Extension and Dissemination of the Electrical and Magnetic Units by the National Bureau of Standards
Related Publications of the National Bureau of Standards

Announcement of Changes in Electrical and Photometric Units

A report of new international agreements, adopted January 1, 1948, on units of electricity and light. International electrical units were in use for over 50 years. Since the latest previous adjustment in 1912, the increasingly close connection between various physical constants has made desirable the use of electrical units derived from the fundamental mechanical units of length, mass, and time.

The system of photometric measurements is admittedly arbitrary because of the psychological factors involved. However, it represents a worldwide agreement on practical units not greatly different from those superseded.

National Bureau of Standards Circular 459, 7 pages, 5 cents.

Precision Resistors and their Measurement

A systematic, practical discussion of the characteristics of precision resistance apparatus and of the National Bureau of Standards' methods for measuring resistance, based largely on substitution procedures providing high accuracy.

The circular contains chapters on resistance materials and construction methods, methods of comparison of resistors, special apparatus for precision measurement, calibration of precision bridges, and resistivity of solid conductors.


Establishment and Maintenance of the Electrical Units

An account of the history of the international system of electrical units, trends of development making them obsolete, and methods used in the measurements forming the basis for the new absolute units. The "international" system of electrical units, officially in operation from 1911 to 1947, was superseded by a system of "absolute" units. Records show that the units of some countries at times drifted nearly 0.01 percent from the mean.

National Bureau of Standards maintenance procedures are reported, and brief descriptions are given of current methods available for the absolute measurement of resistance and of current. This will be used in the future to check on the maintenance of the units.


An Absolute Measurement of Resistance by the Wenner Method

A detailed description of the results of a method devised for independently checking the stability of the standard of electrical resistance in terms of length, time, and the permeability of free space. A change of a few parts in a million in the standards used to maintain the unit can be detected by means of the method.

In 1929 a mutual inductor was constructed and its inductance accurately computed from its dimensions. By use of Wenner's commutated direct-current method, resistance was measured in terms of this inductance and frequency.

Dimensions of the mutual inductor were determined in 1938 and again in 1948. The change in inductance due to drifts in dimensions was compared with the difference in the electrical determinations made on the two dates. During the interval no appreciable change in the value of the Bureau's standard ohm had occurred.


Extension and Dissemination of the Electrical and Magnetic Units by the National Bureau of Standards

Francis B. Silsbee

National Bureau of Standards Circular 531
Issued July 14, 1952
Preface

One of the primary functions of the National Bureau of Standards is the establishment, maintenance, and dissemination of units of measurement that shall be uniform throughout the Nation and constant through the years. To meet the needs of industry in the electrical field, which is developing so rapidly in extent and in complexity, this duty has involved a continuous growth of staff and equipment and the improving of old methods of measurement and the inventing of new ones.

A report of the stewardship of the National Bureau of Standards with respect to the establishment and maintenance of the fundamental electrical units of electromotive force and resistance is given in the Bureau's Circular 475, which describes both the older international system of electrical units and the present absolute system, which was put into effect to replace the older system on January 1, 1948. The present Circular carries the report further by showing how the other electrical and magnetic units are derived from these two, how the range of measurement is extended, and how the Bureau serves the electrical industry by determining the corrections for apparatus submitted for test. It is hoped that these brief descriptions of the methods that have been found effective in the laboratories of the Bureau will be a useful guide in the development of other laboratories that may have the duty of extending the measurement process. This general picture of the procedures by which accurate measurements are actually carried out should be helpful also to educators who wish to give their students a definite and correct understanding of the practical application of their teachings in the theory of electrical measurements.

A. V. Astin, Director.
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Extension and Dissemination of the Electrical and Magnetic Units by the National Bureau of Standards

By Francis B. Silsbee

Starting with the ohm and the volt as maintained by groups of standard resistors and cells, this paper describes the experimental processes by which the other electric and magnetic units, e.g., farad, henry, ampere, watt, joule, gauss, and oersted are derived. It also describes the series of steps by which the scales of measurement of resistance, direct and alternating current, and voltage are derived experimentally. Brief mention is made of the procedures for the dissemination of these standards of measurement throughout the world by the calibration of standard electrical measuring apparatus. An extensive bibliography lists papers describing the measurement procedures in greater detail and serves as a historical report of the work of the National Bureau of Standards in the field of electrical measurements during its first 50 years.

1. Introduction

It is the purpose of this Circular to give an overall picture of the sequence of measuring processes by which a self-consistent system of electrical units is built up in the laboratories of the National Bureau of Standards and thence disseminated throughout the Nation. The branching chains of measurement thus initiated may be considered as extending outward without a break from the groups of primary standard cells and standard resistors by which the units of voltage and of resistance are maintained from year to year, and as ending finally in the myriad measuring operations throughout the Nation, on the basis of which scientific research is conducted, manufactured articles are adjusted and inspected, electric energy is bought and sold, and industrial processes are precisely controlled. It is only as a result of this self-consistency that scientists in different laboratories can talk in the same terms, that apparatus formed by components manufactured in different places at different times can be expected to function properly, that competing sources of energy can be fairly compared, and that complex industrial operations will consistently yield a product of constant quality.

Although many links in this network of measurement are welded in remote laboratories, the National Bureau of Standards has exerted for the past 50 years a very significant influence on the development of the system, both directly by its testing service and by the invention of new methods of measurement, and indirectly by example and precept. The particular experimental methods described herein are primarily those used in the Bureau laboratories, but to a large extent the same or closely similar methods are used elsewhere. The following paragraphs show the points of contact of the Bureau with the industrial world in more detail. The extensive bibliography gives references in the field of electrical measurements, not only to technical papers originating with the Bureau staff, but also, on occasion, to other noteworthy publications in this field.

The extension of measurements into the radio field has not been included in this Circular because the methods used at these higher frequencies constitute a quite distinct discipline.

2. Fundamentals

By international agreement \[4, 6\]^1 the official basis for electrical measurements throughout the world is the system of practical absolute electrical units. These units developed historically as convenient decimal multiples of the units of the centimeter-gram-second electromagnetic system, but they also form the electrical part of the self-consistent meter-kilogram-second-ampere, or Giorgi, system of units.

The experimental processes by which these units are established in terms of length, mass, time, and the conventionally assumed constant \(\mu\), usually designated “permeability of space” are called “absolute measurements.”

Because of the importance in such measurements of ready access to highly accurate standards of length, mass, and time, and because of the

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1 Figures in brackets indicate the literature references at the end of this paper.
long, tedious, and therefore expensive, observational programs required if significant accuracy is to be attained, the burden of periodic determinations of this nature rests almost entirely on the few larger national standardizing laboratories of the world. Of the many theoretically possible forms of absolute electrical measurement, two types of experiment have so far shown the greatest possibilities for high accuracy and, except in certain very special circumstances that will be noted later, are universally used [17, 36].

The first of these experiments involves the construction of a standard of self- or of mutual inductance, the geometrical configuration of which is such that (1) its significant dimensions can be directly and accurately measured [45, 46, 47] and (2) its inductance can be calculated precisely from these measured values [142, 148, 149, 47]. As a preliminary for such procedures, Dr. E. B. Rosa and his colleagues, in the early years of the Bureau, carried on an extensive theoretical examination of the various available formulas for inductance and published a number of papers [132 to 141] in this field, the major results of which are summarized in [135]. A large volume of later work was done by Dr. Chester Snow and others [142 to 159], the results of which are, of course, applicable to the computation of the inductances of various circuit configurations for many other purposes, as well as for absolute measurements. After the value of the inductance of a standard inductor in absolute henries has been computed, an experiment is performed by which the value in absolute ohms of the resistance of a standard resistor is obtained in terms of the reactance of the standard inductor at the known frequency used in the experiment. The net result of the whole process is usually called an "absolute measurement of resistance." At the National Bureau of Standards two completely independent procedures have been used for this purpose, one [38, 39] starting with a self inductor and the other [47] with a mutual inductor. The two final results differed by only 12 ppm (parts per million). The second of these procedures has proved so convenient that it is planned to make fairly frequent (perhaps annual) electrical comparisons between the standard resistors and the standard mutual inductor. It is hoped that any secular changes in the dimensions of the inductor will be so slow that the tedious mechanical measurements of its dimensions need be repeated only at intervals of ten years or more.

The second type of experiment is the absolute measurement of current by the use of a current balance [40, 41, 42]. This is an instrument by which it is possible to weigh accurately the mechanical force that is exerted between two coils as a result of the electric currents in them. The relation between the force and the current can be calculated by fundamental electromagnetic theory and is expressible by formulas that involve only the geometrical shape, but not the absolute size, of the coil system. After such a current balance has been constructed and its shape determined by suitable measurements, it is connected in series with a resistor, the resistance of which is known in absolute units by reference to the former type of experiment.

The current is adjusted until the change in force that results when the current in one coil is reversed is balanced by the change in loading when a known weight is added to one pan of the balance. At the same time the potential drop produced by the current in the known resistor is compared with the electromotive force of a standard cell. By this process, a value in absolute volts is obtained for the electromotive force of the cell. The cell then serves to preserve the result of the experiment for future use. Such an experiment is usually called an "absolute measurement of current," although the final result is the assignment of a value in absolute volts to a standard cell. Coils of a considerable variety of shapes and construction have been used in the current balance at the National Bureau of Standards, and, as a further check against systematic errors, it is planned to use a radically different form, the Pellat [44, 145] balance, in the near future.

Figure 1. Experimental establishment at the National Bureau of Standards of the primary electrical units by absolute measurements.

The rectangles represent physical standards embodying the corresponding units, the circles represent experimental procedures with their associated apparatus used in the measurements. The lines indicate the passage of units and numerical values. The transfers to Washington of the foreign values for \( \gamma \) were made by the U. S. Coast and Geodetic Survey, using "invariable pendulums."
The relations of these absolute measurements to the basic mechanical standards are indicated by the block diagram of figure 1. It will be noted that the local value of the acceleration of gravity, $g$, enters into the computation of force from mass, and that the value used is based upon three independent determinations [31, 32] made at Potsdam, Washington, and Teddington.

These absolute measurements give results in electromagnetic units. An early research was devoted to a precise determination of the ratio of the units of this system to those of the electrostatic system. For this purpose Rosa and Dorsey [28, 29, 30] used a group of capacitors of such shapes that their capacitances could be computed in electrostatic units from their mechanical dimensions. They then measured their capacitances by electromagnetic methods at frequencies of a few hundred cycles per second. The ratio was found to be in very satisfactory agreement (at least to $10^4$) with the velocity of light measured at optical frequencies.

The units realized by these absolute measurements are preserved by a group of 10 manganin standard resistors of the Thomas double-walled type [66, 67] and a group of 25 saturated cadmium standard cells (16 neutral and 9 of the 0.05-normal acid type). The standards of either group are intercompared at intervals, and following each intercomparison slightly revised values are assigned to the individual standards, on the assumption that the mean value of the group has not changed during the interval [6]. Certain of the standards are used during the intervals as the reference standards for the calibration of other resistors and cells that constitute the Bureau's working standards. As shown in the following sections, the values of the other electrical units are derived from the ohm and the volt as thus maintained.

### 3. Derivation of Other Units

The many electric and magnetic quantities used in science and engineering, such as current, inductance, capacitance, and magnetic flux, are all connected with each other and with mechanical quantities by the science of electromagnetism to form a closely linked structure of exact mathematical relations. Similarly, the various units, such as the ampere, henry, farad and weber, in terms of which these quantities are measured, are also linked by a parallel structure of exact definitions. Starting with any group of independent units, chosen as fundamental, it is possible to derive definitions for all the other units by successively applying the appropriate mathematical relations. Both the selection of the fundamental units and of the particular order in which the various derived units are successively defined can be made arbitrarily. The choice is determined by taste or convenience and may well differ according to the purpose for which the definitions are to be used. Thus the sequence chosen by a college professor for teaching a course in electrical measurements may well differ from the sequence offered by the International Committee on Weights and Measures "as a guide for the wording of legislation" [6, p 36].

