.06294	.04241	.02951	.02151	.01665	.01382	.01229	.01159
.003536	.001807	2.305	1.500	.9748	.6325	.4097	.2650	.1715	.1113	.07278	.04818	.03258	.02277	.01668	.01300	.01086	.009729
.002500	.001277	2.788	1.814	1.179	.7649	.4953	.3202	.2069	.1339	.08702	.05699	.03783	.02565	.01799	.01323	.01037	.008716
.001768	.0009034	3.482	2.266	1.472	.9553	.6185	.3998	.2582	.1669	.1081	.07036	.04618	.03072	.02088	.01468	.01083	.008511
.001250	.0006387	4.562	2.968	1.929	1.252	.8105	.5239	.3383	.2185	.1413	.09170	.05982	.03934	.02622	.01785	.01256	.009276
.0008839	.0004517	6.479	4.216	2.740	1.778	1.151	.7444	.4807	.3104	.2007	.1301	.08458	.05529	.03643	.02431	.01656	.01164

Copper

I=377.0 ev, Z/A=0.45619

E_0 / E	mc^2	3.225	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.0800	0.05040	0.03175	0.02000	0.01260
	Mev	1.648	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
mc^2	*Mev*													
2.560	1.308	0.3999												
1.810	0.9250	.3897	0.4121											
1.280	.6540	.3829	.3821	0.4248										
0.9051	.4625	.3727	.3612	.3648										
.6400	.3270	.3586	.3371	.3299	0.3387									
.4525	.2312	.3426	.3105	.2951	.2917	0.3075								
.3200	.1635	.3276	.2843	.2609	.2507	.2502	0.2795							
.2263	.1156	.3168	.2617	.2300	.2138	.2076	.2095							
.1600	.08176	.3124	.2452	.2051	.1827	.1719	.1684	0.1731						
.1131	.05779	.3158	.2361	.1872	.1586	.1433	.1362	.1346	0.1423					
.08000	.04088	.3278	.2349	.1769	.1419	.1219	.1114	.1069	.1067	0.1216				
.05657	.02891	.3486	.2417	.1740	.1321	.1073	.09340	.08633	.08355	.08439				
.04000	.02044	.3789	.2563	.1780	.1289	.09889	.08130	.07160	.06683	.06521	0.06753			
.02828	.01445	.4197	.2792	.1889	.1316	.09593	.07438	.06187	.05507	.05185	.05105	0.05436		
.02000	.01022	.4730	.3113	.2068	.1401	.09800	.07202	.05640	.04740	.04260	.04044	.04028	0.04753	
.01414	.007225	.5421	.3544	.2328	.1547	.1050	.07385	.05468	.04321	.03666	.03323	.03181	.03213	
.01000	.005110	.6325	.4120	.2688	.1764	.1174	.07992	.05651	.04213	.03356	.02871	.02623	.02533	0.02679
.007071	.003613	.7539	.4901	.3185	.2076	.1363	.09084	.06208	.04411	.03308	.02653	.02285	.02103	.02050
.005000	.002555	.9234	.5996	.3890	.2525	.1646	.1082	.07230	.04957	.03535	.02663	.02146	.01859	.01724
.003536	.001807	1.174	.7624	.4941	.3202	.2079	.1356	.08934	.05982	.04111	.02939	.02219	.01794	.01560

TABLE 2. *Electron slowing-down spectra* $y(E_0, E)$ *for various monoenergetic sources in terms of differential track length in mass units* [$g\ cm^{-2}(mc^2)^{-1}$]—Continued

Tin

I=709.0 ev, Z/A=0.42123

E_0 mc^2	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.08000	0.05040	0.03175	0.02000	0.01260
Mev	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
E												
mc^2 *Mev*												
1.810 0.9250	0.4857											
1.280 .6540	.4508	0.5056										
0.9051 .4625	.4282	.4328										
.6400 .3270	.4026	.3931	0.4046									
.4525 .2312	.3742	.3541	.3495	0.3700								
.3200 .1635	.3467	.3160	.3025	.3017	0.3408							
.2263 .1156	.3240	.2820	.2604	.2520	.2545							
.1600 .08176	.3090	.2553	.2253	.2106	.2058	0.2123						
.1131 .05779	.3036	.2376	.1989	.1779	.1682	.1659	0.1761					
.0800 .04088	.3088	.2297	.1817	.1541	.1396	.1332	.1328	0.1537				
.05657 .02891	.3251	.2315	.1735	.1388	.1193	.1092	.1052	.1063				
.04000 .02044	.3534	.2434	.1741	.1316	.1065	.09255	.08560	.08318	0.08672			
.02828 .01445	.3954	.2659	.1835	.1320	.1007	.08232	.07226	.06742	.06617	0.07056		
.02000 .01022	.4541	.3007	.2023	.1400	.1013	.07793	.06432	.05696	.05360	.05333	0.06392	
.01414 .007225	.5354	.3512	.2324	.1565	.1088	.07917	.06133	.05101	.04553	.04319	.04342	
.01000 .005110	.6502	.4242	.2779	.1840	.1242	.08660	.06335	.04935	.04130	.03711	.03555	0.03877
.007071 .003613	.8197	.5332	.3474	.2276	.1509	.1021	.07140	.05240	.04096	.03442	.03108	.02972
.005000 .002555	1.092	.7093	.4608	.3002	.1969	.1307	.08864	.06208	.04559	.03566	.03003	.02762

