NBS CIRCULAR 596

Single-Phase Transfer of Liquefied Gases

UNITED STATES DEPARTMENT OF COMMERCE

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Robert B. Jacobs

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Preface

The greatly increased use of liquefied gases by both governmental and industrial activities has prompted an examination and re-evaluation of the basic problems connected with their handling. The current method of transporting liquefied gases over distances of more than a few hundred feet is by mobile-insulated transport vessels. In this Circular, the feasibility of transporting liquefied gases over appreciable distances through piping systems is discussed, theory for the design of such systems is developed, information required for application of the theory is presented in readily usable graphical form, and illustrative examples are given. The analyses and examples, which are based upon thoroughly verified principles and methods, show that transfer of liquefied gases over appreciable distances can be accomplished with conventional equipment and straightforward techniques.

A. V. Astin, *Director.*

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Single-Phase Transfer of Liquefied Gases

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The problems encountered in the single-phase transfer of liquefied gases are discussed in detail. A general system of equations and the empirical information required for the design of long-distance transfer systems are presented.

A closed-form solution of the mathematical equations for incompressible flows is obtained and discussed. Information required for numerical computations involving helium, hydrogen, nitrogen, and oxygen is presented in graphical form. The numerical solution for three general problems involving hydrogen and oxygen are obtained.

1. Introduction

Recent interest in the large-scale use of liquefied gases has stimulated a re-examination of the fundamental problems associated with their production and transportation. Except for transfers over very short distances, liquefied gases have always been transported in containers; the purpose here is to examine, in detail, the feasibility of transporting liquefied gases over appreciable distances through piping systems, and to present methods for designing such systems.

The problems encountered in the transfer[1] of liquefied gases may be divided into two categories: (1) Those that are predominantly of an unsteady nature and occur before the transfer system has been cooled to its operating condition; (2) those that can be considered as steady state and are applicable after the system has been cooled to its operating condition.

The unsteady cool-down problems are those of two-phase, single-component flow, with boiling heat transfer, which up to the present have apparently not been successfully analyzed; these will not be considered here.

In this paper, the problems occurring after the system has been cooled to its steady operating condition will be discussed. It should be pointed out that it is possible to construct systems which cannot be completely cooled down; liquefied gas cannot be transferred through them. This occurs when the system is relatively long, the pipe diameter is relatively small, and the heat leak is relatively large. The cooling-down difficulties are enhanced by the fact that the propagation velocities in two-phase, single-component fluids are extremely low, permitting the flow to be choked quite readily. However, any system that will transfer a liquefied gas under steady-state conditions can be cooled down. It is only necessary that provision be made to vent the system at a sufficient number of locations so that the system will not be choked.

The broad questions to be discussed here are twofold: (1) What equipment and techniques are required for the long-distance transfer of liquefied gases, and are these now available? (2) are the losses which will occur during these long-distance transfers tolerable?

2. Physical System

Figure 1 is a schematic representation of a transfer system. The primary component may be considered to be a length of insulated pipe (a transfer line) with the state of the fluid being specified at both the inlet and outlet. The energy required to force the fluid through the pipe can be supplied from a pumping system or from a process plant such as a liquefier. It may be desirable not to supply all of the pumping energy at the inlet to the transfer system, but to have pumping stations distributed along the line; in this case the system consists of a series of pipes with the fluid state specified at the inlet and outlet of each pipe. As it is wasteful to carry unavailable energy (as sensible heat) through the system, the fluid is cooled at the intermediate stations by one, or both, of two methods: (1) The liquid can be flashed to atmospheric pressure, the vapor being discarded or piped back to a liquefier and the remaining liquid, which is at its normal boiling point, pumped through the next length of pipe. With this method there is a loss of liquid, due to flashing at each station. (2) The liquid can be cooled by a refrigerator at the intermediate pumping stations. This method introduces the initial expense of refrigerators, the operating expense for power to operate them, and maintenance. However, it does have the advantage of delivering all of the liquid which enters the transfer system to the final destination as liquid.

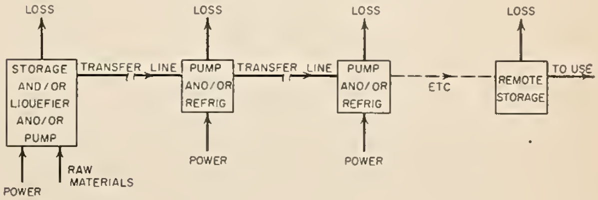

Figure 1. *Schematic of a transfer system.*

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[1] The terms "transfer" or "transfer of liquefied gases", when used here, mean the transportation of liquefied gases through piping systems.

It should be noted that the introduction of energy into the system, either as pumping power, refrigerator power or heat leak, decreases the efficiency of the transfer system, and should therefore be minimized. The losses show up as loss of liquid or use of power.

As the pressure drop (energy dissipation) in single-phase flow systems is very much less than that in comparable two-phase flow systems, it is most desirable to prevent vaporization in the transfer line. For this reason the most efficient systems should involve only single-phase flow in the pipelines. The liquid entering the transfer line must therefore be in a compressed state, and the design must be such that the liquid does not become saturated before it leaves the line.

The present discussion does not analyze systems using refrigerators. Figure 2 can represent the elements of a whole transfer system or just those of one of several sections. In the latter case, the remote storage would become another pump container which would have both a flashing loss and a pumping loss. (Both types of losses will be analyzed in detail later.)

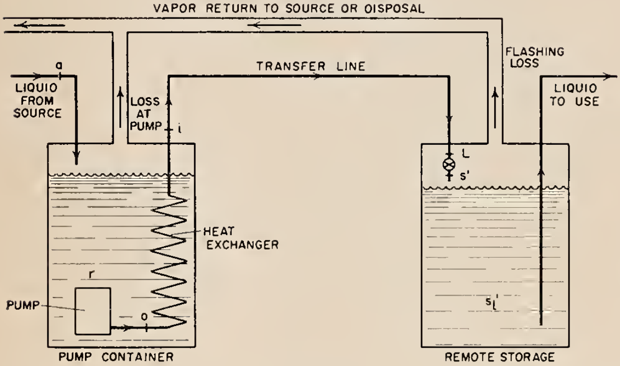

FIGURE 2. *Detailed components of a simple transfer system.*

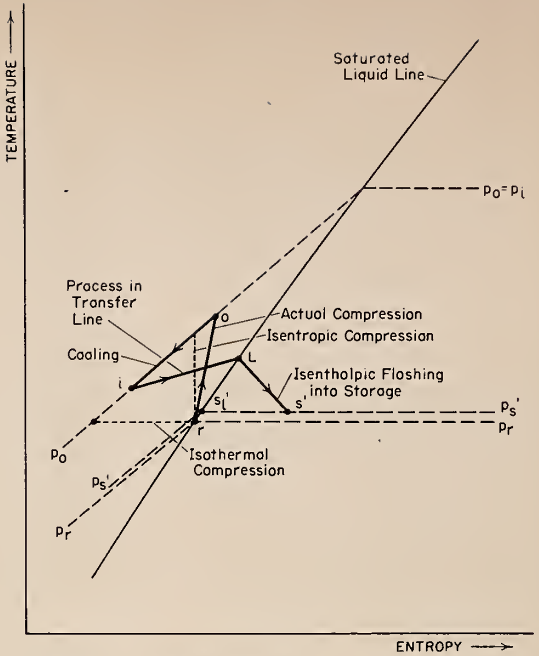

FIGURE 3. *Processes involved in a simple transfer.*

Figure 3 shows the processes which take place in the system of figure 2. The liquid enters the pump container from a source which can be either a liquefier or a preceding section of transfer line (or a refrigerator). It then enters the pump in state r which is assumed to be saturated liquid (in actual cases there will have to be a sufficient net positive suction head to insure adequate pump performance). The liquid increases in pressure and temperature as it passes through the pump and is discharged in state o. After leaving the pump the liquid passes through a heat exchanger where it is cooled to state i; it then enters the transfer line. The energy which is transferred to the liquid in the pump container from the pump casing and the heat exchanger vaporizes liquid which we shall call the "loss at the pump". Note that if $T_i = T_r$, the pump and heat exchanger together comprise an isothermal pump; this is the most desirable situation from the point of view of obtaining maximum transfer-line length with minimum pump pressures. However, as there are situations where $T_i \neq T_r$ (e. g., when the liquid enters the line directly from a process), we shall for the present assume that these temperatures are different.

In the transfer line, the pressure drops because of friction and the temperature rises due to friction and heat leak. For marginal design the line will operate in such a way that the liquid just becomes saturated as it reaches the end of the transfer-line section; the liquid in state L is therefore saturated. In the storage container (or next pump container) the liquid is flashed from state L to state s'. We shall call the resulting vapor "the flashing loss". It is apparent that this loss depends upon the energy introduced by the preceding pump and the heat leak into the transfer line. The liquid in state s_l', which can be the same as state r, is then used, or, in the case of multiple stations, is sent through the next section of line.

3. Mathematical Formulations

3.1. Flow Model

The theoretical bases for the model are the three "conservation laws": the conservation of energy, the conservation of momentum (Newton's second law), and the conservation of mass. The equations expressing these laws in fluid mechanics are the energy equation, the equation of motion, and the continuity equation. Detailed derivations of these equations are given in the appendix; a complete list of symbols is given in section 7.1; the basic assumptions used in these discussions are given in section 7.2.

In addition to the above relations, sufficient assumptions and empirical data will be used to yield a model that is complete and unique.

From the appendix, we have: the energy equation,

$$\frac{dh}{dx}+\frac{d}{dx}\left(\frac{V^2}{2}\right)+g\frac{dz}{dx}=\frac{q}{w}; \tag{1}$$

the equation of motion,

$$\frac{d}{dx}(\rho V^2)+\frac{dp}{dx}+\frac{4}{D}\tau+\rho g\frac{dz}{dx}=0; \tag{2}$$

and the continuity equation,

$$\frac{d}{dx}(\rho V)=0. \tag{3}$$

Although eq (1), (2), and (3) contain only 7 unknown functions [h, V, z, q, $\rho=(1/v)$, p, and τ], the additional relations will introduce 5 more quantities; therefore, 9 more independent equations are required.

The enthalpy is generally expressed as a function of thermostatic variables of the fluid. Here, we shall assume that the fluid is a "pure substance", one whose state can be specified by two independent thermostatic variables. Effects such as those caused by changes in ortho-para composition of liquid hudrogen or by magnetic effects of liquid oxygen are neglected. Assuming that the function $h(T, p)$ is known either graphically, numerically, or in some closed functional form, it immediately follows that

$$dh=C_p dT+\left(\frac{\partial h}{\partial p}\right)_T dp. \tag{4a}$$

Another relation, required for the description of the model, is the equation of state. It too may be in graphical, numerical or functional form; assuming that it relates p, v, and T, we have

$$f(p, v, T)=0. \tag{5}$$

If there is a deficiency of explicit enthalpy data, but the function eq (5) is known, enthalpy changes can be calculated from eq (4b) below, provided sufficient specific heat data are available.

$$dh=C_p dT-T\left(\frac{\partial v}{\partial T}\right)_p dp+v dp. \tag{4b}$$

Isobaric specific-heat data are required. Again choosing pressure and temperature as independent variables, another of our relations is

$$C_p=C_p(T, p). \tag{6}$$

The position of the pipe, as specified by z, becomes a known function of x once the site for the transfer system and its general layout have been determined. We therefore know the function

$$z=z(x). \tag{7}$$

The heat transfer, q, to the fluid as it flows through the pipe is a function of the fluid temperature and other fluid properties, the properties of the insulation and the properties of the environment in which the transfer system is located. Another independent expression is therefore:

$$q=q(\text{fluid properties, insulation characteristics, environmental properties}). \tag{8}$$

An expression that relates the shearing stress, τ, to other parameters of the system is required. Although equations can be determined both theoretically and empirically, we shall use wholly empirical information.

Assume that the shearing stress depends only upon the fluid properties and the mean velocity of the fluid relative to the wall. It should be carefully noted that the equation of motion, as expressed in eq (2), is not being altered by this assumption; it follows that

$$\tau=-\frac{D}{4}\left(\frac{dp}{dx}\right)^*,$$

where

$$\left(\frac{dp}{dx}\right)^*$$

is calculated from empirical data that have been obtained for incompressible, steady flows through round pipes of constant cross-sectional area. It is well known that

$$\left(\frac{dp}{dx}\right)^*=-4f\frac{\rho}{D}\frac{V^2}{2},$$

where f is an empirically-determined "friction factor."

Therefore

$$\tau=f\rho\frac{V^2}{2}. \tag{9}$$

f is a function of the Reynolds number, Re.

$$f=f(\text{Re}), \tag{10}$$

where the Reynolds number is defined as

$$\text{Re}\equiv\frac{VD\rho}{\mu}. \tag{11}$$

In general the absolute viscosity, μ, is a function of both temperature and pressure, although the temperature effect is usually dominant. Thus,

$$\mu=\mu(T,p). \tag{12}$$

The 12 equations, (1) through (12), describe the mathematical system determining the flow characteristics of a transfer line. The solution of these equations together with suitable boundary conditions gives the 12 unknown functions: h, V, z, q, $\rho=(1/v)$, p, τ, C_p, T, f, Re, and μ. The solution of this general system in closed form is probably not possible, or even desirable. However, through the use of numerical methods and high-speed computers, solutions of any desired degree of accuracy (limited by the available data) can be obtained.

This flow model is the basis for the technical design of a transfer system. It relates the pertinent design parameters: Flow rate, line length, line diameter, pumping pressure, heat leak, and fluid properties.

In section 5 a simplified model will be solved in closed form, and the errors introduced by the assumptions necessary to produce the model will be discussed. The general model is presented above so that more accurate solutions may be obtained when necessary.

3.2. Liquid Losses

When the detailed design of a transfer system has been completed through the use of the equations in the preceding section, an economic analysis can be made to determine both the initial and maintenance costs. However, the economic feasibility of a transfer system also depends upon the loss of liquefied gas which will occur. The purpose of this section is to present the general relations required for the computation of the liquid loss; they are derived in the appendix.

The liquid losses arise in four ways: (1) The loss at the pump is caused by energy dissipation in the pump container (due to pump inefficiency) and by the desirability of cooling the liquid before it enters the transfer line. (2) The flashing loss is caused by heat leak into the transfer line and by the energy, introduced to the liquid by the pump, which is not removed by the cooling process in the pump container. (3) The cool-down loss, which occurs only once during each transfer operation, is incurred by the necessity to cool the transfer system down from ambient to operating temperature. (4) The trapped-liquid loss, which also occurs only once during each transfer operation, is the liquid which cannot be removed from the system at the end of a transfer operation and be used.

The cool-down and trapped-liquid losses occur (at most) only once during each transfer operation. It is obvious that, as the total amount of liquid transferred during each operation increases, these losses, expressed as a fraction of the total liquid transferred, decrease. On this basis it is probable that, for well-designed systems and procedures, these losses can be made small compared with the pumping and flashing losses.

a. Loss at the Pump

The loss from the pump container, expressed as a fraction of the liquid which flows out of the container into the transfer line, is

$$\Lambda_p^i=\frac{1}{\lambda}\left\{\frac{1}{\eta}\left[\int_{p_r}^{p_i} v\,dp\right]_{s=s_r}-\left[\int_{T_r}^{T_i} C_p dT\right]_{p=p_r}-\left[\int_{p_r}^{p_i} v(1-T\beta)\,dp\right]_{T=T_r}\right\}. \tag{13}$$

This loss, expressed as a fraction of the liquid which enters the pump container, is

$$\Lambda_p^a=\frac{\Lambda_p^i}{1+\Lambda_p^i}, \tag{14}$$

where the quantity Λ_p^i is computed from eq (13).

b. Flashing Loss

The loss due to flashing into the container at the end of the transfer line, expressed as a fraction of the liquid which flows through the transfer line, is

$$\Lambda_f^i=\frac{1}{\lambda}\left\{\frac{qL}{w}-g(z_L-z_i)+\left[\int_{T_r}^{T_i} C_p dT\right]_{p=p_r}+\left[\int_{p_r}^{p_i} v(1-T\beta)dp\right]_{T=T_r}\right\}. \tag{15}$$

Expressed as a fraction of the liquid that enters the preceding pump container, this loss is

$$\Lambda_f^a=\frac{\Lambda_f^i}{1+\Lambda_p^i} \tag{16}$$

where Λ_f^i is computed from eq (15) and Λ_p^i is calculated from eq (13).

c. Cool-Down Loss

An accurate computation of the cool-down loss involves an analysis of the transient cool-down problem which was mentioned in section 1. As pointed out there, this analysis will not be discussed in this paper. Instead, a maximum value for the cool-down loss will be obtained.

If it is assumed that none of the sensible refrigeration of the vapor formed during cool down is utilized, a high value for this loss is obtained. In many cases this conservative figure is negligible compared with the steady-state losses (pump and flashing), and so no further calculations need be made. Assuming that the insulation is axially symmetric, and that sufficient properties of the metal parts and insulation are known, the cool-down loss can be computed from

$$(\Lambda_{CD})_{\max}=\frac{M_{CD}}{w_a\mathrm{T}+M_{CD}}, \tag{17}$$

$$M_{CD}=M_{T_a-T_2}+M_{T_2-T_1}, \tag{18}$$

$$M_{T_a-T_2}=\frac{\sum_{i=1}^{n} m_i \int_{T_2}^{T_a} C_{p_i} dT + 2\pi \sum_{j=1}^{m} \left\{ L_{\mathrm{ins}} \rho_{\mathrm{ins}} \int_{r_i}^{r_o} r \left[\int_{T(r) \geq T_2}^{T_a} C_{p_{\mathrm{ins}}} dT \right] dr \right\}_j}{\left[\int_{T_1}^{T_2} C_p dT + \lambda \right]}, \tag{19}$$

$$M_{T_2-T_1}=\frac{\sum_{i=1}^{n} m_i \int_{T_1}^{T_2} C_{p_i} dT + 2\pi \sum_{j=1}^{m} \left\{ L_{\mathrm{ins}} \rho_{\mathrm{ins}} \int_{r_i}^{r_o} r \left[\int_{T(r) \geq T_1}^{T(r) \geq T_2} C_{p_{\mathrm{ins}}} dT \right] dr \right\}_j}{\lambda}. \tag{20}$$

If the maximum cool-down loss computed above is too large to be acceptable, a more refined calculation, involving estimates as to how much of the sensible refrigeration of the vapor can be utilized, should be made.

d. Trapped-Liquid Loss

Assuming that a volume of liquid equal to the volume of the transfer system is trapped, a maximum value for the trapped-liquid loss, expressed as a fraction of the total liquid transferred, is

$$(\Lambda_{TL})_{\max}=\frac{V_s \rho}{w_a \mathrm{T} + M_{CD}}, \tag{21}$$

where M_{CD} is computed from eq (18), (19), and (20).

Equations (1) through (21) give all the relations necessary to evaluate the suitability of transfer systems for any potential application. They are presented in this extremely general form so that, regardless of the form of the available data, the designer can make as accurate a computation as desirable.

4. Empirical Information

To obtain numerical results for specific problems, it is necessary to use empirical information. In addition to properties of materials, information that will permit the computation of the heat leak and of the drag of the pipe wall on the liquid is required. These latter quantities will now be considered.

4.1. Heat Leak

Because heat transfer into the pump containers, storage containers, etc., does not directly affect the fluid mechanical design of the transfer system, only heat leak into the transfer lines will be considered here.

Computation shows that for all insulations worth considering, the resistance to heat transfer between the liquid line (pipe) and the liquefied gas is negligible. The liquid line will therefore have essentially the same temperature as the liquid, and the heat transfer will depend only upon the liquid temperature, the insulation characteristics, and ambient conditions.

As we are interested in maximizing transfer efficiencies, only the two best types of insulation (so far developed) will be considered: high vacuum and evacuated powder. Conventional insulations, such as glass wool, foam glass, concentric aluminum shells, etc., have been used for liquid oxygen and liquid nitrogen [1][2]; however, analysis shows that for initial-cost amortizations of about 2 yr and longer, a high-vacuum insulation is superior from the economic, as well as heat transfer, standpoint.

High-vacuum insulation is thermally superior to evacuated powders for smaller-size transfer lines, the breaking point depending upon whether shielding is used and the permissible thickness of insulation. (Refer to section 4.1.b for numerical examples.) However, as there are still many technical details to be worked out regarding the use of powder insulation in long transfer systems, the present tendency has been to recommend high vacuum even for larger size lines. Also, for the same size liquid line, evacuated powder is several times as bulky and several times as heavy as a high-vacuum line. The pressure in high-vacuum insulation should be 10^{-5}-mm Hg, or less, during operation, while a pressure of 10^{-2}-mm Hg is satisfactory for evacuated powder.

a. High-Vacuum Insulation

With high-vacuum insulation, the primary mode of heat transfer is radiation. Figure 4 depicts a

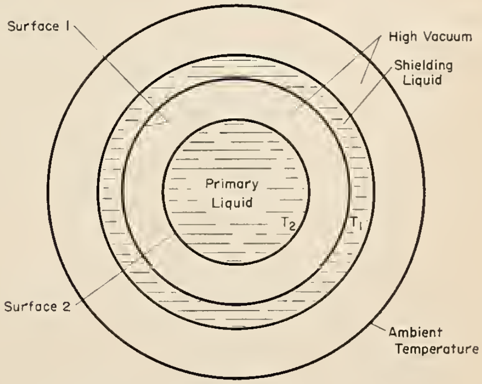

FIGURE 4. *Schematic of transfer line.*

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[2] Figures in brackets indicate the literature references at the end of this Circular.

radial section through a high-vacuum-insulated transfer line; the inner (or primary liquid) line contains the liquefied gas that is being transferred. Separated from the liquid line by a high-vacuum space is a shield refrigerated by an inexpensive liquefied gas. The shield may be desirable when the primary liquid is helium or hydrogen; in these cases the shielding liquid will probably be nitrogen. The shielding liquid is insulated from ambient temperature by high vacuum.

With the shield the ambient temperature radiation is absorbed by the shielding liquid. Only radiation emitted at the temperature (T_1) of the shielding liquid reaches the primary liquid. In a nitrogen shielded line, the heat leak to the primary liquid is only about one two-hundredths that of an unshielded line. However, it is evident that shielding complicates the design of the transfer system. In an unshielded line the two outer walls in figure 4 are omitted and T_1 is essentially ambient temperature.

The net radiant-energy transfer to the primary liquid per unit time per unit length for a transfer line is given by [2].

$$q=\frac{\sigma\pi D_2(T_1^4-T_2^4)}{\frac{1}{\epsilon_2}+\frac{D_2}{D_1}\left(\frac{1}{\epsilon_1}-1\right)},$$

where σ is the Stefan-Boltzman constant, D_1 and D_2 are the diameters of the surfaces enclosing the vacuum space, ϵ_1 and ϵ_2 are the emmissivities of these surfaces, and T_1 and T_2 are the absolute temperatures of the surfaces. A practical value for the diameter ratio, D_2/D_1, is two-thirds. It is justifiable to assume that $\epsilon_1=\epsilon_2=\epsilon$. Therefore,

$$\frac{q}{\Delta(T^4)\times10^{-8}}=0.1365\left(\frac{\epsilon}{5-2\epsilon}\right)D_2 \text{ Btu/hr ft °R}^4,$$

where D_2 is in inches and $\Delta(T^4)\equiv(T_1^4-T_2^4)$.

The right-hand side of the preceding equation does not depend upon the liquefied gas but only upon the materials (and their condition) of which the line is made. Thus, a single set of curves (which are straight lines) of $q/\Delta(T^4)\times10^{-8}$ versus D_2, with ϵ as a parameter will represent the heat-leak equation for all transfer lines; figure 5 gives these curves. For any fluid and shield (or ambient) temperature, the heat leak can then be determined. Table 1 gives the values of $\Delta(T^4)\times10^{-8}$ by which the ordinate in figure 5 must be multiplied to give the heat leak, q, for helium, hydrogen, neon, nitrogen, and oxygen.

