NBS CIRCULAR 584

Simulator for Use in Development of Jet Engine Controls

UNITED STATES DEPARTMENT OF COMMERCE

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Simulator for Use in Development of Jet Engine Controls

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Foreword

This project on the simulation of a turbo-jet engine and its controller was originally proposed by the Power Plant Control Subcommittee of the Accessory and Equipment Technical Committee of the Aircraft Industries Association. Financial support was furnished by the Power Plant Division of the Navy's Bureau of Aeronautics. The primary objective of the project was the development of a simulator whose use would permit controller development to proceed concurrently with the development of a new engine. Emphasis has been placed on the use of simplified simulation techniques and the employment of generally available components to obtain simulation of a number of engine types and over a large range of parameters at minimum cost.

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Contents

Simulator for Use in Development of Jet Engine Controls

Emile S. Sherrard

A method of simulation and cost estimates of simulator components are given for a simulator to be used in the development of turbojet engine control systems for aircraft. The simulator employs typical d-c analog computer components. It is capable of representing a twin-spool, variable-nozzle, afterburning turbojet engine, as well as the engine's controller. The simulator is of moderate accuracy, flexibility, and cost. It is intended to be used in the determination of the stability and performance of the engine control system.

1. Introduction

Recommendations and cost estimates for a simulator capable of representing a twin-spool, afterburning, turbojet engine and its controller are given in this Circular. The simulator is intended as a tool, to be used by a group of engineers engaged in developing engine control systems. The simulator is designed to perform the following tasks:

(1) Determine stability of the engine control system.

(2) Determine performance of the engine control system.

Its intended use requires that this simulator be of moderate accuracy and flexibility, and, insofar as is possible, be simple and low in cost. There may, therefore, be large differences in cost, size, and complexity between this simulator and other simulators intended for: (a) Testing a complete engine control [1],[1] or a component of an engine control system; (b) simulating, testing, and/or developing a wide range of engines and engine control systems [2]; or (c) use in both engine design calculation and control system development. [3].

For maximum usefulness to an engine-controller-development group, a simulator should have the following characteristics:

(1) The simulator must be useful in both early and late stages of the development.

(2) The simulator must operate in real time in order that both actual physical components and simulated components may be operated together.

(3) Transducer equipment must be provided so that a simulated engine may be operated either with the entire physical controller, or with some of its components.

(4) Simulator accuracy must be sufficient for evaluating control system stability and performance.

(5) The cost of the simulator should permit its acquisition by a typical engineering group engaged in control system development.

The first of these requirements has been met by designing the simulator so that it may easily be expanded from a simplified representation of the control system to progressively more detailed representations. This design feature permits simulation with a minimum of data in the first stages of the development period, and full use of all available data in the later stages of development.

Both analog computer and transducer components must have excellent dynamic response for real-time simulation. Electronic computer components easily meet the dynamic requirements. Servo mechanisms, with an advertised frequency response of 40 to 50 cps, may be used to obtain satisfactory dynamic response for the nonelectronic function generators used in simulating the jet engine. The greatest difficulty in obtaining adequate dynamic response will be experienced with the transducer equipment. In particular, the transducer for converting a computer voltage representing speed, to the velocity of a shaft capable of driving a 100 hp load, will require a large improvement in its presently estimated performance [1]. A method for obtaining this performance improvement is suggested in this Circular.

A great deal of space, effort, and expense could be saved by omitting transducer equipment, and relying on complete simulation of the system by analog computer components. This technique has produced satisfactory results for guided missile systems. Apparently it could also be used for jet engine control systems.

Accuracy requirements for the simulation will probably be attained by the employment of simulator components with an accuracy of 1 percent or better. Such components should give sufficiently accurate representation of an engine and controller because design data and experimental data may have a 5 to 10 percent accuracy, and production tolerances on a particular engine or controller may also permit a 5 to 10 percent variation in important characteristics. Analog computer components of 0.1 percent accuracy may be used if desired, with only a moderate increase in simulator cost. If transducer components of this accuracy are required, simulator costs would be considered prohibitive by most controller manufacturers.

[1] Figures in brackets indicate the literature references at the end of this Circular.

2. Description of Simulator

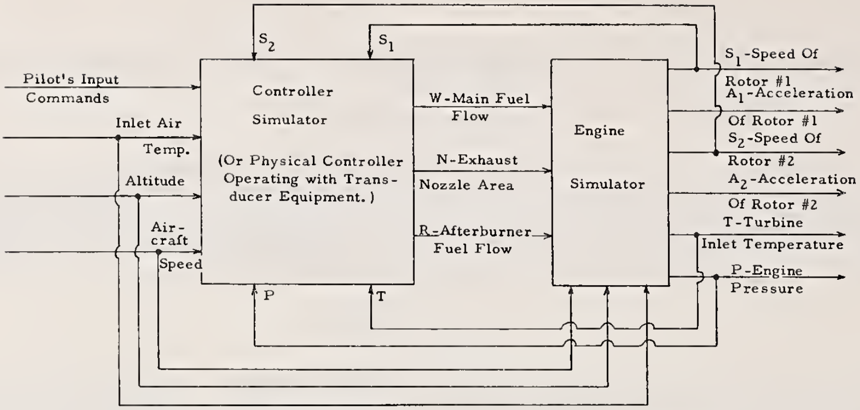


FIGURE 1. *Block diagram for engine-controller simulator.*

The simulator consists of three major parts, an engine simulator, a controller simulator (which may be replaced by a physical controller), and transducer equipment, to be used when a physical controller is operated with a simulated engine. The interconnections between these component parts are shown in the block diagram of figure 1. Each of these components is described in detail in one of the following sections.

2.1. Engine Simulator

In this portion of the simulator, typical analog computer components are interconnected in such a fashion that voltages in the computer are made to satisfy the equations that describe the behavior of the actual jet engine. Inputs to, and outputs from the engine simulator are in the form of d-c voltages. Input voltages, as shown in figure 1, represent: Main fuel flow, W; exhaust nozzle area, N; and afterburner (reheat) fuel flow, R. Output voltages, as shown in figure 1, represent: The speed of rotor 1, S_1; the acceleration of rotor 1, A_1; the speed of rotor 2, S_2; the acceleration of rotor 2, A_2; the turbine inlet temperature, T; and, an engine variable, P, hereafter called engine pressure, which may be a compressor discharge pressure, a compressor pressure ratio, or some other indicator of incipient engine stall.

The engine outputs will respond to altitude and aircraft speed (ram pressure) as well as W, N, and R. These environmental variables have been held constant in this simulator in order to obtain simplicity and economy in engine representation. If engine operation, at different values of altitude, and ram pressure are desired, manual changes may be made in settings of the computer components used to represent the engine. Because there is no simulation of the dynamic behavior of the airplane in which the engine is mounted, provision for automatic changes of engine environment would result merely in convenience and rapidity of adjustment. Because this would entail greatly increased costs and complexity, manual rather than automatic adjustment of engine environment is employed in this simulator.

a. Engine Equations in General Form

For control purposes, the six equations given below may be employed to describe a twin-spool, variable-nozzle, afterburning engine. The first four state that A_1, A_2, T, and P, for constant altitude and ram pressure, are instantaneous functions of five variables, the three engine inputs W, N, and R, and the instantaneous rotor speeds, S_1 and S_2. The last two equations state that A_1 is the derivative of S_1, and A_2 is the derivative of S_2. The first four equations do not involve any derivatives or integrals with respect to time. They result from the assumption that the engine is a quasi-static system [4], i. e., its thermodynamic processes reach equilibrium conditions in such a short time that the fluid flows, temperatures, pressures, and accelerating torques may, for control purposes, be considered to have equilibrium or steady-state values at all times.

The six engine equations are:

$$A_1=A_1\ (W, N, S_1, S_2, R), \tag{1}$$

$$A_2=A_2\ (W, N, S_1, S_2, R), \tag{2}$$

$$T=T\ (W, N, S_1, S_2, R), \tag{3}$$

$$P=P\ (W, N, S_1, S_2, R), \tag{4}$$

$$\frac{dS_1}{dt}=A_1, \tag{5}$$

$$\frac{dS_2}{dt}=A_2. \tag{6}$$

Computer mechanization of the last two equations is very simple on an operational amplifier type of analog computer. Computer mechanization of the first four equations is difficult because:

(1) A_1, A_2, T, and P are not simple functions of W, N, S_1, S_2, and R.

(2) Complete data for A_1, A_2, T, and P as a function of five variables is usually unknown or unavailable.

(3) There are no commercially available function generators that will accept five different input variables, and generate an output that is a complicated function of the input variables.

