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PERMANENT MAGNETS

By

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PREFACE

Some time previous to World War II, a project was undertaken to summarize the available information on the manufacture and properties of permanent magnets, primarily as an aid in answering the numerous inquiries regarding magnets received by the National Bureau of Standards. It was originally intended to supplement the data to be found in the technical literature by a certain amount of experimental work, particularly on the newer types of permanent-magnet alloys. The experimental investigations were interrupted by the necessity of giving full attention to the war effort. However, in view of continued inquiries, it now seems best to publish the data already at hand rather than to defer publication until the experimental investigations can be completed.

LYMAN J. BRIGGS, *Director.*

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ABSTRACT

This Circular gives general information regarding the composition, treatment, and properties of permanent-magnet materials and the design and testing of permanent magnets.

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I. INTRODUCTION

Permanent magnets are employed in various kinds of instruments and devices, such as compasses, electrical measuring instruments, relays, telephone receivers, microphones, magnetos, magnetic chucks, phonograph pick-ups, coin separators, and many others. In some applications, great constancy is not essential, but in others, such as electrical measuring instruments, the highest degree of constancy possible is of primary importance.

The range of usefulness of permanent magnets has been very greatly extended in recent years by the development of new and greatly superior materials, but the advantages to be gained by their use can not be realized unless magnets are properly designed with reference to the magnetic properties of the materials of which they are to be made. *The mere substitution of a different material without an appropriate modification in design is almost certain to give disappointing results.* The older, less expensive materials will often prove to be best for certain applications in which considerations of weight or size are of minor importance. In any event, it is important to exercise considerable care in the choice of materials and design in order to ensure the maximum economy of materials consistent with satisfactory performance.

Data on the composition, treatment, and properties of a great variety of permanent-magnet materials and discussions of design procedures have been published in the technical literature. The object of the present circular is to give a general summary of the available information, together with a brief discussion of the design problem.

II. MAGNETIC QUANTITIES AND UNITS

The characteristics of permanent magnets and the properties of the materials of which they are made are expressed in terms of certain magnetic quantities and units which may be described as follows:

Magnetomotive force.—Magnetization is generally accomplished by means of electric current in windings linked with a magnetic circuit of which the material to be magnetized forms a part. The total measure of the magnetizing effect of such a coil is called the magnetomotive force, $\mathcal{F}$. The cgs unit of magnetomotive force is called the *gilbert* and is defined by the equation

$$\mathcal{F} = 0.4\pi NI,$$

in which

$\mathcal{F}$ = magnetomotive force in gilberts,
N = number of turns in the coil, and
I = current in amperes.

Magnetic flux.—The total measure of the magnetized condition of a magnetic circuit when acted upon by a magnetomotive force is called the magnetic flux, ϕ. It is characterized by the fact that a variation in its magnitude gives rise to an electromotive force in an electric circuit linked with it. The electromotive force thus induced is at any instant directly proportional to the time rate of variation of the flux. The cgs unit of magnetic flux is called the *maxwell* and is defined by the equation

$$e = N(d\phi/dt)10^{-8},$$

in which

e = induced electromotive force in volts,
N = number of turns linked with the flux,
ϕ = magnetic flux in maxwells, and
t = time in seconds.

Magnetic reluctance.—That property of a magnetic circuit which determines the relationship between the magnetic flux and the corresponding magnetomotive force is called the magnetic reluctance, $\mathcal{R}$, of the circuit. The cgs unit[1] is defined by the equation

$$\phi = \mathcal{F}/\mathcal{R},$$

in which
ϕ = magnetic flux in maxwells,
$\mathcal{F}$ = magnetomotive force in gilberts, and
$\mathcal{R}$ = magnetic reluctance in cgs units.

In a magnetic circuit of uniform cross-sectional area and uniform permeability (tc be defined later)

$$\mathcal{R} = l/\mu A,$$

in which
$\mathcal{R}$ = magnetic reluctance in cgs units,
μ = magnetic permeability,
l = length in centimeters, and
A = cross-sectional area in square centimeters.

Magnetic permeance.—Magnetic permeance, $\mathcal{P}$, is the reciprocal of the magnetic reluctance. Thus

$$\phi = \mathcal{F}\mathcal{P},$$

where
ϕ = magnetic flux in maxwells,
$\mathcal{F}$ = magnetomotive force in gilberts, and
$\mathcal{P}$ = magnetic permeance in cgs units.

In a circuit of uniform cross section and permeability

$$\mathcal{P} = \mu A/l.$$

Magnetizing force.—The magnetomotive force acting on a magnetic circuit is distributed along its length in a manner determined by the distribution of the magnetizing winding and of the reluctance of the circuit. The magnetomotive force per unit length along the circuit is called the magnetizing force, H. The cgs unit is called the *oersted* and is defined by the equation

$$H = d\mathcal{F}\ dl,$$

[1]The unit of magnetic reluctance was called the oersted until 1930, when the International Electrotechnical Commission adopted that name for the unit of magnetizing force, thus leaving the unit of reluctance without a name.

in which

H = magnetizing force in oersteds,
$\mathcal{F}$ = magnetomotive force in gilberts, and
l = length in centimeters.

At the center of a very long, uniformly wound solenoid having n turns per centimeter in which a current of I amperes is flowing, the magnetizing force in oersteds is

$$H = 0.4\pi nI.$$

Magnetic induction.—Magnetic induction, B, also called magnetic flux density, is the magnetic flux per unit area of a section normal to the direction of the flux. The cgs unit is called the *gauss* and is defined by the equation

$$B = d\phi/dA,$$

in which

B = magnetic induction in gausses,
ϕ = magnetic flux in maxwells, and
A = area in square centimeters.

Intrinsic induction.—That part of the magnetic induction which is in excess of the induction which would exist in a vacuum under the influence of a given magnetizing force is called the intrinsic induction, B_i. Numerically, in the cgs system $B_i = B - H$.

It is generally conceded that the only known source of magnetic effects is electricity in motion (electric current). According to present theory [38],[2] ferromagnetic effects are due to groups of electrons within a ferromagnetic material called "domains" and consisting of electrons spinning on their own axes. The magnetic axes of the spinning electrons in a single domain are held parallel to each other by mutual forces known as "exchange forces," so that each domain behaves as a single unit. The domains are in effect current-turns and so account for the magnetomotive forces inherent in ferromagnetic materials. The sum total of the magnetomotive forces due to the domains is the quantity which remains "permanent" in a permanent magnet.

In the unmagnetized condition, the domains are so oriented with respect to each other that the net magnetic effect is zero in any direction. Under the influence of a magnetic field applied by means of an external electric current, the magnetic axes of the domains tend to be oriented more or less in the direction of the applied field, so that their

[2] Figures in brackets indicate the literature references at the end of this paper.

effect is added to that of the applied field. The flux density due to the combined effect of the applied field and the domains is the magnetic induction, *B*, and that due to the domains alone is the intrinsic induction, B_i.

Upon removal of the magnetizing force, the orientations of the domains resulting from the applied field tend to persist to a greater or less extent, depending upon their distribution in space, which in turn depends upon the internal structure of the material. In order to reduce the induction further, it is necessary to apply a reverse, or demagnetizing, force. The relationships between induction and the applied field are shown graphically in figure 1. In the figure, the abscissas represent values of magnetizing force, *H*, and the ordinates are values of magnetic induction, *B*. The curve *oabc* is called the normal induction curve, and shows how the induction varies as the magnetizing force applied to a material initially in the demagnetized condition

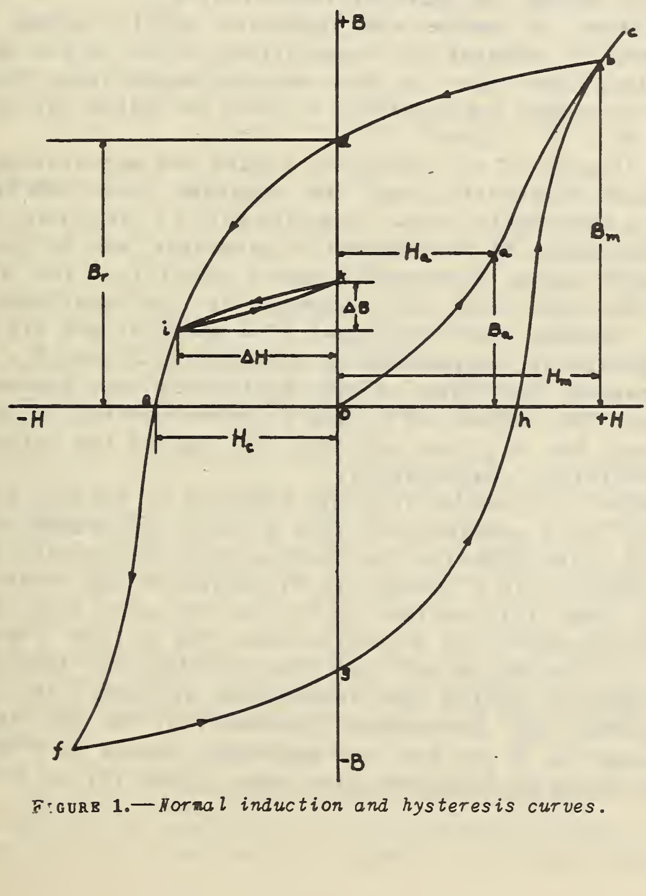

FIGURE 1.—*Normal induction and hysteresis curves.*

is increased from zero. At any point on this curve, the ratio of the value of induction to the corresponding value of magnetizing force is called the *magnetic permeability*, μ. Thus the coordinates of point *a* on the curve are B_a and H_a, respectively, and the permeability is

$$\mu = B_a / H_a$$

Since the line *oabc* is not straight, the permeability is not constant, but varies with the degree of magnetization. This variation is one of the distinguishing characteristics of ferromagnetic materials.

If, after the magnetizing force has reached a certain value (as at point *b*), it is then decreased, the induction does not follow the curve *oabc* in reverse order but lags behind the magnetizing force, as shown by the curve *bd*. Thus, when the magnetizing force has been reduced to zero, point *d*, the induction still has an appreciable value. This value is called the *residual induction*, B_r.

In order to reduce the induction still further, it is necessary to increase the magnetizing force in the opposite direction. The value of the reversed magnetizing force required to reduce the induction to zero is called the *coercive force*, H_c.

The lagging of the induction behind the magnetizing force is called *hysteresis*, and the complete curve *bdefghb*, is called a hysteresis loop. Hysteresis is another of the characteristics of ferromagnetic materials and is the property which makes permanent magnets possible. The size of the hysteresis loop, and consequently the magnitudes of B_r and H_c, depends upon the values of B and H at the tip of the loop, generally designated by the symbols B_m and H_m. As H_m is increased, the size of the hysteresis loop approaches a maximum. The values of B_r and H_c corresponding to the maximum loop for a given material are called the *retentivity* and *coercivity*, respectively.