The limitations and exigencies of practical experience have led to the use of still other sequences in the laboratory. In nearly all electrical measurements, resistance and voltage are among the fundamental quantities chosen, because their units can be readily embodied and preserved in standard resistors and standard cells. In this section of the Circular, the particular, and sometimes rather devious, sequences normally used in the electrical testing work at the Bureau are described in some detail. For some quantities, different parts of the full range of measurement are derived independently by different sequences from the fundamental units.

![Figure 2](image_url)

Figure 2. Establishment and extension of capacitance measurements.

The dotted lines indicate the approximate limits of the range over which the several types of standards and methods of measurement are useful. The total range from $0.0001 \mu F$ to $100 \mu F$ involves a factor of $1,000,000,000,000$.}

3
3.1 Capacitance

The unit of capacitance is derived from the ohm and the second as indicated by the block diagram in figure 2. Standards of capacitance are measured by a Maxwell commutator bridge [108]. Its circuit is shown schematically in figure 3.

![Figure 3. Maxwell commutator bridge.](image)

The capacitance, $C$, is measured in terms of the resistances of the bridge arms and the known frequency of the vibrating contactor, which alternately charges $C$ by way of the resistance, $R$, and discharges it through $r$.

The contactor, which alternately charges and discharges the capacitor, is operated by a tuning fork driven at frequencies of 100 or 1,000 c/s by a multivibrator. This multivibrator is coupled to the 100 kc/s quartz oscillators which constitute the NBS standard of frequency [25]. The three resistance arms of the bridge may range in value from 10 to $10^8$ ohms, depending on the value of the air capacitor to be measured. This process is used to assign values of capacitance to a group of fixed air capacitors shown in figure 4, which can be connected in parallel with each other and with an adjustable capacitor which has a range from 50 to 1,500 $\mu$F. This combination of air capacitors can be used as a standard for any value of capacitance from 50 $\mu$F to 0.26 $\mu$F. They are useful over a wide range of frequency, provided certain corrections for the self-inductance of the capacitors and their leads are made at the higher frequencies.

Occasionally an independent check of the unit of capacitance is obtained by using a Maxwell-Wien a-c bridge to determine capacitance in terms of resistance and the primary computable standards of self- or of mutual inductance. The values derived by the two methods agree within 0.002 percent. This operation is the inverse of the use shown for the Maxwell-Wien bridge at the left in figure 1.

A commercial highly precise capacitance-conductance bridge [116] used in connection with these standard air capacitors serves to measure other capacitors having either air or solid dielectric in the range 10 $\mu$F to 1.11 $\mu$F. This completely shielded bridge has two equal fixed resistance arms and an adjustable admittance arm reading directly in capacitance up to 1.11 $\mu$F and in conductance up to 1,111 micromhos. This bridge may be used either as a comparison bridge, in which case its calibration is checked against the standard air capacitors, or it may be used in a substitution method using the air capacitors as standards. Normally frequencies from 200 c/s to 150 kc/s are used, but with special external detectors, measurements are possible down to 30 c/s.

A number of fixed and decade groups of mica capacitors with a total capacitance of 10 $\mu$F [106, 109] are calibrated from time to time at a particular frequency by the type 12 bridge, and are occasionally used at the same frequency in an improvised Schering bridge for measurements of other solid-dielectric capacitors up to 100 $\mu$F.

To cover the range of extremely low capacitances, such as the interelectrode capacitance of electron tubes, an independent procedure has been developed [112, 115], based on the construction of capacitors of such shape and size that their capacitance can be computed accurately from their measured dimensions. These capacitors are either parallel-plate units of the conventional Kelvin guard-ring type or of a form referred to as the "guard-well" type, in which the guarded "island" electrode lies at the bottom of a cylindrical opening, or "well", in the guard ring (see fig. 5). They range in value from 0.001 to 5 $\mu$F.
Figure 5. Cross section of standard capacitor of the guard-well type.

F, Guarded electrode, supported by an inulator of low expansion glass. J, E, high-voltage electrode; G, H, parts of guard ring. If the depth, \( d \), of the well is greater than its radius, \( a \), the direct capacitance between \( E \) and \( F \) is very much smaller than if \( F \) is flush with the lower surface of \( H \).

Auxiliary standards used in this range include (1) an adjustable capacitor of the guard-well type, in which the island electrode can be adjusted by a micrometer screw to any desired depth in the well, thus securing a working range 0.1 \( \mu \text{F} \) down to 0.001 \( \mu \text{F} \). (2) an adjustable capacitor of a modified Zichner type with a movable septum between the electrodes by which the direct capacitance can be varied from 0.3 \( \mu \text{F} \) down to zero when the aperture in the septum is closed; (3) a decade group of 0.1 \( \mu \text{F} \) units, the direct capacitance of any unit of which can be reduced to zero by inserting a grounded septum between its electrodes; and (4) a similar group of 0.01 \( \mu \text{F} \) units.

By the substitution of these standards in the \( X \)-arm of a commercial direct-capacitance test set having inductive ratio arms with a fixed 1:1 ratio, and a fixed fourth arm, they may be intercompared at 465 kc/s, and other standards of direct capacitance up to 5 \( \mu \text{F} \) can be measured. It is also possible to step down from standards of higher value with a shielded Schering bridge to 5 \( \mu \text{F} \), and at this level the two procedures have been found to agree within 0.05 percent. The value derived by extending this step-down process to a 0.1-\( \mu \text{F} \) capacitor was found to differ from the value derived independently from the computable standards by less than 0.1 percent. Because of these close agreements, in the range of overlap, no significant inconsistencies can arise anywhere in the range of capacitance, in spite of the independent bases on which the two ends rest.

The unit of dielectric constant for solid dielectrics is derived from the standards of capacitance by combining the electrically measured capacitance of a specimen with its mechanical dimensions. The capacitance is measured at frequencies from 60 c/s to 200 kc/s in a shielded Schering bridge (see fig. 6) that has been calibrated in terms of the standard air capacitors. A major complication arises from the fringing of the electric field at the edges of the specimen, and a number of approximate working formulas have been developed for correcting for such effects [114]. With these edge-correction formulas, the accuracy is about 1 percent. For higher accuracy, the guarding technique is used. The area of the specimen is readily computed from its scaled dimensions, and the accuracy is usually limited by the measurement of thickness. When feasible, this is deduced from measurements of the mass, density, and area of the samples. Under the best conditions, results are repeatable to 0.1 percent.

Another method for minimizing edge effects [117] is that of placing a 2-inch-diameter disk of the dielectric in a micrometer electrode holder. A capacitance measurement is made with the specimen in place and another with it removed. Either the electrodes may be moved closer together by a measured amount to give the same value of capacitance for the second measurement, or they may be left at the same spacing and the new value of the capacitance measured.

Standards for the measurement of loss factor in capacitors and dielectrics are usually air capacitors. Studies have shown that such apparatus may have a loss factor as large as 1\( \times \)10\(^{-4}\), and that if accuracies of 1\( \times \)10\(^{-8}\) are needed, a more precise standard is required. By combining

Figure 6. Modified Schering bridge and accessories for measuring dielectric constant and loss factor up to 200 kc/s.
data obtained with (a) a capacitor of constant spacing and adjustable area and (b) a capacitor of constant area and adjustable spacing, a valid determination of the true zero of power factor can be obtained [110, 111].

Capacitors have recently come into use for measuring the very small direct currents encountered in ionization measurements. Here very low absorption and leakage are essential qualities. The usual calibration procedure is to allow a known current to charge the capacitor for a measured time and to note the resulting voltage with an electrometer. The current is chosen so as to make the total charging time and the final voltage approximately the same as they will be in the future use of the capacitor.

3.2 Inductance

In the practical work of inductance measurement at the National Bureau of Standards, it usually has been found more expedient to derive the henry from the ohm and the farad than to use directly the values computed from the dimensions of the large computable inductors constructed for absolute measurements. These inductors are very bulky and hence have relatively large capacitance to ground and considerable electromagnetic coupling to other circuits. They are wound with thick wire (0.7 mm) and would show appreciable skin effect if used at high frequencies. However, they are very valuable for use occasionally to give an independent check on the accuracy of measurement of inductances in the Maxwell-Wien bridge.

The circuit regularly used to measure inductance is that of the Maxwell-Wien bridge [17, p. 113] shown in figure 7. The a-c source is connected across \( bd \) and the detector (a vibration galvanometer for 60 c/s, a telephone receiver with suitable amplifier for 1,000 c/s, or an electronic null detector) is connected across \( ca \). The adjustable capacitor, \( C \), may be either the group of air capacitors shown in figure 4, which has been calibrated directly by the Maxwell commutator bridge (A, figure 2) or a mica capacitor (B, figure 2) calibrated by the type-12 bridge at the particular frequency at which the inductor is to be tested, usually 60 c/s or 1,000 c/s. This procedure is convenient for inductors from 1 \( \mu \)h to 10 h, and gives results good to 0.01 percent over the middle of this range.

Mutual inductances for use with alternating current and of ranges up to about 50 mh are regularly measured by the Curtis method [17, p. 117] with the circuit as shown, including the dotted lines in figure 7. With switch \( S \) closed to the right, capacitor \( C \) is adjusted to a value \( C_L \), which is a measure of \( L_M \). Switch \( S \) is then thrown to the left and \( C \) readjusted to a new balance at \( C_M \). The mutual inductance is then computed from the relation

\[
M = (C_L - C_M) R_3 R_4 (R_3 + R_4)
\]

with further correction terms involving the residual inductances of the resistance arms.

For inductors of larger value, or at higher frequencies where the effects of distributed capacitance are larger, this method becomes impractical. In such cases, the primary and secondary windings are connected in series, first aiding and then opposing, and the self-inductance of each combination is measured with a Maxwell-Wien bridge. If the observed values are \( L_A \) and \( L_D \), respectively, and if \( L_p \) and \( L_s \) are the inductances of the primary and secondary windings measured separately,

\[
M = \frac{L_A - L_D}{4} = \frac{L_A - L_p - L_s}{2} = \frac{L_p + L_s - L_D}{2}
\]

Any inconsistencies among these equations give a valuable indication of the order of magnitude of the effects of stray capacitance between the coils. It has been found that the values by the Curtis method agree most closely with those computed as \( (L_A - L_D)/4 \).

An independent basis for the measurement of mutual inductance is the use of a computable standard (\( M = 6.6 \) mh) mutual inductor formed by two interwound single-layer helices wound in a double thread on the surface of a porcelain form. With the two windings connected in series, "opposing" measurements may be made with good accuracy (0.01%) up to 1 kc/s.
Mutual inductors of the types used to calibrate the ballistic galvanometers employed in measuring the d-c properties of ferromagnetic materials (see sect. 3.4) are tested ballistically. For this purpose such inductors are compared with the large computable mutual inductor that was constructed primarily for the absolute measurement of resistance by the Wenner method [47]. To do this, the primary windings of the standard and test inductor are connected in series. Their secondary windings are connected in opposition and with two resistance boxes also in the series circuit. A galvanometer is bridged across between the junction of the resistance boxes and the junction of the two secondary windings. The resistances are then adjusted until the galvanometer remains balanced as the primary current is reversed. For this condition, the mutual inductances are proportional to the total resistance of the two parts of the secondary circuit. This method avoids any complications that might enter an a-c measurement because of capacitance in the windings.