Lead

I=1180.0 ev, Z/A=0.39571

E_0 mc^2	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.08000	0.05040	0.03175	0.02000
Mev	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022
E											
mc^2 *Mev*											
1.810 0.9250	0.5566										
1.280 .6540	.5172	0.5849									
0.9051 .4625	.4937	.4992									
.6400 .3270	.4673	.4554	0.4698								
.4525 .2312	.4383	.4130	.4071	0.4325							
.3200 .1635	.4108	.3718	.3546	.3536	0.4035						
.2263 .1156	.3893	.3359	.3081	.2972	.3003						
.1600 .08176	.3776	.3088	.2700	.2509	.2445	0.2531					
.1131 .05779	.3783	.2928	.2423	.2148	.2019	.1988	0.2117				
.08000 .04088	.3931	.2893	.2260	.1894	.1700	.1614	.1608	0.1887			
.05657 .02891	.4235	.2990	.2214	.1748	.1483	.1345	.1290	.1303			
.04000 .02044	.4722	.3231	.2288	.1706	.1361	.1167	.1069	.1034	0.1088		
.02828 .01445	.5435	.3640	.2492	.1772	.1331	.1071	.09273	.08571	.08375	0.08895	
.02000 .01022	.6456	.4266	.2855	.1959	.1399	.1058	.08579	.07483	.06972	.06944	0.08286
.01414 .007225	.7946	.5208	.3437	.2301	.1583	.1135	.08627	.07035	.06173	.05787	.05719

TABLE 2. *Electron slowing-down spectra* $y(E_0,E)$ *for various monoenergetic sources in terms of differential track length in mass units* [$g\ cm^{-2}(mc^2)^{-1}$]—Continued

Air

(0.755 N, 0.232 O, 0.013 A by weight)

I=94.9 ev, Z/A=0.49913

E_0 mc^2	12.90	8.127	5.120	3.225	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.08000	0.05040	0.03175	0.02000	0.01260
E Mev	6.592	4.153	2.616	1.648	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
mc^2 Mev																
10.24 5.232	0.2821															
7.241 3.700	.2858	0.3001														
5.120 2.616	.2967	.2963	0.3238													
3.620 1.850	.3114	.3030	.3060													
2.560 1.308	.3291	.3123	.3063	0.3139												
1.810 0.9250	.3493	.3219	.3079	.3047	0.3199											
1.280 .6540	.3715	.3307	.3074	.2966	.2962	0.3242										
0.9051 .4625	.3954	.3382	.3038	.2851	.2776	.2800										
.6400 .3270	.4209	.3446	.2975	.2700	.2560	.2514	0.2571									
.4525 .2312	.4486	.3512	.2900	.2529	.2322	.2224	.2203	0.2305								
.3200 .1635	.4791	.3593	.2833	.2362	.2084	.1936	.1873	.1871	0.2051							
.2263 .1156	.5132	.3707	.2794	.2221	.1873	.1674	.1574	.1536	.1548							
.1600 .08176	.5519	.3864	.2799	.2123	.1703	.1455	.1318	.1253	.1232	0.1260						
.1131 .05779	.5958	.4072	.2855	.2076	.1586	.1287	.1115	.1023	.09817	.09728	0.1020					
.08000 .04088	.6457	.4337	.2964	.2081	.1520	.1172	.09644	.08469	.07863	.07607	.07596	0.08443				
.05657 .02891	.7025	.4661	.3128	.2138	.1505	.1107	.08629	.07195	.06402	.06004	.05852	.05904				
.04000 .02044	.7675	.5051	.3346	.2243	.1534	.1084	.08040	.06346	.05363	.04828	.04570	.04485	0.04606			
.02828 .01445	.8420	.5512	.3622	.2396	.1605	.1100	.07813	.05851	.04676	.04002	.03642	.03475	.03436	0.03628		
.02000 .01022	.9284	.6057	.3959	.2596	.1715	.1149	.07898	.05652	.04277	.03459	.02995	.02752	.02646	.02640	0.03006	
.01414 .007225	1.029	.6704	.4367	.2848	.1865	.1231	.08257	.05701	.04112	.03144	.02572	.02251	.02086	.02021	.02041	
.01000 .005110	1.149	.7476	.4861	.3160	.2057	.1345	.08877	.05972	.04146	.03014	.02328	.01924	.01701	.01590	.01552	0.01607
.007071 .003613	1.294	.8410	.5462	.3544	.2300	.1494	.09763	.06454	.04358	.03043	.02230	.01738	.01451	.01294	.01220	.01201
.005000 .002555	1.471	.9558	.6204	.4022	.2605	.1687	.1095	.07160	.04744	.03216	.02259	.01667	.01311	.01104	.009937	.009438
.003536 .001807	1.693	1.100	.7137	.4625	.2992	.1934	.1251	.08125	.05320	.03535	.02406	.01699	.01263	.01000	.008500	.007711
.002500 .001277	1.978	1.285	.8340	.5403	.3494	.2257	.1457	.09426	.06128	.04021	.02680	.01831	.01299	.009710	.007748	.006634
.001768 .0009034	2.358	1.532	.9943	.6441	.4165	.2689	.1734	.1120	.07251	.04722	.03106	.02075	.01423	.01013	.007613	.006112
.001250 .0006387	2.888	1.876	1.217	.7887	.5100	.3292	.2123	.1369	.08849	.05739	.03745	.02469	.01654	.01136	.008120	.006124
.0008839 .0004517	3.672	2.385	1.548	1.003	.6486	.4187	.2700	.1741	.1124	.07275	.04728	.03092	.02042	.01370	.009435	.006753