Mechanization of eq (1) through (4) on an analog computer requires that these equations be replaced by a much simpler set of equations which still represent the pertinent behavior (for control purposes) of the engine. These simpler equations must also require a minimum amount of engine data, and, if at all possible, the data required should correspond to steady-state operation of the engine, rather than to transient operation of the engine. The reduction of (1) through (4) to simpler equations satisfying these requirements is discussed in the two following sections.

b. Method of Simplifying Engine Equations

Equations (1) through (4) are usually replaced by equations that may be mechanized on an analog computer through the use of either a "component simulation" technique or a "transference simulation" technique. In component simulation, the engine components (compressor, combustor, turbine, nozzle, etc.) are individually simulated, and then connected together to obtain a simulation of the complete engine. This technique requires a large number of equations (two or more for each component simulated), a good understanding of the thermodynamic and fluid-flow characteristics of the simulated components, and a knowledge of permissible approximations that may be worthwhile. This technique simplifies the simulation problem by replacing eq (1) through (4), which involve functions of five independent variables, by a larger number of equations which may involve functions of no more than two variables. References [2, 3] give examples of component simulation.

Examples of transference simulation for single-spool engines are furnished by references [1, 5]. This technique simulates only the dynamic response of the engine's output (controlled) variables to the engine's input variables (output variables of the controller). It does not simulate engine variables that are neither engine outputs nor engine inputs. Transference simulation requires much less background in thermodynamics and engine design than is required for component simulation. It requires an excellent knowledge of useful approximations for describing the controlled responses of the engine. The equations used are fewer in number than those employed for component simulation, but for complex engines they may involve functions of more than two variables.

Transference simulation has been employed in this simulator for the following reasons:

(1) A transference simulator would be preferred by users with a control-system background. Such users would find it easier to understand, modify, or check. It would also avoid the need for intensive study of engine component design or operation by persons principally concerned with engine control.

(2) A transference simulator will require less data than a component simulator. This may be only a small advantage if the required data is obtained from calculations made with a digital computer. If the required data is obtained from experimental testing, this advantage is significant.

(3) A transference simulator for engine-controls development is expected to cost less and require fewer components than a component simulator. This statement is certainly true when a simple engine is to be simulated. It is believed also true for the relatively complex twin-spool, afterburning engine considered in this Circular.

c. Simplified Engine Equations

The first simplified form for eq (1) through (4) is obtained by linearizing them about points of steady-state operation (equilibrium points), i. e., operating conditions of the engine in which all inputs are constant, rotor speeds are constant, and rotor accelerations are zero. This first step replaces (1) through (4) by four simpler equations, each of which is the sum of five terms. These five terms are then reduced to two by a proper choice of the equilibrium points, as detailed in appendix A. The simplified equations for the engine may be written:

$$A_1=\frac{\partial A_1}{\partial W}\,\delta W+\frac{\partial A_1}{\partial S_2}\,\delta S_2, \tag{7}$$

$$A_2=\frac{\partial A_2}{\partial W}\,\delta W+\frac{\partial A_2}{\partial S_2}\,\delta S_2, \tag{8}$$

$$T=T'+\frac{\partial T}{\partial W}\delta W+\frac{\partial T}{\partial S_2}\,\delta S_2, \tag{9}$$

$$P=P'+\frac{\partial P}{\partial W}\delta W+\frac{\partial P}{\partial S_2}\,\delta S_2, \tag{10}$$

$$\frac{dS_1}{dt}=A_1 \tag{5}$$

$$\frac{dS_2}{dt}=A_2. \tag{6}$$

In the above equations, $\delta W=W-W'$ and $\delta S_2=S_2-S_2'$, where W and S_2 represent, respectively, the instantaneous values of main fuel flow and speed of the second rotor, W' written as $W'(N, S_1, R)$ and S_2' written as $S_2'(N, S_1, R)$ are the steady-state fuel flow and steady-state speed corresponding to equilibrium operation with nozzle area N, speed of the first rotor S_1, and afterburner fuel flow, R. All the partial derivatives are evaluated at this same equilibrium point denoted by (N, S_1, R) and their values will vary as the point (N, S_1, R) varies. In (9) and (10), $T'=T'(N, S_1, R)$, and $P'=P'(N, S_1, R)$ are used to denote the equilibrium temperature and equilibrium pressure corresponding to (N, S_1, R). Equations (5) and (6) are repeated here for the sake of completeness.

d. Usefulness of Simplified Engine Equations

Equations (5) through (10) accurately describe the engine for small values of δW and δS_2. Thus, they may be employed in conjunction with corresponding controller equations to determine stability when engine and controller variables execute only small departures from any steady-state operating point. Experience indicates that the engine control system is stable, if it is stable for such small-departure operation. Therefore, these small-departure equations provide an adequate description of the engine for all stability studies of the engine control system.

These equations are also useful in predicting the dynamic performance of the engine-controller system for small or slowly varying pilot commands.

Throttle-burst operation is the principal large-departure performance requiring simulation. Such operation occurs whenever a large and sudden advance of the engine's throttle is made. For such advances (e. g., a change from idle speed to top speed) A_1, A_2, $T-T'$, and $P-P'$ will vary with δW and δS_2 in a nonlinear manner rather than the linear manner indicated by eq (7) through (10). These linear equations require correction if the simulator is to yield, in response to a throttle burst, the maximum engine accelerations, maximum engine temperature, maximum engine pressure, and time for the engine to attain a new steady-state operating condition. For a particular equation, the amount of correction required depends on the magnitude of δW and δS_2. The correction method used in this simulator replaces δW, in (7) through (10) by $\delta W-K_i(\delta W)^2$ $(i=1, 2, T, P)$. K_i is chosen so as to make the particular equation correct for the largest value of δW permitted by the engine's acceleration boundary, which is determined by compressor stall, or maximum temperature, or rich blowout. No correction is made for large values of δS_2, because it is expected that, for most engines, δS_2 will remain small during throttle-burst operation. Details of the δW correction procedure are discussed in appendix A. When corrected in this fashion, the engine equations may be used to determine performance during throttle-burst operation.

e. Mechanization of Engine Equations

Equations (5) and (6) can be simply mechanized on an analog computer. Integrating amplifiers whose respective inputs are A_1 and A_2 are employed to yield S_1 and S_2, respectively, at their outputs.

Equations (7) and (8) will be mechanized by summing two terms, each of which is the product of a partial derivative and an increment of either fuel flow or speed. Each such product is obtained as the output of a multiplier. Because the partial derivatives are functions of the three variables N, S_1, and R, they may be generated by a three-variable function generator (3VFG) whose inputs are N, S_1, and R. The δW and δS_2 increments may be generated by summing W and $-W'$ and S_2 and $-S_2'$, respectively. W is obtainable as an input to the engine simulator, S_2 as an output of the simulated engine, $-W'$ as the output of a 3VFG with inputs N, S_1, and R, and $-S_2'$ as the output of a 3VFG with inputs N, S_1, and R. The block diagram for mechanizing (7) and (8) is given in figure 2. Also shown are two integrators whose outputs are S_1 and S_2, and whose inputs are A_1 and A_2. These integrators mechanize eq (5) and (6). Further details of the mechanization of eq (5), (6), (7), and (8) for three special operating conditions are contained in appendix A.

The mechanization of temperature, eq (9), is the same as for (7) and (8) except that, in addition, T', generated as the output of a 3VFG with N, S_1, and R inputs, is added to the terms composed of a product of a partial derivative and an increment. The block diagram for mechanizing the temperature equation is given in figure 3. Further details are contained in appendix B.

The mechanization of pressure, eq (10), involves exactly the same details as the mechanization of eq (9). Figure 4 gives the block diagram for mechanizing the pressure equation.

Figures 2, 3, and 4 are consolidated in figure 5, which shows the block diagram for the entire engine simulator. Table 1 lists required components and estimated costs.

Correction of eq (7) through (10) for throttle-burst operation is illustrated in figure 6 for eq (7). The figure shows that $(\partial A_1/\partial W)\ \delta W$ has been replaced by $(\partial A_1/\partial W)\ \delta W-K_1(\delta W)^2$, where $-K_1$ is obtained as the output of a function generator whose inputs are N, S_1, and R. During throttle-burst operation, the characteristics of some engines and engine controllers may cause N or R, or both, to be constant or zero. Again $-K_1$ may be insensitive to one or both of these variables, and so may be considered constant or zero. In such cases $-K_1$ may be generated more simply than shown in figure 6. Details of the correction of eq (8) through (10) are the same as for eq (7).

2.2. Data Required for Engine Simulation

The minimum number of engine environments for which engine simulation might be required are considered to be sea level with one or two ram pressures, and a representative altitude with one or two ram pressures. At each engine environment for which system stability and performance are to be determined, the data listed below will be required.