Table 2 gives the values of the absorptivity [3] for some surfaces which might be used in transfer lines; these values may be used as emmissivities. It must be noted that these values were obtained from tests of relatively short duration. Even with pressures of 10^{-6}-mm Hg, the surfaces adsorb a sufficient amount of gas to appreciably

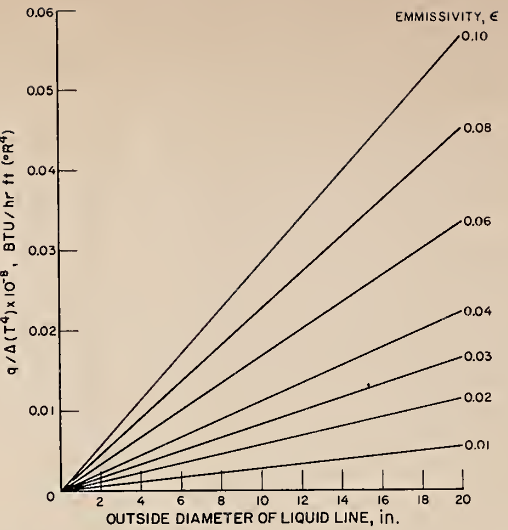

FIGURE 5. *Radiant heat leak in transfer lines with diameter ratios of ⅔.*

TABLE 1. *Factors for computation of heat leaks*

Ambient temperature, 80° F=300° K=540° R. Liquid nitrogen shield temperature, −321° F=77° K=139° R.

Primary liquid	Helium		Hydrogen		Neon		N_2	O_2
	No shield	N_2 shield	No shield	N_2 shield	No shield	N_2 shield	No shield	No shield
Normal boiling point °R	7	7	36	36	49	49	139	162
$\Delta(T^4)\times10^{-8}$ °R⁴	850	3.73	850	3.71	850	3.67	846	843
(ΔT) °R	533	132	504	103	491	90	401	378

TABLE 2. *Total hemispherical absorptivity of metals at 76° K for 300° K thermal radiation*

	Material	Absorptivity
in.		
0.001	Kaiser *aluminum* foil, unannealed	0.018
.0015	Cockron home foil, *aluminum*	.018
.0015	Hurwich home foil, mat side, *aluminum*	.021
.0015	Hurwich home foil, bright side, *aluminum*	.022
.020	*Aluminum*, cold acid cleaned	.028
.020	*Aluminum*, hot acid cleaned, Alcoa process	.029
.020	*Aluminum*, wire brush, emery paper, steel wool, cold acid	.045
.020	*Aluminum*, wire brush	.06
	Aluminum, sprayed onto stainless steel	.07
.001	*Yellow brass*, shim stock (65% Cu, 35% Zn)	.029
.005	Millrun *copper* sheet, annealed	.015
.005	*Copper*, dilute chromic acid dip	.017
.005	*Copper*, wet polished with pumice	.018
.005	*Copper*, electrolytically cleaned	.017
.005	*Copper*, fine emery	.023
	Silver sheet	.008
	Silver plate, careful preparation, nickel strike on stainless steel	.009
	Silver plate, careful preparation, nickel and copper strike on stainless steel	.007
	Allegheny *silver* spray process on stainless steel	.009
.005	Type 302 sheet *stainless steel*	.048
	Commercial ball, type 302 *stainless steel*	.07
	Silver plated on copper	.017 at 76° K .013 at 20° K
	Nickel plated on copper	.033 at 76° K .027 at 20° K

increase the heat-transfer rate in a relatively short time. For example, the heat transfer to aluminum foil at 76° K from a surface at 300° K in a pressure of 10^{-6}-mm Hg, increases by about 35 percent in 1 week. However, data are not yet available to allow the prediction of the deterioration of high-vacuum insulation over longer periods of time.

b. Evacuated Powder Insulation

Figure 4 can also represent a transfer line using evacuated-powder insulation if the regions marked "high vacuum" are supposed filled with powder and evacuated to a pressure of about 10^{-2}-mm Hg.

Those evaluating evacuated-powder insulation have assumed that the steady-state-heat-conduction equation for long cylinders [4] can be used; transposing,

$$\frac{q}{\bar{k}\Delta T}=\frac{2\pi}{\ln\frac{D_1}{D_2}},$$

in which the right side depends only upon the dimensions of the insulation. This relation, plotted in figure 6, may be applicable to all evacuated powders, with all fluids if $\bar{k}$ is obtained from the conduction equation and if the thickness of the powder, $\frac{1}{2}(D_1-D_2)$, is great enough. If the powder is too thin, radiant heat leak will play a significant role. The effective thermal conductivity, $\bar{k}$, has been obtained by Fulk [3] for a few cases.

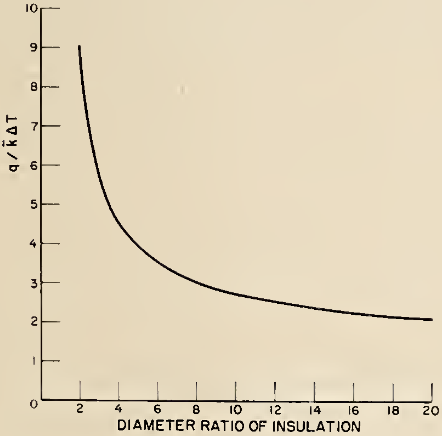

Figure 6. *Heat conduction through axially symmetric insulations.*

As $\bar{k}$ depends upon the temperatures at the boundaries of the insulation as well as the insulation properties, no generality is lost if $(\bar{k}\Delta T)$ is used as a parameter instead of $\bar{k}$. Values of this parameter are listed in table 3 and may be relied upon if the powder thickness is not less than 1 in. To determine the heat leak to a liquefied gas through evacuated-powder insulation, one obtains $q/\bar{k}\Delta T$ by entering figure 6 with the desired diameter ratio, and then obtains q by multiplying by the appropriate value of $\bar{k}\Delta T$ obtained from table 3. It is apparent that more data are needed. Data are not available for either liquid helium or liquid hydrogen without a nitrogen shield. With a nitrogen shield, no data are available for liquid helium, while only a meager amount of data for perlite and silica aerogel is available for liquid hydrogen. The values shown for liquid oxygen were obtained by using $\bar{k}$ values measured for liquid nitrogen; the same technique has been used to determine the heat leak to liquid hydrogen with no shield. Although the results are inaccurate, they lead to conservative designs.

Table 3. *Values of $(\bar{k}\Delta T)$ for various powders and liquefied gases*

(Btu/hr ft)

Insulation	Liquid H_2 with liquid N_2 shield	Liquid N_2 with no shield	Liquid O_2 with no shield
Perlite (−80 mesh), N_2 is interstitial gas, pressure is 10^{-2}-mm Hg	[a] 0.0297	0.285	[b] 0.268
Perlite (−80 mesh), N_2 is interstitial gas, pressure is 10^{-3}-mm Hg	[a] .0173	.255	[b] .240
Perlite (−80 mesh), He is interstitial gas, pressure is $<10^{-5}$-mm Hg	.0126	.2548	[b] .240
SOC perlite with 50-percent Al powder, pressure is $<10^{-3}$-mm Hg	------	.232	[b] .218
Silica Aerogel, N_2 is interstitial gas, pressure is 10^{-2}-mm Hg	.0297	.642	[b] .605
Silica Aerogel, N_2 is interstitial gas, pressure is 10^{-3}-mm Hg	.0173	.533	[b] .503
Silica Aerogel, He is interstitial gas, pressure is $<10^{-5}$-mm Hg	.0126	.463	[b] .437
Silica Aerogel with 50-percent Al powder, pressure is $<10^{-3}$-mm Hg	------	.123	[b] .116
Silica Aerogel with 50-percent Alcoa Al Flitter 660, pressure $<10^{-3}$-mm Hg	------	.290	[b] .273
Diatomaceous earth, N_2 is interstitial gas, pressure is 10^{-2}-mm Hg	------	.306	.288
Diatomaceous earth, N_2 is interstitial gas, pressure is 10^{-3}-mm Hg	------	.278	[b] .262
Diatomaceous earth, N_2 is interstitial gas, pressure is $<10^{-5}$-mm Hg	------	.264	[b] .249
Phenolic spheres, N_2 is interstitial gas, pressure is 10^{-2}-mm Hg	------	.405	[b] .382
Phenolic spheres, N_2 is interstitial gas, pressure is 10^{-3}-mm Hg	------	.313	[b] .295
Phenolic spheres, N_2 is interstitial gas, pressure is $<10^{-5}$-mm Hg	------	.283	[b] .266
Lampblack, N_2 is interstitial gas, pressure is 10^{-2}-mm Hg	------	.413	[b] .389
Lampblack, N_2 is interstitial gas, pressure is 10^{-3}-mm Hg	------	.336	[b] .317
Lampblack, N_2 is interstitial gas, pressure is $<10^{-5}$-mm Hg	------	.303	[b] .286

[a] These values are assumed to be the same as those measured for Silica Aerogel.

[b] These values are based upon the $\bar{k}$ determined for liquid nitrogen.

Numerical computations, based upon figures 5 and 6 and tables 1 and 3, compare the two types of insulations: Consider a 4-in. diameter hydrogen transfer line. Without a nitrogen shield we find: (1) The heat leak through high vacuum with a wall emmissivity of 0.04 is 0.0045×850=3.82 Btu/hr ft. (2) The heat leak though minus 80-mesh perlite at 10^{-2}—mm Hg with a diameter ratio of 5 is less than $0.285\times 280/220\times 3.95=1.43$ Btu/hr ft. However, with a liquid-nitrogen shield we find: (1) The heat leak through high vacuum with a wall emmissivity of 0.04 is $0.0045\times 3.71=0.0167$ Btu/hr ft. (2) The heat leak through minus 80-mesh perlite at 10^{-2}—mm Hg with a diameter ratio of 5 is $3.95\times 0.0297=0.117$ Btu/hr ft. The conclusion, based upon the data presently available, is that if minimization of heat leak is important enough to justify the use of liquid-nitrogen shielding, then high vacuum is superior for even quite large lines. If shielding cannot be justified, however, and techniques for using powders are perfected, evacuated powders should be used for lines over 1 or 2 in. in diameter. It should be pointed out that minimizing heat leak not only increases transfer efficiency (by decreasing liquid losses), but also increases the lengths of transfer lines which will be feasible with given pumping equipment.

4.2. Drag of Pipe Wall on Liquid

An expression [eq (9)], giving the drag of the wall on the liquid and involving a friction factor, was given in section 3.1; the friction factor, f, is an empirically determined function of the Reynolds number, Re, the form of the function depending upon the range of Reynolds number for which the expression is to be valid. To make the area of application of this development as broad as possible, a wide range of validity for the friction factor is desired. The expression of Koo [5], supposed to be accurate in the range $3{,}000<\mathrm{Re}<3{,}000{,}000$, will be used. Thus,

$$f=0.00140+0.125\ (\mathrm{Re})^{-0.32}.$$

Substituting into eq (9),

$$\tau=\frac{\rho}{2}\left[0.00140\ V^2+0.125\ \nu^{0.32}D^{-0.32}\ V^{1.68}\right]. \qquad (22)$$

The drag thus depends upon the fluid velocity, line size, density, and viscosity.

5. Solution to the Incompressible Model

A closed-form solution of the general system presented in the preceding sections is probably not possible. Numerical methods can be used, however, to obtain solutions that are as accurate as the data (describing the fluid properties, friction factors, heat leak, etc.) available; however, this laborious and expensive procedure is not now justified. Only the simplest model, which is adequate for most design problems, will be solved here. Numerical examples based upon this solution are given in section 5.5.

5.1. Determination of the Flow Characteristics

Assume that the pressure changes in the transfer line are such that the effect of pressure on the density and enthalpy is negligible. The error introduced by this assumption depends upon the pressure changes, the fluid, and the average value used for the density. If the density at the normal boiling point is used, conservative conclusions are obtained. For helium and hydrogen the error can become appreciable for moderate pressures, while for liquids like nitrogen and oxygen, quite high pressures can be used with reasonable accuracy. (The designer should evaluate the errors for each particular case; if they are unacceptable more elaborate calculations will have to be made.) It follows from eq (3) that the liquid velocity, V, is constant. Assume that the changes in temperature in the transfer line are small enough so that the changes in viscosity are negligible. Then the Reynolds number, Re, [eq (11)] is constant, and, thus, the friction factor, f, [eq (10)] and the shearing stress at the wall, τ, [eq (9)] are also constant. The above assumptions lead to negligible changes in the specific heat and to q not being a function of x. Assuming that gravity effects are negligible, eq (1) and (2) can be immediately integrated between state i at the inlet of the transfer line (at $x=0$) and state L at the end of the transfer line (at $x=L$):

$$T_L-T_i=\frac{q}{wC_p}L,$$

and

$$p_L-p_i=-\frac{4}{D}\tau L.$$

It is assumed that state i, at the entrance to the transfer line, is known. However, state L at the exit to the line is unknown. As stated in section 2 and indicated in figure 3, the length, L, of the transfer line is determined so that the liquid just becomes saturated at the end of the line; the temperature and pressure at that point are therefore related by the vapor-pressure curve for the liquid.

The most accurate computations should use an empirically-determined vapor-pressure equation or the Clapeyron equation with accurate values of the fluid properties. However, for this computation an approximate form of the Clapeyron equation will be substituted; the designer should evaluate the accuracy of this equation for each particular case. Assuming that the specific volume of saturated liquid is negligible compared with that of saturated vapor at the same pressure, the saturated vapor obeys the equation of state for a perfect gas ($pv=RT$), and changes in the latent heat of vaporization are negligible, the

Clapeyron equation is

$$-\frac{\lambda}{R}\left[\frac{1}{T_L}-\frac{1}{T_r}\right]=\ln\frac{p_L}{p_r},$$

where state r is the reference saturated-liquid state as shown in figure 3. Eliminating p_L and T_L from the three preceding equations, we obtain

$$\pi=\pi_f+\pi_t, \tag{23}$$

where

$$\pi\equiv\frac{p_i}{p_r},$$

$$\pi_f\equiv 4\frac{\tau}{p_r}\frac{L}{D},$$

and

$$\pi_t\equiv\exp\left\{\frac{\lambda}{RT_r}\left[1-\frac{1}{\frac{T_i}{T_r}+\frac{qL}{wC_pT_r}}\right]\right\}.$$

Equation (23) can be considered as the "flow equation" for the transfer system; it relates all of the variables which influence the physical and flow characteristics of the system. The final values given to these variables (pressure ratio, diameter, length, flow rate, heat leak, etc.) depend upon economic considerations (some of which are expressed through the various liquid losses) and application specifications, as well as upon the flow equation.

The pressure ratio, π, should be minimized: If the liquid enters the transfer line from a liquefier, this will increase the yield. If the liquid comes from a pump, then π is the pressure ratio across the pump; in this case, the minimization decreases the pump size, speed, and weight.

Equation (23) shows that for this model the energy (pressure) required at the entrance to the transfer line is determined independently by frictional effects and by thermal effects. The dimensionless frictional parameter, π_f, expresses the former, and the dimensionless thermal parameter, π_t, expresses the latter. The relative importance of these two parameters depends upon the specific application; π_f will be dominant if the flow rate and line length are large and the diameter is small, while π_t will be dominant if the inlet temperature, heat leak, and line length are large, and the flow rate is small.

The dimensionless parameters, π_f and π_t, are themselves composed of dimensionless groups; these subgroups, rather than the individual variables, determine the character of the transfer system. As far as the frictional effects are concerned the length-diameter ratio, L/D, and the Reynolds number, Re, are significant. The thermal effects are determined by the three explicit dimensionless groups, λ/RT_r, T_i/T_r, and qL/wC_pT_r.

The quantity, λ/RT_r, a property of the fluid, is the ratio of the energy required for vaporization to the work required to make the ideal gas flow from a region at absolute zero to one at a temperature T_r. Smaller values of λ/RT_r are desirable if one wants low-pressure ratios. If there were no other considerations, one could minimize this quantity by using high pressures.

The initial temperature ratio, T_i/T_r, should be minimized. With systems in which the motive energy is supplied by pumps, as in figure 2, the ratio can be set at its minimum value, unity. However, if the liquid comes directly from a liquefier where temperature differentials in heat exchangers must be greater than zero, the initial temperature ratio will be greater than unity and the initial pressure will have to be raised.

The parameter which accounts for the effect of heat leak is $[(qL)/w]/C_pT_r$. It can be considered as the ratio of the total heat leak per unit mass of fluid to the thermal energy in a unit mass of liquid that has been isobarically heated from $T=0$ to $T=T_r$. This parameter should be minimized. The numerator, qL/w, is under the control of the designer, while the denominator, C_pT_r, essentially is not. It is apparent that the total heat leak per unit time, qL, must be minimized. However, to minimize the effect of heat leak, the flow rate, w, must be as large as possible. This result is obvious when one realizes that it is the heat leak per unit mass of liquid, not heat leak per unit time, which determines how rapidly the liquid approaches saturation.

Although friction considerations require that the flow rate be minimized, thermal considerations require maximization of flow rate. The compromise, which thus must be made concerning the flow rate, is influenced by line diameter, pressure ratio, line length, heat leak, etc., as well as the liquid loss (system efficiency).

5.2. Liquid Losses

The expressions which were presented in section 3.2 and which are used to compute the liquid losses will now be simplified by the same assumptions as the flow equations in the preceding section.

a. Loss at the Pump

The assumptions of incompressibility and constant specific heat, together with the additional assumption that the thermal expansion coefficient β does not change appreciably, allow eq (13) to be integrated immediately. If averaged values were to be used for the specific volume, the v in the first integral would be averaged along the isentropic between p_r and p_i while the v in the last integral would be averaged along an isotherm between p_r and p_i; these averages would be different. However, v_r, the specific volume of saturated liquid at p_r, will be used in both places. As this procedure leads to a value of the loss that is too large the results will be conservative. Similarly, the use of a constant value of β evaluated at p_r leads to a conservative result. On the other hand, the use of a value for C_p evaluated at T_r tends to give a loss which is too low. However, as T_i can be made essentially equal to T_r in the pump container (see fig. 2), the effect of the second integral can be made negligible. Integrating eq (13) leads to

$$\Lambda_p^i=\frac{1}{\lambda}\left\{\left(\frac{1}{\eta}-1+T_r\beta_r\right)p_rv_r(\pi-1)-C_pT_r\left(\frac{T_i}{T_r}-1\right)\right\}. \tag{24}$$

The loss calculated from eq (24) leads to conservative designs.

Equation (24) shows that the loss at the pump container depends upon the fluid properties, the efficiency and pressure ratio of the pump, and the effectiveness of the heat exchanger. The significances of the various terms become apparent if one follows the detailed derivation of the equation. Although a large temperature ratio, T_i/T_r, will decrease the loss at the pump container, there are detrimental effects which more than balance this saving; these are a decrease in the possible length of the system (for a given pump pressure) and an increase in the flashing loss (see section 5.2.b).

b. Flashing Loss

Assumptions already made and again leading to conservative results, enable us to evaluate the integrals in eq (15); the (z_L-z_i) term is eliminated by the assumption of negligible gravity effects. Equation (15) becomes:

$$\Lambda_f^i=\frac{1}{\lambda}\left\{\frac{qL}{w}+C_pT_r\left(\frac{T_i}{T_r}-1\right)+(1-T_r\beta_r)p_rv_r(\pi-1)\right\}. \tag{25}$$

The only term which appears in this equation that is absent from eq (24) is the heat-leak parameter, qL/w; the significance of this quantity has been discussed in section 5.1. It is interesting to compare eq (24) and (25); both express losses as fractions of the liquid flowing through the transfer line. If these equations are added, the result is the total steady-state loss associated with the transfer line:

$$\Lambda_p^i+\Lambda_f^i=\frac{1}{\lambda}\left\{\frac{1}{\eta}p_rv_r(\pi-1)+\frac{qL}{w}\right\}.$$

This combined loss depends only upon the total work entering the pump shaft and the total heat leak. What fraction appears as a loss at the pump container and how much as a flashing loss depends upon the particular system. In any event it is apparent that the pump efficiency should be as high as possible, while the pressure ratio and heat leak must be kept small.

In this work the losses at the two locations will be considered separately. This has an advantage when considering systems with multiple pumping stations and also simplifies graphical representation of the losses.

c. Cool-Down and Trapped-Liquid Losses

The expressions for these losses, eq (17), (18), (19), (20), and (21), are not simplified by the assumptions made in the preceding sections. There are several simplifications which can be made, depending upon the specific systems being analyzed. For example, with high-vacuum insulation, the energy which must be removed from the insulation (second terms in eq (19) and (20)) is negligible; if the liquid is not greatly subcooled, the integral in the denominator of eq (19) may be neglected.

If the transfer system is very long, only the cool-down and trapped-liquid losses associated with the transfer line need be considered, those associated with the pumping stations, fittings, and tanks being negligible. Neglecting also the sensible refrigeration of the liquid (which produces a conservative result), we obtain

$$\Lambda_{CD}=\frac{M_{CD}}{w_a\mathrm{T}+M_{CD}}, \tag{17}$$

where

$$M_{CD}=\frac{M_m\int_{T_1}^{T_a}C_{p_m}dT+2\pi L_{\mathrm{ins}}\rho_{\mathrm{ins}}\int_{r_i}^{r_o}r\left[\int_{T(r)\geq T_1}^{T_a}C_{p_{\mathrm{ins}}}dT\right]dr}{\lambda}. \tag{26}$$

M_m is the equivalent total mass of the metal in the transfer line that is cooled to liquid temperature, and C_{p_m} is its equivalent specific heat.

It is frequently convenient to express the cool-down and trapped-liquid losses as the time that would be required to supply the actual masses of liquid (M_{CD} and $V_s\rho$) when the liquid flows at the steady-state transfer rate, w_a. Expressed in this way, the losses are: (1) The time required to supply liquid for the cool-down loss

$$\mathrm{T}_{CD}=\frac{M_{CD}}{w_a},$$

where M_{CD} is obtained from eq (26). (2) The time required to supply liquid for the trapped-liquid loss

$$\mathrm{T}_{TL}=\frac{V_s\rho}{w_a}.$$

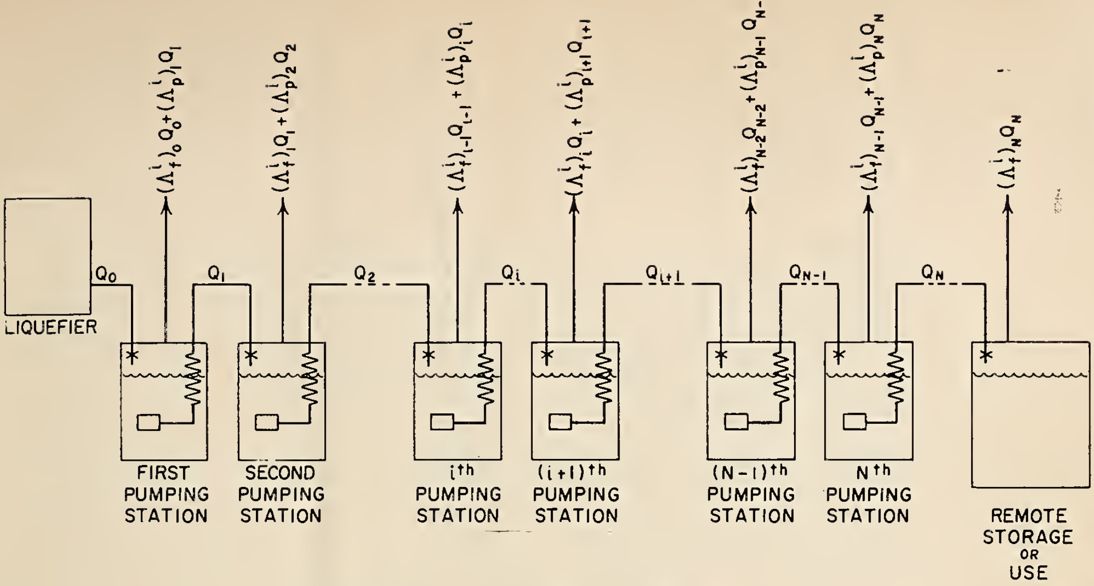

FIGURE 7. *Multiple pump transfer system.*

5.3. Multiple Pumping Stations

The additional relation required for the determination of the effects of the number of pumping stations on the characteristics of transfer systems accounts for the decrease in mass flow as the fluid passes through each pumping station. When it is used with the relations given previously for single-pump systems, possible changes in flow rate, pipe diameter, pump pressure ratio, losses, etc., which can occur from one end of the system to the other can be computed.