A major hysteresis loop is produced by varying the magnetizing force continuously from a positive maximum value to the same value negative and back again to the positive maximum. However, if a change in direction of the variation is made at some intermediate point on the major loop, a minor loop is produced, as shown between the points *i* and *k* in figure 1. The slope of the line joining the tips of such minor loop is called the *incremental permeability*, μ_Δ. In other words, the incremental permeability is the ratio of the change in B to the corresponding change in H when the average value of B differs from zero. That is, $\mu_\Delta = \Delta B / \Delta H$.

That part of the hysteresis loop extending from the residual induction, B_r, to the coercive force, H_c, i.e., from *d* to *e*, is called the *demagnetization curve*. Points on this curve are designated by the coordinates B_d and H_d. Most of the important characteristics of a permanent-magnet material can be indicated by points on this curve or within the area between it and the axes of coordinates. The value of H_m necessary to produce the maximum loop depends upon the type of material and ranges from about 300 oersteds for most of the older types of material to about 3,000 oersteds or more for the newer types. Figure 2 represents a typical demagnetization curve, together with the so-called *energy-product*

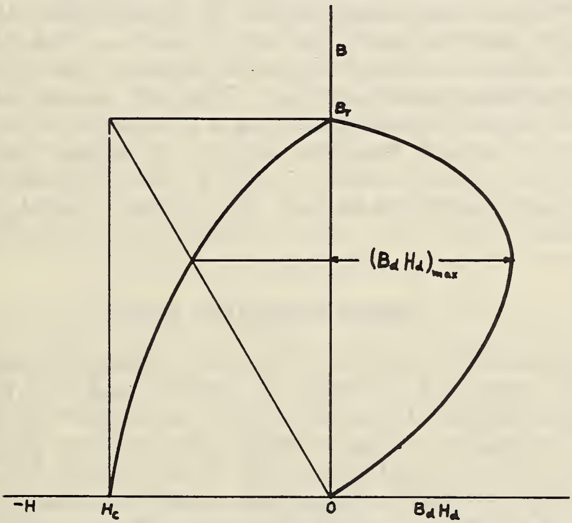

FIGURE 2.—*Demagnetization and energy-product curves.*

curve obtained by plotting the product of corresponding values of B_d and H_d on the demagnetization curve against B_d. The maximum value of this product, $(B_dH_d)_{max}$, can be considered as a criterion of quality and is useful for the comparison of various materials. It is generally not necessary to plot the B_dH_d curve in order to determine the maximum energy product. A line drawn from the origin of coordinates to the intersection of horizontal and vertical lines through B_r and H_c, respectively, intersects the demagnetization curve at the point where the product B_dH_d is a maximum for most materials.

III. PERMANENT-MAGNET MATERIALS

Although it is possible to make permanent magnets of almost any kind of steel that is capable of being hardened by heat treatment, it is best to use materials specially produced for the purpose. Magnets made from other types of material are likely to be either inferior in quality or unnecessarily expensive. Before the development of the special magnet steels, magnets were generally made of plain high-carbon tool steel. This type of steel is relatively inexpensive, but its magnetic properties are so greatly inferior to those of the special steels that now it is practically never used for making magnets.

High-grade permanent-magnet materials are either quench-hardening steels of high alloy content or carbon-free alloys of the dispersion-hardening type. The quench-hardening steels are so called because they are hardened by rapid cooling from a high temperature by "quenching" in a suitable cooling medium, such as water or oil. The dispersion-hardening alloys contain no intentionally added carbon and are not properly called steels. They are called dispersion-hardening because the hardening procedure brings about a dispersion of one of the two phases of which they are composed in the other phase. They were originally called precipitation-hardening alloys, but since actual precipitation probably does not occur, dispersion-hardening is the better term.

1. QUENCH-HARDENING STEELS

Carbon-manganese steel.—A low-cost permanent-magnet steel, containing about 0.6 percent of carbon and 0.8 percent of manganese is used widely if weight or volume is not important and the magnetic requirements are not too severe.

Tungsten steel.—Prior to the first World War, tungsten steel was the standard high-grade permanent-magnet material. The optimum tungsten content is between 5 and 6 percent, with about 0.6 percent of carbon. This type of steel is water-hardening, but can be made oil-hardening by the addition of about 0.5 percent of chromium. As a rule, producers make several different grades having various percentages of tungsten and carbon besides small quantities of other elements, such as chromium or molybdenum. Each producer has his own preferred compositions, so that, although the various grades are quite similar, the actual compositions may differ considerably.

Chromium steels.—Chromium magnet steels were developed at the time of the first World War, when tungsten was scarce. There are two principal grades, one containing about 1 percent of chromium and 0.6 percent of carbon, and the other containing about 3.5 percent of chromium and 0.9 percent of carbon. The chromium steels require somewhat closer control in heat treatment, but have properties practically as good as the more expensive tungsten steels, and now have practically superseded them.

Cobalt steels.—In 1920, the Japanese metallurgist Honda, [2] announced the development of a new permanent-magnet steel having cobalt as the principal alloying element, which he named "KS" magnet steel. This steel also contained substantial percentages of tungsten, chromium, and molybdenum. The coercive force of this material is about four times as great as that of the tungsten or chromium magnet steels. This development stimulated further investigations which led to the commercial production of several types of cobalt magnet steel having various percentages of cobalt. High-cobalt magnet steels have from 35 to 41 percent of cobalt, and low-cobalt steels have from 8 to 9 percent. These steels also have tungsten and chromium in addition to the cobalt.

Cobalt-chromium steels.—The cobalt-chromium steels have from 15 to 17 percent of cobalt and about 9 percent of chromium. Their magnetic properties are intermediate between those of the high-cobalt and the low-cobalt steels. Tungsten is sometimes substituted for part of the chromium.

2. DISPERSION-HARDENING ALLOYS

Aluminum-nickel-iron alloys.—The fact that high values of coercive force can be obtained with alloys containing no carbon was announced in 1932 by Seljesater and Rogers [14] in the United States, Koster [15] in Germany, and Mishima [13] in Japan. The first dispersion-hardening permanent-magnet alloys contained aluminum, nickel, and iron. Although the residual induction was relatively low, the coercive force was so high, about 475 oersteds, that new applications of permanent magnets were made possible. These alloys are very hard and brittle, so that they can not be formed by the usual methods of forging and machining but must be either cast or molded in powder form under great pressure and then sintered. Final finishing is done by grinding.

Aluminum-nickel-cobalt-iron alloys.—Cobalt added to the aluminum and nickel has been found to be beneficial, and a series of alloys has been developed which are manufactured under the trade name Alnico. The various compositions are identified by roman numerals as Alnico I, Alnico II, etc. At the present writing, Alnico V is the latest development.

Aluminum-nickel-titanium-iron alloy.—This alloy, manufactured under the trade name Nipermag, is somewhat similar in composition and properties to the Alnico series but contains a small percentage of titanium in place of the cobalt.

Cobalt-molybdenum alloy.—This alloy produced under the trade name Comol [35] can be machined after the first stage of its heat treatment, which consists in quenching from a high temperature. Its properties are similar to those of high-cobalt steel, but since it is dispersion hardening, it is structurally more stable.

Demagnetization and energy-product curves for a number of typical permanent-magnet materials are shown in figure 3, and table 1 gives approximate compositions, magnetic characteristics and customary heat treatments for the principal commercial types of permanent-magnet materials.

TABLE 1.—*Typical commercial permanent-magnet materials*[1]

Type	Percentage composition[2]	B_r	H_c	$(B_dH_d)_m \times 10^{-6}$	Heat treatment
Mn........	0.8 Mn; 0.6 C......	10,000	43	0.18	Quench, 800°C (1,475°F)—water.
W.........	5.5 W; 0.6 C.......	10,000	65	.27	Quench, 840°C (1,540°F)—water.
W-Cr......	6.0 W; 0.5 Cr; 0.7 C	9,000	70	.26	Quench, 840°C (1,540°F)—oil.
1% Cr.....	1.0 Cr; 0.6 C......	10,000	50	.21	Quench, 790°C (1,450°F)—oil.
3 1/2% Cr..	3.5 Cr; 0.9 C......	9,500	63	.25	Quench, 840°C (1,540°F)—oil.
Low Co....	8.5 Co; 5.0 Cr; 1.0 W; 0.9 C.	7,500	120	.38	Quench, 900°C (1,650°F)—oil.
Co-Cr.....	16.0 Co; 9.0 Cr; 1.0 C.	8,000	180	.61	Triple treatment[4].
Co-W......	17.0 Co; 9.0 W; 3.5 Cr; 0.9 C.	9,000	165	.62	Quench, 950°C (1,740°F)—oil.
High Co...	36.0 Co; 5.0 W; 2.0 Cr; 0.8 C.	9,000	230	.87	Quench, 950°C (1,740°F)—oil.
Alnico I..	12 Al; 20 Ni; 5 Co..	7,300	440	1.40	Dispersion hardening.
Alnico II[3]	10 Al; 17 Ni; 12.5 Co; 6 Cu.	7,350	560	1.60	Treatment generally consists of a quench in oil from 1,200°C (2,190°F), followed by long heating at 600°C (1,110°F).
Alnico III	12 Al; 25 Ni.......	6,900	475	1.38	
Alnico IV	12 Al; 28 Ni; 5 Co..	5,300	730	1.30	
Alnico V..	8 Al; 14 Ni; 24 Co; 3 Cu[5].	12,500	550	4.50	
Nipermag..	12 Al; 32 Ni; 0.4 Ti	5,500	675	1.34	
Comol.....	12 Co; 17 Mo.......	10,500	250	1.10	

[1] To be understood as typical only. Composition and properties vary in practice.

[2] Plus Fe, together with the usual small amounts of Si, P, S, and Mn as impurities.

[3] Cast. Sintered material has somewhat lower magnetic properties.

[4] Heat to 1,150°C (2,100°F), cool in air; 620°C (1,150°F), cool in air; 1,010°C (1,850°F), cool in air.

[5] Heat treated in a magnetic field and has directional properties.