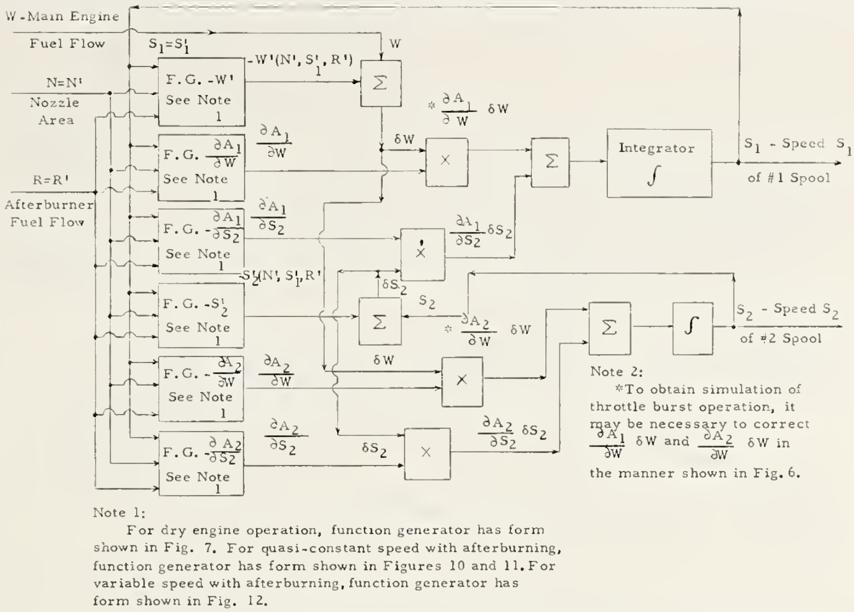
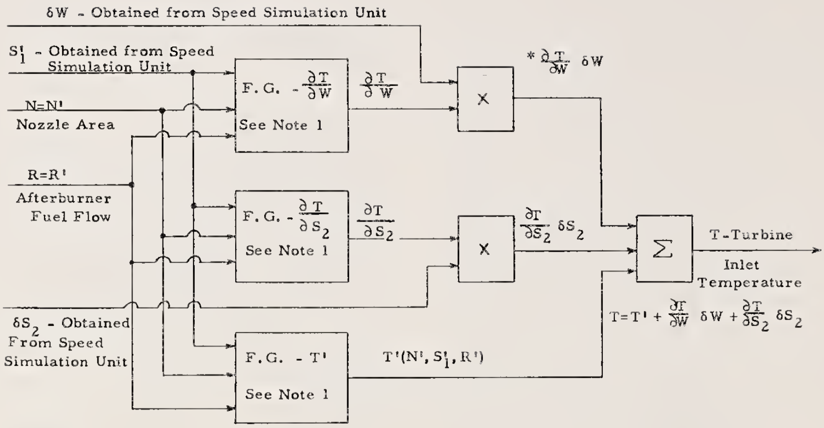


FIGURE 2. *Block diagram for simulation of engine speed.*

TABLE 1. *Estimated computer components required for simulating a twin-spool engine*

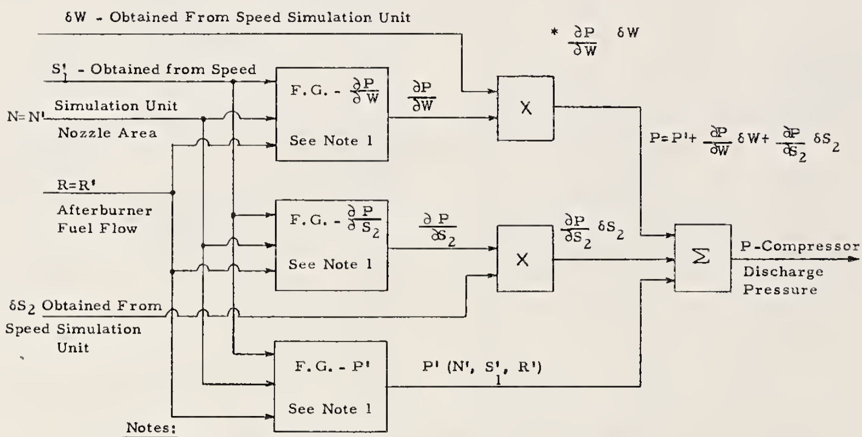
Type of equipment	Number of units required for small departure simulation			Additional units required for throttle-burst simulation
	Dry engine operation	After burning		
		Quasi-constant speed	Variable speed	
Summing amplifiers	30	30	30	3 to 12.
Integrating amplifiers	2	2	2	
Potentiometers	50	50	50	
Fast response servos	3	3	3	1.
Tapped servo potentiometers	12	12	12	1 to 4.
Diode function generators	36 to 60 (3 to 5 for each tapped servo potentiometer).	108 to 180 (3 times number required for dry engine).		3 to 5 to 12 to 20 (3 to 5 for each tapped servo potentiometer.
Electronic multipliers	8	20	8	1 to 4.
Two-variable function generators			36 to 60 (3 to 5 for each tapped servo potentiometer).	
Estimated cost	$40,000 to $70,000	$100,000 to $150,000	$150,000.	$5,000 to $15,000.
Number racks of computing equipment.	4	6	6.	½ to 1.



Notes:

1. For dry engine operation, function generator has form shown in Fig. 7. For quasi-constant speed with afterburning, function generator has form shown in Fig. 10 and Fig. 11. For variable speed with afterburning, function generator has form shown in Fig. 12.
2. * To obtain simulation of throttle burst operation, it may be necessary to correct $\frac{\partial T}{\partial W}\,\delta W$ in the manner shown in Fig. 6.

FIGURE 3. *Block diagram for simulation of turbine inlet temperature.*



Notes:

1. For dry engine operation function generator has form shown in Fig. 7. For quasi-constant speed with afterburning, function generator has form shown in Fig.10 and 11. For variable speed with afterburning, function generator has form shown in Fig. 12.
2. * To obtain simulation of throttle burst operation, it may be necessary to correct $\frac{\partial P}{\partial W}\,\delta W$ in the manner shown in Fig. 6.

FIGURE 4. *Block diagram for simulation of compressor discharge pressure.*

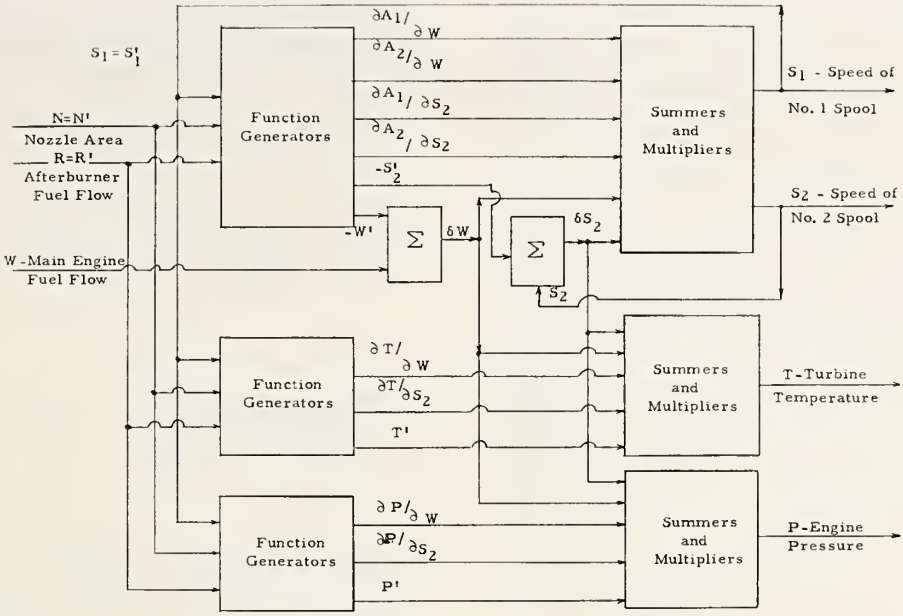


FIGURE 5. *Schematic diagram for engine simulator.*

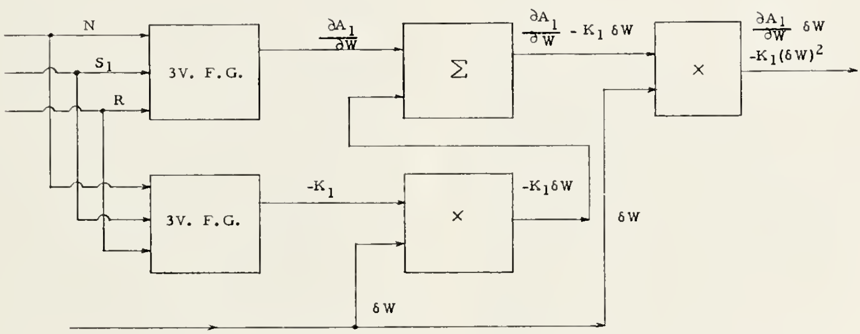


FIGURE 6. *Correction of* $(\partial A_1/\partial W)$ *for throttle-burst operation.*

a. Data Required for Small-Departure Equations

(1) *Acceleration Equations.* Mechanization of the acceleration eq (7) and (8) will require the following steady-state characteristics: Steady-state fuel flow, W', as a function of nozzle area, N, speed of No. 1 rotor, S_1, and afterburner fuel flow, R; steady-state speed of No. 2 rotor, S_2', as a function of N, S_1, and R.

