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PHOTOELECTRIC TRISTIMULUS COLORIMETRY WITH THREE FILTERS

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U. S. DEPARTMENT OF COMMERCE
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PHOTOELECTRIC TRISTIMULUS COLORIMETRY WITH THREE FILTERS

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PREFACE

The photoelectric cell is finding ever-widening usage in scientific apparatus. The present Circular describes the development and use of an "artificial eye for color measurement", which consists of a barrier-layer photocell and three selected spectral filters. The distinguishing feature of a photoelectric tristimulus colorimeter is the approximate spectral equivalence of the filter-photocell combinations of the apparatus and the tristimulus specifications of the spectrum which characterize the color vision of the average normal observer. Because of this approximate equivalence, the new apparatus will respond to color differences in much the same manner as the normal human eye. Excepting cases in which the present lack of ideal spectral equivalence introduces errors which cannot be tolerated, the photoelectric tristimulus colorimeter described herein can be used to measure both the size and the character of color differences. The potential speed and relative simplicity of the apparatus suggest its use in quantitative studies of color tolerance. Whenever suitable color standards are available, the same advantages recommend the use of the apparatus for tests of materials for compliance with color specifications.

LYMAN J. BRIGGS, *Director.*

PHOTOELECTRIC TRISTIMULUS COLORIMETRY WITH THREE FILTERS[1]

By Richard S. Hunter

ABSTRACT

The term "Photoelectric colorimetry" is commonly employed to designate both photoelectric tristimulus colorimetry, used to evaluate the appearance of materials, and abridged spectrophotometry, often used to assist in chemical analyses. This paper is devoted to the first type of measurement.

For a photoelectric tristimulus colorimeter, it is desired to find three or more source-filter-photocell combinations of such spectral character that they duplicate the standard ICI observer for colorimetry. With an instrument having these combinations, tristimulus values would be obtained by direct measurement. Although no one has duplicated the ICI observer perfectly, several investigators have obtained source-filter-photocell combinations suitable for the measurement of color differences between spectrally similar samples.

To measure color differences as small as those which the trained inspectors of paint, textile, plastic, paper, and ceramic products can see, an instrument must have high precision. If the needed precision is available, a photoelectric tristimulus colorimeter may be used to measure: (1) ICI colorimetric values, x, y, and Y, relative to those of a spectrally similar, calibrated standard; (2) relative values of α and β, components of the chromaticity departure from neutral in a new uniform-chromaticness-scale mixture diagram for representing surface colors; (3) amounts of color difference between pairs of spectrally similar samples; (4) amounts of color change accompanying fading; and (5) whiteness of white and near-white surfaces.

In giving examples of the measurement of some of these different properties and in describing the errors of color measurement to which the tristimulus method is subject, reference is made to operations with the author's recently developed multipurpose photoelectric reflectometer.

CONTENTS

[1] A preliminary brief account of this project was presented as part of the ASTM-ISCC joint Symposium on Color held in Washington, March 5, 1941 [19].

I. INTRODUCTION

The possibility of using photocells to measure color has attracted widespread attention ever since the photoelectric cell became available in practical form. Photoelectric cells have now been used for a number of years in spectrophotometry to measure the spectral reflective and transmissive properties of materials. From the spectrophotometric curves of samples, one may obtain the tristimulus values of color by multiplying the separate spectral values of any sample by the three distribution functions of the standard observer [2] and totaling the three sets of products throughout the visible spectrum (see eq. 3, below). Although the use of photocells has considerably diminished the time needed to obtain spectrophotometric curves, and although devices such as the one developed by Sears [43] [3] are available to facilitate the integration, a considerable expenditure of time and effort is still required [29 p. 259] to obtain the accurate colorimetric specification of a sample by the method of spectrophotometry and integration.

Photoelectric tristimulus colorimetry is direct and rapid, because the result of integration with respect to wavelength is found automatically by the use of specially chosen source-filter-photocell combinations (see eq. 5, below). With this type of photoelectric colorimeter, the tristimulus specification of a sample is found by settings upon it using in succession each of the three or more filters in an instrument. This direct method of color measurement has at present somewhat limited application, because the best available source-filter-photocell combinations fail to be spectrally equivalent to the desired combination of the ICI standard observer and standard illuminant for colorimetry. Nevertheless there are many problems of color measurement to which the shorter procedure may be advantageously applied, and these will doubtless increase in number as research progresses.

Photoelectric tristimulus colorimetry is frequently confused with photoelectric abridged spectrophotometry [11], and both are popularly called photoelectric colorimetry. Therefore, a short discussion

[2] For descriptions of the ICI standard observer for colorimetry, see references [15, 20, 22, 25, 31, 33 and 45].
[3] Numbers in brackets, either with or without specific page numbers, indicate references listed at the end of this Circular.

of the two experimental methods and of the differences between them is appropriate. Instruments for both procedures require a source, several filters, and a photocell. Instead of the prism or grating used in a true spectrophotometer, three to about eight spectral filters are used in an abridged spectrophotometer to isolate spectral bands. These bands are of wider wavelength ranges and are fewer in number than the bands employed in true spectrophotometry, consequently accurate spectrophotometric curves are not obtained [21]. Nevertheless, an abridged spectrophotometer is often a useful instrument. Frequently, it is known that some particular chemical or physical property of a substance may be studied by using a single spectral range or several spectral ranges which may be isolated by filters. An abridged spectrophotometer may serve adequately for the measurements needed.

A tristimulus colorimeter also requires filters, but it does not primarily measure the variation of any property of samples with respect to wavelength. Instead samples are measured with source-filter-photocell combinations which spectrally duplicate, as nearly as possible, the three distribution functions characterizing the standard observer combined with some standard illuminant. With one source and photocell, three or at most four filters are needed. However, the filters for tristimulus colorimetry differ from those used for abridged spectrophotometry in that they transmit bands of wide wavelength range.

Ives described a thermopile tristimulus colorimeter in 1915 [23], which made use of a spectrum selectively projected through a series of carefully cut templates in place of the filters employed in the more recent devices. Twyman and Perry showed in 1928 [46] that an instrument which would yield direct tristimulus measurements could be obtained by finding source-filter-photocell combinations spectrally similar[4] to the standard-observer, illuminant combination for colorimetry.[5] However, it was not until 1938 that Perry described an actual instrument for photoelectric tristimulus colorimetry. He called his device the Blancometer [40] because of its special suitability for white and near-white samples. Guild described the experimental model of a tristimulus colorimeter in 1934 [14], and Winch and Palmer [50], Dresler and Frühling [4], and Barnes [2], have described devices for the same purpose since that time. Gibson [11] and Van den Akker [47] have discussed the problem of obtaining the source-filter-photocell combinations giving the closest spectral equivalence to the standard observer.

The present paper describes a three-filter method for approximate photoelectric tristimulus colorimetry which has so far been used chiefly in conjunction with the author's multipurpose reflectometer [18] to measure surface color. Tristimulus measurements with this instrument have proved to be valuable in studies of the colors of paints, ceramic products, textiles, papers, pigments, inks and other materials which reflect light. The major part of the present paper is devoted

[4] For a discussion of what is meant by spectrally similar, see footnote 7.

[5] The three ideal source-filter-photocell combinations would each duplicate in spectral response the relative spectral distribution of one of the three functions of the ICI standard observer combined with chosen standard illuminant (as expressed in eq 1, below). A standard illuminant is, in general, relatively nonselective and plays a secondary role in determining the spectral character of the ideal source-filter-photocell combinations. In the remainder of the paper it has sometimes been convenient to speak of spectral duplication of, or spectral similarity to the standard observer when actually spectral duplication of, or spectral similarity to, the standard observer combined with a standard illuminant is meant.

to tristimulus measurements of surface colors, but it should be noted that a number of the methods suggested are equally useful for the study of volume colors by measurements of transmitted energy and for the study of illuminant colors by measurements of emitted energy.

II. TERMS, SYMBOLS, AND DEFINITIONS

It is the purpose of photoelectric tristimulus colorimetry to give by measurement quantities which provide useful information about the appearance of objects and lights. Three different classes of properties must be recognized in the study of the subject: physical, psychological, and psychophysical. Physical properties have no necessary connection with an observer; psychological properties deal directly with the impressions of an observer and are therefore not subject to exact physical measurement; psychophysical properties are an intermediate class which are capable of measurement, but which relate to the impressions of an observer. It is important to distinguish between these three classes of properties and identify the terms which refer to each.

To assist in this differentiation, table 1, which classifies a number of the terms used herein, has been prepared. This table is modeled after figure 1 in the Preliminary Draft of a Report on Nomenclature and Definitions for Colorimetry [24] by L. A. Jones, chairman of the Colorimetry Committee of the Optical Society of America. One term in table 1, chromaticness, has been added to accord with the present draft of the uncompleted report of this committee.

TABLE 1.—*Tabular arrangement of some of the terms used to describe the color of a surface*

Category	Physical	Psychophysical (Physically measured, psychologically significant).	Psychological.
Instrument	Spectrophotometer	Tristimulus colorimeter	Visual mechanism of the observer.
Product of the instrument.	Spectral apparent-reflectance curve.	Designation of the surface color by a set of tristimulus values for it; X, Y, and Z; A, G, and B; or other.	The perceived surface color.
The separate attributes of the surface color, and the perceived surface color.		(1) Luminous apparent reflectance, A_{δ_i}, δ_i (chiefly designated below by Y), the lightness index, L.	(1) Lightness.
		(2) Dominant wavelength, Λ, or hue angle, ϕ.	(2) Hue.
		(3) Purity, p, or saturation index, S	(3) Saturation, or "strength."
		(2 and 3 combined) Chromaticity, indicated either by Λ and p, ϕ and S, or by a pair of tri-linear coordinates, x and y, α and β, etc.	(2 and 3 combined) Chromaticness.

The definitions, and in some cases symbols, for a number of the terms used in the present paper are separated below according to the three categories.

1. PHYSICAL

A *spectrophotometer* is an instrument which measures transmission or apparent reflectance as a function of wavelength.

A *spectrophotometric curve* is a curve giving transmission, T_λ, or apparent reflectance, A_λ, of a sample as a function of wavelength.

2. PSYCHOLOGICAL

A perceived surface color is color experienced as a property of a surface [30]; the three attributes of a perceived surface color are lightness, hue, and saturation.

The lightness of a perceived surface color is that attribute which permits it to be classed as equivalent to some member of the series of grays ranging from black to white [30]. (In the Munsell system, value correlates closely with lightness under usual observing conditions.)

The hue of a perceived color is that attribute which permits it to be classed as red, yellow, green, blue, purple, or an intermediate [30].

Chromatic surfaces are those perceived to possess hue (not to be confused with Munsell chroma) [30].

Achromatic surfaces are those such as whites, grays, and blacks which are perceived to possess no hue [30].

Saturation is that attribute of a perceived surface color which determines the degree of its difference from the gray of the same lightness [30]. (Munsell chroma correlates closely with saturation under usual visual conditions.)

The chromaticness of a perceived color is determined by its hue and saturation taken together.

3. PSYCHOPHYSICAL

A *tristimulus colorimeter* is a device which measures a color stimulus in terms of three selected stimuli called primaries. The ideal source-filter-photocell combinations for a photoelectric tristimulus colorimeter are those which are spectrally equivalent to the 1931 ICI standard observer for colorimetry in combination with an appropriate illuminant.

The 1931 ICI standard observer for colorimetry embodies the average colorimetric characteristics of 17 normal observers studied by Guild [13] and Wright [51]. The definition of the standard observer consists of three functions of wavelength, λ, showing the relative amounts ($\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$) of three primary stimuli required to color-match the various parts of the equal-energy spectrum [15, 22, 25, 31, 33, 45].

A tristimulus designation of an unknown stimulus consists of the amounts (such as X, Y, Z) [6] of the three primary stimuli required to produce a color match for it.

Apparent reflectance is the reflectance which a perfectly diffusing surface would need to have in order to produce, under the same angular conditions of illuminating and viewing, the same instrumental effect as the specimen actually measured. Apparent reflectance given as a function of wavelength is designated *spectral apparent reflectance*, $A_{\lambda(\theta i,\theta v)}$, and is most frequently represented by a spectrophotometric curve. Apparent reflectance weighted according to the luminosity function is designated *luminous apparent reflectance*, $A_{(\theta i,\theta v)}$, or Y. For apparent-reflectance values weighted according to other spectral functions, such as those of the source-filter-photocell combinations described below, symbols which identify the functions are used.

The Y component of the tristimulus designation of a color repre-

[6] It should be noted that there has not been universal agreement on the use of these symbols. The National Bureau of Standards has in the past usually used $\bar{x}$, $\bar{y}$, and $\bar{z}$ to represent the tristimulus designation of an unknown stimulus and X, Y, and Z to represent the trichromatic coefficients of the spectrum. In the present paper, however, this former use of symbols has been discarded in favor of the above usage, which is recommended in the present unpublished draft of the OSA Colorimetry Committee report.

sents the luminous component of that color [31, 33]. Thus Y designates the luminous apparent reflectance of a surface color, the luminous transmission of a volume color, and the brightness of an aperture or illuminant color. The methods of measurement described below are in most cases applicable to problems involving volume, aperture, and illuminant colors as well as to the surface colors referred to in the text. For this reason, the symbol Y has been used below instead of A_{θ_i,θ_v} where the text refers to luminous apparent reflectance. It should be noted that the symbol A is used herein to designate settings with the multipurpose reflectometer and amber filter.

The settings, A, G, and B, with the multipurpose reflectometer employing, respectively, the amber, green, and blue filters, are, for a surface color, apparent reflectances for illumination at 45° and viewing at 0° relative to MgO. In practice the instrument is adjusted by means of a working standard calibrated in terms of MgO. Illumination at 45° and viewing at 0° have been internationally adopted as standard conditions for the colorimetry of opaque surfaces [22], because they represent a satisfactory average of the directional conditions under which surfaces are observed in everyday life. Because the combination of source, green filter, and photocell nearly duplicates the standard luminosity ($\bar{y}$) function times ICI illuminant C, readings obtained with the green-filter combination closely duplicate Y. Because the three source-filter-photocell combinations are nearly equivalent spectrally [7] to the ICI standard observer, measurements of A, G, and B of a sample constitute an approximate tristimulus description of its color.

The chromaticity of a color is its characterization by either (1) dominant wave length and purity, (2) hue angle and saturation index, or (3) a pair of trilinear coordinates (x and y, α and β, etc.).

The dominant wavelength, λ[41], and *the hue angle*, ϕ, of a color are both computed from the tristimulus specification to indicate hue.