Figure 7 is a schematic of a transfer system with N pumping stations. At each station there is a loss consisting of a "flashing loss" and a "loss at the pump"; the liquid leaving a pumping station is less than that entering by an amount equal to the loss. Q_i is the flow rate from the i^{th} pumping station. Then from the discussion in section 3.2.b, we see that the flashing loss at the $(i+1)^{th}$ station is $(\Lambda_f^i)_i Q_i$, where the subscript outside of the parenthesis indicates the transfer-line section with which the loss is associated. From section 3.2.a we see that the "loss at the pump" from the $(i+1)^{th}$ station is $(\Lambda_p^i)_{i+1} Q_{i+1}$. The total loss from a station is merely the sum of these as indicated in the figure.

The calculations must start at a location where sufficient information is given. In the usual case, the flow entering the system, Q_o, is known; the step-wise computations therefore proceed from the inlet end to the discharge end of the system. It is conceivable that the flow that must be delivered, Q_N, may be specified; in this case the calculations will proceed in the reverse direction. For the usual computation, therefore, Q_{i+1} must be computed from Q_i. Conservation of mass considerations lead to the relation:

$$Q_{i+1} = Q_i \left[\frac{1 - (\Lambda_f^i)_i}{1 + (\Lambda_p^i)_{i+1}} \right].$$

The computation of $(\Lambda_f^i)_i$ requires a knowledge of the length, L_i, the heat leak, q, the flow rate, Q_i, the pressure ratio in the i^{th} pumping station, π_i, and fluid properties. The computation of $(\Lambda_p^i)_{i+1}$ requires a knowledge of the pressure ratio in the $(i+1)^{th}$ station, the pump efficiency at that point, and fluid properties. As all of these quantities are either known or can be used as independent variables, Q_{i+1} can be computed from the preceding expression.

The detailed design of a multiple station system can be greatly simplified by assuming that the maximum amount of liquid (Q_o) flows through the whole system. This will overdesign the system as far as pump discharge pressure and pipe diameters are concerned and will lead to flashing losses that are too small. However, if the length of each of the N sections of transfer line is L/N, where L is the total length of the system, this approximation reduces the amount of computation by a factor of more than $1/N$.

5.4. Numerical Computations

In this section general numerical solutions to the incompressible model are calculated for liquid helium, liquid hydrogen, liquid nitrogen, and liquid oxygen, and are presented as readily usable graphs. These substances have been chosen because of the interest in their transfer at the present time and in the foreseeable future. All of the calculations required for any design are readily made with the aid of the graphs.

To facilitate the computation, and to simplify the presentation, two further assumptions are made:

(1) The reference state, indicated by the subscript r, is saturated liquid at one atmosphere.

(2) The temperature at the inlet to the transfer line, T_i, is equal to the reference temperature, T_r; it is therefore the normal boiling point. Thus,

$$\frac{T_i}{T_r}=1.$$

As mentioned previously, this assumption can be completely valid for a pump setup such as that shown in figure 2. However, when the liquid is transferred directly from a process, such as a liquefier, practical considerations will frequently require that T_i be appreciably greater than T_r. In such cases, the curves in figure 10, will have to be recalculated, and the values obtained for the loss at the pump and the flashing loss may have to be corrected. Note that the sum of the losses computed in this section does not depend upon the temperature ratio; it is therefore accurate even if assumption 2 above is violated.

The numercial information required for these calculations is tabulated in table 4. The accuracy of these data is more than adequate considering the approximations that have been used in constructing the mathematical formulation.

a. Flow Equation

The dimensionless frictional parameter can be rewritten

$$\pi_f=3.266\left(\frac{L}{D^3}\right)(D^2\tau), \tag{27}$$

where L is in feet, D is in inches, and $(D^2\tau)$ is in pounds. By manipulation, eq (22) can be written in the form

$$D^2\tau=C_1\left(\frac{Q}{D}\right)^2+C_2\left(\frac{Q}{D}\right)^{1.68}, \tag{29}$$

where Q is the flow rate in gallons per minute, and C_1 and C_2 are functions of the fluid properties only.

We see that for any fluid the quantity $(D^2\tau)$ is a function of the parameter (Q/D) only. Equation (29) is plotted in figure 8 for the liquids in which we are interested. The ranges of the parameter which correspond to the accepted valid ranges of the equations are indicated; the lower limits correspond to a Reynolds number of 3,000 and the upper limits correspond to one of 3,000,000. The curves are extended for use outside of these ranges, but decrease in accuracy of the resulting computation must be borne in mind.

Once a value for $(D^2\tau)$ has been obtained from figure 8, the determination of π_f is straightforward. Figure 9 is a plot of eq (27), with $(D^2\tau)$ plotted along the axis of the abscissa, π_f along the ordinate axis, and (L/D^3) as a parameter. The use of these curves will be explained shortly.

Table 4. *Properties of saturated liquids at their normal boiling points*

Property	Helium	Hydrogen	Nitrogen	Oxygen
Boiling point, T_r ° K	4.21	20.4	77.32	90.13
Latent heat of vaporization, λ cal/g	4.9	106.5	47.6	50.8
Gas constant, R cal/g ° K	0.496	0.985	0.0709	0.0621
Isobaric specific heat, C_p cal/g ° K	1.24	2.50	.489	.400
Density, ρ g/cm³	0.1255	0.0709	.807	1.149
Absolute viscosity, μ cp	.0047	.0130	.158	0.190
Thermal expansion coefficient, β_r 1/° K	.1690	.0175	.00580	.00738
Specific volume, v_r cm³/g	7.97	14.11	1.237	.871
Ratio of specific volume of saturated vapor to that of saturated liquid, v_g/v_r	~10.84	50	~183	~265
Slope of vapor-pressure curve, dT/dp ° K/mm Hg	0.0016	0.00422	0.0122	0.0107

Note that the dimensionless thermal parameter can now be written:

$$\pi_t=\exp\frac{C_3}{1+C_4\dfrac{Q}{qL}}, \tag{28}$$

where C_3 and C_4 are functions of the fluid properties only. The thermal pressure ratio, π_t, is a function of the heat leak parameter, Q/qL, only. Equation (28) is plotted in figure 10 for the four liquids under consideration.

The flow equation is thus represented by the three curve sheets whose detailed use depends upon what quantities are known and thus upon the particular problem at hand. Usually trial and error or equivalent graphical methods are required. For example, the flow rate, Q, the line length, L, and the insulation may be specified, while the pressure ratio, π, and the line diameter, D, are to be determined. One method of solution would be as follows: (1) Select a range of values of the diameter, which, it is felt, contains the design value. (2) For each D, compute (Q/D) and obtain $D^2\ \tau$ from figure 8. (3) For each D, compute L/D^3 and obtain the friction pressure ratio, π_f, from figure 9. (4) For each D, compute the heat leak, q, from information such as that given in section 4.1. (5) For each D, compute the reciprocal of the heat leak parameter, Q/qL, and obtain the thermal pressure ratio, π_t, from figure 10. (6) For each D, add the π_f from step 3 to π_t from step 5, the sum being the pressure ratio, π. (7) Plot the π's versus the D's; the resulting curve represents the flow equation for the given flow, length, and insulation. This curve is then used, along with the results of design calculations for the various losses and costs, to determine the

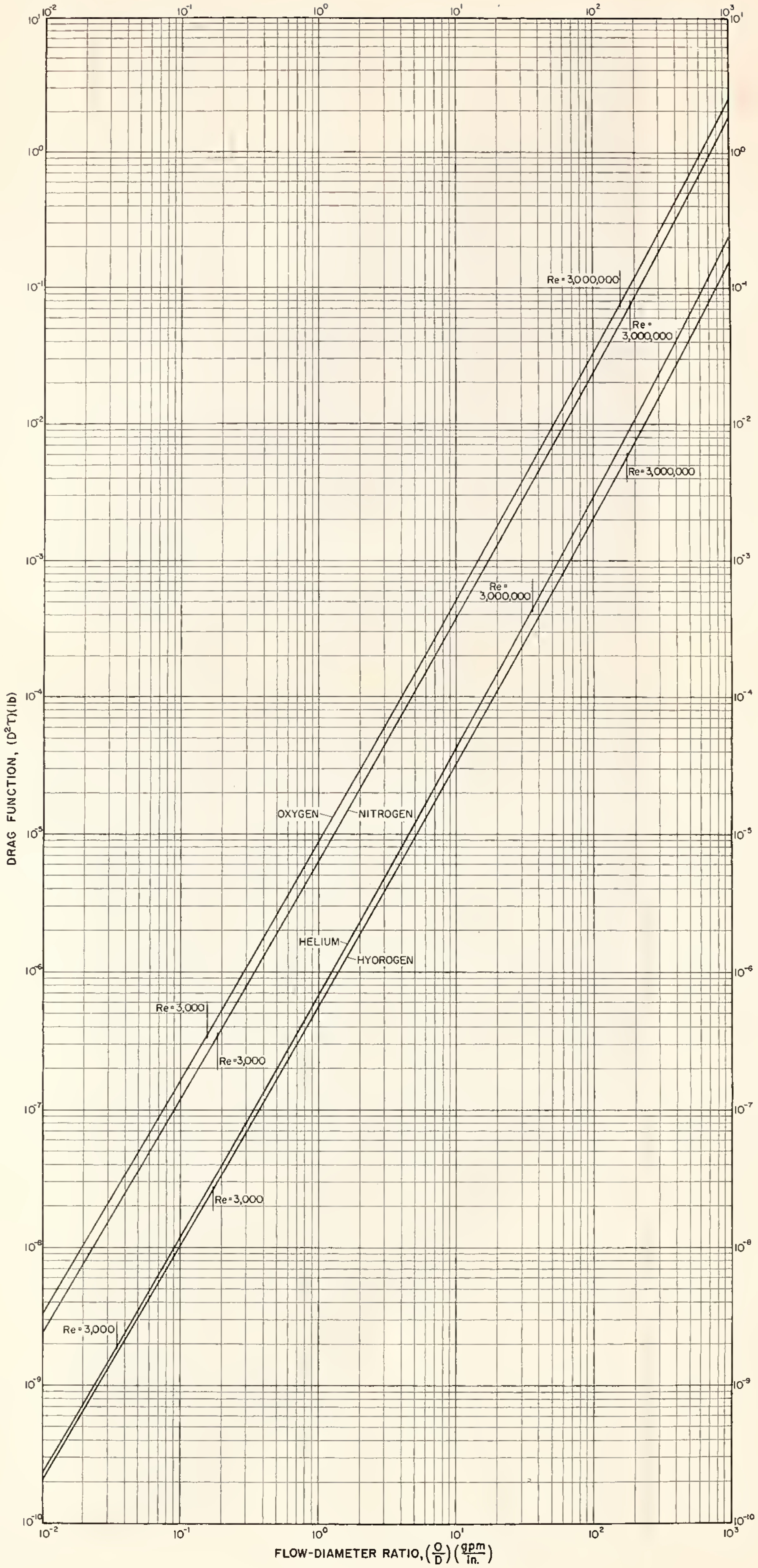

FIGURE 8. *Drag function.*

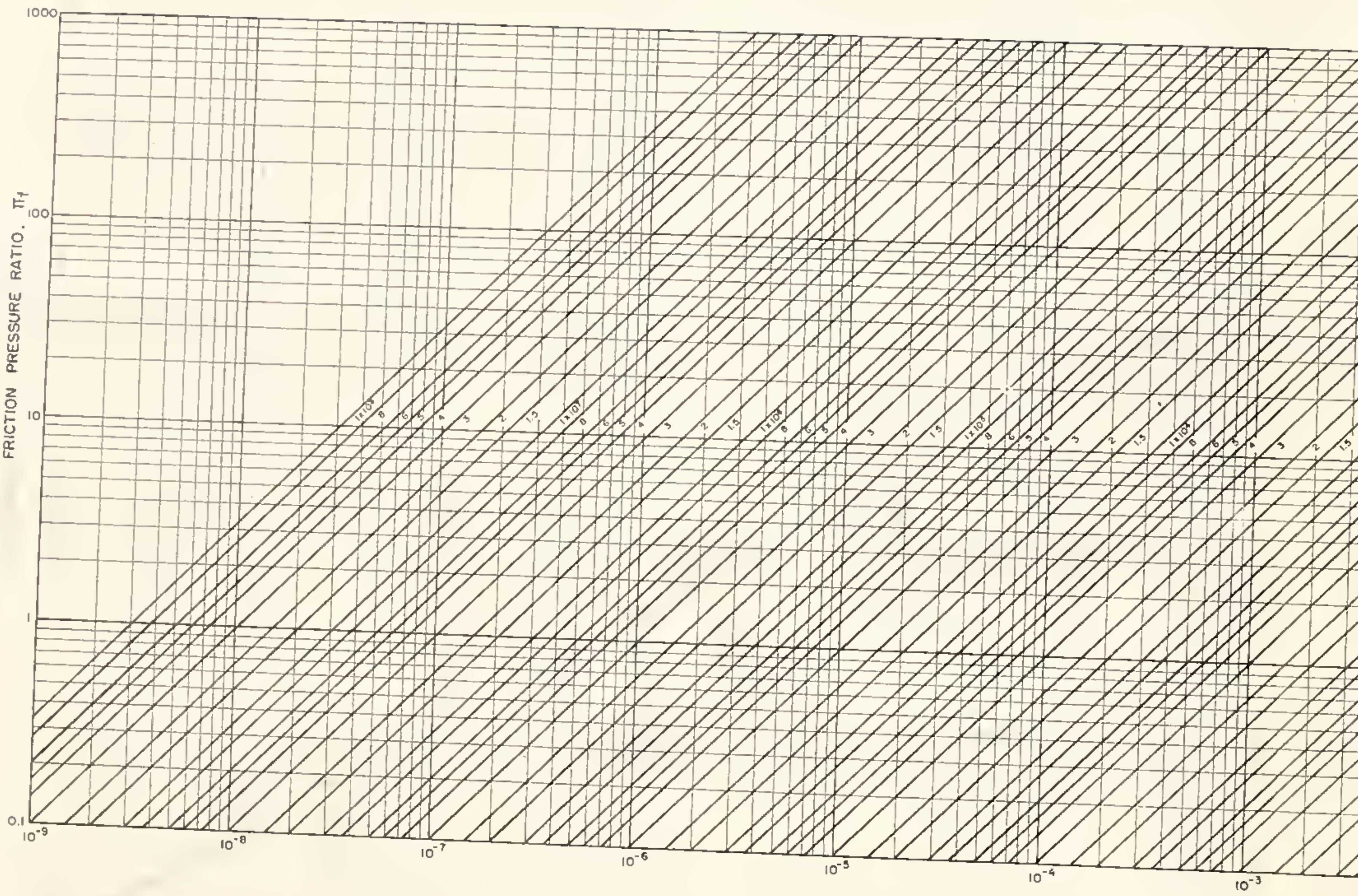

FIGURE 9. *Friction pressure ratio.*

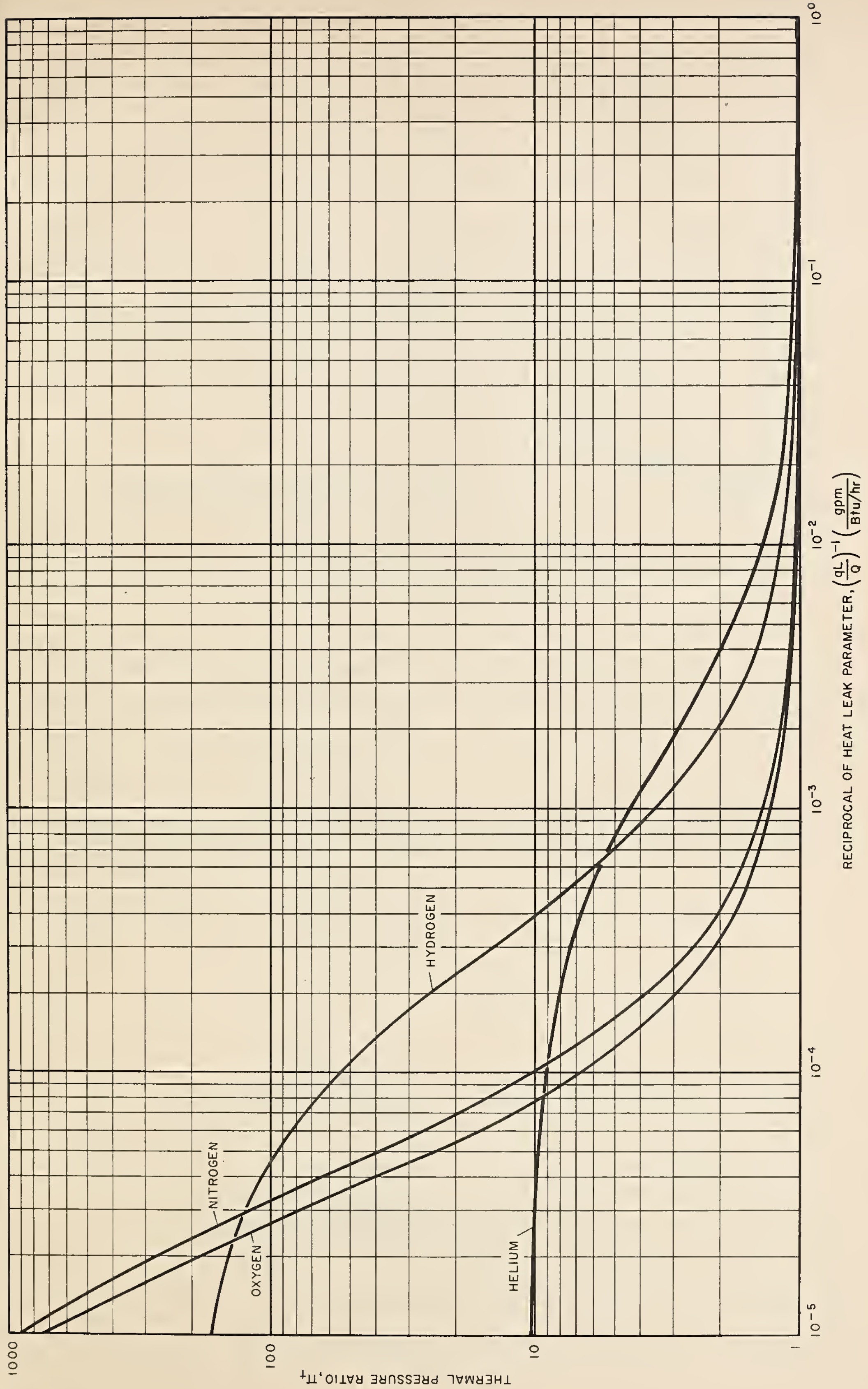

FIGURE 10. *Thermal pressure ratio.*

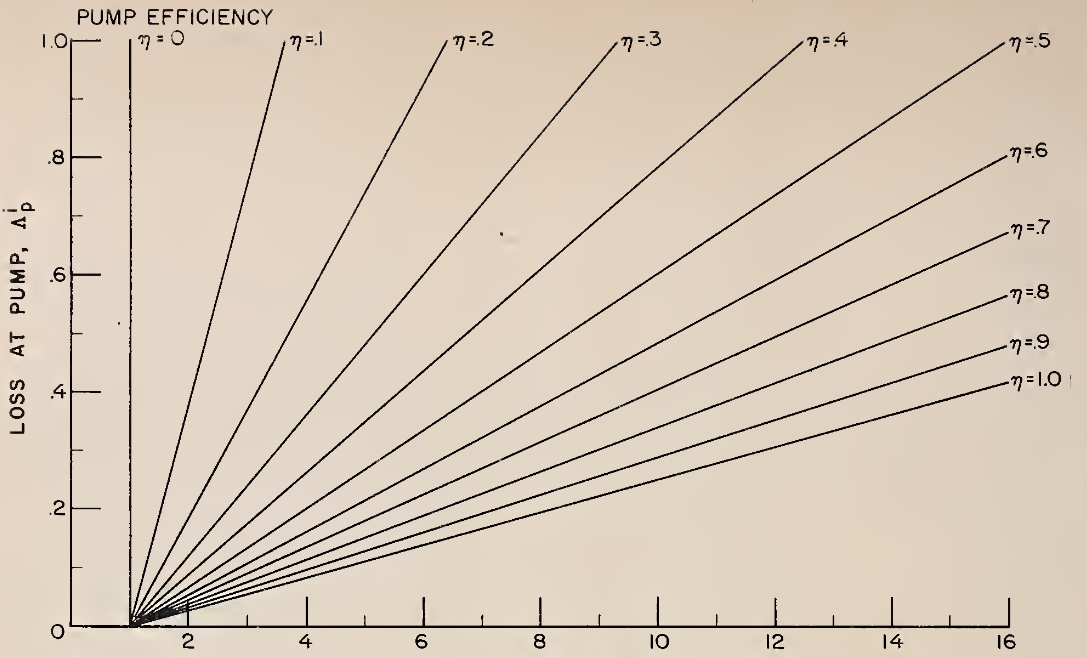

Figure 11. *Loss at pump for helium.*

desired combination of pressure ratio and diameter. Several working curves of this type (only treating the length as a variable also) are presented in section 5.5.

b. Loss at the Pump

Subjected to the two assumptions at the beginning of this section, the loss at the pump eq (24) becomes

$$\Lambda_p^i=\frac{1}{\lambda}\left\{\left(\frac{1}{\eta}-1+T_r\beta_r\right)p_r v_r(\pi-1)\right\}. \qquad (30)$$

If this loss is plotted against the pressure ratio, π, with the pump efficiency, η, as a parameter, the curves are straight lines. For the liquids with which we are concerned, eq (30) is plotted in figures 11 through 15.

It should be stressed that the loss at the pump is but a fraction of the total loss, and in many cases a small fraction. The pressure ratio, π, that is acceptable cannot therefore be determined on the basis of this loss alone, but the flashing loss must also be evaluated and taken into consideration.

Figure 11 shows that the loss at the pump for liquid helium is very large, even with highly efficient pumps; this situation is primarily due to the extremely small latent heat of vaporization. Because of this, an effort must be made to minimize the pressure ratio in a liquid-helium transfer system. One direction toward which this effort can lead is to large line diameters. In order to counteract the detrimental effects of large lines, the insulation will have to be optimized. This may well lead to the mandatory use of liquid-nitrogen shielded lines for all liquid-helium transfers over appreciable distances.

Figures 12 and 13 give the liquid loss at the pump for liquid hydrogen. Figure 12 is presented because there are applications in which hydrogen is used at these high-pressure ratios. However, the range of π covered in figure 13 is the more likely range for long-distance transfers. For moderate pressure ratios and pump efficiencies, the pump loss with liquid hydrogen may be quite acceptable.

Figures 14 and 15, which are for liquid nitrogen and liquid oxygen respectively, show that the losses at the pump for moderate pressures and efficiencies are very small; even at very high pressures these losses remain moderate and probably quite tolerable.