The following partial derivatives will be required:

$$\frac{\partial A_1}{\partial W}, \frac{\partial A_2}{\partial W}, \frac{\partial A_1}{\partial S_2}, \text{ and } \frac{\partial A_2}{\partial S_2}$$

These derivatives must be known at the steady-state operating points, as functions of the three independent variables, N, S_1, and R.

(2) *Turbine Temperature Equation.* At and near rated engine speed, the engine temperature, eq (9), will appear among the equations of the control system. For this mode of operation, eq (9) will be linearized about steady-state values, and the following data will be required: Steady-state turbine inlet temperature, T', as a function of nozzle area, N, speed of No. 1 rotor, S_1, and afterburner fuel flow, R. Also, the partial derivatives:

$$\frac{\partial T}{\partial W} \text{ and } \frac{\partial T}{\partial S_2},$$

must be known at the steady-state operating points, as functions of the three independent variables, N, S_1, and R.

(3) *Pressure Equation Requirements.* If an engine pressure is used in the steady-state control of the engine, the steady-state pressure, P', as a function of N, S_1, and R will be required. In addition $\partial P/\partial W$ and $\partial P/\partial S_2$ evaluated at steady-state operating points will be required.

b. Data Required for Throttle-Burst Simulation

(1) *Acceleration Equations.* In order that the small-departure equations may be corrected for throttle-burst operation, both K_1 and K_2 must be furnished as a function of N, S_1, and R.

(2) *Turbine Temperature Equation.* During throttle-burst operation, the engine controller may act to hold temperature at its maximum permissable value. Because tight control is required to prevent excursions beyond the temperature-fixed acceleration boundary, it would seem advantageous to linearize temperature eq (9) about its maximum permissible value. In such a case, $\partial T/\partial W$ and $\partial T/\partial S_2$ would have to be evaluated at off-steady-state operating points. Experimentally, such evaluation is impractical. It may also be expected that calculated values of these derivatives at fractional engine speeds and off-steady-state operating points will be more inaccurate than similar values for steady-state operating points.

These difficulties render it advisable to linearize the temperature equations about a steady-state value. If necessary, this equation may be corrected for large values of δW, as were the acceleration equations. Such correction will require that a correction factor, K_t, be furnished as a function of N, S_1, and R.

(3) *Pressure Equation.* During throttle-burst operation, engine acceleration may occur with the controller acting to hold that value or function of P, which indicates the stall boundary of the compressor. As in the case of the temperature equation, it is considered advisable to linearize the pressure eq (10) around steady-state values, and correct, if necessary, by the method used to correct the acceleration equations. This will require that another correction factor K_p be given as a function of N, S_1, and R. If the terms of pressure eq (10), have not already been evaluated for use in steady-state control, throttle-burst operation will require that P', $\partial P/\partial W$, and $\partial P/\partial S_2$ be furnished as a function of N, S_1, and R.

2.3. Description of Controller Simulator

As in the engine simulator, typical d-c analog computer components are proposed for simulating the engine's controller. In the complete system, as shown in figure 1, output voltages of this simulated controller serve as input voltages to the simulated engine, and output voltages of the simulated engine serve as input voltages to the simulated controller.

No particular difficulties are expected in simulating the controller. Present simulation techniques are considered adequate both for its linear representation and, if necessary, for the representation of its principal nonlinearities. The estimated equipment requirements, and costs, are listed in table 2, which shows a two-variable function generator used to generate the acceleration boundary of the engine, and three single-variable function generators used to generate scheduled values of N, S_1, and R as a function of throttle position. A very complex controller might require appreciably more function generators and linear computing components than are listed in table 2; on the other hand, a simple controller will require appreciably fewer. Therefore, it would be advisable to reestimate equipment and costs for controller simulation whenever a very complex or a simple controller is to be developed.

TABLE 2. *Estimated requirements for simulating the controller of an engine-controller system*

Type of component		Number required
Computing amplifiers	Integrators	12
	Summers	24
One-variable function generators		3
Potentiometers		54
Two-variable function generator		1

The above equipment, including power supplies, can probably be obtained in two racks of computing equipment. For 0.1 percent accuracy components, estimated total cost is $30,000. For 1 percent accuracy components, estimated total is $15,000.

In the early stages of development, simulation of a linearized controller should be adequate. In an intermediate stage, a control designer may wish to deliberately introduce a nonlinear element into his speed controller, or his temperature controller, or both. Because, for reasons of simplicity, this nonlinearity will probably be a function of only one variable, two, or perhaps three function generators for a single variable will be the principal nonlinear components required for intermediate development. For the later stages of development, transducer equipment may be required for operating a physical controller with a simulated engine, but may be omitted if complete simulation of the engine-controller system is employed. If reliance is placed on complete simulation, the later development period will require simulation of the essential nonlinearities of controller components, and this will call for additional computer equipment. The variety of presently used controller components and the differing types of nonlinearities exhibited by mechanical, electrical, electronic, and hydraulic controllers prevent description of a general simulator for nonlinear engine controllers. Therefore, no estimate of equipment required for nonlinear controller simulation can be given in this Circular.

2.4. Data Required for Controller Simulator

a. Controller Component Data

Data describing the static and dynamic behavior of each controller component will be required. For initial development studies, this data need not be extensive and may be of only moderate accuracy. For intermediate development studies, data will be required that will permit a computer study of the effect on performance and stability of such nonlinear effects as backlash, deadband, variable gain, and saturation. For advanced development work, data giving an accurate description of each component over its full range of operation will be required.

b. Engine Data

Complete schedules for all variables scheduled by throttle position will be required. The acceleration boundary of the engine will also be required.

2.5. Description of Transducer Equipment

Transducer equipment is required whenever a physical controller is to be operated with a simulated engine. As shown in figure 1, its function is, (1) to convert voltage outputs of the simulated engine to physical variables which are used as inputs to the physical controller, and (2) to convert outputs of the physical controller into voltages which are used as inputs to the simulated engine.

It is not considered necessary or desirable to transduce all of the outputs of the simulated engine and the physical controller. Ordinarily, it is convenient and economical to simulate thermocouple response directly. Afterburner fuel flow, which is very large, is usually omitted for economical reasons. Other transducers are considered both feasible and necessary; the minimum judged adequate for a development simulator include those for:

(1) Converting the voltage representing engine speed to the velocity of a shaft capable of driving a load of 100 hp at rated engine speed.

(2) Converting voltage to air pressure.

(3) Converting main fuel flow to voltage.

An ideal transducer would convert a voltage to a physical variable, or a physical variable to a voltage, instantaneously and accurately. It should also be reasonable in cost. Practical transducers exhibit neither instantaneous response, absolute accuracy, nor insignificant cost. An acceptable transducer equipment for this simulator has been developed by Vickers, Inc., for testing J–34 engine controllers [1]. Development has been completed on this unit, and the whole test stand, or parts of it, are available on relatively short-term delivery. This unit is capable of producing the principal static environmental conditions under which a controller would operate. It is also of moderate size, moderate cost, and moderate power requirements.

That unit, if employed in this simulator, may exhibit two deficiencies. One is the lack of a voltage-to-air pressure transducer, which may be remedied by the addition of a recently developed voltage-to-air pressure transducer [2] whose preliminary performance seems to be excellent. The second, and more serious deficiency, is the closed-loop frequency response of the voltage-to-speed transducer, which is believed inadequate for this simulator. Design calculations indicate it will be unsatisfactory for input frequencies greater than about 1 cps. If used with this simulator, the closed-loop response of this transducer will require considerable improvement over its calculated response, to prevent the introduction of excessive phase lags into the speed control system when a physical controller is operated with a simulated engine. A satisfactory frequency response will be such that it introduces no more than a 20° phase lag at the crossover frequency of the open-loop transfer function of the engine-speed control system. Crossover frequencies for various control systems will vary, but it is estimated that if the crossover for the voltage-to-speed transducer is made to occur at 10 cps, instead of 1 cps (as presently estimated), this transducer will prove adequate for controller development purposes.

This improvement in transducer response might be accomplished in several ways. One method might be to insert a filter, consisting of analog computer components, between the input to the transducer and the voltage representing engine speed. The frequency response of this filter might be chosen so as to compensate the undesirable parts of the transducer's frequency response. Another method is to employ a model technique [6] to obtain a satisfactory transducer frequency response.

3. Use of Simulator

As noted in the introduction, this simulator is intended to be useful throughout the entire controller development program. In a typical program, it might first be used for preparing proposals on a new control system, and shortly thereafter for investigating the relative merits of different control schemes. As the program proceeds, the simulator may be used for determining system stability, system performance, and optimum, or necessary values of controller components. In the last stages of development, the simulator might be used for evaluation of controller hardware prior to tests with a prototype engine.

This variety of usage requires that the simulation be easily simplified, that it be capable of using "generalized" engine data, and that the engine be simulated for several different operating environments.