At low values of inductance, errors from the residual phase defects of the bridge arms may become appreciable. Hence, for the measurement of the residual inductances of nominally "non-inductive" two-terminal resistors [125, 126] it becomes essential to use a substitution method. In this a standard, the inductance of which can be computed and the resistance of which is nominally equal to that of the impedance under test, is substituted for the unknown in the X-arm of the bridge. Each of the computable standards for 10,000 and 1,000 ohms consists of a fine wire stretched down the axis of a brass tube about 5 cm in diameter. For 100- and 10-ohm standards, parallel wires are used. The time constants \((L/R)\) of these standards can be computed to \(1 \times 10^{-3}\) sec. For still lower resistances four-terminal standards are used, either of the parallel-wire, reflexed-strip, or coaxial-tube type [128, 129]. Some of the coaxial tube type have their potential leads inside the inner tube, whereas, others have them outside the outer tube. These four-terminal standards of inductance range in resistance from 0.1 to 0.0002 ohm and in current rating from 10 to 2,500 amp.

3.3 Current, Power, and Energy

The block diagram of figure 8 shows the derivation of the units of current, power, and energy from those of resistance, voltage, and time. An electric current (if one excepts the persistent current in a superconducting circuit) is in the nature of things, a transitory affair, and a "standard ampere" cannot be preserved on a laboratory shelf as can a standard ohm. For the precise measurement of direct current the normal procedure is to insert a known resistance, \(R\), in the circuit and to compare the potential drop produced in this resistance by the unknown current with the emf of a standard cell. This comparison is usually made with a potentiometer, but in rare cases, where extreme accuracy is needed, special resistors are used of such value that the IR drop at the desired current is very closely equal to the emf of the standard cell. Thus the circuit becomes very simple, and errors from contacts, thermal electromotive forces, and drift in an auxiliary potentiometer current are minimized or eliminated. With a potentiometer and suitable standard resistor, it is easily possible to measure a direct current much more accurately than with an ammeter. Thus indicating ammeters of the 1/10-, 1/100-, and 1/1000-percent-accuracy classes are readily tested, and the true values of current corresponding to a number of scale points are determined. Such ammeters can in turn be used to check ammeters of lower accuracy and also to check recording instruments.

The measurement of power in a d-c circuit, as for instance, in calibrating a calorimeter, can be made most accurately by using a potentiometer with an accessory shunt and volt box to measure the current and the voltage of the circuit separately and by then taking their product. The checking of a wattmeter is done in the same way, except that in this case separate storage batteries are used to supply the current and the voltage circuits, as shown in figure 9. With switch \(S_{ab}\) closed in either direction and with switch \(S_{ad}\)
Figure 9. Circuits for transfer testing a wattmeter.

With switch $S_{AD}$ closed to the left, both the test wattmeter and the standard transfer wattmeter are energized with direct currents from batteries $B_1$ and $B_2$. With switch $S_{AD}$ closed to the right, both wattmeters are energized with alternating currents from the two coupled generators, $G_1$ and $G_2$. Ammeters and voltmeters $A_1, A_2, V_1, V_2$ indicate approximate values of current and voltage for control and for setting power factor. Potentiometer gives more exact measures of d-c voltage and current for d-c calibrations. Switch $S_{em}$, by reversing direction of currents in wattmeter circuits permits measurement of current and voltage for control and for setting power factor. Potentiometer (with switch $S_{em}$ closed upward) is used to test the d-c standardization of the wattmeter.

The test of an a-c watthour meter is similar except that not only the magnitudes of the current and the voltage, but also the phase angle between them must be held constant. This is accomplished by using the special astatic electrodynamic transfer wattmeter described in section 4. This wattmeter is first standardized for a particular value of power using direct current and voltage, which are measured with a potentiometer. It is then connected with its current coil in series with the current circuit of the a-c watthour meter under test and with its voltage circuit in parallel with that of the watthour meter. The two circuits are then energized from separate transformers that are supplied from two coupled generators driven by a common motor. The current and voltage measured by an auxiliary ammeter and voltmeter are adjusted separately to the desired values by control of the generator fields, and the stator of one generator is then shifted in angular position until the desired relative phase of current and voltage is secured. This condition is indicated by the reading of the transfer wattmeter, which then gives the same reading as on the earlier d-c calibration. One observer then holds the transfer wattmeter at this constant reading by occasional slight readjustments of the field rheostats while a second observer counts and times the revolutions of the watthour meter as described above for d-c watthour meters. At the end of each “run” the d-c standardization of the wattmeter is repeated, and a small correction is applied if any drift in its calibration has occurred.

This procedure is much more elaborate and time-consuming than those used by power companies in their routine testing of large numbers of meters. Its use is justified (a) because the NBS normally receives for test only master standards of high quality, by means of which many other meters are standardized, and (b) because the relatively small number but considerable variety of meters submitted would not justify a large investment in special automatic test equipment.

In a project, currently active, it is planned to set up a group of standard a-c watthour meters that are to be standardized at long intervals by the method just described and used as working standards for the more convenient testing of other meters. No definite data are yet available on the practicability of this procedure.

### 3.4 Magnetic Units

The unit of magnetic flux, the weber, is derived from the henry and the ampere, the latter in turn being derived from the volt and the ohm. The
procedure is to change, by a known amount, the current in the primary winding of a mutual inductor and to note the resulting deflection of a ballistic galvanometer, which, together with the test coil that is to be used in later measurements, is connected to the secondary winding of the mutual inductor. The value of the mutual inductance in henries is obtained by comparing the working inductor with a large standard computable mutual inductor. This computable standard has an inductance of about 11 mh, and the working standard inductors range from about 2 to 50 mh. The value in amperes of the change in current is determined by a potentiometer, which compares the emf of a standard cell with the change in $IR$ drop in a known resistor. The change, $N\Delta \phi$, in flux linkages with the $N$ turns of the secondary winding of the working standard inductor, which results from the change, $\Delta I$, in the primary current is given simply by

$$N\Delta \phi = M\Delta I.$$  

For convenience, the primary current is merely reversed so that $\Delta I = 2I$. If $I$ is in amperes and $M$ is in henries, $N\Delta \phi$ is in webers. Normally the resistance of the complete secondary circuit (including all test coils) is adjusted so as to make the galvanometer scale direct reading [244].

The weber as thus established is used to measure the flux in test specimens in various types of permeameter. The test specimen in a permeameter is surrounded by a “B-coil,” of a known number of turns, which is connected in series with the secondary winding of the calibrating mutual inductor in the galvanometer circuit. A measured change in the magnetizing current in the permeameter produces a change in the flux linking the galvanometer circuit, and the resulting deflection measures the flux change.

A measurement of the cross-sectional dimensions of the test specimen then permits the computation of the change in magnetic induction, $\Delta B = \Delta \phi / A$. If $A$ is in square meters, $\Delta B$ is in webers/meter$^2$ (myriagausses). This result multiplied by $10^4$ gives the induction in gausses. Normally the area of the specimen and number of turns in the $B$-coil are lumped with the other factors in the calibration of the galvanometer, so that the induction is read directly.

Two distinct methods are used for establishing the unit of magnetizing force. In the older method, exemplified by the Burrows Permeameter [236, 249], the magnetomotive force is distributed around the magnetic circuit in proportion to the reluctance of the various portions so that all portions carry the same flux and are at substantially the same magnetic potential. If this condition is attained, the magnetizing force, $H$, at any section, such as the central portion of the test specimen, is directly computable from the current-turns per unit length at that section. At the surface of the specimen the tangential component of $H$ is, of course, the same inside the metal as outside. Hence, the value thus computed applies to the interior of the specimen also. To approximate the desired distribution of magnetizing forces, the currents in auxiliary coils near the joints and yokes as well as the current in the main magnetizing coils are adjusted until the flux in the central part of the specimen is equal to the average of the fluxes at two sections at the quarter points of its length. For high accuracy it is essential that the test specimen be very uniform throughout its length [237].

The second method for realizing the unit of magnetizing force, $H$, is to measure the tangential component of magnetic induction, $B$, in the air closely adjacent to a test specimen. This is numerically equal, in the centimeter-gram-second-electromagnetic system, to $H$ in the air, and by a fundamental theorem the tangential component of $H$ is continuous across the boundary and into the specimen. The measurement of $B$ in the air is preferably made by quickly rotating a pair of “flip coils” of known area and number of turns through 180°. The axes of these coils are parallel to the axis of the specimen at the beginning and end of the rotation. This method is used in the High-$H$ [241, 242] and the $MH$ permeameters [243]. The latter is shown in figure 10.

Because the value of $H$ may vary with distance out from the surface of the specimen, by reason of leakage flux from other parts of the circuit, it is necessary to apply a suitable correction to the value observed by the “flip coil” nearest the surface. The difference between the flux linked by this flip coil and the second located at a known greater distance from the surface of the specimen gives a basis for extrapolating to the value at the surface. Such multiple coils are provided in the High-$H$ and $MH$ permeameters. By applying such corrections and by careful calibration of the galvanometer, accuracies of 0.5 percent in $B$ or $H$ can be obtained. In ordinary testing practice, the accuracy is about 1 percent for either $B$ or $H$.

Alternatively a fixed $H$-coil, adjacent to the specimen may be used and the magnetizing force

![Figure 10. The MH permeameter.](image)

The bar specimen lies along the axis of the central coil. The four auxiliary coils supply magnetomotive force for the joints and yokes. The plunger of the iron-chal coil solenoid at the left “flips” the $H$-coils, which are located just below the specimen near its center.
deduced from the change in flux linkage in this coil when the magnetizing current of the permeameter is reversed. This is the normal procedure with the widely used Fahy Simplex Permeameter [248, 250, 252].

Here the $H$-coil extends between two blocks of soft iron clamped to the ends of the specimen and responds to the average value of the magnetic potential gradient between the blocks. For accurate work a correction factor must be applied to relate this to the true magnetizing force acting on the specimen. This factor is determined by the use of standard test bars that have been first calibrated in an absolute permeameter.

With permeameters of these types, measurements of the normal induction curve, hysteresis loops, and of residual induction and coercive force [244] are regularly made on sample bars submitted for test by the laboratories of steel companies and other organizations. The circulation of such bars insures uniformity and accuracy in this branch of industry. Also by such comparisons the performances of permeameters of other types have been studied in detail [246 to 252]. A closely related service is the calibration of search coils to determine magnetically their effective area-turns. This is done by placing the coil near the center of a long, uniformly wound, single-layer solenoid with the axis of the search coil parallel to that of the solenoid. The mutual inductance, $M$, between the coil and the solenoid winding is measured ballistically. The area-turns, $AN$, of the coil are then computed by the equation

$$AN = M I / B.$$

The ratio of the central flux density, $B$, to the current, $I$, is a constant that is computed from the measured dimensions and pitch of the solenoid winding.

Measurements of magnetic properties with alternating currents involve either power measurements with wattmeters or the use of a-c bridges or potentiometers. The maximum cyclic change in flux is measured by using a voltmeter of high resistance (400 ohms/$\gamma$) or more with a copper-oxide rectifier to indicate the average value of the induced voltage. The derivation of the corresponding units is given elsewhere.