Water

I=74.1 ev, Z/A=0.55556

E_0 mc^2	12.90	8.127	5.120	3.225	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.08000	0.05040	0.03175	0.02000	0.01260
E Mev	6.592	4.153	2.616	1.648	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
mc^2 Mev																
10.24 5.232	0.2482															
7.241 3.700	.2514	0.2637														
5.120 2.616	.2606	.2603	0.2840													
3.620 1.850	.2731	.2660	.2685													
2.560 1.308	.2881	.2737	.2686	0.2751												
1.810 0.9250	.3050	.2815	.2696	.2668	0.2798											
1.280 .6540	.3235	.2885	.2686	.2594	.2590	0.2829										
0.9051 .4625	.3431	.2942	.2649	.2489	.2425	.2446										
.6400 .3270	.3639	.2988	.2586	.2352	.2232	.2194	0.2242									
.4525 .2312	.3863	.3032	.2511	.2196	.2020	.1937	.1919	0.2006								
.3200 .1635	.4109	.3090	.2444	.2044	.1808	.1683	.1630	.1628	0.1780							
.2263 .1156	.4385	.3174	.2400	.1914	.1619	.1451	.1366	.1334	.1345							
.1600 .08176	.4698	.3295	.2393	.1821	.1466	.1256	.1141	.1086	.1069	0.1093						
.1131 .05779	.5053	.3458	.2429	.1771	.1358	.1107	.09616	.08846	.08498	.08424	0.08822					
.08000 .04088	.5457	.3668	.2511	.1767	.1295	.1003	.08279	.07294	.06787	.06574	.06565	0.07271				
.05657 .02891	.5916	.3927	.2638	.1806	.1274	.09405	.07365	.06167	.05504	.05173	.05047	.05090				
.04000 .02044	.6438	.4237	.2809	.1885	.1292	.09156	.06818	.05405	.04587	.04143	.03929	.03859	0.03959			
.02828 .01445	.7035	.4605	.3026	.2003	.1344	.09228	.06580	.04949	.03974	.03415	.03117	.02980	.02948	0.03108		
.02000 .01022	.7723	.5038	.3292	.2160	.1428	.09581	.06603	.04743	.03606	.02931	.02549	.02349	.02262	.02257	0.02557	
.01414 .007225	.8522	.5547	.3613	.2356	.1543	.1019	.06853	.04746	.03437	.02641	.02172	.01909	.01775	.01722	.01739	
.01000 .005110	.9462	.6151	.3998	.2598	.1691	.1106	.07312	.04930	.03434	.02508	.01948	.01619	.01437	.01347	.01317	0.01361
.007071 .003613	1.058	.6874	.4462	.2895	.1878	.1220	.07978	.05282	.03575	.02506	.01845	.01447	.01215	.01089	.01029	.01014
.005000 .002555	1.194	.7752	.5029	.3259	.2110	.1366	.08871	.05804	.03852	.02618	.01847	.01371	.01084	.009193	.008310	.007916
.003536 .001807	1.361	.8837	.5731	.3712	.2400	.1551	.1003	.06517	.04271	.02843	.01941	.01377	.01030	.008215	.007025	.006404
.002500 .001277	1.572	1.021	.6620	.4286	.2770	.1788	.1154	.07467	.04856	.03189	.02130	.01460	.01041	.007837	.006301	.005433
.001768 .0009034	1.846	1.199	.7774	.5033	.3253	.2099	.1353	.08735	.05656	.03685	.02426	.01625	.01118	.008009	.006063	.004909
.001250 .0006387	2.216	1.439	.9331	.6041	.3904	.2519	.1623	.1047	.06761	.04385	.02862	.01889	.01268	.008755	.006296	.004790
.0008839 .0004517	2.738	1.778	1.153	.7466	.4825	.3113	.2006	.1293	.08343	.05398	.03507	.02294	.01517	.01021	.007065	.005095

TABLE 2. *Electron slowing-down spectra* $y(E_0,E)$ *for various monoenergetic sources in terms of differential track length in mass units* [$g\ cm^{-2}(mc^2)^{-1}$]—Continued

Bone

(0.064 H, 0.278 C, 0.027 N, 0.410 O, 0.002 Mg, 0.070 P, 0.002 S, 0.147 Ca by weight)