Consideration should be given to the accuracy of any computation made for high-pressure systems. It is known that the analysis is less accurate at high pressures, and, in fact, is not directly applicable if the pressure at the downstream end of the transfer line is above critical. However, it should be borne in mind that most of the pressure ratio is required to overcome the frictional pressure drop in the transfer line. It is therefore probable that, in all cases, the pressure at the downstream end of a transfer line will be low, the condition of incipient vaporization will

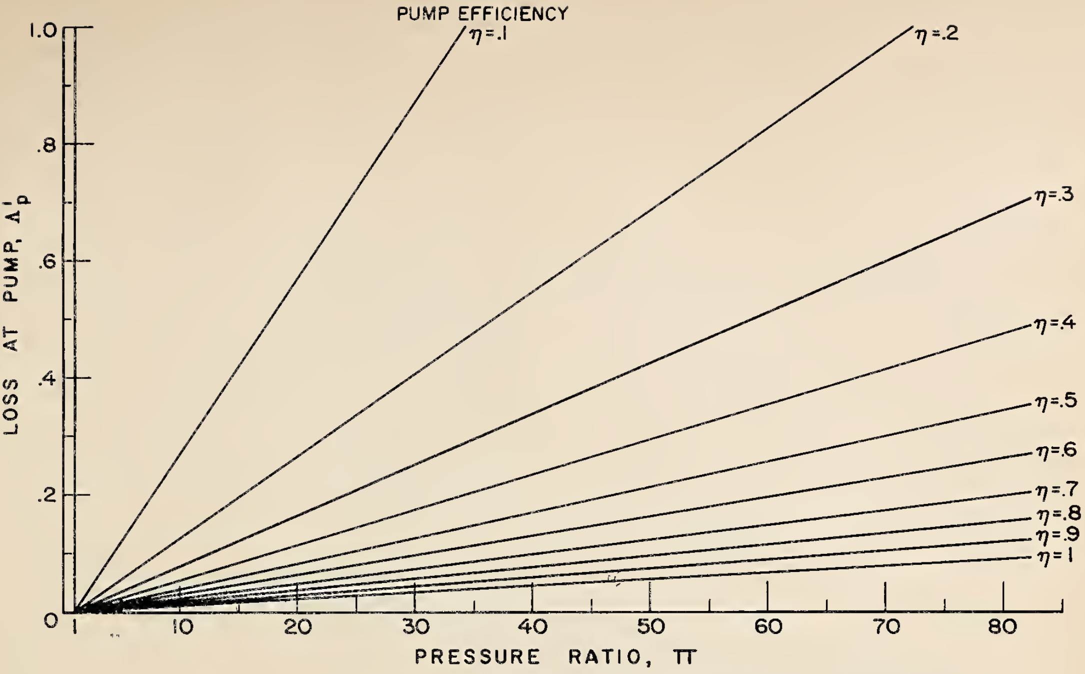

Figure 12. *Loss at pump for hydrogen.*

Figure 13. *Loss at pump for hydrogen.*

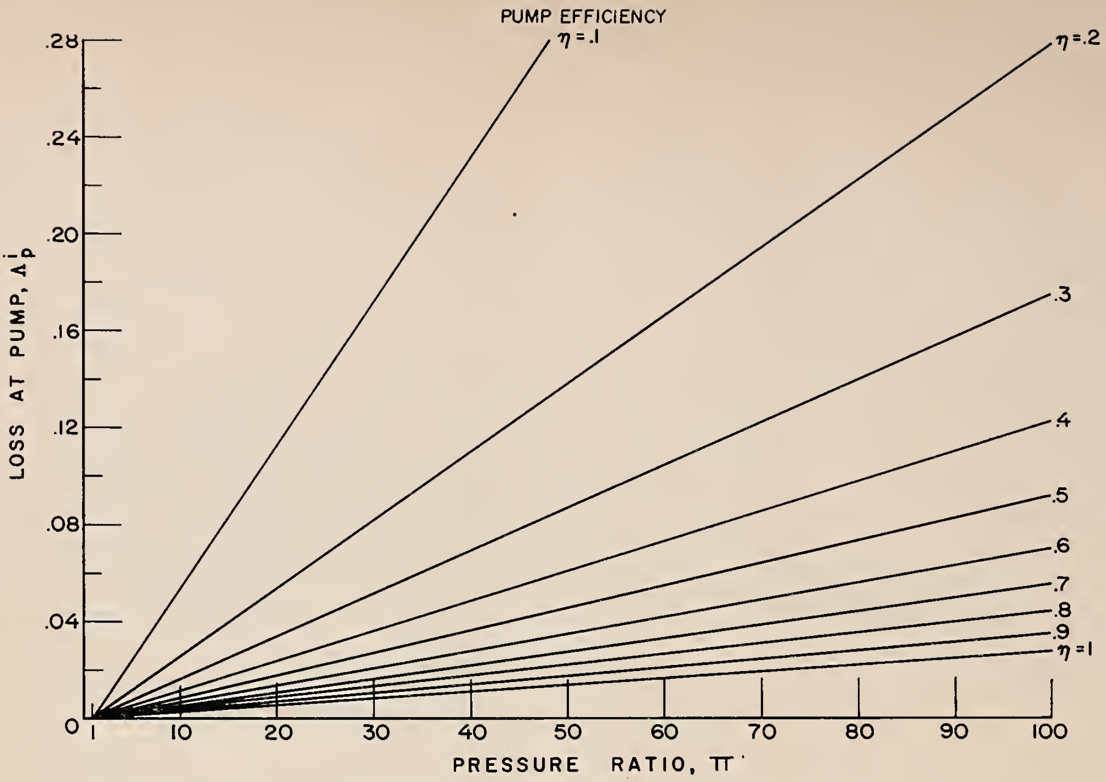

FIGURE 14. *Loss at pump for nitrogen.*

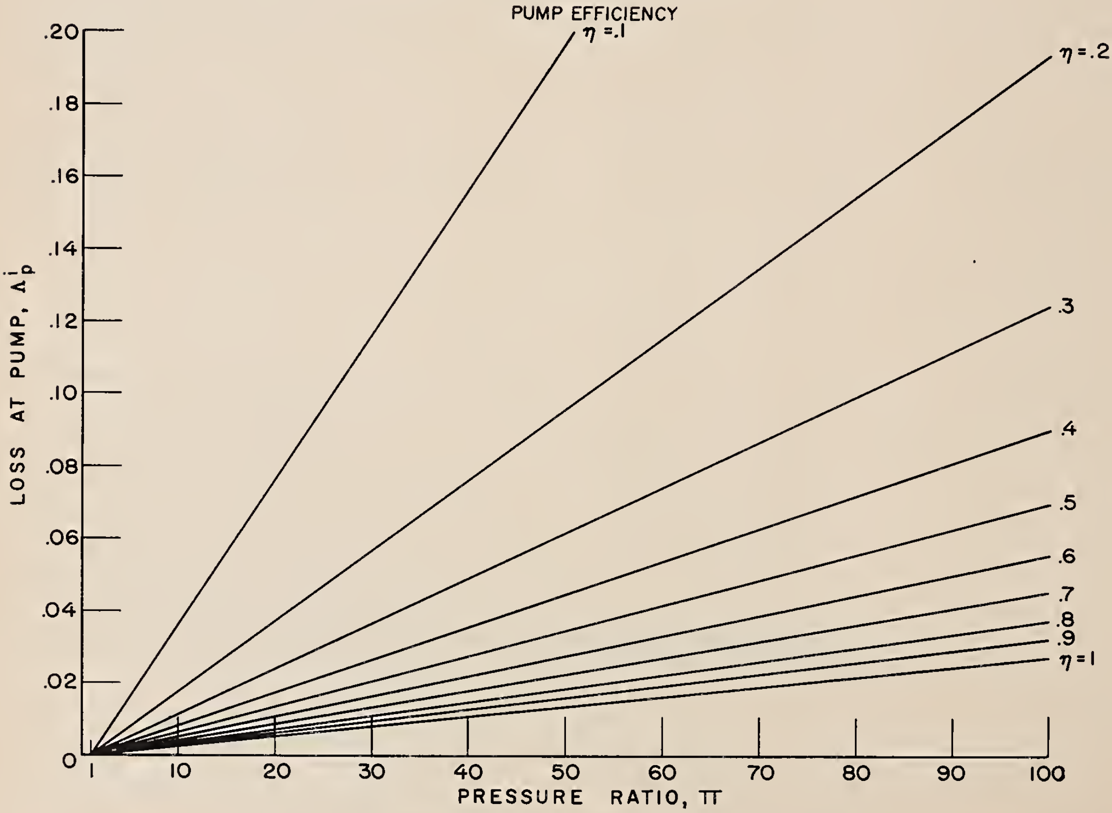

FIGURE 15. *Loss at pump for oxygen.*

exist, and the analysis will be applicable. This situation will extend the range of applicability of the flow equation to high pressures.

The effect of pump efficiency on the liquid loss in a transfer system is well illustrated by the preceding computation. However, another effect of pump efficiency, which can theoretically ruin the performance of a transfer system, should be mentioned. Refer to figure 16. Assume that a pump compression starts at the saturated liquid state, r. For isentropic compression (efficiency equals unity), there is always pure liquid in the pump; on the other hand, for isobaric compression (efficiency equals zero), vapor immediately forms in the pump, causing cavitation. It can be implied that there is a critical efficiency, η_c, above which vapor will not form in the pump, and below which vapor will form; for the latter case the performance of the transfer system will be unsatisfactory.

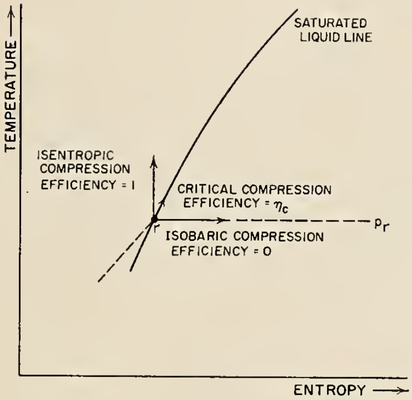

FIGURE 16. *Process for critical efficiency.*

The critical pump efficiency will now be determined to ascertain whether or not it should be of concern. It is apparent from the figure that the critical efficiency depends upon the slope of the saturated liquid line, and thus upon the fluid properties.

The efficiency of a pump for an infinitesimal process is

$$\eta \equiv \frac{v\,dp}{dh}.$$

As

$$dh = C_p dT + v(1 - T\beta)dp,$$

$$\eta = \frac{1}{\frac{C_p}{v}\left(\frac{dT}{dp}\right) + (1 - T\beta)}.$$

When the efficiency is critical, the process is along the saturated liquid line (for small pressure ratios) and $(dT/dp) = (dT/dp)_{\text{sat}}$, the slope of the vapor pressure curve. It follows that

$$\eta_c = \frac{1}{\frac{C_p}{v}\left(\frac{dT}{dp}\right)_{\text{sat}} + (1 - T\beta)}.$$

Using values given in table 4 the critical efficiencies for helium, hydrogen, nitrogen, and oxygen at their normal boiling points have been calculated; they are given in table 5. At first glance, the efficiencies appear to be too low to be of concern (i. e., relative to usual water pump practice). However, because of low viscosity, uncertainty about clearances due to differential thermal contraction, relative high losses in bearings and seals, etc., an efficiency of 12.4 percent in a helium pump may not be too readily exceeded. For most cases, it should not be difficult to construct a hydrogen pump with an efficiency appreciably greater than 4.2 percent. However, as high-pressure, positive-displacement pumps may have very low efficiencies, the notion of critical efficiency should be kept in mind when pumping hydrogen. If the liquid can be subcooled or compressed before it enters the pump, this effect of low efficiency can be inhibited. The values in table 5 show that the critical efficiencies for nitrogen and oxygen are too low to be of concern. If the pressure rise across the pump is large, the critical efficiencies will be higher than the values obtained from the above expression.

TABLE 5. *Critical pump efficiencies*

Liquid	Critical pump efficiency, η_c
	Percent
Helium	12.4
Hydrogen	4.2
Nitrogen	0.66
Oxygen	.65

c. Flashing Loss

Subjected to the two assumptions at the beginning of this section, the flashing loss, eq (25) becomes:

$$\Lambda_f^i = \frac{1}{\lambda}\left\{\frac{qL}{w} + (1 - T_r\beta_r)\,p_r v_r(\pi - 1)\right\}. \qquad (31)$$

Thus, for a given fluid the flashing loss (in this simplified model) is a function of only two parameters: the heat-leak parameter, qL/w, and the pressure ratio, π. Furthermore, the functions are linear; therefore the equation can be very simply represented graphically by two sets of curves as follows: Let

$$\Lambda_f^i \equiv (\Lambda_f^i)^q + (\Lambda_f^i)^\pi, \qquad (32)$$

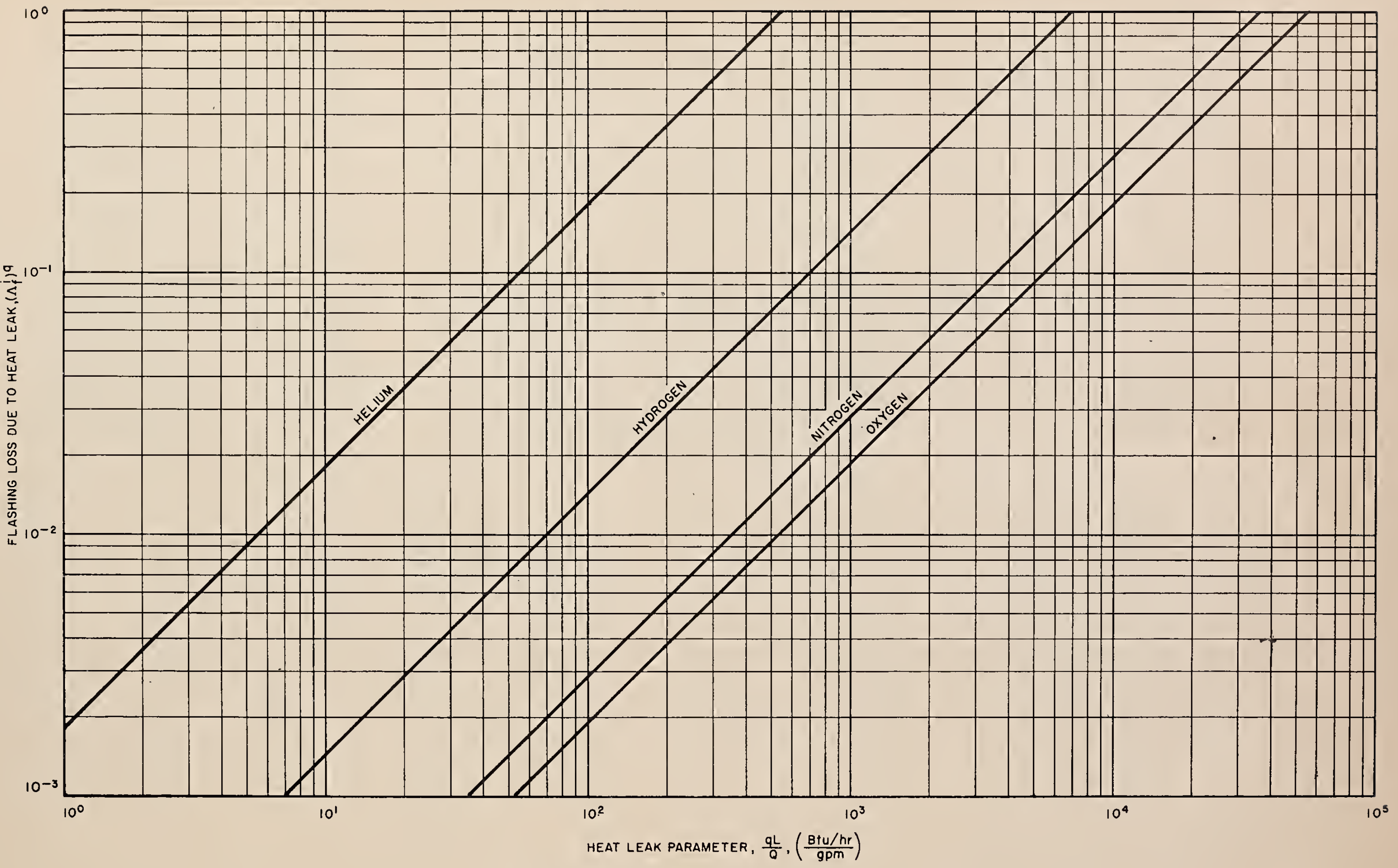

FIGURE 17. *Flashing loss due to heat leak.*

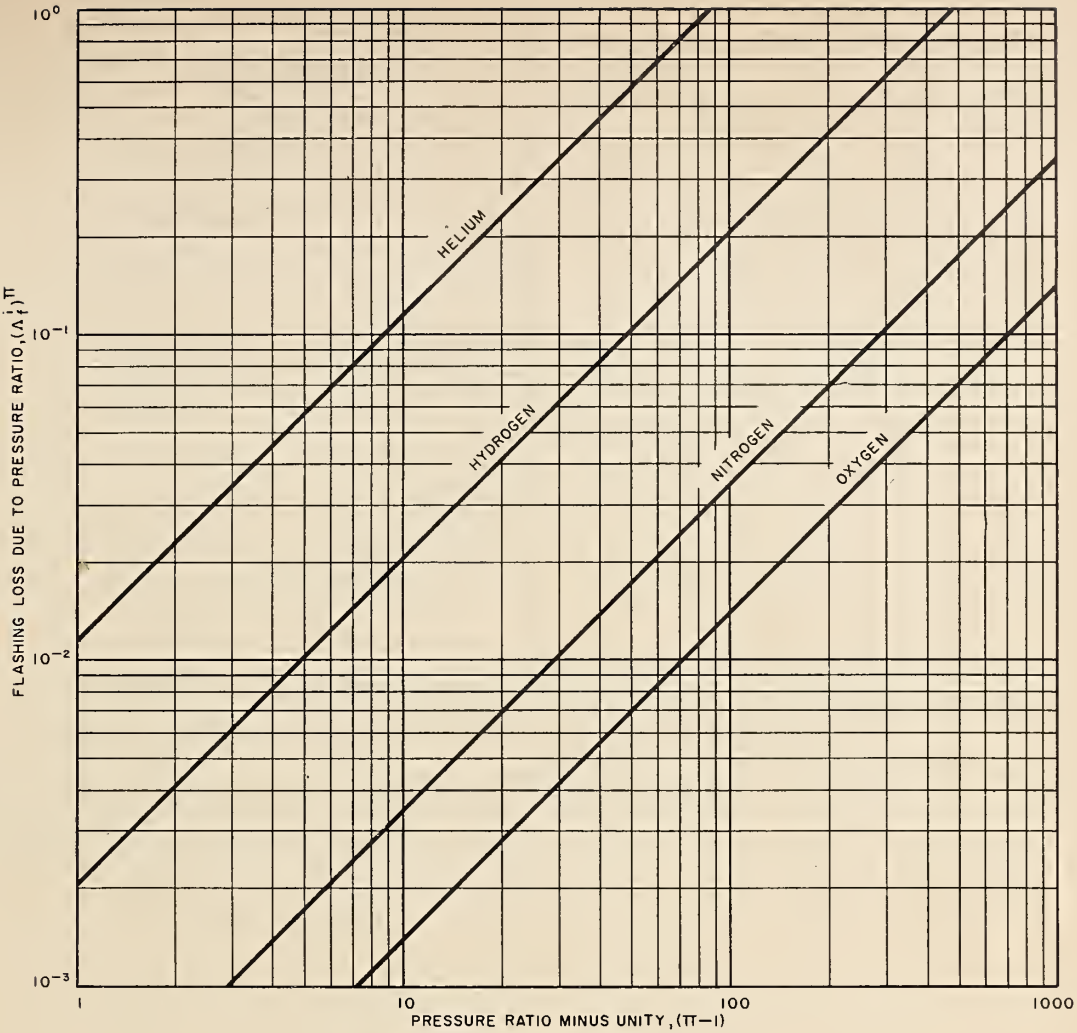

FIGURE 18. *Flashing loss due to pump work.*

where the flashing loss due to heat leak is

$$(\Lambda_f^i)^q \equiv \frac{1}{\lambda}\frac{qL}{w}, \tag{33}$$

and the flashing loss due to pressure is

$$(\Lambda_f^i)^\pi \equiv \frac{1}{\lambda}(1-T_r\beta_r)p_rv_r(\pi-1). \tag{34}$$

Both eq (33) and (34) plot as single straight lines for each fluid. The two parameters, qL/Q and π, are not independent but are related through the flow equation. As the flashing loss cannot be greater than unity, these variables have definite upper limits which depend upon the fluid. From figure 10, we see that a range of the heat leak parameter from 10^0 to 10^5 is satisfactory; this range is also satisfactory for plotting eq (33). For the purposes of plotting eq (34) the range of the pressure ratio, π, will arbitrarily be taken as from 1 to 1,000. For liquid helium, liquid hydrogen, liquid nitrogen, and liquid oxygen, eq (33) and (34) are plotted in figures 17 and 18.

The use of figures 17 and 18 is straightforward: In previous calculations the heat-leak parameter and the pressure ratio have been determined. One merely enters the figures with these values and determines the components of the flashing loss.

d. Cool-Down and Trapped-Liquid Losses

It is not feasible to construct curves for the computation of the cool-down and trapped-liquid losses; the computations are straightforward, and should be made for each design after it has been ascertained that the flow parameters and other losses are acceptable.

5.5. Examples

In this section three examples, based upon the solutions given in the preceding four sections, are given. The purpose is to give the reader a feeling for the numbers (lengths, pipe diameters, pump pressures, etc.) which can arise, and to provide some design curves for anticipated applications.

In all three examples, the same type of insulation is used: Silica Aerogel at a pressure of less than 10^{-5}-mm Hg with a diameter ratio of 5. This material is the poorest insulation included in table 3; it is the only powder, however, for which data were available when the computations were made. As shown in section 4.1.b, very much more effective insulations are available for liquid hydrogen systems, leading to systems with much smaller losses, much greater possible lengths, etc. The values in table 3 show that, for liquid oxygen, powders are available that will permit only about one-quarter the heat leak experienced with Silica Aerogel at less than 10^{-5}-mm Hg. Thus the examples given here do not presuppose the use of optimum insulation; on the contrary, they are systems with relatively poor insulation.

The first two examples are general treatments of liquid hydrogen and liquid oxygen transfers with only one pumping station. As the insulation (and therefore the heat leak) is fixed, only four variables remain in the flow equation: The inside diameter of the liquid line, D; the length of the transfer line, L; the flow rate, Q; and the pressure ratio across the pump, π. The losses at the pump for both examples have already been computed and are given in figure 13 for hydrogen and figure 15 for oxygen. The flashing losses are functions of three variables: The flow rate, Q, the line length, L, and the pressure ratio across the pump, π. For a detailed picture of the influences of all of the variables, the flow equation and the flashing loss require several sheets of curves.