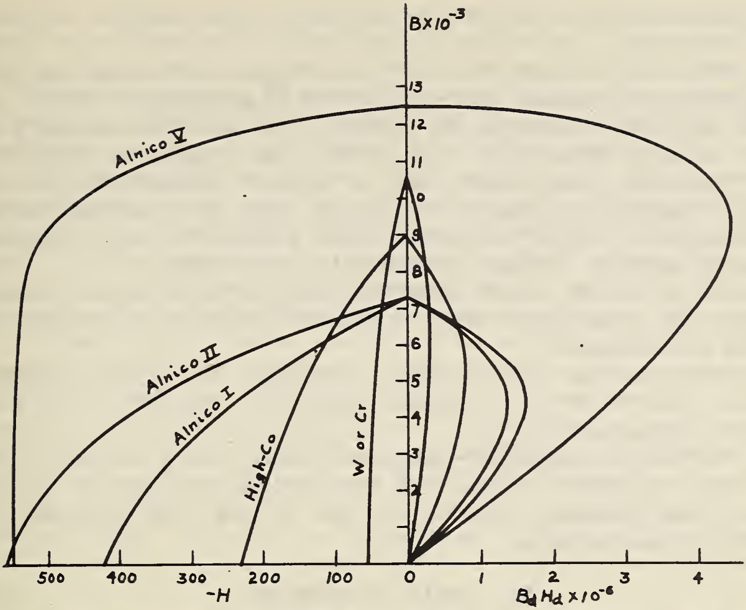

FIGURE 3.—*Demagnetization and energy-product curves for typical permanent magnet materials.*

3. MISCELLANEOUS ALLOYS

Several permanent-magnet alloys have been produced which, although not in general commercial production, are of considerable interest and value for special applications. The

TABLE 2.—*Miscellaneous special permanent-magnet materials*[1]

Type	Percentage composition[2]	B_r	H_c	$(B_d H_d)_m \times 10^{-6}$	Heat treatment
New KS....	27 Co; 18 Ni; 7 Ti; 3.7 Al.	7,150	785	2.03	Prolonged heating 660°C (1,220°F).
Fe-Ni-Cu..	20 Ni; 60 Cu.....	5,280	460	1.07	(3)
Vicalloy..	32 to 62 Co; 6 to 16 V.	9,600	400	2.8	(4)
Oxide.....	16 CoO; 34 Fe_2O_3; 50 Fe_3O_4.	3,500	600	1.03	Compress-sinter 980°C (1,800°F).[5]
Fe-Pt.....	77 Pt; 23 Fe.....	5,830	1,570	3.07	Quench, 1,200°C (2,190°F)—oil.
Co-Pt.....	23 Co; 77 Pt.....	4,530	2,650	3.77	Quench, 1,200°C (2,190°F)—oil.
Ag-Mn-Al..	87 Ag; 9 Mn; 4 Al	500	5,500	0.06	

[1]These materials are not regularly produced commercially but may be useful for special applications.
[2]Plus Fe and unavoidable impurities.
[3]Prolonged heating 1,040°C (1,900°F), quench in oil, prolonged heating at 600°C (1,110°F), cold roll and repeat.
[4]Cast, hot swage, cold draw, 1 hour at 600°C (1,110°F).
[5]Best magnetized at about 315°C (600°F).

compositions and properties of some of these alloys are shown in table 2.

The alloy called "New KS" is a dispersion-hardening alloy developed by Honda. It has a very high coercive force, but the residual induction is rather low, and the energy product is only about half that of the less expensive Alnico V.

Vicalloy, like Comol, can be worked and machined and has better magnetic properties. It is used to a certain extent for magnetic sound recording and should be more generally useful when it becomes commercially available.

The sintered oxide material [16] has interesting properties, but it is quite fragile and therefore is of rather limited usefulness.

The platinum alloys have very high values of coercive force, but their low residual induction and high cost limit their usefulness.

The silver-manganese-aluminum alloy has a coercive force of about 5,500 oersteds, but the residual induction is only about 500 gausses. The alloy has a limited applicability for special purposes.

IV. HEAT TREATMENT

The quality of a permanent magnet depends not only upon its chemical composition but also upon the thermal treatment to which it is subjected at each stage of manufacture from the original melting of its constituent elements to the final hardening and aging process.

Although the methods of melting, rolling, and forming magnet steels are, in general, similar to those employed in the production of high-grade tool steels, certain of the usual treatments are detrimental to the magnetic properties and must be modified or omitted altogether. With the exception of the carbon-manganese and low-chromium steels which are produced in large tonnages in open-hearth furnaces, magnet steels are almost universally melted in electric furnaces. The quality of the final product is affected by the pouring temperature, the rate at which the ingots are cooled, and the temperatures at which the structure is "broken down" by rolling or forging. It is usually preferable to break down the ingots without cooling and subsequent reheating. Surface defects are removed by grinding. In view of the fact that proper mill practice is developed by experience and differs somewhat from that customarily employed for other types of alloy steels, it is usually best to procure magnet steels from a well-established mill experienced in the production of permanent-magnet materials.

Considerable care is necessary in the heat-treatment incident to forming and final hardening, because it is easily possible to ruin an originally acceptable material by improper heat treatment. One common source of trouble is the attempt to improve the machinability by annealing. Annealing should be avoided if possible, because it always impairs the magnetic quality. If the material is held too long at a temperature in the neighborhood of 900°C (1,650°F), the deterioration becomes so great that "spoiling" takes place, the result of which is to reduce the value of coercive force that can be obtained by subsequent hardening. The exact nature of the phenomenon is not definitely known, but it probably results from the decomposition of some of the carbides or more complex compounds, thus partially releasing the intense local strains which are responsible for the high coercive force. All types of quench-hardening steels are subject to spoiling. Spoiled steel can be restored, at least partially, by heating quickly to a temperature in the neighborhood of 1,200°C (2,190°F), and holding the temperature just long enough to ensure that the material is heated through. Cooling should be sufficiently rapid through the spoiling range to avoid a repetition of the spoiling process. It is better, of course, to avoid spoiling in the first place.

If annealing is found to be absolutely necessary in order to carry out machining operations, the material should be heated rapidly, held at temperature only long enough to heat through, and cooled as rapidly as possible without hardening. Another reason for avoiding prolonged heating is to prevent surface decarburization. A decarburized surface layer acts as a magnetic shunt on the magnet and thus decreases its effective strength. Similar precautions should be taken to avoid spoiling when heating for forging or forming operations.

The conditions associated with "magnetic hardness" are not well understood, but it seems certain that one essential condition, probably the most important one, is a high degree of localized internal strain. The treatment which brings about this condition in quench-hardening steels consists in rapid cooling from an appropriate high temperature by "quenching" in a suitable medium, such as water or oil. The proper temperature and quenching medium depend upon the composition of the steel. It is usually best to follow the practice recommended by the producer of the steel.

The usual quenching temperature for tungsten or chromium steels is from 840°C to 850°C (1,540°F to 1,560°F). Tungsten steels are generally quenched in water but sometimes contain enough chromium to make them oil-hardening. Chromium steels are quenched in oil.

The high-cobalt steels are quenched in oil from about 950°C (1,740°F). Steels of the cobalt-chromium type require a more complicated treatment, such as cooling in air from 1,150°C (2,100°F), reheating to 620°C (1,150°F) and cooling in air, followed by cooling in air from about 1,010°C (1,850°F).

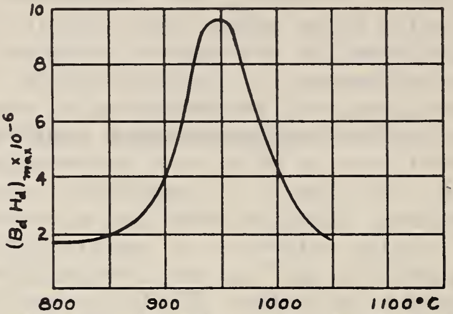

FIGURE 4.—*Effect of quenching temperature on maximum energy product for high-cobalt magnet steel.*

The use of too high a temperature in hardening is to be avoided, as this may lead to the retention of some of the solid solution known as austenite, which is practically nonmagnetic. This brings about a low value of residual induction and consequently a weak magnet. If the quenching temperature is too low, the desired high value of coercive force is not developed. The influence of quenching temperature on quality is illustrated in figure 4, which shows the relation between the maximum energy product and quenching temperature for a high-cobalt steel.

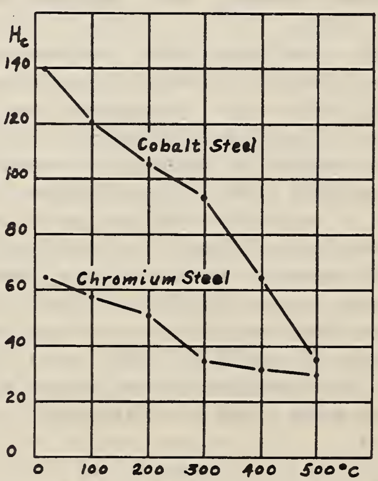

FIGURE 5.—*Effect of tempering on coercive force.*

Magnet steels are not tempered after hardening except at a relatively low temperature for stabilization, which will be discussed in a subsequent section. Tempering at higher temperatures, such as is customary with other types of steel, is detrimental to the magnetic properties of magnet steels. This is illustrated in the curves of figure 5, which show how the coercive force decreases with tempering temperature.

Finish-grinding after hardening has to be done carefully to avoid excessive local heating, which may produce grinding cracks. Cracks do not always appear immediately but may develop some time after grinding. Cracks can usually be avoided by wet-grinding and very light cuts.

It has been found that magnet steel can be cast to shape with satisfactory results, especially for intricate shapes. Cast magnets are somewhat more brittle than forged magnets, but their magnetic properties are fully as good. It has also been found that there is no improvement in magnetic quality if magnets are made of several pieces or laminations unless the aggregate cross section is so great that they can not be satisfactorily hardened if made in one piece.

The mechanism by which the intense local strains responsible for high coercive force are produced in the dispersion-hardening alloys is to some extent similar to that of the quench-hardening steels, but the heat treatment which brings it about is different. As in the steels, a solid solution is formed at high temperatures, and this is retained by rapid cooling. Upon subsequent reheating, the dissolved phase tends to precipitate. In practice, however, the reheating is carried only to the point of incipient precipitation, the resulting distortion of the crystal lattice producing the strains responsible for the high coercive force.

In view of the fact that these alloys are generally cast to shape in the mill in which they are originally made, the final heat treatment is usually carried out by the producer. Practice varies to a considerable extent, but a common method consists in quenching in oil from 1,200°C (2,190°F), followed by long heating at 600°C (1,110°F). An alternative method is to omit the quenching operation and cool from the solution temperature at a controlled rate to about 700°C (1,300°F) (the rate depending upon the composition), holding at this temperature for about 15 minutes, and then cooling in air or sometimes in the furnace.

Alnico V is heat treated in a magnetic field. This produces definite directional properties and, consequently, it is important that it be magnetized during heat treatment in the direction in which it is to be magnetized for use.