I=92.13 ev, Z/A=0.53018

E_0 mc^2	12.90	8.127	5.120	3.225	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.0800	0.05040	0.03175	0.02000	0.01260
E *Mev*	6.592	4.153	2.616	1.648	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
mc^2 *Mev*																
10.24 5.232	0.2649															
7.241 3.700	.2684	0.2818														
5.120 2.616	.2785	.2782	0.3040													
3.620 1.850	.2923	.2845	.2873													
2.560 4.308	.3089	.2931	.2876	0.2947												
1.810 0.9250	.3277	.3021	.2890	.2859	0.3002											
1.280 .6540	.3484	.3103	.2885	.2783	.2779	0.3041										
0.9051 .4625	.3707	.3171	.2850	.2675	.2604	.2627										
.6400 .3270	.3944	.3231	.2790	.2533	.2401	.2359	0.2412									
.4525 .2312	.4201	.3290	.2718	.2371	.2177	.2086	.2066	0.2162								
.3200 .1635	.4485	.3365	.2654	.2213	.1954	.1816	.1757	.1754	0.1923							
.2263 .1156	.4802	.3469	.2616	.2080	.1755	.1569	.1475	.1440	.1452							
.1600 .08176	.5162	.3615	.2619	.1987	.1595	.1363	.1236	.1174	.1155	0.1181						
.1131 .05779	.5570	.3808	.2670	.1942	.1484	.1206	.1044	.09588	.09201	.09117	0.09559					
.08000 .04088	.6034	.4053	.2771	.1946	.1422	.1097	.09030	.07932	.07367	.07128	.07118	0.07908				
.05657 .02891	.6562	.4354	.2922	.1998	.1406	.1035	.08073	.06735	.05995	.05624	.05482	.05530				
.04000 .02044	.7165	.4716	.3125	.2095	.1433	.1013	.07516	.05936	.05019	.04520	.04279	.04200	0.04313			
.02828 .01445	.7858	.5144	.3380	.2236	.1498	.1027	.07298	.05468	.04373	.03744	.03408	.03253	.03217	0.03396		
.02000 .01022	.8659	.5650	.3692	.2421	.1600	.1072	.07371	.05277	.03995	.03234	.02801	.02574	.02476	.02470	0.02811	
.01414 .007225	.9596	.6248	.4070	.2655	.1738	.1147	.07699	.05318	.03837	.02936	.02403	.02104	.01951	.01890	.01909	
.01000 .005110	1.071	.6963	.4527	.2943	.1916	.1252	.08269	.05565	.03865	.02811	.02172	.01797	.01589	.01486	.01451	0.01502
.007071 .003613	1.204	.7826	.5083	.3298	.2140	.1391	.09086	.06007	.04057	.02834	.02078	.01621	.01354	.01209	.01139	.01122
.005000 .002555	1.368	.8886	.5768	.3739	.2421	.1568	.1018	.06656	.04411	.02991	.02102	.01553	.01222	.01030	.009273	.008811
.003536 .001807	1.572	1.021	.6627	.4294	.2778	.1796	.1162	.07544	.04940	.03283	.02235	.01579	.01174	.009314	.007921	.007189
.002500 .001277	1.835	1.192	.7733	.5009	.3239	.2092	.1351	.08738	.05681	.03728	.02485	.01698	.01206	.009021	.007205	.006174
.001768 .0009034	2.183	1.418	.9201	.5960	.3854	.2488	.1605	.1036	.06708	.04369	.02873	.01921	.01317	.009389	.007061	.005674
.001250 .0006387	2.666	1.732	1.124	.7279	.4706	.3038	.1959	.1263	.08164	.05295	.03456	.02278	.01526	.01050	.007505	.005666
.0008839 .0004517	3.376	2.193	1.423	.9221	.5962	.3849	.2482	.1600	.1033	.06685	.04345	.02841	.01877	.01260	.008680	.006218

Muscle

(0.102 H, 0.123 C, 0.035 N, 0.729 O, 0.0008 Na, 0.0002 Mg, 0.002 P, 0.005 S, 0.003 K by weight)

I=71.53 ev, Z/A=0.55014

E_0 mc^2	12.90	8.127	5.120	3.225	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.08000	0.05040	0.03175	0.02000	0.01260
E *Mev*	6.592	4.153	2.616	1.648	1.038	.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
mc^2 *Mev*																
10.24 5.232	0.2499															
7.241 3.700	.2531	0.2654														
5.120 2.616	.2624	.2620	0.2858													
3.620 1.850	.2749	.2677	.2703													
2.560 1.308	.2899	.2754	.2703	0.2768												
1.810 0.9250	.3068	.2832	.2712	.2685	0.2815											
1.280 .6540	.3252	.2902	.2702	.2610	.2606	0.2845										
0.9051 .4625	.3448	.2958	.2663	.2503	.2439	.2460										
.6400 .3270	.3655	.3002	.2599	.2365	.2245	.2206	0.2255									
.4525 .2312	.3878	.3045	.2523	.2207	.2031	.1948	.1930	0.2017								
.3200 .1635	.4123	.3102	.2454	.2053	.1817	.1692	.1638	.1636	0.1789							
.2263 .1156	.4397	.3184	.2408	.1921	.1626	.1458	.1373	.1341	.1352							
.1600 .08176	.4708	.3303	.2400	.1827	.1472	.1262	.1146	.1091	.1074	0.1098						
.1131 .05779	.5062	.3465	.2435	.1776	.1362	.1111	.09655	.08885	.08538	.08463	0.08861					
.08000 .04088	.5464	.3673	.2515	.1770	.1298	.1005	.08308	.07322	.06816	.06602	.06594	0.07299				
.05657 .02891	.5920	.3930	.2640	.1808	.1276	.09425	.07384	.06186	.05524	.05194	.05067	.05111				
.04000 .02044	.6440	.4238	.2810	.1886	.1293	.09167	.06829	.05418	.04601	.04157	.03943	.03873	0.03973			
.02828 .01445	.7033	.4603	.3025	.2003	.1344	.09230	.06585	.04956	.03982	.03424	.03127	.02989	.02958	0.03118		
.02000 .01022	.7716	.5033	.3289	.2158	.1426	.09576	.06601	.04745	.03610	.02936	.02555	.02355	.02269	.02264	0.02563	
.01414 .007225	.8509	.5538	.3606	.2352	.1540	.1018	.06845	.04742	.03437	.02643	.02175	.01913	.01779	.01726	.01743	
.01000 .005110	.9440	.6136	.3988	.2592	.1687	.1104	.07295	.04920	.03429	.02506	.01947	.01620	.01439	.01350	.01319	0.01363
.007071 .003613	1.055	.6851	.4447	.2885	.1871	.1216	.07952	.05265	.03565	.02500	.01843	.01446	.01215	.01089	.01030	.01015
.005000 .002555	1.189	.7718	.5007	.3244	.2100	.1360	.08831	.05779	.03836	.02609	.01841	.01367	.01083	.009187	.008310	.007918
.003536 .001807	1.353	.8788	.5699	.3690	.2386	.1542	.09975	.06480	.04247	.02828	.01931	.01371	.01026	.008194	.007014	.006398
.002500 .001277	1.561	1.014	.6572	.4255	.2750	.1775	.1146	.07412	.04820	.03166	.02115	.01451	.01035	.007800	.006278	.005418
.001768 .0009034	1.830	1.188	.7704	.4987	.3223	.2079	.1341	.08653	.05603	.03650	.02404	.01610	.01109	.007949	.006024	.004883
.001250 .0006387	2.191	1.422	.9223	.5971	.3858	.2489	.1604	.1034	.06680	.04332	.02828	.01867	.01254	.008660	.006234	.004748
.0008839 .0004517	2.698	1.752	1.136	.7353	.4752	.3065	.1975	.1273	.08214	.05314	.03453	.02259	.01494	.01006	.006964	.005028