The purity, p[41], and *the saturation index*, S, of a color are obtained from the tristimulus specification to indicate what is variously called saturation or strength.

The trilinear coordinates (x, y, and z, or α, β, and γ) are the amounts of the three primary stimuli expressed as fractions of their total. Because the three trilinear coordinates of a color are fractions of a total and must necessarily sum to unity, two of them (either x and y, or α and β) are sufficient to define the *chromaticity* of a stimulus.

A chromaticity diagram is a plot according to trilinear coordinates in which position of a point indicates the chromaticity of the stimulus represented.

III. THE SOURCE-FILTER-PHOTOCELL COMBINATIONS

The distinguishing feature of apparatus for photoelectric tristimulus colorimetry is that spectral elements nearly equivalent to the ICI standard observer for colorimetry must be provided. If the device is to be used for the colorimetry of light sources, television screens, or other self-luminous color stimuli, the apparatus will require filters and photocells only. If the samples are to be non-self-luminous

[7] By "nearly equivalent spectrally to the ICI standard observer" is meant that the spectral distributions of the three source-filter-photocell combinations can be changed by transformation equations which are linear and homogeneous [31, p. 345] to spectral functions which nearly duplicate the ICI standard observer combined with illuminant C (see figure 1, below).

objects, such as opaque surfaces or transparent media, a source for illuminating the objects must be added to the filter-photocell combinations. The resulting sets of source, filter, and photocell must then be nearly equivalent in spectral response to the standard observer combined with the desired standard illuminant.

The combinations of elements described in the present paper are those which have been used for tristimulus colorimetry with the author's multipurpose reflectometer [18]. Although used for the most part in this one instrument, the source-filter-photocell combinations described below are applicable to photoelectric tristimulus colorimetry with other photoelectric devices [8] of suitable precision.

If the three spectral response functions of the ICI observer are denoted by $\bar{x}$, $\bar{y}$, and $\bar{z}$ and the spectral energy of the illuminant for equivalent visual analysis is denoted by E_v, then the ideal source-filter-photocell combinations for photoelectric colorimetry according to the ICI observer will satisfy the following equations:

$$\left.\begin{aligned} E_I T_1 s K_1 &= \bar{x} E_v \\ E_I T_2 s K_2 &= \bar{y} E_v \\ E_I T_3 s K_3 &= \bar{z} E_v. \end{aligned}\right\} \tag{1}$$

The source in the instrument is represented by its spectral energy, E_I, the ideal filters by their spectral transmissions, T_1, T_2, and T_3, the photocell by its spectral response, s, and the constants of proportionality between the two sides of the respective equations by K_1, K_2, and K_3.

An instrument was desired which would give numbers corresponding to those furnished by the ICI observer rating objects under ICI illuminant C [22],[9] a standard illuminant which is similar in spectral quality to overcast sky. Accordingly E_C representing ICI illuminant C, was substituted for E_v in eq 1. Since a source of high efficiency and small area is needed in the apparatus, a Mazda projection-type incandescent lamp was chosen. This is operated at approximately 3,100° K, and hence the relative spectral irradiance corresponding to 3,100° K [44] was used for E_I. The General Electric light-sensitive cell [10] was chosen because [10] of the high current it generates for a given illumination and the relatively high response to light of the blue region of the spectrum, and because data giving the spectral response of the cells available for the author's instrument were at hand.[11]

It was recognized that ideal filters satisfying eq 1 would not be found. No attempt was made to duplicate with a single source-filter-photocell combination the complete $\bar{x}$ function with its two components. Instead, the two components were separated as suggested by Gibson [11], and the short-wave component was taken to be spectrally similar to the $\bar{z}$ function.[12] A combintion was then sought

[8] These combinations, for example, have been used in the Lumetron Colorimeter, Photovolt Corporation, 95 Madison Avenue, New York, N. Y.

[9] By using one of the color-temperature altering filters described by Gage [9] or Estey [5] with each of the three source-filter-photocell combinations, it is possible to obtain the effect of raising or of lowering the color temperature of E_v. Thus it is possible to simulate not only visual observations made under ICI illuminant C, but visual observations made under light from a yellow incandescent lamp, or from blue sky.

[10] For a more detailed account of the selection of the photocells used for tristimulus colorimetry in the multipurpose reflectometer, see [18, p. 584, and 18a, p. 538].

[11] Kindly supplied by B. T. Barnes, of the General Electric Co., Nela Park, Cleveland, Ohio.

[12] Guild [14], Perry [40], Dresler and Frühling [4], Barnes [2], and Van den Akker [47] have all pointed out that a fourth source-filter-photocell combination is preferable to using the $\bar{z}$ combination in order to simulate suitably the short-wave component of the $\bar{x}$ function. However, the author is chiefly interested in the measurement of small differences in color with the simplest procedures possible. If three filters will suffice, a fourth adds undue complexity to the measurements. Until important improvements are made in the degree to which the three present filters duplicate the ICI observer, the use of a fourth filter is unwarranted.

to duplicate the long-wave component only. This component of the $\bar{x}E_C$ function is approximated by using an amber filter having a spectral transmission, T_A. The $\bar{y}E_C$ and $\bar{z}E_C$ functions are approximated by using green (T_G) and blue (T_B) filters, respectively.

TABLE 2.—*Spectral character of the individual components and of the source-filter-photocell combinations used for photoelectric tristimulus colorimetry with the multipurpose reflectometer [18]*

Wavelength	Spectral transmissions (Corning filters)			E_I, relative spectral irradiance of source at 3,100° K [44]	s, spectral response of GE cell No. 55 (in microamperes per milliwatt)	Spectral specifiation of the three source-filter-photocell combinations. (Each value of k has been chosen so that the sum of the column is 100,000.)		
	T_A, amber filter of No. 330 yellow glass and No. 978 green glass	T_G, green filter of No. 423 green glass and No. 330 yellow glass	T_B, blue filter of No. 554 blue glass and No. 038 yellow glass			$E_I T_A s k_A$ (amber)	$E_I T_G s k_G$ (green)	$E_I T_B s k_B$ (blue)
mμ								
380	-------	0.010	0.000	0.1188	94.5	-------	31	-------
90	-------	.010	.000	.1425	101.5	-------	39	-------
400	-------	.011	.000	.1688	107.8	-------	56	-------
10	-------	.011	.020	.1979	113.9	-------	70	337
20	-------	.012	.298	.2294	119.5	-------	92	6,109
30	-------	.013	.443	.2634	124.7	-------	120	10,885
40	-------	.017	.478	.2998	129.7	-------	185	13,901
450	-------	.023	.482	.3383	134.2	-------	291	16,368
60	-------	.035	.445	.3790	138.8	-------	514	17,511
70	-------	.050	.338	.4215	143.0	-------	842	15,237
80	-------	.077	.195	.4658	147.2	-------	1,476	10,001
90	0.004	.107	.090	.5114	151.3	96	2,315	5,208
500	.010	.138	.045	.5582	155.5	269	3,350	2,922
10	.020	.173	.015	.6061	159.1	596	4,664	1,082
20	.033	.209	.003	.6548	162.3	1,248	6,210	239
30	.063	.235	.000	.7040	164.9	2,258	7,628	-------
40	.095	.249	-------	.7536	167.0	3,695	8,763	-------
550	.128	.251	-------	.8033	168.4	5,351	9,496	-------
60	.164	.243	-------	.8531	168.7	7,291	9,778	-------
70	.196	.222	-------	.9025	167.4	9,148	9,378	-------
80	.224	.193	-------	.9516	162.9	10,727	8,366	-------
90	.237	.163	-------	1.0000	156.3	11,443	7,125	-------
600	.238	.135	-------	1.0476	147.5	11,363	5,833	-------
10	.227	.109	-------	1.0945	135.4	10,393	4,516	-------
20	.206	.086	-------	1.1403	117.3	8,511	3,216	-------
30	.181	.067	-------	1.1850	97.7	6,472	2,170	-------
40	.153	.053	-------	1.2284	79.1	4,594	1,440	-------
650	.122	.040	-------	1.2704	60.8	2,910	864	-------
60	.099	.032	-------	1.3111	43.7	1,752	512	-------
70	.077	.025	-------	1.3503	28.5	914	268	-------
80	.057	.020	.000	1.3880	18.0	439	140	-------
90	.043	.017	.002	1.4240	11.9	226	81	25
700	.030	.014	.003	1.4584	8.8	117	50	28
10	.022	.012	.004	1.4912	7.2	74	36	32
20	.016	.010	004	1.5223	6.1	46	25	28
30	.011	.009	.005	1.5517	5.8	31	22	34
40	.008	.008	.004	1.5793	5.7	22	20	27
750	.005	.007	.004	1.6055	5.5	12	17	26
					Σ-------	99,998	99,999	100,000

Spectral specifications for the source-filter-photocell combinations chosen, $E_I T_A s k_A$, $E_I T_G s k_G$, and $E_I T_B s k_B$, are given in table 2. Included in the same table are the spectral transmissions of the three filters,[13]

[13] The author is indebted to H. J. Keegan, of this Bureau, for numerous spectral-transmission curves obtained during the course of this work.

the spectral irradiance from a source at 3,100° K., and the spectral response of GE light-sensitive cell No. 55. The values of k were adjusted so that each of the final three columns would total 100,000. Each filter consists of two components, all six components chosen being from stock manufactured by the Corning Glass Works.[14]

The success with which the source-filter-photocell combinations duplicate the ICI standard observer combined with illuminant C is shown graphically in figure 1, in which $\bar{x}E_C$, $\bar{y}E_C$, and $\bar{z}E_C$, which

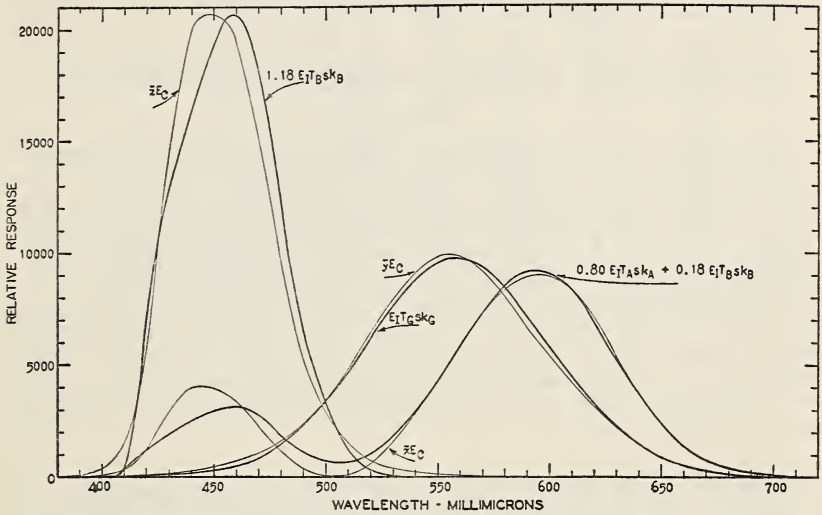

FIGURE 1.—*The ICI observer, represented by the curves $\bar{x}E_C$, $\bar{y}E_C$, and $\bar{z}E_C$, compared to its approximate duplication by the source-filter-photocell combinations used in the multipurpose reflectometer.*

enclose areas of approximately 98,000, 100,000, and 118,000, respectively, are plotted as functions of wavelength together with

$$0.80E_IT_Ask_A+0.18E_IT_Bsk_B,$$

$$1.00E_IT_Gsk_G,$$

and

$$1.18E_IT_Bsk_B,$$

which enclose respectively equal areas. The duplication is far from perfect and can be improved by further work. Also, as already noted,

[14] It is possible to obtain suitable tristimulus filters from Corning glass available at the present writing (October 1941), by using the following designations and thicknesses:

Component	Catalog number	Melt designation	Thickness
			mm
Yellow component of amber filter	326	190	2.42
Green component of amber filter	978	*β_{640}=0.338	1.3
Blue-green component of green filter	428	194	3.65
Yellow component of green filter	330	17	3.30
Blue component of blue filter	554	211	5.00
Yellow component of blue filter	038	206	1.5

*This Corning β value bears no relation to the trilinear coordinate β.

the use of a fourth filter would improve the duplication. The following approximate equations designate the relationships shown in figure 1 and serve as the basis for comparison of ICI tristimulus values and photoelectric tristimulus values:

$$\left.\begin{aligned}\overline{x}E_C &\doteq 0.80E_I T_A s k_A + 0.18E_I T_B s k_B \\ \overline{y}E_C &\doteq 1.00E_I T_G s k_G \\ \overline{z}E_C &\doteq 1.18E_I T_B s k_B.\end{aligned}\right\} \quad (2)^{15}$$

IV. REDUCTION OF PHOTOELECTRIC TRISTIMULUS DATA

Values of A, G, and B obtained from a photoelectric tristimulus colorimeter may be converted to useful information about the colors of the samples measured in a number of different ways. Each procedure for reducing data gives numbers which are useful for a particular purpose. In table 3 the quantities which can be computed are identified by the equations for finding them and by their principal uses.

As previously noted, the methods of measurement suggested herein refer chiefly to surface colors. It is believed, however, that several of these suggested methods will prove to be equally applicable to the evaluation of volume colors from transmission measurements and to the evaluation of the colors of self-luminous objects from measurements of radiant flux.

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[15] The author is indebted to Miss N. J. Hendley (now Mrs. E. M. Furness) for assistance in finding transformations of the source-filter-photocell combinations which were closely equivalent to the standard observer. After experiment with several transformations giving slightly closer equivalence to the standard observer, it was decided that the simple transformations indicated by eq 2 were more practical for ordinary use.