3.1. Representation of Simplified System

The problem of simplifying the controller simulation will not be discussed here except to say that it may be easily accomplished by techniques frequently employed by control designers. The most convenient way to simplify the engine simulation is to replace the 3 VFG's by two-variable, or one-variable function generators. Thus, if the effect of afterburner fuel flow R is slight, all partial derivatives, W', and S'_2 may be considered functions of only two variables, N and S_1. If, in addition, the engine is assumed to be always operating near scheduled values of N and S_1, then only their values at scheduled operating points are required for W', S'_2 and the partial derivatives. In such a case, these quantities may be regarded as functions only of N or only of S_1. This last simplification would result in replacing each of the 3 VFG's by a one-variable function generator. Such a simplified representation markedly reduces the requirements both for engine data and for computer equipment, also for time required to set up a simulated engine.

3.2. Use of Generalized Variables

If all the physical variables in eq (5) through (10) are suitably transformed [7], they will appear in the equations in a form called "generalized". The mechanization of these generalized equations will cause the simulated engine to operate in generalized time. Operation in real time may be obtained by converting the engine accelerations A_1 and A_2 to their real-time values, and then integrating to obtain S_1 and S_2 in real time. S_1 and S_2 may then be connected to either the transducer equipment or the simulated controller. However, S_1 and S_2 must be reconverted to generalized values before they may be used in eq (7) through (10), when these employ generalized values. These conversions merely require multiplying by constants given in [7], and are easily performed. Generalized values of P and T must similarly be multiplied by a constant wherever P and T is connected to a transducer or controller element; real-time values of P and T must also be multiplied by a conversion constant before they can be used in (7) through (10), when these employ generalized values.

3.3. Simulation for Different Engine Environments

The simulator may be required to represent the engine at several altitudes and air speeds These environmental changes may cause significant changes in the numerical value of the functions in (7) through (10), and so require a number of settings to be changed in the engine simulator. These settings will be changed manually rather than automatically, as the simulated engine is not to "fly" in an airplane traveling at varying speeds and altitudes. The number of manual changes required will not be so large that the time to make them would be excessive. Because controller development concentrates upon a relatively few operating conditions, these changes, although a nuisance, will not be a significant handicap for this simulator.

4. Estimated Costs for Engine-Controller Simulator

Table 1 lists the components required for the engine simulator, and their estimated cost. The components are electronic except for the fast-response servomechanisms used in the function generators. Components of 1 percent accuracy should prove adequate for the electronic components. Amplifiers and multipliers of 0.1 percent accuracy may be used, if desired, with an attendant increase in cost; 0.1 percent diode function generators may not be obtainable. The cost of the servomechanisms will be largely determined by their speed of response. Servos with an advertised closed-loop frequency response of 40 to 60 cps should prove adequate for meeting the requirements. Each servo should be capable of mounting 3 or 4 of the tapped potentiometers used in the 3VFG's. Static accuracy of 1 percent or better is acceptable, and should easily be obtained.

Table 2 lists the components required for the controller simulator, and their estimated cost. The equipment listed should prove adequate for simulating a large majority of engine controllers. Equipment requirements for very simple controllers or extremely complex controllers may differ from those given in table 2.

The cost of transducer equipment is estimated at \$50,000 to \$100,000, depending on the amount of instrumentation and computer components that are purchased with the test stand. It is assumed each stand will be individually ordered with varying deviations from the original J–34 design. An additional transducer for converting a voltage to an air pressure may have to be procured separately.

The total cost of engine simulator, controller simulator, and transducer equipment for a twin-spool engine with afterburning is estimated at a minimum of \$165,000 and a maximum of \$280,000. The minimum figure consists of \$100,000 for engine simulation equipment, \$15,000 for controller simulation equipment, and \$50,000 for transducer equipment. The maximum figure consists of \$150,000 for engine simulation equipment, \$30,000 for controller simulation equipment, and \$100,000 for transducer equipment.

5. References

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6. Appendix A. Simulation of Engine Acceleration

A.1. Equation for Engine Accelerations

The engine is assumed to have two independent tandem spools whose speeds are S_1 and S_2, and whose corresponding accelerations are A_1 and A_2. Main fuel flow of the engine is represented by W, the variable area of the nozzle by N, and afterburner fuel flow (reheat fuel flow) by R. Denoting steady-state values by primes, A_1', and A_2' are zero and S_1' and S_2' are determined by the steady-state values W', N', and R', which are outputs from the engine's control system. During transient operation, A_1 and A_2 are assumed to be "quasi-static" variables, whose values are determined by the instantaneous values W, N, S_1, and R [4]. Thus, we may write:

$$A_1 = A_1\,(W, N, S_1, S_2, R), \tag{11}$$

$$A_2 = A_2\,(W, N, S_1, S_2, R). \tag{12}$$

Equation (11) may be written:

$$A_1 = A_1 - A_1' = A_1\,(W, N, S_1, S_2, R) - A_1'\,(W', N', S_1', S_2', R'). \tag{13}$$

For small departures of W, N, S_1, S_2, and R from a set W', N', S_1', S_2', R' of steady-state values, A_1 may be approximated by

$$A_1 = \frac{\partial A_1}{\partial W}\,\delta W + \frac{\partial A_1}{\partial N}\,\delta N + \frac{\partial A_1}{\partial S_1}\,\delta S_1 + \frac{\partial A_1}{\partial S_2}\,\delta S_2 + \frac{\partial A_1}{\partial R}\,\delta R \tag{14}$$

where

$$\delta W = W - W',\ \delta N = N - N',\ \delta S_1 = S_1 - S_1',\ \delta S_2 = S_2 - S_2',\ \text{and}\ \delta R = R - R',$$

and where the partial derivatives are evaluated at the steady-state operating point W', N', S_1', S_2', R'. A similar equation may be written for A_2.

The engine simulator will be furnished with instantaneous values of W, N, and R. It will be required to generate S_1 and S_2. Steady-state engine data may be expected to furnish $W'\,(N_1', S_1', R')$, i.e., the steady-state fuel flow required at a given nozzle area N', reheat fuel flow R', and rotor speed S_1'. Also, knowledge of $S_2'\,(N_1', S_1', R')$, the steady-state speed of the second spool, may be expected. In eq (14), if the steady-state values N_1', S_1', and R' vary, the partial derivatives will, in general, also vary. Thus, the partial derivatives may be regarded as functions of N', S_1', and R', and it will be assumed that these functions are known.

The mathematical form of (14) may be simplified by choosing the steady-state values N', S_1', and R' equal to the corresponding instantaneous values N, S_1, and R. Such a choice makes $\delta N = 0 = \delta S_1 = \delta R$, and (14) becomes:

$$A_1 = \frac{\partial A_1}{\partial W}\left\{W - W'\,(N', S_1', R')\right\} + \frac{\partial A_1}{\partial S_2}\left\{S_2 - S_2'\,(N', S_1', R')\right\}. \tag{15}$$

Similarly, A_2 may be written:

$$A_2 = \frac{\partial A_2}{\partial W}\left\{W - W'\,(N', S_1', R')\right\} + \frac{\partial A_2}{\partial S_2}\left\{S_2 - S_2'\,(N', S_1', R')\right\}. \tag{16}$$

A.2. Computer Representation of Engine Acceleration

The further discussion of eq (15) and (16) is conveniently divided into two separate cases: "dry engine" operation in which there is no afterburning, and "reheat" operation in which afterburning occurs.

a. **Dry Engine Operation**

In this case, $R=0=R'$, and W' and S_2' become functions of N' and S_1' only. We therefore write:

$$W'=W'(N', S_1') \tag{17}$$

$$S_2'=S_2'(N', S_1'). \tag{18}$$

The function $W'(N', S_1')$ may be generated by the function generator shown in figure 7. The voltage output representing $W'(N', S_1')$ appears at the arm of a tapped potentiometer. Each tap of the potentiometer is supplied by a function generator whose output voltage represents $W'(N', S_1')$ for a particular value of N', such as $\beta_1, \beta_2, \ldots, \beta_5$. The arm of the potentiometer moves in correspondence to the nozzle area, to select the output of one of the function generators, or to interpolate linearly between two of the function-generator outputs. The generation of a voltage representing $S_2'(N', S_1')$ is accomplished in the same manner.