TABLE 2. *Electron slowing-down spectra y* (E_0, E) *for various monoenergetic sources in terms of differential track length in mass units* $[g\ cm^{-2}(mc^2)^{-1}]$—Continued

Polyethylene

(C_2H_4)

$I = 54.9$ ev, $Z/A = 0.57143$

E_0 mc^2	20.48	12.900	8.127	5.120	3.225	2.032	1.280	0.8064	0.5080	0.3200	0.2016	0.1270	0.08000	0.05040	0.03175	0.02000	0.01260
E *Mev*	10.46	6.592	4.153	2.616	1.648	1.038	0.6540	.4120	.2596	.1635	.1030	.06489	.04088	.02575	.01622	.01022	.006438
mc^2 *Mev*																	
20.48 10.46	0.2319																
14.48 7.400	.2242																
10.24 5.232	.2301	0.2354															
7.241 3.700	.2406	.2383	0.2497														
5.120 2.616	.2550	.2468	.2465	0.2683													
3.620 1.850	.2734	.2581	.2515	.2539													
2.560 1.308	.2961	.2717	.2584	.2537	0.2597												
1.810 0.9250	.3235	.2868	.2652	.2542	.2517	0.2636											
1.280 .6540	.3555	.3031	.2710	.2528	.2443	.2440	0.2658										
0.9051 .4625	.3919	.3202	.2754	.2485	.2339	.2281	.2300										
.6400 .3270	.4323	.3382	.2786	.2418	.2205	.2096	.2060	0.2105									
.4525 .2312	.4762	.3574	.2815	.2339	.2052	.1891	.1816	.1800	0.1879								
.3200 .1635	.5231	.3784	.2855	.2265	.1901	.1688	.1574	.1526	.1524	0.1661							
.2263 .1156	.5730	.4020	.2918	.2214	.1772	.1505	.1353	.1276	.1247	.1257							
.1600 .08176	.6259	.4288	.3014	.2196	.1677	.1356	.1166	.1063	.1013	.09976	0.1019						
.1131 .05779	.6824	.4593	.3148	.2217	.1622	.1249	.1022	.08915	.08224	.07912	.07846	0.08205					
.08000 .04088	.7432	.4940	.3324	.2279	.1608	.1183	.09203	.07635	.06752	.06299	.06108	.06101	0.06729				
.05657 .02891	.8094	.5333	.3542	.2382	.1634	.1157	.08576	.06748	.05677	.05085	.04791	.04679	.04718				
.04000 .02044	.8822	.5780	.3805	.2524	.1696	.1165	.08288	.06201	.04942	.04214	.03819	.03630	.03568	0.03656			
.02828 .01445	.9633	.6288	.4115	.2705	.1792	.1204	.08292	.05937	.04488	.03624	.03129	.02866	.02745	.02718	0.02860		
.02000 .01022	1.055	.6869	.4479	.2927	.1921	.1271	.08547	.05909	.04264	.03260	.02665	.02329	.02153	.02077	.02074	0.02335	
.01414 .007225	1.159	.7539	.4905	.3194	.2083	.1365	.09027	.06083	.04228	.03078	.02379	.01968	.01738	.01621	.01575	.01590	
.01000 .005110	1.280	.8320	.5406	.3512	.2282	.1486	.09723	.06436	.04351	.03043	.02235	.01746	.01461	.01303	.01226	.01200	0.01237
.007071 .003613	1.423	.9239	.5999	.3892	.2524	.1637	.1064	.06961	.04617	.03134	.02207	.01635	.01291	.01091	.009822	.009312	.009189
.005000 .002555	1.592	1.034	.6709	.4350	.2817	.1823	.1180	.07668	.05022	.03339	.02278	.01614	.01206	.009612	.008204	.007454	.007122
.003536 .001807	1.798	1.167	.7574	.4909	.3177	.2054	.1327	.08581	.05576	.03658	.02441	.01673	.01193	.008984	.007225	.006224	.005705
.002500 .001277	2.052	1.333	.8646	.5603	.3625	.2342	.1511	.09750	.06307	.04104	.02699	.01807	.01244	.008923	.006769	.005490	.004770
.001768 .0009034	2.375	1.542	1.000	.6483	.4194	.2709	.1747	.1126	.07264	.04704	.03066	.02021	.01357	.009389	.006769	.005170	.004225
.001250 .0006387	2.796	1.815	1.718	.7633	.4938	.3189	.2056	.1324	.08535	.05512	.03574	.02335	.01543	.01639	.007211	.005226	.004015
.0008839 .0004517	3.366	2.186	1.418	.9190	.5946	.3840	.2476	.1594	.1027	.06623	.04283	.02783	.01822	.01207	.008149	.005671	.004126