The third example illustrates the effects of the number of pumping stations on the various parameters: Diameter, pump pressure ratio, total liquid loss, etc. In addition the cool-down and trapped-liquid losses are discussed as a function of the number of pumping stations.

a. Liquid Hydrogen

The heat leak, q, is obtained from the data given for liquid nitrogen in table 3 by making the conservative assumption that the mean thermal conductivity, $\bar{k}$, is the same when insulating a liquid hydrogen line as a liquid nitrogen line. Thus, using figure 6 and table 3, we obtain:

$$q_{H_2}=\left(\frac{q}{\bar{k}\Delta T}\right)(\bar{k}\Delta T)_{N_2}\frac{\Delta T_{H_2}}{\Delta T_{N_2}}$$

$$=3.95\times 0.463\times\frac{280}{220}=2.32\ \text{Btu/hr ft.}$$

The eight figures, 19 through 26, give a picture of the flow characteristics. On each figure the liquid-line diameter, D, is plotted against the transfer-

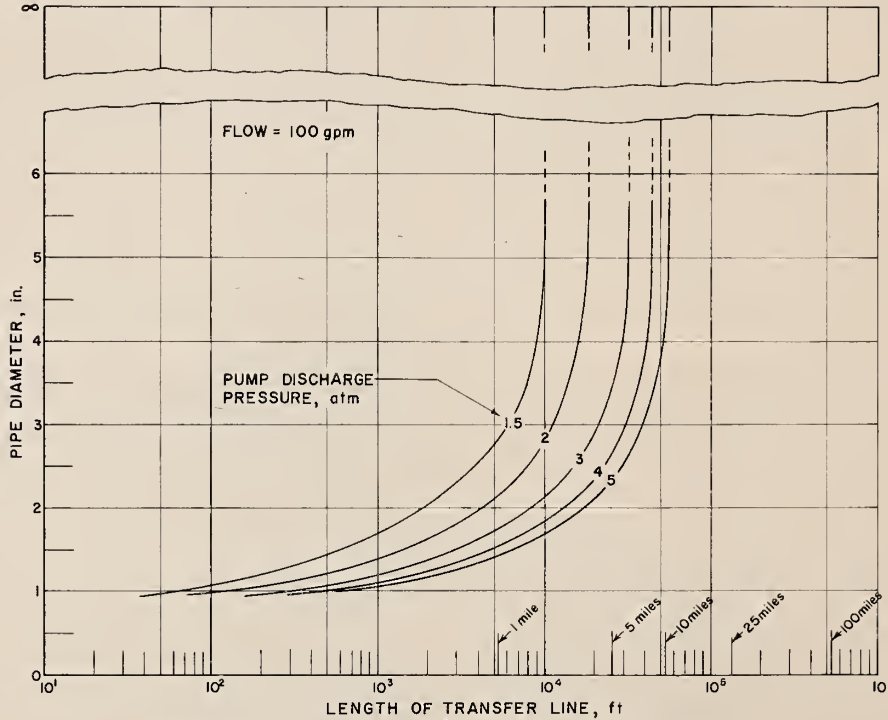

FIGURE 19. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

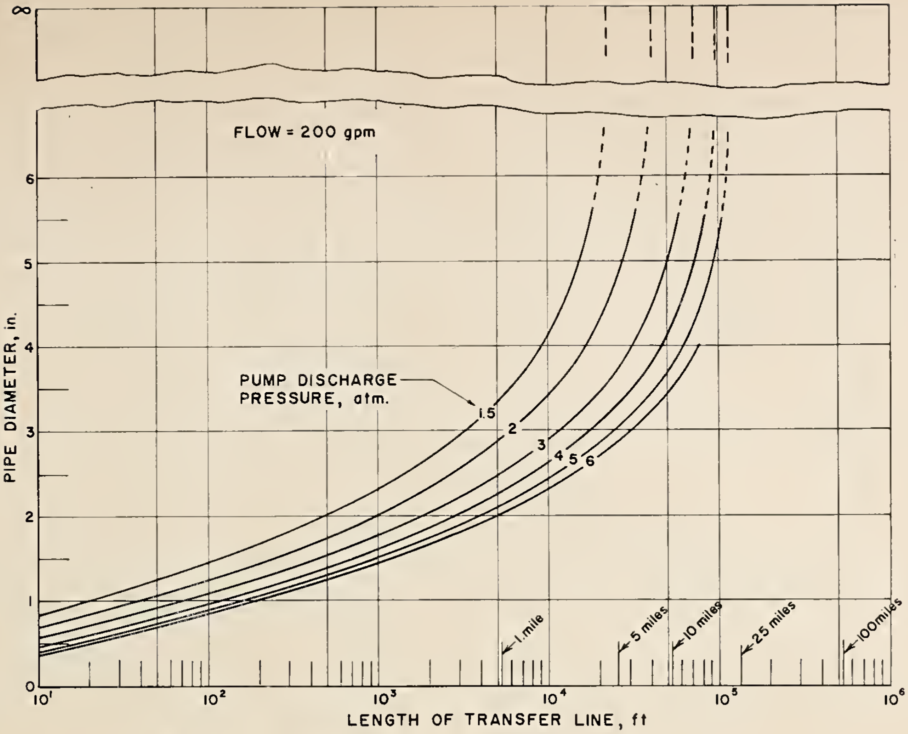

FIGURE 20. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

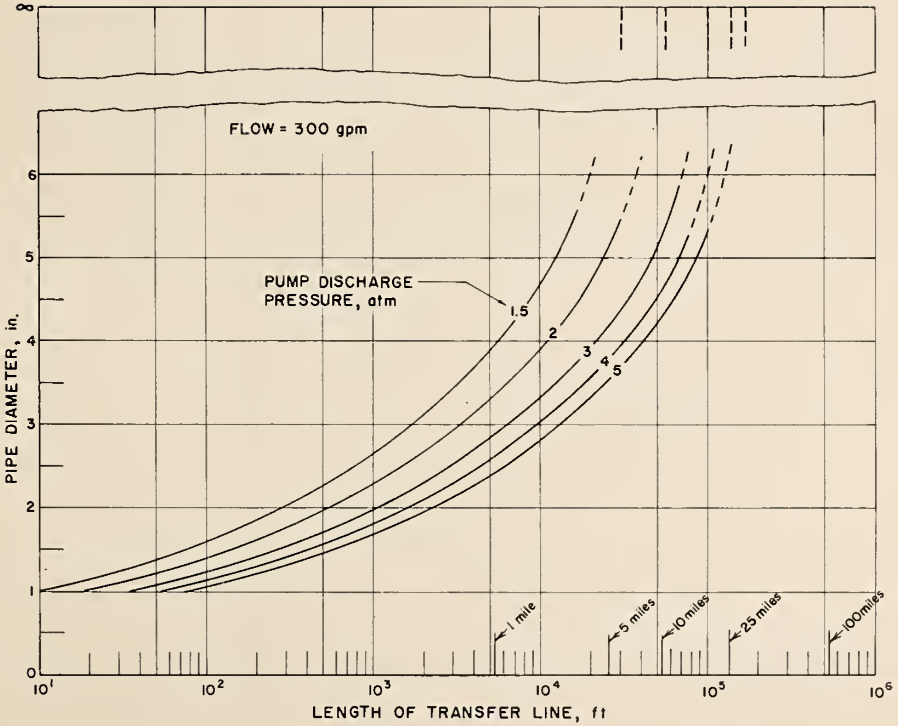

FIGURE 21. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

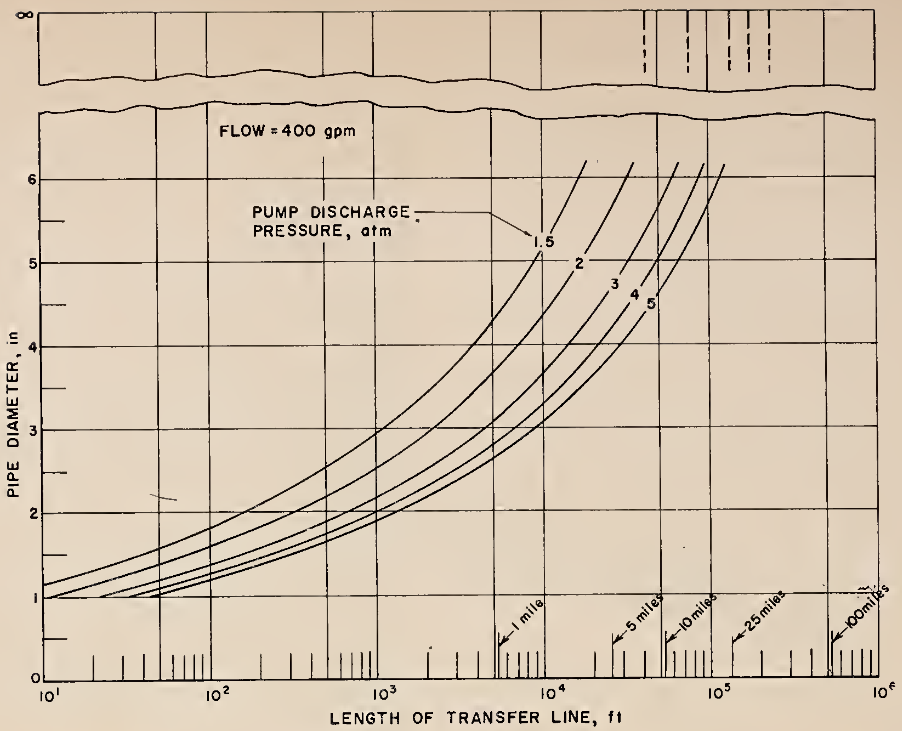

FIGURE 22. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

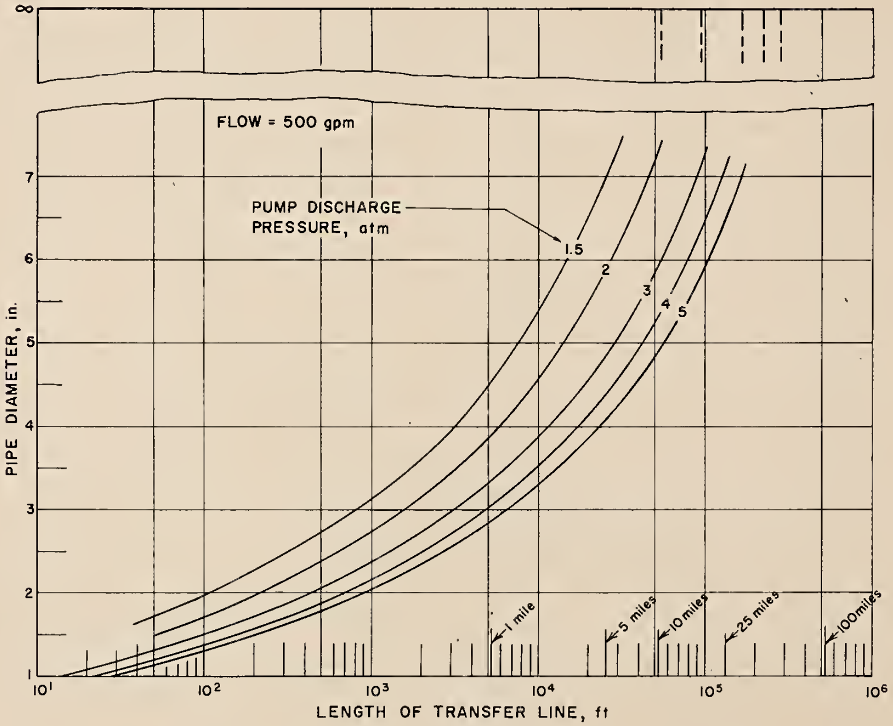

FIGURE 23. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

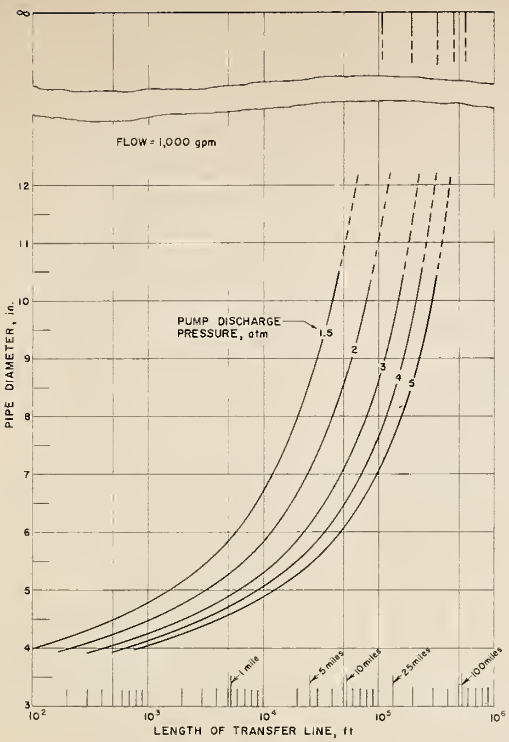

FIGURE 24. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

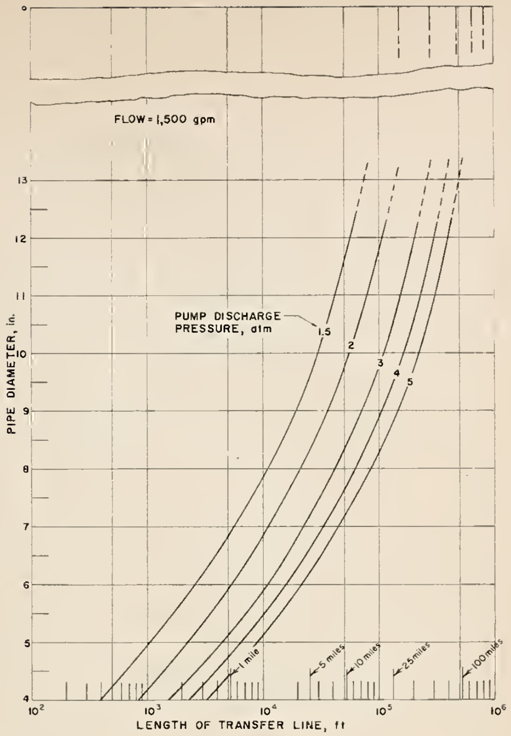

FIGURE 25. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

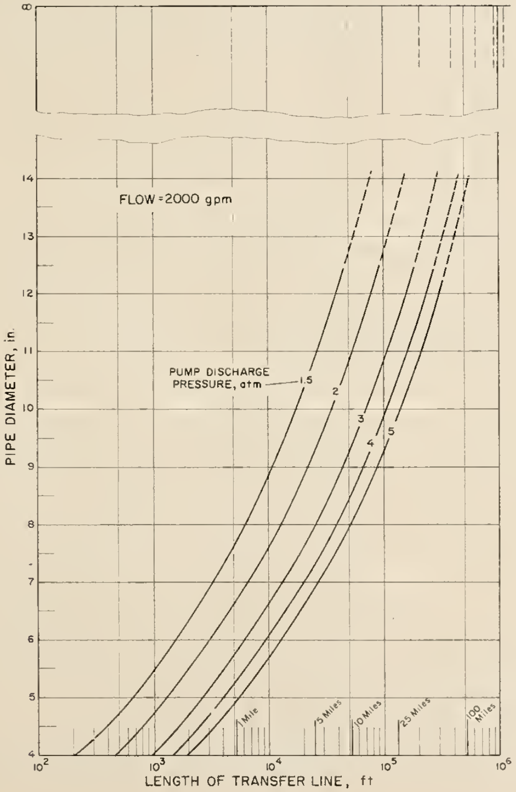

FIGURE 26. *Liquid hydrogen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

line length, L, with the pump-discharge pressure as a parameter; each figure is for a different flow rate, Q.

The range of values for the pump-discharge pressure (i. e., up to 5 atm) was chosen because work done by the author on the pumping of liquid hydrogen indicates that these pressures can be attained, for this application, by altering conventional equipment.

The range of values for the length depends, of course, upon the flow rate and the pump-discharge pressure, the maximum length for any set of conditions being when the friction effects are zero (i. e., the diameter is infinite). These maximums are shown at the top of each figure. They range from about 1.9 miles for a flow of 100 gpm at a pump pressure of 1.5 atm to over 200 miles for a flow of 2,000 gpm and a pump pressure of 5 atm.

The range of values for the flow rate (100 to 2,000 gpm) was selected because these flows are anticipated for various applications, and show that very long-distance transfers may be feasible if high enough rates are used. The advantages of transferring at high-flow rates can be obtained by employing intermittent-transfer procedures: store the liquid and then transfer rapidly. The storage interval should not be so long that the system is permitted to warm appreciably, for then the repeated cool-down and trapped-liquid losses will become too large.

The pipe diameter is the dependent variable in these figures. It should be noticed that when pumping at 100 gpm, a liquid-line diameter of 4 to 5 in. is effectively infinite. However, with a flow of 2,000 gpm, quite large pipes are necessary to approach the maximum line lengths. Of course, losses are just as, or more, important than the initial cost as influenced by equipment and material size; pipe diameter is therefore not the sole consideration.

The loss at the pump is given in figure 13; if the efficiency is over 40 percent, this loss (for pressure ratios of 5 and less) is quite reasonable.

The flashing loss is plotted in figures 27 through 30 as a function of the transfer-line length, with the pump-discharge pressure as a parameter. The right-hand ends of the curves are at the lengths corresponding to zero friction effects: infinite diameter. For short lines, the energy supplied by the pump is the dominant factor influencing the flashing loss; thus the flashing loss depends only upon the pump-discharge pressure. As the lines become longer, the energy added to the liquid through heat leak gains in importance until it becomes dominant.

The maximum loss, which occurs when the line length is a maximum, depends only upon the pump pressure; this is demonstrated by the following reasoning: In this case the pump-discharge pressure equals the thermal pressure ratio, π_t, given by eq (28); therefore for a given pump-discharge pressure, the heat-leak parameter, qL/Q, is fixed. Since π and qL/Q are fixed, it follows from eq (31) that the flashing loss is fixed.

Figures, such as 19 through 30 and 13, can form the technical basis for the design of long transfer lines. Consider, for example, the problem of transferring liquid hydrogen over a distance of 25 horizontal miles with one pumping station. Figure 23 shows that, for a flow of 500 gpm, a 7-in. line is required with a 4-atm pump while a 6¼-in. line is required with a 5-atm pump. If smaller line sizes are desired, higher pumping pressures must be used (for the same flow). For a flow of 500 gpm, figure 27 shows that the flashing losses are about 8 percent; thus, depending upon the pump efficiency, the total losses would be between 9 and 15 percent.

Let us reconsider the 25-mile transfer, but now at a 2,000 gpm rate. Figure 26 shows that a 2-atm pump can perform the transfer in a 13½-in. pipe, but that with a 5-atm pump, only a 9¾-in. pipe is required. The main disadvantage of a high-flow rate is thus illustrated: it requires a larger line, leading to greater initial cost, greater cool-down loss, and the possibility of a greater trapped-liquid loss. The primary advantage of the high rate is seen from figure 30; at 2,000 gpm, the flashing loss for the 25-mile transfer is less than 3 percent, a decrease of more than 5 percent under the 500 gpm loss.

b. Liquid Oxygen

The heat leak for this example is also obtained from figure 6 and table 3. Recalling that the insulation is Silica Aerogel at a pressure of less than 10^{-5}-mm Hg with a diameter ratio of 5,

$$q_{O_2}=\frac{q}{\bar{k}\Delta T}(\bar{k}\Delta T)_{O_2}=3.95\times 0.437=1.727 \text{ Btu/hr ft.}$$

The flow equation is plotted in figures 31 through 35. The range for the pump-discharge pressure now involves moderately high pressures. Although these are probably readily attained with liquid oxygen, the high pressures have many drawbacks and multiple pumping stations should be seriously considered for very long lines.

The range of flows included in the figures was chosen through consideration of present and predictable applications. The range of values for the maximum lengths (i. e., with no friction effects), is fantastic. It varies from over 250 miles at 200 gpm and a pump-discharge pressure of 10 atm to over 6,600 miles at 2,000 gpm at 60-atm pressure. Of course to approach this upper limit is unrealistic from both initial-cost and liquid-loss consideration.

Figure 15 gives the loss at the pump for oxygen. It shows that, for moderately efficient pumps, this loss will always be less than 4 percent and in most cases less than 2 percent.

Figures 36 through 40 give the flashing losses for transferring liquid oxygen over long distances, the ranges of variables being those discussed above.

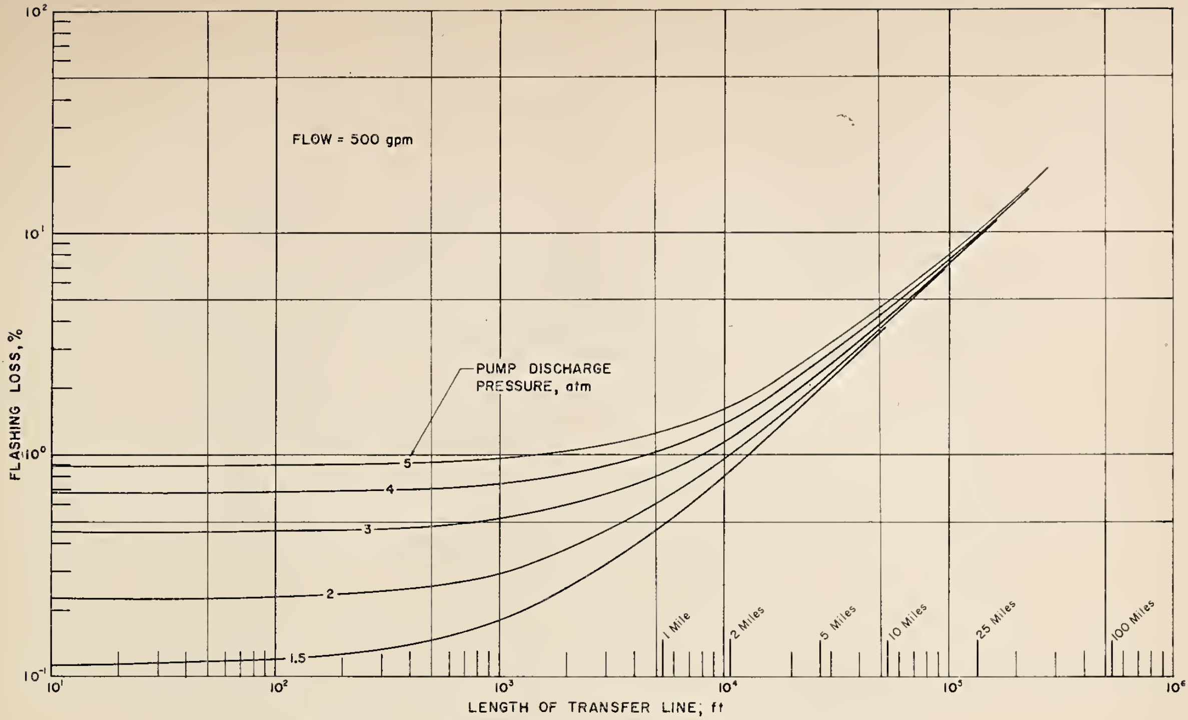

Figure 27. *Flashing losses liquid hydrogen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

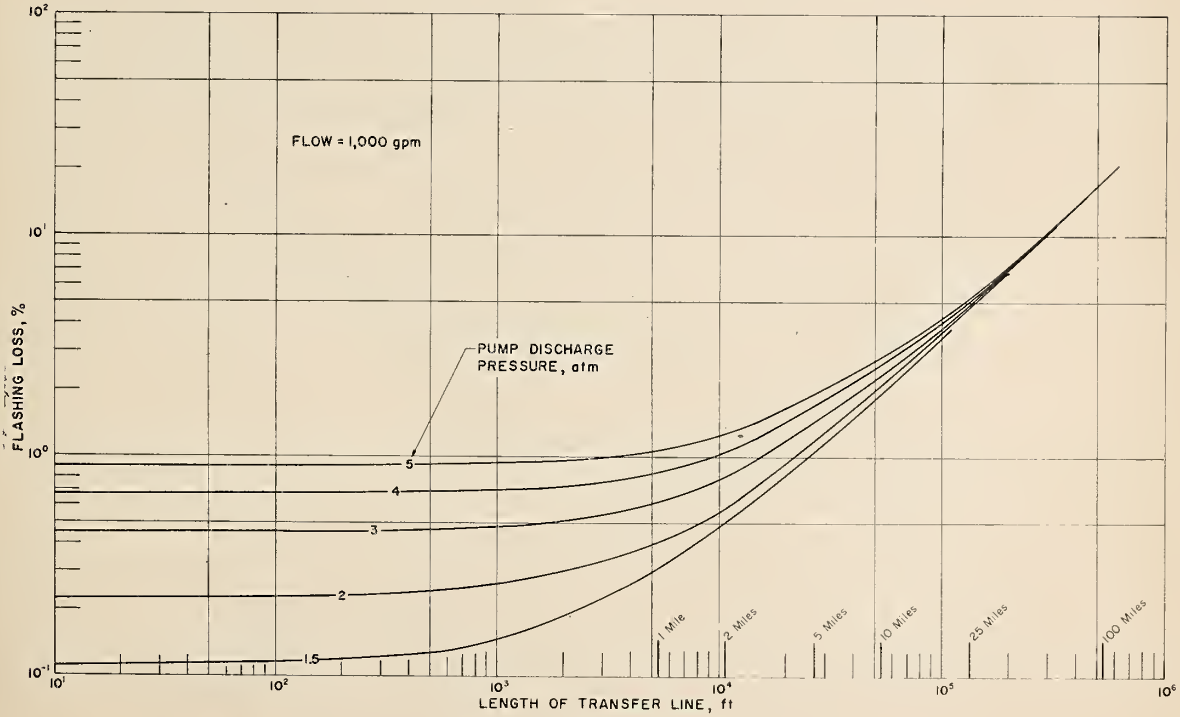

Figure 28. *Flashing losses in liquid hydrogen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg; Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