TABLE 3.—*Uses of photoelectric tristimulus measurements*

Quantities computed from settings	Equations used	Uses of results
The coordinates, x, y, and Y of the ICI standard system.	*From eq 5 and 6:* $x \doteq (0.80A+0.18B)/\Sigma$ $y \doteq G/\Sigma$ $\Sigma = 0.80A+G+1.36B$ $Y \doteq G$	To find whether sample meets color requirements specified by x, y, and Y. To represent colors in the standard ICI system.
The chromaticity coordinates, α and β, of a uniform-chromaticness-scale system.	*Eq 8:* $\alpha \doteq (A-G)/(A+2G+B)$ $\beta \doteq 0.4(G-B)/(A+2G+B)$	To designate chromaticity in a coordinate system yielding approximately uniform-chromaticness scales for surface color. To indicate changes of chromaticness of specimens with time, exposure, change of composition, change of method of preparation, or other treatment.
The amount of color difference, ΔE, between two surface colors.	*Eq 13:* $\Delta E = f_0\{[7Y^{1/4}\sqrt{\overline{\Delta\alpha}^2+\overline{\Delta\beta}^2}\cdot 10^2]^2+[k_1\Delta(Y^{1/2})]^2\}^{1/2}$	To measure the amount of color difference between two surface colors in NBS units. To find whether a sample differs from a standard by more or less than a specified color tolerance. To measure the amount of color change resulting from exposure, change of composition, change of method of preparation, or other treatment of the sample.
The coordinates, α', β', and L' of a uniformly-spaced surface-color solid.	*Eq 14:* $\alpha' = 700\ Y^{1/4}\alpha$ $\beta' = 700\ Y^{1/4}\beta$ $L' = k_1 Y^{1/2}$	To designate surface color using coordinate scales which give perceptually nearly uniform measures of stimulus difference.
The hue angle, ϕ, the saturation index, S, and the lightness index, L.	*Eq 16:* $\phi \equiv$ angle whose tangent is β/α $S \equiv \sqrt{(\alpha^2+\beta^2)}\,Y^{1/2}$ $L \equiv Y^{1/2}$	To approximate the hue, the saturation, and the lightness of colors by separate numbers.
The hue, saturation, and lightness components, $\Delta H'$, $\Delta S'$, and $\Delta L'$, of a surface-color difference in NBS units.	*Eq 17:* $\Delta H' = 12.2Y^{1/4}\sqrt{\alpha^2+\beta^2}\cdot\Delta\phi$ ($\Delta\phi$ in degrees) $\Delta S' = 700Y^{1/4}\cdot\Delta(\sqrt{\alpha^2+\beta^2})$ $\Delta L' = k_1\Delta(Y^{1/2})$	To separate a surface-color difference into the hue, saturation, and lightness components of the difference in NBS units.
The whiteness, W, of a white or near-white surface.	*Eq 19:* $W = 1-\left\{[30\sqrt{\alpha^2+\beta^2}]^2+\left[\frac{1-Y}{2}\right]^2\right\}^{1/2}$	To designate a white or near-white surface with a number on a scale of whiteness which gives an MgO surface a value of 1.00 and a black surface a value of zero.
Yellowness	*Eq 20:* Yellowness $= (A-B)/G$	To give a white or near-white surface a number which indicates degree of yellowness if positive, degree of blueness if negative.

1. TO ICI VALUES, x, y, AND Y

The ICI standard system [22] for designating colors has international recognition and is therefore widely used in the specification of colors. The tristimulus specification of a surface color according to the ICI observer is the summation through the spectrum of the spectral apparent reflectance of the sample relative to MgO($A_{\lambda(\theta i,\theta v)}$ being measured by a spectrophotometer), times the spectral energy, E_C, of the illuminant, times the respective spectral distribution functions, $\bar{x}$, $\bar{y}$, and $\bar{z}$ of the ICI observer:

$$\left.\begin{aligned} X &\equiv \sum_{\lambda=0}^{\lambda=\infty} A_{\lambda(\theta i\ \theta v)} E_C \bar{x} \Delta\lambda \\ Y &\equiv \sum_{\lambda=0}^{\lambda=\infty} A_{\lambda(\theta i,\theta v)} E_C \bar{y} \Delta\lambda \\ Z &\equiv \sum_{\lambda=0}^{\lambda=\infty} A_{\lambda(\theta i,\theta v)} E_C \bar{z} \Delta\lambda. \end{aligned}\right\} \tag{3}$$

If the same sample is compared with MgO in a photoelectric tristimulus colorimeter, the following summations through the spectrum are made automatically:

$$\left.\begin{aligned} \sum_{\lambda=0}^{\lambda=\infty} A_{\lambda(\theta i,\theta v)} E_I T_A sk_A \Delta\lambda &\equiv A \\ \sum_{\lambda=0}^{\lambda=\infty} A_{\lambda(\theta i,\theta v)} E_I T_G sk_G \Delta\lambda &\equiv G \\ \sum_{\lambda=0}^{\lambda=\infty} A_{\lambda(\theta i,\theta v)} E_I T_B sk_B \Delta\lambda &\equiv B \end{aligned}\right\} \tag{4}$$

A, G, and B are, by definition, the settings of the sample relative to those of standard MgO with the amber, green, and blue filters, respectively.

By multiplying both sides of eq 2 by $A_{\lambda(\theta i,\theta v)}$ and making summations through the spectrum, we find by way of eq 3 and 4 that:

$$\left.\begin{aligned} X &\doteq 0.80A + 0.18B, \\ Y &\doteq 1.00G, \\ Z &\doteq 1.18B. \end{aligned}\right\} \tag{5}$$

The equations are only approximate, because the source-filter-photocell combinations are spectrally somewhat different from the standard ICI observer, as shown in figure 1. However, photoelectric tristimulus measurements according to a substitution method using a calibrated standard spectrally similar to the samples are, as is shown below, suitably accurate for many purposes.

Since the $\bar{y}$ function of the ICI observer is the luminosity function, approximate values of luminous apparent reflectance are given directly by settings with the green filter. To indicate chromaticity in the ICI system, the trilinear coordinates of a color are computed as follows:

$$\left.\begin{aligned} x &\equiv X/(X+Y+Z), \\ y &\equiv Y/(X+Y+Z). \end{aligned}\right\} \tag{6}$$

Figure 2 shows the ICI (x,y)-diagram and the corresponding photoelectric approximation to the diagram plotted in the same coordinates. In each diagram are the spectrum locus and points representing 12 differently colored porcelain-enamel plaques,[15] each illuminated by

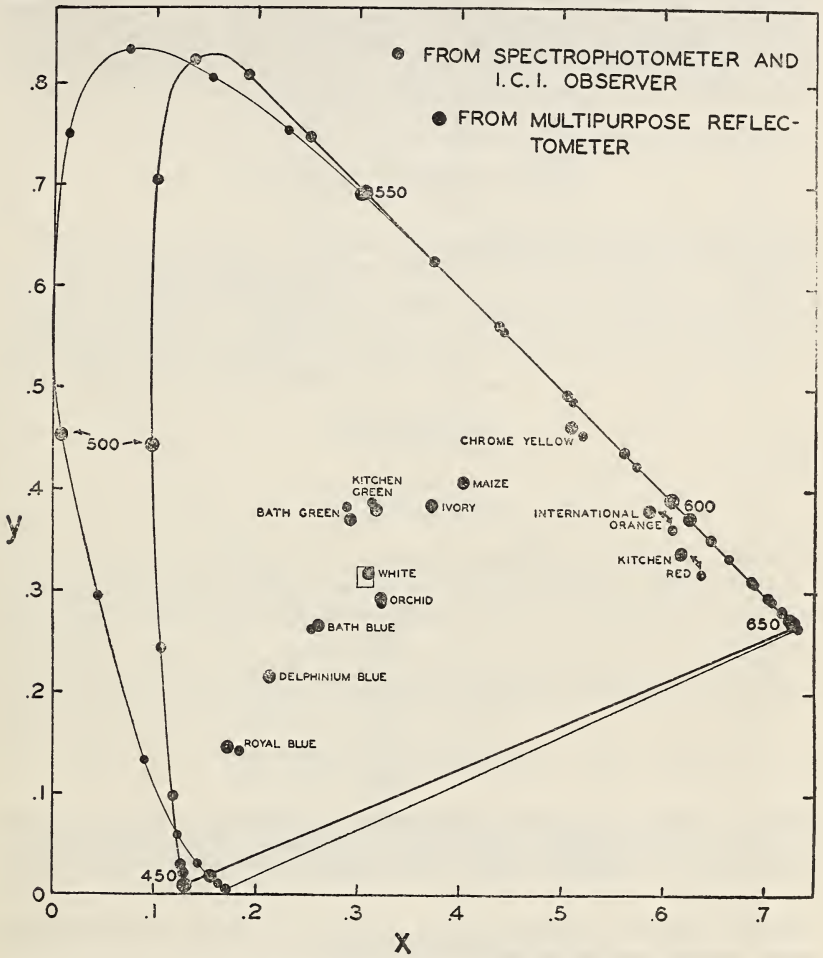

FIGURE 2.—*ICI (x,y)-diagram and corresponding photoelectric approximation to the diagram with the point for white* (MgO) *common to both diagrams.*

The pairs of points for the 11 chromatic porcelain-enameled plaques show the errors in chromaticity measured by the photoelectric tristimulus method. The small rectangle near the center gives boundaries shown in figure 7, below.

ICI illuminant *C*. The point $x=0.3101$ and $y=0.3163$, representing a freshly prepared magnesium-oxide surface similarly illuminated is common to both diagrams. Each of the twelve plaques was measured relative to MgO, first with a spectrophotometer and then with a multipurpose reflectometer; the values of x and y obtained from the two sets of data were computed and plotted in figure 2.

[15] The standards used are 10 kitchen and bathroom colors [36], and plaques representing medium chrome yellow [6] and international orange [7].

Coordinates of the spectrum locus of the ICI mixture diagram are published in many places [15, 22, 25, 31, 33, 45]; coordinates of the boundary of the photoelectric approximate chromaticity diagram were computed by applying eq. 5 and 6 to the values of $E_I\bar{T}_A sk_A$, $E_I T_G sk_G$ and $E_I T_B sk_B$ given in table 2. At the ends of the visible spectrum (380 to 420 mμ and 670 to 750 mμ) the source-filter-photocell combinations imperfectly duplicate the ICI standard observer as the distributions diminish to zero. Therefore the trilinear coordinates which would result from photoelectric measurement of the spec-

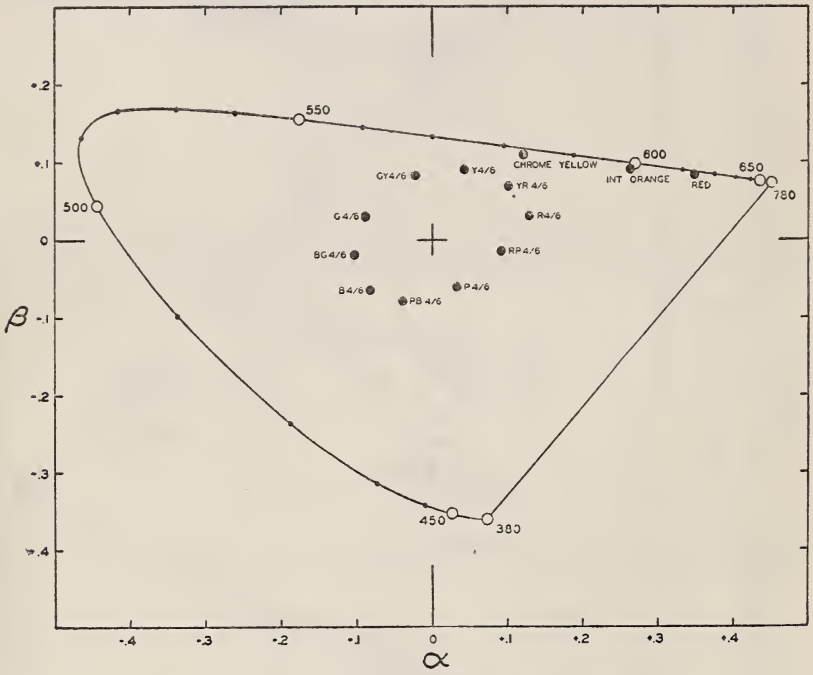

FIGURE 3.—*The* (α,β)*-uniform-chromaticness-scale diagram for surface colors showing the spectrum locus and points representing ten Munsell colors of four-value and six-chroma* [38], *medium chrome yellow, international orange, and toluidine red.*

trum for these wavelengths of low response are highly inaccurate and have been left out of figure 2.

2. TO (α, β)-UNIFORM-CHROMATICNESS-SCALE COORDINATES

The (α, β)-chromaticity diagram shown in figure 3 has been planned especially for representing surface colors measured with the multipurpose reflectometer and tristimulus filters. Cartesian coordinates and an origin located at the point representing MgO were used because of the greater convenience for plotting chromaticity, and because of the simple relation afforded thereby between position of point in the diagram and saturation index and hue angle of the color represented. The convenience of these two features was recognized by Breckenridge and Schaub, who used them in their earlier RUCS triangle [3]. In

addition to these features and the advantage of easy computation from settings with a reflectometer, values of α and β have an important advantage as uniform-chromaticness scale coordinates for representing the colors of paints, papers, textiles, and opaque ceramic products. The equations defining α and β were chosen, as pointed out by Scofield, Judd, and Hunter [42] to give measures of chromaticity in improved accord with visual estimates of the chromaticness of opaque surfaces.

The coordinates of the chromaticity diagram are defined by:

$$\left.\begin{aligned}\alpha &\equiv \frac{2.4266x-1.3631y-0.3214}{1.0000x+2.2633y+1.1054},\\ \beta &\equiv \frac{0.5710x+1.2447y-0.5708}{1.0000x+2.2633y+1.1054}.\end{aligned}\right\} \quad (7)\ [17]$$

Prior to the selection of these equations defining the (α,β)-diagram, equations were sought which would make the conversion of multipurpose reflectometer settings to values in the new coordinate system a simple matter. After trial with a number of possible sets,[18] the following simple equations were chosen to give approximate values of α and β in the chromaticity diagram:

$$\left.\begin{aligned}\alpha &\doteq \frac{A-G}{A+2G+B},\\ \beta &\doteq \frac{0.4(G-B)}{A+2G+B}\end{aligned}\right\} \quad (8)$$

Neither eq 8a nor 8b gives as close approximation to uniform-chromaticness spacing for surface colors as eq 8. Closest accord with the spacing in Judd's UCS diagram [26] is, however, afforded by the use of eq 8b.