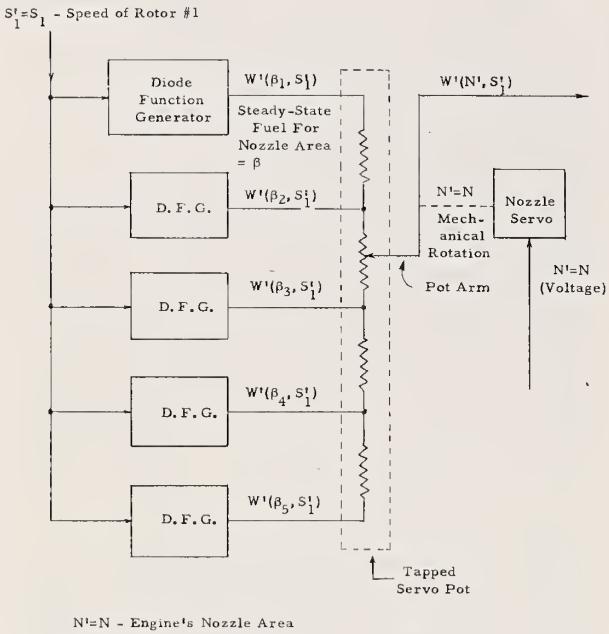

FIGURE 7. *Two-variable function generator for generating $W'(N', S')$.*

Because the partial derivatives are evaluated at a steady-state operating point, they also are functions of N' and S_1'. They may be generated in the same manner as W' and S_2'. In figure 8, the four variables W', S_2', $\partial A_1/\partial W$, and $\partial A_1/\partial S_2$ are shown as the output of the corresponding two-variable function generators $FG\text{–}W'$, $FG\text{–}S_2'$, $FG\text{–}\partial A_1/\partial W$, and $FG\text{–}\partial A_1/\partial S_2$. Inputs to these function generators are N' and S_1'. In figure 8, A_1 is obtained by summing $(\partial A_1/\partial W)\delta W$ and $(\partial A_1/\partial S_2)\delta S_2$. Each of these terms is obtained as the output of a multiplier whose inputs are a partial derivative factor and an increment factor. Each partial derivative is obtained as the output of one of the function generators. The increments are obtained by subtracting from the instantaneous fuel input W, and the instantaneous speed S_2, the steady-state values W' and S_2'. The latter are in turn obtained from two-variable function generators whose inputs are N' and S_1'.

The acceleration A_2 may be generated in the same manner as A_1. Figure 9 shows the generation of both A_2 and A_1. In addition, it shows the integration of A_1 and A_2 to yield S_1 and S_2 which in turn are fed back to assist in the computation of A_1 and A_2. In figure 9, the inputs to the engine are W and N. The outputs are shown as S_1 and S_2, the instantaneous engine speeds.

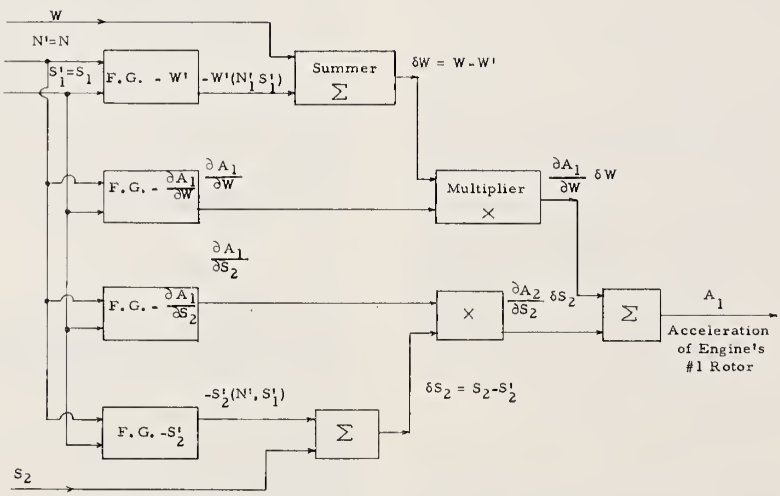

FIGURE 8. *Block diagram for simulation of engine acceleration.*

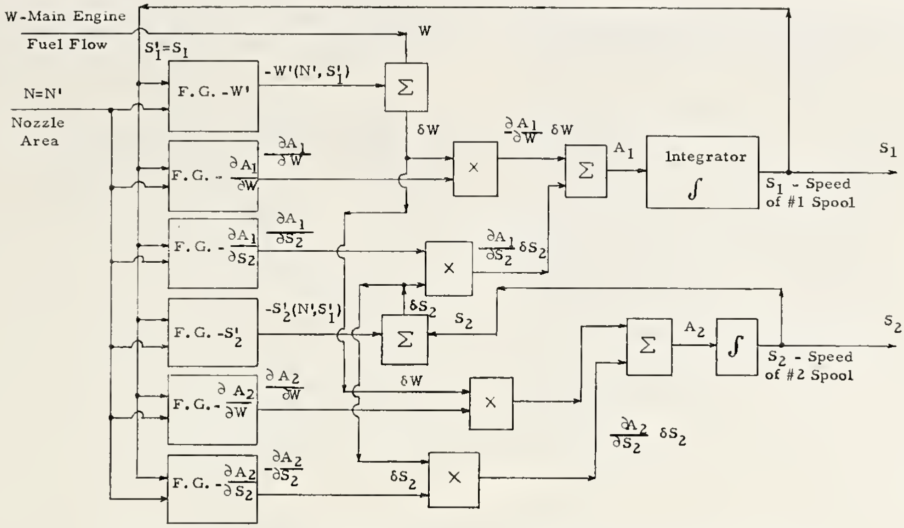

FIGURE 9. *Generation of engine spool speeds for dry engine operation.*

b. Reheat Operation

Two possible modes of reheat operation will be separately considered. The first occurs when afterburner operation is prohibited below rated engine speed. In this case, steady-state values of afterburner fuel flow and nozzle area are such that the steady-state engine speed and turbine inlet temperature are both maintained at their rated values. The second mode of operation permits afterburner operation at steady-state speeds below the engine's rated speed. The first mode will be called "quasi-constant engine speed," the second mode "variable engine speed."

For both modes, we shall choose $N'=N$, $R'=R$, and $S_1'=S_1$ in order to obtain the simplified eq (15). Since R is no longer identically zero, the steady-state fuel flow W', the steady-state speed S_2', and the partial derivatives $\partial A_1/\partial W$ and $\partial A_1/\partial S_2$ are now functions of three variables, N', R', and S_1'. Because the first mode is the simpler to simulate, it is discussed first.

(1) *Quasi-Constant Engine Speed.* In this case, $S_1'=S_1$ is nearly equal to rated engine speed, S_{10}. Since the engine speed control and the inertia of the rotor act to maintain S_1 nearly equal to S_{10} at all times, only small departures of S_1 from S_{10} need be considered. This restriction allows W' (N', S_1', R') and the corresponding values of S_2' and the partial derivatives to be generated rather simply for large ranges of N' and R'. The method is the same for each of these four functions. It is described below for W' (N', S_1', R'), and diagramed in figures 10 and 11.

At some constant value β of N', the quantity W' (β, S_1', R') is a function only of S_1' and R'. Because $S_1'-S_{10}$ is small, we write as a good approximation:

$$W'(\beta, S_1', R')=W'(\beta, S_{10}, R')+\frac{\partial W'}{\partial S_1'}(S_1'-S_{10}) \quad (19)$$

where

W' (β, S_{10}, R') is a function only of R', and where $\partial W'/\partial S_1'$ is evaluated at $N'=\beta$, $S_1'=S_{10}$, and at R'. Consequently, this derivative is a function only of R'.

The function W' $(\beta, S_1', R'=0)$ has already been generated for the case of dry engine operation. This function may be written, for S_1' near S_{10}, as

$$W'(\beta, S_1', R'=0)=W'(\beta, S_{10}, R'=0)+D(S_1'-S_{10}) \quad (20)$$

where D is the partial derivative of W' $(\beta, S_1', R'=0)$ with respect to S_1' evaluated at $S_1'=S_{10}$, and W' $(\beta, S_{10}, R'=0)$ is a constant. Adding and subtracting eq (20) from (19) yields:

$$W'(\beta, S_1', R')=W'(\beta, S_1', R'=0)+W'(\beta, S_{10}, R')-$$
$$W'(\beta, S_{10}, R'=0)+\left\{\frac{\partial W'}{\partial S_1'}-D\right\}\left\{S_1'-S_{10}\right\}. \quad (21)$$

Equation (21) shows that $W'(\beta, S_1', R')$ may be obtained by the addition of the steady-state fuel for dry engine operation, $W'(\beta, S_1', R'=0)$, the output of a function generator of one variable,

$$W'(\beta, S_{10}, R')-W'(\beta, S_{10}, R'=0)$$

whose input is R', and the output of a multiplier whose inputs are $(S_1'-S_{10})$ and the output of a function generator, $(\partial W'/\partial S_1'-D)$, whose input

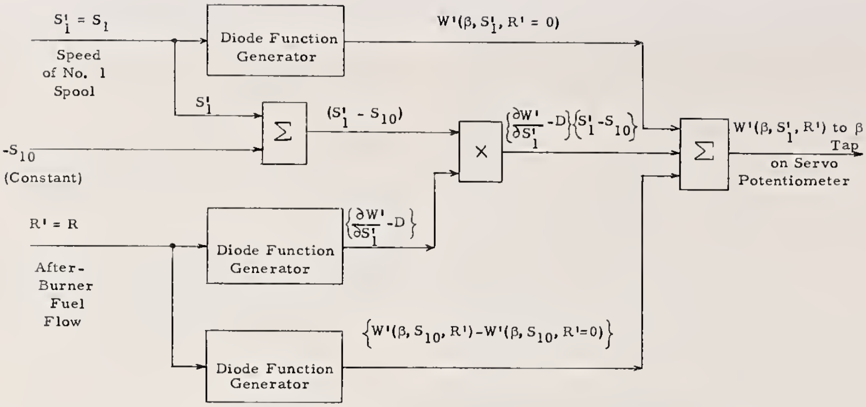

FIGURE 10. *Generation of* W' $(N'=\beta.\,,\ S_1'\cong S_{10},\ R')$.