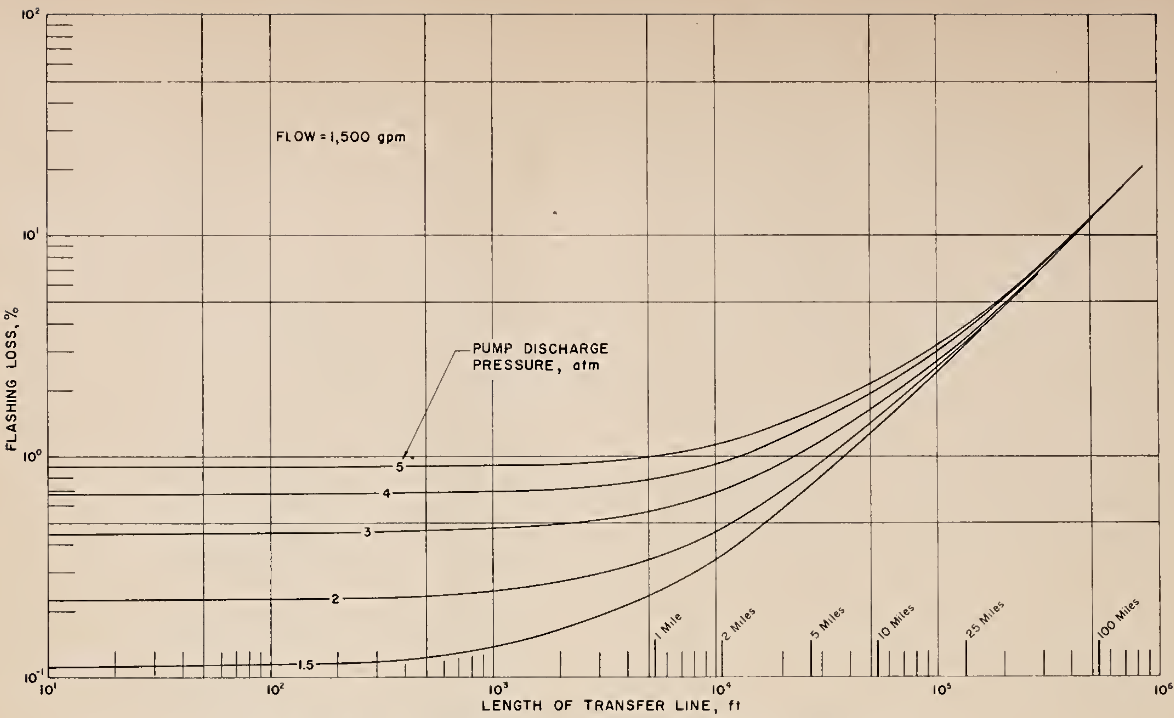

FIGURE 29. *Flashing losses in liquid hydrogen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

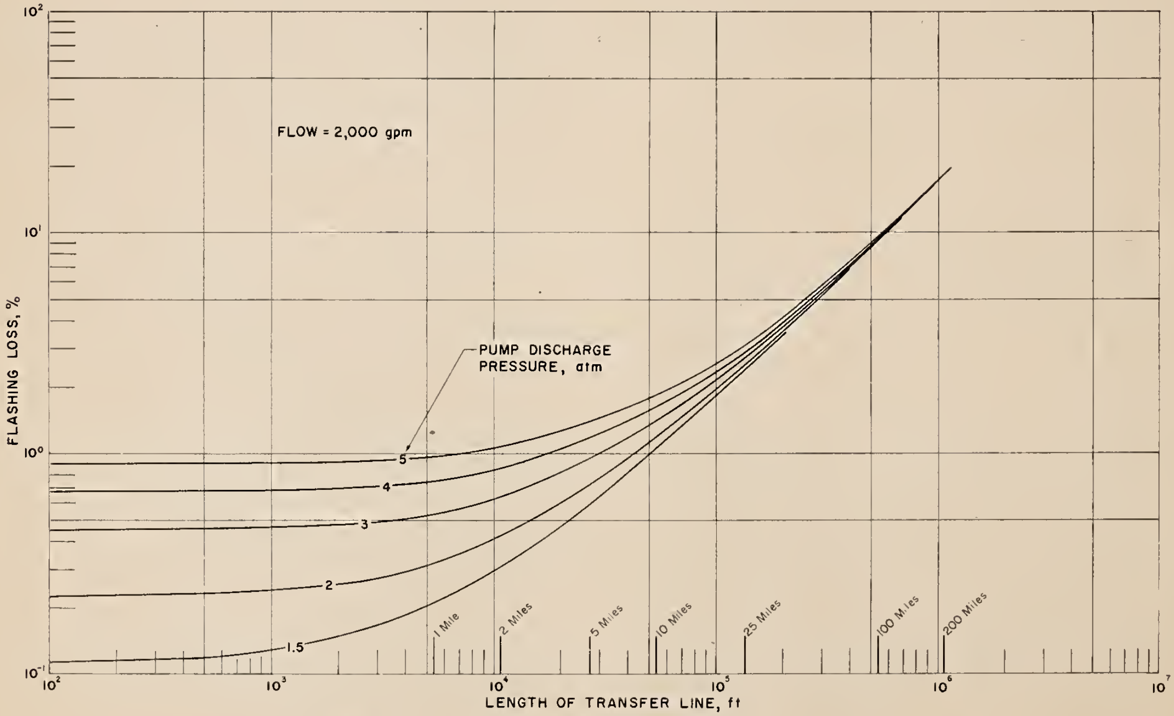

FIGURE 30. *Flashing losses in liquid hydrogen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 20.4° K.

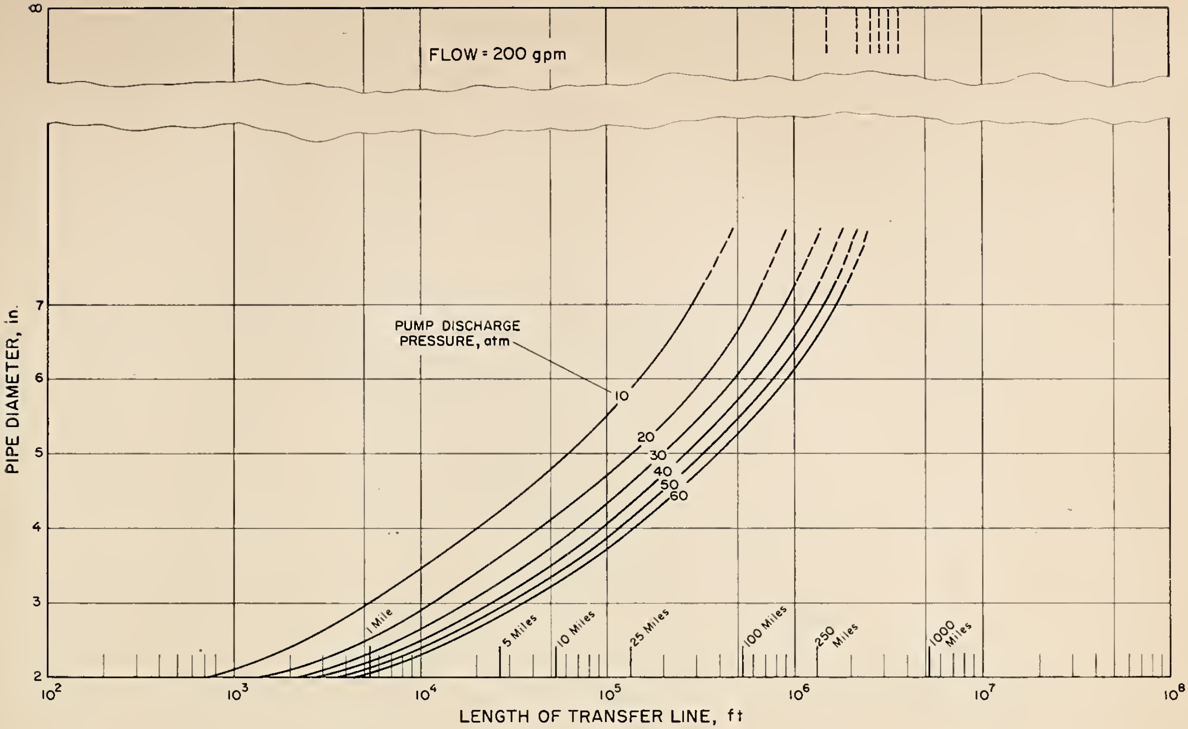

FIGURE 31. *Liquid oxygen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

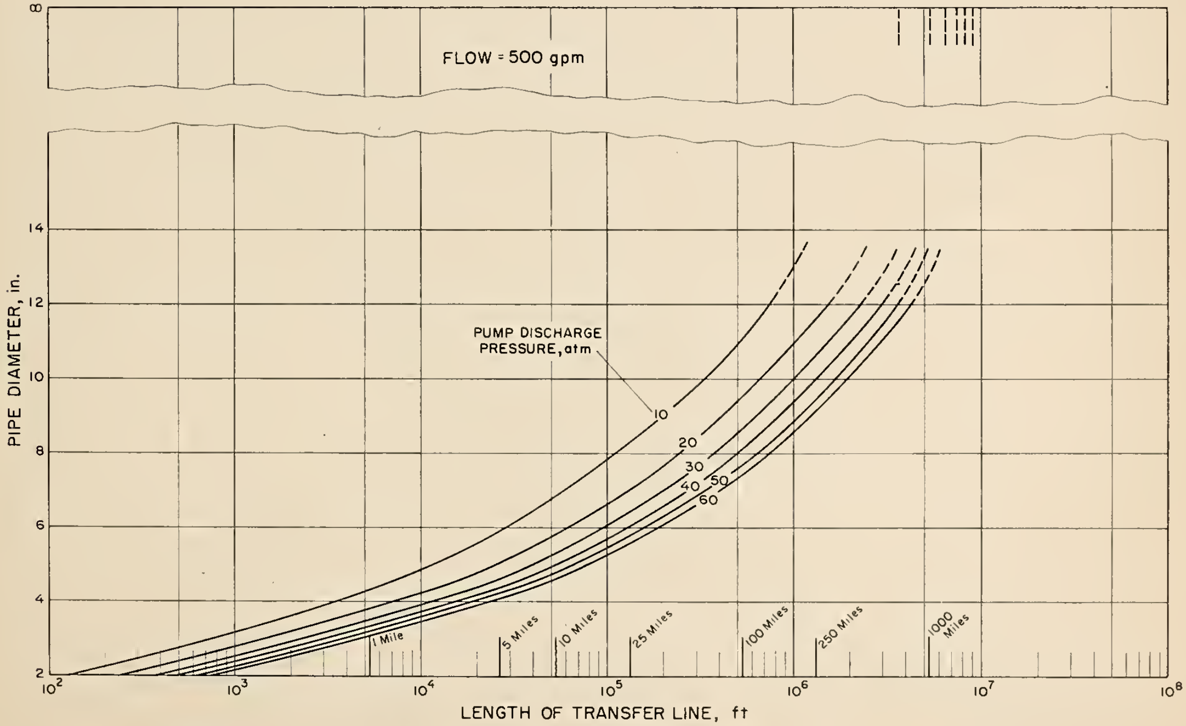

FIGURE 32. *Liquid oxygen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K

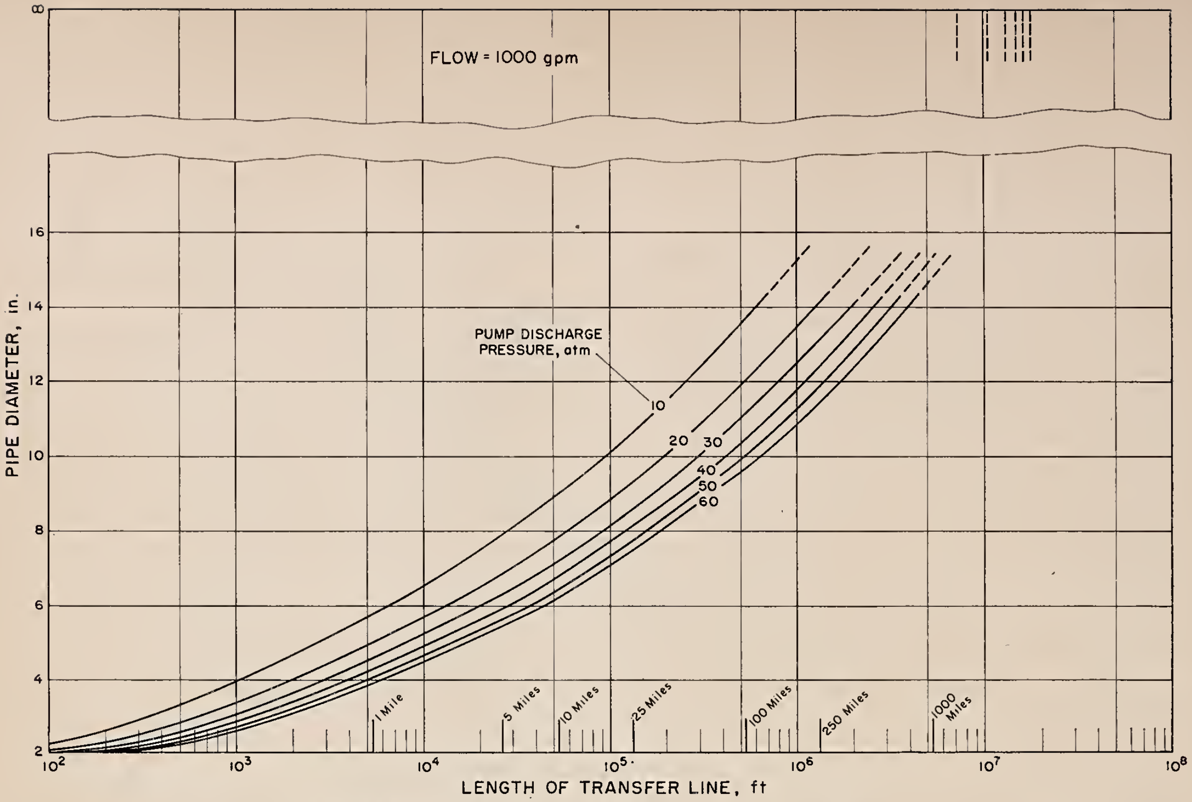

FIGURE 33. *Liquid oxygen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

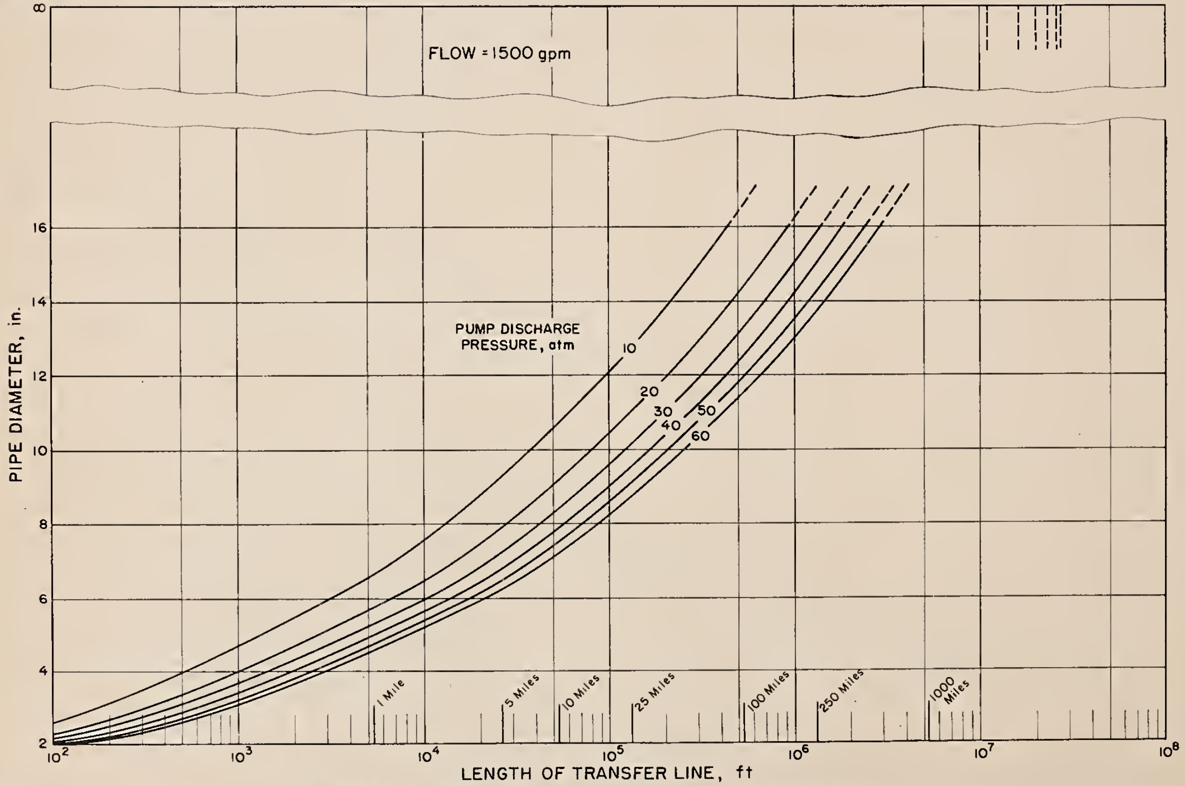

FIGURE 34. *Liquid oxygen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

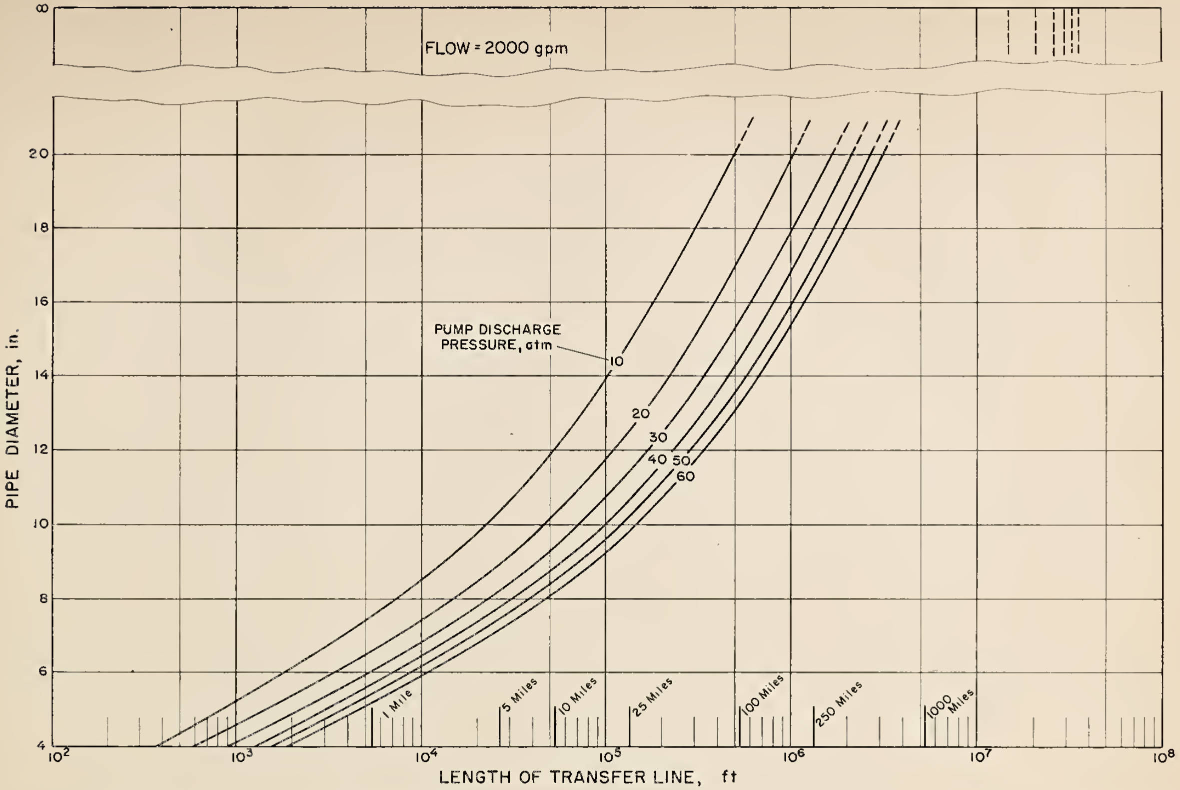

FIGURE 35. *Liquid oxygen transfer line characteristics.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

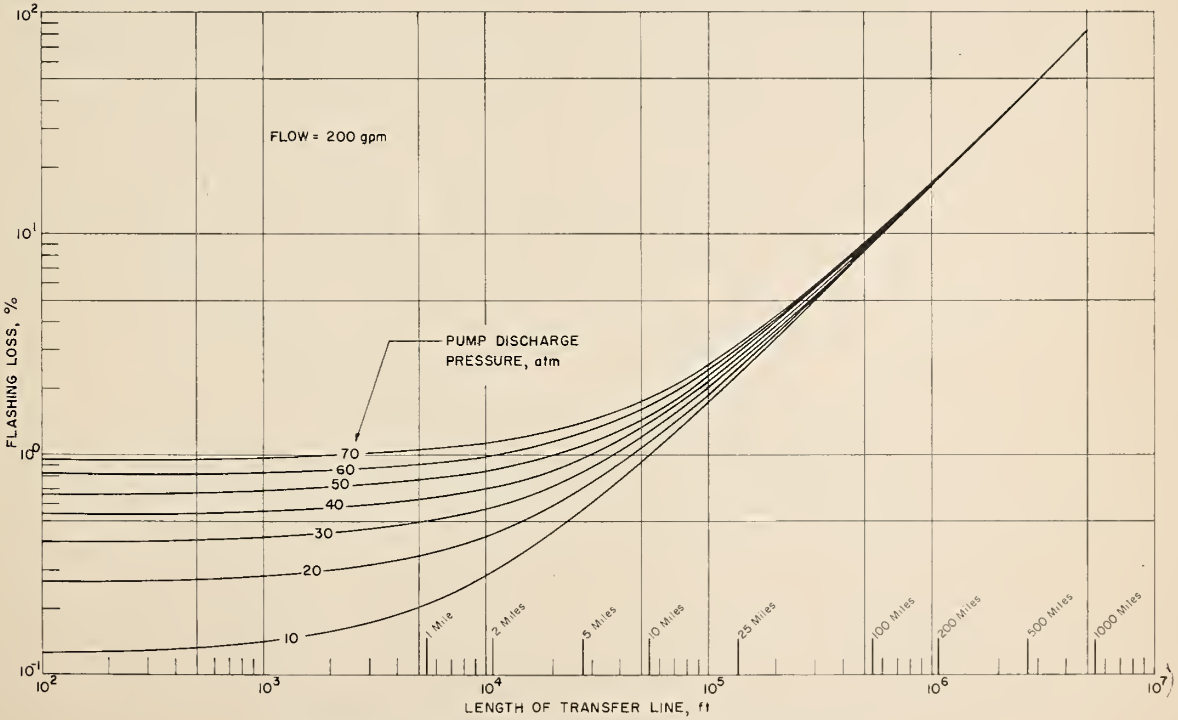

FIGURE 36. *Flashing losses in liquid oxygen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

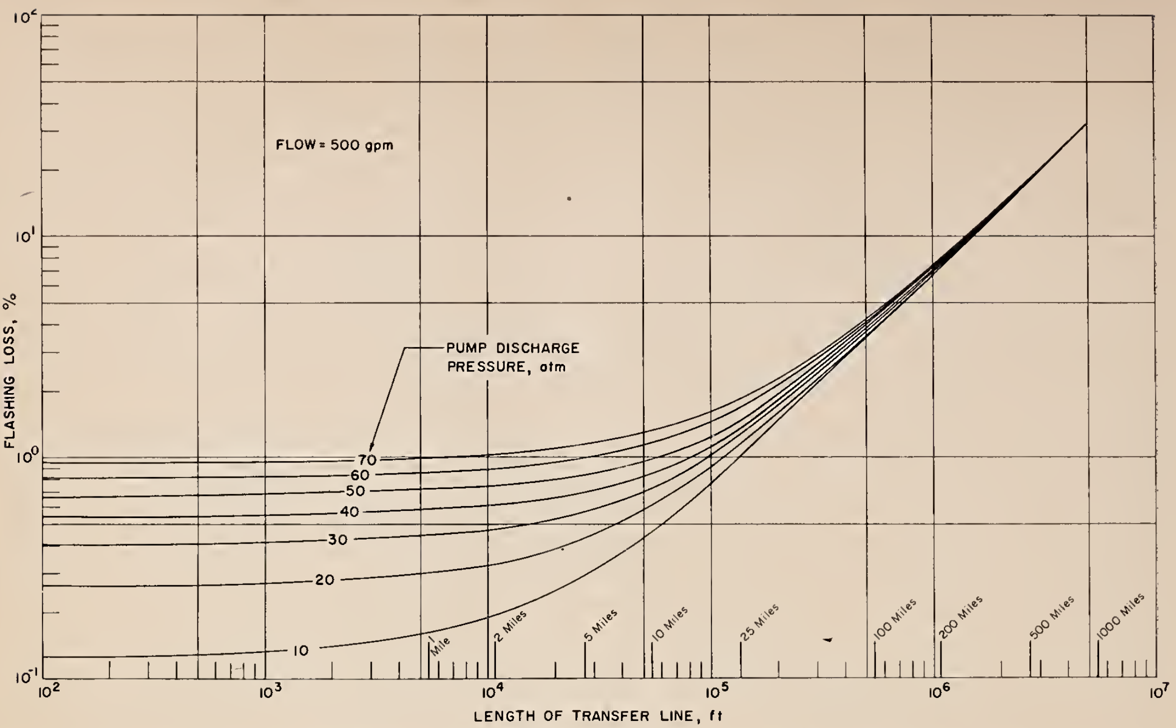

FIGURE 37. *Flashing losses in liquid oxygen transfer lines.*

Insulation: Evacuated silia aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