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V. MAGNETIZATION

The old methods of "single touch" and "double touch" by which magnets were magnetized by stroking with other magnets have long since been abandoned. Modern permanent-magnet materials require the application of very much more magnetomotive force than can be obtained by contact with a magnet.

For best results, the required magnetomotive force should be applied in such a way that every part of the piece is magnetized to the point of practical saturation. The value of magnetomotive force required depends upon the material of which the magnet is made and the length of the magnet. A good general rule is that the magnetomotive force should be at least five times the coercive force of the material multiplied by the length in centimeters. The magnetomotive force in gilberts is 0.4π times the number of ampere-turns. As an example, a magnet 25 cm long of chromium steel having a coercive force of 60 oersteds would require at least $5 \times 60 \times 25 = 7,500$ gilberts, or about 6,000 ampere-turns.

The ideal way of applying the required magnetomotive force would be by means of current in a winding uniformly distributed along the length of the magnet, with a "keeper" applied

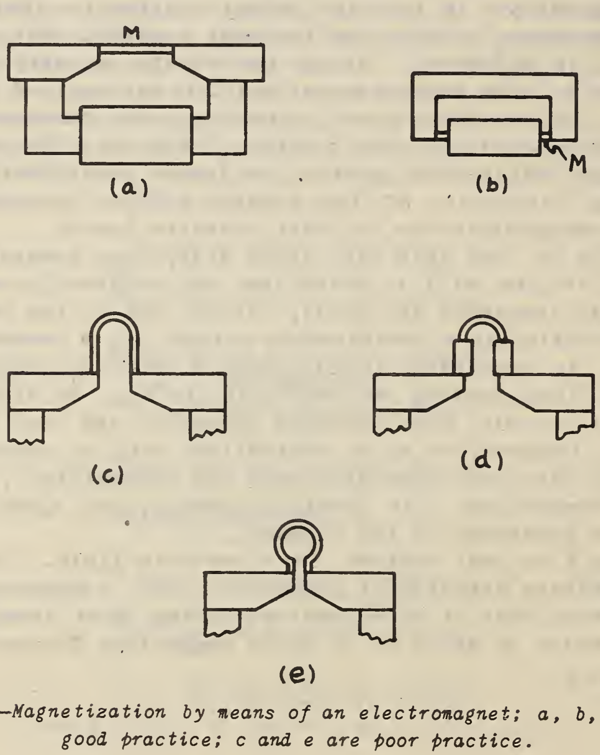

FIGURE 6.—*Magnetization by means of an electromagnet; a, b, and d are good practice; c and e are poor practice.*

to close the magnetic circuit. In practice, this is seldom feasible. Instead, the magnetomotive force is often applied by momentarily passing a current of several thousand amperes through a single conductor linked with the magnet. The necessary current may be obtained from large storage batteries, a heavy-current generator or from a thyratron-controlled heavy-current transformer so arranged that only a single half wave of current passes through the winding. Such an arrangement has been described by H. W. Lord [23].

Short, straight magnets can be magnetized between the poles of an electromagnet, as indicated at (*a*) in figure 6. However, bars more than about 15 cm long should be magnetized by a solenoid surrounding them, shown at (*b*), preferably with an external soft-iron yoke in contact with their ends. *M* indicates the magnet.

U-shaped magnets are often magnetized between the poles of an electromagnet as shown at (*c*) in the figure. It is practically impossible, however, to obtain complete saturation in this way. The leakage between the limbs of the magnet and between the pole pieces of the electromagnet is so great that the neutral zone in the curved part of the magnet does not reach saturation no matter how many ampere-turns there are in the winding of the electromagnet. It was found experimentally, for instance, that 50,000 ampere-turns in the electromagnet winding produced a magnetizing force of only about 123 oersteds at the neutral zone of a chromium-steel magnet having a total length of 25cm. Doubling the number of ampere-turns, making the value 100,000, only increased the value of magnetizing force at the neutral zone to about 126 oersteds. For this type of steel, which has a coercive force of about 60 oersteds, the magnetizing force required for practical saturation is at least 300 oersteds. Better results can be obtained by means of auxiliary coils on the limbs of the magnet, as shown at (*d*) in the figure. For magnets having their poles very close together, as shown at (*e*), the electromagnet is entirely inadequate, and a single conductor or winding on the magnet itself must be used.

The magnetizing process is one of the most important elements in the production of permanent magnets because without proper magnetization it is impossible to obtain satisfactory results even though properly treated material of the best quality is used.

VI. STABILIZATION

In certain applications of permanent magnets, such as electrical measuring instruments, it is important that the magnets be so treated as to have the greatest possible constancy. The influences tending to weaken a magnet are the passage of time, heat, mechanical shock or vibration, and magnetic fields. In order to minimize the effects of these influences certain treatments are applied to bring about stabilization.

Structural stabilization.—The sudden cooling from a high temperature incident to the hardening of quench-hardening steels suppresses certain structural transformations which, however, tend to proceed at a very slow rate even at ordinary room temperatures. Corresponding changes in magnetic properties accompany the structural changes so that the material becomes magnetically softer. In other words, the coercive force is decreased and the residual induction is increased, as shown in figure 7. Consequently, a magnet magnetized immediately after hardening becomes progressively weaker with time. This change is called aging. As shown in figure 8, the rate of change is relatively rapid at first but becomes less as time goes on and eventually becomes practically negligible. The amount and rate of change and the degree to which constancy is finally attained depend primarily upon the composition and heat treatment of the material and the dimensions of the magnet and its associated magnetic circuit.

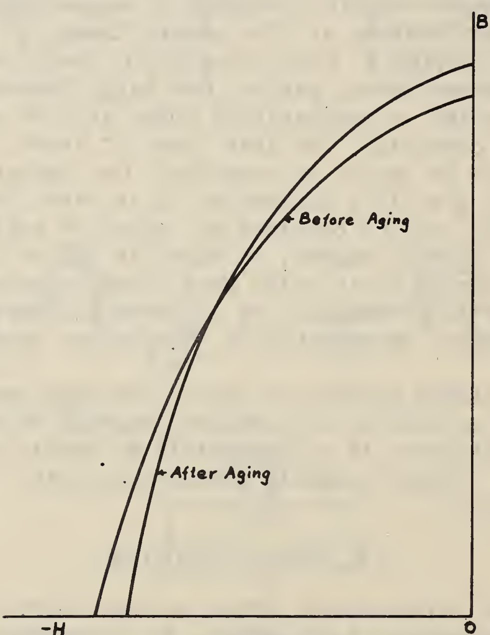

FIGURE 7.—*Effect of aging on the demagnetization curve.*

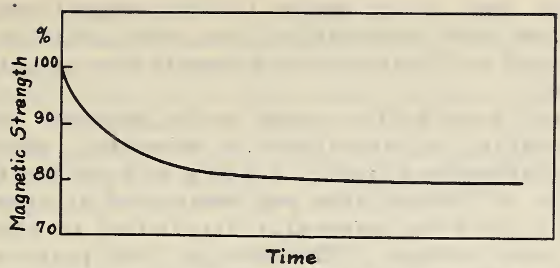

FIGURE 8.—*Decay of magnetization with time.*

The approach to a stable structural condition can be greatly accelerated by artificial aging, or maturing, applied either before or after magnetization. A common method of maturing a magnet is to heat it to 100°C (212°F) the temperature of boiling water, and keep it at this temperature for several hours. The time required depends upon circumstances, but ordinarily 10 or 12 hours is considered to be sufficient. This temperature is convenient to maintain and will not cause serious deterioration which might result from the use of higher temperatures. There is some difference in practice in applying the maturing process. Some instrument makers prefer to subject magnets to temperature cycles such as alternate heating to 100°C (212°F) and cooling to 0°C (32°F) the temperature of melting ice. Others place no reliance in artificial aging and store their magnets after magnetization for periods of from one to two years before use

The degree to which constancy can be attained by either natural or artificial aging depends upon a number of factors not fully understood. The fact that high-grade electrical measuring instruments as a rule maintain their original calibrations over long periods of time would indicate that a high degree of constancy is produced by a suitable maturing process. Experimental data have been published [6], however, which show that aging may proceed at a perceptible rate for as long as ten years even in materials originally subjected to a stabilizing treatment. It seems likely that the aging characteristics of permanent-magnet materials depend to a considerable extent upon the original hardening treatment as well as upon the subsequent stabilizing procedure.

Aging effects are more pronounced in short magnets than in long ones because the point representing the magnetization of a short magnet is in the lower part of the demagnetization curve in which the percentage change due to aging is greatest. Above the crossing point the induction is raised by aging, and the question might be asked as to why a long magnet does not become stronger as the result of aging. This would result, of course, if the magnet were remagnetized after aging, but if the magnet is not remagnetized, the demagnetization curve representing the aged condition does not apply, so that no increase in strength due to aging should be expected.

Structural instability, such as is observed in quench-hardening steels, is very slight or completely absent in the dispersion-hardening alloys. A very slight weakening of magnets made of these alloys has been noted by some investigators, but this is generally attributed to other causes than structural changes. The data on this point are as yet very meager. In general, the dispersion-hardening alloys, if properly treated, withstand deteriorating influences of all kinds much better than do the quench-hardening steels.

Mechanical stabilization.—The effect of mechanical shock or vibration is very slight in the dispersion-hardening alloys. In the quench-hardening materials, however, shock or vibration applied immediately after magnetization tends to reduce the magnetization. Tests are usually made by dropping a magnet a definite distance on to a hard surface, usually wood. The first drop produces the greatest effect, and the effect decreases with each successive impact. After a certain number of impacts, depending upon the material and its structural condition, further impacts do not produce a perceptible change. The total loss due to mechanical shock may amount to from 10 to 25 percent, but depends upon the type and conditions of the material. In general, the harder a material is in the magnetic sense, that is, the higher the coercive force, the less sensitive it is to the effect of mechanical shock. Aging, either natural or artificial, tends to decrease the effect. In the production of magnets in which the highest possible degree of stability is required, it is good practice after final magnetization and aging to subject them to a sufficient number of impacts to bring about a stable condition. The proper number depends upon the nature of the impacts and the type and condition of the material and is best determined by experiment.

Magnetic stabilization.— Structural stabilization does not protect a magnet against the effect of an external magnetic field. It is possible, however, to provide a certain degree of magnetic stabilization by partial demagnetization. The procedure which consists in the application and removal of a demagnetizing force is described in detail later. (See p. 24.)

Thermal stabilization (temperature coefficient).—In common with other physical properties of materials, the strength of a permanent magnet is affected by ordinary changes in temperature; in other words, a magnet has a temperature coefficient. The temperature coefficient is approximately the same for all permanent-magnet materials and amounts to about

-0.02 percent per degree C. B_n decreases with increase in temperature. In order that the coefficient shall be constant for a given magnet, it is necessary to apply a thermal stabilizing treatment after all other stabilizing treatments have been applied. This consists in subjecting the magnet to several cycles of temperature between the limits for which constancy is required. If a magnet is remagnetized for any reason, the temperature cycles must be repeated.