Because the settings of A, G, and B for any white or nearly achromatic color are of roughly the same magnitude, suitable values of α and β for these colors may often be found by

$$\left.\begin{aligned}\alpha &\doteq 2.5(A-G)/10G,\\ \beta &\doteq (G-B)/10G.\end{aligned}\right\} \quad (9)$$

Values of α and β computed according to eq 7 for spectrum colors at 10-millimicron intervals are recorded in table 4 and plotted in figure 3 to form the boundary of the (α,β)-diagram. Points repre-

[17] The reverse equations are

$$\left.\begin{aligned}x &= \frac{0.5583\alpha+0.1631\beta+0.2466}{0.0100\alpha-1.4347\beta+0.7951},\\ y &= \frac{-0.2515\alpha+0.6285\beta+0.2515}{0.0100\alpha-1.4347\beta+0.7951}.\end{aligned}\right\} \quad (7a)$$

[18] Prior to the choice of eq 8, two earlier pairs were used by the author. The first pair, suggested in 1939 [16], was

$$\left.\begin{aligned}\alpha &= \frac{2A-2G}{2A+5G+3B},\\ \beta &= \frac{G-B}{2A+5G+3B}.\end{aligned}\right\} \quad (8a)$$

and the second pair, suggested in 1940 [17], was

$$\left.\begin{aligned}\alpha &= \frac{2.5\,(A-G)}{A+6G+3B},\\ \beta &= \frac{G-B}{A+6G+3B}.\end{aligned}\right\} \quad (8b)$$

senting three colors widely used on signs and markers have also been plotted in figure 3. In addition, points have been plotted to show the degree of agreement between the chromaticity scales of the new system and the chromaticness spacing of the Munsell system. Using Nickerson "smoothed" values of x and y for Munsell papers [38], values of α and β were computed by eq 7 for the 10 papers representing the hues and intermediate hues of 4-value and 6-chroma under ICI illuminant C. The points representing these 10 papers would be spaced at equal intervals along the circumference of a circle if the Munsell system and the (α,β)-diagram both gave perfectly uniform chromaticness spacings.

TABLE 4.—*Values of α and β, computed according to eq 7, of spectrum colors at 10-millimicron intervals*

Wavelength in millimicrons	α	β	Wavelength in millimicrons	α	β
380	+0.0731	−0.3604	550	−0.1793	+0.1557
90	+.0725	−.3608	60	−.0924	+.1451
			70	+.0001	+.1330
400	+.0718	−.3612	80	+.0952	+.1204
10	+.0705	−.3617	90	+.1879	+.1082
20	+.0679	−.3622			
30	+.0613	−.3611	600	+.2688	+.0974
40	+.0484	−.3579	10	+.3320	+.0891
			20	+.3754	+.0833
450	+.0265	−.3528	30	+.4035	+.0796
60	−.0095	−.3430	40	+.4229	+.0769
70	−.0729	−.3146			
80	−.1876	−.2361	650	+.4352	+.0753
90	−.3373	−.0977	60	+.4423	+.0744
			70	+.4458	+.0740
500	−.4440	+.0446	80	+.4483	+.0736
10	−.4651	+.1317	90	+.4501	+.0734
20	−.4166	+.1661			
30	−.3387	+.1688	700 to 780 incl	+.4506	+.0733
40	−.2605	+.1641			
			MgO under ICI illuminant C	.0000	.0000

Since the chromaticness spacing of the Munsell system is known to be fairly good [37], the departure of the actual 10 points plotted in figure 3 from equal spacing on the circumference of a circle can be considered a rough measure of the degree to which the (α,β)-diagram fails to give uniform chromaticness spacing of surface colors. Comparison of the spacing of points in figure 3 with the similar spacings of points representing Munsell papers in the (x,y)-diagram as shown by Newhall [37, p. 640] indicates a considerable advantage in favor of the (α,β)-system.

3. TO THE AMOUNT OF PERCEIVED COLOR DIFFERENCE, ΔE, BETWEEN TWO SURFACE COLORS

Expressions for estimating the amount of difference perceived between two colors and for estimating the whiteness of a perceived color have recently been set up by Judd [28, 29, 32]. Visual estimates of these two properties are widely used in commerce, where it must frequently be decided whether the perceived color of one object is a suitable match for that of another, or whether a material meets a whiteness specification. By making minor changes in Judd's two expressions, methods are available for rapidly converting photoelectric tristimulus settings to values of ΔE and W which are in good accord with perceived color difference and whiteness, respectively.

To form his empirical equation for ΔE, the approximate amount of perceived color difference, Judd first assumed that color can be represented by a three-dimensional figure in which distance is a close measure of the perceptibility of the corresponding surface-color difference. In this Euclidian solid, the chromaticity and reflectance components of color difference, ΔE_C and ΔE_Y, are vectors at right angles. By expressing these two components in a unit which gives uniform measure of perceived color difference throughout the color solid, the definition of ΔE may be written

$$\Delta E \equiv \sqrt{\overline{\Delta E_C}^2 + \overline{\Delta E_Y}^2} \qquad (10)$$

Because the square root of luminous apparent reflectance is known to correlate well with observers' estimates of lightness under usual observing conditions, Judd wrote the reflectance component [28, 29]:

$$\Delta E_Y = k_1 \Delta(Y^{1/2}), \qquad (11)$$

where Y is the luminous apparent reflectance.

The chromaticness difference corresponding to a given chromaticity difference varies with the luminous apparent reflectance of the specimens represented. Judd assumed this variation to be proportional to $Y^{1/4}$:

$$\Delta E_C = k_2 Y^{1/4} \Delta S,$$

where the chromaticity difference, ΔS, is expressed in the units of Judd's own uniform-chromaticness-scale diagram [26].

For the present work, use is made of chromaticity differences measured in the (α,β)-coordinate system, represented by $\sqrt{\overline{\Delta\alpha}^2 + \overline{\Delta\beta}^2}$. In order to find the factor relating chromaticity differences in Judd's UCS diagram and chromaticity differences in the author's (α,β)-diagram, points representing eight selected stimuli were plotted in each system. The stimuli were chosen so that the points representing them were distributed throughout each diagram and lines were drawn connecting various pairs of these points. It was found by measurement that the lines in Judd's diagram averaged 1.16 (from 1.10 to 1.27) times the lengths of the corresponding lines in the (α,β)-diagram:

$$\Delta E_C = 1.16 k_2 Y^{1/4} \sqrt{\overline{\Delta\alpha}^2 + \overline{\Delta\beta}^2}. \qquad (12)$$

To establish the "NBS Unit of Color Difference",[19] numbers were assigned to the constants k_1 and k_2 in the expressions representing the lightness and chromaticness components of perceived color difference, respectively. These numbers were intended to make the unit of color difference so small that measured differences of less than one unit would represent perceptually unimportant color differences in most commercial transactions. Measured differences of more than one unit, however, were expected to represent color differences which are commercially important. For k_2 the number 600 was chosen [28, 29]. The quantity k_1 is called the proximity factor; the number assigned is properly chosen on the basis of the visual proximity of the surface areas compared for color difference (see table 5 and accompanying text, below).

By replacing 1.16 times k_2 by its equivalent, 7 times 10^2, in eq 12, substituting in eq 10, and adding a factor (f_s), explained below, to

[19] This unit has since been called by Balinkin [1] the "judd", after its originator.

account for the masking of the perceived color difference by specularly reflected light from glossy surfaces, the equation for ΔE becomes

$$\Delta E = f_g \left\{ \left[7 Y^{1/4} \sqrt{\overline{\Delta\alpha}^2 + \overline{\Delta\beta}^2} \cdot 10^2 \right]^2 + \left[k_1 \Delta(Y^{1/2}) \right]^2 \right\}^{1/2}. \quad (13)$$

As given, eq 13 is not in the most conventional form, because the chromaticity difference, $\sqrt{\overline{\Delta\alpha}^2 + \overline{\Delta\beta}^2}$, is within a bracket which is itself squared. One would think that the indicated squaring should be carried out. However, the expression is given in the above form because of the convenience of evaluating it graphically. Twice in computing a value of ΔE according to eq 13, it is necessary to find the square root of the sum of two squares. These two operations were purposely provided because they can easily be done graphically with sufficient precision for color-difference measurement by using a pair of dividers and a piece of rectangular cross-section paper to get the length of the hypotenuse of the right triangle whose other two sides are known.

The constant, k_1, and the gloss factor, f_g, remain to be discussed. With respect to k_1, experience has shown that the expression $\Delta(Y^{1/2})$ is proportional to lightness difference only so long as the conditions for observation of the samples remain unchanged. To obtain the best agreement between instrumentally obtained data substituted in eq 13 and the average of visual estimates, it has proved necessary to vary values of k_1 to accord with the proximity of the areas visually compared. Thus k_1 has been called the proximity factor. When specimens are compared by holding their colored areas in immediate juxtaposition, differences in $Y^{1/2}$ are found to be readily apparent. The value of k_1 representing this manner of observation must therefore be relatively large and is, in practice, taken as about 100. If, however, specimens must be examined with a border or pattern separating the areas compared, the same differences in $Y^{1/2}$ are found to correspond to less discriminable lightness differences. The value of k_1 representing this manner of observation is therefore smaller, perhaps as low as 40. The values of the proximity factor, k_1, suggested by Judd [28, p. 425] are listed in table 5.

TABLE 5.—*Values of k_1, the proximity factor, suggested by Judd [28, p. 425]*

Conditions of observation giving equivalent visual estimates of color difference	k_1
Samples separated by a very narrow or nonexistent dividing line	120
Samples separated by a contrasting, but narrow dividing line	90
Samples separated by a broad patterned area different from the areas being compared	40
Samples evaluated for whiteness without visual reference to other samples	20
However a convenient value for use when samples are separated by a narrow line is	100

The factor f_g in eq 13 was added to account for the masking of perceived color difference by specularly reflected light [28, p. 426]. For matte surfaces, f_g is unity; for glossy surfaces it is assumed to be

$$f_g = Y/(Y + K_g),$$

where K_g is a constant referring to the conditions under which the samples are examined visually. In situations yielding considerable

admixture of specularly reflected light, K_g is made large to give correspondingly small values of ΔE. If, on the other hand, care is taken in the corresponding visual situation to illuminate the samples so that only a small portion of the incident light is specularly reflected into the eyes of the observer, K_g should be small. A value of 0.025 was found by Judd to give satisfactory correspondence between visual estimates under the latter conditions and measured values of ΔE for a number of pairs of glossy porcelain-enameled plaques. In figure 4, f_g is plotted against apparent reflectance for $K_g=0.025$.

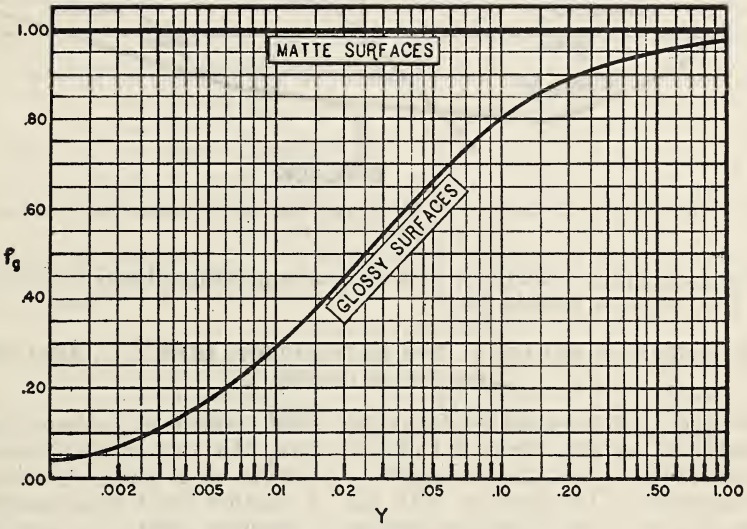

FIGURE 4.—*The gloss factor, f_g, plotted as a function of Y for $K_g=0.025$.*

4. TO THE COORDINATES, α', β', L', OF A SOLID WITH AN APPROXIMATELY UNIFORM COLOR-PERCEPTION SPACING

From eq 13 rectangular coordinates can be derived for a color solid designed to give spacings which accord with perceptual surface-color differences observed with light-gray backgrounds. If the unit of length along each of the rectangular axes is made the NBS unit of color difference, if the point representing black is made the origin for these axes, and if the components of color parallel to the α, the β, and the $Y^{1/3}$ axes are designated by α', β' and L', respectively, then from eq 13 it may be shown that

$$\left.\begin{aligned}\alpha' &= 700\,Y^{1/3}\alpha,\\ \beta' &= 700\,Y^{1/3}\beta,\\ L' &= k_1 Y^{1/3}.\end{aligned}\right\} \tag{14}$$

A value of 100 is frequently substituted for k_1.

The (α', β', L')-uniform-surface-color solid, in which

$$\Delta E=\sqrt{\overline{\Delta\alpha'}^2+\overline{\Delta\beta'}^2+\overline{\Delta L'}^2}, \tag{15}$$

is illustrated in figure 5. White is at the top of this solid, with a value of L' equal to $k_1=100$. The chromaticity plane at an L' level of approximately 45 has been drawn. Its boundary was determined by finding the points representing colors of maximum saturation index

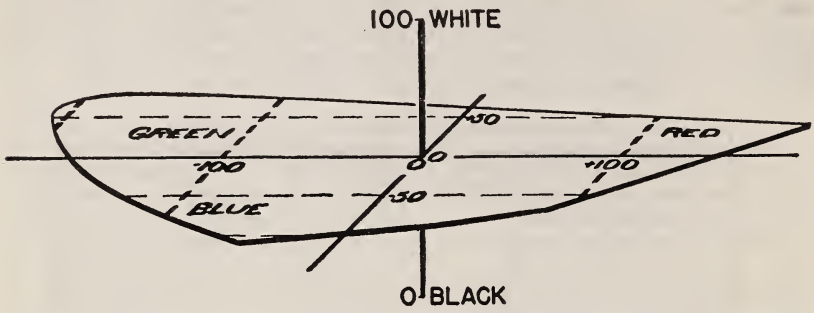

FIGURE 5.—*Diagram of the* (α', β', L')*-surface-color solid showing the chromaticity plane for* $Y=0.20$.

[34] for apparent reflectance of 0.20 (corresponding closely to $L'=45$ for $k_1=100$, or a midlde gray).

5. TO THE HUE ANGLE, ϕ, THE SATURATION INDEX, S, AND THE LIGHTNESS INDEX, L

In the psychological surface-color solid, which is perhaps best exemplified in the Munsell Color System, the two components of chromaticness, hue and saturation, combine in a polar-coordinate arrangement. To correlate with hue, a number must be a measure of angle; to correlate with saturation, a number must be a measure of radius [31, p. 353 and 33, p. 11]. To obtain numbers from photoelectric tristimulus settings which will correlate well with hue, saturation, and lightness of surface-color perceptions, the following definitions have proved useful:

$$\left.\begin{aligned} \phi &\equiv \text{ angle whose tangent is } \beta/\alpha, \\ S &\equiv \sqrt{(\alpha^2+\beta^2)}\,Y^{1/2}, \\ L &\equiv Y^{1/2}. \end{aligned}\right\} \tag{16}$$

The above quantities have been designated the hue angle, the saturation index and the lightness index, respectively. It can be seen that the signs of both α and β must be known to place ϕ in the proper quadrant.