Recent work at the National Bureau of Standards has made possible an alternative method for measuring with extremely high accuracy the magnetic induction in fields above 1,000 gauss. The studies of Bloch [253] and of Purcell and his colleagues [254] have shown that if a substance is placed in a strong fixed magnetic field of intensity $B_0$, on which is superposed a weak radio-frequency field, the material will show a resonance absorption at a particular frequency, $f_0$. This resonant frequency is given by $f_0 = \gamma B_0 / 2\pi$, where $\gamma$ is the gyromagnetic ratio of the atomic nuclei of the substance used. At the Bureau, Thomas, Driscoll, and Hippie [255] have performed this experiment with great care, using a material (water) rich in protons and have determined the gyromagnetic ratio of the proton to be

$$\gamma = (2.67528 \pm 0.00006) \times 10^4 \text{ sec}^{-1} \text{ gauss}^{-1}.$$

In this work the gauss was established by observing the mechanical force exerted by the field on the conductors that formed one side of a rectangular "force coil" hanging in the field and carrying a measured current. The current was measured in terms of a standard cell and standard resistor. The force was measured by balancing it against the weight of a known mass that could be added to or removed from the pan of the precision balance from which the force coil was suspended. The effective length of the conductors in the bottom side of the coil was measured by a special micrometer in comparison with a known end standard.

By using this value of $\gamma$ and a resonance probe similar to the one developed for this work, other experimenters can measure the induction in an unknown magnetic field by a simple determination of the resonant frequency in terms of the standard radio-frequency emissions from NBS radio station WWV [25]. Such a resonance detector can also be used (1) to study the distribution of flux density in a magnetic field and (2) with suitable additional circuits as a monitor to hold constant the induction at a given place in the field.

### 4. Transfer from D-C to A-C Measurements

A standard cell and the units of voltage and of current realized from it directly or in conjunction with a standard resistor are applicable to the measurement of d-c quantities only. On the other hand, by far the more frequent and important measurements in the power and communication fields involve alternating currents. The procedure used to transfer from the fundamental d-c standards to a-c measuring apparatus, as indicated in the middle of figure 8, therefore constitutes an essential link in the chain of electrical measurement.

Of the three fundamental methods for relating the root-mean-square value of the alternating current to an equivalent direct current, viz: electrodynamic, electrostatic, or thermal, the NBS normally uses an electrodynamic method for wattmeters for frequencies up to about 1,000 c/s. The same instrument can be used with a shunt as an ammeter up to 10 amp. Methods using thermal converters, which have been confirmed by comparisons with this instrument, are normally used in testing ammeters and voltmeters at frequencies up to 20 kc/s.\(^2\)

\(^2\) It may be noted that at the British National Physical Laboratory [178] an electrostatic method is used as fundamental. A very carefully constructed quadrant electrometer itself calibrated on direct current serves to calibrate in turn other voltmeters directly on alternating current. It is also used to hold a known alternating voltage at the terminals of a noninductive resistor, and a second highly precise quadrant electrometer connected so as to function as a wattmeter measures the power dissipation in the resistor. With the electrostatic wattmeter thus calibrated at unity power factor, it can be used to calibrate ordinary wattmeters at any desired power factor and frequency.
The basic electrodynamic transfer instrument for current and power at power frequencies [177] is shown in figure 11. It is an astatic electrodynamic wattmeter, the moving coils of which are carried by a taut suspension of phosphor-bronze strip. The fixed coils are rated at 2.5, 5, or 10 amp, according as they are connected in series, series-parallel, or parallel. They are supported in slabs of plate glass to which adjustable supports and a torsion head for the suspension are firmly attached. The angular position of the pair of movable coils is read by a mirror and scale at a radius of 2 m. The movable coils are used in the position where the mutual inductance between them and the fixed coils is substantially zero. The deflection is usually about 60°. A capacitor shunts part of the series resistor and thus compensates for the self-inductance of the movable coils. A change in deflection of 1 mm corresponds to 0.025 percent in the product of the power and (with the fixed coils in series) to a change of $10^{-6}$ (amp)$^2$ in the product of the currents in the fixed and moving coils.

Even though the electrodynamic transfer instrument was carefully designed to be as free as practicable from such sources of a-c error as eddy currents, capacitance, and inductance, a great many checks were made on its performance before it was put into commission as a transfer instrument. These included (1) a comparison up to 3,000 c/s when connected as an ammeter, with a Hartman and Braun expansion-type hot-wire ammeter, (2) a comparison at 60 c/s and full volt-amperes but zero power factor with a sensitive quadrant electrometer, and (3) the measurement at 500, 1,000, and 2,000 c/s of the loss in a 10-µf mica capacitor, the phase defect of which had been previously measured in an a-c bridge. These check tests showed the existence of two small departures from an ideal electrodynamic instrument. The first was a spurious torque due to eddy currents in the aluminum damping vane. This effect is less than 2 ppm at 60 c/s but increases as the square of the frequency and has to be corrected for at frequencies above 1,000 c/s. The second is an effective phase defect that increases as the first power of the frequency and at 60 c/s amounts to only 0.38 minute, i.e., to an error of 0.011 percent of the volt-amperes, even at zero power factor.

This transfer instrument is used principally in checking the accuracy of electrodynamic wattmeters of the 1/1000-percent or the 1/2000-percent class, which in turn are used as standard transfer instruments in other laboratories for the checking of other wattmeters or watthour meters using alternating current. In the usual test, the unknown and the standard transfer wattmeter are connected with their current circuits in series and their voltage circuits in parallel, as shown in figure 9. With switch $S_{ad}$ closed to the right, alternating current and voltage are applied from separate transformers, $T_2$ and $T_1$, and their relative phase is adjusted by shifting the stator of $G$ to give the desired power factor (usually 0.5). This can be calculated accurately enough from the readings of the wattmeter under test and of the auxiliary ammeter, $A_n$, and voltmeter, $V_n$. The series resistor of the standard instrument is reduced until the spot of light comes to a convenient position near the center of the scale, where the axes of the moving coils are at right angles to those of the fixed coils and the mutual inductance between the two circuits is zero. One observer then adjusts the current to bring the pointer of the instrument under test as exactly as possible on some scale mark, usually using either a projection equipment or a magnifying glass. The other observer reads as $A_1$ the position of the spot of light on the instrument scale. The four-pole double-throw switch, $S_{ad}$, is then closed to the left to substitute previously adjusted direct current and voltage in place of the a-c quantities. Current is again adjusted until the instrument under test reads the same, and the standard instrument reading, $D_1$, is recorded for this setting. Both current and voltage are then quickly reversed by switch $S_{mn}$ and a second d-c reading, $D_2$, is obtained. The original connections are then restored and a check a-c reading, $A_2$, is recorded. The average of $D_1$ and $D_2$ is free from any effect of the earth's magnetic field. The average of $A_1$ and $A_2$ corresponds to substantially the same time of observation as does the mean of $D_1$ and $D_2$. Hence any linear drifts like the effects of spring fatigue or of heating in either instrument cancel out, and their difference, when divided by the mean equivalent deflection ($\beta$), $B$.

$^3$ The standard transfer instrument is used with a uniform millimeter scale, the divisions of which do not exactly correspond with equal increments in the measured power over the whole range. Hence to obtain relative changes in power from the observed changes in deflection, it is convenient to divide the latter by an "effective deflection." $\beta$ is defined as $\beta = \frac{P(D_1/D_2)}{D_1}$, where $D_1/D_2$ is the slope of the graph of deflection, $D$, versus power, $P$, at the part of the scale used.
\[ A_1 + A_2 - D_1 - D_2 \]

is a measure of the real difference in the performance of the instrument under test on alternating current as compared with direct current. In practice three such sets of four readings each are taken in rapid succession, and with a wattmeter of good quality the mean difference thus obtained and divided by the effective deflection corresponding to the d-c reading will be found repeatable to \( \pm 0.02 \) percent.

The performance of many wattmeters on alternating-current can often be represented approximately by an equation of the form

\[ P_a = P_d (1 + a) + bEI \sin \theta, \tag{1} \]

where \( P_a \) is the reading on alternating current, and \( P_d \) is the reading on direct current for the same true power, \( E \) and \( I \) are voltage and current, and \( \cos \theta \) is the power factor. A transfer test first made at unity power factor (\( \sin \theta = 0 \)) gives a measure of \( a \), which is usually very small. This combined with a second test at \( \cos \theta = 0.5 \) (\( \sin \theta = 0.866 \)) gives a measure of \( b \).

If eq (1) represented completely the performance of a wattmeter, it would be more convenient to make the second test on alternating current at zero power factor only. This can be done by adjusting the phase difference until the wattmeter under test indicates zero with rated current and voltage and then noting the reading, \( P_a' \), of the standard instrument, after which the latter is calibrated using direct current. This procedure gives

\[ b = \frac{P_a'}{EI} \tag{2} \]

For such a test at zero power factor the torsion head of the transfer instrument is shifted so as to bring the zero of the instrument near the center of the scale.

However, wattmeters are frequently found not to obey eq (1). This condition is best revealed by a pair of tests at low power factor with the current first leading and then lagging by the same angle. If the formula applies, the changes in error when going from unity power factor to the other two conditions should be equal in amount and opposite in sign. Half of the algebraic difference in such a pair of relative ac-dc differences, when divided by \( \tan \theta \), is a good measure of the effective phase defect coefficient, \( b \), but half their algebraic sum is often quite different from the value of \( a \) derived from the test at unity power factor. The reasons for such departures of performance from eq (1) are obscure, but in part may be associated with small vibrations in the moving system.

The same standard electrodynamic instrument serves also as the basis for the transfer testing of ammeters at power frequencies. For this purpose it is used as a wattmeter to measure the rate of energy dissipation in a noninductive standard resistor that is in series with the ammeter under test. Readings are made with alternating and direct current alternately, as described above. It has been found more convenient, however, in the normal testing of a-c ammeters to use, as an intermediate standard, an ammeter of the composite-coil type [180]. The a-c performance of this instrument has been carefully compared with that of the fundamentally simpler astatic wattmeter, and it is regularly used on alternating current, often with a standardized current transformer, to check ammeters on alternating current.

The basic instrument for the transfer of voltage from direct current to alternating current at power frequencies is a suppressed-zero electrodynamic voltmeter [179] in which the movable system is carried by a taut suspension and its position is read with a telescope and scale at a scale distance, as now installed, of 3 m. A short boom attached to the moving system is free to play between two stops, only a few degrees apart and insures that the suspension is held at all times with substantially the same twist as that present when readings are made. As normally used in the transfer testing of voltimeters, the double optical lever and temperature compensating coils described in [179] have been found unnecessary. An adjustable noninductive resistance in series with the instrument is set at such a value as to bring the cross hair somewhere near the center of the scale when the operating voltage is applied. Sets of alternately d-c and a-c readings are then obtained as described above for wattmeter testing. A measurement of the instrument sensitivity is also made by applying two successive values of direct voltage and noting the change in deflection which results.
An alternative procedure for transferring from d-c to a-c standards is the use of electrothermic instruments. Such apparatus of high precision has recently been installed [182, 207] and is regularly used for testing voltmeters from 0.2 to 750 v and ammeters from 1 ma to 50 amp at frequencies up to 20,000 c/s. Figure 12 shows some of these instruments. In spite of the fundamental simplicity of this method, a number of possible sources of error exist such as Peltier temperature shifts in the thermal converter, losses in the insulating bead and resistance in common between heater and couple circuits. With some individual thermal converters, these effects may not be negligible when an accuracy of 0.01 percent is sought. As finally worked out, however, the electrothermic transfer for current and for voltage has been found to agree with the electrodynamic transfer to an accuracy of 0.01 percent at frequencies up to 500 c/s, and it is believed to be superior to the latter and to be accurate to 0.02 percent for higher frequencies up to 20 kc/s.

Still another unit, definable only in a-c circuits, is the var, the unit of reactive power. It is the reactive power corresponding to the maintenance of 1 amp in quadrature with a potential difference of 1 v. The test of a single-phase varmeter is normally made by supplying the instrument with sinusoidal current and voltage from two generators coupled to a common driving motor. The power factor is adjusted to zero by mechanically shifting the stator of one generator relative to the other until a wattmeter indicates zero. The reactive power in vars is then equal to the product of the voltage and current, as measured by a voltmeter and ammeter. A further check at unity power factor by connecting the varmeter at the terminals of a load of substantially nonreactive resistors, gives an indication of any phase defect in the varmeter circuit.