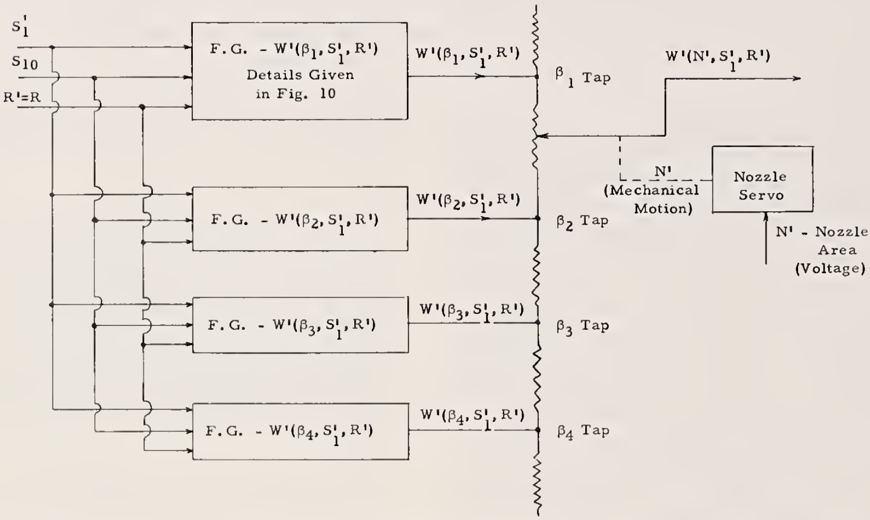

FIGURE 11. *Generation of* W' $(N',\ S_1'\cong S_{10},\ R')$*—steady-state fuel flow for afterburning with quasi-constant speed.*

is R'. A schematic diagram is shown in figure 10. As shown in the diagram, the function $W'(\beta,\ S_1',\ R')$ may be generated by three one-variable function generators and a multiplier. The value of $W'(\beta,\ S_1',\ R')$ for a particular value of β is connected to a particular tap on the servo pot, as shown in figure 11. The other taps correspond to other values of β. The servo then moves in accordance with N' to select the function for which $N'=\beta_1\ .\ .\ .,\ \beta_4$ or to interpolate between the two values of β nearest N'.

Details of the generation of S_2', $\partial A_1/\partial W$, and $\partial A_1/\partial S_2$ are the same as for W', whose generation is discussed above. Except for these additional complexities in the function generators, the block diagram is the same as for dry engine operation.

(2) *Variable Engine Speed.* The only change in the block diagram occurs in the function generators which produce W', S_2', $\partial A_1/\partial W$, and $\partial A_1/\partial S_2$. If these functions must be generated over large ranges of N', S_1', and R' a two-variable function generator must be connected to each tap of the tapped potentiometers driven by the nozzle servo. A schematic drawing is given in figure 12 for $W'(N',\ S_1',\ R')$. A suitable function generator might be that produced by G. A. Philbrick [8]. Or, the particular form of the engine data might allow it to be generated by a special technique given by D. A. Elliot [9].

It is questionable whether these four functions, when each is a function of three variables, will be furnished the engine-control designer. It is very unlikely that they will be completely defined by test data. Unless these engine data are available, there is little use in simulating the engine for this mode of operation. The difficulty of obtaining sufficient data may cause this complex simulation to be very infrequently employed. However, for the sake of completeness, the case of variable engine speed has been discussed in this section.

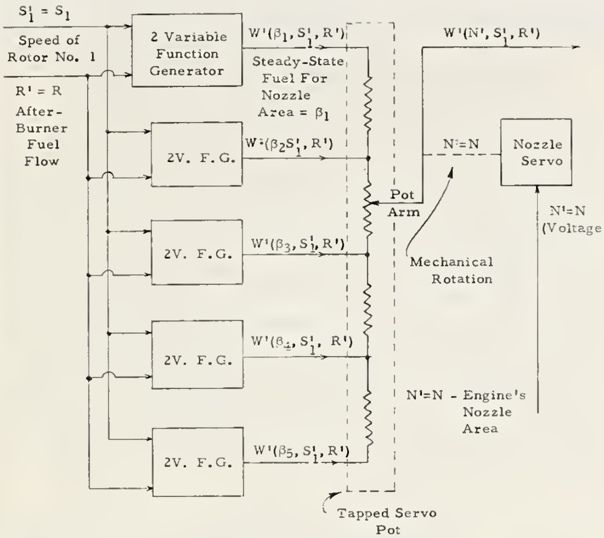

FIGURE 12. *Three-variable function generator for generating W' (N', S', R').*

A.3. Simulation of Throttle Bursts

If δW and δS_2 are both small, an excellent approximation for a typical engine acceleration is, from (15) and (16), given by:

$$A=\frac{\partial A}{\partial W}\delta W+\frac{\partial A}{\partial S_2}\delta S_2. \tag{22}$$

In conjunction with the other small-departure equations for the engine-controller system, these acceleration equations may be used to determine small-departure stability of the engine control system, i. e., the engine and its controller execute only small excursions from steady-state operating points. Because experience indicates the control system is stable if it is stable for small departures, these engine equations are adequate for determining control system stability. They are also adequate for determining responses to small or slow changes in throttle position, because, for small or slow changes in throttle position, the departures of the engine from steady state are kept small by the engine control system.

If large and sudden changes in throttle position are also to be simulated, eq (22) requires alteration. This small-departure equation, if used to simulate large throttle bursts, will yield a higher acceleration than actually occurs in the engine. Therefore, the "response time" (the time to accelerate the engine from one steady-state operating point to another steady-state operating point) for large throttle bursts will be smaller than that of the actual engine. To obtain accurate simulation of this response time, the small-departure equation for acceleration will be corrected for large values of δW, the "excess fuel".

During the large-departure operation, resulting from throttle bursts, δW is, for the major portion of the large-departure period, equal to δW_m, the maximum excess fuel which may be furnished the engine without the occurrence of stall, blowout, or engine over-temperature. The acceleration produced by δW_m is, according to eq (22), $(\partial A/\partial W)\ \delta W_m$. The actual engine acceleration, A_m, produced by δW_m is shown in figure 13 as the ordinate of the solid line curve (acceleration characteristic) corresponding to an abscissa δW_m. The horizontal dashed line in this figure represents $(\partial A/\partial W)\ \delta W_m$. For small δW, the acceleration characteristic and $(\partial A/\partial W)\ \delta W$ are in good agreement with each other. However, as δW increases, the slope of the acceleration characteristic decreases from $\partial A/\partial W$ at $\delta W=0$ towards a much smaller value at $\delta W=\delta W_m$. The exact shape of the acceleration characteristic may vary significantly with the operating point of the engine. Accurate calculations of the exact acceleration characteristic are ordinarily not available. Accurate experimentally determined acceleration characteristics also are ordinarily not available. Hence, it seems advisable to compute an approximate value of A_m by correcting $(\partial A/\partial W)\ \delta W_m$ and to make this correction a flexible one.

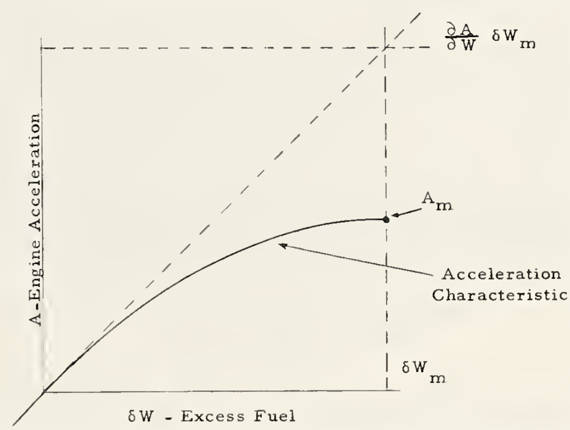

FIGURE 13. *Typical variation of engine acceleration with excess fuel.*

For most engines, the slope of the acceleration characteristic at δW_m is greater than, or equal to zero. The approximation of the acceleration characteristic by $(\partial A/\partial W)\ \delta W-K(\delta W)^2$ will have slope $\partial A/\partial W$ at $\delta W=0$ and a slope greater than zero at $\delta W=\delta W_m$ if K is positive and less than $\frac{1}{2}\ (\partial A/\partial W)/\ \delta W_m$. Therefore, for throttle bursts the acceleration characteristic will be approximated not by $(\partial A/\partial W)\ \delta W$ but by $(\partial A/\partial W)\ \delta W-K\ (\delta W)^2$ where $0<K\leq\frac{1}{2}\ \partial A/\partial W/\ \delta W_m$. The exact value of K to be used will be determined by calculated data, experimental data, or the experience and discretion of the control system designer. The value of A_m will, according to the value of K used, be between $(\partial A/\partial W)\ \delta W_m$ and $\frac{1}{2}\ (\partial A/\partial W)\ \delta W_m$. The correction of $(\partial A/\partial W)\ \delta W$ for throttle bursts is shown in figure 6 for rotor No. 1.