6. TO ESTIMATES IN NBS UNITS OF THE HUE, SATURATION, AND LIGHTNESS COMPONENTS, $\Delta H'$, $\Delta S'$, AND $\Delta L'$, OF A PERCEIVED SURFACE-COLOR DIFFERENCE

It is frequently valuable to compute not only an estimate of the size of the difference perceived between two surface colors, but also estimates of sizes of the separate hue, saturation, and lightness components of the difference.

To compute these separate components of color difference, it is necessary to convert the rectangular coordinates of the color solid

defined by eq 14 to cylindrical coordinates. The saturation attribute in this solid is measured by radius from the lightness axis as center:

$$S' \equiv \sqrt{\alpha'^2 + \beta'^2}.$$

The hue-difference component of color difference in this solid, for hue-angle differences not too large, is measured by distance along the circumference which passes through the point midway between the points representing the two colors:

$$\Delta H' \equiv \frac{S_1' + S_2'}{2} \Delta\phi$$

From these relations and eq 14,

$$\left.\begin{aligned} \Delta H' &= 12.2 Y^{1/4} \sqrt{\alpha^2 + \beta^2} \cdot \Delta\phi, \\ &\text{where } \Delta\phi \text{ is measured in degrees;} \\ \Delta S' &= 700 Y^{1/4} \Delta(\sqrt{\alpha^2 + \beta^2}), \\ \Delta L' &= k_1 \Delta(Y^{1/3}), \end{aligned}\right\} \quad (17)$$

where the values which do not refer to difference measurements are taken as averages of the corresponding values for the two specimens concerned.

7. TO WHITENESS, *W*

The whiteness of a sample is assumed, for purposes of its measurement, to be proportional to the perceived degree of approach of its color to that of an ideal white surface. If MgO is assumed to approximate sufficiently well the ideal white, the whiteness of the unknown surface may be quantitatively defined by a relation suggested by Judd [32]:

$$W \equiv 1 - \frac{\Delta E_{\text{MgO to specimen}}}{\Delta E_{\text{MgO to black}}}, \quad (18)$$

which gives a scale of whiteness from 1.00 for MgO to zero for black. Judd found that a proximity factor (k_1) of 20 yielded numerical ratings for the whiteness of surfaces in good accord with visual estimates. By substituting this value in eq 13, the color difference between MgO and black for the assumed conditions of whiteness grading is found to be 20 NBS units. The above equations for whiteness may therefore be written

$$W = 1 - \frac{1}{20} \Delta E_{\text{MgO to specimen}}.$$

Since the use of this equation is to be restricted to white and near-white surfaces, eq 13 may be somewhat simplified when incorporated into the relation for whiteness. Within the range of apparent reflectances (0.60 to 1.00) covered by whites and near whites, $(7 \cdot Y^{1/4} \cdot 10^2)/20$ does not vary greatly from 30, and, for the $\Delta(Y^{1/2})$ component, $(1.00 - Y^{1/2})$ does not differ greatly from the more easily computed $(1.00 - Y)/2$. Since, in addition, the values of α and β for MgO are both zero, an approximate expression for whiteness of a white or near-white surface may be written:

$$W = 1 - \left\{ \left[30\sqrt{\alpha^2 + \beta^2} \right]^2 + \left[\frac{(1.00 - Y)}{2} \right]^2 \right\}^{1/2}. \quad (19)$$

Whiteness evaluated by this expression differs from that by eq 18 by less than 0.025 for W greater than 0.80. This difference is less than the uncertainty of estimates of whiteness equality by direct visual inspection. In correlating visual estimates of whiteness of white papers, finishes, and other materials with colorimetric values of the samples examined, one frequently finds that magnesium oxide is not a suitable standard for whiteness. Judd [27] had 15 experienced paper technologists each rate 29 pieces of paper for whiteness. As a result of the ratings supplied, he found that in the paper industry there are recognized apparently not one, but at least two "ideal whites." One ideal white is "natural paper white," a yellowish color representing the best grade of undyed paper. Another ideal white seems to possess almost the chromaticity of MgO; it is typical of papers "whitened" with blue dye. Some observers judge whiteness by one ideal, others by another.

The fact that some observers use ideal whites differing in color from MgO does not destroy the usefulness of relations for rating whiteness numerically. If the ideal white used by the graders of the material in question can be identified, eq 18 can be used by substituting the colorimetric coordinates of this ideal white for those of MgO.

8. TO YELLOWNESS

White and near-white surfaces are so frequently rated for degree of yellowness that it is convenient to have a scale which provides approximate psychophysical measures of yellowness. The equation

$$\text{Yellowness} = \frac{A-B}{G} \tag{20}$$

is similar to others which have been used for the same purpose [48, 49] and gives a scale of yellowness increasing from zero for MgO or any equally nonselective surface to positive values for yellowish surfaces and negative values for bluish surfaces.

V. STANDARDS FOR PHOTOELECTRIC TRISTIMULUS COLORIMETRY

For almost all photoelectric tristimulus measurements, regardless of the purpose for which they are made, a substitution method should be employed. That is, the specimens being measured should be exposed under the same circumstances as the standard with which they are compared. This standard may be either the actual reference standard, which for surface colors is usually a freshly smoked magnesium-oxide surface, or it may be a secondary or working standard, such as a porcelain-enameled plaque which has been calibrated in terms of the reference standard. If simply the differences in color between the test specimens and a specimen designated as standard are of interest, this arbitrary standard often need not be calibrated.

The present photoelectric tristimulus method is subject to error because of the above-mentioned spectral inaccuracy of the source-filter-photocell combinations. In the following section of this paper, it will be shown that this error tends to increase in amount with the spectral dissimilarity between the specimens compared. Because of this tendency, it is advantageous to use wherever possible in photo-

electric tristimulus colorimetry a working standard which is spectrally similar [20] to the samples being measured.

To calibrate a working standard for photoelectric tristimulus colorimetry without reference to a spectrally similar, previously calibrated standard, the spectrophotometric curve of the new standard is first obtained. From this curve, x, y, z, and Y are computed according to one of the customary methods [15, 25, 31, 45]. From these computed values, A_{std}, G_{std}, and B_{std} may be derived using eq 21, which is derived from eq 5 and 6:

$$\left.\begin{aligned} A_{std} &= \frac{Y}{y}(1.25x - 0.1907z) \\ G_{std} &= Y \\ B_{std} &= \frac{Y}{y}0.8475z \end{aligned}\right\} \tag{21}$$

As a specific example of how values of A, G, and B may be assigned a working standard, the pink plaque used for the measurements demonstrated in example 2 below, was given the following ICI colorimetric values by spectrophotometry and computation:

$$\begin{aligned} x &= 0.3585 \\ y &= 0.3345 \\ z &= 0.3070 \\ Y &= 0.606 \end{aligned}$$

Accordingly values of A, G, and B were computed for this working standard according to eq 21:

$$\begin{aligned} Y/y &= 1.81166 \\ 1.25x &= 0.448125 \\ 0.1907z &= 0.058545 \\ \text{Difference} &= 0.389580 \\ A = \text{Difference} \cdot (Y/y) &= 0.7058 \\ G = Y &= 0.606 \\ 0.8475z &= 0.260182 \\ B = 0.8475z(Y/y) &= 0.4714 \end{aligned}$$

When, instead of ICI values, values of α, β, and Y of the working standard are given, values of A, G, and B are computed by using equations which are essentially eq 8 reversed:

$$\left.\begin{aligned} A &= Y\left(\frac{1.0 + 3.0\alpha + 2.5\beta}{1.0 - \alpha + 2.5\beta}\right) \\ G &= Y \\ B &= Y\left(\frac{1.0 - \alpha - 7.5\beta}{1.0 - \alpha + 2.5\beta}\right) \end{aligned}\right\} \tag{22}$$

To find the colors of a series of spectrally similar specimens, none of which can be used as a working standard because of unstable color,

[20] By spectrally similar specimens are meant specimens which are not metamerically different (see text, p. 25).

a stable plaque is assigned values of A, G, and B which relate to the impermanent specimens rather than to its own color. To calibrate such a working standard, one of the unstable specimens must be measured on a spectrophotometer and then compared immediately with the stable plaque on the tristimulus instrument.

VI. ERRORS IN PHOTOELECTRIC TRISTIMULUS MEASUREMENTS

In the paper describing the multipurpose reflectometer [18], 18 potential sources of instrument error are listed. These 18 sources include most of those which will be found in photoelectric tristimulus instruments, and it is believed, therefore, that no further study of the errors of instruments is needed here. With special reference to the errors of photoelectric tristimulus colorimetry, however, there are several matters which merit attention. These matters are considered herein with particular reference to measurements with the author's three-filter multipurpose reflectometer.

It should be pointed out first that high precision is needed for useful photoelectric tristimulus measurements. In order to identify by an instrument, chromaticity differences as small as can be distinguished by an experienced observer, relative values of A, G, and B must be highly precise. To show what this precision must be, changes of 0.001 in the instrument settings on a hypothetical white surface and on a hypothetical yellow surface have been converted to corresponding changes in x, y, α, β, ΔE, and W by using the relations above. These are shown in table 6.

TABLE 6.—*Amounts of change in chromaticity and color difference introduced by changes in instrument settings of 0.001*

		For a white surface reflecting 80 percent	For a yellow surface reflecting 35 percent
If settings of the instrument should be	A	0.800	0.500
	G	.800	.350
	B	.800	.050
But instead the instrument gives	A	.801	.501
	G	.799	.349
	B	.801	.051
Changes of the following magnitudes will result	in x	+.00024	+.00049
	in y	−.00054	−.00182
	in α	+.00062	+.00160
	in β	−.00025	−.00064
	in $\sqrt{\alpha^2+\beta^2}$	.00067	.00172
	in ΔE (maximum, NBS units)	±.45	±.94
	in W	−.0025	

Under favorable observing conditions, a trained inspector can usually distinguish as different the colors of two light-colored surfaces if

$$\Delta E = 0.3,$$

or if

$$\sqrt{\overline{\Delta\alpha}^2+\overline{\Delta\beta}^2}=0.0005.$$

In general, it can therefore be said that a tristimulus colorimeter giving apparent reflectances imprecise by 0.001 will not serve to iden-

tify color differences as small as those which the trained color inspector can distinguish.

Although the individual settings of a multipurpose reflectometer are usually uncertain by more than 0.001, a technique is used to obtain settings with this instrument which ordinarily reduces the errors of the resulting values of x, y, α, β, ΔE, and W to considerably less than those given in table 6. With this instrument the settings on each specimen with the three filters are made consecutively. By thus taking settings with the three filters almost simultaneously and without moving the specimen, the errors ordinarily introduced by temperature drift and by change of the position of the specimen between exposures are minimized in the computed *differences* between the settings.

It may be seen from eq 8 above that the chromaticity measurement of a sample depends chiefly upon these differences between settings. These differences must be uncertain by less than 0.001 for precise photoelectric colorimetry, since changes in the differences between A, G, and B are relatively more important in determining chromaticity than changes in their magnitudes. The three readings of a sample would have to be simultaneously high, or simultaneously low by four times 0.001 (for a near-white sample) to cause a lightness-component error in ΔE as large as the chromaticity-component error caused by an inaccuracy in $A-G$ of 0.001. Resort to eq 13 above will verify this conclusion.

1. ERRORS IN TRISTIMULUS MEASUREMENTS OF CHROMATICITY DIFFERENCE DUE TO SPECTRAL INACCURACY OF THE SOURCE-FILTER-PHOTOCELL COMBINATIONS

The spectral inaccuracy of the source-filter-photocell representation of the ICI standard observer has been demonstrated above. Although the errors from this source were shown [18] to be not serious in measurements of luminous apparent reflectance, the errors from the same source are serious in measurements of chromaticity.

It was pointed out by Perry [40] that these chromaticity errors are small and can usually be tolerated when the photoelectric tristimulus method is used to measure differences in chromaticity between spectrally similar, or as he said, "spectrophotometrically similar" specimens. Under other circumstances, however, the chromaticity errors are frequently larger than can be tolerated. It can only be recommended, therefore, that the present method be used to measure color differences between spectrally similar specimens.

Before undertaking quantitative evaluations of chromaticity errors, it seems desirable to explain in some detail what Perry meant when he spoke of "spectrophotometrically similar" specimens. Actually a word suggested by Ostwald [39], "nonmetameric", seems to apply more specifically than either spectrally similar or spectrophotometrically similar to the property of the differences which is important. The stimuli produced by two illuminated specimens are metamers when there is a spectral difference between them which is of such a nature that they nevertheless have the same color. As an example, there is the frequently cited case of two specimens which have the same color under one illumination, but different colors under a second illumination. Under the first illumination, the two stimuli are metamers

for the specimens have to be spectrally different to appear different in color under the second illumination.

In the present discussion, reference is made to "degree of metamerism." It is supposed that the spectral difference for two speci-

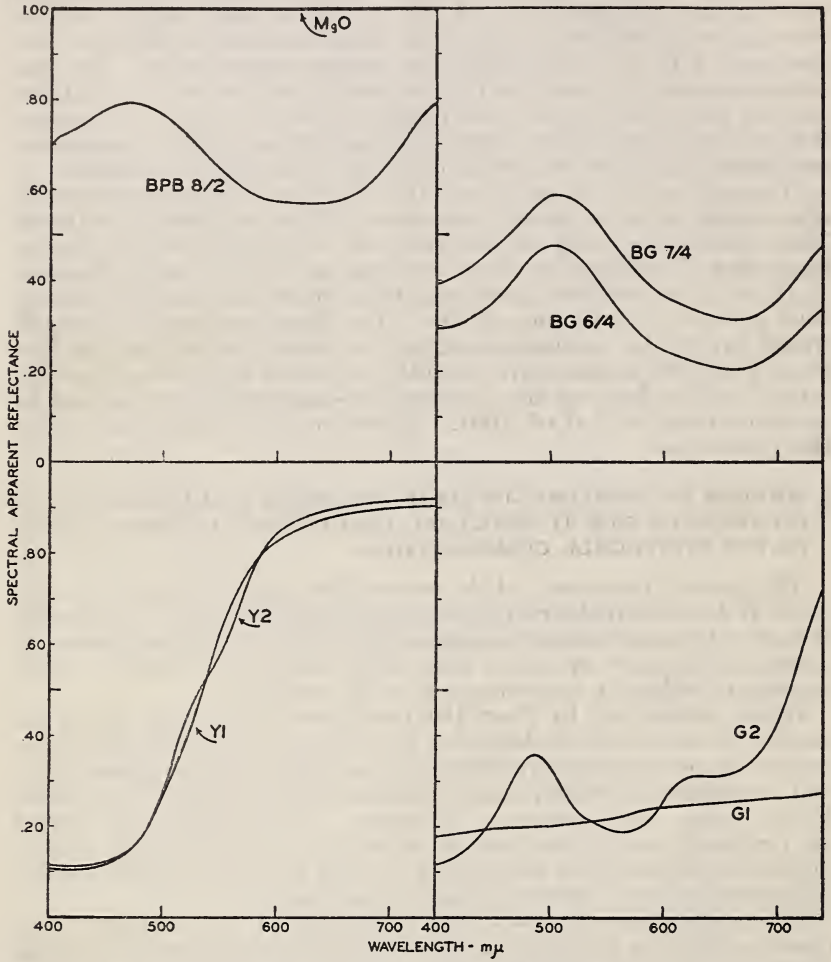

FIGURE 6.—*Spectral curves of four pairs of specimens used for the study of amounts of error resulting from failure of the source-filter-photocell combinations to be spectrally equivalent to the ICI standard observer.*

Upper two pairs—curves representing essentially nonmetameric differences; lower two pairs—curves representing differences in which there is appreciable metamerism.

mens under a given illumination can be separated into three components: (1) a factor of constant proportionality throughout the spectrum corresponding to the lightness difference, (2) a component which is responsible for the chromaticity difference, and (3) a metameric component yielding no color difference.