VII. DESIGN

The function of a permanent magnet is to maintain magnetic flux in a magnetic circuit of which it is a part. In general, the useful part of the flux is that in the portion of the magnetic circuit which is external to the magnet.

The principal difficulty in the design of a permanent magnet is due to the lack of a material which can act as an insulator with respect to magnetic flux. Consequently, the main problem is the estimation of the magnetic leakage, or flux maintained by the magnet which does not traverse the gap and thus is not useful. Except in very special and unusual cases, it is not feasible to calculate the leakage with high accuracy. However, it is desirable to make as good an estimate as possible to serve as a guide in determining the best dimensions of magnet to use for a given application. If magnets of a given kind are required in considerable quantity, the calculations should be supplemented by experimental development. On the other hand, if only one or at most only a very few magnets are required, it is likely that more material would be consumed in experimental development than would be wasted by making too liberal an allowance for the uncertainties involved.

In discussing the general principles of permanent-magnet design, the following symbols will be employed:

B_d = induction at a point on the demagnetization curve
B_n = induction at the neutral zone of a magnet
B_{ng} = that portion of the induction at the neutral zone which traverses the gap
$B_{n\lambda}$ = that portion of the induction at the neutral zone which traverses the leakage paths
H_d = magnetizing force at a point on the demagnetization curve
H_n = magnetizing force at the neutral zone
H_g = field strength in the gap
H_s = magnetizing force representing degree of magnetic stability
A_m = area of the magnet
A_g = area of the gap
l_m = length of the magnet

l_g = length of the gap
$\mathscr{P}_g$ = permeance of the gap
$\mathscr{P}_m$ = permeance of the space occupied by the magnet
$\mathscr{P}_{\lambda m}$ = permeance between the limbs of the magnet
$\mathscr{P}_{\lambda p}$ = permeance between the surfaces of the pole pieces not including the gap
Λ = leakage factor
$\tan \delta$ = slope of the shearing line (demagnetizing effect)
$\tan \delta_\lambda$ = slope of the shearing line due to leakage
$\tan \delta_{\lambda m}$ = slope of the shearing line due to leakage from the magnet
V_m = volume of the magnet
a = a parameter used in calculating $\mathscr{P}_{\lambda p}$

The magnetic state of a permanent magnet is determined by the magnetic properties of the material of which it is made, the dimensions of the magnet and its pole pieces, and the magnetizing procedure. The magnetic properties of the material are represented by the demagnetization curve with its associated minor hysteresis loops. The effect of the dimensions of the magnet and its pole pieces is represented by a line called a shearing line, as indicated by OS in figure 9. The way in which $\tan \delta$, the slope of the shearing line depends upon the dimensions can be explained most easily by

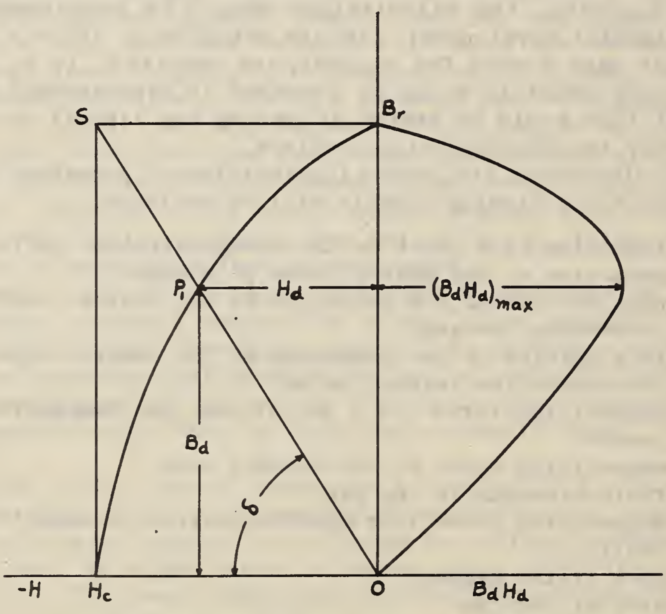

FIGURE 9.—*Shearing line and demagnetization curve.*

considering a magnet with an air gap between pole pieces, as shown in figure 10. For the sake of simplicity, it may be assumed at first that there is no leakage, that is, that all the flux in the magnet also traverses the air gap.

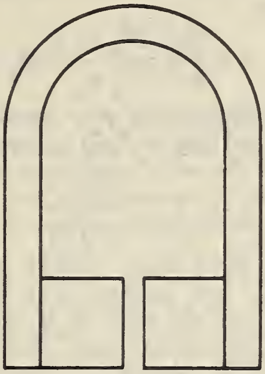

FIGURE 10.—*Ordinary U-shaped magnet with pole pieces.*

Since the magnetomotive force inherent in the magnet is due to the current-turns of the domains within its structure, the only permeability that need be considered is that of free space, which in the cgs system is arbitrarily taken to be unity. If there were no gap in the magnetic circuit, the total inherent magnetomotive force would act to maintain the flux in the magnet material, and the induction would be equal to the residual induction, B_r. However, if a gap is introduced into the magnetic circuit, part of the inherent magnetomotive force is required to maintain the flux in the gap, and that much less is available for the magnet. The effect of the gap, therefore, is to reduce the induction to some value lower than B_r, such as B_d, the ordinate of P_1 in figure 9. If there were no gap in the circuit, the demagnetizing force that would have to be applied by means of an external coil to reduce the induction from B_r to the same value B_d is H_d. The total reverse magnetomotive force then is $H_d l_m$. From this it can be seen that the effect of inserting a gap into the circuit such as would reduce the induction from B_r to B_d is equivalent to the effect of a reverse magnetomotive force equal to $H_d l_m$.

The magnetomotive force required to maintain the flux in the gap is $H_g l_g$, and since its effect is equivalent to the application of the reverse magnetomotive force $H_d l_m$, we can say that

$$H_d l_m = H_g l_g .$$

Since all of the flux in the magnet also traverses the gap,

$$B_d A_m = H_g A_g .$$

The slope of the shearing line, $B_d H_d$, is calculated from these equations to be

$$\tan \delta = \frac{B_d}{H_d} = \frac{A_g l_m}{l_g A_m} = \mathcal{P}_g / \mathcal{P}_m$$

The volume of the magnet, $A_m l_m$, obtained from the equations by solving for A_m and l_m and multiplying is

$$V_m = A_m l_m \frac{H_g^2 A_g l_g}{B_d H_d} ,$$

from which it can be seen that, under the hypothetical condition of no leakage, the volume is a minimum if the dimensions are so chosen as to make $B_d H_d$ a maximum, as shown in the figure. In the practical case, however, this is not a satisfactory basis for design, because it takes account neither of the effect of magnetic stabilization nor of the effect of leakage, which may comprise a considerable part of the total flux in the magnet.

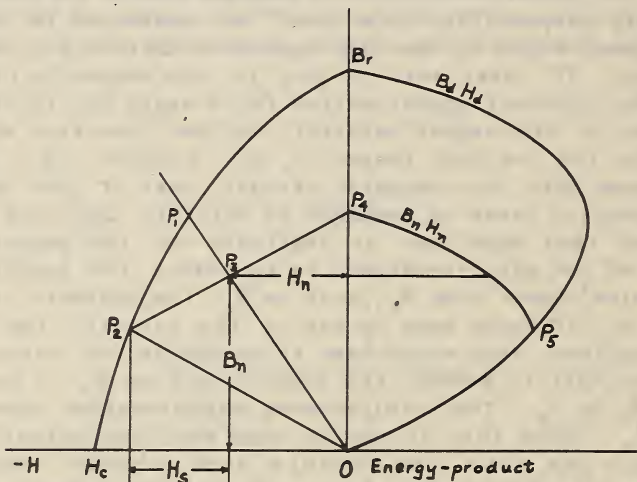

FIGURE 11.—*Effect of partial demagnetization.*

The effect of stabilization by partial demagnetization is indicated in the diagram of figure 11. If a magnet of such proportions that its equilibrium condition would be represented by the point P_1 on the demagnetization curve were to be subjected to an additional demagnetizing force such as to bring the induction down to point P_2 and then the demagnetizing force were removed, the induction would rise from P_2 along a minor hysteresis loop (represented in the figure by a straight line) to the point P_3, where the minor loop and the shearing line intersect. Since the equilibrium point is no longer on the demagnetization curve, its coordinates are not designated as B_d and H_d but by the symbols B_n and H_n. The volume of the magnet is now,

$$V_m = \frac{H_g^2 A_g l_g}{B_n H_n}.$$

The volume is a minimum when the product $B_n H_n$ is a maximum. The energy-product curve corresponding to the minor loop is the line $P_4 P_5$. The maximum value is at P_5. But this corresponds to a point on the demagnetization curve, so that the magnet can have a minimum volume only for a point on the demagnetization curve. What this means is that magnetic stabilization must be paid for by adding to the volume of the magnet. The greater the degree of magnetic stabilization required, the greater must be the volume of the magnet over and above what would be needed if there were to be no stabilization.

It is not necessary to apply a reverse magnetizing force by means of an external coil to obtain magnetic stabilization. The slope of the shearing line for a magnet before the pole pieces are installed is less than the slope of the shearing line, when the pole pieces are in place. Referring to figure 11, if the shearing line for the magnet without pole pieces is OP_2, the equilibrium point when the magnet is magnetized before the pole pieces are added is at P_2. Adding the pole pieces changes the shearing line to OP_1 and the corresponding equilibrium point is at P_3, the intersection of line OP, and the minor loop $P_2 P_4$. Thus the magnetic stability, H_s, is obtained automatically by magnetizing the magnet before putting the pole pieces in place. The degree of magnetic stability thus obtained is determined by the dimensions of the magnet and the air gap.

In the practical design of a permanent magnet it is necessary to take into account the effect of magnetic leakage. The amount of leakage depends upon the dimensions and form of the magnetic circuit. It may comprise anywhere from 25 to 90 percent of the total flux in the neutral zone of the magnet, according to circumstances. The principal difficulty in the way of calculating the amount of the leakage is that the leakage paths are not of uniform section, so that it is necessary to have recourse to certain empirical relationships and simplifying assumptions which have been found to yield sufficiently close approximations for practical purposes.