Throttle bursts may also cause the value of δS_2 to be large. δS_2 is equal to $S_2 - S_2'$ (N', S_1', R'). The rotor inertias prevent S_1 and S_2 from changing rapidly during a throttle burst. The rapidity of nozzle area change is limited by the size of the nozzle actuator, the inertia of the nozzle, and the forces exerted by the exhaust gases upon the nozzle. Permissible changes in afterburner fuel flow, R', are usually made small during throttle bursts in order that the permissible value of excess fuel, δW_m, will not have to be varied with R'. Therefore, throttle bursts will not immediately produce large changes in S_1', N', R', S_2' (N', S_1', R'), S_2, and $\delta S_2 = S_2 - S_2'$ (N', S_1', R'). Henee, at the beginning of a throttle burst transient, δS_2 will be small. It will also be small near the end of this transient. If S_2 responds much more rapidly to the excess fuel than does S_1, δS_2 will probably be positive during the throttle burst transient. If S_2 responds much more slowly to the excess fuel than does S_1, δS_2 will probably be negative during the throttle burst transient. If the response of both S_2 and S_1 to the excess fuel is approximately the same, δS_2 will probably remain small and may reverse sign during the throttle burst transient. The values δS_2 may assume during a throttle burst seem to vary widely according to the particular engine that is being simulated. It is felt that for the majority of engines, both rotors will exhibit approximately the same response to excess fuel, and that δS_2 will be small during a throttle burst transient as well as at the beginning and the end of this transient. Therefore, in eq (22), $(\partial A/\partial S_2)\,\delta S_2$ will be used in computing acceleration for both small-departure and throttle burst operation of the engine.

A.4. Experimental Determination of the Partial Derivatives

The general equation for small departure acceleration has the form:

$$A = \frac{\partial A}{\partial W}\delta W + \frac{\partial A}{\partial S_2}\delta S_2 \tag{22}$$

where the partial derivatives are to be determined at a steady-state operating point, (N', S_1', R'). The value of $\partial A/\partial W$ may be determined by experimental testing. Experimental determination of $\partial A/\partial S_2$ may be difficult or impractical. If this is the case $\partial A/\partial S_2$ may be determined from the value of $\partial A/\partial W$ and steady-state data by eq (25) below.

To obtain this relation, we may write:

$$\left.\begin{aligned} \delta W &= W - W' = W - W'' + W'' - W' = W - W'' + \delta W', \\ \delta S_2 &= S_2 - S_2' = S_2 - S_2'' + S_2'' - S_2' = S_2 - S_2'' + \delta S_2' \end{aligned}\right\} \tag{23}$$

where W' and S_2' are steady-state values corresponding to the steady-state operating point N', S_1', R', and W'' and S_2'' are steady-state values corresponding to another steady-state operating point N'', S_1'', R'' near N', S_1', R'. If, in (23), W is chosen equal to W'', S_2 is chosen equal to S_2'', A is zero at the second steady-state operating point, and (22) may be written:

$$0 = \frac{\partial A}{\partial W}\delta W' + \frac{\partial A}{\partial S_2}\delta S_2'. \tag{24}$$

From (24), we have:

$$\frac{\partial A}{\partial S_2} = -\frac{\partial A}{\partial W}\delta W'/\delta S_2'. \tag{25}$$

The value of $\partial A/\partial W$ is assumed to be known at N', S_1', and R' from experimental testing, $\delta W'$ and $\delta S_2'$ may be evaluated from steady-state data by choosing convenient steady-state increments for $\delta N' = N'' - N'$, $\delta S_1' = S_1'' - S_1'$, and $\delta R' = R'' - R'$. For rotor 1, (25) becomes:

$$-\frac{\partial A_1}{\partial S_2} = \frac{\partial A_1}{\partial W}\frac{\delta W'}{\delta S_2'}. \tag{26}$$

Convenient values to choose are $\delta R' = 0$ and $\delta N' = 0$, and $\delta W'$ and $\delta S_2'$ as the increments resulting from a small value of $\delta S_1'$.

For rotor 2 (25) becomes:

$$-\frac{\partial A_2}{\partial S_2} = \frac{\partial A_2}{\partial W}\frac{\delta W'}{\delta S_2'}. \tag{27}$$

where $\delta R'$ and $\delta N'$ have the same values as are used in eq (26). As a check and a guard against errors in determining $\delta W'$ and $\delta S_2'$, (26) and (27) should be used to determine values of $\partial A_1/\partial S_2$ and $\partial A_2/\partial S_2$ for $\delta W'$ and $\delta S_2'$ equal to values obtained by choosing $\delta R' = 0 = \delta S_1'$, and $\delta N' \neq 0$, but small.

7. Appendix B. Simulation of Turbine Inlet Temperature

B.1. Derivation, Simplification and Mechanization of Temperature Equation

The turbine inlet temperature T is assumed to be a quasi-static variable, and is taken as an instantaneous function of W, N, S_1, S_2, and R. We denote this relation by $T=T\ (W, N, S_1, S_2, R)$, and use the primed quantity, T', to indicate a steady-state temperature corresponding to steady-state values W', N', S_1', and R'. Subtracting T' from T yields:

$$\delta T=T-T'=\frac{\partial T}{\partial W}(W-W')+\frac{\partial T}{\partial N}(N-N')+\frac{\partial T}{\partial S_1}(S_1-S_1')+\frac{\partial T}{\partial S_2}(S_2-S_2')+\frac{\partial T}{\partial R}(R-R'). \quad (28)$$

As in the case of the acceleration eq (14), N', S_1', R' are chosen equal to the corresponding instantaneous values N, S', and R. Equation (28) then reduces to:

$$\delta T=\frac{\partial T}{\partial W}\delta W+\frac{\partial T}{\partial S_2}\delta S_2 \quad (29)$$

where

$$\delta W=W-W'=W-W'\ (S_1', N', R'),\ \delta S_2=S_2-S_2'\ (S_1', N', R'), \quad (30)$$

and the partial derivatives in (29) are evaluated at a steady-state operating point determined by N', S_1', and R'.

The mechanization of eq (29) and (30) requires generation of $W'\ (S_1', N', R')$, $S'_2\ (S'_1, N', R')$, and the partial derivatives that are likewise functions of N', S_1', and R'. The generation of δT is shown in schematic form in figure 3. It is similar in detail to the generation of an engine acceleration. To obtain T, the instantaneous temperature, T' is added to δT. T' is also a function of N', S_1', and R', and is generated by a function generator whose details are identical with the function generator which generates $W'\ (N', S_1', R')$.

If desired, the temperature equation may be corrected for throttle-burst operation by the method used for correcting the acceleration equations. The amount of correction required should, for most engines, be relatively small.

B.2. Experimental Determination of Partial Derivatives

The two partial derivatives in (29) are to be evaluated at steady-state operating points, and should therefore be much easier to determine by experimental testing than if their value were required at transient operating points. Experimental determination of $\partial T/\partial W$ will probably be practical. Experimental determination of $\partial T/\partial S_2$ may be difficult or impractical, in which case this derivative may be calculated in terms of $\partial T/\partial W$ and steady-state data by eq (32) below.

This equation is developed by taking the difference of two steady-state temperatures $T''\ (N'', S_1'', R'')$ and $T'\ (N', S_1', R')$. The values $\delta N'=N''-N'$, $\delta S_1'=S_1''-S_1'$, and $\delta R'=R''-R'$ may be chosen small so that we may write as a good approximation:

$$\delta T'=T''-T'=\frac{\partial T}{\partial W}(W''-W')+\frac{\partial T}{\partial S_2}(S_2''-S_2')=\frac{\partial T}{\partial W}\delta W'+\frac{\partial T}{\partial S_2}\delta S_2'. \quad (31)$$

By a suitable choice of $\delta N'$, $\delta S_1'$, and $\delta R'$, and with not more than one of them equal to zero, the steady-state temperatures may be made the same. For such a choice, $\delta T'=0$ and (31) becomes:

$$-\frac{\partial T}{\partial S_2}=\frac{\partial T}{\partial W}\delta W'/\delta S_2' \quad (32)$$

where $\delta W'$ and $\delta S_2'$ are steady-state increments about W' and S_2' corresponding to steady-state increments $\delta N'$, $\delta S_1'$, and $\delta R'$ about N', S_1', and R$'$.

In (32) $\partial T/\partial W$ is supposedly known at N', S_1', and R' from experimental values. A convenient choice of $\delta N'$, $\delta S_1'$ and $\delta R'$ which makes $\delta T'=0$ will permit $\delta W'$ and $\delta S_2'$ to be determined from steady-state data.

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