In general, the chromaticity error in photoelectric tristimulus measurement increases with the size of the latter two components. For pairs yielding no appreciable metameric component, therefore, the size of the chromaticity error is roughly proportional to the size of the chromaticity difference.

In order to find the approximate factor relating chromaticity error to size of nonmetameric chromaticity difference, and to show also that this factor fails to evaluate the amount of error when there is appreciable metamerism, four chromaticity differences were carefully studied. The errors of these four differences were computed from the spectral curves of the eight specimens involved. With each of the four pairs, one specimen was assumed to represent a standard and the other an unknown sample. True (α, β) and approximate (photoelectric) values of chromaticity were computed, using the spectral specifications of the ICI observer and the source-filter-photocell combinations, respectively. True chromaticity differences were then compared with the corresponding photoelectric-tristimulus chromaticity differences to obtain the errors of the latter values.

The spectral curves of the four pairs of specimens are shown in figure 6. With the top two pairs there is relatively little metamerism; with the bottom two there is significant metamerism. For the first pair, MgO and Munsell paper BPB 8/2 were chosen. The Munsell paper was chosen to represent a specimen which differed in chromaticity from MgO by about as much as any specimen which would be called "near white." Perry pointed out that because white and near-white specimens were relatively nonselective, significantly metameric differences between them were rare. Realizing that a photoelectric tristimulus colorimeter thus possessed special advantages for use on white materials, Perry named the device which he designed the Blancometer [40].

The two yellow papers, *Y1* and *Y2*, are very nearly alike in color; in fact the perceived color difference between them will change in character with change from daylight to incandescent illuminant. Thus they represent a case in which the spectral difference is chiefly a metameric difference. The degree of metamerism is even greater for the difference between the two gray samples, *G1* and *G2*.

The true chromaticity differences and the computed errors of the photoelectric tristimulus method are given for the four pairs of specimens in table 7. Since these data are computed from the spectral curves of the specimens, the errors shown are those due solely to the spectral inaccuracy of the source-filter-photocell combinations. From table 7 it can be seen that if sample and standard are not metamerically different, photoelectric tristimulus measurements of chromaticity difference may be in error by roughly 10 percent of the difference measured. If there is appreciable metamerism, however, the error is not related to the size of the chromaticity difference. For this reason parentheses have been placed about the latter two "percent errors" in table 7.

TABLE 7.—*Errors in measured chromaticity differences which result from substitution of photoelectric tristimulus combinations for the ICI standard observer*

Specimens compared	Size of the chromaticity difference $\sqrt{\overline{\Delta\alpha}^2+\overline{\Delta\beta}^2}$	Error in the photoelectrically measured chromaticity difference	
		Actual	Relative to size of difference
			%
Munsell BPB 8/2 and MgO	0.0229	0.0013	6
BG 6/4 and BG 7/4	.0150	.0018	12
Yellow *Y1* and yellow *Y2*	.00187	.00043	(23)
Gray *G1* and gray *G2*	.0160	.0061	(38)

In figure 7 there is experimental confirmation of the conclusion that errors are small in the measurement of the chromaticities of white samples. Points representing twelve white porcelain-enameled plaques which were measured with both the multipurpose photoelectric reflectometer and the visual, subtractive colorimeter [29] are plotted in this figure. The plaques were compared in both instruments with a Vitrolite standard, for which the assigned chromaticity is shown. For none of the plaques is the disparity as great as 0.001 in $\sqrt{\overline{\Delta x}^2+\overline{\Delta y}^2}$, even though it includes the errors of both instruments as well as the error due to spectral inaccuracy of the source-filter-photocell combinations. To show how large a part of the total (x,y)-diagram is covered by figure 7, its boundaries were plotted as a small rectangle near the center of figure 2.

The errors represented in table 7 and figure 7 have involved only small chromaticity differences. In figure 2, above, there is evidence which extends to large chromaticity differences the findings about errors given in table 7 for small differences. If distances between the different pairs of points representing the same colors are measured in figure 2, it will be found that they are, on the average, roughly one-tenth the respective distances from these points to the white point common to both diagrams. Since magnesium oxide, represented by the white point, was the standard against which the several specimens were measured, this average error of one-tenth the chromaticity difference is in good agreement with the errors of roughly 10 percent indicated in table 7. Thus one may expect errors which will average about 10 percent of the measured chromaticity differences for large as well as small nonmetameric differences.

2. CHANGES IN MEASURED CHROMATICITY RESULTING FROM CHANGE OF PHOTOCELL IN AN INSTRUMENT

Of the three components of each source-filter-photocell combination, the photocell is in practice the most difficult to duplicate spectrally. In the absence of an evaluation of errors possible from all these components, it may be instructive to evaluate the changes in measured chromaticity which can result from change from one photocell in an instrument to another of the same manufacture. Of the several GE cells for which data were available, the cell most different in spectral response from cell No. 55 was cell No. 2. The relative spectral responses for these two cells are compared in figure 8.

The photoelectrically measured chromaticity difference for each pair of samples identified in figure 6 would change with substitution of

cell No. 2 for cell No. 55. The amounts of these changes were computed in the same way as the errors reported in table 7, and are given in table 8. It will be noted that the changes in computed chromaticity difference are somewhat smaller than the errors for the corresponding pairs reported in table 7, but the changes are nevertheless large enough to be important.

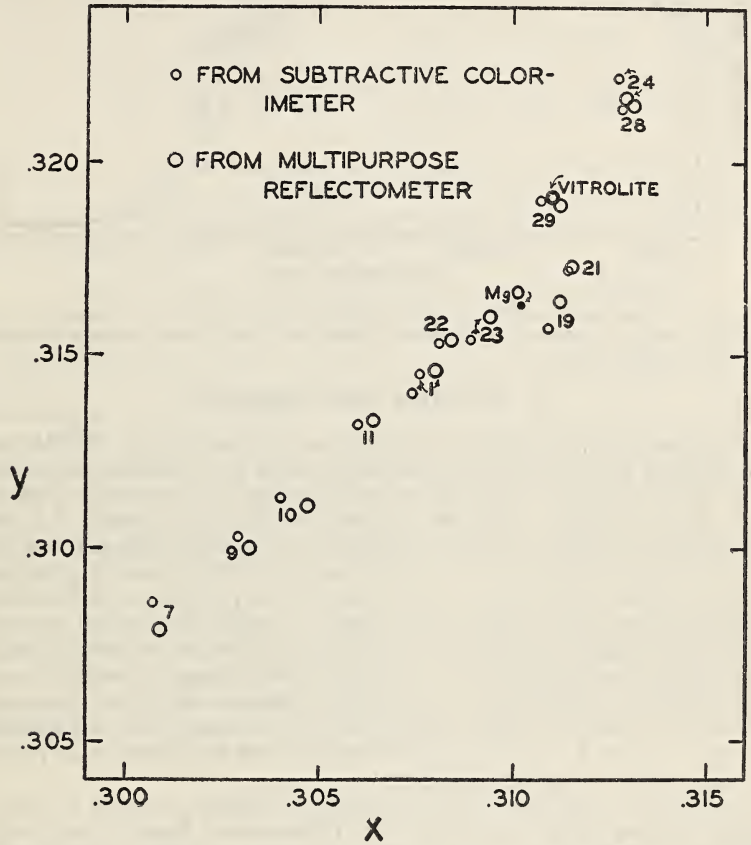

FIGURE 7.—*Part of (x,y)-diagram (outlined by rectangle in fig. 2, above) showing x and y of a number of white porcelain-enameled plaques measured both with the visual subtractive colorimeter [29] and with the multipurpose reflectometer [18].*

TABLE 8.—*Changes in measured chromaticity difference which would result from the substitution of one photocell for another of the same type, but of different spectral response*

Specimens compared	Measured size of the chromaticity difference (with cell 55) $\sqrt{\Delta\alpha^2+\Delta\beta^2}$	Change in the measured chromaticity difference with substitution of cell	
		Actual	Relative to size of difference
			%
Munsell BPB 8/2 and MgO	0.0261	0.0016	6
BG 6/4 and BG 7/4	.0136	.0004	3
Yellow *Y1* and yellow *Y2*	.00224	.00035	(16)
Gray *G1* and Gray *G2*	.0184	.0024	(13)

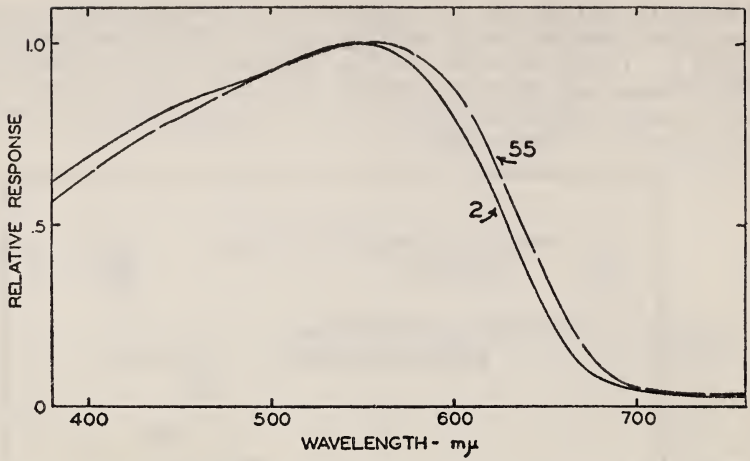

FIGURE 8.—*Relative spectral responses of GE cells No. 55 and No. 2 used to demonstrate change in measured chromaticity which may result from change of the photocell in the instrument.*

3. FLUORESCENCE ERRORS

The Institute of Paper Chemistry has shown [20, p. TS294] that fluorescence of a sample may cause error in any colorimetric instrument in which the energy striking the sample is spectrally different from that ordinarily used to observe the sample visually. A fluorescent sample has the power to change the spectral character of part of the energy striking it. In any instrument in which the incident energy does not possess the same spectral composition as the ordinary noninstrument illuminant, the energy leaving the sample may be altered in a manner different from that in the noninstrument situation.

If, as in the multipurpose reflectometer, the energy is passed through a filter before incidence on the sample, fluorescence may cause the photocell exposed to the sample to receive energy of wavelengths wholly excluded by the filter and therefore not supposed to be present for the particular measurements.

To illustrate the fluorescence error, the author selected a piece of phenol-formaldehyde resin found by Gibson and Keegan [12] to give a spectral curve significantly in error because of fluorescence when measured with the GE recording spectrophotometer. These authors showed that this pink plastic gave a strong reddish fluorescence when illuminated by the yellow-green part of the spectrum. On visual inspection, the pink plastic appeared red relative to Munsell paper RYR 5/6. This visual estimate, which was substantiated by measurement with the Judd subtractive colorimeter, is properly represented by the black dots in figure 9.

Measurements with the multipurpose instrument, represented by the open circles in figure 9, erroneously indicate, however, that the resin would be perceived as greener and weaker in color than the Munsell paper. The error, when evaluated by eq 13, amounted to 14 NBS units of color difference and is about half the fluorescence error found by Gibson and Keegan when they measured the same sample with the GE spectrophotometer.

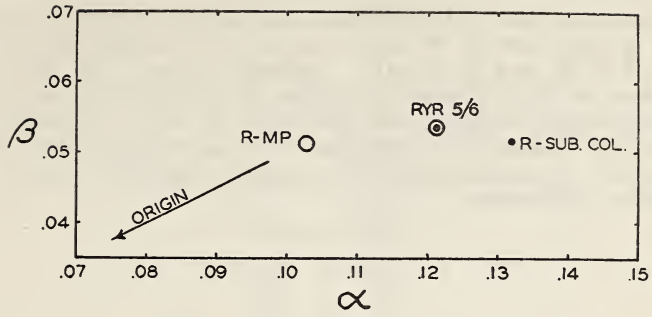

FIGURE 9.—*Error in photoelectrically measured chromaticity due to fluorescence of the resin specimen, R.*

When compared in the subtractive colorimeter [29] with Munsell paper RYR 5/6, the resin is properly assigned the chromaticity represented by the black dot, rather than the erroneous chromaticity assigned by the multipurpose reflectometer [18] and represented by the open circle.

4. SHORT CUTS IN COMPUTATION WHICH PRODUCE ONLY SMALL ERRORS

There are several short cuts which at times may be used to reduce the labor of computing tristimulus quantities from settings with the multipurpose reflectometer. The shortened procedures result in errors which diminish with the computed color difference between the specimens compared. With many small color differences, therefore, the short-cut computational errors are smaller than the errors introduced by lack of precision and are therefore unimportant. Even larger errors from this cause are unimportant if they are appreciably less than the maximum error permitted for a successful solution of the problem.

As an example, short-cut errors are seldom serious when one is following the day-to-day change of the color of a paint or textile material due to fading or other cause. The highest accuracy is not necessary for such measurements so long as the day-to-day reproducibility is fairly good. The short-cut computational errors have negligible effect on this reproducibility.

(a) OMITTING SCALE CORRECTIONS

In practice, the apparent-reflectance scale supplied with a multipurpose reflectometer is not exact. Correction curves, which are found by calibration after the instrument is completed, are necessary to obtain accurate values with the device [18, p. 601; 18a, p. 548]. The corrections do not change value rapidly with the scale reading for most parts of the scales employed. Therefore the omission of scale corrections usually produces only small changes in the computed color differences between samples of similar color. In all but one of the examples of tristimulus color measurement reproduced below, in section VIII, the scale corrections were omitted.