The calibration of polyphase varmeters and the use of varmeters with nonsinusoidal currents introduces theoretical complications that are, in general, more serious than the purely instrumental errors. The Bureau has endeavored to bring some order out of the earlier confused practices by cooperating in the formulation of standard definitions for the numerous concepts involved [205].

5. Extension of Measurement Ranges

The procedures and apparatus described in the foregoing sections have indicated how the various electric and magnetic units are derived from the primary volt and ohm. It is not enough, however, to be able to measure one ampere or one watt. Methods must be developed for extending the range of all electrical quantities to cover the magnitudes encountered in science and industry. Such methods commonly involve the use of bridge circuits having unequal ratio arms, multirange instruments having different values of series, or parallel, resistances, or other procedures that utilize resistances of widely different values. The establishment of the scale of resistance values is therefore of fundamental importance.

5.1 The Resistance Scale

The primary tool used at the National Bureau of Standards for the measurement of resistance is a precision bridge built in the Bureau shops in 1918, which incorporates many features developed as a result of the Bureau's first 15 years of experience in the field of precise measurements [79]. The more important resistance sections consist of hermetically sealed coils; the dial adjustments are of the Waidner-Wolff type, [75] made by changing the shunting around fixed resistors so that the effects of contact resistance and emf are greatly reduced; all resistors and contacts operate under the oil of a thermostatically controlled bath; and the circuit is shielded from stray currents. The bridge may be used either as a Wheatstone or as a Kelvin bridge, and is commonly used with a ratio that is nominally either 1:1 or 10:1. This ratio can be adjusted over a limited range (±0.5 percent) in the vicinity of the nominal value employed. The adjustment is made by means of four decade dials, one step on the lowest-dial corresponding to a shift of 1 ppm in ratio. The step that is fundamental to extending the resistance scale is the establishment of an accurate 10:1 ratio between ratio arms of this bridge. This is done by using a special assemblage of resistors [79] (figure 13). Six of these resistors are nominally of 150 ohms each, and the seventh is of 50 ohms. By successive substitution in the precision bridge, the relative values of the six resistors can be measured. Also by substitution the values of the resistances of nominally 50 ohms each, formed by connecting in parallel each of the two sets of three 150-ohm units, is determined relative to the 50-ohm resistor with high accuracy. The three 150-ohm units of one set are then connected in series with each other and with the 50-ohm unit to form a 500-ohm group. The three 150-ohm units of the other set are then connected in parallel to form a 50-ohm group.

**Figure 13. Circuit for establishing 10:1 ratio.**

With links joining A to C and B to D, the tap at g', ratio is 1:10. With links joining K to M and L to N, and tap at g, ratio is 10:1.
These two groups, having a ratio close to 10:1 are connected to form two arms of the high-precision bridge, the remaining two arms being the adjustable ratios referred to above. The bridge is then balanced by manipulating the four dials and their setting, \( N_1 \), is noted. The paralleling links are then shifted so that the 150-ohm coils of the first set are put in parallel and form a 50-ohm group, and those of the second set are in series, and, with the central 50-ohm coil, form a 500-ohm group. These groups are turned end for end and again connected in the bridge, and a second dial setting, \( N_2 \), at balance is noted. The mean of the two observed ratio settings, \( N_1 \) and \( N_2 \), is then adjusted by applying a small correction derived from the observed differences between the 50-ohm coil and the means of the groups of three in parallel. A further correction for the resistance of the paralleling links is applied, and after these several corrections are taken into account, the setting of the bridge dials for which the ratio is exactly 10:1 is known within about 1 part in 3 million.

An alternative, but more laborious, procedure sometimes used for obtaining an accurate 10:1 ratio is to use a group of six 100-ohm standard resistors. Each in turn is used as one arm of a bridge; the other five in series constitute an adjacent arm. The other two arms are the adjustable 100-ohm arm of the precision bridge and a fixed 20-ohm arm. The average of the six successive settings on the adjustable arm then corresponds to an exact 100:20 ratio to an order of accuracy given by the sum of the squares of the departures of the individual 100-ohm standards from their mean, provided proper corrections are applied for the resistances of the contacts and connectors. This ratio is then used to compare each of the two 50-ohm sections of a divided 100-ohm standard resistor with a 10-ohm standard, and from these two results the ratio of the divided 100-ohm to the 10-ohm is computed. The results obtained by this procedure are usually found to agree with those by the first-mentioned procedure to a few parts in 10^7.

By using the bridge, thus calibrated, it is easy by successive steps to measure the values of wire-wound standard resistors of 10, 100, 1,000, or 10,000 ohms, and similarly to step down to 0.1-, 0.01-, 0.001-, and 0.0001-ohm standards. Resistances of intermediate value are occasionally measured by using analogous series or parallel combinations of nominally equal resistors to evaluate other bridge ratios that involve small integers.

In contrast to the foregoing methods for testing individual standard resistors while immersed in a thermostated oil bath, the procedure normally used for calibrating a decade dial of a bridge arm or resistance box is to substitute a known standard resistor successively for each of the steps of the dial [80]. This is done, as shown in figure 14, by connecting in series (1) a resistance box that has the dial \( X \) to be tested, (2) an auxiliary box that has a dial \( A \) of the same nominal value as \( X \), and (3) a pair of mercury cups, \( C \), into which may be placed either (a) a short-circuiting link, \( L \), or (b) a known standard resistor, \( S \), of the same nominal value as one step of dial \( X \) or \( A \). The series combination is connected to form one arm of a Wheatstone bridge, the other arms of which are formed by a direct-reading ratio set \( A-B \) and a fixed resistor, \( R \). The diodes are so set at each stage in the process that the total resistance of the arm is nominally 10 times the resistance of one dial step. Successive pairs of bridge balances are then made. In the first balance of each pair the standard resistor is in the circuit. In the second balance it is replaced by the link, and the \( X \) dial is at a setting one step higher than during the first balance. This constitutes in effect a substitution of the dial step for the standard resistor, and their difference is read from the small change required in the balancing arm of the bridge. The auxiliary dial is then shifted one step, and a second pair of similar balances gives a calibration for the next step of \( X \). Incidentally, the data thus obtained suffice for the calibration of the auxiliary dial also.

When making precise measurements with resistors of 1 ohm or less, it is essential to use resistors of the four-terminal type to eliminate the effects of contact resistance. The Kelvin bridge or some equivalent circuit has proved to be in general the most suitable for the comparison of such resistors [76]. Of the various procedures [77] that can be used to balance out the effects of

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Figure 14. Circuit for calibrating resistance decades.
the resistance in the potential leads, that which involves applying a short-circuit across the ends of the main ratio arms is commonly used.

The Kelvin double bridge is usually employed at the NBS in all measurements of low resistance, but remarkably good results can often be obtained with simpler apparatus, provided appropriate methods and precautions are used [78].

At the other extreme the measurement of high resistance merges into the measurement of the resistivity of insulating materials [81]. High-grade wire-wound resistance elements are available up to 1,000,000 ohms. Groups of such 1-megohm resistors may be measured in a high-grade Wheatstone bridge while connected in parallel and can be used singly or in series to form a 10-megohm standard. Units of higher individual value consisting of heterogenous compositions or of very thin film are likely to show changes in resistance with changes in temperature, voltage, and, unless very carefully protected, in ambient humidity. Great precision in their measurement is therefore not required, and a simple direct deflection method using a shunted galvanometer of high current sensitivity (10¹⁰ mm/μA) is often satisfactory.

If the voltage is limited to a few volts, this procedure fails at resistances exceeding 10⁹ ohms. For higher resistances the charge carried by the test resistor is accumulated on one electrode of an adjustable air capacitor. The rate of accumulation is measured by either changing the capacitance or the potential applied to the other electrode while using an electrometer as a null indicator. When higher voltages can be used, correspondingly higher resistances can be measured [82, 86]. An accuracy of 0.2 percent can be obtained up to 10¹³ ohms at 1.5 v.

5.2 The Scale of Direct Voltage

The starting point of the voltage scale is of course the primary group of saturated cadmium standard cells [61], which maintain the volt notive force of the unknown cell directly. One in electromotive force between the cell under test and a standard reference cell. A simple automatic computing device incorporated in the apparatus enables the observer to read the electromotive force of the unknown cell directly. One division on the instrument corresponds to 1 μv. Great pains were taken in the design of this comparator to minimize spurious thermoelectric effects, which usually constitute the most serious source of error in this type of measurement. It is used regularly in testing the unsaturated standard cells that are sent to the Bureau periodically for check from a large number of scientific and industrial laboratories, as well as for the comparison of saturated cells.

When an unsaturated standard cell is received at the Bureau for test it is placed with others in a large box that is thermally insulated, but not thermostatically controlled, and its electromotive force is measured daily for at least 10 days. If, as is the usual case, the emf soon settles down and remains within ±50 μv of a steady value, a certificate is issued giving this mean value to the nearest 10 μv. However, if the fluctuations continue to exceed 0.01 percent or if the cell has an abnormally low value, below 1.0183 v, or if it shows an abnormally high internal resistance, no formal "certificate" is issued, but instead the data observed are sent to the customer in the form of a "report." If properly used, unsaturated cadmium standard cells can constitute very accurate and reliable standards of voltage. They are, however, easily damaged by excessive current drain, and their electromotive force may be temporarily affected by sudden changes in temperature and also by any temperature difference that may exist between the two electrodes [52]. Modern cell containers are made with copper liners to minimize this latter effect.

The extension of the d-c voltage scale to lower values is usually made by the use of a potentiometer. For the normal precision testing of low-range voltimeters and millivoltmeters at the Bureau a well-seasoned null potentiometer of the Feussner type is used. The lowest dial has steps of 10 μv. For more rapid and less exacting calibrations a Brooks deflection potentiometer [96 to 99], developed at the Bureau primarily for the rapid calibration testing of electric indicating instruments, and for incandescent lamp testing, is used. It is, however, not suited for measurement of emf in circuits of high internal resistance. In this apparatus the bulk of the voltage to be measured is balanced by the IR drop in the resistance of the main dial. Any unbalanced residue causes current to flow in the calibrated galvanometer and produce a corresponding deflection. The value of the total voltage is then indicated directly by the one (or two) digits on the dial followed by three digits read from the galvanometer scale, the last digit corresponding to 0.1 scale division.

A number of other types of potentiometer have been developed at the Bureau for special purposes. The Wenner type [94, 95] is quite generally used for precise temperature measurement with thermocouples. On its lower range, one step on the lowest dial corresponds to 0.1 μv. A special multirange potentiometer [90] has proved convenient for measuring small temperature differences. For the measurement of very small intensities of radiation by a physical photometer,
another special potentiometer [93] reading to 0.001 \( \mu \text{V} \) is used.

Potentiometers, in effect, measure merely the ratio of the unknown electromotive force to that of the standard cell. Their calibration therefore needs involve no direct measurement of resistance, but merely the determination of ratios of resistances. Hence, for the testing of potentiometers [87] and also for other applications, it has been found convenient to use a universal ratio set, which is equivalent to a slide wire having a total resistance of 2111.1 ohms, with a tap adjustable in steps of 0.01 ohm.