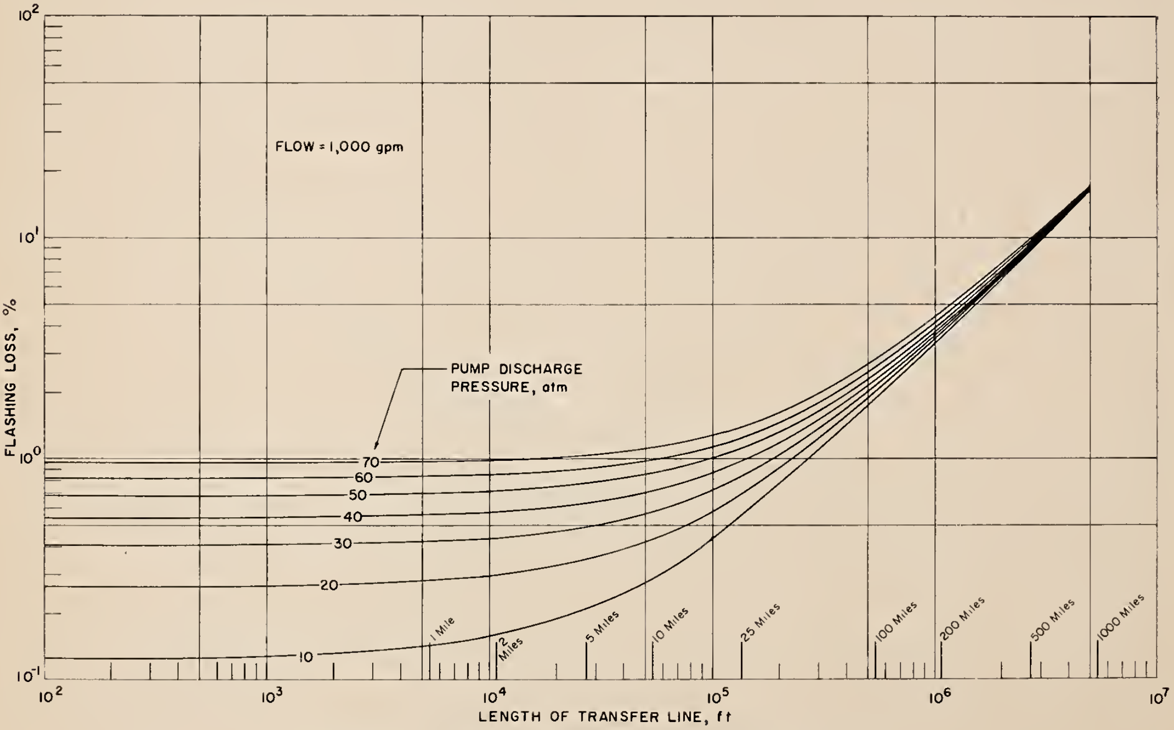

FIGURE 38. *Flashing losses in liquid oxygen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

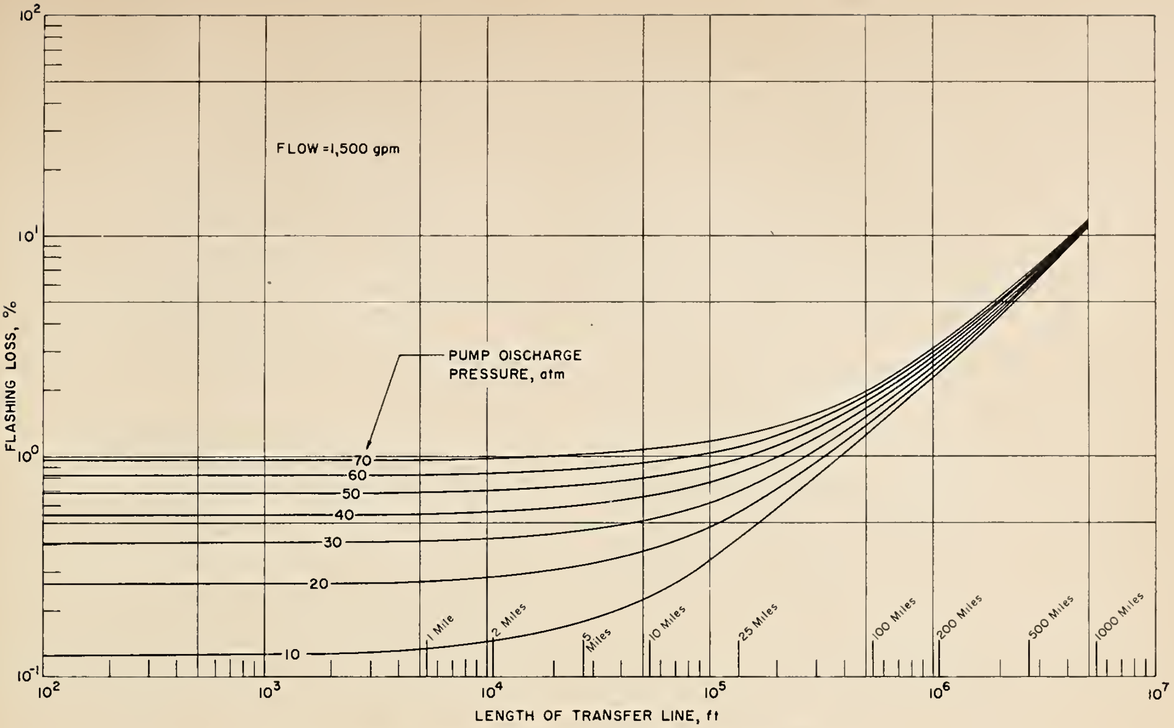

FIGURE 39. *Flashing losses in liquid oxygen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

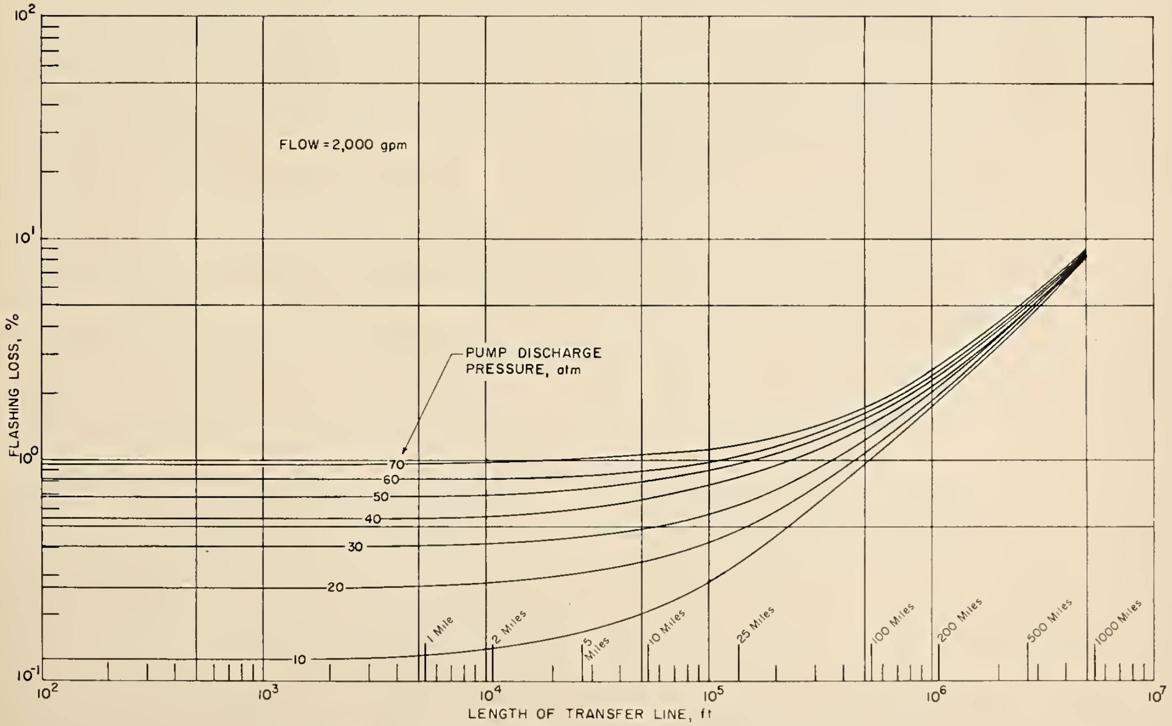

FIGURE 40. *Flashing losses in liquid oxygen transfer lines.*

Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Fluid properties: Assumed constant. Values are those of saturated liquid at 90.13° K.

We see that even for the relatively low flow of 200 gpm, the flashing losses are less than 10 percent for a 100-mile transfer, while, at a flow of 2,000 gpm, less than 2 percent is lost through flashing after a 100-mile transfer. Even for low flows, the flashing loss is less than 2 percent for transfers under 10 miles in length.

In order to obtain an idea of pipe sizes and liquid losses, let us consider the example of a 25-mile transfer with one moderate pressure pump (10 atm). For a flow of 200 gpm, the pipe diameter would have to be 6 in., while the total loss (pump plus flashing) would be less than 3 percent. At a rate of 2,000 gpm, however, a 15 in.-diameter pipe would be required, while the total loss would be less than 1.2 percent. It must be emphasized that the saving in steady-state losses with the higher flow rate can be significantly decreased by the larger initial cost of the installation, and the greater cool-down and trapped-liquid losses. Of course, if liquid *had to be removed* from a plant at the rate of 2,000 gpm, there would be no alternative to the 15-in. line except to use multiple pumping stations and/or higher pump-discharge pressures.

c. Multiple Stations

Assume that the problem is to transfer liquid hydrogen horizontally over a distance of 10 miles at an initial rate of 500 gpm; this amount of liquid enters the first pumping station in a saturated state at a pressure of 1 atm. The insulation is Silica Aerogel at a pressure of less than 10^{-5}-mm Hg; in section 5.5.a it was concluded that the heat leak into the transfer line would be 2.32 Btu/hr ft. For the cool-down loss calculations it is assumed that the liquid line is copper with 1/16-in. wall, and that heat capacity of all equipment is negligible compared with that of the liquid line; for the trapped-liquid loss calculations it is assumed that the internal volume of all equipment is negligible compared with that of the liquid line; and that none of the liquid can be salvaged from the line.

The independent variables are the number of pumping stations and the pump-discharge pressure. It is assumed that the pumping stations are equally spaced, the line length between two adjacent stations then being $10/N$ miles where N is the number of pumping stations. Thus $10/N$ miles is to be used for the length (L) when determining the flow characteristics and losses associated with each portion of the system. Stations need not be equally spaced; in some cases, the terrain may not permit this. However, in any case the lengths required for the step-by-step computations are readily ascertained. Finally, it is assumed that all pumps have efficiencies of 50 percent.

Figure 41 gives the average liquid line diameter as a function of the number of pumping stations and the pump-discharge pressure. The average is the arthmetic mean of the diameters computed for the various sections. For minimum line size a large number of stations and high pressures should be used: A 3-in. line is adequate if 10 stations with 5-atm pumps are used; for 1 station with a 2-atm pump, a 7.3-in. line is required. Note that one 1½-atm pump cannot possibly fulfill the specifications.

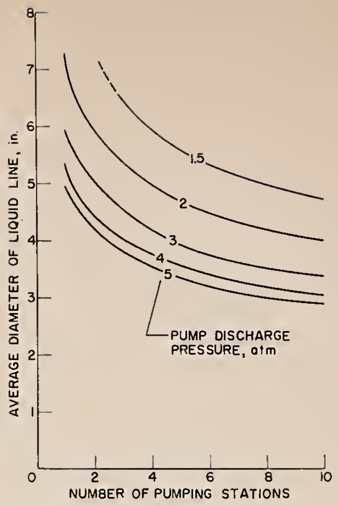

FIGURE 41. *Effect of multiple pumping stations on transfer line size.*

Fluid: Liquid hydrogen. Properties assumed to be those of saturated liquid at 20.4° K. Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Flow rate: 500 gpm.

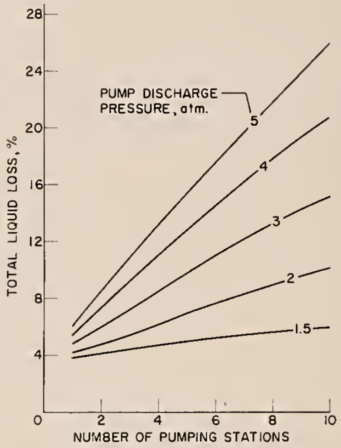

FIGURE 42. *Effect of multiple pumping stations on total steady-state liquid loss.*

Fluid: Liquid hydrogen. Properties assumed to be those of saturated liquid at 20.4° K. Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Flow rate: 500 gpm.

Figure 42, which gives the effects of the number of pumping stations and pump-discharge pressure upon the total steady-state liquid losses (losses at the pumps plus flashing losses), shows that the small diameters are associated with the large losses. This is because much more energy is put in by the pumps and because the heat-leak parameter, qL/Q, is greater due to the smaller average flow Q. The steady-state loss for the 3-in. line would be 26 percent while it would be only 4 percent for the 7.3-in. line. It is apparent that each problem

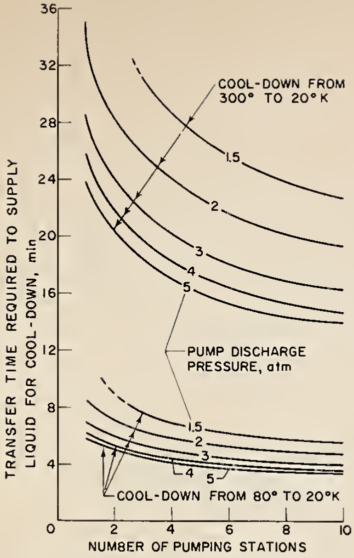

Figure 43. *Effect of multiple pumping stations on cool-down losses.*

Fluid: Liquid hydrogen. Properties assumed to be those of saturated liquid at 20.4° K. Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Flow rate: 500 gpm.

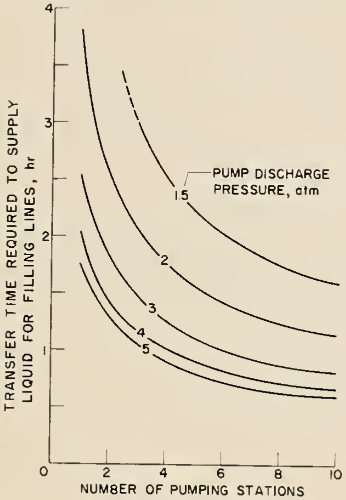

Figure 44. *Effect of multiple pumping stations on trapped-liquid losses.*

Fluid: Liquid hydrogen. Properties assumed to be those of saturated liquid at 20.4° K. Insulation: Evacuated silica aerogel at less than 10^{-5} mm Hg. Diameter ratio: 5. Flow rate: 500 gpm.

should be analyzed to obtain an optimum combination of line size, number of pumping stations, pump-discharge pressure, etc.

The results of the computations of the cool-down loss are given in figure 43. It was assumed that the refrigeration for the cool-down was supplied by the latent heat of vaporization plus one-half of the sensible energy available between liquid-hydrogen boiling point (20° K) and the temperature at which cooling with liquid hydrogen was started. Two sets of computations were made: one assumed that liquid hydrogen was used to cool the system through the whole range from room temperature (300° K). The other assumed that the system was precooled to 80° K with liquid nitrogen; hydrogen was used to cool from 80° to 20° K only. The cool-down loss is expressed as the time that it would take to supply the liquid required for cooling down if the liquid were supplied at the steady-state transfer rate, 500 gpm. (This time is not related to the actual time required to cool the system to its operating temperature.)

It is seen, for example, that the ordinate (in fig. 43) for a nitrogen precooled system with two 2-atm pumps is about 7 min; if the total run were only 4.86 days (7,000 min), the cool-down loss would be equivalent to only one-tenth of 1 percent of the total liquid hydrogen transferred. Without precooling the loss would be only four times greater. It thus appears that the cool-down loss can readily be made negligible.

The trapped-liquid loss, however, can be significant. Figure 44 presents the results of this calculation, the ordinate being similar to that used in the preceding figure. We have assumed that a volume of liquid equal to the total volume of the transfer line is lost. The figure indicates that the system with two 2-atm pumps would require 2.7 hr of liquid production. In order for this trapped-liquid loss to be less than 1 percent of the total liquid transferred, a run would have to last at least 11.25 days (270 hr). As it is anticipated that plants of this capacity will be run on a continuous basis, this loss can be made negligible also.

6. Conclusions

Because it has not been experimentally verified, the material presented here should be considered as tentative. However, because the principles upon which these developments are based have led to accurate conclusions for similar problems for many years, it is felt that this work is probably valid. Of course, experimental verification should be sought; work of this type is now being undertaken. In addition, some large transfer systems are currently being built; the operating data obtained from these can provide some of the required empirical verification.

If the material developed here is valid, then there are three conclusions which can be drawn:

(1) It is feasible, with conventional pumping equipment and known insulation techniques, to pump liquefied gases over long distances.

(2) Design calculations for single-phase transfer systems for liquefied gases are straightforward and uncomplicated.

(3) For some applications, transfer systems can be designed so that the transfer losses (pumping, flashing, cool down, and trapped liquid) are acceptable, while for others the use of transfer systems is not practical. Preliminary design computations will therefore be required for many applications before the practicability of using a transfer system can be ascertained.

7. Appendix. Derivation of General Equations

In this section, the general equations required in the body of this paper are derived. They are the three conservation laws (energy, momentum, and mass), and equations expressing each of the four liquid losses that occur in a transfer system.

7.1. List of Symbols

The symbols conform with convention whenever possible. Several subscripts have more than one meaning; however, as the different meanings are in different sections of the text, no confusion should result.

C_p = isobaric specific heat,
D = inside diameter of liquid line of transfer line,
E = energy to be removed from parts of a transfer system during cool-down,
f = friction factor,
g = acceleration of gravity,
h = enthalpy,
$\bar{k}$ = a mean thermal conductivity,
L = length of transfer line,
M = mass of liquid lost,
M_{CD} = cool-down loss, in mass units,
m = mass of a metallic part of a transfer system,
N = number of pumping stations,
p = pressure,
q = heat leak per unit time per unit length,
Q = flow rate, in gallons per minute,
r = radial coordinate,
R = gas constant,
Re = Reynolds number,
s = entropy,
T = temperature,
T = total time of transfer after steady state is attained,
u = internal energy of fluid in the absence of macroscopic motion, capillarity, and external force fields,
V = mean flow velocity in a transfer line,
v = specific volume,
V_s = total internal volume of transfer system,
w = mass rate of flow,
W = shaft power delivered to pump,
x = a coordinate which locates position along a transfer line,
z = a coordinate in the vertical direction,
β = volume coefficient of expansion,
ΔT = the difference between two temperatures,
$\Delta(T^4)$ = the difference of the fourth powers of two temperatures,
ϵ = emissivity,
η = pump efficiency,
η_c = critical pump efficiency,
λ = latent heat of vaporization,
Λ_{CD} = cool-down loss, expressed as a fraction of the total amount of liquid that enters a transfer system,
Λ_f^a = flashing loss, expressed as a fraction of the fluid that enters the preceding pump container,
Λ_f^i = flashing loss, expressed as a fraction of the liquid that flows through the preceding transfer line section,
Λ_p^a = loss at a pump, expressed as a fraction of the fluid which enters the pump container,
Λ_p^i = loss at a pump, expressed as a fraction of the liquid which enters the following transfer line section,
Λ_{TL} = trapped-liquid loss, expressed as a fraction of the total amount of liquid that enters a transfer system,
μ = absolute viscosity of liquid,
ν = kinematic viscosity,
π = pressure ratio across the pump,
π_f = dimensionless frictional pressure parameter,
π_t = dimensionless thermal pressure parameter,
ρ = liquid density,
σ = Stefan-Boltzman constant,
τ = shearing stress exerted by the pipe wall on the liquid.

Subscripts:

a	refers to conditions at the entrance to a transfer system,
a	when used with T, refers to ambient conditions,
CD	refers to cool down,
f	refers to flashing,
H_2	refers to hydrogen,
He	refers to helium,
i	refers to fluid state at entrance to transfer line,
i	refers to inside surface of insulation,
i	a running index to distinguish various parts of a transfer system,
ins	refers to insulation,
j	a running index to identify various parts of the insulation,
l	refers to vapor loss from pump container,
L	refers to fluid state at exit of transfer line, usually saturated liquid,
N_2	refers to nitrogen,
o	refers to fluid state at pump discharge,
o	refers to outside surface of insulation,
o'	refers to discharge state of an isentropic pump,
0_2	refers to oxygen,
p	refers to pump,
r	refers to reference state, usually saturated liquid at a pressure of 1 atm,
s'	refers to state of fluid at exit of throttle valve in remote storage or pump container,
TL	refers to trapped-liquid,
t	refers to a fluid state defined by $p_t = p_i$ and $T_t = T_r$,
1	refers to state of liquid entering transfer system,
2	refers to saturated liquid,
3	refers to saturated vapor state,
4	refers to superheated vapor state.

7.2. Basic Assumptions

Five general assumptions are used in the following derivations. Additional assumptions which apply only in specific instances are made at appropriate places.

(1) The fluid is a continuum.

(2) Steady state exists; at no place in the system does a property, parameter, or variable, change with time. It follows that the value of any variable is a function of its position in the system only.

(3) The system is one-dimensional. Position in the system is determined by a single coordinate (x in these discussions).

(4) The effects of surface tension and all force fields, except gravity, are negligible. The acceleration of gravity is constant.

(5) The cross-sectional areas of the transfer lines are constant and circular.

7.3. Energy Equation

Consider the element of pipe of length Δx shown in figure A.1. The length Δx is small enough so that the change of any variable between the positions x and $(x+\Delta x)$ is negligible compared with the value of the variable.

Next consider a region bounded by a fixed control surface that consists of the radial planes at x and $(x+\Delta x)$, and the pipe wall. Then, as steady state exists, the net energy transported into the region per unit time must be zero. Energy transport takes place by three modes: (1) Convection, (2) heat transfer, (3) work effect.