(b) NEGLECT OF THE DIFFERENCE BETWEEN THE KNOWN *A*, *G*, AND *B* VALUES FOR THE STANDARD AND THE APPROXIMATE VALUES READ DIRECTLY FROM THE INSTRUMENT

Because of the usual drift of instrument readings with time, the difficulty of making minute adjustments of photometric balance, and

the frequent necessity of correcting readings for scale errors, the readings of a photoelectric tristimulus instrument seldom give accurate values of A, G, and B directly. Instead, values of A, values of G, and values of B each relative to those of the standard are obtained from instrument settings, after the effect of drift is balanced out in the manner illustrated in examples 1, 3, 5, and 6, below. Thus if one exposes a standard and two samples for which the accurate tristimulus values are

$$A_{\text{std}},\ G_{\text{std}},\ B_{\text{std}}$$
$$A_1,\quad G_1,\quad B_1$$
$$A_2,\quad G_2,\quad B_2,$$

he actually obtains from the corrected settings

$$k_a A_{\text{std}},\ k_g G_{\text{std}},\ k_b B_{\text{std}}$$
$$k_a A_1,\quad k_g G_1,\quad k_b B_1$$
$$k_a A_2,\quad k_g G_2,\quad k_b B_2.$$

For many cases in which the highest accuracy is not required, it is not worthwhile to evaluate k_a, k_g, and k_b; their departures from unity are found to be negligible. In examples 3 and 4, section VIII, use was made of this short-cut B.

One might suppose that a simple additive method could be used to compute accurate tristimulus values from instrument readings. If such a procedure were possible, one could take settings on a standard and sample, and from these settings compute values of x and y. From the computed and known values of x and y for the standard, the supposed additive corrections, Δx and Δy, would be found. That such corrections applied additively are inexact, however, may easily be shown. An "uncorrected" value of x for the standard would be computed from the readings on it by:

$$x_{\text{std}(u)}=\frac{0.80k_aA_{\text{std}}+0.18k_bB_{\text{std}}}{0.80k_aA_{\text{std}}+k_gG_{\text{std}}+1.36k_bB_{\text{std}}},$$

whereas the true value of x would be

$$x_{\text{std}}=\frac{0.80A_{\text{std}}+0.18B_{\text{std}}}{0.80A_{\text{std}}+G_{\text{std}}+1.36B_{\text{std}}}.$$

It will be seen that there is no simple function of $x_{\text{std}(u)}$ and x_{std} by which the factors k_a, k_g, and k_b can be eliminated.

(c) USING 0.5 AND 0.1 INSTEAD OF THE ACCURATE VALUES FOR THE NEUTRAL-FILTER TRANSMISSION FACTORS

To extend its photometric range, the multipurpose reflectometer makes use of two neutral filters transmitting approximately 0.5 and 0.1, respectively, of the normally incident energy. Actually the transmissions of the two filters used in the author's instrument are

not 0.5 and 0.1, nor are the filters ideally neutral. Instead the transmissions are as follows:

Nominal transmissions of "neutral" filters	Exact transmissions of "neutral" filters		
	For amber tristimulus filter	For green tristimulus filter (giving luminosity function)	For blue tristimulus filter
0. 5 . 1	0. 480 . 088	0. 479 . 086	0. 427 . 065

When the color differences between the specimens being compared are very small, or when accuracy is not needed, use of 0.5 and 0.1 instead of the proper filter factors is frequently permissible. It should be obvious, however, that these approximate factors should not be used for the computations if the neutral filter employed in conjunction with any one of the tristimulus filters was changed during the settings on the group of specimens. Example 4, section VIII, illustrates this third short cut.

(d) AMOUNTS OF ERROR FROM USING SHORT-CUT METHODS

To give an idea of the sizes of the errors which would be found in actual practice, instrument settings for six typical color differences were converted to amounts of color difference, first by the accurate computational procedure and then by using the short cuts. Two white, two delphinium-blue, and two international-orange pairs of samples were chosen, so that for each color there was one difference of about 1.5 NBS units and one of about 7 units. For each color, pairs of samples were sought which seemed likely to give the most serious errors from the use of the short-cut methods. For these pairs, amounts of color difference were computed five times; first they were computed accurately, then each of the three short cuts was used separately, and finally they were used together.

TABLE 9.—*Percentage errors in computed values of color difference from using the suggested short cuts*

Short cut	Errors in percentage of total difference—					
	For two pairs of white samples		For two pairs of blue samples		For two pairs of orange samples	
	ΔE=1.8	ΔE=6.6	ΔE=1.0	ΔE=7.6	ΔE=1.4	ΔE=6.0
Scale corrections omitted	0.6	1.1	0	4.1	4.2	4.8
Use of instrument values of *A*, *G*, and *B*, having only relatively correct magnitudes	0	0	1.0	2.9	26.6	6.3
Use of 0.5 and 0.1 instead of true neutral-filter factors			3.9	3.6	7.7	1.0
Combination of above three short cuts	0.6	0.3	0	8.7	27.3	4.8

The errors are given in table 9 as percentages of the accurately computed differences. The size of each error depends upon the nature of the color difference involved as well as upon the spectral selectivity

of the samples, and hence some of the differences are surprisingly small. In general, it will be seen that the errors become larger as the samples become more strongly colored. It will also be seen that the errors from the use of more than one short cut are not always additive. By comparison of the percentage errors in table 9 with those in table 7, it will be seen that most of these short-cut errors are smaller than the errors from spectral inaccuracy of the source-filter-photocell combinations. For this reason the short cuts can be recommended for use in many photoelectric tristimulus comparisons of color in which the highest accuracy possible is not needed.

VII. THE RELATION OF PERCEPTUAL IMPRESSIONS AND MEASURED VALUES OF COLOR

Because the ICI standard observer was created from the color matches made by a number of normal observers [13, 51], the numerical description of a color difference in the ICI or a related color system will, if possible interfering circumstances are eliminated, indicate the character of the color difference perceived and described by the normal observer. With some of the methods of measuring color suggested in the present paper, however, numbers are given which, in addition to indicating the character of the difference, are intended to designate quantitatively the perceptual impression of the difference. Unfortunately there is at present little information on the success with which quantitative perceptual impressions are duplicated by these photoelectric tristimulus measurements. One bit of information may be noted. In a study of several methods for measuring color difference, Balinkin [1] compared the photoelectrically measured values of ΔE for 10 pairs of green porcelain tiles with the averages of the visual estimates of the same differences by 60 observers. He found fairly good agreement between measured and estimated values, with a correlation according to ranks of 0.86.

A number of the circumstances which may disturb the correspondence between visual estimates and photoelectric determinations of color and color difference are well known. Some of these circumstances relate to imperfections in the measuring method, others to imperfections in the observing situation. There follows a two-part list in which a number of the circumstances are given. It will be seen that each of those listed under "imperfections in the measuring method" was discussed above.

CIRCUMSTANCES WHICH MAY INTERFERE WITH CORRELATION BETWEEN VISUALLY ESTIMATED AND PHOTOELECTRICALLY MEASURED VALUES OF COLOR AND COLOR DIFFERENCE

Imperfections in the Measuring Method:

Error from spectral inaccuracy of source-filter-photocell combinations.
Error from fluorescence of sample.
Error from photometric inaccuracy of measuring instrument.
Failure of the empirically chosen scales for chromaticity, surface-color difference, and lightness to represent appropriate scales for these quantities.

Imperfections in the Observing Situation:

Spectral difference between illuminant used for observations and ICI illuminant *C*.
Angular difference between directions of illuminating and viewing used for the visual observations and those used for the measurements. (This factor frequently affects the correlation if appreciable specularly reflected light is present in one case, but is eliminated in the other.)
Differences in texture and shape of the specimens being compared.
Nonuniformity of color from point to point on some of the specimens. (For instrument measurements of pairs of specimens, central areas are usually used; for visual comparisons, the colors of the immediately adjacent areas of the specimens have the greatest influence on estimated color difference.)
Failure to take proper account in the measured values of the character of the dividing line, separating pattern, or background used during visual observation.
Abnormality of the observer, either temporary as from fatigue, or permanent as with an observer having abnormal color vision.

VIII. EXAMPLES

To demonstrate uses of the photoelectric tristimulus method, and to illustrate forms for recording the corresponding data, six examples have been prepared. These examples represent multipurpose reflectometer measurements made at the National Bureau of Standards for purposes which have been identified on each of the forms which is shown. All of the steps leading from the initially recorded settings to the desired values for color have been duplicated. The number of settings taken on each sample and the short cuts permitted in the reduction of the data were decided in each case by a consideration of the needs for precision and accuracy of the problem at hand. Whenever two readings were made with each filter for every specimen, the order of readings was: Standard, No. 1, No. 2, No. 3, * * * No. 3, No. 2, No. 1, Standard. This symmetrical order, which was used to eliminate some of the error of drift, was mentioned above. The equations used in the following examples are given in table 3 and in the text above.

Example 1.—Measurement of the ICI values, x, y, z, and Y of four white painted plaques (see eq 5 and 6, and fig. 10.)

Purpose—To test samples for compliance with color requirements given in Federal Specification TT–P–23a [8], namely,

	x	y	z	Y
Minimum			0. 355	0. 85
Maximum	0. 324	0. 331		

Computational short cut—Scale corrections omitted.
Working standard—vitrolite No. 1 (A=0. 906, G=0. 909, B=0. 890).

Specimen	Standard	No. 1	No. 2	No. 3	No. 4
Blue settings	{0. 8985	0. 8480	0. 7900	0. 7810	0. 7080
	{ . 8955	. 8450	. 7880	. 7790	. 7070
Mean	. 8970	. 8465	. 7890	. 7800	. 7075
B=mean $\times \frac{0.890}{0.8970}$		. 8398	. 7828	. 7739	. 7020
Amber settings	{0. 9055	0. 8785	0. 8720	0. 8420	0. 8530
	{ . 9030	. 8770	. 8680	. 8400	. 8520
Mean	. 9042	. 8778	. 8700	. 8410	. 8525
A=mean $\times \frac{0.906}{0.9042}$		. 8795	. 8717	. 8427	. 8542
Green settings	{0. 9100	0. 8770	0. 8620	0. 8340	0. 8320
	{ . 9070	. 8740	. 8585	. 8320	. 8310
Mean	. 9085	. 8755	. 8602	. 8330	. 8315
$Y \doteq G$=mean $\times \frac{0.909}{0.9085}$		. 8760	. 8607	. 8335	. 8320
0.18 B		0. 1512	0. 1409	0. 1393	0. 1264
1.18 B		. 9910	. 9237	. 9132	. 8284
0.80 A+0.18 B		. 8548	. 8383	. 8135	. 8098
Denominator (G+0.80 A+1.36 B)		2. 7218	2. 6227	2. 5602	2. 4702
$x \doteq \frac{0.80\ A+0.18\ B}{\text{denominator}}$		0. 3141	0. 3196	0. 3177	0. 3278
$y \doteq G$/denominator		. 3218	. 3282	. 3256	. 3368
$z \doteq 1.18B$/denominator		. 3641	. 3522	. 3567	. 3354
Result of test		Pass	Fail (z)	Fail (Y)	Fail (all)

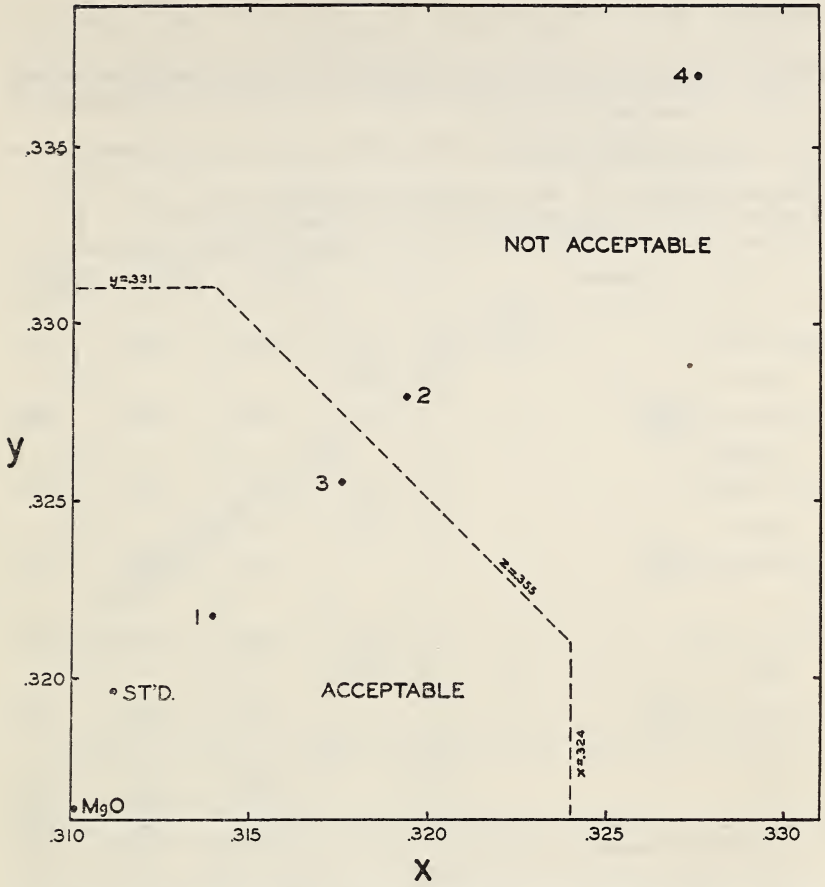

FIGURE 10.—*Part of the (x,y)-diagram showing chromaticities acceptable and chromaticities not acceptable according to Federal Specification TT–P–23a for white casein paints.*

The plotted points show values obtained with actual samples.

Example 2.—Measurement of the change of α and β of three painted plaques with exposure (see eq 8 and fig. 11)

Purpose—To follow the chromaticity changes of panels of the same paint exposed out of doors and in two machines which accelerate the changes produced by weathering.

Note—Only one of the many sets of computations used to obtain values of α and β is reproduced below.

Computational short cut—Scale corrections omitted.
Working standard—pink plaque ($A=0.7058$, $G=0.606$, $B=0.4714$).