It has been shown above that the magnetomotive force in the part of the magnetic circuit external to the magnet is equal to $H_d l_m$ if the equilibrium point is on the demagnetization curve, or $H_n l_m$ if it is on a minor loop. In order to take into account the effects of leakage, it will be convenient to consider the external part of the magnetic circuit as consisting of three parallel branches jointly acted upon by the magnetomotive force $H_n l_m$. One branch is the air gap, another is the path from one limb of the magnet to the other, and the third is the path from the surface of one pole piece to the surface of the other, excluding the gap itself. The corresponding permeances are designated as $\mathcal{P}_g$, $\mathcal{P}_{\lambda m}$, and $\mathcal{P}_{\lambda p}$, respectively.

The total flux in the neutral zone of a magnet is equal to the sum of the fluxes in the gap and in the leakage paths. Since we are considering the gap and the leakage paths as parallel circuits, the total flux will be equal to the magnetomotive force multiplied by the sum of the permeances. That is,

$$\phi_n = B_n A_m = H_n l_m (\mathcal{P}_g + \mathcal{P}_{\lambda m} + \mathcal{P}_{\lambda p}) .$$

We have seen previously that the slope of the shearing line, tan δ, is equal to the ratio of B_n to H_n. Hence

$$\tan \delta = \frac{B_n}{H_n} = \frac{l_m}{A_m} (\mathcal{P}_g + \mathcal{P}_{\lambda m} + \mathcal{P}_{\lambda p})$$

or since

$$\frac{A_m}{l_m} = \mathcal{P}_m$$

we can write

$$\tan \delta = \frac{B_n}{H_n} = \frac{\mathcal{P}_g}{\mathcal{P}_m} + \frac{\mathcal{P}_{\lambda m}}{\mathcal{P}_m} + \frac{\mathcal{P}_{\lambda p}}{\mathcal{P}_m} ,$$

from which it is seen that tan δ consists of three parts, $\mathcal{P}_g/\mathcal{P}_m$, $\mathcal{P}_{\lambda m}/\mathcal{P}_m$, and $\mathcal{P}_{\lambda p}/\mathcal{P}_m$. This is illustrated graphically in figure 12.

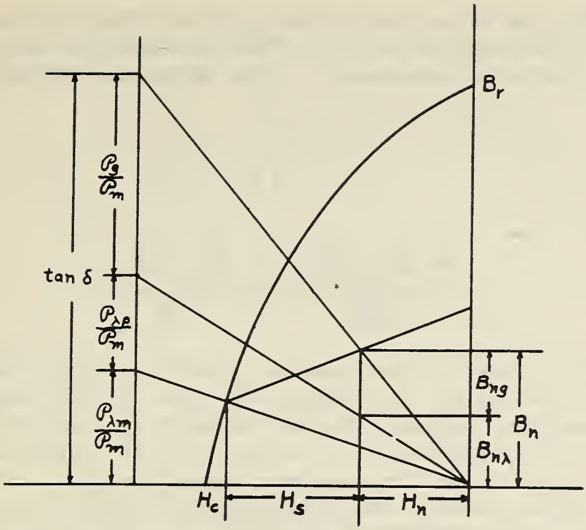

FIGURE 12.—*Permanent-magnet diagram.*

From the equation, it can be seen that the total flux, and consequently the induction, in the neutral zone is divided between the gap and the leakage paths in direct proportion to the respective permeances. This is shown graphically in the figure. Since the areas of the magnet and gap are not necessarily equal, the field strength in the gap has to be calculated from the equation

$$H_g = \frac{B_{ng} A_m}{A_g},$$

or it could be obtained from the equation

$$H_g = \frac{H_n l_m}{l_g}.$$

The figure illustrates the condition existing if magnetic stabilization is automatic. If the magnet were magnetized with the pole pieces in place and then stabilized by the application of a suitable demagnetizing force, the value of B_n would be determined by the intersection of a different minor loop with the shearing line whose slope is tan δ, and the magnetic stabilization, H_s, would have a different value. The relative proportions of the flux in the gap and the leakage paths, however, would be the same.

The ratio of the total flux to the flux in the gap is called the leakage factor, Λ. This is equal to the ratio of the total external permeance to the permeance of the gap.

$$\Lambda = \frac{B_n}{B_{ng}}$$

or

$$\Lambda = \frac{\mathcal{P}_g + \mathcal{P}_{\lambda m} + \mathcal{P}_{\lambda p}}{\mathcal{P}_g} = 1 + \frac{\mathcal{P}_{\lambda m} + \mathcal{P}_{\lambda p}}{\mathcal{P}_g}.$$

In order to apply the equation quantitatively, it is necessary to determine the values of the various permeances. If the so-called fringing flux is included in the pole-piece leakage, the permeance of the gap can be taken simply as $\mathcal{P}_g = A_g / l_g$. Since the leakage paths are not of uniform cross-sectional area, the calculation is not so simple and it is necessary to have recourse to certain empirical methods of approximation. Scott [12] has found experimentally that, provided the poles are not too close together, the self-demagnetizing effect of a magnet of curved form is practically the same as that of a straight bar having the same length and cross-sectional area. This is equivalent to saying that the permeance of the external part of the magnetic circuit is the same in both cases. Also, Picou [9] has shown that the leakage permeance associated with the magnet alone is not changed by the addition of the pole pieces. It has been found experimentally that to a sufficiently close approximation for the present purpose,

$$\mathcal{P}_{\lambda m} = \frac{l_m}{2}.$$

Although various methods of flux mapping or empirical equations may be employed for determing $\mathcal{P}_{\lambda p}$, the equations developed by Finnis and published by Evershed [3] appear to be as convenient as any. The equations were developed originally to give the permeance between the cylindrical surfaces of two cylinders placed end to end with a short air gap between the ends. They can be adapted to pole pieces of rectangular section by substituting for the rectangle a circle having the same perimeter. In the case of the usual pole-piece arrangement for an electrical measuring instrument in which there is a cylindrical core inside a cylindrical gap, a fair approximation can be obtained by considering the length of one pole piece to be half the distance between the ends of the magnet minus half the total length of the gap. If we denote the perimeter of the rectangle as p and the length of one pole piece as l_p, the equations are

$$a = \frac{p}{2\pi} \log_e \left[1 + 2\frac{\sqrt{(l_p^2 + l_p l_g)}}{l_g} \right]$$

If $a > l_p$,

$$\mathscr{P}_{\lambda p} = \frac{\pi\sqrt{(a^2 - l_p^2)}}{\cos^{-1}(l_p/a)}$$

if $a = l_p$,

$$\mathscr{P}_{\lambda p} = \pi l_p$$

if $a < l_p$,

$$\mathscr{P}_{\lambda p} = \frac{\pi\sqrt{(l_p^2 - a^2)}}{\log_e \left[\frac{l_p + \sqrt{(l_p^2 - a^2)}}{a} \right]}$$

An approximate check on this method of analysis has been obtained by applying it to several actual magnets in commercial electrical measuring instruments. The demagnetization curves for these magnets were not available, but, since the magnets were made during the period when 5.5-percent-tungsten steel was the standard permanent-magnet material, a typical demagnetization curve for this type of material was used. It was also assumed that magnetic stabilization was automatic. The results of this check were quite reassuring. The procedure for one of these magnets was as follows:

The magnet had the following dimensions in centimeters:

$A_m = 3.02.$ $A_g = 11.2.$

$l_m = 34.7.$ $l_g = 0.255.$ $V_m = 104.8\ cm^3.$

Since part of the length of the magnet which is in contact with the pole pieces is not effective, l_m was reduced by the length of the pole piece in contact with the magnet, 3.5 cm, so that the value used for l_m was 31.2 cm. From these dimensions, the following values were calculated:

$\mathscr{P}_m = A_m/l_m = 0.0968$	$\mathscr{P}_g/\mathscr{P}_m = 453.5$
$\mathscr{P}_g = A_g/l_g = 43.9$	$\mathscr{P}_{\lambda p}/\mathscr{P}_m = 103.3$
$\mathscr{P}_{\lambda m} = l_m/2 = 15.6$	$\mathscr{P}_{\lambda m}/\mathscr{P}_m = \underline{161.2}$
$\mathscr{P}_{\lambda p} = 10$	$\tan\delta = 718.0$
	$\tan\delta_\lambda = 264.5$

$\mathcal{P}_{\lambda p}$ was calculated from the Finnis formula given on the preceding page.

The graphical solution is illustrated in figure 13. The slope of the line OS is tan δ, or 718. The slope of line OS_λ is $\mathcal{P}_{\lambda p}/\mathcal{P}_m + \mathcal{P}_{\lambda m}/\mathcal{P}_m$, or $\tan \delta_\lambda = 264.5$. The slope of the line $OS_{\lambda m}$ is 161.2. The induction in the magnet on open circuit after magnetization is at point $\mathcal{P}_1$, the intersection of $OS_{\lambda m}$ with the demagnetization curve.

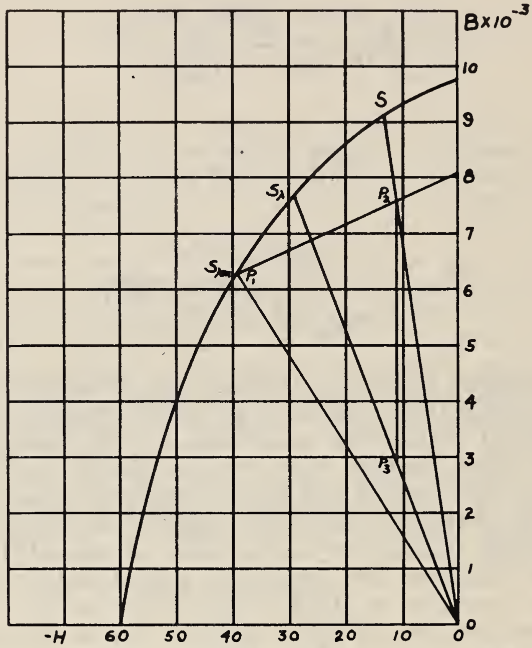

FIGURE 13.—*Permanent-magnet diagram for an instrument magnet.*

When the pole pieces are put in place, the induction rises along the minor loop to the point $\mathcal{P}_2$, where the minor loop intersects the line OS. At this point, $H_n = 10.6$. From this we can calculate H_g directly from the relation,

$$H_g = \frac{H_n l_m}{l_g},$$

or

$$H_g = \frac{10.6 \times 31.2}{0.255} = 1{,}297 \text{ oersteds.}$$

As a check, we can calculate from the corresponding values of induction

$$B_n = 7{,}610$$
$$B_{n\lambda} = 2{,}800$$
$$B_{ng} = 4{,}810$$

$$H_g = \frac{B_{ng} A_m}{A_g} = \frac{4{,}810 \times 3.02}{11.2} = 1{,}297 \text{ oersteds.}$$

The degree of magnetic stability is

$$H_s = H_d - H_n = 39 - 10.6 = 28.4 \text{ oersteds,}$$

and the leakage factor is

$$\Lambda = \frac{B_n}{B_{ng}} = \frac{7{,}610}{4{,}810} = 1.58.$$

The observed value of H_g for this magnet was 1,450 oersteds, or about 10 percent greater than the calculated value, which is fully as good agreement as could be expected under the circumstances.