TABLE 3. *Table of 0.300* mc^2/β^2 *as a function of kinetic energy*

E		$\frac{1}{\beta^2}$	$\frac{0.300\ mc^2}{\beta^2}$	E		$\frac{1}{\beta^2}$	$\frac{0.300\ mc^2}{\beta^2}$
mc^2	*Mev*			mc^2	*Mev*		
20.48	10.46	1.002	0.3012	0.1131	0.05781	5.183	1.558
14.48	7.400	1.004	.3018	.08000	.04088	7.010	2.107
10.24	5.232	1.008	.3030	.05657	.02891	9.596	2.884
7.241	3.700	1.015	.3051	.04000	.02044	13.26	3.984
5.120	2.616	1.027	.3088	.02828	.01445	18.43	5.540
3.620	1.850	1.049	.3153	.02000	.01022	25.75	7.740
2.560	1.308	1.086	.3263	.01414	.007226	36.11	10.85
1.810	0.9250	1.145	.3442	.01000	.005110	50.75	15.25
1.280	.6541	1.238	.3722	.007071	.003613	71.46	21.48
0.9051	.4625	1.380	.4149	.005000	.002555	100.8	30.28
.6400	.3270	1.592	.4785	.003536	.001807	142.2	42.73
.4526	.2312	1.901	.5714	.002500	.001277	200.8	60.34
.3200	.1635	2.347	.7054	.001768	.0009033	283.6	85.24
.2263	.1156	2.985	.8973	.001250	.0006387	400.8	120.5
.1600	.08176	3.894	1.170	.0008839	.0004517	566.4	170.3

TABLE 4. *Probability for production of secondary electrons*

$K(E,\epsilon)$ is the probability for production of knock-on secondary electrons of energy greater than ϵ, but less than 1/2 E, per unit track length of a primary of kinetic energy E. The table gives values of the scaled probability $(\beta^2/2C mc^2)\ K(E,\epsilon)$, which is the same for all materials.

ϵ mc^2 / E Mev	28.28	20.00	14.14	10.00	7.071	5.000	3.536	2.500	1.768	1.250	0.8839	0.6250	0.4419	0.3125	0.2210	0.1562
	14.45	10.22	7.225	5.110	3.613	2.555	1.807	1.277	0.9034	0.6387	.4517	.3194	.2258	.1597	.1129	.07982
mc² *Mev*																
80.00 40.88	0.01762	0.03604	0.05900	0.08969	0.1320	0.1912	0.2744	0.3918	0.5576	0.7919	1.123	1.592	2.255	3.192	4.517	6.392
56.57 28.91		.02479	.05074	.08311	.1264	.1862	.2698	.3874	.5533	.7878	1.119	1.588	2.250	3.187	4.513	6.387
40.00 20.44			.03480	.07129	.1169	.1780	.2624	.3805	.5467	.7812	1.113	1.581	2.244	3.181	4.506	6.380
28.28 14.45				.04871	.09992	.1641	.2502	.3693	.5360	.7707	1.102	1.571	2.233	3.170	4.495	6.368
20.00 10.22					.06791	.1396	.2296	.3508	.5187	.7539	1.085	1.554	2.216	3.152	4.476	6.349
14.14 7.225						.09420	.1941	.3201	.4904	.7267	1.058	1.526	2.188	3.123	4.446	6.318
10.00 5.110							.1298	.2683	.4442	.6829	1.015	1.482	2.142	3.076	4.397	6.267
7.071 3.613								.1773	.3681	.6127	.9465	1.413	2.070	3.000	4.317	6.183
5.000 2.555									.2398	.5009	.8393	1.305	1.958	2.882	4.192	6.049
3.536 1.807										.3207	.6753	1.141	1.788	2.702	3.998	5.841
2.500 1.277											.4243	.9024	1.541	2.438	3.714	5.532
1.768 .9034												.5566	1.198	2.072	3.314	5.093
1.250 .6387													.7276	1.587	2.783	4.502
.8839 .4517														.9539	2.110	3.749
.6250 .3194															1.262	2.828
.4419 .2258																1.694