The energy convected through the plane at x per unit time is

$$w\left(u+\frac{V^2}{2}+gz\right).$$

(It has been assumed that the energy due to position in the gravitational field is zero when $z=0$.) As there is no energy convected through the pipe wall, it follows that the net energy convected into the region per unit time is

$$-\frac{\Delta}{\Delta x}\left[w\left(u+\frac{V^2}{2}+gz\right)\right]\Delta x.$$

The energy transported into the region per unit time due to heat transfer is

$$q\Delta x.$$

The only work effect that must be considered is that of the pressure forces at the radial planes at x and $(x+\Delta x)$. Because shaft work takes place only at the liquefier, pumps, etc., which are not considered to be part of the transfer line (in this discussion), it need not be accounted for here; as the work done by the friction force exerted by the wall upon the fluid does not act at the control surface, it need not be considered here either. The work done per unit time on the fluid entering the radial plane at x is

$$p\left(\frac{\pi}{4}D^2\right)V=p\left(\frac{\pi}{4}D^2\right)\frac{wv}{\frac{\pi}{4}D^2}=wpv.$$

It follows then that the net work entering the region per unit time is

$$-\frac{\Delta}{\Delta x}(wpv)\,\Delta x.$$

As the energy transported into the region per unit time must be zero, we have

$$-\frac{\Delta}{\Delta x}\left[w\left(u+\frac{V^2}{2}+gz\right)\right]\Delta x+q\Delta x-\frac{\Delta}{\Delta x}(wpv)\,\Delta x=0.$$

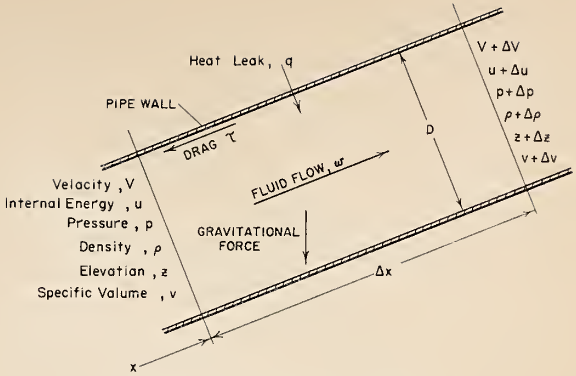

Figure A.1. *Element of transfer line.*

Allowing the elementary length Δx to approach zero in the usual manner of differential calculus, and noting that the flow rate (w) is constant, we have

$$\frac{du}{dx}+\frac{d}{dx}\left(\frac{V^2}{2}\right)+g\frac{dz}{dx}+\frac{d}{dx}(pv)=\frac{q}{w}.$$

Noting that the enthalpy (h) is defined as $(u+pv)$, the energy equation becomes

$$\frac{dh}{dx}+\frac{d}{dx}\left(\frac{V^2}{2}\right)+g\frac{dz}{dx}=\frac{q}{w}. \tag{1}$$

7.4. The Equation of Motion

Consider the element of pipe of length Δx, shown in figure A.1, and the fixed region bounded by the radial planes at x and $(x+\Delta x)$ and by the pipe wall. Then Newton's second law, for steady state, requires that the increase of the x component of momentum of fluid per unit time, as it passes through the region, must equal the x component of the forces acting on the fluid in the region. Momentum is transported through the control surface at the planes at x and $(x+\Delta x)$. The rate of entry of the x component of momentum through the surface at x is

$$\left(\frac{\pi}{4}D^2V\right)\rho V=\frac{\pi}{4}D^2\rho V^2.$$

As the diameter is constant, the net increase of the x component of momentum per unit time as fluid passes through the region is

$$\frac{\pi}{4}D^2\frac{\Delta}{\Delta x}(\rho V^2)\,\Delta x.$$

Three forces, which have x components, act on the fluid as it passes through the region: (1) The pressure force, (2) the friction (drag) force exerted by the pipe wall, (3) the gravitational force.

The pressure force acts on the radial planes only; on the surface at x it is $p\left(\frac{\pi}{4}D^2\right)$. Thus, the net pressure force in the x direction is

$$-\frac{\Delta}{\Delta x}\left[p\left(\frac{\pi}{4}D^2\right)\right]\Delta x=-\frac{\pi}{4}D^2\frac{\Delta p}{\Delta x}\Delta x.$$

The x component of the friction force exerted by the pipe wall upon the fluid is

$$-\tau(\pi D)\Delta x.$$

The x component of the gravitational force exerted on the fluid in the region is

$$-\frac{\pi}{4}D^2(\Delta x)\rho g\frac{\Delta z}{\Delta x}.$$

Arranging the preceding expressions in accordance with Newton's second law gives

$$\frac{\pi}{4}D^2\frac{\Delta}{\Delta x}(\rho V^2)\ \Delta x=-\frac{\pi}{4}D^2\frac{\Delta p}{\Delta x}\Delta x-\tau\ (\pi D)\Delta x-\frac{\pi}{4}D^2(\Delta x)\rho g\frac{\Delta z}{\Delta x}.$$

Simplifying and allowing the length Δx to approach zero as a limit yields the equation of motion:

$$\frac{d}{dx}(\rho V^2)+\frac{dp}{dx}+\frac{4}{D}\tau+\rho g\frac{dz}{dx}=0. \qquad (2)$$

7.5. Continuity Equation

For the steady-state, one-dimensional flow system shown in figure A.1, the principle of the conservation of mass merely requires that the mass rate of flow, w, be constant. But

$$w=\frac{\pi}{4}D^2\rho V.$$

Therefore, $\rho V=a$ constant. The equation of continuity in differential form is therefore

$$\frac{d}{dx}(\rho V)=0. \qquad (3)$$

7.6. Loss at the Pump

The loss at the pump is the difference between the mass of liquid which enters the pump container and that which enters the transfer line. Refer to figure A.2. It is assumed that the liquid entering the container is saturated; if this is not the case, there may be a flashing loss associated with the pump container. (The calculation of flashing losses will be analyzed in the next section.) We shall further assume that the heat leak into the pump container is negligible compared with the power, W, introduced through the pump shaft.

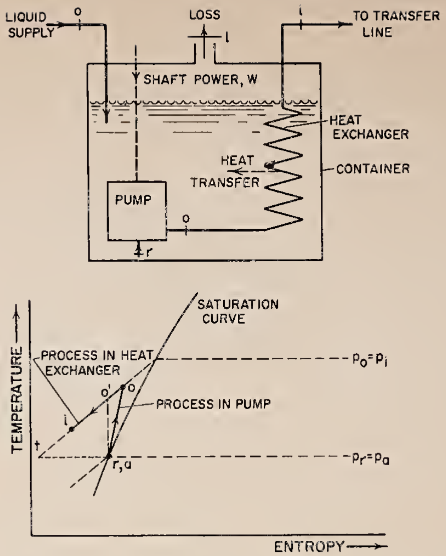

FIGURE A.2. *Layout and processes in a pumping station.*

As steady state exists, we can write the steady-flow energy equation for the pump container. Assuming that gravity effects and the difference between the kinetic energies of the entering and leaving streams are negligible,

$$w_a h_a+W=w_l h_l+w_i h_i.$$

Continuity requires that

$$w_a=w_l+w_i.$$

Recalling that $h_a=h_r$,

$$(w_l+w_i)h_r+W=w_l h_l+w_i h_i.$$

Therefore the liquid loss per unit time is

$$w_l=\frac{1}{(h_r-h_l)}[w_i(h_i-h_r)-W].$$

If we assume that the vapor, in state l, leaving the pump container is saturated, then (h_l-h_r) is the latent heat of vaporization, λ.

Thus,

$$w_l=\frac{1}{\lambda}[W-w_i(h_i-h_r)].$$

Assuming that all of the shaft work which enters the pump container reaches the pump, we may write

$$W=\frac{w_i}{\eta}(h_{o'}-h_r),$$

where η is the efficiency of the pump and $(h_{o'}-h_r)$ is the isentropic enthalpy change between state r and the pump discharge pressure p_i. This assumption is quite reasonable because the only shaft work which does not reach the pump is that dissipated by the shaft turning in the liquid and that dissipated in intermediate shaft bearings which may be located in the pump container. Therefore, the liquid loss can be written

$$w_l=\frac{w_i}{\lambda}\left\{\frac{1}{\eta}(h_{o'}-h_r)-(h_i-h_r)\right\}.$$

Recall that $Tds=dh-vdp$. As the process between states r and o' is isentropic, it follows that

$$(h_{o'}-h_r)=\left[\int_{p_r}^{p_i}vdp\right]_{s=s_r}.$$

Define state t so that $p_t=p_i$ and $T_t=T_r$. Then (h_i-h_r) can be computed from the identity

$$(h_i-h_r)=(h_i-h_t)+(h_t-h_r).$$

Noting that $dh=C_pdT+\left(\frac{\partial h}{\partial p}\right)_T dp$, we can immediately write

$$(h_i-h_t)=\left[\int_{T_r}^{T_i}C_pdT\right]_{p=p_r},$$

and

$$(h_t-h_r)=\left[\int_{p_r}^{p_i}\left(\frac{\partial h}{\partial p}\right)_T dp\right]_{T=T_r}$$

The liquid loss at the pump can thus be written

$$w_l=\frac{w_i}{\lambda}\left\{\frac{1}{\eta}\left[\int_{p_r}^{p_i}vdp\right]_{s=s_r}-\left[\int_{T_r}^{T_i}C_pdT\right]_{p=p_r}-\left[\int_{p_r}^{p_i}\left(\frac{\partial h}{\partial p}\right)_T dp\right]_{T=T_r}\right\}.$$

We can therefore write that the liquid loss at the pump, expressed as a fraction of the liquid which is pumped into the transfer line, is

$$\Lambda_p^i\equiv\frac{w_l}{w_i}=\frac{1}{\lambda}\left\{\frac{1}{\eta}\left[\int_{p_r}^{p_i}vdp\right]_{s=s_r}-\left[\int_{T_r}^{T_i}C_pdT\right]_{p=p_r}-\left[\int_{p_r}^{p_i}\left(\frac{\partial h}{\partial p}\right)_T dp\right]_{T=T_r}\right\}. \qquad (13)$$

In many cases it is more meaningful to express the liquid loss at the pump as a fraction of the fluid which enters the pump container. This can be calculated from eq (13) by means of the equation:

$$\Lambda_p^a\equiv\frac{w_l}{w_a}=\frac{\Lambda_p^i}{1+\Lambda_p^i}. \qquad (14)$$

Noting again that $dh=Tds+vdp$, an alternate form of the last integral in eq (13) can be obtained.

$$\left(\frac{\partial h}{\partial p}\right)_T=T\left(\frac{\partial s}{\partial p}\right)_T+v.$$

Through the use of a Maxwell equation this becomes

$$\left(\frac{\partial h}{\partial p}\right)_T=v-T\left(\frac{\partial v}{\partial T}\right)_p,$$

and the last integral in eq (13) becomes,

$$\left\{\int_{p_r}^{p_i}\left[v-T\left(\frac{\partial v}{\partial T}\right)_p\right]dp\right\}_{T=T_r}$$

$$=\left\{\int_{p_r}^{p_i}v[1-T\beta]\,dp\right\}_{T=T_r}.$$

This integral is readily evaluated if (p, v, T) data are available.

7.7. Flashing Loss

Energy is introduced into the liquid as it passes through the pump and through the transfer line; the former is the pump work and the latter is heat leak. A part of the pump work and all of the heat leak cause liquid to vaporize (flash) as it is isenthalpically throttled from state L to state s' in the remote storage. We shall now evaluate this flashing loss.

Refer to figure A.3. For the flashing process, we may write,

$$h_L=h_{s'}=h_r+\Lambda_f^i\lambda,$$

where, λ_f^i is the fraction (mass) of liquid, flowing in the transfer line, that flashes to vapor. Thus

$$\Lambda_f^i=\frac{1}{\lambda}(h_L-h_r)=\frac{1}{\lambda}[(h_L-h_i)+(h_i-h_r)].$$

Neglecting changes in kinetic energy in the transfer line,

$$(h_L-h_i)=\frac{qL}{w}-g(z_L-z_i).$$

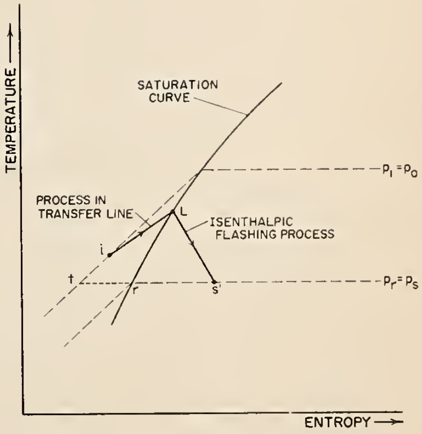

Figure A.3. *Processes in a transfer line.*

From the preceding section,

$$(h_i-h_r)=\left[\int_{T_r}^{T_i} C_p dT\right]_{p=p_r}+\left[\int_{p_r}^{p_i}\left(\frac{\partial h}{\partial p}\right)_T dp\right]_{T=T_r}.$$

Thus,

$$\Lambda_f^i=\frac{1}{\lambda}\left\{\frac{qL}{w}-g(z_L-z_i)+\left[\int_{T_r}^{T_i} C_p dT\right]_{p=p_r}+\right.$$

$$\left.\left[\int_{p_r}^{p_i}\left(\frac{\partial h}{\partial p}\right)_T dp\right]_{T=T_r}\right\}. \qquad (15)$$

Equation (15) gives the flashing loss, the fraction of the liquid, which flows through the transfer line, that flashes to vapor in the remote storage.

It may be desirable to express this loss as a fraction of the liquid that enters preceding pump container. Note that $\Lambda_f^i w_i$ is the flashing loss per unit time. Thus, $\Lambda_f^i\left(\frac{w_i}{w_a}\right)$ is the flashing loss expressed as a fraction of the liquid (w_a) which is supplied to the pump container. But

$$\frac{w_i}{w_a}=\frac{1}{1+\Lambda_p^i}.$$

Thus, the flashing loss, expressed as a fraction of the liquid supplied to the pump container, is

$$\Lambda_f^a=\frac{\Lambda_f^i}{1+\Lambda_p^i}, \qquad (16)$$

where Λ_f^i is to be computed from eq (15) and Λ_p^i is to be computed from eq (13).

7.8. Cool-Down Loss

The cool-down loss is caused by the necessity to cool a transfer system down to its steady-state operating condition. In most cases the system will have to be cooled from ambient temperature; however, if the system has not had sufficient time to completely warm up since the preceding transfer operation, the cool-down loss will be less.

This loss occurs (at most) only once during each transfer operation. It is obvious that, as the total amount of liquid transferred during each transfer operation increases, the cool-down loss, expressed as a fraction of the total liquid transferred, decreases. On this basis it is probable that, for well-designed systems and procedures, the cool-down loss can be made small compared with the pumping and flashing losses.

An accurate calculation of the cool-down loss requires a detailed analysis of the entire transient cool-down process; as stated in section 1, this will not be attempted here. Another difficulty is the decision concerning just how the cool-down loss is to be expressed. It is easy to determine the energy which causes the loss. But while some of the energy evaporates liquid, another portion of it may only be absorbed as sensible energy. In the latter case the liquid may possibly be useful and thus should not be considered as complete loss. If we consider a metallic line surrounded by insulation of appreciable mass, it is apparent that the metal will cool very rapidly, but that pure liquid may flow through the line long before the insulation is cooled down. How much of this pure liquid should be considered as loss? It is probable that the sensible heating loss will eventually show up as flashing at the downstream end of the transfer line; the loss will therefore be computed on this basis.

Let us first compute the energy which must be removed from the system during cool down. It comes from the metallic parts which are in intimate contact with the liquefied gas, and from the insulation.

The metallic parts must be cooled from ambient temperature, T_a, to the liquid temperature, T_1. As the effect of pressure on the amount of energy that must be removed is negligible, the energy which must be removed from a homogeneous piece of metal of mass m_i with isobaric specific heat C_{p_i} is

$$E_i=m_i\int_{T_1}^{T_a} C_{p_i} dT.$$

Thus, if there are n different metallic parts in a transfer system, each having a mass m_i ($i=1, 2, 3, \ldots n$), which must be cooled from T_a to T_1, then the total energy which must be removed from these parts is

$$E_{\text{metal}}=\sum_{i=1}^{n} E_i=\sum_{i=1}^{n} m_i\int_{T_1}^{T_a} C_{p_i} dT.$$

Now let us compute the energy which must be removed from the insulation to cool it from ambient temperature, T_a, to its operating temperature. It should be noted that the operating temperature of the insulation is not uniform, but varies from approximately liquid temperature on the inside to about room temperature on the outside. (In high-vacuum insulation, essentially no energy must be removed from the insulation. A negligible amount of energy may be extracted to condense some residual gas and enough energy will have to be removed from the vacuum jacket to establish the required temperature difference (a degree or two) between the jacket and the atmosphere.)

Refer to figure A.4. We shall assume that the insulation is axially symmetric; while this computation is thus directly applicable to a transfer line, the same procedure can be applied to any configuration. It is assumed that the steady-state temperature distribution in the insulation, $T(r)$, is known, its determination requiring a very simple experiment. It is important to note that the cooling down of some insulation at a radius, r, involves a change in temperature from T_a to $T(r)$.

Consider the annular portion of the insulation (see fig. A.4) bounded by the cylindrical surfaces

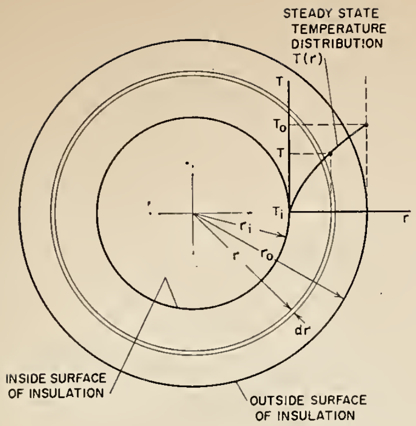

FIGURE A.4. *Diagram for cool-down analysis of insulation.*

r and $(r+dr)$, and of length L_{ins}. If the density of the insulation, ρ_{ins}, is constant and uniform, the energy which must be removed from the annular portion in cooling it from T_a to $T(r)$ is

$$dE_{\text{ins}}=2\pi r dr L_{\text{ins}}\rho_{\text{ins}}\int_{T(r)}^{T_a} C_{p_{\text{ins}}}dT.$$

$C_{p_{\text{ins}}}$ is the isobaric specific heat of the insulation, the pressure effects having been assumed negligible. It has been assumed that $C_{p_{\text{ins}}}$ is a known function of temperature. This information requires the performance of careful experiments upon the insulation. The total energy which must be removed from the insulation is merely the summation of the energies removed from all the annular regions:

$$E_{\text{ins}}=\int dE_{\text{ins}}=2\pi L_{\text{ins}}\rho_{\text{ins}}\int_{r_i}^{r_o} r\left[\int_{T(r)}^{T_a} C_{p_{\text{ins}}}dT\right]dr.$$

The integration inside of the brackets is to be performed first; then the integration with respect to r can be performed.

It should be borne in mind that several expressions of the above type, one for each part of the system, may have to be evaluated. If there are m such parts of the system, we have

$$E_{\text{ins}}=2\pi\sum_{j=1}^{m}\left\{L_{\text{ins}}\rho_{\text{ins}}\int_{r_i}^{r_o} r\left[\int_{T(r)}^{T_a} C_{p_{\text{ins}}}dT\right]dr\right\}_j.$$

The total energy which must be removed from the transfer system during cool down is

$$E_{\text{cool down}}=E_{\text{metal}}+E_{\text{ins}}.$$

Thus,

$$E_{\text{cool down}}=\sum_{i=1}^{n} m_i\int_{T_i}^{T_a} C_{p_i}dT+$$

$$2\pi\sum_{j=1}^{m}\left\{L_{\text{ins}}\rho_{\text{ins}}\int_{r_i}^{r_o} r\left[\int_{T(r)}^{T_a} C_{p_{\text{ins}}}dT\right]dr\right\}_j.$$

As mentioned previously, the cool-down loss, as such, is not unique, but depends upon the procedures used. For example, in systems where the liquefied gas is relatively expensive (e. g., hydrogen, neon, helium), a less expensive liquid such as nitrogen may be used to partially cool the system. Then the loss would consist of precoolant used from ambient down to its steady-state condition plus an amount of primary liquefied gas required to cool the rest of the way down. The relations developed here apply to all cases, and are applied in steps when the cooling takes place in steps. It should be noted that no calculations are made here concerning the time required to cool the system; it is obvious however, that higher losses are associated with faster cool-downs, since complete utilization of the sensible heat of the coolant vapor will be more difficult.

The difficulty in calculation of the cool-down loss is in the determination of just how much of the refrigeration capacity of the liquid is utilized. Refer to figure A.5. Assume that the liquid is supplied to the transfer system in the compressed state, indicated by 1. At the beginning of cool down, if the fluid passes through the system slowly enough, the fluid will leave the system at ambient temperature. All of the available refrigeration is used. However, as the system is cooled, the temperature at which the gas leaves the system (state 4) decreases. Eventually, none of the sensible energy of the vapor is utilized. And, after $T_2=T_3$ is reached, no latent heat of vaporization is utilized.

The least economical cool down will waste all of the sensible refrigeration of the vapor. We shall now compute the loss for this case. In cooling the system from T_a to $T_3=T_2$, each unit mass of liquid will be capable of absorbing an amount of energy equal to

$$\left[\int_{T_1}^{T_2} C_p dT+\lambda\right].$$

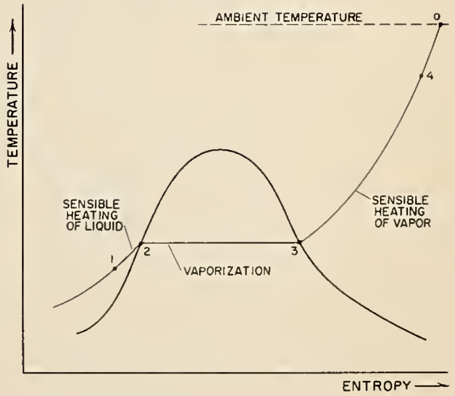

FIGURE A.5. *States of refrigerant during cool down.*

It is once again assumed that pressure effects are negligible. The mass of liquid lost during this process is therefore

$$M_{T_a-T_2}=\frac{\sum_{i=1}^{n} m_i \int_{T_2}^{T_a} C_{p_i} dT + 2\pi \sum_{j=1}^{m} \left\{ L_{\text{ins}}\, \rho_{\text{ins}} \int_{r_i}^{r_o} r \left[\int_{T_{(r)} \geq T_2}^{T_a} C_{p_{\text{ins}}} dT \right] dr \right\}_j}{\left[\int_{T_1}^{T_2} C_p dT + \lambda \right]}. \tag{19}$$

In cooling the system down from T_2 to T_1, no vaporization takes place (due to cool down); however as stated previously the loss will show up as flashing in the remote storage. We may therefore consider that the refrigeration is affected through the latent heat of vaporization. Thus, the loss in cooling from T_2 to T_1 is

$$M_{T_2-T_1}=\frac{\sum_{i=1}^{n} m_i \int_{T_1}^{T_2} C_{p_i} dT + 2\pi \sum_{j=1}^{m} \left\{ L_{\text{ins}}\, \rho_{\text{ins}} \int_{r_i}^{r_o} r \left[\int_{T(r) \geq T_1}^{T(r) \geq T_2} C_{p_{\text{ins}}} dT \right] dr \right\}_j}{\lambda}. \tag{20}$$

The maximum total cool-down loss is thus

$$M_{CD}=M_{T_a-T_1}=M_{T_a-T_2}+M_{T_2-T_1}. \tag{18}$$

If w_a is the steady-state rate at which liquid enters the transfer system and T is the total duration of the transfer, then w_aT is the total mass of liquid transferred. The maximum fraction lost during cool down is thus

$$(\Lambda_{CD})_{\max}=\frac{M_{CD}}{w_a \mathrm{T}+M_{CD}}, \tag{17}$$

where M_{CD} is computed from the preceding relations. This quantity is directly comparable to those given by eq (14) and (16).

In practice, the maximum loss will probably be close to the actual loss. This is because it will usually be desirable to establish single-phase flow as quickly as possible. As the flow limitation during cool down is imposed by the necessity to push a large volume of vapor through the system, it is advantageous to bypass the vapor out of the system as soon as feasible.

If cases arise where $(\Lambda_{CD})_{\max}$ is too large to permit acceptance of a transfer system, a more accurate computation might be desirable.

Boulder, Colo., December 13, 1957.

7.9. The Trapped-Liquid Loss

The actual mass of liquid trapped in the system depends upon the detailed layout, including elevations; however, the upper limit to this loss is based upon the assumption that the whole system is filled with trapped liquid.

Let V_s be the total internal volume of the transfer system. Then $V_s\rho$ is the maximum mass that can be trapped. The trapped-liquid loss, expressed as a fraction of the total liquid transferred is thus

$$(\Lambda_{TL})_{\max}=\frac{V_s\rho}{w_a \mathrm{T}+M_{CD}}. \tag{21}$$

It is apparent from this relation that, for transfer of long duration (large T) and/or high-flow rate (large w_a), this loss will be small, and can be made negligible for well-designed systems and procedures.

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