When leakage is taken into account, the volume of the magnet is

$$V_m = \frac{\Lambda H_g^2 A_g l_g}{B_n H_n}$$

The minimum value of V_m for a given value of H_g now depends not only upon the product $B_n H_n$ but also on the value of Λ. If automatic stabilization is employed, the value of Λ tends to increase if the dimensions are altered so as to increase the value of $B_n H_n$, so that if a magnet is so proportioned as to give the required value of H_g, any other set of dimensions which give the same H_g will lead to about the same volume. However, if a given value of magnetic stabilization, H_s, is adopted and stabilization is by the application of the required value of demagnetizing force after magnetization with the pole pieces in place, it is possible to design for minimum volume. Moreover, the design procedure is considerably more simple.

In figure 14 are shown a demagnetization curve and a series of minor loops represented by their axes. Underhill [37] has shown that, to a sufficiently close approximation for the purpose, the slope of the minor loops can be taken to be equal to the slope of demagnetization curve at the

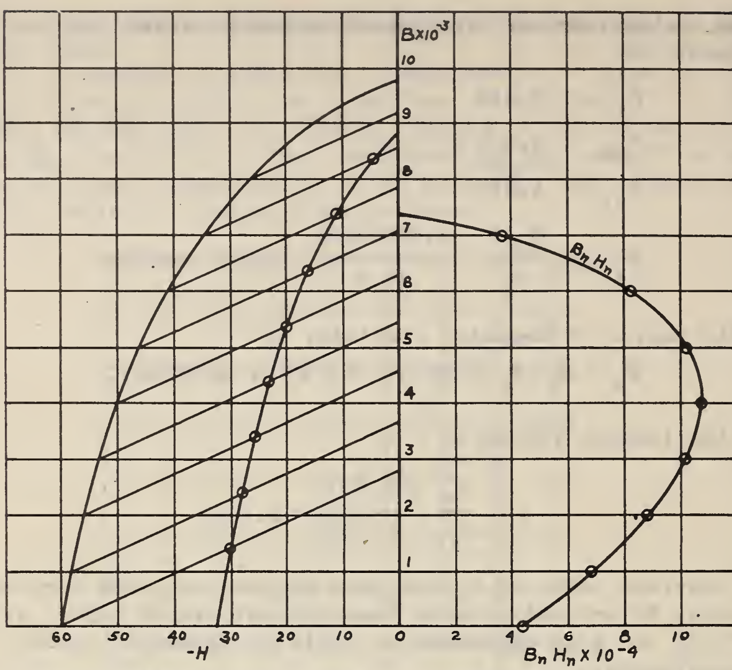

FIGURE 14.—*Energy-product curve for constant magnetic stability.*

point B_r. An auxiliary curve is drawn through the points on the minor loops at which H_s has the same value. In this case, H_s is 30 oersteds. The corresponding energy-product curve is plotted at the right. The ordinates are the values of B_d on the demagnetization curve at which the corresponding minor loops originate. It can be seen that for a given value of H_s, the energy product is a maximum for a certain value of B_d. In figure 15 are curves showing the values of the product B_nH_n for different values of H_s plotted against the corresponding values of B_d. From these curves, the curve of figure 16 is plotted. This shows the relation between B_d and H_s for maximum product B_nH_n. The values of B_d on this curve are the values to which partial demagnetization must be carried to give the corresponding values of H_s with the minimum volume of material. Similar curves could be constructed for other materials having different demagnetization curves.

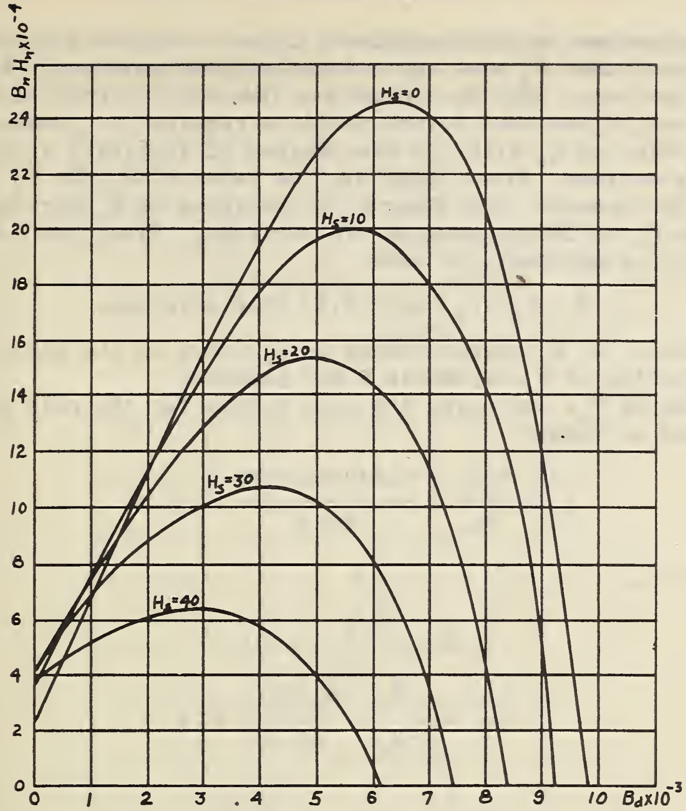

FIGURE 15.—*Energy-product curves for various degrees of magnetic stability.*

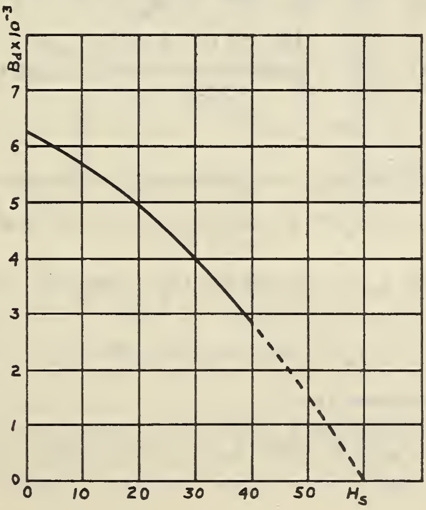

FIGURE 16.—*Relation between magnetic stability and lower tip of minor hysteresis loop.*

In the case of the instrument magnet considered above, it was found that H_g was approximately 1,300 oersteds, H_s was 28.4 oersteds, and the volume was 104.8 cm^3. It will be of interest to see what volume would be required to produce the same value of H_g with the same degree of stability if B_nH_n is made a maximum. From figure 16, the value of B_d for $H_s=28.4$ is 4,250 gausses. From figure 17, the value of H_d corresponding to $B_d=4{,}250$ gausses is 49 oersteds. From this, since H_s is 28.4 oersteds, we have

$$H_n=H_d-H_s=48-28.4=20.6 \text{ oersteds.}$$

The value of B_n corresponding to $H_n=20.6$ on the minor loop originating at $B_d=4{,}250$ is 5,580 gausses.

Knowing H_n, and using the same values for the pole pieces and gap as before,

$$l_m=\frac{H_g l_g}{H_n}=\frac{1{,}300\times 0.255}{20.6}=16.1.$$

from this

$$\mathcal{P}_{\lambda m}=\frac{l_m}{2}=8.05$$

$$\tan\delta=\frac{B_n}{H_n}=\frac{5{,}580}{20.6}=271$$

but

$$\tan\delta=\frac{1}{\mathcal{P}_m}(\mathcal{P}_g+\mathcal{P}_{\lambda p}+\mathcal{P}_{\lambda m})=\frac{1}{\mathcal{P}_m}(43.9+10+8.05)$$

$$\mathcal{P}_m=\frac{A_m}{l_m}=\frac{43.9+10+8.05}{271}=0.229$$

and

$$A_m=0.229l_m=0.229\times 16.1=3.68 \text{ cm}^2$$

$$V_m=A_m l_m=3.68\times 16.1=59.2 \text{ cm}^3.$$

Adding 3.5 cm to l_m to account for inactive material as was done before,

$$l_m=16.1+3.5=19.6$$

and the total volume is

$$V_m=3.68\times 19.6=72.1 \text{ cm}^3.$$

Thus we have the same values of H_g and H_s as before but use about 31 percent less material.

As a check on these calculations, we have

$$\mathcal{P}_g/\mathcal{P}_m = 192 \qquad B_n = 5{,}580$$
$$\mathcal{P}_{\lambda p}/\mathcal{P}_m = 43.8 \qquad B_{n\lambda} = \underline{1{,}620}$$
$$\mathcal{P}_{\lambda m}/\mathcal{P}_m = \underline{35.2} \qquad B_{ng} = 3{,}960$$
$$\tan\delta = 271.0 \qquad H_g = \frac{3{,}960 \times 3.68}{11.2} = 1{,}300$$
$$\tan\delta_\lambda = 79.0$$
$$\Lambda = \frac{5{,}580}{3{,}960} = 1.41$$

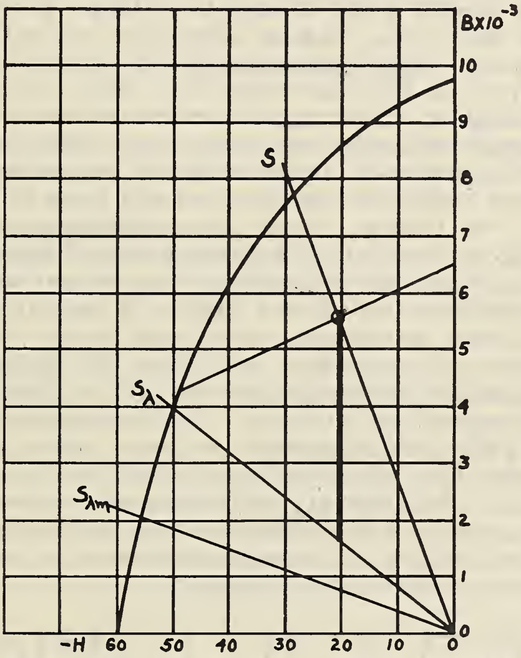

FIGURE 17.—*Permanent-magnet diagram for alternative design of instrument magnet.*

Although for a given performance this method of design required a smaller volume of material than that based upon automatic stabilization, magnetization and stabilization by partial demagnetization must be done after the pole pieces are placed in position. If they should subsequently be removed and then replaced, the field strength in the gap would be permanently reduced because the material would then be working on a lower minor hysteresis loop. The original strength can be restored only by repeating the magnetization and stabilizing treatment with the pole pieces in place. This is an important point to consider in connection with manufacturing procedure.