In the measurements of voltages of extremely low values, as well as in the highly precise measurement of resistance, one limiting factor may be the sensitivity of the galvanometer. This was recognized early, and considerable effort has been devoted to theoretical studies and practical designs of galvanometers for both direct [168, 169, 92] and alternating [163, 164, 165, 166] current. The galvanometers used at the Bureau for precision resistance measurements are of a special Bureau design, with taut suspensions and with the center of gravity slightly out of line with the suspensions. With this construction, it is possible to adjust the effective restoring torque by properly tilting the instrument. With a 10-second period, these have a sensitivity of 40 mm/\( \mu \text{V} \). In most of the other electrical measurements, commercial galvanometers of appropriate design are used. The more complex amplifying combinations, which can push the sensitivity to the limit set by Brownian motion (0.001 \( \mu \text{V} \) for a 10-ohm circuit and 2-second period), are seldom needed. The Coblentz [167] moving-magnet galvanometer has been superseded by moving-coil galvanometers, as they are not similarly affected by external magnetic fields.

The extension of the scale of direct voltage to higher values while still using the potentiometer is made by using instrument voltage dividers (more commonly called “volt boxes”) [100]. Such a divider consists of two resistors, \( R_a \) and \( R_b \), which are connected in series, across the voltage to be measured. A potentiometer is connected to measure the voltage, \( E_b \), at the terminals of \( R_b \), and the total voltage, \( E_{a+b} \), is then calculated by the relation

\[
E_{a+b} = E_b \frac{R_a + R_b}{R_b}.
\]

In principle the ratio of resistances might be computed from measurements of \( R_a \) and of \( R_b \) separately, such measurements being based on the 10:1 resistance ratio described in section 5.1. This method is not desirable, however, because of the two principal sources of error that arise in the use of volt boxes: (1) the possibility that at the high values of voltage applied and hence larger values of current in \( R_b \), it may be so heated that its value becomes significantly different from that observed in a resistance-bridge measurement at smaller current, and (2) the possibility that leakage currents in the insulating supports may bypass part or all of \( R_b \) but flow through \( R_a \), thus invalidating the assumption implicit in eq (3) that the same current exists throughout the entire series circuit. To minimize such errors, the extension of the voltage scale at the Bureau is made by using an especially constructed standard volt box rated for 750 \( \text{V} \), but satisfactory for use as high as 1,500 \( \text{V} \) [100].

In the design of this apparatus (1) the resistance has been chosen so high (333 1/3 ohms/\( \text{V} \)) that heating effects are negligible, and (2) guard plates have been provided to intercept any leakage paths over the insulating supports and bushings. The guard plates are connected to taps on an auxiliary guard circuit so that the potential difference across any insulator is never more than 75 \( \text{V} \), and at most bushings is nominally zero.

The measuring circuit is composed of groups of five coils of nominally equal resistances alternating with single coils, each having a resistance nominally equal to the total resistance between it and the grounded end of the circuit. To determine the ratio of the divider with high accuracy, it is necessary merely to substitute its successive equal coils or groups of coils in one arm of a sensitive and stable, but uncalibrated, Wheatstone bridge. The relative values of the coils are noted, and the value of the total resistance to any tap point is computed from these relative values, and an assumed nominal value for the lowest step, on the assumption that the resistances are truly additive. In taking the ratio of any total resistance to the resistance up to a tap point, the assumed nominal value cancels out leaving only the ratio. With reasonable care, an accuracy of 0.005 percent can be obtained.

In use the volt box under test is connected in parallel with the standard volt box, using on the latter a tap giving the same nominal ratio. The difference in voltage, if any exists, between the tap points of the two dividers is measured with a small potentiometer of the Lindeck type. The ratio of this difference to the total applied voltage, measured with a voltmeter, gives the ratio error of the volt box under test relative to the standard volt box. Corrections for lead resistance, etc., are derived by auxiliary measurements with the potentiometer. The equipment used for such tests is shown in figure 15.

In the measurement of the higher direct voltages used in X-ray generators a new source of error enters the picture. This is the corona discharge from points and edges that are at a considerable potential above their surroundings. This can result in a significant current leakage to ground directly through the air and can also cause trouble from the chemical action of the ozone and oxides of nitrogen formed and from the electrostatic precipitation of dust. To obviate these troubles, a construction has been developed at the Bureau [102] in which the resistance, \( R_a \), is subdivided into units of \( 5 \times 10^6 \) ohms. Each is surrounded with a
metal shield formed with surfaces of such large radius of curvature that no external corona develops. Each shield is connected to one end of the group of five 1-megohm resistors that it surrounds, and the voltage of 7.500 between resistor and case is so low that no corona occurs inside the shield. A divider of this construction, having 20 units of 5 megohms each, serves to measure direct voltages up to 150 kv with an accuracy of 0.01 percent.

For measurements in connection with X-rays of still higher voltage, a voltage divider has been constructed for 1,400 kilovolts. It is contained in the left-hand column shown in figure 16. It consists of 10 sections, each enclosed in an outer shield which is connected to an appropriate tap on the cascade transformer-rectifier set of 10 units, which produces the high direct voltage. Within each of these 10 sections the divider consists of 14 subsections, each of 10 megohms. Each subsection is enclosed by an inner corona shield, which is connected to one end of the subsection, but insulated from the outer shield. The normal current is 1 ma. If this current is measured by a potentiometer, and if corrections for self-heating are applied, the total voltage can be determined with an accuracy of about \( \frac{1}{20} \) percent.

5.3 The Scale of Direct Current

The measurement of a current by measuring with a potentiometer, the voltage drop in a resistor in which the current is flowing suffices to cover a very wide range of direct currents. At the low current end of the scale the accuracy begins to be limited by the current sensitivity of the galvanometer when the resistance of the resistor materially exceeds that of the potentiometer and galvanometer. For a galvanometer sensitivity of 1 mm for \( 10^{-8} \) amp 1\( \mu \) could be measured in this way to 1 percent. To calibrate instruments at lower currents it is necessary only to connect a large resistance, \( R_1 \), in series with the instrument and to shunt the combination with a smaller resistance, \( R_2 \). The total current can then be measured as before, and the range of possible calibration is thus extended by the ratio of \( R_1 \) to \( R_2 \). This procedure, however, is not applicable to the measurement of an unknown small current, and this latter problem is identical with that of measuring very high resistances, already discussed. The techniques used for this are equivalent to the measurement of currents down to \( 10^{-15} \) amp.

At the high-current end of the scale the limiting factors are the heating and thermoelectric effects in the series resistor. At the National Bureau of Standards a Wolff precision resistor of 0.0001 ohm is available for measurements at high currents. This is immersed in stirred oil, which in turn is cooled by a coil of tubing through which water is circulated. At currents up to 1,000 amp the resistance of this resistor is a definite and repeatable function of the cooling-oil temperature.
The function has the usual parabolic form characteristic of manganin, with a maximum resistance at 32° C. At higher currents, even though the temperature is held at the same value by vigorous cooling, the resistor shows an increase in resistance. This effect amounts to 100 ppm at 3,000 amp, and if proportional to the square of the current, would be 0.1 percent at 10,000 amp. It seems probable that the effect is the result of mechanical strain set up in the resistance alloy sheets by their differential expansion relative to the cooler rigid copper terminal blocks. Pending the construction of an improved high-current resistor free from such effects, the upper end of the precise scale of direct currents must be considered as tapering off from an accuracy of 0.01 percent at 3,000 amp to 0.1 percent at 10,000 amp. It appears that there would be no serious difficulty in constructing a high-current resistor that could extend the high-accuracy limit well above 10,000 amp, if a need for such measurements should arise. The capacity of the storage batteries presently available at the National Bureau of Standards permits a current to be held at this latter value for about ½ hour. Figure 17 shows this high-current circuit.

5.4 The Scale of Alternating Current

The electrothermic transfer ammeters offer a convenient means for measuring with high accuracy alternating currents down to 1 ma at frequencies up to 20 kc/s. Lower values are readily derived by using a resistive shunting network, the ratio of which is deduced from the resistance of its parts. With such networks very low currents can be produced and used to calibrate a-c amplifiers. The amplifiers in turn can be used to measure unknown currents of low value. The principal limitations in such work arise from superposed noise and instability in amplifier gain rather than from any difficulty in correlating the results with the ampere.

In the early years of the Bureau the scale of alternating current was extended upward by using a group of astatic electrodynamic instruments [175] having suspended moving coils and fixed coils of cable formed of insulated strands of fine wire (litzendraht). The highest rated (5,000 amp) instrument was a “tubular” electrodynamometer [176] in which the fixed “coil” consisted of two coaxial copper tubes. With the coming into general use of current transformers [227], the number of high-range, self-contained ammeters, wattmeters, and watthour-meters submitted for test has fallen to almost zero. The old high-range standard instruments have been discarded, and when occasionally a high-range instrument is tested, the measurements are made with the 5-amp standard transfer instrument and a carefully calibrated current transformer.

The ratio and phase angle of a current transformer depend to a very appreciable extent on the conditions of burden and frequency at which it is operated and, to a much less extent, on the arrangement of the primary leads. The extension of accurate measurements to large alternating cur-

Figure 17. Equipment for large direct currents.

Large switches connect cell groups in series or parallel. The water-cooled rheostat in background and fine rheostat in front adjust current. Standard resistors are in cylindrical oil tanks. Black transite box at left is used for tests at elevated temperatures.
For primary currents exceeding 2,500 amp use is made of a general principle in instrument transformer design, namely, if the primary winding of an instrument transformer consists of two or more sections, all having the same number of turns, then at the same frequency, secondary burden, and secondary current (or voltage) the ratio correction factor (i.e., the quotient of true ratio divided by nominal ratio) and the phase angle will be the same, to a high degree of precision, whether the primary sections are connected in series or in parallel. In other words, the core and secondary winding cannot distinguish between the additional magnetomotive force caused by the same current linking the core in an additional series turn and that caused by additional current linking the core in a parallel turn. Deviations from this principle will arise, for a current transformer, only in proportion to the product of two factors each of which is unlikely to be large: (a) the relative difference in the distribution of current among the several primary sections when they are in parallel, and (b) the relative difference in the coupling (mutual inductance) between each of the several primary sections and the secondary winding. If their relative differences do not exceed 1 percent each, the principle will be valid to 0.01 percent.

To apply this principle, a special high-range standard current transformer having eight primary ranges has been constructed. This has 24 sections of primary winding, and with these in series, can be calibrated by the usual resistance method at ratings from 500 to 2,500 amp. It is then used with its primary sections in parallel as a standard up to 12,000 amp. A careful study [224] of its performance has shown that the equality of the coupling of the various sections of the secondary and the equality of the division of current among the primary sections are both satisfactorily close. When the transformer is measured with only 500 secondary turns (i.e., 2,500 amp turns at full current) and with each of the eight possible connections of its primary turns, its ratio correction factor at any secondary current was constant to 0.02 percent and its phase angle to 1 minute, and when operating at its normal 12,000 amp turns, the performance of the transformer is presumably even better. Even if the flux distribution is greatly distorted by omitting one of the primary turns, the ratio correction factor is affected by less than 0.01 percent.

It seems probable that serious difficulty will be encountered in an attempt to construct non-inductive primary resistors for sustained currents exceeding 2,500 amp because the lack of chemical uniformity in the resistance alloy will affect the distribution of current and thus the inductance. However, there is every indication that the measurement range at power frequencies can be extended well above 12,000 amp by the use of multirange standard transformers.

Figure 18. Circuit for testing a current transformer by the resistance method.
Figure 19. Equipment for testing high-range current transformers.

Current is supplied by the step-down transformer, $K$, and measured either by the standard multirange transformer $S$ or by using tubular standard resistors in the oil tank $M$. The transformer under test is at $O$ in the center of the "cage" at the right. The heavy copper tubes comprise the outer conductors of the coaxial secondary leads.