ϵ mc^2 / E Mev	0.1105	0.07812	0.05524	0.03906	0.02762	0.01953	0.01381	0.009765	0.006905	0.004882	0.003453	0.002441	0.001726	0.001220	0.0008627
	.05646	.03992	.02823	.01996	.01411	.009980	.007057	.004990	.003528	.002495	.001764	.001247	.0008820	.0006234	.0004408
mc² *Mev*															
80.00 40.88	9.042	12.79	18.09	25.59	36.19	51.19	72.40	102.4	144.8	204 8	289.6	409.6	579.2	819 2	1159
56.57 28.91	9.038	12.79	18.09	25.59	36.19	51.19	72.39	102.4	144.8	204.8	289.6	409.6	579.2	819.2	1158
40.00 20.44	9.031	12.78	18.08	25.58	36.18	51.18	72.38	102.4	144.8	204.8	289.6	409.6	579.2	819.2	1158
28.28 14.45	0.019	12.77	18.07	25.57	36.17	51.16	72.37	102.4	144.8	204.8	289.6	409.6	579.2	819.2	1158
20.00 10.22	8.999	12.75	18.05	25.54	36.15	51.14	72.35	102.3	144.8	204.7	289.6	409.5	579.2	819.1	1158
14.14 7.225	8.966	12.71	18.01	25.51	36.11	51.10	72.30	102.3	144.7	204.7	289.5	409.5	579.1	819.1	1158
10.00 5.110	8.912	12.66	17.95	25.44	36.04	51.03	72.23	102.2	144.6	204.6	289.4	409.4	579.0	819.0	1158
7.071 3.613	8.824	12.56	17.85	25.34	35.93	50.92	72.11	102.1	144.5	204.5	289.3	409.2	578.9	818.8	1158
5.000 2.555	8.681	12.41	17.69	25.17	35.75	50.73	71.92	101.9	144.3	204.2	289.1	409.0	578.6	818.6	1158
3.536 1.807	8.458	12.17	17.44	24.90	35.47	50.42	71.59	101.5	143.9	203.9	288.7	408.6	578.2	818.1	1157
2.500 1.277	8.123	11.81	17.05	24.48	35.02	49.95	71.09	101.0	143.4	203.3	288.0	407.9	577.5	817.4	1157
1.768 .9034	7.642	11.28	16.48	23.86	34.35	49.24	70.33	100.2	142.5	202.4	287.1	406.9	576.5	816.3	1156
1.250 .6387	6.984	10.56	15.68	22.99	33.41	48.22	69.23	99.04	141.3	201.1	285.7	405.5	574.9	814.7	1154
.8839 .4517	6.134	9.603	14.61	21.82	32.12	46.82	67.72	97.41	139.5	199.2	283.7	403.4	572.7	812.4	1151
.6250 .3194	5.087	8.407	13.26	20.30	30.45	44.98	65.72	95.24	137.2	196.7	281.1	400.6	569.7	809.2	1148
.4419 .2258	3.835	6.966	11.61	18.42	28.33	42.63	63.13	92.42	134.1	193.4	277.5	396.8	565.7	805.0	1144
.3125 .1597	2.306	5.262	9.630	16.14	25.72	39.69	59.86	88.82	130.2	189.1	272.9	391.8	560.5	799.3	1138
.2210 .1129		3.180	7.296	13.42	22.57	36.08	55.79	84.28	125.2	183.6	267.0	385.4	553.6	792.0	1130
.1562 .07982			4.427	10.19	18.80	31.70	50.77	78.61	118.9	176.7	259.3	377.1	544.6	782.4	1120
.1105 .05646				6.204	14.31	26.45	44.66	71.60	110.9	167.8	249.6	366.4	533.0	769.8	1106
.07812 .03992					8.729	20.16	37.29	63.02	101.1	156.7	237.1	352.7	517.9	753.5	1088
.05524 .02823						12.31	28.45	52.66	89.02	142.8	221.5	335.2	498.6	732.3	1065
.03906 .01996							17.39	40.19	74.40	125.8	201.9	313.1	473.9	705.0	1035
.02762 .01411								24.57	56.80	105.2	177.9	285.5	442.7	670.1	996.9
.01953 .009980									34.73	80.31	148.7	251.5	403.7	626.0	947.6
.01381 .007057										49.11	113.6	270.3	355.6	570.8	885.2
.009765 .004990											69.44	160.6	297.4	502.9	807.3
.006905 .003528												98.20	227.1	420.5	711.2
.004882 .002495													138.9	321.1	594.7
.003453 .001764														196.4	454.2
.002441 .001247															277.7

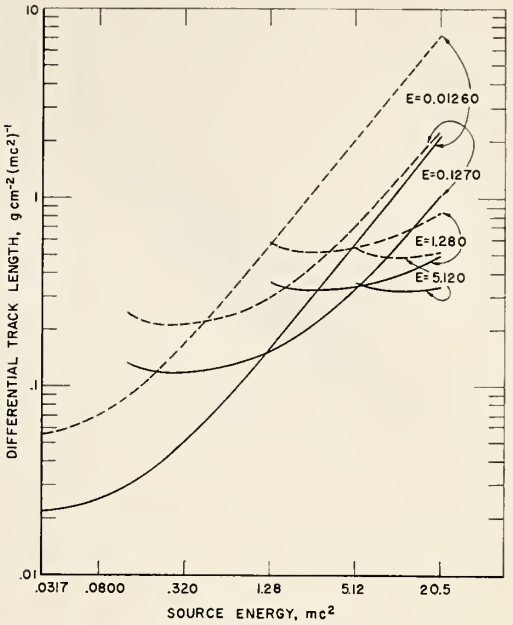

FIGURE 1. *Differential track length in mass units for electrons in lead and aluminum as a function of the source energy for various degraded energies.*

— — — Lead, I—1180.0 ev, and ——— Aluminum, I—16 .0 ev.