Specimen	Standard	Roof (4 days)	Machine 1 (43 hours)	Machine 2 (41 hours)
Blue settings	0. 4995	0. 300	0. 685	0. 597
	. 501			
Mean	. 5002			
$B=\text{setting}\times\frac{0.4714}{0.5002}$		. 2827	. 6453	. 5626
Amber settings	. 704	. 510	. 7075	. 675
	. 704			
Mean	. 704			
$A=\text{setting}\times\frac{0.7058}{0.704}$		. 5113	. 7093	. 6767
Green settings	. 620	. 424	. 708	. 656
	. 619			
Mean	. 6195			
$G=\text{setting}\times\frac{0.606}{0.6195}$		. 4148	. 6926	. 6417
$A-G$		+. 0965	+. 0167	+. 0350
$G-B$		+. 1321	+. 0473	+. 0791
$0.4\ (G-B)$		+. 0528	+. 0189	+. 0316
$B+A+2G=$denominator		1. 6236	2. 7398	2. 5227
$\alpha \doteq (A-G)/$denominator		+0. 0594	+0. 0061	+0. 0139
$\beta \doteq 0.4(G-B)/$denominator		+. 0325	+. 0069	+. 0125

One fact is worthy of comment in connection with the changes of chromaticity accompanying fading of the paint illustrated in figure 11. At the start of exposure, a change in the paint from moderate pink to weak orange took place. This pronounced hue change is unusual in

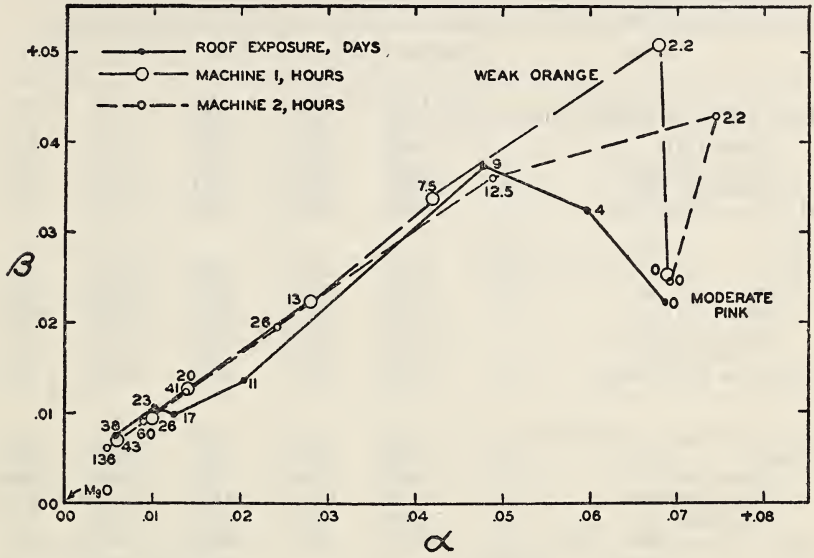

FIGURE 11.—*Part of the* (α,β)*-diagram used to show the changes of chromaticity of a pink-colored paint with exposure outdoors and in two machines designed to accelerate the changes produced in weathering.*

fading; paints, textile, and other materials are usually perceived to become merely weaker in color as the result of exposure to light, heat, and humidity. The usual curve of chromaticity change with time of exposure is therefore merely a curve directed toward the origin of the (α,β)-diagram, similar to the curves of figure 11 beyond the points showing the initial hue change.

Example 3.—Measurement of the amount of color difference, ΔE, between a standard and each of four similarly colored brown automobile pile fabrics (see eq 13).

Purpose—To find which fabrics comply with the requirement that they differ from the standard by no more than one NBS unit of color difference.

Computational short cut—No calibrated standard used.
$k_1 = 100$, $f_g = 1.00$.

Fabric	Standard	No. 1	No. 2	No. 3	No. 4
Blue settings (with 0.427 filter)	0. 1700	0. 1744	0. 1620	0. 1690	0. 2024
	. 1688	. 1738	. 1618	. 1686	. 2018
Mean (M)	. 1694	. 1741	. 1619	. 1688	. 2021
Scale correction	−. 0139	−. 0138	−. 0138	−. 0136	−. 0121
Corrected M	. 1555	. 1603	. 1481	. 1552	. 1900
B=Above times .427	. 0664	. 0684	. 0632	. 0663	. 0811
Amber settings (with 0.480 filter)	. 2098	. 2144	. 2022	. 2042	. 2524
	. 2082	. 2142	. 2018	. 2036	. 2520
Mean (M)	. 2090	. 2143	. 2020	. 2039	. 2522
Scale correction	−. 0120	−. 0120	−. 0120	−. 0120	−. 0113
Corrected M	. 1970	. 2023	. 1900	. 1919	. 2409
A=above times .480	. 0946	. 0971	. 0912	. 0921	. 1156
Green settings (with 0.479 filter)	. 1916	. 1946	. 1848	. 1882	. 2300
	. 1900	. 1944	. 1844	. 1876	. 2302
Mean (M)	. 1908	. 1945	. 1846	. 1879	. 2301
Scale correction	−. 0127	−. 0126	−. 0130	−. 0129	−. 0122
Corrected M	. 1781	. 1819	. 1716	. 1750	. 2179
$Y \doteq G$=Above times .479	. 0853	. 0871	. 0822	. 0838	. 1044
$A-G$	+. 0093	+. 0100	+. 0090	+. 0083	+. 0112
$G-B$	+. 0189	+. 0187	+. 0190	+. 0175	+. 0233
0.4 $(G-B)$	+. 0076	+. 0075	+. 0076	+. 0070	+. 0093
Denominator $=B+A+2G$	. 3316	. 3397	. 3188	. 3260	. 4055
$\alpha \doteq (A-G)$/denominator	+. 0280	+. 0294	+. 0282	+. 0255	+. 0276
$\beta \doteq 0.4(G-B)$/denominator	+. 0229	+. 0221	+. 0238	+. 0215	+. 0229
$\Delta\alpha = \alpha - \alpha_{std.}$		+. 0014	+. 0002	−. 0025	−. 0004
$\Delta\beta = \beta - \beta_{std.}$		−. 0008	+. 0009	−. 0014	. 0000
$\sqrt{\overline{\Delta\alpha}^2+\overline{\Delta\beta}^2}\cdot 10^2$		. 16	. 09	. 29	. 04
$Y^{1/2}$	. 292	. 295	. 287	. 289	. 323
$7(Y^{1/4})$*	3. 78				3. 98
$\Delta C = 7(Y^{1/4})\sqrt{\overline{\Delta\alpha}^2+\overline{\Delta\beta}^2}\cdot 10^2$		. 60	. 34	1. 10	0. 16
$\Delta Y^{1/2}$		+. 003	−. 005	−0. 003	+. 031
$\Delta L = k_1(\Delta Y^{1/2})$		. 3	. 5	. 3	3. 1
$\Delta E/f_g = \sqrt{\overline{\Delta C}^2+\overline{\Delta L}^2}$		.67	. 60	1. 14	3. 11
ΔE		0. 7	0. 6	1. 1	3. 1

*Compute $7(Y^{1/4})$ for standard and for any samples for which the value of Y differs from that of the standard by more than 10 percent. For the latter samples, use the mean of the two values of $7(Y^{1/4})$; for others, use the value for the standard.

Example 4.—Measurement of the hue-difference estimate, $\Delta H'$, the saturation-difference estimate, $\Delta S'$, and the lightness-difference estimate, $\Delta L'$, between an actual earth sample and two modified earths (see eq 17).

Purpose—To find how the modified earths differ in color from the actual earth.

Computational short cuts—Scale corrections omitted, no calibrated standard used, and the factor 0.5 used instead of the actual transmissions of the neutral filter.

$k_1 = 100$, $f_o = 1.00$.

Specimen	Earth mixture	Modification No. 1	Modification No. 2
Blue settings	0. 295	0. 260	0. 370
B=Above times 0.5	. 147_5	. 130	. 185
Amber settings	. 496	. 446	. 575
A=Above times 0.5	. 248	. 223	. 287_5
Green settings	. 428	. 386	. 508
$Y \doteq G$=Above times 0.5	. 214	. 193	. 254
$L = Y^{1/2} \doteq G^{1/2}$	. 463	. 439	. 504
$\Delta L' = k_1 \Delta(Y^{1/2})$		−2. 4	+4. 1
$A - G$ (note sign)	+. 034	+. 030	+. 0335
$G - B$ (note sign)	+. 0665	+. 063	+. 069
0.4 $(G - B)$	+. 0266	+. 0252	+. 0276
Denominator $= B + A + 2G$	. 8235	. 739	. 9805
$\alpha \doteq (A - G)$/denominator	+. 0413	+. 0406	+. 0342
$\beta \doteq 0.4(G - B)$/denominator	+. 0323	+. 0341	+. 0281
$\sqrt{\alpha^2 + \beta^2}$	. 0525	. 0528	. 0443
$\Delta(\sqrt{\alpha^2 + \beta^2})$		+. 0003	−. 0082
$700\, Y^{1/4}$ *		470.	487.
$\Delta S' = 700\, Y^{1/4} \Delta(\sqrt{\alpha^2 + \beta^2})$		+0. 14	−3. 99
β/α	+. 782	+. 840	+. 822
ϕ=angle whose tan is β/α	38. 03°	40. 03°	39. 42°
$\Delta\phi$		+2. 00°	+1. 39°
$12.2\, Y^{1/4}$ *		8. 19	8. 48
$12.2\, Y^{1/4} \sqrt{\alpha^2 + \beta^2}$ *		0. 431	0. 411
$\Delta H' = 12{\cdot}2\, Y^{1/4} \sqrt{\alpha^2 + \beta^2} \cdot \Delta\phi$		+0. 86	+0. 57
Color of modification relative to that of actual earth, according to the measurements.		Darker, greener, slightly stronger.	Lighter, weaker, slightly greener.

*Values which are average for the 2 specimens being compared should be entered in those rows marked by an asterisk.

Example 5.—Measurement of the whiteness, W, of one new and two laundered sheeting specimens. (See eq 19)

Purpose—To measure the effect of repeated laundering on the whiteness of new sheeting material.

Computational short cut—Scale corrections omitted.

$k_1=20$, $f_o=1.00$.

Working standard—vitrolite No. 1 ($A=0.906$, $G=0.909$, $B=0.890$).

Specimen	Standard	New	1L	20L
Blue settings	0.898	0.837	0.845	0.765
	.892	.850	.852	.787
Mean	.895	.8435	.8485	.776
$B=\text{mean}\times\frac{0.890}{0.895}$		.8388	.8438	.7717
Amber settings	.9085	.892	.912	.826
	.906	.904	.912	.836
Mean	.9072	.898	.912	.831
$A=\text{mean}\times\frac{0.906}{0.9072}$		.8968	.9108	.8299
Green settings	.9055	.884	.900	.812
	.903	.894	.900	.824
Mean	.9042	.889	.900	.818
$G=\text{mean}\times\frac{0.909}{0.9042}$		.8937	.9048	.8223
$A-G$		.0031	.0060	.0076
$2.5(A-G)$		.0078	.0150	.0190
$G-B$		.0549	.0610	.0506
$\alpha\doteq 2.5(A-G)/10G$		.0009	.0017	.0023
$\beta\doteq (G-B)/10G$		.0061	.0067	.0062
$\sqrt{\alpha^2+\beta^2}$		.0062	.0069	.0066
$W_C=30\sqrt{\alpha^2+\beta^2}$		.186	.207	.198
$1.00-G$		.106	.095	.178
$W_L=(1.00-G)/2$		.053	.048	.089
$\sqrt{W_C^2+W_L^2}$		.193	.213	.217
$W=1-\sqrt{W_C^2+W_L^2}$		.807	.787	.783

Example 6.—Measurement of the yellowness of four near-white porcelain-enameled specimens (eq 20).

Purpose—To find which of several enamel specimens are yellowish (+ values) and which are bluish (− values).
Computational short cut—scale corrections omitted.
Working standard—vitrolite No. 1 ($A=0.906$, $G=0.909$, $B=0.890$).

Plaque No.	Standard	1	10	22	24
Blue settings	0.8955	0.7540	0.7760	0.8238	0.7804
	.8920	.7490	.7716	.8222	.7780
Mean	.8938	.7515	.7738	.8230	.7792
$B=\text{mean}\times\frac{0.890}{0.8938}$		.7483	.7705	.8195	.7759
Amber settings	.8945	.7276	.7236	.7990	.7980
	.8936	.7222	.7176	.7962	.7962
Mean	.8940	.7249	.7206	.7976	.7971
$A=\text{mean}\times\frac{0.906}{0.8940}$		.7346	.7303	.8083	.8078
Green settings	.9037	.7370	.7364	.8094	.8046
	.9030	.7316	.7324	.8074	.8030
Mean	.9034	.7343	.7344	.8084	.8038
$G=\text{mean}\times\frac{0.909}{0.9034}$		.7389	.7390	.8134	.8088
$A-B$		−.0137	−.0402	−.0112	+.0319
Yellowness $=(A-B)/G$ (eq 20)		−.0185	−.0544	−.0138	+.0394

IX. SUMMARY

Photoelectric tristimulus measurements are well suited for comparisons of the colors of spectrally similar specimens. In commerce there seem to be many situations in which such measurements may prove to be useful: (1) in the day-to-day control of the colors of materials in production, (2) in obtaining compliance of materials with color-tolerance requirements, (3) in making modifications of color to produce matches, and (4) in the measurements of color changes accompanying fading. For problems of this general character, it is believed that photoelectric tristimulus measurements will prove to be useful.

The advantages and disadvantages of photoelectric tristimulus colorimetry have been summarized as follows:

ADVANTAGES

Small number of settings required for each sample.
Small number of computational steps required to obtain x, y, and Y, or α, β, and Y.
Simplicity and inexpensiveness of apparatus.
Opportunity to convert settings rapidly to numbers giving size and character of color difference perceived between samples.
Correspondence between values of α, β, and Y, and positions of points representing colors in the uniformly spaced surface-color solid.

DISADVANTAGES

Errors resulting from the inaccurate spectral response of the source-filter-photocell combinations used.

The necessity for working standards which are spectrally similar to the samples measured.

The necessity for special precautions to attain the high precision required if the tristimulus method is to equal the eye in power to discriminate colors.

Failure of tristimulus measurements to provide the physical description of a color stimulus such as is given by a spectrophotometric curve.

The directness of the method and the small number of computational steps necessary to give numbers indicative of color recommend the method. Its inaccuracy because of failure of the source-filter-photocell combinations to be spectrally equivalent to the ICI standard observer is its most important defect. Nevertheless it can be reasonably expected that the use of photoelectric tristimulus colorimetry will increase as the combinations of elements used are improved in spectral character.

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