Figure 19 shows the circuit used in measurements from 500 to 12,000 amp. The transformer under test is placed at the right in the "cage" formed by the four 10- by ½-inch copper bars, which, in parallel, constitute the "return" lead of the circuit. The central "outgoing" lead either passes through the window of the transformer or is bolted to the transformer primary terminals if it is of the bar type. This circuit geometry [187] offers two advantages. The first is that the components of magnetic field at the transformer in the center, produced by the action of the four return bars cancel one another so that the resultant effect is closely the same as if the return circuit were at infinity. This situation constitutes a logical basis with which the effects of other configurations can be compared [225, 226]. The second advantage is that the resultant magnetic field of the entire circuit drops off very rapidly with distance from the axis (roughly as $r^{-2}$) and is negligible in those parts of the laboratory where the measuring circuits are located.

Most of the transformers submitted for test are intended to be used as standards with which to compare the performance of other current transformers. Such comparisons are usually made by means of the Silsbee current transformer testing set, the circuit of which was developed at the Bureau in 1918 [216]. In the use of this set the measuring elements are required to measure merely the difference in magnitude and phase of the secondary currents of the two transformers being compared. Therefore, a high accuracy can be obtained in their relative ratios, although only moderate accuracy is required in the adjustment of the circuit constants. Nevertheless, as a check on possible incorrect adjustment at the factory or on major drifts in calibration, such transformer test sets are frequently submitted to the National Bureau of Standards for test. The test consists in the measurement of the resistance of the slide wire at 13 points on the ratio scale and of the mutual inductance of the inductor at 15 points on the phase-angle scale. The test set is also experimentally checked in actual operation at midscale and one other point. The true values of transformer ratio and phase angle are then computed and tabulated against the corresponding settings used in the test.

5.5. The Scale of Alternating Voltage

Between 1 v and 300 v, two alternative bases are available for the measurement of alternating voltages. The first, applicable at power frequencies, is to use the electrodynamic transfer voltmeter, described in section 4, to transfer from direct to alternating voltage at the 100- or 200-v level, and then to use a three-winding step-down transformer. The ratios of the voltages of the various sections of the two secondary windings to each other are checked by comparison with a
special resistive voltage divider. Normally, one winding supplies the standard voltage transformer, already calibrated with direct voltage, and the other tapped secondary supplies the a-c voltmeter under test. Such a transformer has a very constant ratio over a wide range of primary voltages.

The second procedure, applicable also at audio as well as at power frequencies, is to use the electrothermic transfer voltimeters directly to fill the scale from 1 to 750 v.

Below 1 v, known voltages can be obtained by using resistive voltage dividers or attenuators that have been checked at somewhat higher voltages against the standard volt box or by measuring the resistances of their component parts. The low voltages thus derived, down to about 20 µv, can be used to check low-range electronic voltimeters and as standard signals for testing amplifier gain. To measure an unknown voltage in this low range, the known low voltage with properly adjusted phase, can be bucked against the unknown. The difference, after amplification, is applied to a detector, and the known low voltage is adjusted to balance the unknown.

From about 300 up to 250,000 v almost all precise a-c voltage measurements make use of step-down voltage (potential) transformers [227]. The Bureau has contributed to the establishment of this alternating-voltage scale by developing methods and equipment for measuring the ratio and phase angle of voltage transformers, by suggesting methods for the comparison of the performance of a voltage transformer with a standard transformer of the same nominal rating [214, 215], and by its testing service for calibrating such standard transformers and comparison test sets [229].

The use of standard voltage transformers as a basis for the checking of others of the same nominal ratio is even more desirable than the analogous use of standard current transformers both (1) because very simple apparatus suffices for the comparison [214, 228] and (2) because of the greater safety when the operator is isolated from the high-voltage circuit by the transformer insulation and need handle only circuits connected to the low-voltage secondary windings.

It is almost universal practice to use voltage transformers with one end of the primary winding at ground potential. However, some unpublished experimental studies at the Bureau have indicated that, with transformers of normal construction, the ratio and phase angle are affected very little, even if the transformer is used with its primary across the lines of a three-phase, grounded neutral system while its secondary is kept grounded.

It is common practice in the electric-power industry to use portable commercial voltage-transformer test sets [228] to compare the secondary voltages of the standard and test transformers. These sets in turn are often submitted to the National Bureau of Standards for test to verify the accuracy of their adjustment. This is done by supplying both the set under test and a special voltage transformer comparator from a pair of 60-cycle voltages (a-c and d-c) which differ in magnitude and phase by adjustable amounts (see fig. 20). The set under test is set on a succession of 11 points on its ratio scale and 13 points on its phase-angle scale, and also as a check at the four “corner” points corresponding to combinations of the extreme settings of the two scales. The true voltage relations are read on the comparator for each point, and the resulting corrections are reported in tabular form.

The comparator is also used to compare transformers submitted for test with one of the standard transformers owned by the Bureau. These seven standard transformers have the ratings shown in table 1. Their ratios and phase angles are determined with great care at intervals of about a year by the methods outlined in the following paragraphs, and have been found to be very stable.

By using them as intermediate working standards, the need for operations involving direct metallic connections between the observer and the high-voltage circuit is limited to the annual performance checks.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Primary volts</th>
<th>Secondary volts</th>
<th>Kilovolt-ampere rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A, 2B</td>
<td>5000/2500/1250</td>
<td>255/127.5</td>
<td>2</td>
</tr>
<tr>
<td>3A, 3B</td>
<td>25,000/12,500/6,250</td>
<td>255/127.5</td>
<td>3</td>
</tr>
<tr>
<td>10A, 10B</td>
<td>100,000/50,000/25,000</td>
<td>255/127.5</td>
<td>4</td>
</tr>
<tr>
<td>100</td>
<td>250,000/125,000/62,500</td>
<td>255/127.5</td>
<td>5</td>
</tr>
</tbody>
</table>

Up to 30 kv the standard transformers are measured by the use of a shielded resistor [220] that has a total resistance in its working circuit of 512,000 ohms. All, or part, of this may be connected in parallel with the primary of the transformer under test, as shown in figure 21. The
secondary voltage of the transformer is then balanced by the potential drop in an adjustable portion, $R_g$, of the resistor circuit at its grounded end. A mutual inductor, $M$, which has its primary in series with the resistor and its secondary in series with the detector, serves to balance out any phase displacement between primary and secondary voltages. The principal source of error in such a method is the capacitance to ground of the various sections of the resistor. To minimize this error, the working circuit is divided into sections of 20,000 ohms each, and each section is enclosed in a metal shield. Each shield is connected to an appropriate tap point on an auxiliary high-resistance guard circuit, $R_g$. This guard circuit, also of about 500,000 ohms, is in parallel with the working circuit, and its taps are adjusted to be at the same potentials as the midpoints of the sections of the working circuit they shield. The capacitance currents from the shields to ground therefore flow in the guard circuit only and produce only a second-order error in the working circuit. With such an arrangement the second-order error increases roughly as the fourth power of the primary voltage, with the result that the method becomes rather impractical above 30 kV.

Above 30 kV several procedures are possible. One is the use of a capacitance voltage divider as used by Sharp and Crawford [222] and in more convenient form, by Bousman and Ten Broeck [223]. Such a circuit is now under development at the National Bureau of Standards as a further check on the performance of the standard multirange voltage transformers that are currently in use to carry the testing service to 250 kV.

These multirange voltage transformers constitute another application of the general principle enunciated earlier, which forms the basis for extending the range of alternating current. Each is made with its high-voltage winding in several sections that can be connected in parallel for calibration and in series for use as a high-voltage standard transformer. Ideally, such a transformer would have the same phase angle on all ranges, and its ratio would be strictly proportional to the number of turns in series on each range. Departures from this ideal might result either: (1) from inequalities in the magnetic coupling from the several primary sections to the secondary or (2) from the presence of capacitance currents flowing between portions of the primary circuit in a different manner when different connections are used. The applicability of this principle has been demonstrated in a large number of multirange transformers in which both the higher and the lower ranges could be tested by the use of the 30-kv shielded resistor [193, p. 327]. Auxiliary measurements of equality of sections and of capacitance between sections indicate that the departures from the ideal are negligible in the two higher-range transformers used in the extension of the Bureau's scale of alternating voltage. The first of these transformers is calibrated with its four sections in parallel up to 30 kV and used with them in series up to 120 kV. The second is compared with the first up to 120 kV and used with its two sections in series up to 240 kV. Figure 22 shows some of the standard transformers and the shielded resistor.

The chain of measurement of voltage from the electromotive force of a standard cell to an alternating voltage of 240,000 is a long one of many links, and it was felt desirable to check the result by some independent method. For this purpose

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* By using an autotransformer of high rating (75 kVA) to supply the shields Weller [221] found it possible to push this method to 132 kV.
an absolute electrometer of the attracted-disk type suitable for voltages up to 275 kv was designed and constructed [192]. In this instrument a light Duralumin disk (16 cm in diameter) hangs from one arm of a sensitive balance and is centered with small clearance in an opening in the center of a large guard ring. A flat circular plate is supported below the disk and guard ring, parallel to them and at a distance that can be adjusted over a range from 2 cm for low voltages (10 kv) to a maximum of 110 cm for 275 kv. The rms value of the voltage applied between the disk and the lower plate can be computed in electrostatic units from the measured diameter of the disk, the spacing between it and the plate, and the force of attraction, which is weighed by the balance. Figure 23 shows the general arrangement of the electrometer, and figure 24 shows some details at the balance.

A long series of experiments [193] was performed in 1936 in which an alternating voltage (usually of 60 c/s) was measured simultaneously with the electrometer and with a sensitive electromagnetic voltmeter fed by one of the calibrated standard voltage transformers. The conditions were varied from one experiment to another by changes that involved different transformer connections, electrometer spacings and disk diameters, frequencies (60 c/s and 25 c/s), wave forms and magnitudes (10,000 to 100,000 v) of the voltage measured, and changes in the potentiometer circuit used to calibrate the voltmeter. The two methods were found to agree with an average discrepancy without regard to sign of only 0.01 percent. The correctness of the extension of the a-c voltage scale by the use of voltage transformers is thus confirmed by an entirely independent method. Because of the much greater convenience of the transformer method, it will be used almost exclusively in the future. It is planned, however, to reassemble the absolute electrometer again and to make a similar cross check between the two methods at the 275-kv level.

The interrelations of the two methods in terms of the present absolute systems of units is shown in figure 25. It will be seen that both methods use a mass subject to local gravity as the measure of force (at F) and that the electrometer and the current balance occupy corresponding positions, each involving only the ratios of its significant dimensions.

In the electrostatic method, the conventional constant, \( \varepsilon_0 \), usually designated as “the permittivity of space,” is arbitrarily chosen as 1 (in the centimeter-gram-second-electrostatic system). On the other hand, in the electromagnetic method, it is the conventional constant, \( \mu_0 \), usually designated as “the permeability of space,” that is chosen arbitrarily either as 1 (in the cgs-emu system) or as \( 4\pi \times 10^{-7} \) (in the rationalized mksa system).

The absolute ohm is evaluated by a separate process, from the units of length and time and the chosen value of \( \mu_0 \). Combining the absolute ohm with the absolute ampere, evaluated by the current balance, fixes the absolute volt, in terms of which values are assigned to NBS standard cells. The long chain of step-up and d-c to a-c transfer procedures then yields the electromagnetic value for a high alternating voltage shown at A. The electrometer yields directly at B the value of the same voltage in statvolts. According to electromagnetic theory \( \varepsilon_0 \) and \( \mu_0 \) are not independent, but are connected (as indicated by the dot-dash lines in figure 25) by the relation

\[
\mu_0 \varepsilon_0 = \frac{1}{c^2},
\]

where \( c \) is the velocity of light. This velocity has been measured by optical experiments at very high frequencies and at the NBS by Rosa and Dorsey [28, 29, 30] at low frequency. The two very different types of experiment gave results in satisfactory agreement and are indicated by the vertical dashed line in figure 25. Accordingly, in the verification of the high-voltage scale, the electrometer results (at B) were multiplied by this factor (299,805 megameters/sec) and compared with the transformer results (at A). The optical experiments are presumably of materially higher
accuracy, but the slight observed residual difference in the voltage measurements might be regarded as an independent determination of $c$ as being 299.783.