If the air-gap permeance is not constant, as in magnetos for instance, the quantities will have to be determined for the two conditions, maximum $\mathcal{P}_g$ and minimum $\mathcal{P}_g$, and the performance calculated in terms of the change in H_b in going from one value of $\mathcal{P}_g$ to the other. If, in addition to changing reluctance, a back magnetomotive force results from current induced in a coil rotating in the gap, this must also be taken into consideration.

Regardless of the particular method employed for predetermining the performance of a permanent magnet, the accuracy of the result will depend to a large extent upon the accuracy to which the leakage permeances are estimated. It is at this point that experience is most essential. This is particularly true if some other form than the conventional U-shape is adopted. The high coercive forces of the newer types of permanent-magnet materials often lead to designs in which short lengths and large areas are indicated. In such cases most of the magnetic circuit may have to be made of soft iron. The leakage under these circumstances will depend not only on the relative proportions of magnet and soft iron but also upon the location of the magnet material, as indicated in figure 18. In the figure, *M* represents the magnet and the rest of the circuit is soft iron. The arrangement shown at (*a*) obviously will have the largest leakage factor because of the large surface of the soft iron parts between the magnet and the gap. The arrangement at (*b*) is better because leakage from the lower piece of soft iron does not take flux from the gap. The best arrangement is shown at (*c*). In general, in a composite circuit of this kind the magnet is more effective the nearer it is to the gap. The estimation of leakage permeances in the different cases requires considerable judgment.

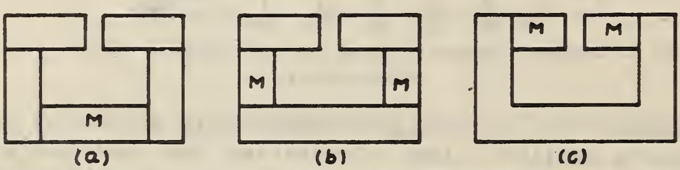

FIGURE 18.—*Alternative arrangements of magnet and soft iron in a magnetic circuit.*

Since the leakage factor is a function of mechanical dimensions only, this point can be checked experimentally, provided that the construction is such that the total flux in the neutral zone and that in the gap can be measured. If this test shows a satisfactory agreement between the calculated and measured values of leakage factor, any discrepancy between the calculated and measured value of field strength in the air gap must be attributed to other factors, such as differences between the demagnetization curves of the test specimen and the actual material in the magnet, failure to produce practical saturation throughout the whole of the magnet in the magnetizing process (a common difficulty) or the use of the wrong slope for the minor hysteresis loops. In any event, it should be remembered that any method of predetermination can only be expected to give a fair approximation. However, even a fair approximation should be a valuable guide in the practical design of a permanent magnet for a given purpose.

VIII. TESTING METHODS

The testing of permanent-magnet materials for determining their demagnetization curves is pretty well standardized [25] and need not be discussed here. However, there are some principles involved in the testing of finished magnets which merit some consideration.

It is a quite common practice to judge the strength of a magnet roughly by the force of attraction between the magnet and a piece of soft iron. If the purpose for which the magnet is to be used is to attract or hold an armature, this is a logical method of test, and the attractive force may be measured quantitatively for the purpose of comparing magnets of the same size and shape. However, if the magnet is to be used for any other purpose, the application of such a test may not only give unsatisfactory results but also may prove to be detrimental to the magnet. The characterization of a permanent-magnet material by stating the weight a magnet can lift as related to its own weight is usually misleading, because this relationship is a function not only of the magnetic quality of the material but also of the size and shape of the magnet and the nature of the contact between the magnet and the mass lifted.

Magnets are customarily tested by either one of two methods. One method consists in determining the flux in the neutral zone by quickly removing a test coil connected to a ballistic galvanometer or fluxmeter. The other method consists in determining the field strength in the gap of special pole pieces to which the magnet is applied. The field strength may be measured in terms of the deflection of a pivoted coil in which a definite current is flowing, the emf induced in an armature rotated at a definite speed, or by a test coil and ballistic galvanometer. A bismuth spiral might also be used, but this is less satisfactory than any of the other methods.

Unless the special pole pieces have the same permeance as the pole pieces to which the magnet is to be applied in use, neither of these methods duplicates the conditions under which the magnet is to function and, consequently, the results must be interpreted with care. If the magnet is designed in such a way as to require magnetization with pole pieces in place, with subsequent magnetic stabilization by partial demagnetization, it will be necessary to remagnetize after testing. Otherwise, the required strength will not be obtained in the finished apparatus. Furthermore, a single test will give no indication of whether or not the magnet has been properly aged. Thermal stabilization must be applied to the finished apparatus after all other treatments have been applied and tests as to its effectiveness must be made by temperature tests on the finished apparatus.

IX. REFERENCES

The following list of references (in some cases not cited in the text) is by no means complete but includes the majority of the sources from which the information summarized in the foregoing was obtained.

[1] S. P. Thompson, *The magnetism of permanent magnets*, J. Inst. Elec.. Engrs. (London) **50**, 80, (1913).

[2] K. Honda and S. Saito, *On KS magnet steel*, Phys. Rev. **16** (pt.II) 495, (1920).

[3] S. Evershed, *Permanent magnets in theory and practice*, J. Inst. Elec. Engrs. (London) **58**, 780, (1920); **63**, 725 (1925).

[4] E. A. Watson, *Permanent magnets and the relation of their properties to the constitution of magnet steel*, J. Inst. Elec. Engrs. (London) **61**, 641 (1923).

[5] W. Cramp and N. I. Calderwood, *The calculation of air-space flux*, J. Inst. Elec. Engrs. (London) **61**, 1061 (1923).

[6] R. L. Dowdell, *Investigation of the treatment of steel for permanent magnets*, Trans. Am. Soc. Steel Treating **5**, 27, (1924); Trans. Am. Soc. Metals **22**, 19 (1934).

[7] J. R. Adams and F. E. Goeckler, *Some factors affecting coercive force and residual induction of some magnet steels*, Trans. Am. Soc. Steel Treating **10**, 173 and 213 (1926).

[8] R. L. Sanford, *Some principles governing the choice and utilization of permanent magnet steels*, BS Sci. Pap. **22**, 557 (1927) S567.

[9] R. V. Picou, *Calculdes appereils magnetiques*, Rev. gen. elec. **23**, 259 (1928).

[10] P. H. Brace, *Cobalt magnet steel*, Elec. J. **26**, 111 (1929).

[11] J. E. Gould, *The ageing of permanent magnet steels*, Proc. World Eng. Cong., Tokyo, **34**, 273 (1929).

[12] K. L. Scott, *Magnet steels and permanent magnets*, Elec. Eng. **51**, 320 (1932).

[13] T. Mishima, *Magnetic properties of iron-nickel-aluminum alloys*, Ohm (July 1932). Abstracted in Iron Age **130**, 346 (1932).

[14] K. S. Seljesater and B. A. Rogers, *Magnetic and mechanical hardness of dispersion hardened iron alloys*, Trans. Am. Soc. Steel Treating **19**, 553 (1932).

[15] W. Koster, *Dauermagnetwerkstoffe auf der grundlage der ausscheidungs hartung*, Stahl U. Eisen **53**, 849 (1933).
[16] Y. Kato and I. Tokei, *Permanent oxide magnet and its characteristics*, J. Inst. Elec. Engrs. (Japan) **53**, 408 (1933).
[17] E. Hughes, *Design of permanent magnets*, Elec. Rev. **112**, 410 (1933).
[18] W. C. Ellis and E. E. Schumacher, *A survey of magnetic materials in relation to structure*, Metals & Alloys **5**, 269 (1934); **6**, 26 (1935).
[19] K. Honda, H. Masumoto and Y. Shirakawa, *On new KS permanent magnet steel*, Science Reps. Tohoku Imperial Univ. (S) **23**, 365 (1934).
[20] J. L. Gregg, Alloys of Iron and Tungsten, (McGraw-Hill Book Co. Inc., New York, N. Y., 1934).
[21] K. L. Scott, *Steels and alloys for permanent magnets*, Metal Progress **30**, 64 (1932).
[22] A. C. Whiffen, *Effect of change of temperature on the strength of permanent magnets, with special reference to modern magnet steels*, J. Inst. Elec. Engrs. (London) **81**, 727 (1937).
[23] H. W. Lord, *A Half-cycle magnetizer with thyratron control*, Gen. Elec. Rev. **40**, 418, (1937).
[24] A. B. Kinzel and W. Crafts, Alloys of Iron and Chromium (McGraw-Hill Book Co. Inc., New York, N. Y., 1937).
[25] R. L. Sanford, *Magnetic testing*, NPS Cir. C415 (1937).
[26] C. E. Webb, *Recent developments in magnetic materials*, J. Inst. Elec. Engrs. (London) **82**, 303 (1938).
[27] J. Q. Adams, *Alnico—Its properties and possibilities*, Gen. Elec. Rev. **41**, 518 (1938).
[28] A. J. Hornfeck and R. F. Edgar, *The output and optimum design of permanent magnets subjected to demagnetizing forces*, Trans. Am. Inst. Elec. Engrs. **59**, 1017 (1940).
[29] Anonymous, *Important new magnetic alloy (viçalloy)*, Bell. Labs. Record **19**, 36 (1940).
[30] J. F. Kayser, *Permanent magnets*, Engineer **170**, 183 (1940).
[31] I. F. Kinnard and J. H. Goss, *Braking magnets for watt-hour meters*, Trans. Am. Inst. Elec. Engrs. **60**, 431 (1941).
[32] C. Wainwright, *Spoiling of tungsten magnet steels*, J. Sci. Instruments **18**, 97 (1941).
[33] J. H. Goss, *Permanent magnets*, Metals & Alloys **15**, 576 (1942)
[34] B. M. Smith, *Alnico, properties and equipment for magnetization and test*, Gen. Elec. Rev. **45**, 210 (1942).
[35] M. S. Wilson and J. M. Whittenton, *Influence of improved magnetic alloys on design trends of electrical instruments*, Am. Inst. Elec. Engrs. Tech. Paper, p. 44-31, (Dec. 1943).
[36] E. M. Underhill, *Permanent magnet design*, Electronics **16**, 126 (1943).
[37] E. M. Underhill, *Designing stabilized permanent magnets*, Electronics **17**, 118 (1944).
[38] R. M. Bozorth, *Present status of ferromagnetic theory*, Elec. Engng. **54**, 1251 (1935); The physical basis of ferromagnetism, Bell System Tech. J. **19**, 1 (1940).

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