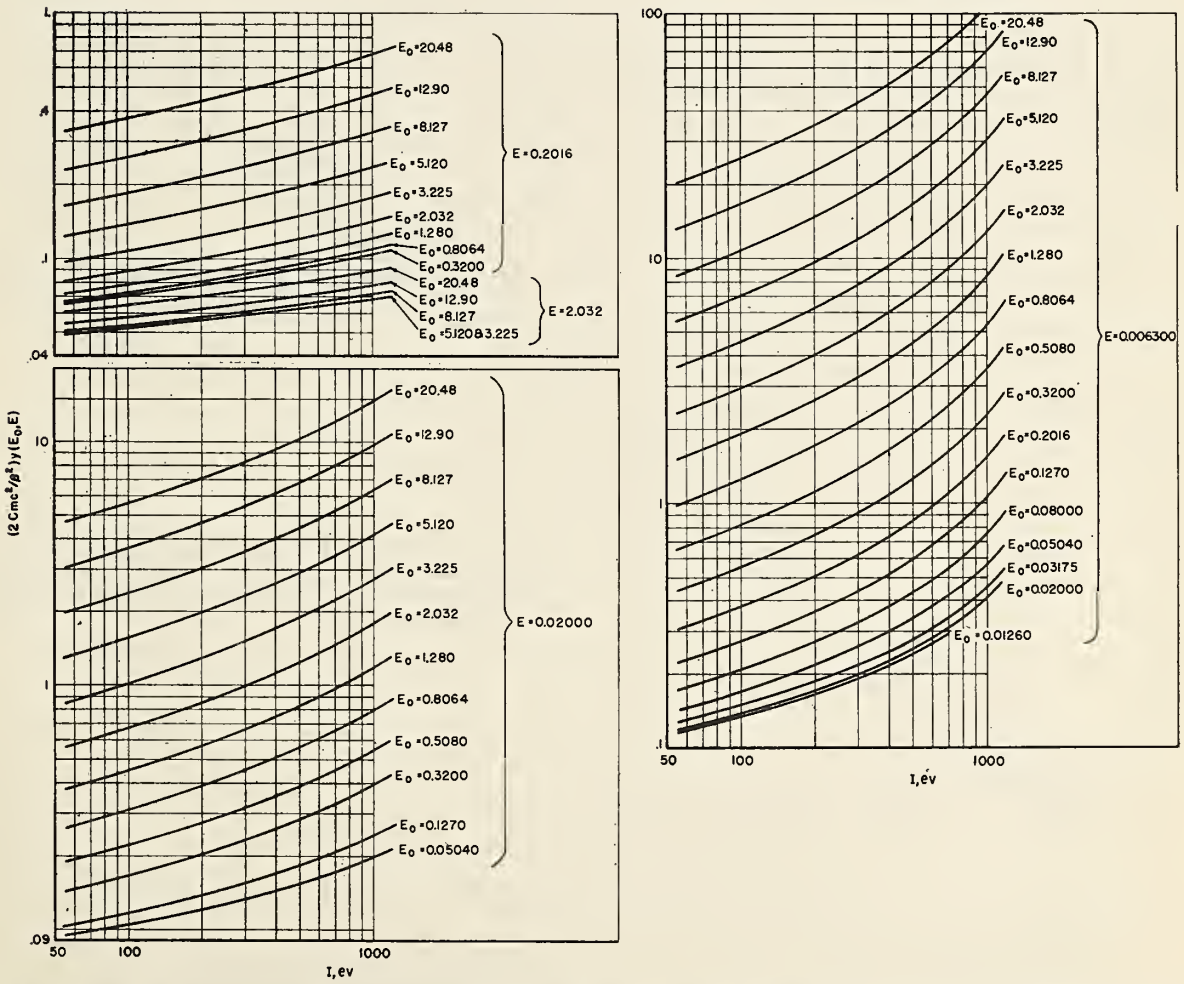

FIGURE 2. *Scaled energy spectra* $(2Cmc^2/\beta^2)y(E_0,E)$ *as a function of average excitation energy* I *for given energies* E_0 *and* E.

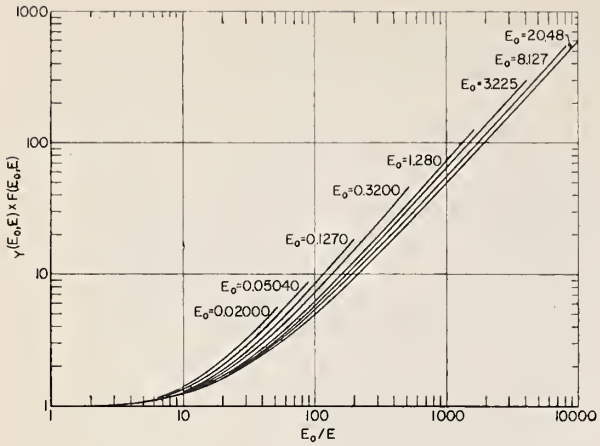

FIGURE 3. *The $\bar{R}$-factor $y(E_0,E)\times F(E_0E)$ for polyethylene ($I=54.9$ ev) as a function of E_0/E for various starting energies E_0.*

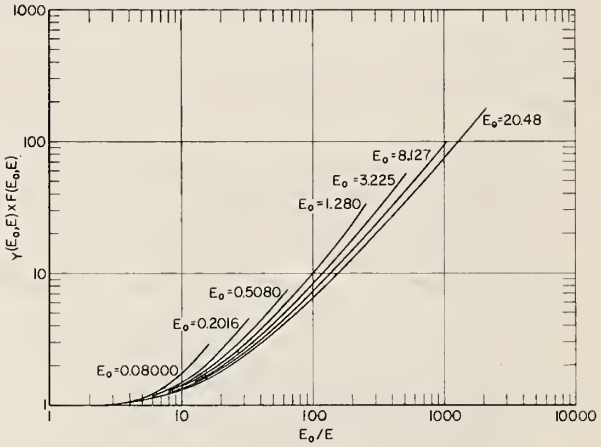

FIGURE 4. *The $\bar{R}$-factor $y(E_0,E)\times F(E_0,E)$ for lead ($I=1180.0$ ev) as a function of E_0/E for various starting energies E_0.*

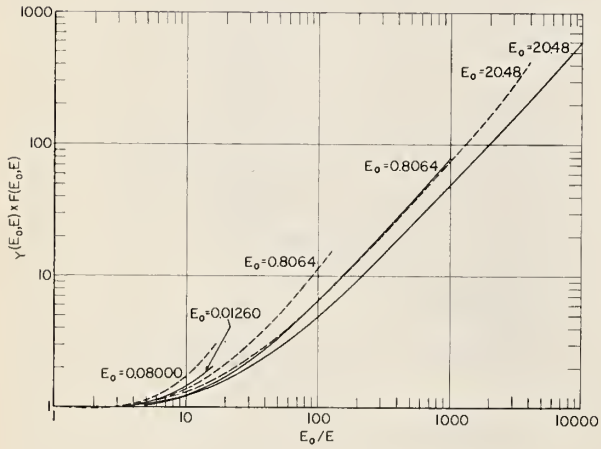

FIGURE 5. *The $\bar{R}$-factor $y(E_0,E)\times F(E_0,E)$ for lead and polyethylene as a function of E_0,E showing the dependence of this factor on the starting energy E_0 and the average excitation energy I.*

— — — Lead, I—1180.0 ev, and ——— Polyethylene, I—54.9 ev.

WASHINGTON, April 9, 1958.

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