5.6 Measurement of Crest and Surge Voltage

In the higher voltage range, engineers are usually more concerned with crest values than with rms values of alternating voltages. This is because the breakdown of insulation is to a large extent dependent upon the crest value of the voltage applied to it, and the testing of insulation is the principal application of voltages above the normal operating levels. The commonly used measurement technique in this field is to note the spark-over voltage between metal spheres of known diameter and spacing in air under standard test conditions. The standard currently recognized in the United States is that of the American Institute of Electrical Engineers [202], which is based on the pioneer work of the large electric manufacturing companies. The values in it differ somewhat from those in the corresponding standard of the International Electrotechnical Commission, which is generally used in Europe. One of the projects still ahead of the National Bureau of Standards is a study of the spark-over voltage of standard sphere gaps with a view to reconciling the differences and perhaps improving the accuracy of measurement. Preliminary experiments have indicated the feasibility of determining the crest factor of an alternating voltage by using a capacitance potential divider and a point-by-point measurement of wave form with a quadrant electrometer as a detector. This method is slow but capable of an accuracy better than 0.1 percent; it will form the basis for calibrating other techniques.

An alternative method for measuring crest volt-
Figure 25. The high-voltage scale and its confirmation by the absolute electrometer.

Figure 26. View in High-Voltage Laboratory at National Bureau of Standards.

The four columns of capacitors at the right constitute the 2,000,000-volt surge-voltage generator; the three 60-cycle 330,000-v transformers (one in pit with only its bushing visible) can be cascaded to give 1,000,000 v; the large transformer in the rear (with smooth tank) is the 125,000/250,000-v standard voltage transformer.

Figure 27. Surge current generator.

This bank of 40 capacitor units mounted in five tiers (only two of which are visible in the photograph) is shown delivering a surge having a crest value of about 100,000 amp. The outer conductor of the tubular shunt used in measuring such currents is visible below the table. The 100-kv charging line and the triggering circuit are visible at the top.
The unit of time as represented by standard radio carrier and modulation frequencies can be disseminated conveniently and with extreme accuracy by broadcasting. National Bureau of Standards radio stations WWV and WWVH perform this service [23 to 26], and anyone interested can just tune in. The electrical units cannot be transmitted so easily, and hence the values have to be disseminated by the more laborious physical transportation of electric instruments, meters, and other measuring apparatus. Electric measuring apparatus to be standardized is sent to the Bureau from manufacturers of electric instruments and machinery, public utility companies, State public utility commissions, university laboratories, industrial research laboratories, as well as from Federal agencies and private individuals.

Some organizations send in groups of saturated standard cells and 1-ohm standard resistors and carry on the rest of the measurement chain in their own laboratories. A number of foreign governments that are currently in the process of founding or expanding their national standardizing agencies come in this class. More often, unsaturated standard cells, sets of standard resistors, capacitors, inductors, and instrument transformers covering a considerable range of values are submitted. Many power companies submit their standard transfer wattmeters for comparison with the NBS instrument to verify the relation of its a-c performance to that on d-c, although they are well equipped to calibrate it on direct current themselves. Manufacturers of electric measuring devices submit for test their own "laboratory tools," by the use of which they adjust their product, and thus disseminate the units to their ultimate customers, who therefore may not need any direct contact with the Bureau. Indicating instruments are submitted not only by small laboratories that use them as primary standards, but also by larger organizations as a check on the steps in the measurement sequence as performed in their own laboratories. As a result a stream of about 2000 high-grade electric instruments and pieces of related apparatus flows through the electrical laboratories of the Bureau annually.

The Bureau is required by law [1, 2] to charge appropriate fees for such testing service unless it is for a branch of the Federal or State Government. Regular fee schedules [5] list these charges for the more usual types of test, and arrangements can often be made by correspondence for special unlisted tests at appropriate fees.

A considerable fraction of the apparatus submitted to the Bureau for test is newly manufactured, and in some cases (e.g., standard resistors) it is desirable to have it held in the laboratory for several months to enable successive measurements to insure that secular drifts in value, originating in the process of manufacture, have steadied down to a negligible rate. Other standard apparatus, however, is usually submitted at regularly scheduled intervals to insure the sustained accuracy of measurements based on it. The desirable frequency of such periodic checks depends on a great number of factors, including (1) the accuracy required, (2) the extent to which intercomparisons are made among the standards in the local laboratory, (3) the ruggedness and stability of the particular type of apparatus, and (4) the carefulness and skill of those handling the apparatus.

A standard cell is relatively delicate and can be ruined in a few seconds by an excessive drain of current, yet if a group of at least three cells is available and if its members are intercompared frequently, a change of any one relative to the rest is quickly evident, and the volt as fixed by the mutually consistent remaining cells may be trusted to an accuracy of 0.01 percent for 6 months or a year. Most standard cells of the unsaturated

6. Dissemination of Units
type show a gradual decrease in emf, which on the average is about 80 µv/year.

By contrast, a standard resistor, intended for the measurement of currents of several thousand amperes, is a very rugged structure and is not at all likely to be damaged by carelessness in the laboratory. Barring corrosion from acids in its cooling oil, if this is allowed to become rancid, or very slow metallurgical changes, such a resistor may be trusted to an accuracy of 0.01 percent for 5 or even 10 years. Resistors made with fine wires are decidedly more prone to change as a result of corrosion and mechanical strains, and should be rechecked every 2 or 3 years. This periodicity also applies to resistance boxes, measurement voltage dividers, bridges, and potentiometers.

In many cases an initial test at the Bureau may be very desirable, and further periodic checks may be unnecessary. An example is the transfer test of a high-grade wattmeter, which determines the difference, if any, between its a-c and its d-c performance. This difference depends upon such factors as eddy currents, inductance, and capacitance in its windings and is very unlikely to change appreciably with time or use. The d-c calibration as a wattmeter, which depends also on its springs and the dimensions and relative positions of its coils, may show secular changes.

This d-c performance, however, can be adequately checked by measurements made at the local laboratory with a potentiometer and standard cell.

Recognizing the desirability of having precise apparatus checked initially at the Bureau, several manufacturers of electric measuring apparatus make it a practice to submit for test groups of usually 10 or 20 similar standards. After the test these are returned to the maker's stock and later sold "with a National Bureau of Standards Certificate" at an increase in price to cover the Bureau's fees and the handling costs. By this process, the necessary seasoning time does not introduce a delay between the sale of the standard and its ultimate use.

The results of calibration tests of measuring instruments and apparatus are issued in one of two forms: "Certificates" and "Reports." A Certificate is usually issued if the results have relatively permanent validity, and if the apparatus tested meets certain standards of precision and tolerance.

Reports are issued when unusually long and complex measurement programs are involved, when the apparatus is such that it can be too readily thrown out of adjustment, or when it shows symptoms of instability.

7. International Relations

By the Convention of the Meter, as amended in 1921 [6], the International Bureau of Weights and Measures at Sèvres, France, is authorized to coordinate the work of the various national standardizing laboratories in the electrical field. The National Bureau of Standards cooperates actively with the International Bureau and sends groups of standard cells and of standard resistors to Sèvres every 2 years for comparison with the groups of standards kept at the International Bureau. Other nations do the same, and in this way the relative values of the units as maintained in the different countries are derived. Graphs of these values are given in [6, p. 13, 16]. In abstract theory, all electrical units are supposed to be derived by an unbroken chain from those maintained at the International Bureau. In the final analysis, however, because of errors in comparison, changes during transportation, and drifts with time, the unit actually used in practice in any individual laboratory is bound to differ somewhat from the ideal. The relative differences between countries seldom exceed ±20 parts per million. The international comparisons of high precision serve to give early warning to any laboratory in case its units should show a drift relative to those of the other laboratories. The appearance of such relative drifts in the German standards led in 1931 to cooperative experiments in Berlin in which representatives of the British National Physical Laboratory and the National Bureau of Standards took part [12]. As a result of these experiments the German laboratory changed the values of its electrical units [6]. Also, the results of any new and improved absolute measurements can be disseminated with high precision to the rest of the world by these biennial intercomparisons.

Most countries derive their electrical units by sending standards to the International Bureau for certification. As a matter of practical convenience, however, a number of national laboratories have preferred to have their standards tested at the National Bureau of Standards. This is particularly expedient when the standard in question is manufactured in the United States and can readily be standardized in Washington before shipment abroad.

Occasionally, when a standard capacitor or indicating instrument of extreme accuracy happens to be shipped, say, from England to the United States, it is found feasible to arrange to have it tested at the National Physical Laboratory before shipment and again at the National Bureau of Standards on its arrival in this country [109]. The agreements found in such comparisons are usually not as precise as those based on the shipment of standard resistors and cells, but give a very valuable assurance that the methods used in the two laboratories for deriving other units from the more fundamental ones lead to results that are consistent to an accuracy well within the needs of commercial operations.

The dissemination of scientific units of measurement throughout the world offers an almost unique example of harmonious international relations which have persisted for many years. The gain
to society from world-wide uniformity in the field of measurement is very great, and the sacrifices required to maintain uniformity, at least in the electrical fields, are small. The workers in the national standardizing laboratories, confronted as they are with similar problems, have learned a mutual respect for their conferees. The international organization which deals with weights and measures, established in 1875, has filled such a definite need and was devised with such farsighted wisdom that it has survived two world wars and has grown in prestige, scope, and influence. The precise unification of electrical measurements is one of its outstanding accomplishments.

8. Bibliography

The following list gives references to a large number of technical papers, standards, and books originating for the most part at the National Bureau of Standards and dealing more or less directly with the extension and development of methods for electrical and magnetic measurements. A number of papers on absolute measurements have also been included because the references listed in Bureau Circular 475 included only a few selected papers in this field. Many NBS papers that give the results of applying these methods of measurement in various fields, such as to the properties of insulating and of magnetic materials or to the performance of engineering devices, have not been included. A few of the references are included for their historic value only, but in those cases where the content of an older paper has been included as part of a later publication, the older reference has been omitted.

The following letter-symbol designations are used in combination with the Bureau publication numbers, and these letters should be included with the numbers in all references to Bureau publications: S, Scientific Papers; T, Technologic Papers; RP, Research Papers; C, Circulars; H, Handbooks; M, Miscellaneous Publications; LC, Letter Circulars (mimeographed).

When a price is stated, the publication may be purchased from the Superintendent of Documents, Government Printing Office, Washington 25, D. C. Many of the older papers are now out of print, but can be consulted in large public or university libraries which maintain sets of Bureau publications.

A complete catalog of Bureau publications covering the period from 1901 to June 30, 1947, with author and subject indexes, NBS Circular C460, is available from the Superintendent of Documents for $1. A mimeographed supplement to this is included. Copies of articles published in other technical journals are not obtainable in the Government. The standards mentioned may be purchased from the American Standards Association, 70 East 45th Street, New York, 17, N. Y., or from the American Institute of Electrical Engineers, 30 West 39th Street, New York 18, N. Y.

In the following references, to save space, the code letters listed below have been used preceding the volume number as abbreviations to indicate the corresponding series of Bureau publications: B, Bulletin of the Bureau of Standards; S, Scientific Papers of the Bureau of Standards; BJ, Bureau of Standards Journal of Research; J, Journal of Research of the National Bureau of Standards; C, Circular (of the National Bureau of Standards).

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